Numerical Solution of Eigenvalue Problems Arising in the Analysis of Disc Brake Squeal

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Mathematics for key technologies

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Outline

1. Introduction
2. Numerical Linear algebra, Model reduction.
3. Adaptive Finite Elements for evp
4. Conclusions

Challenges in MSO

- Key technologies require **M**odeling, **S**imulation, and **O**ptimization (MSO) of complex dynamical systems.
- Modeling, analysis, numerics, control and optimization techniques should go hand in hand.
- The quantification of errors and uncertainties is lagging behind.
- **Are we able to solve problems in industrial practice?**
- **Do we have a rigorous mathematical background?**
- **Can we analyze errors, uncertainties?**
- **Can we put this into mathematical software?**

Numerical Linear Algebra is a key factor in this.

Model based approach

Interdisciplinary project with car manufacturers + SMEs

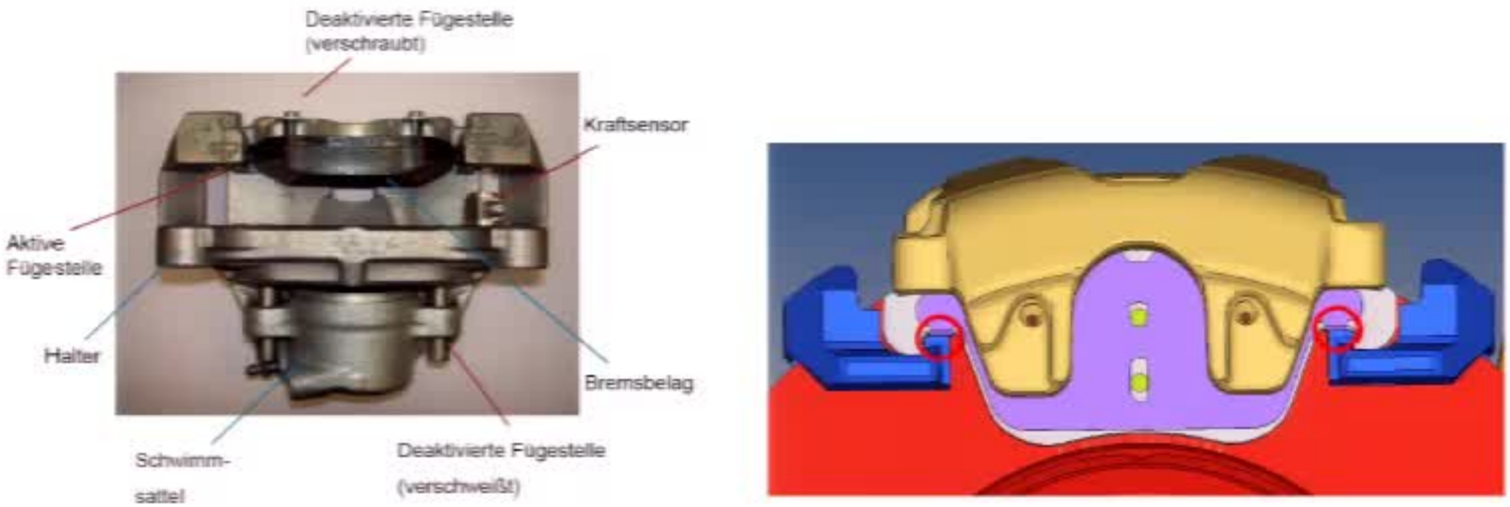
Supported by German Minist. of Economics via AIF foundation.

University: N. Gräbner, U. von Wagner, TU Berlin, Mechanics,
N. Hoffmann, TU Hamburg-Harburg, Mechanics,
S. Quraishi, C. Schröder, TU Berlin Mathematics.

Goals:

- Develop model of brake system with all effects that may cause squeal. (Friction, circulatory, gyroscopic effects, etc).
- Simulate brake behavior for many different parameters (disk speed, material and geometry parameters).
- Lin. Alg. tasks: Detection of instability, model reduction, solution of large scale parametric eigenvalue problems.
- Passive (optimization) and active (control) remedies.
- Future: Stability/bifurcation analysis for a parameter region.

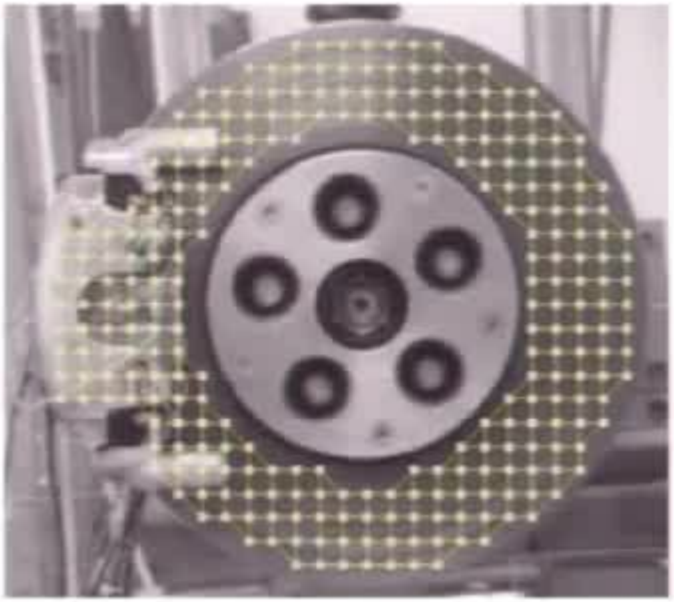
Brake pad



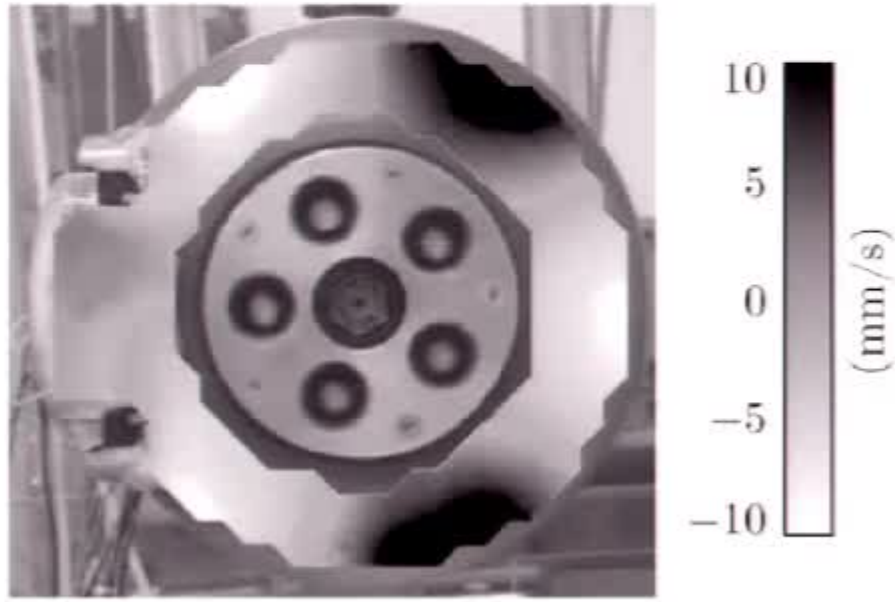
[1]

Figure: View of the brake model

[1] Institute f. Mechanics TU HH



Gitter der Messpunkte



Betriebsschwingform (1750 Hz)

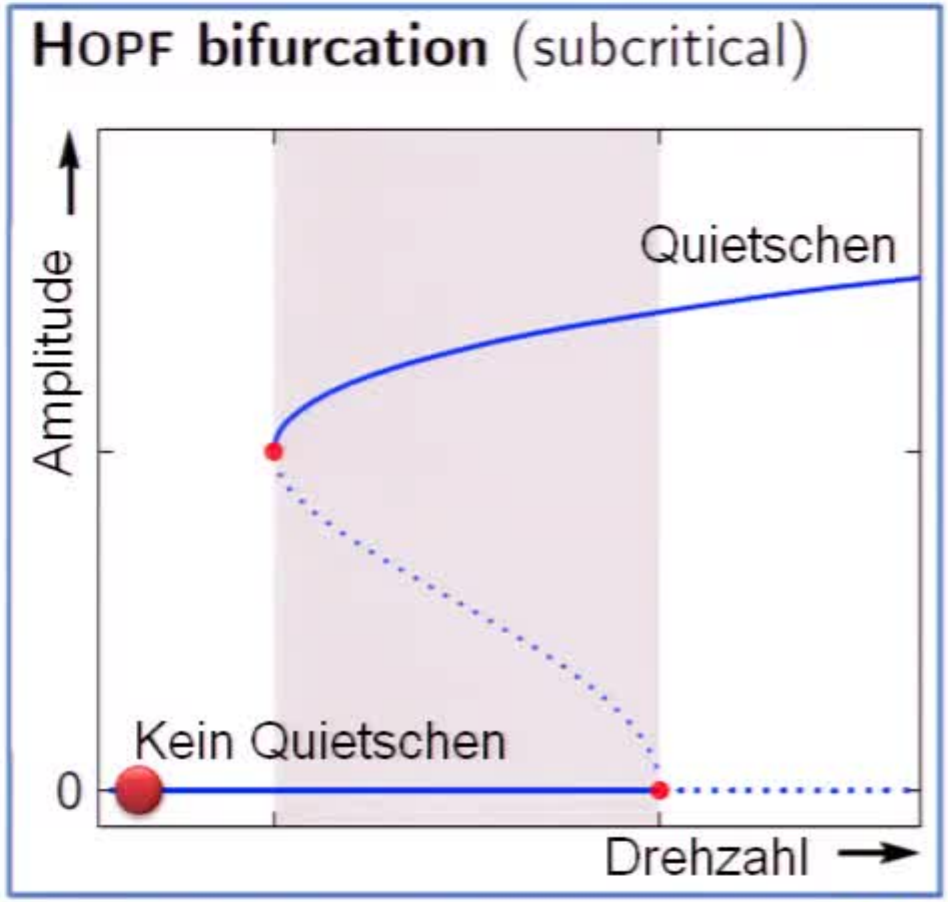
[2]

- Experiments indicate nonlinear behavior (subcritical Hopf bifurcation) → film.

[2] Institute f. Mechanics, TU Berlin

Einfluss von Nichtlinearitäten

Start

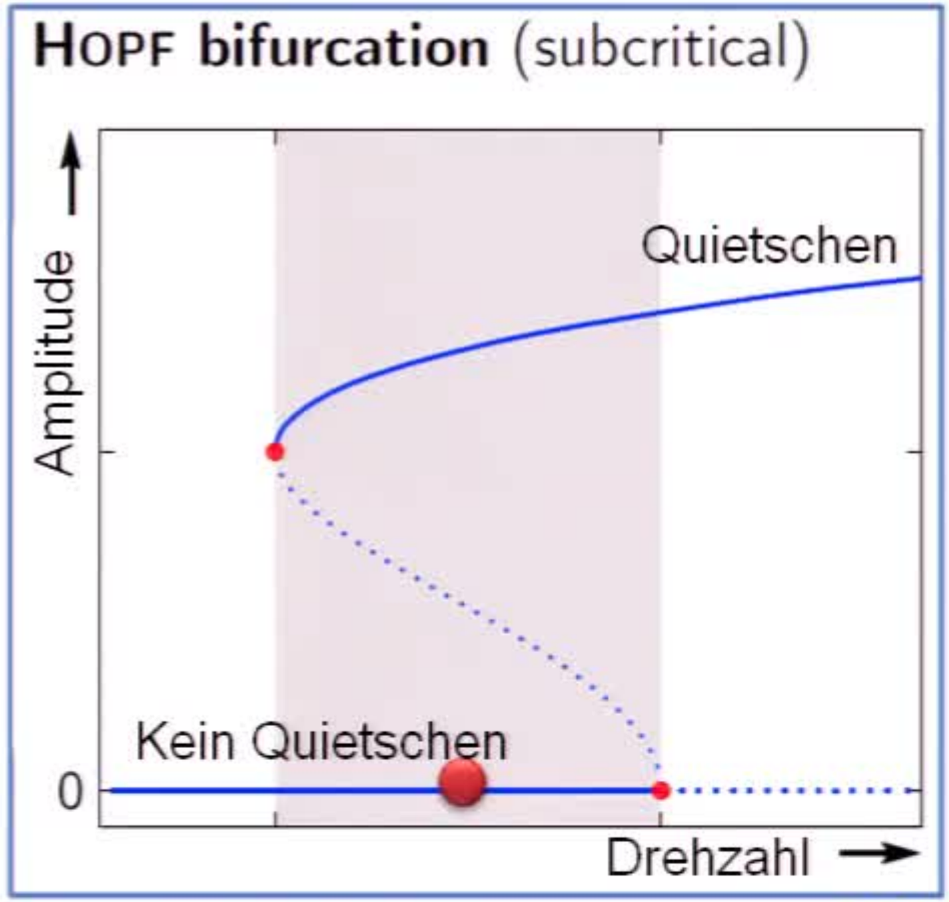


MMD

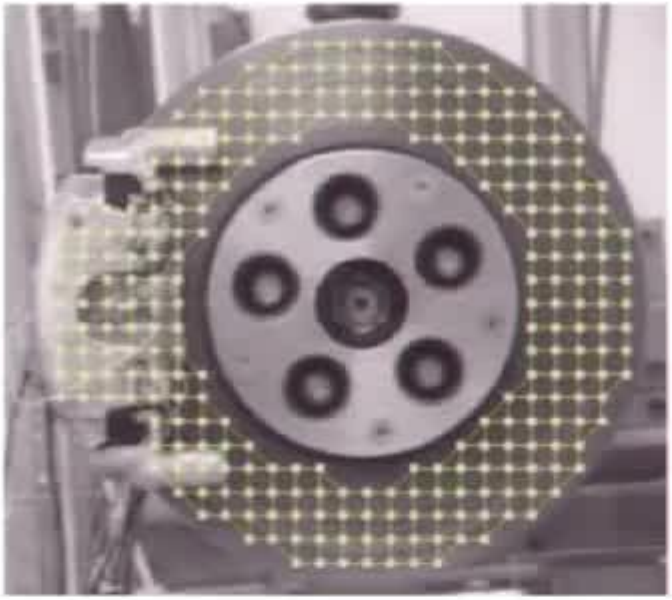
Einfluss von Nichtlinearitäten

Technische Universität Berlin

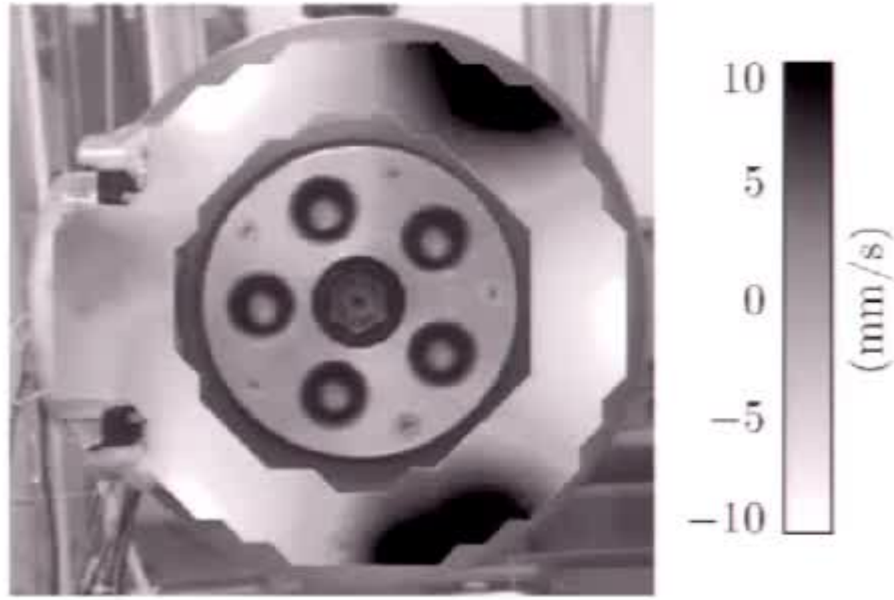
Start



Experiment



Gitter der Messpunkte



Betriebsschwingform (1750 Hz) [2]

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[2] Institute f. Mechanics, TU Berlin

Modeling in industrial practice

Multi-body system based on Finite Element Modeling (FEM)

- Write displacements of structure $z(x, t)$ as linear combination of basis functions (e.g. piecewise polynomials),

$$z(x, t) \approx \sum_{i=1}^{N} q_i(t)\phi_i(x, t).$$

- Integrate against test functions (Petrov Galerkin) $\rightarrow$ discretized model for the vibrations in weak form.
- Add friction and damping as macroscopic surrogate model fitted from experimental data.
- Simplifications: Remove some nonlinearities, asymptotic analysis for small parameters, etc.

Mathematical model details

Large differential-algebraic equation (DAE) system and evp depend. on parameters (here only disk speed displayed).

$$M\ddot{q} + (C_1 + \frac{\omega_r}{\omega} C_R + \frac{\omega}{\omega_r} C_G)\dot{q} + (K_1 + K_R + (\frac{\omega}{\omega_r})^2 K_G) q = f,$$

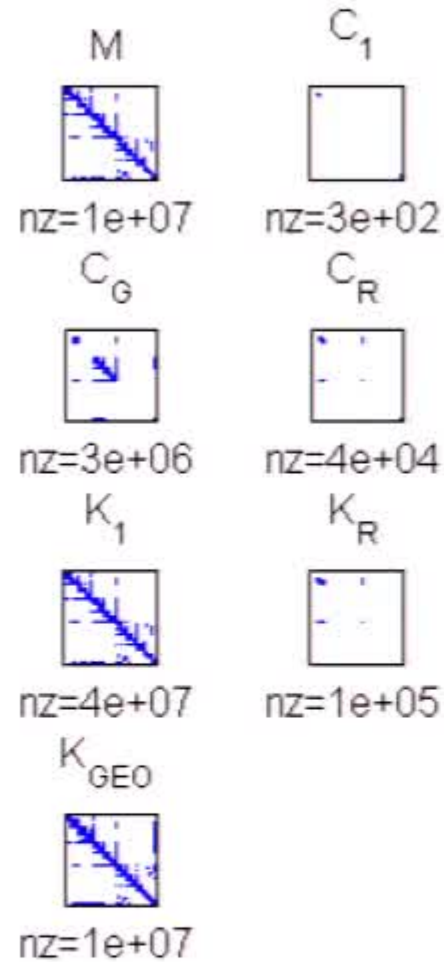
- M symmetric, pos. semidef., singular matrix (constraints),
- C_1 symmetric matrix, material damping,
- C_G skew-symmetric matrix, gyroscopic effects,
- C_R symmetric mat., friction induced damping, (phenomenological)
- K_1 symmetric stiffness matrix,
- K_R nonsymmetric matrix, circulatory effects,
- K_G symmetric geometric stiffness matrix.
- ω rotational speed of disk with reference velocity ω_r.
- Other parameters, material, geometry, etc.

Nature of FE matrices

Industrial model

$C = C_1 + \frac{\omega_r}{\omega} C_R + \frac{\omega}{\omega_r} C_G,$
$K = K_1 + K_R + (\frac{\omega}{\omega_r})^2 K_G$
$n = 842,638,\ \omega_r = 5,\ \omega = 17 \times 2\pi$

matrix	structure	2-norm	rank
M	symm	5e-2	842,623
C_1	symm	1e-19	160
C_G	skew	1.5e-1	217500
C_R	symm	7e-2	2120
K_1	symm	2e13	full
K_R	-	3e4	2110
K_G	symm	40	842,623



Model evaluation, challenges

This is really a hierarchy and mixture of models.

- FE Model hierarchy: grid hierarchy, type of ansatz functions, component and domain decomposition.
- Coupled with surrogate model for friction and damping?

Challenges

- Are the simplifications nonlinear/linear, expansions justified?
- We do not have a PDE, error estimates, adaptivity?
- How can we get a reduced model for optimization.
- How can we solve the parametric eigenvalue problem.

Can we analyze the model and quantify the errors?

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Complex eigenvalue analysis

- Ansatz $q(t) = e^{\lambda(\omega)t} v(\omega)$ gives quadratic evp (QEP):

$$P_\omega(\lambda)v(\omega) = (\lambda(\omega)^2 M + \lambda(\omega)C(\omega) + K(\omega))v(\omega) = 0.$$

- Want evs with positive real part and corresponding evecs. These are few, ideally one, since squeal is mono-frequent.
- Want problem to be robustly away from instability for all disk speeds. (Distance to instability.)
- Want efficient method to compute evs/ pseudospectra in right half plane for many parameter values.
- Want subspace associated with all the unstable evs for model reduction.

Is there anything to do? Why did the companies ask for help?

Projection approach

- Project QEP: $P_\omega(\lambda)v(\omega) = (\lambda^2 M + \lambda C(\omega) + K(\omega))v(\omega) = 0$ into small subspace spanned by columns of Q independent of ω.
- Projected QEP

$$\tilde{P}_\omega(\lambda) = Q^T P_\omega(\lambda) Q = \lambda^2 Q^T M Q + \lambda Q^T C(\omega) Q + Q^T K(\omega) Q$$

- How to choose Q?
  - to get sufficiently good approximation of evs with pos. real part;
  - ideally Q should contain good approximations to the desired evecs for all parameter values;
  - be able to construct Q in a reasonable amount of computing time.

Traditional approach

Traditional (heuristic) approach: Q_{TRAD}:=dominant evecs (ass. with smallest evs) of generalized eigenvalue problem (GEVP) $L(\mu) = (\mu M - K_E)$ $(\mu = -\lambda^2)$

Advantage:

- One only has to solve a large, sparse, symmetric, definite GEVP.

Disadvantages:

- Subspace does not take into account damping and parameter dependence.
- Often poor approximation of evs/evecs of the full model.

Solution of full Problem

Spectral transformation Consider full problem $P_\omega(\lambda)v(\omega) = 0$.

- Set $\lambda_\tau(\omega) = \lambda(\omega) - \tau$, where τ is such that $\det(P_\omega(\tau)) \neq 0$.
- New parametric QEP

  $$P_{\omega,\tau}(\lambda(\omega))x(\omega) = (\lambda_\tau(\omega)^2 M_\tau + \lambda_\tau(\omega) C_\tau(\omega) + K_\tau(\omega))v(\omega) = 0,$$

  where $M_\tau = M$, $C_\tau = 2\tau M + C$ and $K_\tau = \tau^2 M + \tau C + K$ is nonsingular.
- Shift point τ is chosen in the right half plane, ideally near the expected eigenvalue location.
- Consider reverse polynomial, then evs near τ become large in modulus, while evs far away from τ become small.

Linearization, first order form.

We use classical companion linearization (first order form)

$$A_\tau(\omega)v(\omega) = \mu_\tau B_\tau(\omega)v(\omega)$$

with

$$\begin{bmatrix} K_\tau(\omega) & 0 \\ 0 & I_n \end{bmatrix} \begin{bmatrix} v(\omega) \\ \mu_\tau(\omega)v(\omega) \end{bmatrix} = \mu_\tau(\omega) \begin{bmatrix} -C_\tau(\omega) & -M_\tau \\ I_n & 0 \end{bmatrix} \begin{bmatrix} v(\omega) \\ \mu_\tau v(\omega) \end{bmatrix}.$$

Structured linearizations. Mackey/Mackey/Mehl/M. 2006, Dopico, de Teran, Mackey 2011-2015

Shift and invert Arnoldi

- Compute ev and evec approximations near shift τ via shift-and-invert Arnoldi method ARPACK Lehouq/Sorensen/yang
- Given $v_0 \in \mathbb{C}^n$ and $W \in \mathbb{C}^{n\times n}$, the Krylov subspace of $\mathbb{C}^n$ of order k associated with W is

$$\mathcal{K}_k(W, v_0) = span\{v_0, Wv_0, W^2 v_0 \dots, W^{k-1} v_0\}.$$

- Arnoldi obtains orthonormal basis V_k of this space and

$$WV_k = V_k H_k + fe_k^*,$$

- Columns of V_k approx. k-dim. invariant subspace of W.
- Evs of H_k approximate evs of W associated to V_k.
- Apply with shift τ and frequency ω to $W = B_\tau(\omega)^{-1} A_\tau(\omega)$. Per step we multiply with $A_\tau(\omega)$ and solve system with $B_\tau(\omega)$.

Parametric Projection (POD)

New proper orthogonal decomposition (POD) approach

- Construct a measurement matrix $V \in \mathbb{R}^{n,km}$ containing 'unstable' evecs for a set of ω_i,

$$V = [V(\omega_1), V(\omega_2), V(\omega_3), \dots V(\omega_k)]$$

- Perform (partial) SVD $V = U\Sigma Z^H$

$$V = [\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_{km}] \begin{bmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \sigma_3 & & \\ & & & \ddots & \\ & & & & \sigma_{km} \end{bmatrix} [\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_{km}]^H$$

with U, Z unitary.

Compression

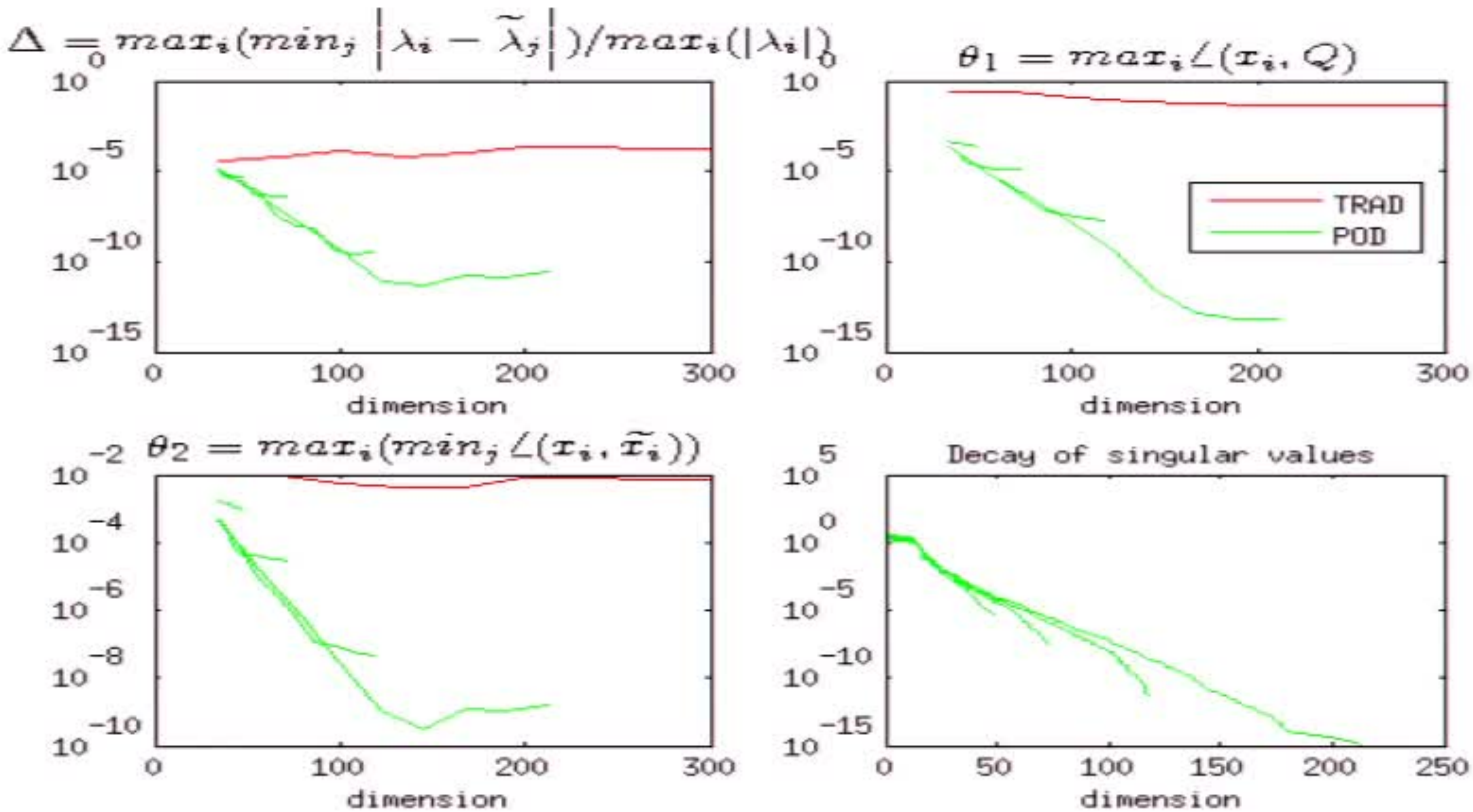
- Use approximation

$$\tilde{V} \approx [\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_d] \begin{bmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \sigma_3 & & \\ & & & \ddots & \\ & & & & \sigma_d \end{bmatrix} [\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_d]^H$$

  by deleting $\sigma_{d+1}, \sigma_{d+2}, \ldots\sigma_{km}$ that are small. (Actually these are not even computed).
- Choose $Q = [\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_d]$ to project $P_\omega(\mu)$.

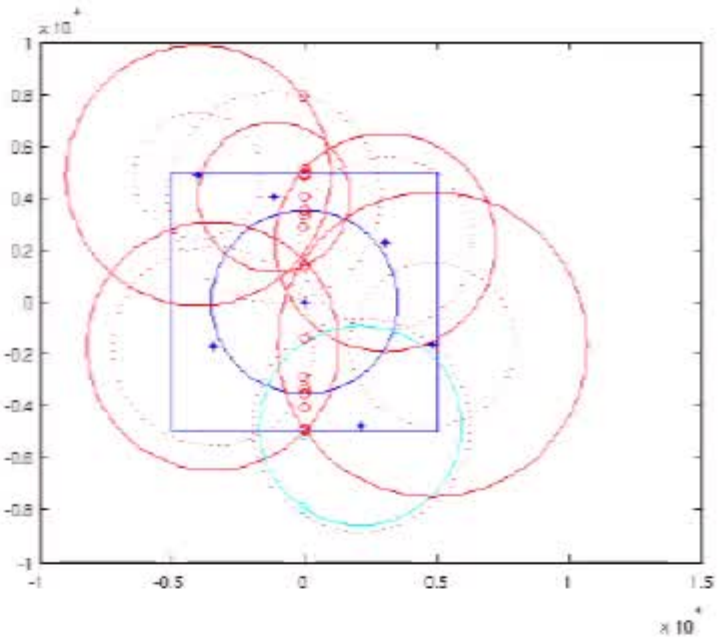
Results for toy problem $n \approx 5000$

- SVD reduction for uniformly spaced $\omega_j = 2^j + 1, j = 0, 1, 2, \dots$



- Increasing dimension does not improve traditional approach

Algorithm for choosing shifts



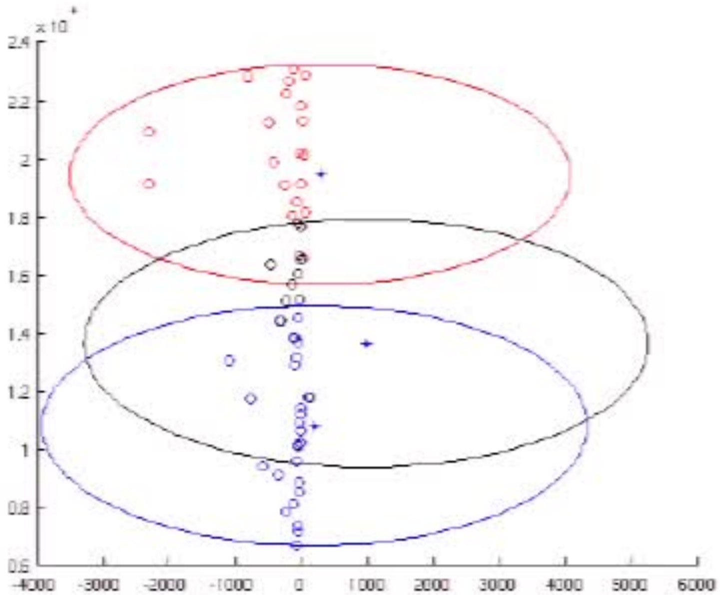
- Use ARPACK/eigs to compute evs with shift at center of rectangle.
- Compute covered area A_c

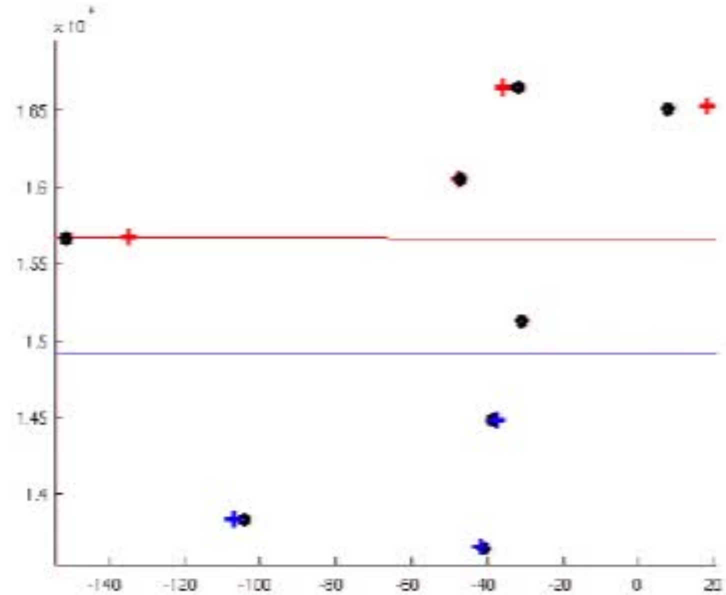
while $(A_c < 1)$
  - select a large number (e.g. 500) of circles with random radius, outside covered area
  - choose center which gives maximum A_c

end

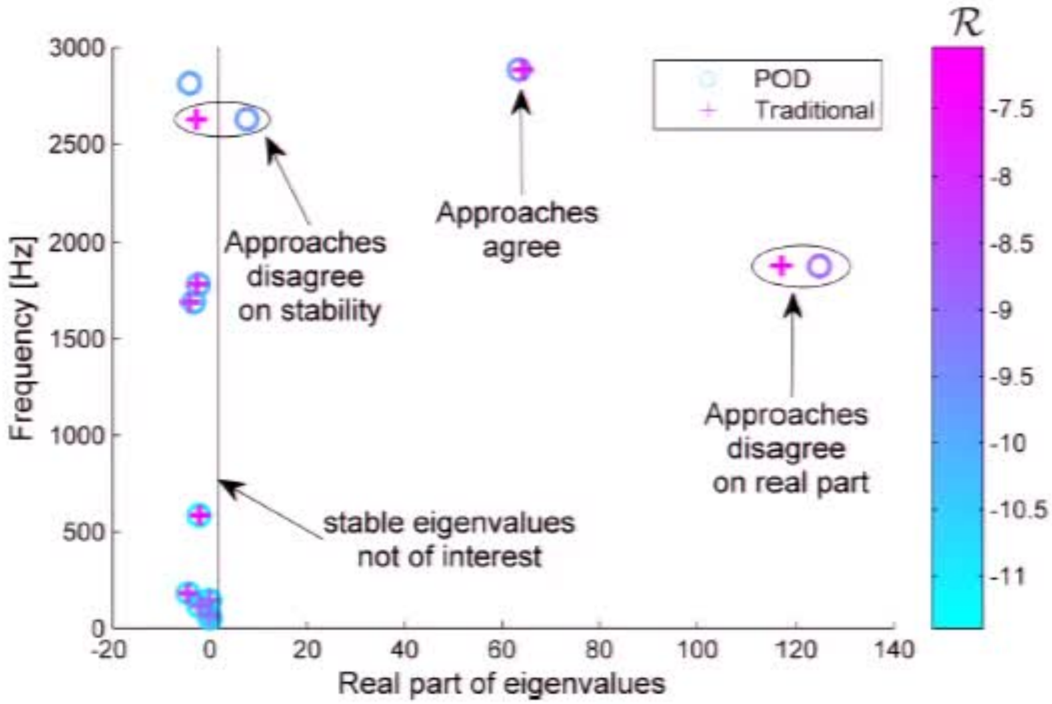
Mismatch of evs from different shifts

- Mismatch from different shifts
- o and + should agree





Mismatch of evs in different approaches



Problem in industrial models

- Shifted matrix $\tau^2 M + \tau C + K$ which has to be inverted at every step has condition number $\sim 10^{14}$ for a large range of shift points τ.
- Optimal scaling of three matrices and also diagonal scaling of system matrix has still condition number $\sim 10^{10}$ for a range of shift points.
- This is still too large to trust the results!

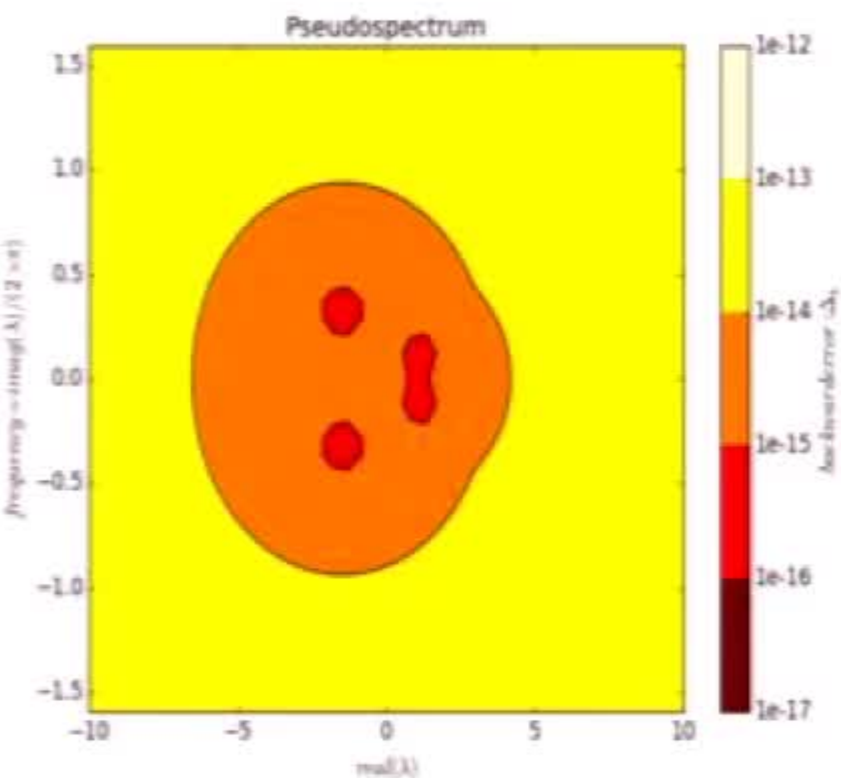
Assessing 'accuracy of evs'

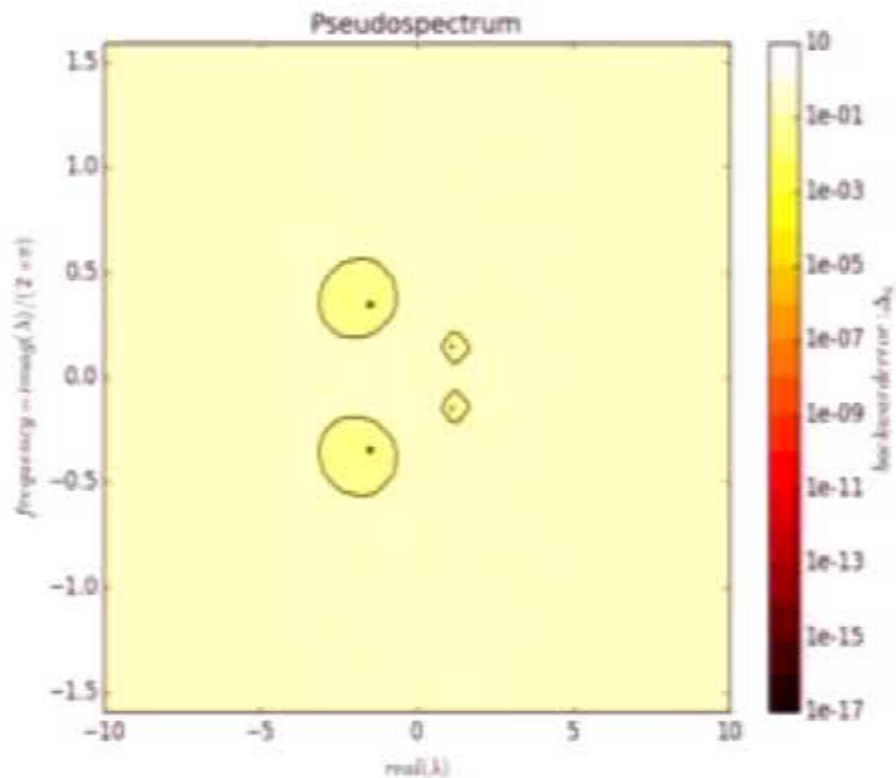
- Forward error: $\Delta_f = |\lambda_{exact} - \lambda_{computed}|$
- Backward error: smallest in norm perturbation Δ_b to M, C, K such that $\tilde{v}, \tilde{\lambda}$ satisfies QEVP defined by perturbed matrices $\tilde{M}, \tilde{C}, \tilde{K}$
- Computation of backward error: $\Delta_b(\lambda) = \frac{\|(\lambda^2 M + \lambda C + K)\|}{|\lambda|^2 \|M\| + |\lambda| \|C\| + \|K\|}$
- The pseudospectrum gives the level curves of $\Delta_b(\lambda)$).

Stiff springs are the reason for high sensitivity, see also Kannan/Hendry/Higham/Tisseur '14

Pseudospectrum of toy brake model

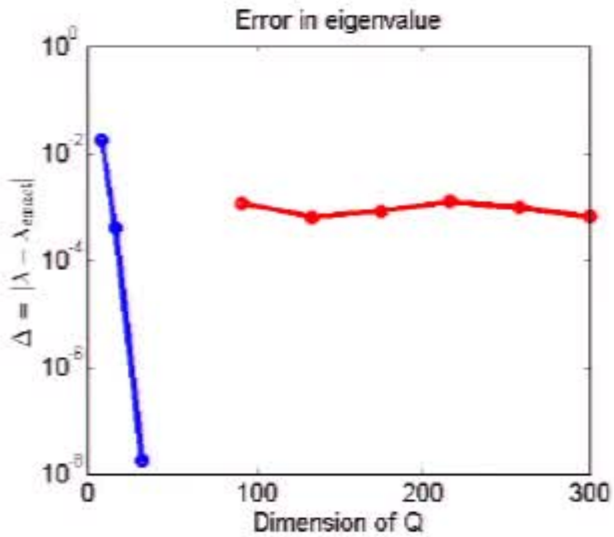
Brake model with 5000 dof, with stiff springs and with stiff springs replaced by rigid connections.

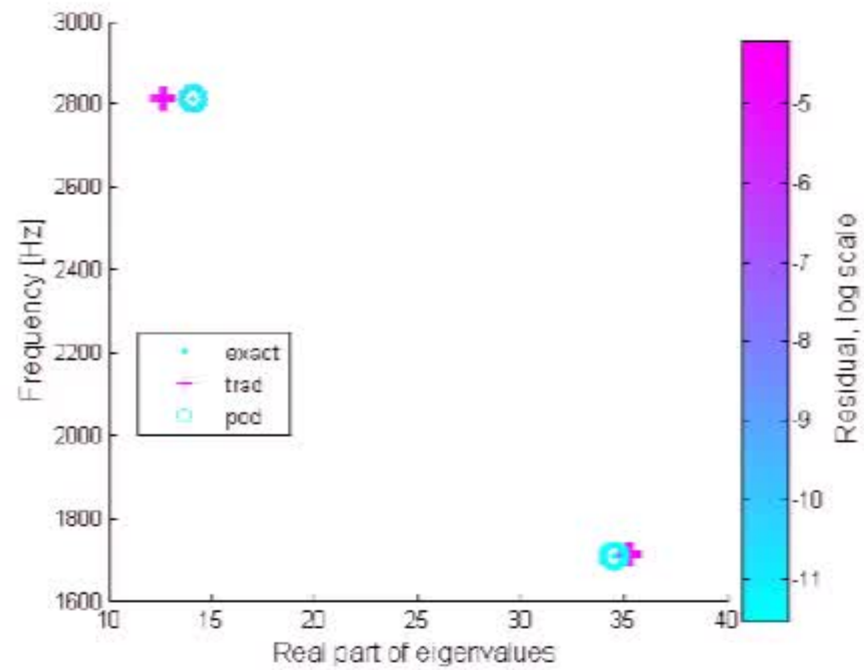




Results with new POD method

Industrial model 1 million dof





- Solution for every ω
  - Solution with 300 dimensional TRAD subspace $\sim$ 30 sec
  - Solution with 100 dimensional POD subspace $\sim$ 10 sec

Intermediate Conclusions

- Modeling with very stiff springs is not advisable.
- New POD approach better than traditional one but not satisfactory.
- Discrete FE and quasi-uniform grids followed by expensive model reduction is really a waste.
- Can we get error estimates and adaptivity? (AFEM, AMLS)
- Can we do better than uniform mesh and brute force linear algebra.

Model problem: Elliptic PDE evp

Consider a model problem like the disk brake without damping, gyroscopic, circulatory terms and reasonable geometry.

$$\begin{aligned} \Delta u &= \lambda u \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

Adapative Finite Element Method

- Adaptive Finite Element methods refine the mesh where necessary, and coarsen it where the solution is well represented.
- They use a priori and a posteriori error estimators to get information about the discretization error.
- They are well established for PDE boundary value problems.
- But here we want to use them for PDE eigenvalue problems, which is much harder.

Solve $\rightarrow$ **Estimate** $\rightarrow$ **Mark** $\rightarrow$ **Refine**

Solve: Weak formulation

Weak formulation:
Determine ev/e.-function pair $(\lambda, u) \in \mathbb{R} \times V := \mathbb{R} \times H^1(\Omega; \mathbb{R})$ with $b(u, u) = 1$ and

$$a(u, v) = \lambda b(u, v) \quad \text{for all } v \in V,$$

where the bilinear forms $a(\cdot, \cdot)$ and $b(\cdot, \cdot)$ are defined by

$$a(u, v) := \int_\Omega \nabla u \cdot \nabla v \, dx, \; b(u, v) := \int_\Omega uv \, dx \quad \text{for } u, v \in V.$$

Induced norms $|||\cdot||| := |\cdot|_{H^1(\Omega)}$ on V and $\|\cdot\| := \|\cdot\|_{L^2(\Omega)}$ on $L^2(\Omega)$.

Solve: Discrete Formulation

Discrete evp: Determine ev./e.-function pair $(\lambda_\ell, u_\ell) \in \mathbb{R} \times V_\ell$ with $b(u_\ell, u_\ell) = 1$

$$a(u_\ell, v_\ell) = \lambda_\ell b(u_\ell, v_\ell) \quad \text{for all } v_\ell \in V_\ell.$$

Algebraic eigenvalue problem: Use coordinate representation to get finite-dim. generalized evp

$$A_\ell x_\ell = \lambda_\ell B_\ell x_\ell$$

stiffness matrix $A_\ell = [a(\varphi_i, \varphi_j)]_{i,j=1,\dots,N_\ell}$, mass matrix $B_\ell = [b(\varphi_i, \varphi_j)]_{i,j=1,\dots,N_\ell}$, assoc. with nodal basis $V_\ell = \{\varphi_1, \dots, \varphi_{N_\ell}\}$.
Discrete eigenvector: $x_\ell =: [x_{\ell,1}, \dots, x_{\ell,N_\ell}]^T$.
Approximated eigenfunction:

$$u_\ell = \sum_{k=1}^{N_\ell} x_{\ell,k}\varphi_k \in V_\ell.$$

Error estimation

This approach includes several errors:

- Model error (PDE model vs. Physics)
- Discretization error (finite dim. subspace)
- Error in eigenvalue solver (iterative method)
- Roundoff errors in finite arithmetic.

Estimate the error a posteriori via

$$|\lambda - \lambda_\ell| + \|u - u_\ell\|^2 \lesssim \eta_\ell^2 := \|u_{\ell-1} - u_\ell\|^2.$$

Here $\lesssim$ denotes inequality up to a multiplicative constant.
A posteriori error estimators for Laplace eigenvalue problem
Grubisic/Ovall 2009, M./Miedlar 2011, Neymeyr 2002

Marking strategy

Employ an edge residual a posteriori error estimator
Duran et al 2003, Carstensen/Gedicke 2008.

$$\eta_\ell^2 := \sum_{E \in \mathbb{E}_\ell(\Omega)} \eta_\ell^2(E) \quad \text{with} \quad \eta_\ell^2(E) := |E| \| [\nabla u_\ell] \cdot \nu_E \|_{L^2(E)}^2,$$

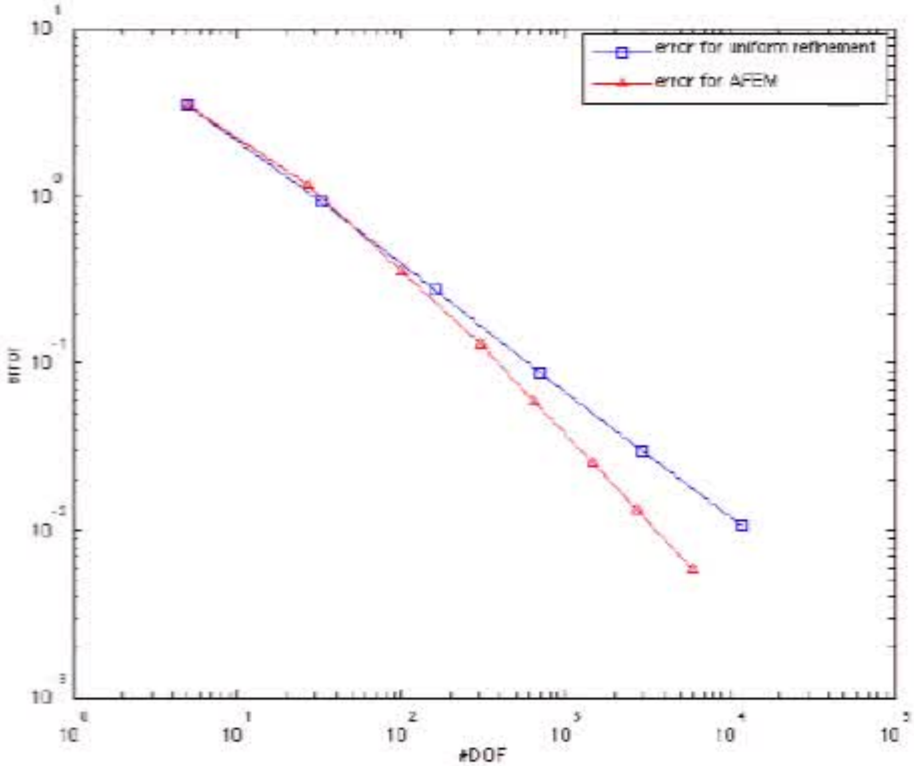
which is reliable and efficient for sufficiently small mesh-size H_0

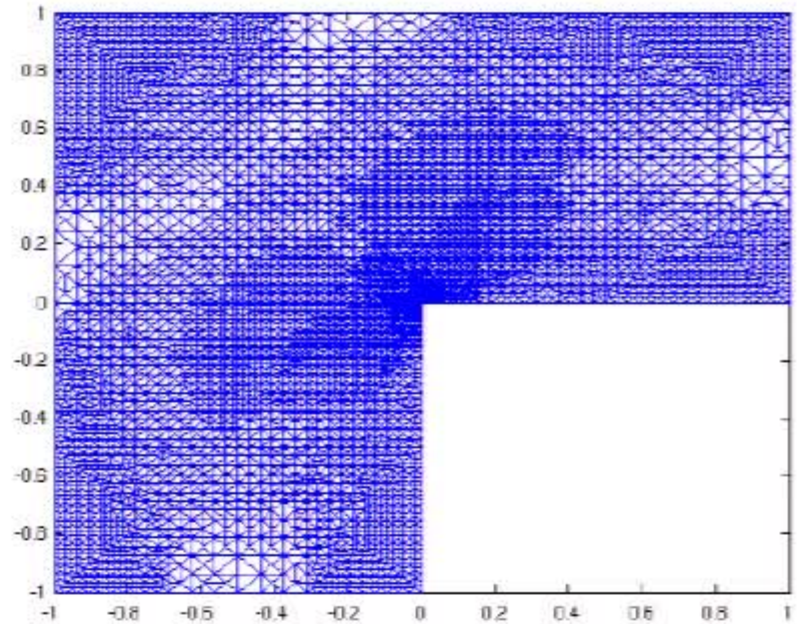
$$|\!|\!| u - u_\ell |\!|\!| \approx \eta_\ell.$$

Let $\mathbb{M}_\ell \subseteq \mathbb{N}_\ell(\Omega)$ be the minimal set of refinement nodes such that for $0 < \theta \leq 1$

$$\theta \sum_{z \in \mathbb{N}_\ell(\Omega)} \eta_\ell^2(\mathbb{E}_\ell(z)) \leq \sum_{z \in \mathbb{M}_\ell} \eta_\ell^2(\mathbb{E}_\ell(z)).$$

Convergence on L-shape domain.





Evaluation of AFEM

- AFEM works nicely for elliptic self-adjoint evps, even with complicated domains.
- For the analysis in most AFEM methods it is assumed that the algebraic evp is solved exactly.
- The high accuracy solution of the algebraic evps requires most of the computing time.
- The solution of the algebraic evp is only used to determine where the grid is refined. This is a complete waste of computational work.
- How can we incorporate the approximate solution of the algebraic evp into the adaptation process?

AFEMLA M./Miedlar 2011

Solve:

- compute approx. eigenpair $(\tilde{\lambda}_H, \tilde{\mathbf{u}}_H)$ on the coarse mesh,
- use iterative solver, i.e. Krylov subspace method,
- but do not solve very accurately, stop after a few steps or when tolerance *tol* is reached.

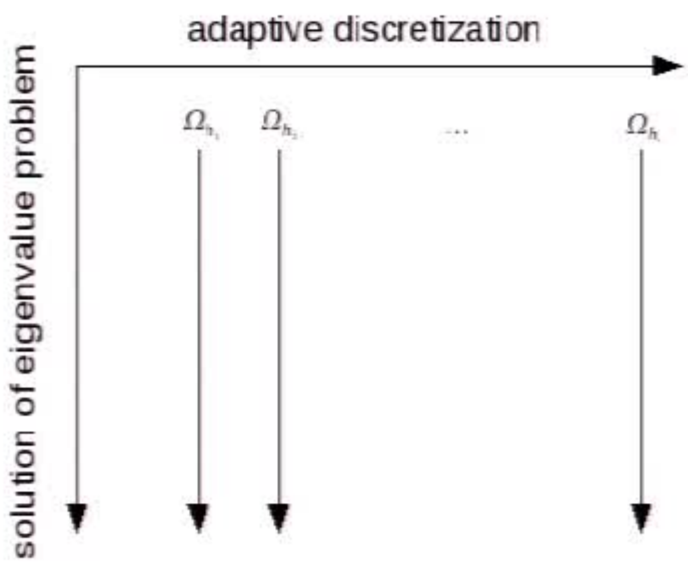
Estimate:

- prolongate $\tilde{\mathbf{u}}_H$ from the coarse mesh $\mathcal{T}_H$ to the uniformly refined mesh $\mathcal{T}_h$,
- Balance residual vector $\hat{\mathbf{r}}_h$ and error estimate Miedlar 2011.

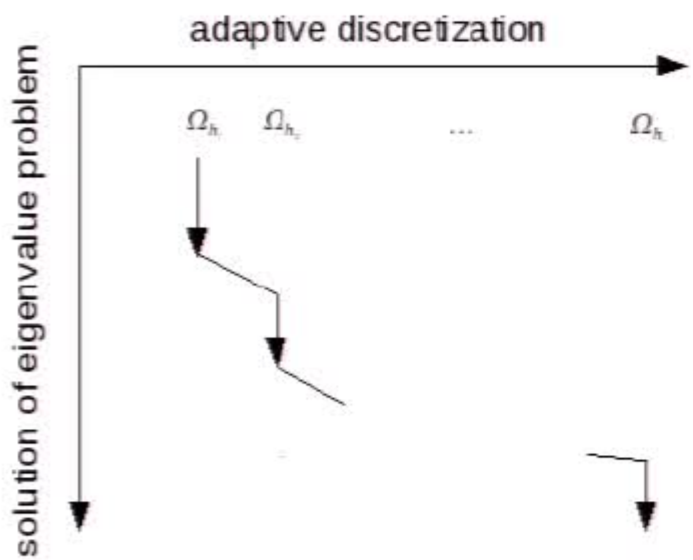
Mark and Refine: mark elements and refine the mesh.

Standard AFEM versus AFEMLA

Solve → Estimate → Mark → Refine



Standard AFEM



AFEMLA

Evaluation of AFEMLA

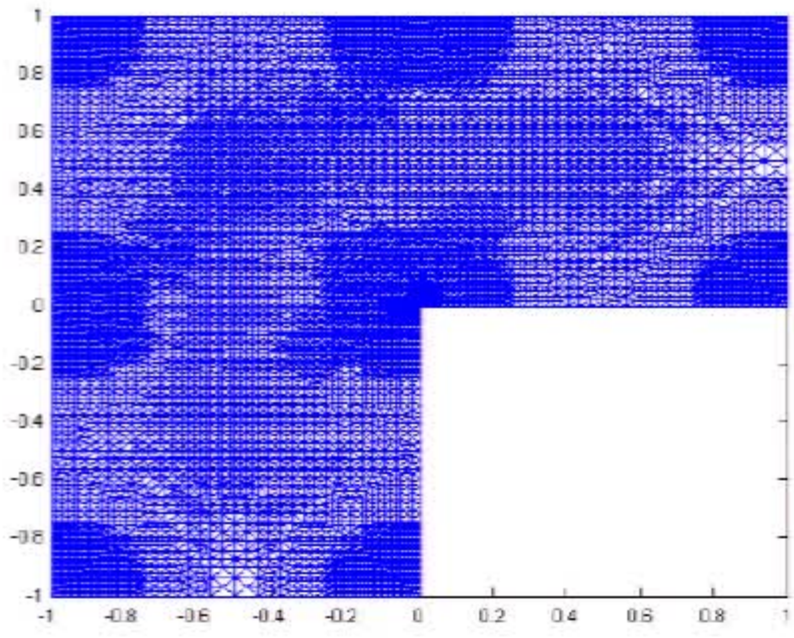
- AFEMLA works nicely for elliptic self-adjoint evps.
- It significantly reduces the computing time.
- Balancing of discretization and LA error, Miedlar 2011.
- Proof of convergence M./Miedlar 2011 if saturation property holds, i.e., there exist $\beta < 1$ such that $|\lambda_h - \lambda| \leq \beta |\lambda_H - \lambda|$.

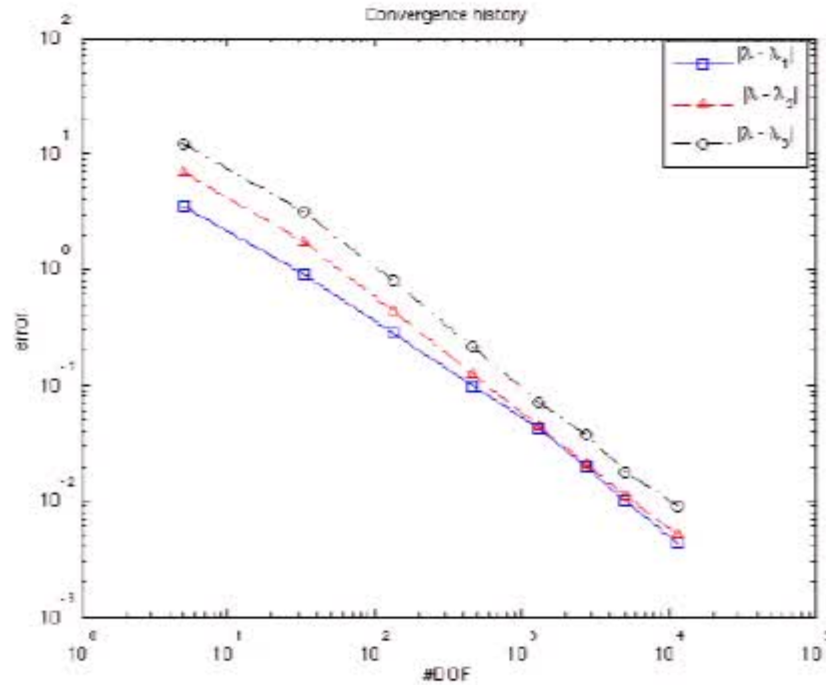
Theorem (Carstensen/Gedicke/M./Miedlar 2013)

Suppose that the initial triangulation $\mathcal{T}_0$ has sufficiently small maximal mesh-size H_0. Then there exists $0 \leq \varrho < 1$ such that for all $\ell \in \mathbb{N}_0$ the following inequalities hold

$$\begin{aligned} |\!|\!| u - u_{\ell+1} |\!|\!|^2 &\leq \varrho |\!|\!| u - u_\ell |\!|\!|^2 + \lambda_{\ell+1}^3 H_\ell^4; \\ |\lambda - \lambda_{\ell+1}| &\leq \varrho |\lambda - \lambda_\ell| + \lambda_{\ell+1}^3 H_\ell^4. \end{aligned}$$

Conv. first 3 evs, L-shape domain.





Another approach: AMLS

Compute smallest evs of self-adjoint evp $(\lambda M - K)x = 0$ with M, K pos. def. as in trad. approach. Bennighof-Lehouq 2004

- Use symmetric reordering of matrix to block form or use directly domain decomposition partition. $(\lambda \tilde{M} - \tilde{K})x = 0$, with structure 
- Compute block Cholesky factorization of $\tilde{M} = LDL^T$ and form $\hat{K} = L^{-1}\tilde{K}L^{-T}$.
- Compute smallest evs and evecs of 'substructure' evps $(\lambda D_{ii} - \hat{K}_{ii})x_i$ and project large problem (modal truncation).
- Solve projected evp.

Analysis of AMLS

- This produces locally global (spectral) ansatz functions in substructure.
- This is a domain decomposition approach, where efunctions are used in substructures.
- Substructure efunctions are sparsely represented in FE basis.
- Analysis only for self-adjoint case and real simple evs.
- Works extremely well for mechanical structures with little damping.
- How can we modify the ideas of AFEM/AMLS to deal with the general problem?

A non-self-adjoint model problem

Carstensen/Gedicke/M./Miedlar 2012

Convection-diffusion eigenvalue problem:

$$-\Delta u + \gamma \cdot \nabla u = \lambda u \text{ in } \Omega \qquad \text{and} \qquad u = 0 \text{ on } \partial\Omega$$

Discrete weak primal and dual problem:

$$\begin{aligned} a(u_\ell, v_\ell) + c(u_\ell, v_\ell) &= \lambda_\ell b(u_\ell, v_\ell) \quad \text{for all } v_\ell \in V_\ell, \\ a(w_\ell, u_\ell^\star) + c(w_\ell, u_\ell^\star) &= \overline{\lambda_\ell^\star} b(w_\ell, u_\ell^\star) \quad \text{for all } w_\ell \in V_\ell. \end{aligned}$$

Generalized algebraic eigenvalue problem:

$$(A_\ell + C_\ell)\mathbf{u}_\ell = \lambda_\ell B_\ell \mathbf{u}_\ell \qquad \text{and} \qquad \mathbf{u}_\ell^\star (A_\ell + C_\ell) = \lambda_\ell^\star \mathbf{u}_\ell^\star B_\ell$$

Smallest real part ev. is simple and well separated Evans '00.

Consider

$$\mathcal{H}(t) = (1-t)\mathcal{L}_0 + t\mathcal{L}_1 \quad \text{for } t \in [0,1],$$

where $\mathcal{L}_0 u := -\Delta u$ and $\mathcal{L}_1 u := -\Delta u + \beta \cdot \nabla u$.
Discrete homotopy for the model eigenvalue problem:

$$\mathcal{H}_\ell(t) = (A_\ell + C_\ell)(t) = (1-t)A_\ell + t(A_\ell + C_\ell) = A_\ell + tC_\ell.$$

Errors

Homotopy error:

$$|\lambda(1) - \lambda(t)| \lesssim (1-t)\|\gamma\|_{L^\infty(\Omega)}\|u\| = \nu,$$

Discretization error:

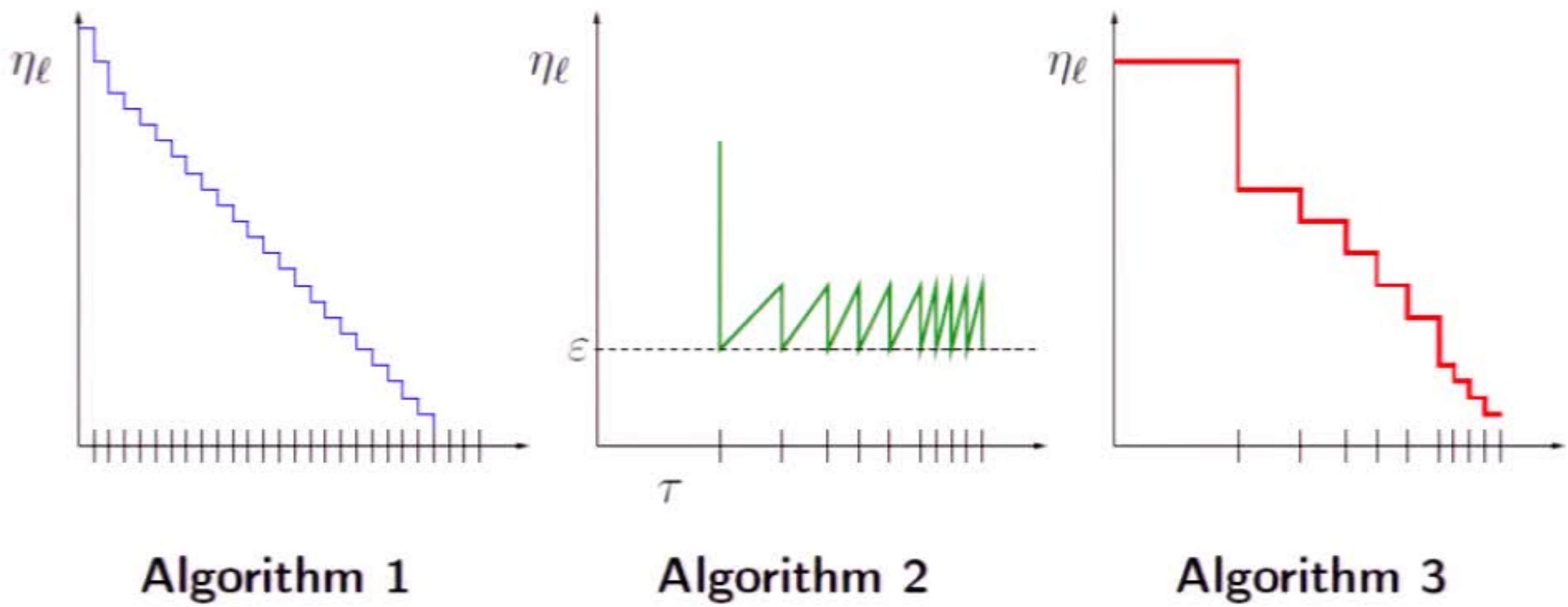
$$\|\lambda(t) - \lambda_\ell(t)\| \lesssim \sum_{T \in \mathcal{T}_\ell} \left(\eta_\ell^2(T) + \eta_\ell^{\star 2}(T)\right).$$

Approximation error:

$$|\lambda_\ell(t) - \tilde{\lambda}_\ell(t)| + |\lambda_\ell^\star(t) - \tilde{\lambda}_\ell^\star(t)| \leq \mu_\ell.$$

A posteriori error estimator: Carstensen/Gedicke/M./Miedlar '12

Error dynamics



Outline

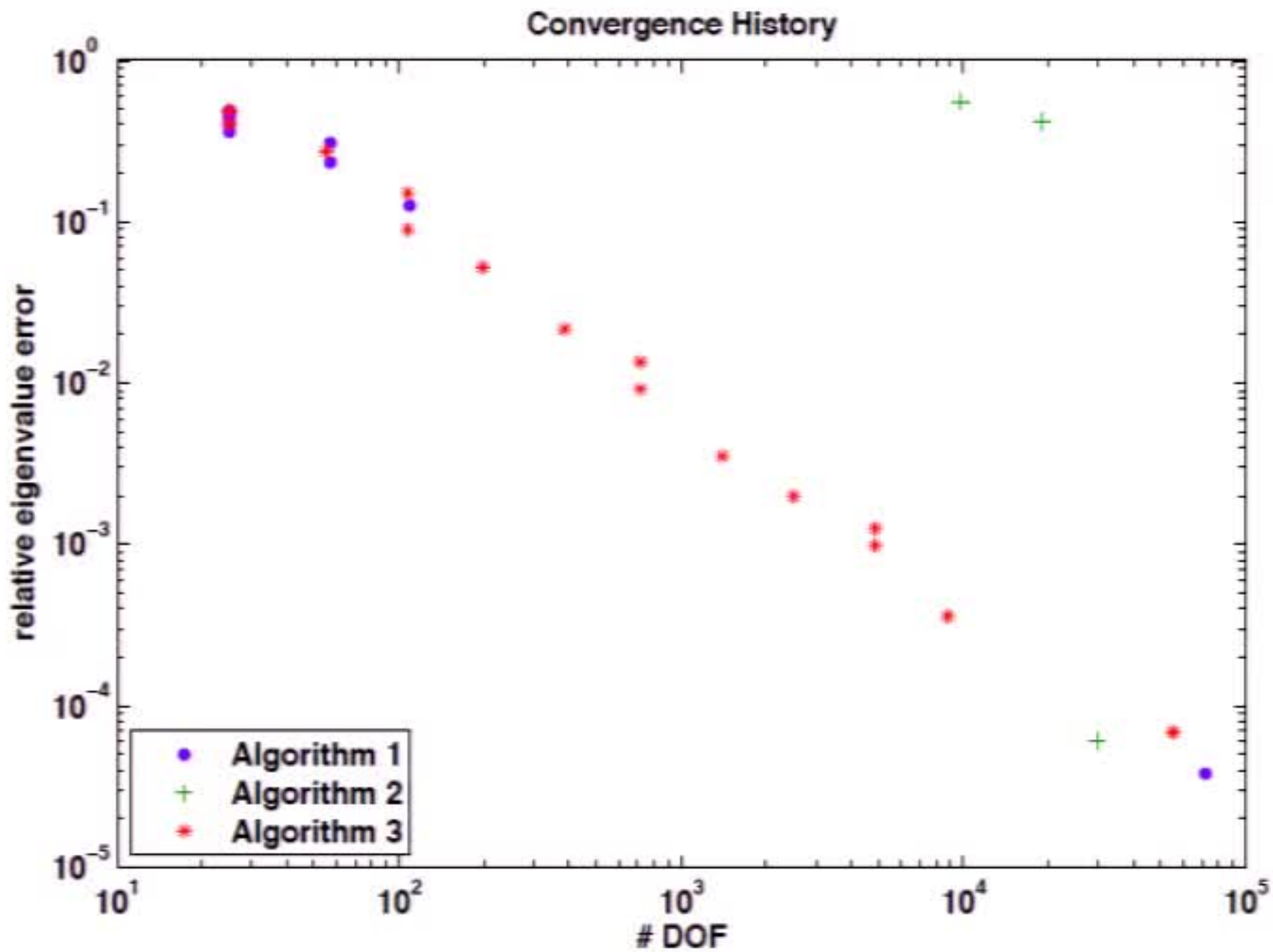


Figure: Conv. history of Algorithm 1, 2 and 3 with respect to #DOF.

1. Introduction
2. Numerical Linear algebra, Model reduction.
3. Adaptive Finite Elements for evp
4. Conclusions

Conclusions

- Eigenvalue methods are important in industrial practice.
- Using fine mesh and model reduction usually works, but hardly any error estimates exist.
- Current numerical linear algebra methods (in particular those in commercially available codes) are not satisfactory. AFEMLA is an alternative, it gives error bounds.
- Extension of backward error analysis to infinite dimensional case Miedlar 2011/2014
- A posteriori error estimates for hp-finite elements for non-self-adjoint PDE evps Giani/Grubisic/Miedlar/Ovall 2014
- Multiple evs self-adjoint case Galistil 2014
- **No results on multiple, complex evs, Jordan blocks in non-self-adjoint case**.
- Nonlinear effects, bifurcation, computation of limit cycle.

Thanks

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Details: http://www.math.tu-berlin.de/~mehrmann/
Video from MOR school in Pilsen: http://slideslive.com/t/more

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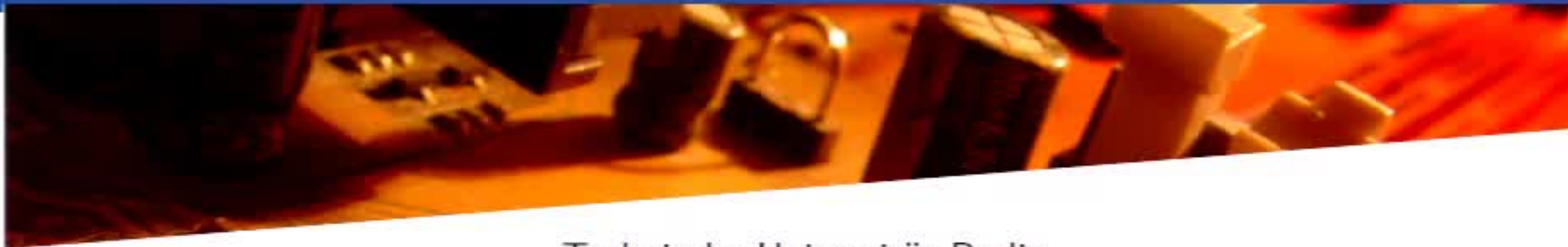
School and workshop announcement



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