Accounting for Model Error in EnKF by Stochastic Parameterization

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Kalman-Bucy Filter

Assumption: linear functions and Gaussian random variables

$$\begin{aligned} \mathbf{x}^{n+1} &= F^n \mathbf{x}^n + \mathbf{w}^n \\ \mathbf{z}^{n+1} &= H^{n+1} \mathbf{x}^{n+1} + \mathbf{v}^{n+1} \end{aligned}$$

$$\left.\begin{aligned} p(\mathbf{x}^n | \mathbf{z}^{1:n}) &\sim N(\mathbf{x}^n;\ \boldsymbol{m}_{n|n}, P_{n|n}) \\ p(\mathbf{x}^{n+1} | \mathbf{z}^{1:n}) &\sim N(\boldsymbol{m}_{n+1|n}, P_{n+1|n}) \\ &\quad\ \text{(prior density)} \\ \mathbf{z}^{n+1} &= H^{n+1} \mathbf{x}^{n+1} + N(0, R_{n+1}) \end{aligned}\right\}$$

$$\Rightarrow p(\mathbf{x}^{n+1} | \mathbf{z}^{1:n+1}) \sim N(\boldsymbol{m}_{n+1|n+1}, P_{n+1|n+1})$$

(posterior density)

Simple: only need to estimate the mean and covariance matrix to present the conditional probability density

KU KANSAS

Ensemble Kalman filters (EnKF): nonlinar models

- An ensemble X at time n: an $m \times N$ matrix

$$X = [\mathbf{x}_1^n, \cdots, \mathbf{x}_N^n], \quad \mathbf{x}_i^n \sim p(\mathbf{x}^n | \mathbf{z}^{1:n})$$

- Time $n+1$: forward the ensemble members using the model:

$$\mathbf{x}_i^{n+1} = f(\mathbf{x}_i^n) + \mathbf{v}^n$$

and replace the mean and covariance $m_{n+1|n}$ and $P_{n+1|n}$ by the sample mean and covariance from the ensemble members

$$X = [\mathbf{x}_1^{n+1}, \cdots, \mathbf{x}_N^{n+1}], \quad \mathbf{x}_i^{n+1} \sim p(\mathbf{x}^{n+1} | \mathbf{z}^{1:n})$$

- using Kalman formula to update the ensemble members to present the conditional probability density

Inflation and Localization for EnKF

- **Covariance localization:** to remove poorly estimated long-range spatial correlations due to insufficient ensemble size
- **Covariance inflation:** to account for the underestimation in the covariance of the forecast ensemble
- These techniques have been found to compensate the model error effectively