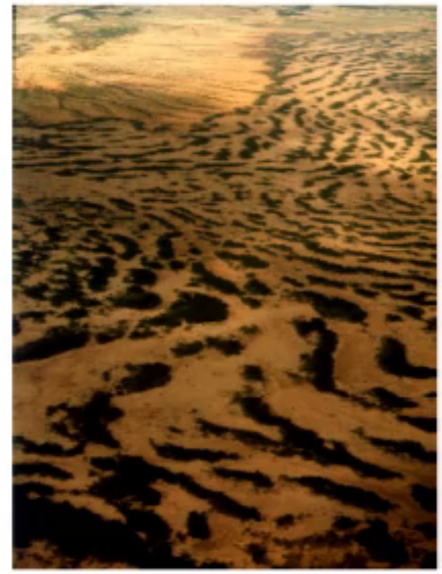
# ‘Spatio-temporal patterns in ecology’

Spatial patterns in phytoplankton
Lotte Sewalt

Pattern-formation in Semiarid Vegetation
Sarah Iams

Stripe Patterns in Vegetation
Eric Siero



Patterns in Musselbeds

# Patterns in animal communities

Coherent spatial patterns in animal communities are found

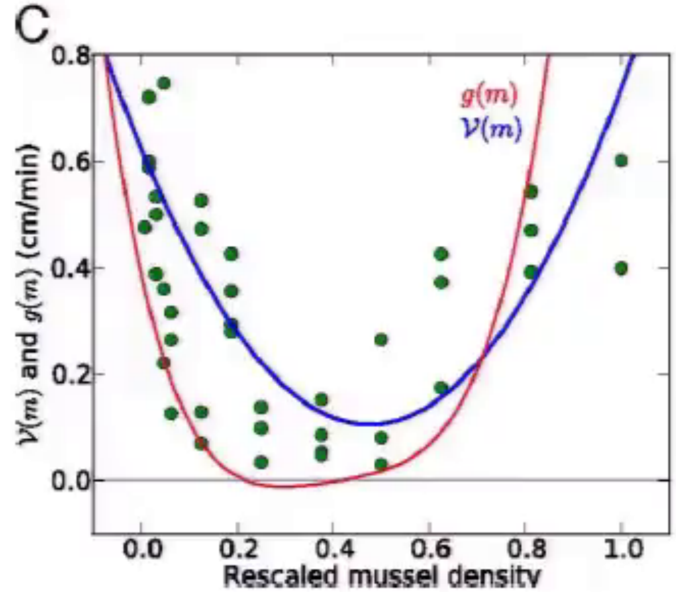
Our model ecosystem: **mussel beds**

Patterns **do not** arise from activation/inhibition or chemotaxis

# Laboratory experiments

Similar patterns are observed in the lab

from Van de Koppel et al. (Science 2008)




Movement speed of mussels depends on the local density

Photo Aniek van der Berg


Leave Fullscreen
Time:
Seek to 0:00 / 0:08
musselmov_s1.mov


Leave Fullscreen
Pause
Time:
Seek to 0:00 / 0:08
musselmov_s1.mov


Leave Fullscree
Pause
Time:
Seek to 0:00 / 0:08
musselmov_s1.mov


Leave Fullscreer
Pause
Time:
0:01 / 0:08
musselmov_s1.mov


Leave Fullscree
Pause
Time:
0:01 / 0:08
musselmov_s1.mov


Leave Fullscree
Pause
Time:
0:02 / 0:08
musselmov_s1.mov


Leave Fullscreen
Time:
0:07 / 0:08
musselmov_s1.mov


Leave Fullscreen
Time:
Seek to 0:00 / 0:08
musselmov_s1.mov

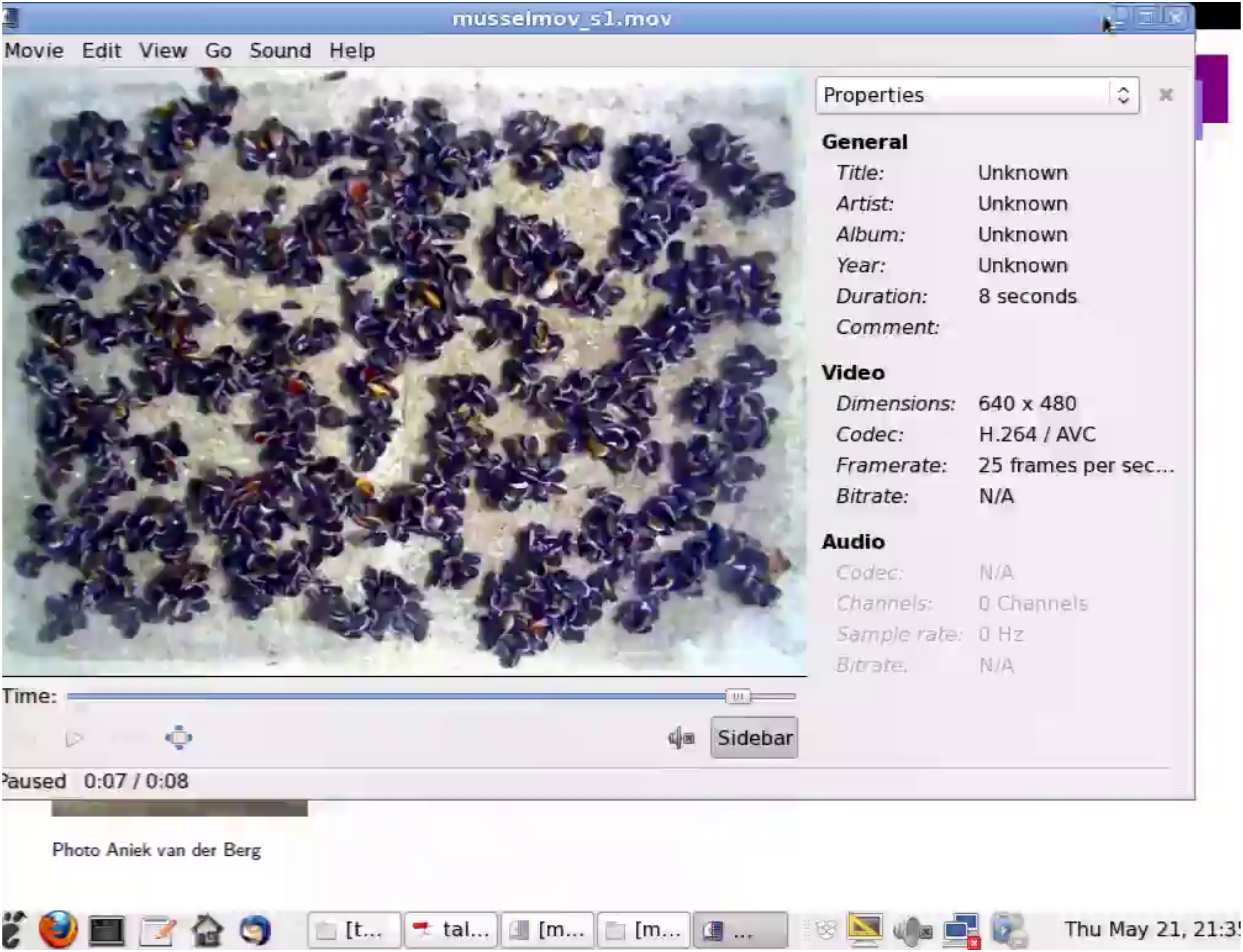


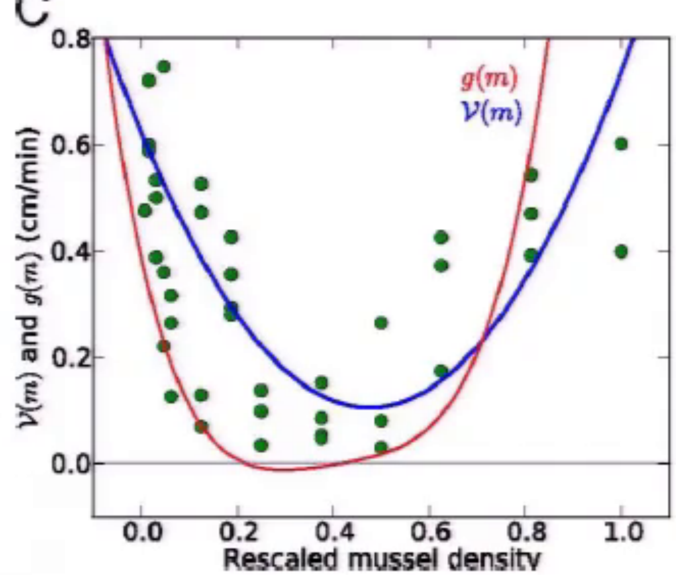

Photo Aniek van der Berg

# Laboratory experiments

Similar patterns are observed in the lab

from Van de Koppel et al. (Science 2008)

Photo Aniek van der Berg




Movement speed of mussels depends on the local density

# Aim: derive model

Based on density-dependent movement derive model

$m(x,t)$ local density of mussels; $v(m)$ speed of mussels

Mussel pattern formation is a fast process: independent of birth or death

So, total **mass** of mussels **conserved**

$$\int_{domain} m(s,t)ds = \int_{domain} m(s,0)ds = \text{ constant}$$

and, generic **conservation equation** is satisfied

$$\frac{\partial m}{\partial t} = -\nabla \cdot J = -\nabla \cdot (J_v + J_{nl})$$

$J$ net flux of mussels

$J_v$ flux related to speed $v$

$J_{nl}$ flux related to nonlocal interactions

# The model

Population of organisms performing random walk in 2 dimensions

with speed $v(m)$ and turning rate $\tau(m)$ [Schnitzer]

$$J_v = -\frac{v(m)}{2\tau(m)}\nabla\Big(v(m)m\Big)$$

Long range interactions

$$J_{nl} = \kappa\nabla(\Delta m)$$

for some $\kappa > 0$

Then

$$\frac{\partial m}{\partial t} = \nabla\left[f(m)\nabla m - \kappa\nabla(\Delta m)\right],$$

with

$$f(m) = \frac{v}{2\tau}\Big(v + m\frac{dv}{dm}\Big)$$

# Surprise: a Cahn-Hilliard equation

The model can be written as the Cahn-Hilliard equation

$$\frac{\partial m}{\partial t} = \nabla^2 \left[ F(m) - \kappa \nabla^2 m \right], \text{ for some } \kappa > 0$$

with

$$F'(m) = f(m) = \frac{v}{2\tau}\left(v + m\frac{dv}{dm}\right)$$

The CH was proposed in 1958 to model phase separation in binary alloys

# Phase separation

The Cahn-Hilliard equation $\frac{\partial m}{\partial t} = \nabla^2 \left[F(m) - \kappa \nabla^2 m\right]$
was proposed in 1958 to model phase separation in binary alloys

CH can be derived from energy functional

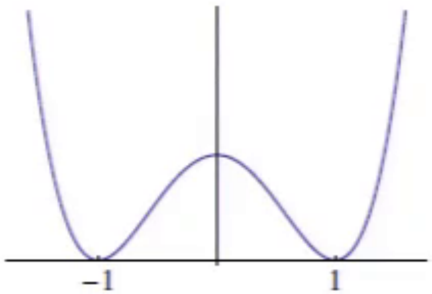
$$E(m) = \int_\Omega F(m) + \frac{1}{2}\kappa |\nabla m|^2 dx$$

In the standard setting, $W(m)$ is a **symmetric double-well potential** $(W' = F)$

In standard setting, $W(m)$ is a
**symmetric double-well**
**potential** $(W' = F)$

$$W(m) = \frac{1}{4}(m^2 - 1)^2,$$

$$F(m) = W'(m) = m^3 - m.$$



# Phase separation

The Cahn-Hilliard equation $\frac{\partial m}{\partial t} = \nabla^2 \left[F(m) - \kappa \nabla^2 m\right]$
was proposed in 1958 to model phase separation in binary alloys

CH can be derived from energy functional

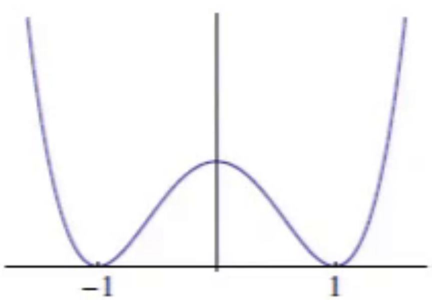
$$E(m) = \int_\Omega F(m) + \frac{1}{2}\kappa|\nabla m|^2 dx$$

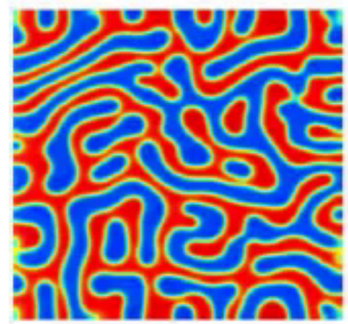
In the standard setting, $W(m)$ is a **symmetric double-well potential** $(W' = F)$

In standard setting, $W(m)$ is a **symmetric double-well potential** $(W' = F)$

$$W(m) = \frac{1}{4}(m^2 - 1)^2,$$

$$F(m) = W'(m) = m^3 - m.$$



Generates patterns consisting of
2 phases $m_+ = +1$ and $m_- = -1$
$=$ minima of $W(m)$ $(W' = F)$

More general: CH-type patterns are formed when $W$ has 2 minima

# Back to mussels

In mussels the CH equation **does not** come from energy conservation

$$\frac{\partial m}{\partial t} = \nabla^2 \left[ F(m) - \kappa \nabla^2 m \right],$$

with

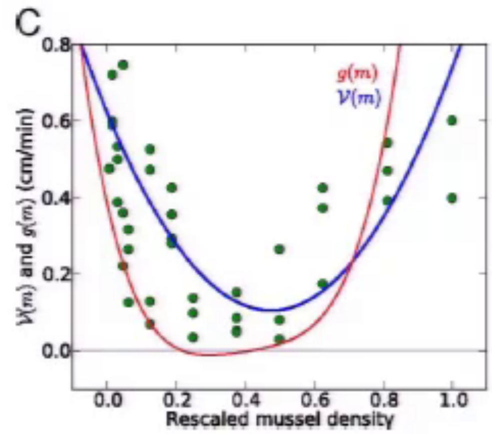
$$F'(m) = f(m) = \frac{v}{2\tau}\left(v + m\frac{dv}{dm}\right) = W''(m)$$

Patterns when $f$ has **one negative minimum**

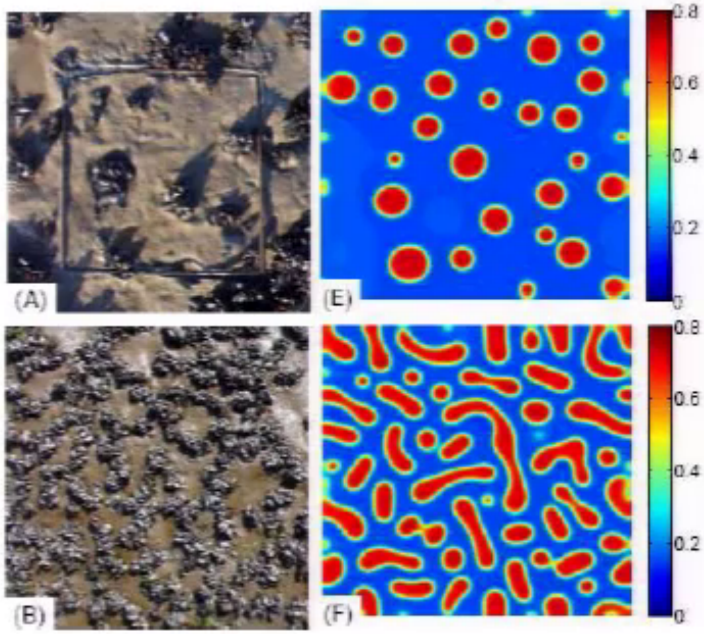
Fit data:

$$v(m) = am^2 + bm + c,$$

$a, b$ and $c$ can be obtained from data



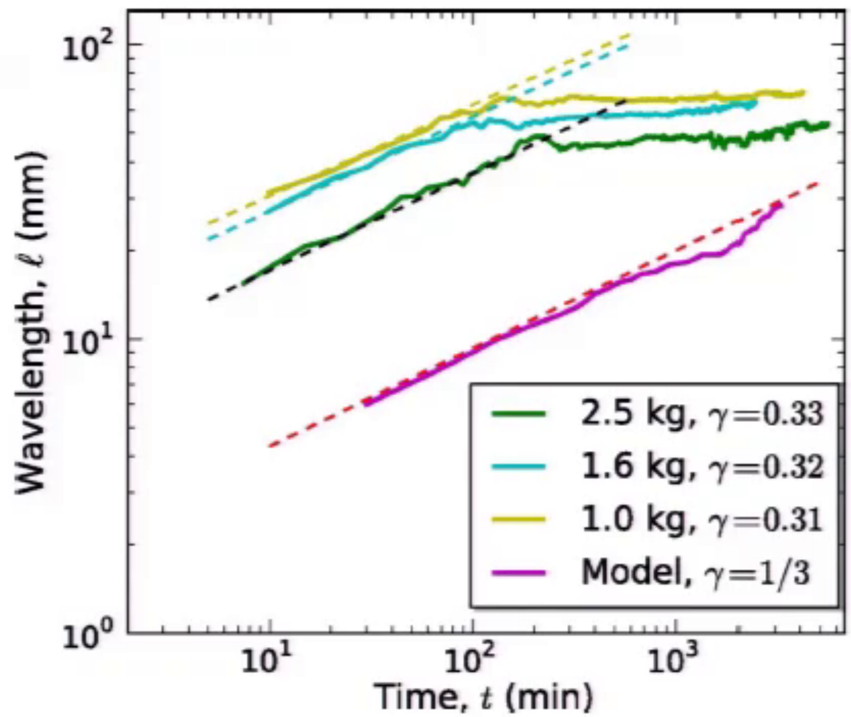
# Comparison




Increasing total mass of mussels in domain

**Coarsening**: wavelength of pattern grows in time

Experiments versus model



# Discussion

## Phase separation explains a new class of self-organized spatial patterns in ecological systems

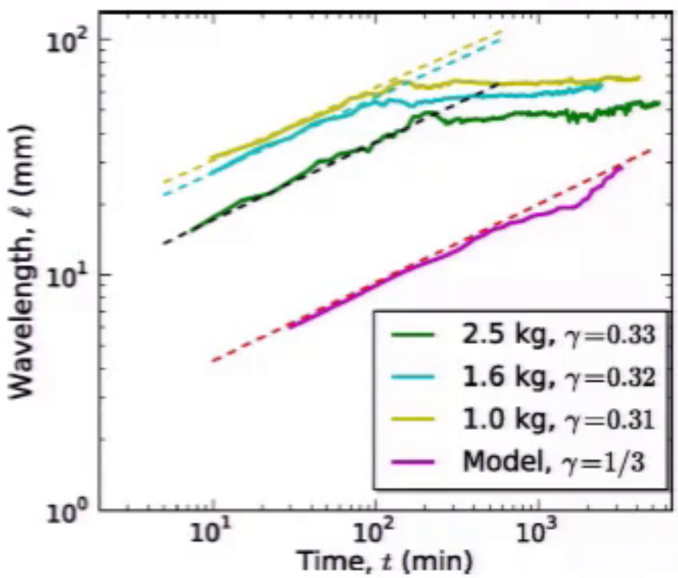
**Quan-Xing Liu[a,b,1], Arjen Doelman[c], Vivi Rottschäfer[c], Monique de Jager[a], Peter M. J. Herman[a], Max Rietkerk[d], and Johan van de Koppel[a,e]**

[a]Department of Spatial Ecology, Royal Netherlands Institute for Sea Research, 4400 AC Yerseke, The Netherlands; [b]Aquatic Microbiology, Institute for Biodiversity and Ecosystem Dynamics, University of Amsterdam, 1090 GE Amsterdam, The Netherlands; [c]Mathematical Institute, Leiden University, 2300 RA Leiden, The Netherlands; [d]Department of Environmental Sciences, Copernicus Institute, Utrecht University, 3508 TC Utrecht, The Netherlands; and [e]Community and Conservation Ecology Group, Centre for Ecological and Evolutionary Studies, University of Groningen, 9700 CC Groningen, The Netherlands

- Model can also be used for other animal populations;
  density dependent movement also observed in grazing herds (elk)

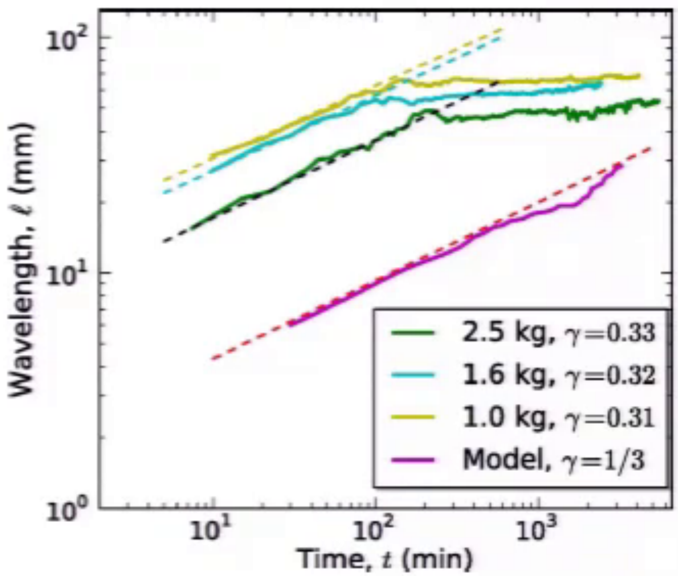
# Future/Discussion

- Long term dynamics (Turing patterns): add growth and mortality
- Couple to variation and availability of food resources



# Future/Discussion

- Long term dynamics (Turing patterns): add growth and mortality
- Couple to variation and availability of food resources




- Model assumes Brownian motion but animals perform a **Lévy walk** when their density is low


Time:
0:13 / 0:40
LevyMussel-large.wmv

Leave Fullscreen
Time:
0:04 / 0:40
LevyMussel-large.wmv