Stable, asymmetric ice belts in an energy balance model of Pluto

Alice Nadeau and Emma Jaschke
University of Minnesota

SIAM Conference on Applied Dynamical Systems
May 22, 2019

IMG: NASA

IMG: NASA

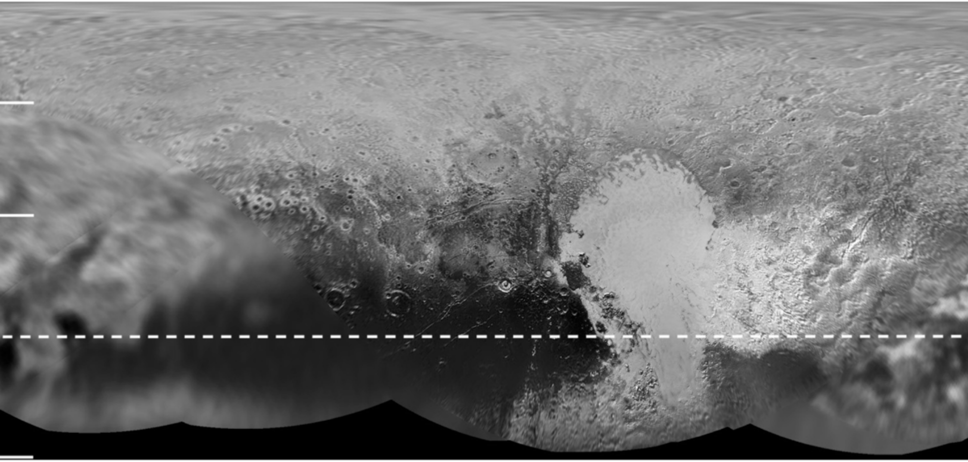
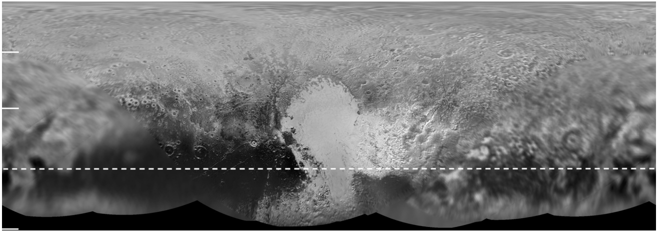


Image: NASA/JPL

Motivations: What feedback mechanisms are at play?

- Hamilton et al (2016) and Earle et al (2018) suggest a runaway albedo effect

- Earle et al (2017) suggest SP's location within diurnal zone



Today's Talk

- Description of the Budyko-Widiasih energy balance model
- Approximation with System of ODEs
- Results and comparison with observations

Nondimensional Budyko-Widiasih-type Energy Balance Model

$$R\frac{\partial T}{\partial t} = Qs(y,\zeta)(1-\alpha(y,\eta)) - (A + BT(y,t)) - C\left(T - \overline{T_\eta^*}\right), \qquad \frac{d\eta}{dt} = \epsilon(T_\eta^*(\eta) - T_c)$$

$$\downarrow$$

$$\frac{\partial \varphi}{\partial \tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta\left(\varphi(y,\tau) - \overline{\varphi}\right), \qquad \frac{d\eta}{d\tau} = \lambda\varphi(\eta)$$

Nondimensional Budyko-Widiasih-type EBM

$$\frac{\partial \varphi}{\partial \tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta\left(\varphi(y,\tau) - \overline{\varphi}\right)$$
$$\frac{d\eta}{d\tau} = \lambda \varphi(\eta)$$

$\varphi(y,\tau) = Q(T(y,t) - T_c)/B$ normalizes temperature based on critical temperature

Nondimensional Budyko-Widiasih-type EBM

$$\frac{\partial\varphi}{\partial\tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta(\varphi(y,\tau) - \overline{\varphi})$$
$$\frac{d\eta}{d\tau} = \lambda\varphi(\eta)$$

$\varphi(y,\tau) = Q(T(y,t) - T_c)/B$ normalizes temperature based on critical temperature

$\mu = \frac{A+BT_C}{Q}$ relates incoming and outgoing radiation

Nondimensional Budyko-Widiasih-type EBM

$$\frac{\partial\varphi}{\partial\tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta(\varphi(y,\tau) - \overline{\varphi})$$
$$\frac{d\eta}{d\tau} = \lambda\varphi(\eta)$$

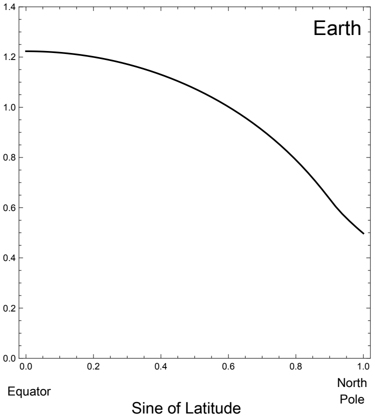
$\varphi(y,\tau) = Q(T(y,t) - T_c)/B$	normalizes temperature based on critical temperature
$\mu = \frac{A+BT_C}{Q}$	relates incoming and outgoing radiation
$\delta = C/B$	relates heat transport and outgoing radiation

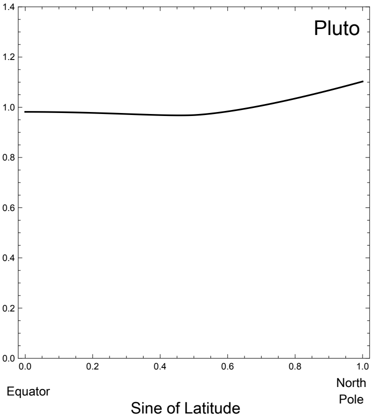
Nondimensional Budyko-Widiasih-type EBM

$$\frac{\partial\varphi}{\partial\tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta\left(\varphi(y,\tau) - \overline{\varphi}\right)$$
$$\frac{d\eta}{d\tau} = \lambda\varphi(\eta)$$

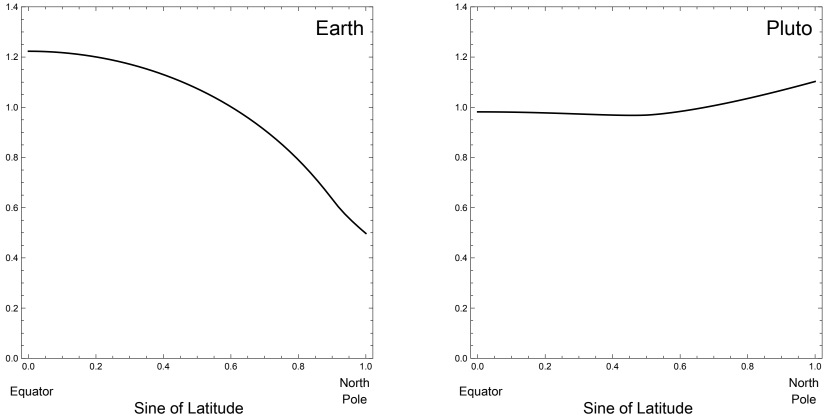
$\varphi(y,\tau) = Q(T(y,t) - T_c)/B$	normalizes temperature based on critical temperature
$\mu = \frac{A+BT_C}{Q}$	relates incoming and outgoing radiation
$\delta = C/B$	relates heat transport and outgoing radiation
$s(y,\zeta)$	annual average solar radiation

Incoming **sol**ar radi**ation**: $s(y, \zeta)$





Incoming **sol**ar radi**ation**: $s(y, \zeta)$



$$s(y, \zeta) \approx 1 - s_2 p_2(\zeta) p_2(y) - s_4 p_4(\zeta) p_4(y) - s_6 p_6(\zeta) p_6(y)$$

$p_{2i}(y)$: $2i$-th Legendre polynomial

ζ : cos(obliquity)

Nondimensional Budyko-Widiasih-type EBM

$$\frac{\partial \varphi}{\partial \tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta\left(\varphi(y,\tau) - \overline{\varphi}\right)$$

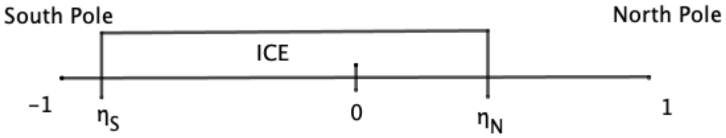
$$\frac{d\eta}{d\tau} = \lambda \varphi(\eta)$$

$\varphi(y,\tau) = Q(T(y,t) - T_c)/B$	normalizes temperature based on critical temperature
$\mu = \frac{A+BT_C}{Q}$	relates incoming and outgoing radiation
$\delta = C/B$	relates heat transport and outgoing radiation
$s(y,\zeta)$	annual average solar radiation
$\alpha(y,\eta)$	surface albedo

Ice Line Assumption

There are two ice lines, η_S and η_N, between which there is always ice and $\eta_S < \eta_N$.

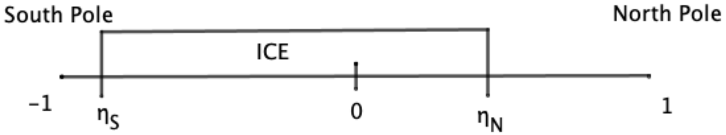
$$\alpha(y,\eta) = \begin{cases} \alpha_1 & -1 \le y < \eta_S \\ \alpha_2 & \eta_S < y \le \eta_N \\ \alpha_1 & \eta_N < y \le 1 \end{cases}, \qquad \alpha 1 < \alpha_2$$



Ice Formation Assumption

Permanent ice forms if the annual average temperature is below a critical temperature T_c and sublimates if the annual average temperature is above T_c.

$$\frac{d\eta_S}{dt} = \lambda\varphi(\eta_S)$$
$$\frac{d\eta_N}{dt} = -\lambda\varphi(\eta_N)$$



Budyko-Widiasih Model Summary

$$\frac{\partial \varphi}{\partial \tau} = s(y,\zeta)(1-\alpha(y,\eta)) - \mu - \varphi(y,\tau) - \delta\left(\varphi(y,\tau) - \overline{\varphi}\right)$$

$$\frac{d\eta_S}{dt} = \lambda \varphi(\eta_S)$$

$$\frac{d\eta_N}{dt} = -\lambda \varphi(\eta_N)$$

Ice Line Assumption: There are two ice lines, η_S and η_N, between which there is always ice.

No symmetry assumption: Do not require $\eta_S = -\eta_N$.

Approximate System of ODEs

Following framework given in McGehee and Widiaish (2014), let X be the space of functions of the form

$$\varphi(y) = \begin{cases} \sum_{i=0}^{3}(u_{2i} + v_{2i})p_{2i}(y) & y < \eta_S \\ \sum_{i=0}^{3} v_{2i}p_{2i}(y) & \eta_S < y < \eta_N \\ \sum_{i=0}^{3}(w_{2i} + v_{2i})p_{2i}(y) & y > \eta_N \end{cases}$$

where $u_{2i}, v_{2i}, w_{2i} \in \mathbb{R}$ for each i,p_{2i} is the $2i$-th Legendre polynomial, and have

$$\varphi(\eta_S) = \frac{\lim_{y\to\eta_S^+}\varphi(y) + \lim_{y\to\eta_S^-}\varphi(y)}{2}, \text{ and}$$
$$\varphi(\eta_N) = \frac{\lim_{y\to\eta_N^+}\varphi(y) + \lim_{y\to\eta_N^-}\varphi(y)}{2}.$$

Approximate System of ODEs

$$
\begin{aligned}
\dot{u}_0 &= \alpha_1 - \alpha_2 - (1+\delta)u_0 \\
\dot{v}_0 &= 1 - \alpha_1 - \mu - (1+\delta)v_0 + \delta\bar{\varphi} \\
\dot{w}_0 &= \alpha_1 - \alpha_2 - (1+\delta)w_0 \\
\dot{u}_{2i} &= -(\alpha_1 - \alpha_2)s_{2i}p_{2i}(\zeta) - (1+\delta)u_{2i} \\
\dot{v}_{2i} &= -(1 - \alpha_1)s_{2i}p_{2i}(\zeta) - (1+\delta)v_{2i} \\
\dot{w}_{2i} &= -(\alpha_1 - \alpha_2)s_{2i}p_{2i}(\zeta) - (1+\delta)w_{2i}
\end{aligned}
$$

Approximate System of ODEs

$$\dot{u}_0 = \alpha_1 - \alpha_2 - (1+\delta)u_0$$
$$\dot{v}_0 = 1 - \alpha_1 - \mu - (1+\delta)v_0 + \delta\bar{\varphi}$$
$$\dot{w}_0 = \alpha_1 - \alpha_2 - (1+\delta)w_0$$
$$\dot{u}_{2i} = -(\alpha_1 - \alpha_2)s_{2i}p_{2i}(\zeta) - (1+\delta)u_{2i}$$
$$\dot{v}_{2i} = -(1 - \alpha_1)s_{2i}p_{2i}(\zeta) - (1+\delta)v_{2i}$$
$$\dot{w}_{2i} = -(\alpha_1 - \alpha_2)s_{2i}p_{2i}(\zeta) - (1+\delta)w_{2i}$$

$\downarrow$

after u_0, w_0, u_{2i}, and w_{2i} $(i = 1, 2, 3)$ decay to their equilibria:

$\downarrow$

$$\dot{v}_0 = -v_0 + \underbrace{1 - \alpha_1 - \mu + \frac{\delta(\alpha_1 - \alpha_2)}{2(1+\delta)}\left[2 + \eta_S - \eta_N - \sum_{i=1}^{3}(P_{2i}(\eta_S) - P_{2i}(\eta_N))s_{2i}p_{2i}(\zeta)\right]}_{F(\eta_S,\eta_N)}$$

Approximate System of ODEs

Assuming that the u_{2i}'s and the w_{2i}'s have decayed to their equilibria, we have

$$\varphi(\eta_S) = v_0 + \underbrace{\frac{(\alpha_1 - \alpha_2) + (\alpha_1 + \alpha_2 - 2)\sum_{i=1}^{3} s_{2i} p_{2i}(\zeta) p_{2i}(\eta_S)}{2(1+\gamma)}}_{-G(\eta_S)}$$

$$\varphi(\eta_N) = v_0 + \underbrace{\frac{(\alpha_1 - \alpha_2) + (\alpha_1 + \alpha_2 - 2)\sum_{i=1}^{3} s_{2i} p_{2i}(\zeta) p_{2i}(\eta_N)}{2(1+\gamma)}}_{-G(\eta_N)}$$

Approximate System of ODEs

Assuming that the u_{2i}'s and the w_{2i}'s have decayed to their equilibria, we have

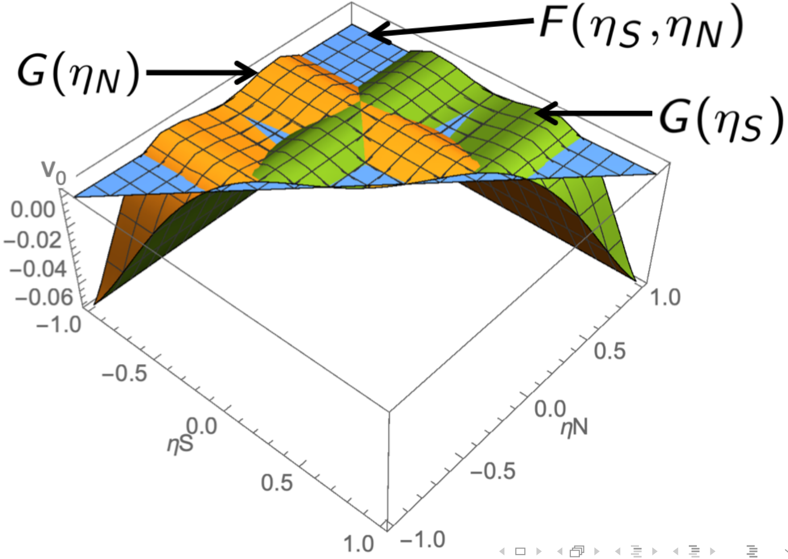
$$\varphi(\eta_S) = v_0 + \underbrace{\frac{(\alpha_1 - \alpha_2) + (\alpha_1 + \alpha_2 - 2)\sum_{i=1}^{3} s_{2i} p_{2i}(\zeta) p_{2i}(\eta_S)}{2(1+\gamma)}}_{-G(\eta_S)}$$

$$\varphi(\eta_N) = v_0 + \underbrace{\frac{(\alpha_1 - \alpha_2) + (\alpha_1 + \alpha_2 - 2)\sum_{i=1}^{3} s_{2i} p_{2i}(\zeta) p_{2i}(\eta_N)}{2(1+\gamma)}}_{-G(\eta_N)}$$

$$\downarrow$$

$$\begin{aligned} \dot{v}_0 &= -(v_0 - F(\eta_S, \eta_N)) \\ \dot{\eta}_S &= \lambda(v_0 - G(\eta_S)) \\ \dot{\eta}_N &= -\lambda(v_0 - G(\eta_N)). \end{aligned}$$

Invariant Surfaces



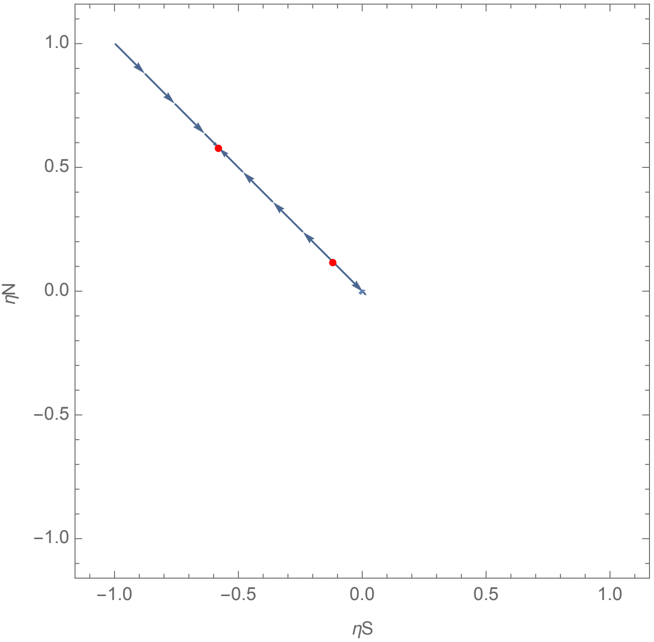
Reduction for small λ

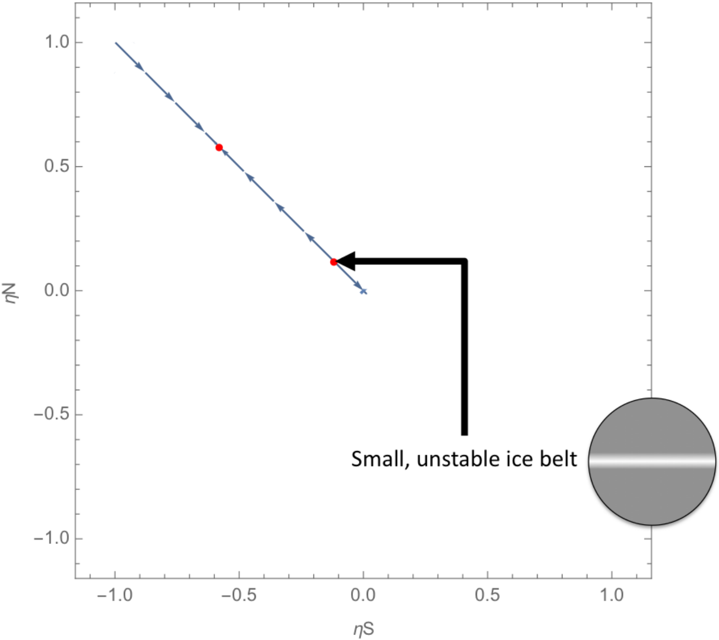
$$\dot{\eta}_S = \lambda(F(\eta_S, \eta_N) - G(\eta_S))$$
$$\dot{\eta}_N = -\lambda(F(\eta_S, \eta_N) - G(\eta_N)).$$

With Symmetry Assumption!

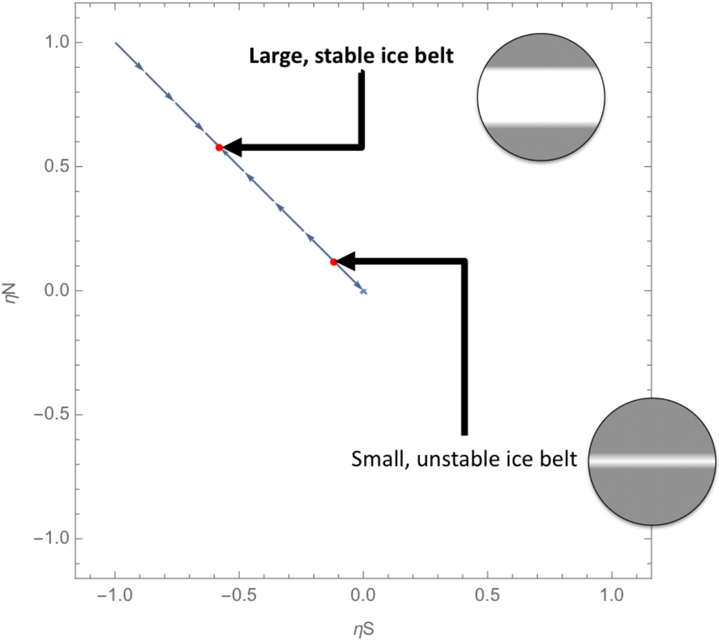
Equator $= 0 \leq y = \sin(\text{latitude}) \leq 1 =$ North Pole

$$\eta_S = -\eta_N$$


1.0
0.5
0.0
−0.5
−1.0
ηN
−1.0
−0.5
0.0
0.5
1.0
ηS



1.0
0.5
0.0
−0.5
−1.0
ηN
−1.0
−0.5
0.0
0.5
1.0
ηS
Small, unstable ice belt

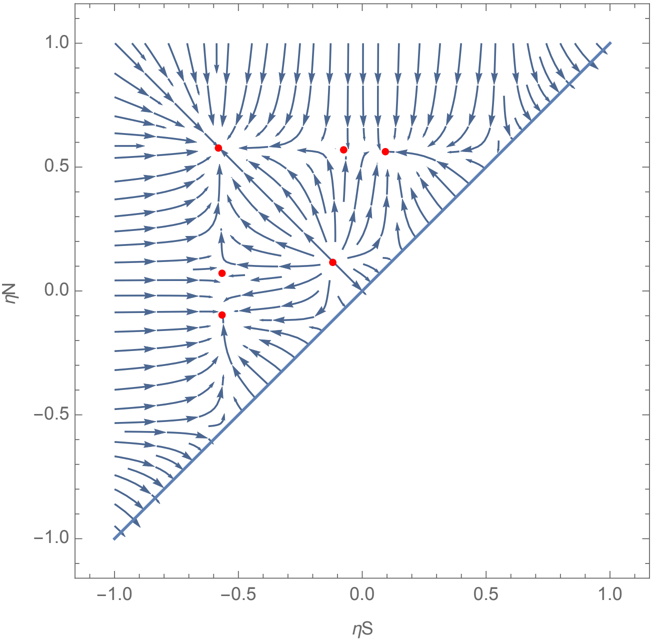

Large, stable ice belt
Small, unstable ice belt
1.0
0.5
0.0
−0.5
−1.0
ηN
−1.0
−0.5
0.0
0.5
1.0
ηS

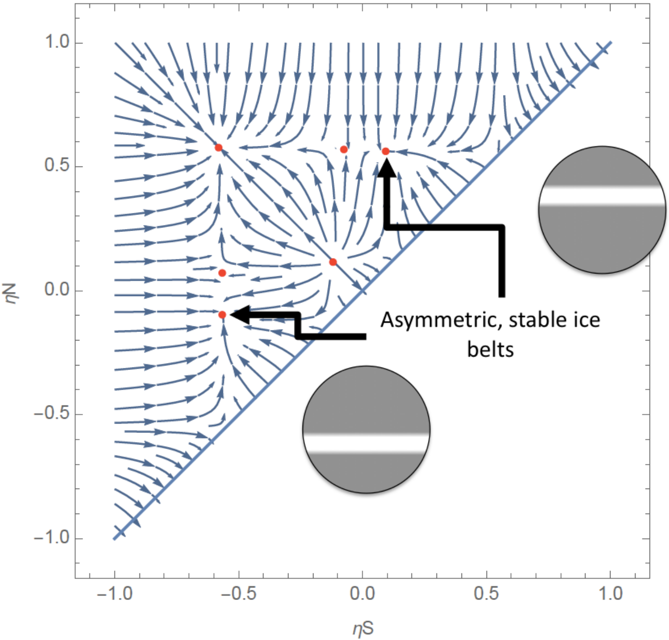
Reduction for small λ

$$\dot{\eta}_S = \lambda(F(\eta_S, \eta_N) - G(\eta_S))$$
$$\dot{\eta}_N = -\lambda(F(\eta_S, \eta_N) - G(\eta_N)).$$

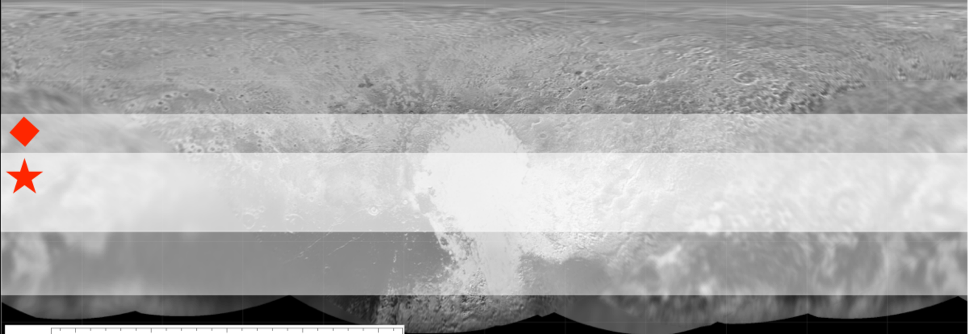
No Symmetry Assumption!

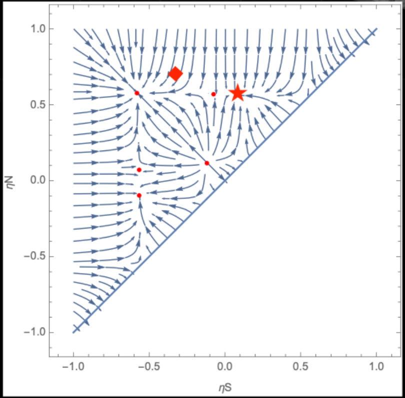
South Pole $= -1 \leq \eta_S < \eta_N \leq 1 =$ North Pole


1.0
0.5
0.0
−0.5
−1.0
ηN
−1.0
−0.5
0.0
0.5
1.0
ηS



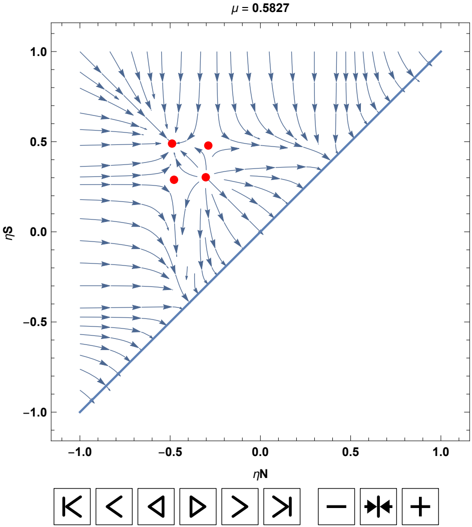
1.0
0.5
0.0
−0.5
−1.0
ηN
−1.0
−0.5
0.0
0.5
1.0
ηS
Asymmetric, stable ice belts



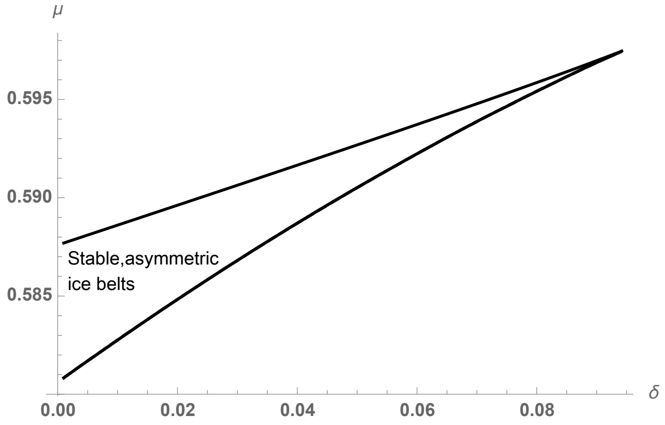


NASA/Johns Hopkins Univ. APL/Southwest Research Institute

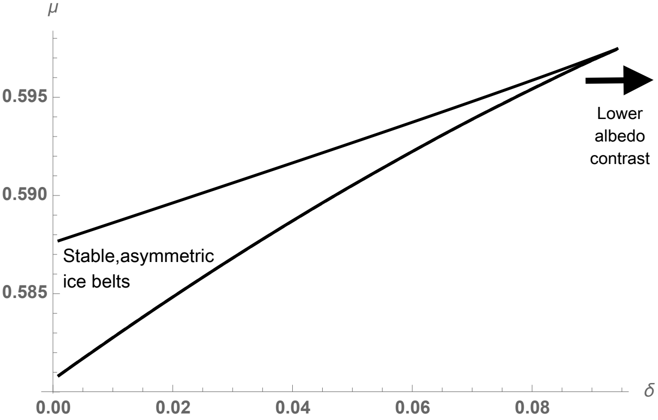
Persistence of Asymmetric Ice Belts



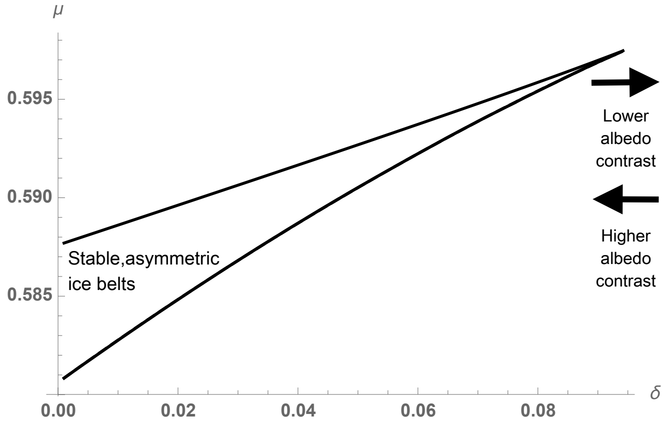
Persistence of Asymmetric Ice Belts



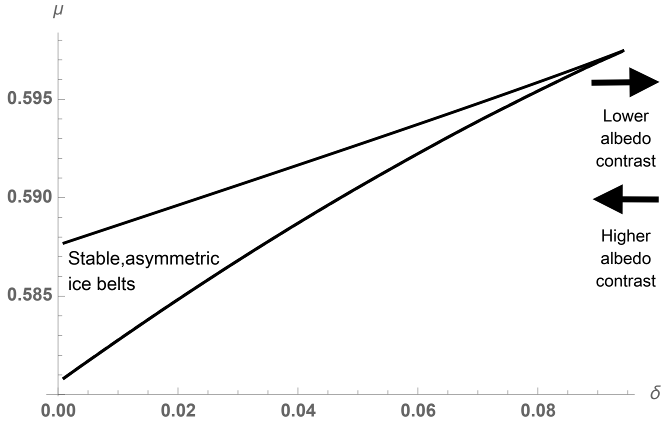
Persistence of Asymmetric Ice Belts



Persistence of Asymmetric Ice Belts



Persistence of Asymmetric Ice Belts

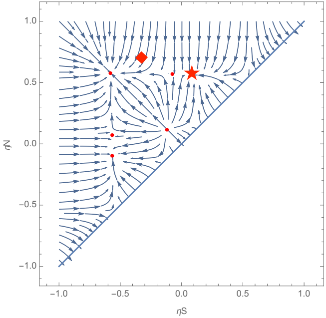


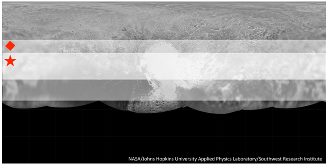
Lemma

Stable, asymmetric ice belts are possible for any albedo contrast.

AN and EJ. (2019).

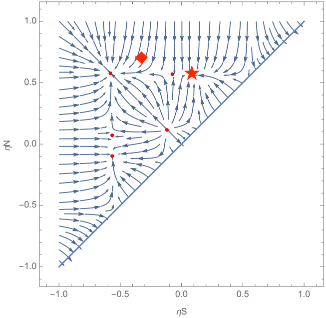
Caveats:

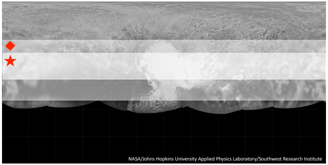




- We don't really know what values to pick for μ and δ

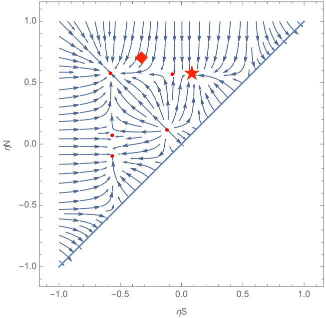
Caveats:

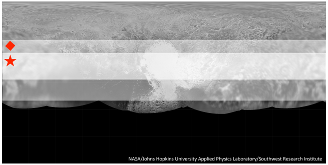




- We don't really know what values to pick for μ and δ
- Basin of attraction of the "*Sputnik Panitia* ice belt" is highly dependent on μ and δ

Caveats:





- We don't really know what values to pick for μ and δ
- Basin of attraction of the "*Sputnik Panitia* ice belt" is highly dependent on μ and δ
- Pluto's albedo has large longitudinal differences

Summary

- Stable, asymmetric ice line equilibria are present in the Budyko-Widiasih EBM

Summary

- Stable, asymmetric ice line equilibria are present in the Budyko-Widiasih EBM
- Albedo contrasts do not seem to be the driving factor for this asymmetry

Summary

- Stable, asymmetric ice line equilibria are present in the Budyko-Widiasih EBM
- Albedo contrasts do not seem to be the driving factor for this asymmetry
- The model might be able to tell us about Pluto's *Spunik Planitia*...

Summary

- Stable, asymmetric ice line equilibria are present in the Budyko-Widiasih EBM
- Albedo contrasts do not seem to be the driving factor for this asymmetry
- The model might be able to tell us about Pluto's *Spunik Planitia*...
  - its location is correlated with annual average sunlight distribution,

Summary

- Stable, asymmetric ice line equilibria are present in the Budyko-Widiasih EBM
- Albedo contrasts do not seem to be the driving factor for this asymmetry
- The model might be able to tell us about Pluto's *Spunik Planitia*...
  - its location is correlated with annual average sunlight distribution,
  - but we don't know if the glaciers should be growing or shrinking...

Summary

- Stable, asymmetric ice line equilibria are present in the Budyko-Widiasih EBM
- Albedo contrasts do not seem to be the driving factor for this asymmetry
- The model might be able to tell us about Pluto's *Spunik Planitia*...
  - its location is correlated with annual average sunlight distribution,
  - but we don't know if the glaciers should be growing or shrinking...
  - so more scientific investigations are needed!

Planetary Motion and its Effects on Climate

MS149: Wednesday, May 22nd, 05:00PM

The Snowball Bifurcation on Tidally Influenced Planets
Jade Checlair, University of Chicago

Ice Caps and Ice Belts: The Effects of Obliquity on Ice-Albedo Feedback
Brian Rose, State University of New York

Modeling Martian Climate with Low-Dimensional Energy Balance Models
Gareth Roberts, College of the Holy Cross

Effects of a Rogue Star on Earth's Climate
Harini Chandramouli, University of Minnesota

MS162: Thursday, May 23rd, 08:30AM

The Geological Orrery: Mapping the Chaotic History of the Solar System using Earth's Geological Record
Paul Olsen, Columbia University

Forcing-Induced Transitions in a Paleoclimate Delay Model
Courtney Quinn, University of Exeter

Modeling the Mid Pleistocene Transition in a Budyko-Sellers Type Energy Balance Model using the LR04 Benthic Stack
Somyi Baek, University of Minnesota

A Conceptual Glacial Cycle Model with Diffusive Heat Transport
James Walsh, Oberlin College

Thank you!