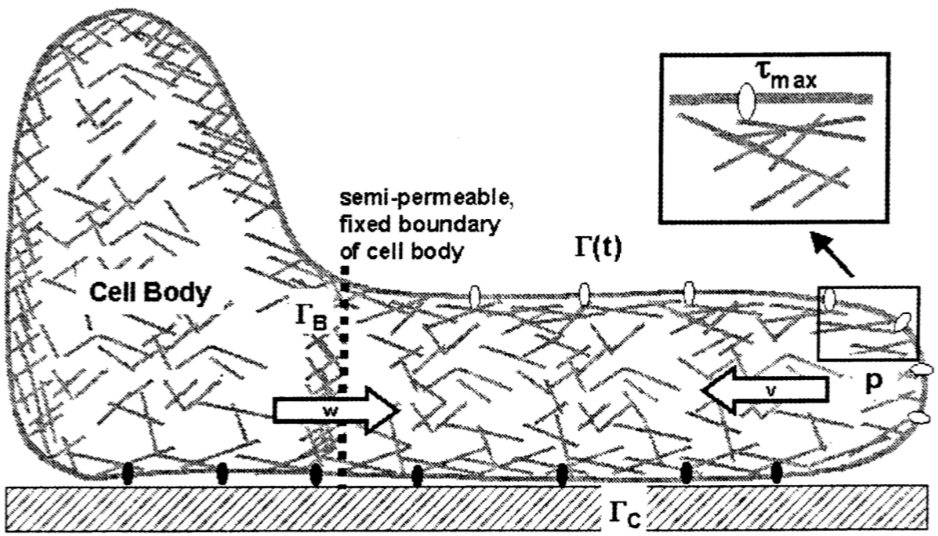
# A two-phase flow model for a soft poroelastic drop in linear flows

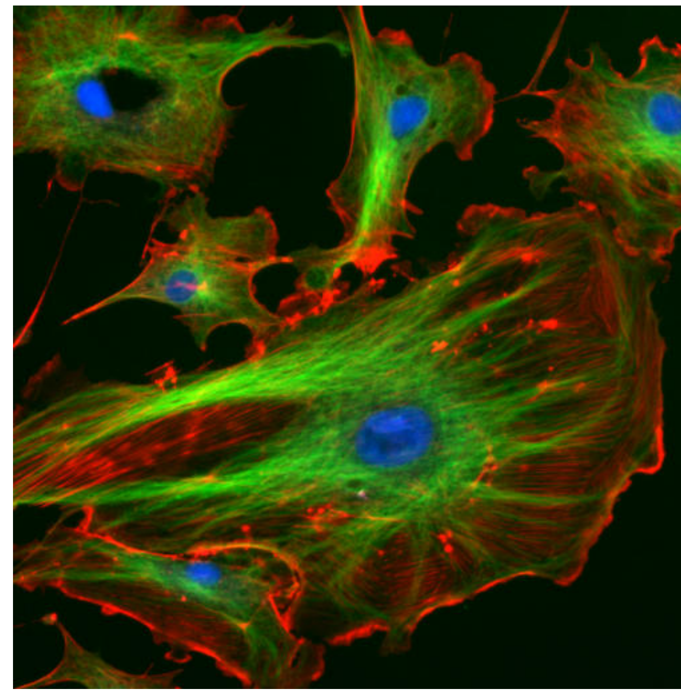
Yuan-Nan Young
New Jersey Institute of Technology

Yoichiro Mori, University of Minnesota
Michael Miksis, Northwestern University

SIAM Life Sciences 2018
YNY Supported by NSF-DMS-1614863 and NSF-DMS-1412789

# Motivation: cytoplasm consists of a viscous fluid, (monomers of) elastic fibers, and everything else (other than the nucleus)



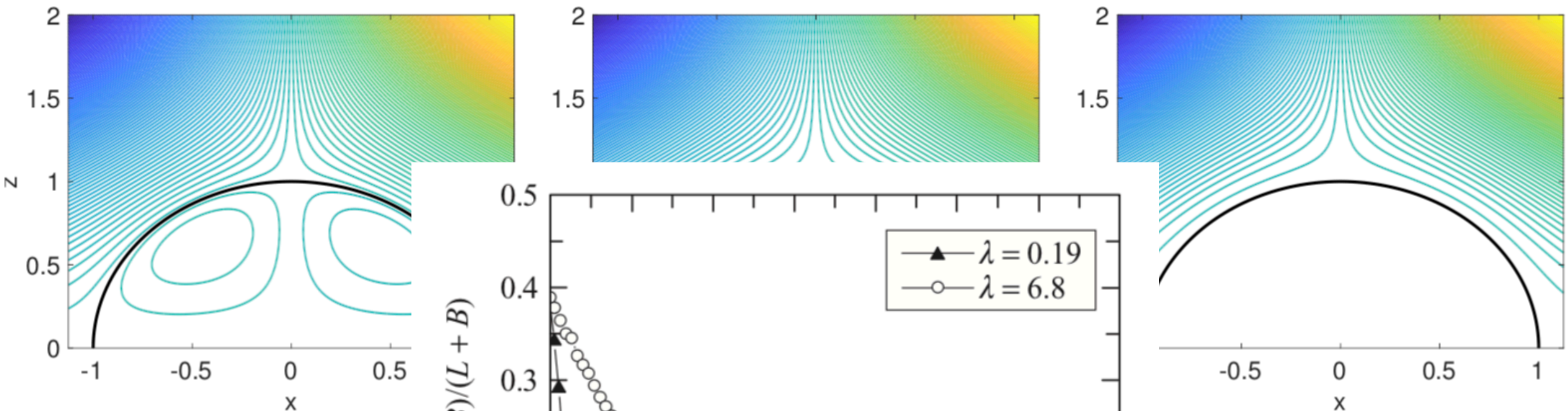



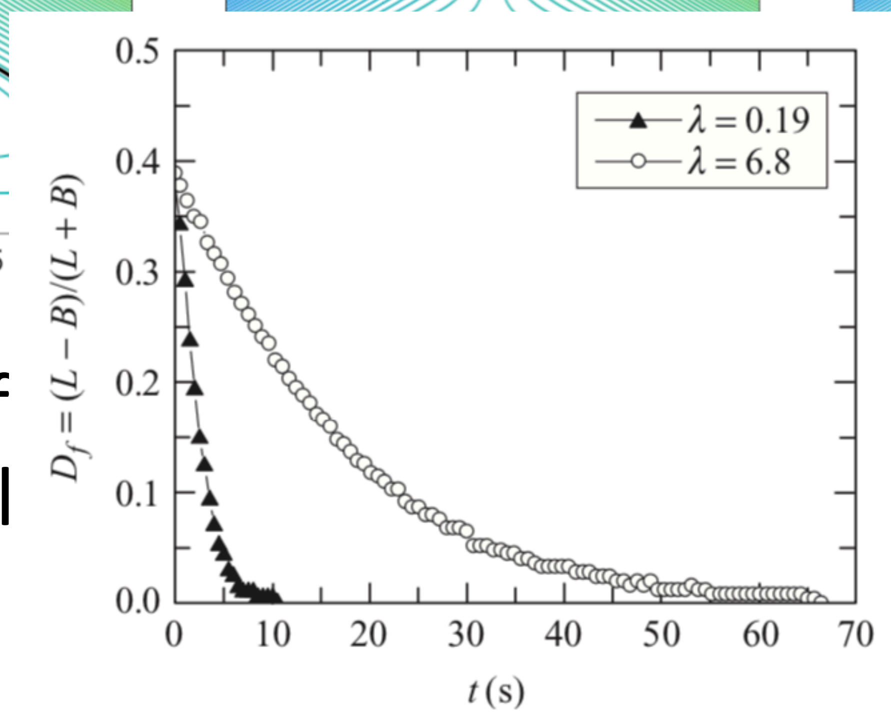
- Alt and Dembo, Math. Biosci. (1999)
- Strychalski *et al*, JCP (2014).

- imagej.nih.gov

# outline

- Modeling cytoplasm as a viscous fluid filling a deformable elastic network **->** a poroelastic




- Small-def … oroelastic
drop in pl

# Mathematical modeling of cytoplasm:
## Two-phase flow model for a network (s) phase in a viscous fluid (f)

- Cogan and Keener (Math. Med. and Biol., 2004)

$$\nabla \cdot \left(\phi_f \mu_i (\nabla \mathbf{u}_f + (\nabla \mathbf{u}_f)^T)\right) - \phi_f \nabla p - \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$

$$\nabla \cdot (\phi_s (\sigma_v + \sigma_e)) - \phi_s \nabla p + \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$

$$\phi_f + \phi_s = 1, \quad \frac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \mathbf{u}_s) = 0, \quad \nabla \cdot (\phi_s \mathbf{u}_s + \phi_f \mathbf{u}_f) = 0,$$

$$\sigma_v = \mu_s \left(\nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T\right)$$

elastic stress $\sigma_e$ neglected in this work

# Mathematical modeling of cytoplasm:
## Two-phase flow model for a network (s) phase in a viscous fluid (f)

- Mori *et al.* (SIAM J. Applied Math., 2013)

$$\nabla \cdot \left(\phi_f \mu_i (\nabla \mathbf{u}_f + (\nabla \mathbf{u}_f)^T)\right) - \phi_f \nabla p - \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$

$$\nabla \cdot \left(\phi_s \left(\sigma_v + \sigma_e\right)\right) - \phi_s \nabla p + \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$

$$\phi_f + \phi_s = 1, \quad \frac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \mathbf{u}_s) = 0, \quad \nabla \cdot (\phi_s \mathbf{u}_s + \phi_f \mathbf{u}_f) = 0,$$

$$\sigma_v = \mu_s \left(\nabla \mathbf{u}_s + (\nabla \mathbf{u}_s)^T\right) \qquad \sigma_e = \phi_s \frac{\partial W_{elastic}(F)}{\partial F} F^T,$$

- This work also considers the polyelectrolytic solutions in the formulation.

# Boundary conditions in Mori *et al.*

(SIAM J. Applied Math., 2013)

- Different from those in Cogan and Keener

$$(\mathbf{U} - \mathbf{u}_s) \cdot \mathbf{n} = \phi_f (\mathbf{u}_f - \mathbf{u}_s) \cdot \mathbf{n} \equiv w,$$

$$(\mathbf{U} - \mathbf{u}_s) \cdot (\mathbf{I} - \mathbf{nn}) = (\mathbf{u}_f - \mathbf{u}_s) \cdot (\mathbf{I} - \mathbf{nn}) \equiv \bar{\mathbf{q}},$$

$$\Sigma \cdot \mathbf{n} - (\sigma_f + \sigma_s) \cdot \mathbf{n} + [p]\mathbf{n} = 0,$$

- More boundary conditions needed

$$\eta_\perp w = \Pi_\perp = \mathbf{n} \cdot \Sigma \cdot \mathbf{n} - \mathbf{n} \cdot \left( \frac{\sigma_f}{\phi_f} \right) \cdot \mathbf{n} + [p],$$

$$\eta_{\|} \bar{\mathbf{q}} = \Pi_{\|} = (\sigma_s \cdot \mathbf{n})_{\|} \,,$$

- MacMinn *et al.* (Phys. Rev. Appl., 2016)

$$-\phi_f \nabla p - \xi \phi_s \phi_f \left(\mathbf{u}_f - \mathbf{u}_s\right) = 0,$$

$$\nabla \cdot \left(\phi_s \left(\sigma_v + \sigma_e\right)\right) - \phi_s \nabla p + \xi \phi_s \phi_f \left(\mathbf{u}_f - \mathbf{u}_s\right) = 0,$$

$$\nabla \cdot \left(\phi_s \left(\sigma_v + \sigma_e\right)\right) - \nabla p \equiv \nabla \cdot \left(\phi_s \sigma'\right) - \nabla p = 0,$$

$$\phi_f + \phi_s = 1, \quad \frac{\partial \phi_s}{\partial t} + \nabla \cdot \left(\phi_s \mathbf{u}_s\right) = 0, \quad \nabla \cdot \left(\phi_s \mathbf{u}_s + \phi_f \mathbf{u}_f\right) = 0,$$

$$\text{viscous dissipation } \sigma_v = 0, \quad \sigma' = \sigma_e = \Lambda \mathrm{tr}\left(\mathbf{H}\right) \mathbf{I} + \left(M - \Lambda\right) \mathbf{H},$$

$$\mathbf{H} = \frac{1}{2} \ln\left(\mathbf{F}\mathbf{F}^T\right), \quad \mathbf{F}^{-1} = \mathbf{I} - \nabla \mathbf{v}_s,$$

$$\mathbf{u}_s = \mathbf{q} - \frac{\phi_f}{\xi \phi_s} \nabla p, \quad \mathbf{u}_f = \mathbf{q} + \frac{1}{\xi} \nabla p,$$

# Small-deformation and linear poroelasticity

- Small deformation: $\frac{\phi_f - \phi_{f,0}}{1 - \phi_{f,0}} \approx \nabla \cdot \mathbf{v}_s \ll 1,$

- Linear poroelasticity:

$$\sigma' = \Lambda \mathrm{tr}(\varepsilon) + (M - \Lambda)\varepsilon, \quad \text{with } \varepsilon \equiv \frac{1}{2}\left[\nabla \mathbf{v}_s + (\nabla \mathbf{v}_s)^T\right],$$

- Coupled to an external Stokes flow:

$$\mu_e \nabla^2 \mathbf{U} - \nabla P = 0, \quad \nabla \mathbf{U} = 0,$$

# Summary of two-phase flow model

- Conservation of mass:

$$\phi_f + \phi_s = 1, \quad \frac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \mathbf{u}_s) = 0, \quad \nabla \cdot (\phi_s \mathbf{u}_s + \phi_f \mathbf{u}_f) = 0,$$

- Momentum balance:

$$\nabla \cdot (\phi_f \mathbf{e}_f) - \phi_f \nabla p + \mathcal{N}^{s \to f} + \phi_f \rho_f \mathbf{g} = 0, \quad \mathcal{N}^{s \to f} \equiv -\xi \phi_f \phi_s (\mathbf{u}_f - \mathbf{u}_s),$$

$$\nabla \cdot (\phi_s \mathbf{e}_s) - \phi_s \nabla p + \mathcal{N}^{f \to s} + \phi_s \rho_s \mathbf{g} = 0, \quad \mathcal{N}^{f \to s} \equiv -\mathcal{N}^{s \to f} = \xi \phi_f \phi_s (\mathbf{u}_f - \mathbf{u}_s),$$

- Incompressible Darcy equations are derived as a limit of dominant solid stress over fluid stress.

$$-\phi_f \nabla p - \xi \phi_f \phi_s (\mathbf{u}_f - \mathbf{u}_s) = -\rho \mathbf{g} \text{ when } \sigma' = \phi_f \mathbf{e}_f + \phi_s \mathbf{e}_s \approx \phi_s \mathbf{e}_s,$$

- Solid structure is allowed to deform (soft solid), and the usual Darcy equations are obtained when the strain is fixed in time ($\mathbf{u}_s$=0).

# Summary of two-phase model (cont.)

- Exterior Stokesian fluid

$$\nabla \cdot (\mu_e \mathbf{E}) - \nabla P = \mu_e \nabla^2 \mathbf{U} - \nabla P = 0,$$
$$\nabla \cdot \mathbf{U} = 0,$$

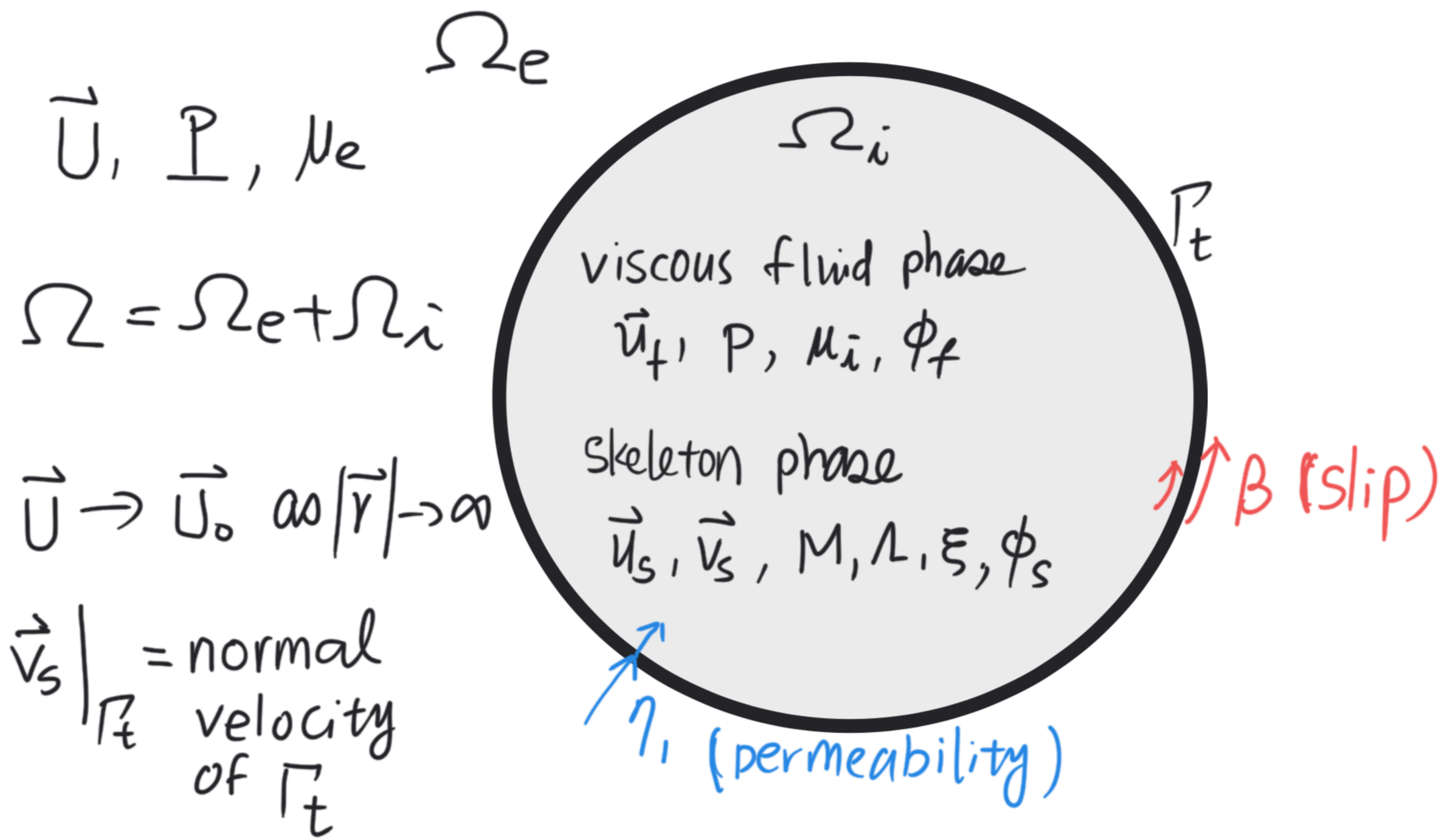
- Interior poroelastic Brinkman fluid

$$\nabla \cdot (\phi_f \mu_i \mathbf{e}_f) - \phi_f \nabla p - \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$
$$\nabla \cdot (\phi_s (\sigma_v + \sigma_e)) - \phi_s \nabla p + \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$
$$\frac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \mathbf{u}_s) = 0,$$
$$\phi_f + \phi_s = 1, \quad \nabla \cdot (\phi_f \mathbf{u}_f + \phi_s \mathbf{u}_s) = 0.$$

- Interior poroelastic Darcy fluid

$$-\phi_f \nabla p - \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$
$$\nabla \cdot (\phi_s (\sigma_v + \sigma_e)) - \phi_s \nabla p + \xi \phi_s \phi_f (\mathbf{u}_f - \mathbf{u}_s) = 0,$$
$$\frac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \mathbf{u}_s) = 0,$$
$$\phi_f + \phi_s = 1, \quad \nabla \cdot (\phi_f \mathbf{u}_f + \phi_s \mathbf{u}_s) = 0.$$

# Boundary conditions: a simple derivation



# Boundary conditions: a simple derivation

$$\int_\Omega \mathbf{U} \cdot [\nabla(\mu_e \mathbf{E}) - \nabla P]\, d^3x$$

$$+ \int_\Omega \mathbf{u}_f \cdot [\nabla(\phi_f \mu_i \mathbf{e}_f) - \phi_f \nabla p - \xi \phi_f \phi_s (\mathbf{u}_f - \mathbf{u}_s)]\, d^3x$$

$$+ \int_\Omega \mathbf{u}_s \cdot [\nabla \cdot (\phi_s \sigma_s) - \phi_s \nabla p + \xi \phi_f \phi_s (\mathbf{u}_f - \mathbf{u}_s)]\, d^3x$$

$$= \int_{\Gamma_t} -\mathbf{U} \cdot (\mu_e \mathbf{E} - P\mathbf{I}) \cdot \hat{\mathbf{n}} ds - \int_\Omega \mu_e \mathbf{E} : \nabla \mathbf{U} d^3x$$

$$+ \int_{\Gamma_t} \mathbf{u}_f \cdot (\phi_f \mu_i \mathbf{e}_f - \phi_f p \mathbf{I}) \hat{\mathbf{n}} ds - \int_\Omega \phi_f \mu_i \mathbf{e}_f : \nabla \mathbf{u}_f d^3x$$

$$+ \int_{\Gamma_t} \mathbf{u}_s \cdot (\phi_s \sigma_s - \phi_s p \mathbf{I}) \hat{\mathbf{n}} ds - \int_\Omega \phi_s \sigma_s : \nabla \mathbf{u}_s d^3x$$

- Velocities relative to the skeleton velocity: $\overline{\mathbf{U}} = \mathbf{U} - \mathbf{u}_s, \quad \overline{\mathbf{u}}_f = \phi_f(\mathbf{u}_f - \mathbf{u}_s),$

- Surface integrals after the integration by parts:

$$\int_{\Gamma_t} -(\overline{\mathbf{U}} + \mathbf{u}_s)(\mu_e \mathbf{E} - P\mathbf{I})\hat{\mathbf{n}} + \left(\frac{\overline{\mathbf{u}}_f}{\phi_f} + \mathbf{u}_s\right)(\phi_f \mu_i \mathbf{e}_f - \phi_f p\mathbf{I})\hat{\mathbf{n}} + \mathbf{u}_s\left(\phi_s \sigma_s - \phi_s p\mathbf{I}\right)\hat{\mathbf{n}} ds$$

$$= \int_{\Gamma_t} \mathbf{u}_s \cdot (-\mu_e \mathbf{E} + P\mathbf{I} + \phi_f \mu_i \mathbf{e}_f + \phi_s \sigma_s - p\mathbf{I})\hat{\mathbf{n}} ds +$$

$$\int_{\Gamma_t} -\overline{\mathbf{U}} \cdot (\mu_e \mathbf{E} - P\mathbf{I})\hat{\mathbf{n}} + \overline{\mathbf{u}}_f \cdot (\mu_i \mathbf{e}_f - p\mathbf{I})\hat{\mathbf{n}} ds$$

- Decomposition: $\overline{\mathbf{U}} = \overline{\mathbf{U}}_\perp + \overline{\mathbf{U}}_\parallel, \quad \overline{\mathbf{u}}_f = \overline{\mathbf{u}}_{f\perp} + \overline{\mathbf{u}}_{f\parallel}$

- Normal component: $\overline{\mathbf{U}}_\perp = \overline{\mathbf{u}}_{f\perp} \;\rightarrow\; (\mathbf{U} - \mathbf{u}_s) \cdot \hat{\mathbf{n}} = \phi_f \left(\mathbf{u}_f - \mathbf{u}_s\right) \cdot \hat{\mathbf{n}}.$

$$\overline{\mathbf{U}}_\perp = \overline{\mathbf{u}}_{f\perp} = \eta_1 \hat{\mathbf{n}} \cdot (\mu_e \mathbf{E} - P\mathbf{I} - \mu_i \mathbf{e}_f + p\mathbf{I}) \cdot \hat{\mathbf{n}}$$

# Boundary conditions (cont.)

- Tangential component:

$$\overline{\mathbf{U}}_{\parallel} = \frac{\beta}{2}\left(\mu_e \mathbf{E}\cdot\hat{\mathbf{n}}\right)_{\parallel} - \gamma\left(\mu_i \mathbf{e}_f\cdot\hat{\mathbf{n}}\right)_{\parallel} = \frac{\beta}{2}\left(\mu_e \mathbf{E}\cdot\hat{\mathbf{n}}\right)_{\parallel},$$

$$\overline{\mathbf{u}}_{f\parallel} = \alpha\left(\mu_e \mathbf{E}\cdot\hat{\mathbf{n}}\right)_{\parallel} - \frac{\beta}{2}\left(\mu_i \mathbf{e}_f\cdot\hat{\mathbf{n}}\right)_{\parallel} = -\frac{\beta}{2}\left(\mu_i \mathbf{e}_f\cdot\hat{\mathbf{n}}\right)_{\parallel},$$

- Interior Darcy flow: Beavers-Joseph slip boundary condition is recovered (Beavers and Joseph, JFM, 1967)

- Interior Brinkman flow: Stokes-Brinkman slip boundary condition is recovered (Angot, Goyeau, and Ochoa-Tapia, PRE, 2017)

- Roughness of the Stokes-Brinkman (or Stokes-Darcy) boundary gives rise to small slip and permeability

# Boundary conditions: summary

$$[(\mathbf{U}-\mathbf{u}_s)-\phi_f(\mathbf{u}_f-\mathbf{u}_s)]\cdot\hat{\mathbf{n}}=0,$$

$$(\mathbf{U}-\mathbf{u}_s)\cdot\hat{\mathbf{n}}=\eta_1\hat{\mathbf{n}}\cdot[(\mu_e\mathbf{E}-P\mathbf{I})-(\mu_i\mathbf{e}_f-p\mathbf{I})]\cdot\hat{\mathbf{n}},$$

$$[(\mathbf{U}-\mathbf{u}_s)]\cdot\hat{\mathbf{t}}=\frac{\beta}{2}\hat{\mathbf{n}}\cdot\mu_e\mathbf{E}\cdot\hat{\mathbf{t}},$$

$$[\phi_f(\mathbf{u}_f-\mathbf{u}_s)]\cdot\hat{\mathbf{t}}=-\frac{\beta}{2}\hat{\mathbf{n}}\cdot\mu_i\mathbf{e}_f\cdot\hat{\mathbf{t}},$$

$$\hat{\mathbf{n}}\cdot[(\mu_e\mathbf{E}-P\mathbf{I})-(\phi_s\sigma_s+\phi_f\mu_i\mathbf{e}_f-p\mathbf{I})]\cdot\hat{\mathbf{n}}=\kappa\gamma,$$

$$\hat{\mathbf{t}}\cdot[(\mu_e\mathbf{E}-P\mathbf{I})-(\phi_s\sigma_s+\phi_f\mu_i\mathbf{e}_f-p\mathbf{I})]\cdot\hat{\mathbf{n}}=0,$$

$$-\phi_f\nabla p-\xi\phi_s\phi_f(\mathbf{u}_f-\mathbf{u}_s)=0,$$

$$\nabla\cdot(\phi_s(\sigma_v+\sigma_e))-\phi_s\nabla p+\xi\phi_s\phi_f(\mathbf{u}_f-\mathbf{u}_s)=0,$$

$$\frac{\partial\phi_s}{\partial t}+\nabla\cdot(\phi_s\mathbf{u}_s)=0,$$

$$\phi_f+\phi_s=1,\quad\nabla\cdot(\phi_f\mathbf{u}_f+\phi_s\mathbf{u}_s)=0.$$

# A Darcy drop in a linear flow: small-deformation

$$r = 1 + \delta r(t,\theta) = 1 + \mathbf{v}_s \cdot \hat{\mathbf{r}}, \quad |\delta r| = |\mathbf{v}_s \cdot \hat{\mathbf{r}}| \ll 1.$$

$$\mathbf{u}_s = \frac{d\mathbf{v}_s}{dt} = \frac{\partial \mathbf{v}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{v}_s \approx \frac{\partial \mathbf{v}_s}{\partial t}$$

- Consequence of small-deformation

$$\frac{\partial \phi_f}{\partial t} + \nabla \cdot \left( \phi_f \frac{d\mathbf{v}_s}{dt} \right) = (1-\phi_0) \frac{\partial}{\partial t} (\nabla \cdot \mathbf{v}_s) + \nabla \cdot \left( \phi_0 \frac{d\mathbf{v}_s}{dt} \right) + h.o.t.$$

$$0 \approx (1-\phi_0) \left( \nabla \cdot \frac{\partial \mathbf{v}_s}{\partial t} \right) + \nabla \cdot \left( \phi_0 \frac{\partial \mathbf{v}_s}{\partial t} \right) + h.o.t.$$

$$\nabla \cdot \left( \frac{\partial \mathbf{v}_s}{\partial t} \right) \approx 0$$

# A Darcy drop in a linear flow: small-deformation

$$r = 1 + \delta r(t, \theta) = 1 + \mathbf{v}_s \cdot \hat{\mathbf{r}}, \quad |\delta r| = |\mathbf{v}_s \cdot \hat{\mathbf{r}}| \ll 1.$$

$$\mathbf{u}_s = \frac{d\mathbf{v}_s}{dt} = \frac{\partial \mathbf{v}_s}{\partial t} + \mathbf{u}_s \cdot \nabla \mathbf{v}_s \approx \frac{\partial \mathbf{v}_s}{\partial t}$$

$$\begin{aligned} -\phi_0 \nabla p - \xi \phi_0 (1 - \phi_0) \left( \mathbf{u}_f - \frac{\partial \mathbf{v}_s}{\partial t} \right) &= 0, \\ \nabla \cdot ((1 - \phi_0) (\sigma_e(\mathbf{v}_s) + \alpha_v \sigma_v(\mathbf{u}_s))) - \nabla p &= 0, \\ \nabla \cdot (\phi_0 \mathbf{u}_f + (1 - \phi_0) \mathbf{u}_s) &= 0. \end{aligned}$$

- Uniaxial extensional flow

- Planar shear flow

# Small-deformation of a Darcy drop in a uniaxial extensional flow

$$\begin{pmatrix} \mathbf{u}_f \\ \mathbf{v}_s \\ p \end{pmatrix} = \begin{pmatrix} \mathbf{u}_{f,e} \\ \mathbf{v}_{s,e} \\ p_e \end{pmatrix} + e^{\lambda t} \begin{pmatrix} \mathbf{u}_{f,1} \\ \mathbf{v}_{s,1} \\ p_1 \end{pmatrix}$$

$$p_e = -\frac{21}{10}(1-\phi_0)(1-\Lambda)d_1 r^2 \left(1+3\cos(2\theta)\right),$$

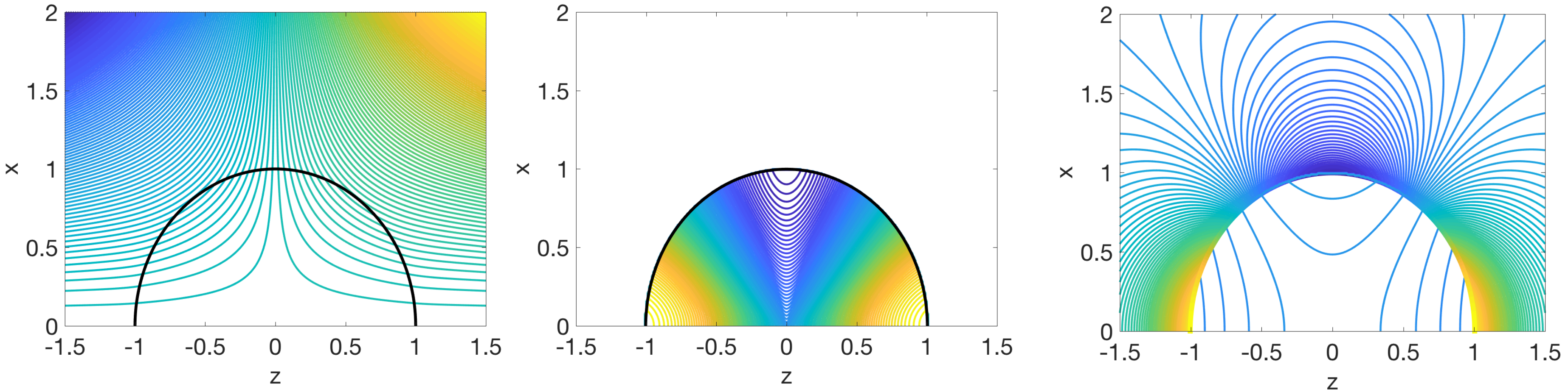
$$\mathbf{v}_{s,e}\cdot\hat{\mathbf{r}} = \left(-\frac{3(5-2\Lambda)}{25}d_1 r^3 - d_2 r - \frac{2\Lambda}{7+3\Lambda}d_3 r^3\right)\left(1+3\cos(2\theta)\right),$$

$$\mathbf{v}_{s,e}\cdot\hat{\theta} = \left(\frac{3(6-\Lambda)}{25}d_1 r^3 + d_2 r + \frac{1}{3}d_3 r^3\right)\sin(2\theta),$$

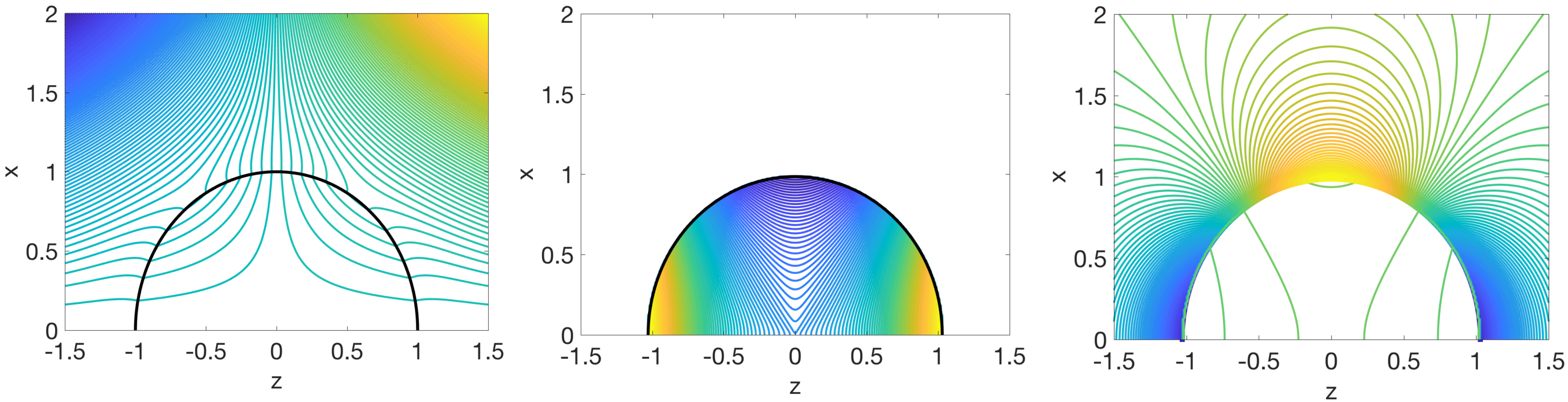
$$\psi_e = E\left(r^3 - \frac{A_1}{2} - \frac{A_2}{2r^2}\right)\cos\theta\sin^2\theta,$$

$$P = -\frac{\alpha_e}{2r^3}A_1\left(1+3\cos(2\theta)\right).$$

# Flow around a soft Darcy drop in a uniaxial extensional flow: equilibrium




- $\Lambda/M=1/3$, $\phi_0=0.8$, $\beta=10^3$, $\eta_1=10$, $\xi=1$




- $\Lambda/M=1/3$, $\phi_0=0.8$, $\beta=0$, $\eta_1=10$, $\xi=1$

# Small-deformation of a Darcy drop in a planar shear flow

$$p_e = \frac{G}{2} d_1 r^2 \sin^2\theta \sin(2\phi),$$

$$\mathbf{v}_{e,s} \cdot \hat{r} = \left( \frac{G}{7(1-\Lambda)(1-\phi_0)} d_1 r^3 + d_3 r + \frac{2\Lambda}{7+3\Lambda} d_4 r^3 \right) \sin^2\theta \sin(2\phi),$$

$$\mathbf{v}_{e,s} \cdot \hat{\theta} = \left( \frac{5G}{21(1-\Lambda)(1-\phi_0)} d_1 r^3 + d_3 r + \frac{1}{3} d_4 r^3 \right) \sin\theta \cos\theta \sin(2\phi),$$

$$\mathbf{v}_{e,s} \cdot \hat{\phi} = \left( -\frac{Gt}{2} r + \left( \frac{5G}{21(1-\Lambda)(1-\phi_0)} d_1 r^3 + d_3 r + \frac{1}{3} d_4 r^3 \right) \cos(2\phi) \right) \sin\theta,$$

$$\mathbf{U}_e \cdot \hat{r} = \frac{G}{20 r^4} \left( 6C_3 + 5C_1 r^2 + 10 r^5 \right) \sin^2\theta \sin(2\phi),$$

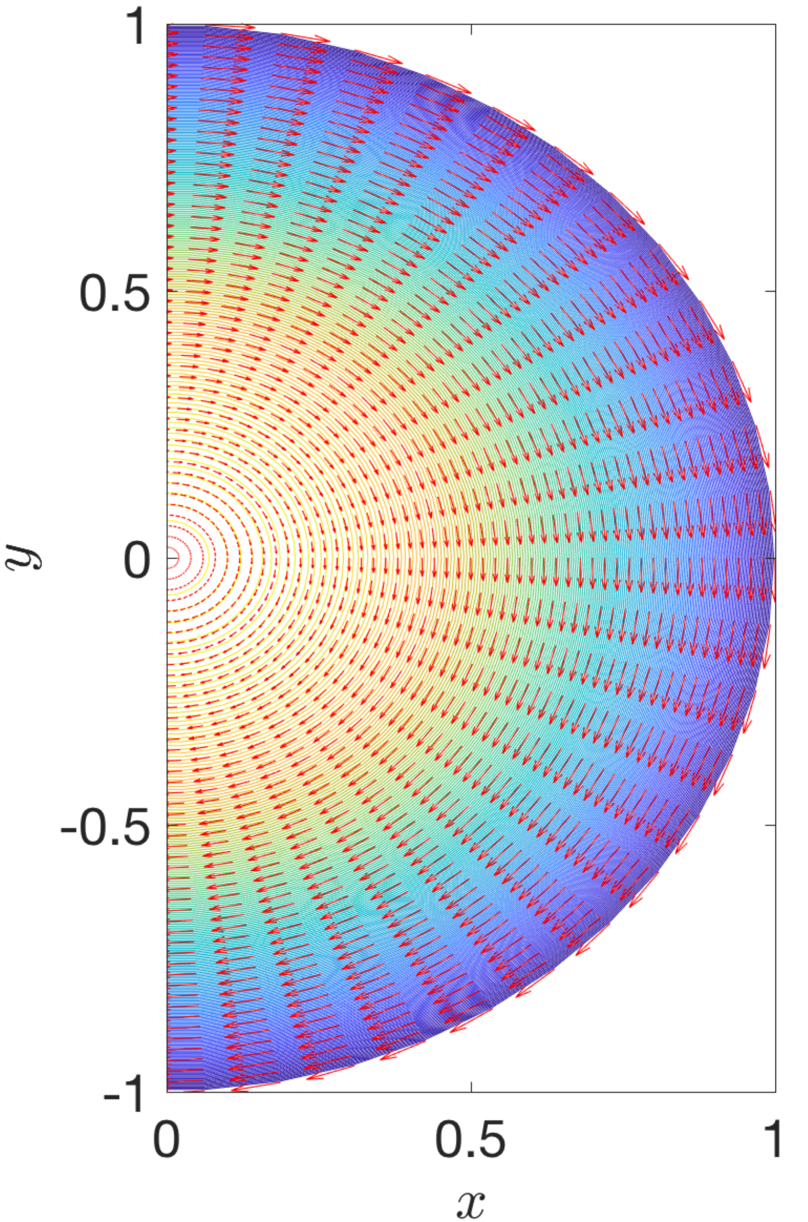
$$\mathbf{U}_e \cdot \hat{\theta} = \frac{G}{20 r^4} \left( -2C_3 + 5 r^5 \right) \sin(2\theta) \sin(2\phi),$$

$$\mathbf{U}_e \cdot \hat{\phi} = -\frac{G}{10 r^4} \left( 5(2C_4 r^2 + r^5) + (2C_3 - 5r^5)\cos(2\phi) \right) \sin\theta,$$

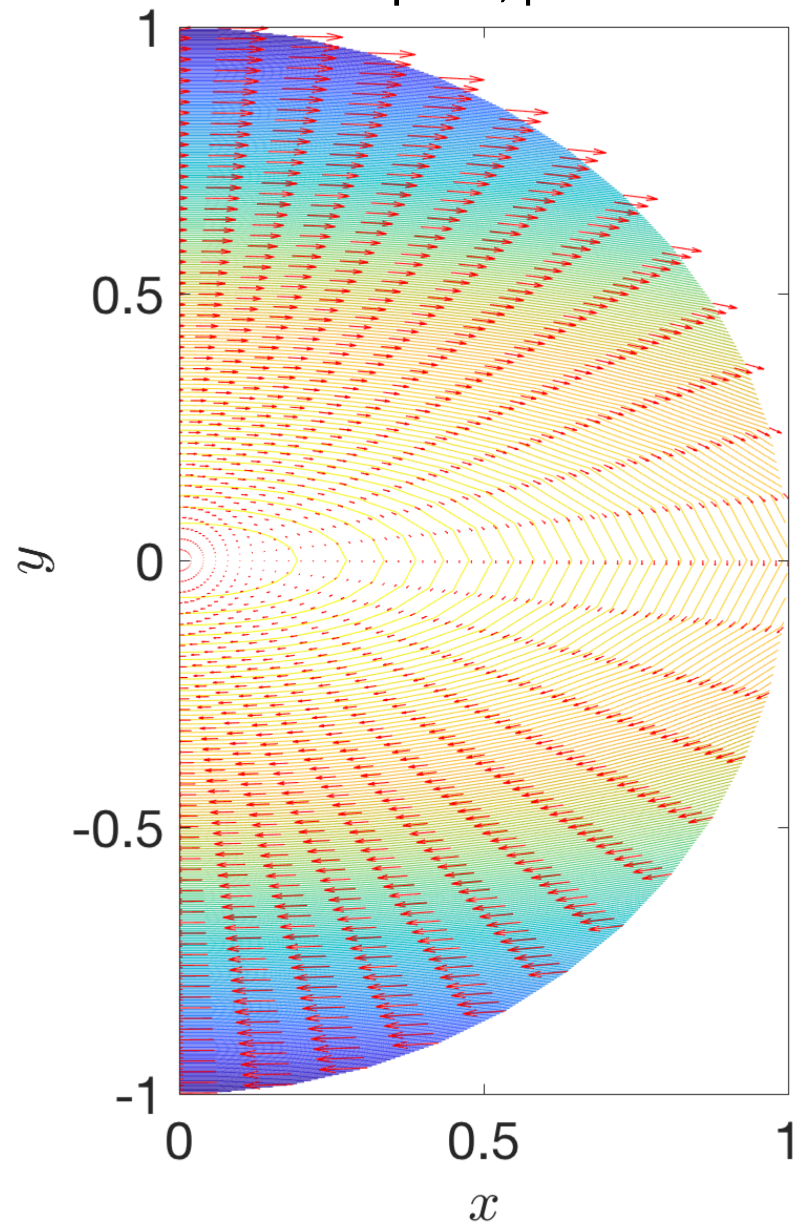
$$P_e = \frac{G\alpha_e}{2 r^3} C_1 \sin^2\theta \sin(2\phi).$$

# Small-deformation of a Darcy drop in a planar shear flow

- $\eta=0$, $\beta=0$




- $\eta=100$, $\beta=0$



# Linear dynamics of a Darcy drop in a uniaxial extensional flow

$$\mathbf{v}_{s,1} \cdot \hat{\mathbf{r}} = \frac{h_0(r)}{3} \left(1 + 3\cos(2\theta)\right) e^{\lambda t},$$

$$\mathbf{v}_{s,2} \cdot \hat{\theta} = h_1(r) \sin(2\theta) e^{\lambda t},$$

$$p_1 = \frac{(1-\phi_0)}{12r} \left[ (8(1+\lambda\mu_s)h_0 + 2(1+\Lambda+\lambda\mu_s) r h_0') + \right.$$

$$( \quad \ldots \quad \theta)) \, e^{\lambda t},$$

$$h_0 = g_1(r) = r^5 + \frac{2\bar{\lambda}}{99} r^7 + \frac{5\bar{\lambda}^2}{20592} r^9 + \frac{\bar{\lambda}^3}{514800} r^{11} + \cdots,$$

$$h_1 = g_2(r) = \frac{28}{\bar{\lambda}} r^3 - \frac{1}{3} r^5 - \frac{\bar{\lambda}}{198} r^7 - \frac{\bar{\lambda}^2}{20592} r^9 - \frac{\bar{\lambda}^3}{3088800} r^{11} + \cdots,$$

$$\psi_{e,1} =$$

$$P_1 = -\frac{\alpha_e}{2r^3} a_1 \left(1 + 3\cos(2\theta)\right) \ldots$$

$$\bar{\lambda} \equiv \frac{\lambda \xi}{(1+\lambda\mu_s)\phi_0}.$$

# Linear dynamics of a Darcy drop in a planar shear flow

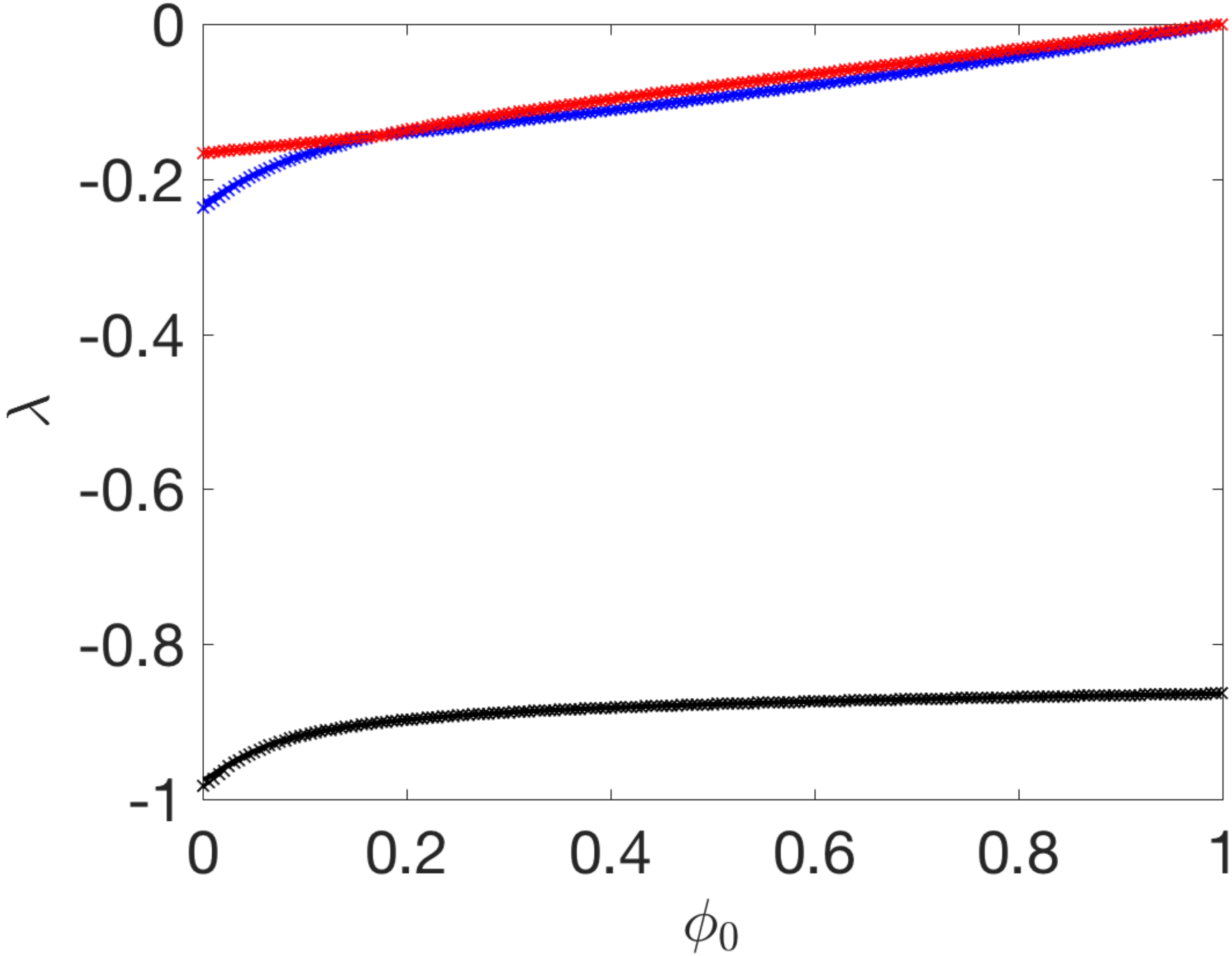
$$\mathbf{v}_{s,1}\cdot\hat{\mathbf{r}} = f_0(r)\sin^2\theta\sin(2\phi)e^{\lambda t},$$
$$\mathbf{v}_{s,1}\cdot\hat{\theta} = \frac{f_1(r)}{2}\cos(2\theta)\sin(2\phi)e^{\lambda t},$$
$$\mathbf{v}_{s,1}\cdot\hat{\phi} = f_1(r)\sin\theta\cos(2\phi)e^{\lambda t},$$

$$p_1 = \frac{(1-\phi_0)}{4r}\left[(8(1+\lambda\mu_s)f_0 + 2(1+\Lambda+\lambda\mu_s)rf_0') - \left(12(1+\lambda\mu_s)f_1 - 2(1-\Lambda+\lambda\mu_s)rf_1' - 2(1-\Lambda+\lambda\mu_s)r^2f_1''\right)\right](1+3\cos(2\theta))\,e^{\lambda t},$$
$$f_0 = \alpha_1 r + \alpha_3 r^3 + \alpha_5 g_1(r),$$
$$f_1 = \alpha_1 r + \frac{5}{3}\alpha_3 r^3 - \alpha_5 g_2(r),$$
$$\mathbf{U}_1\cdot\hat{r} = \frac{G}{20r^4}\left(c_3 + 5c_1r^2\right)\sin^2\theta\sin(2\phi)e^{\lambda t},$$
$$\mathbf{U}_1\cdot\hat{\theta} = -\frac{G}{10r^4}c_3\sin(2\theta)\sin(2\phi)e^{\lambda t},$$
$$\mathbf{U}_1\cdot\hat{\phi} = -\frac{G}{10r^4}\left(10c_4r^2 + 2c_3\cos(2\phi)\right)\sin\theta e^{\lambda t},$$
$$P_1 = \frac{G\alpha_e}{2r^3}c_1\sin^2\theta\sin(2\phi)e^{\lambda t}.$$

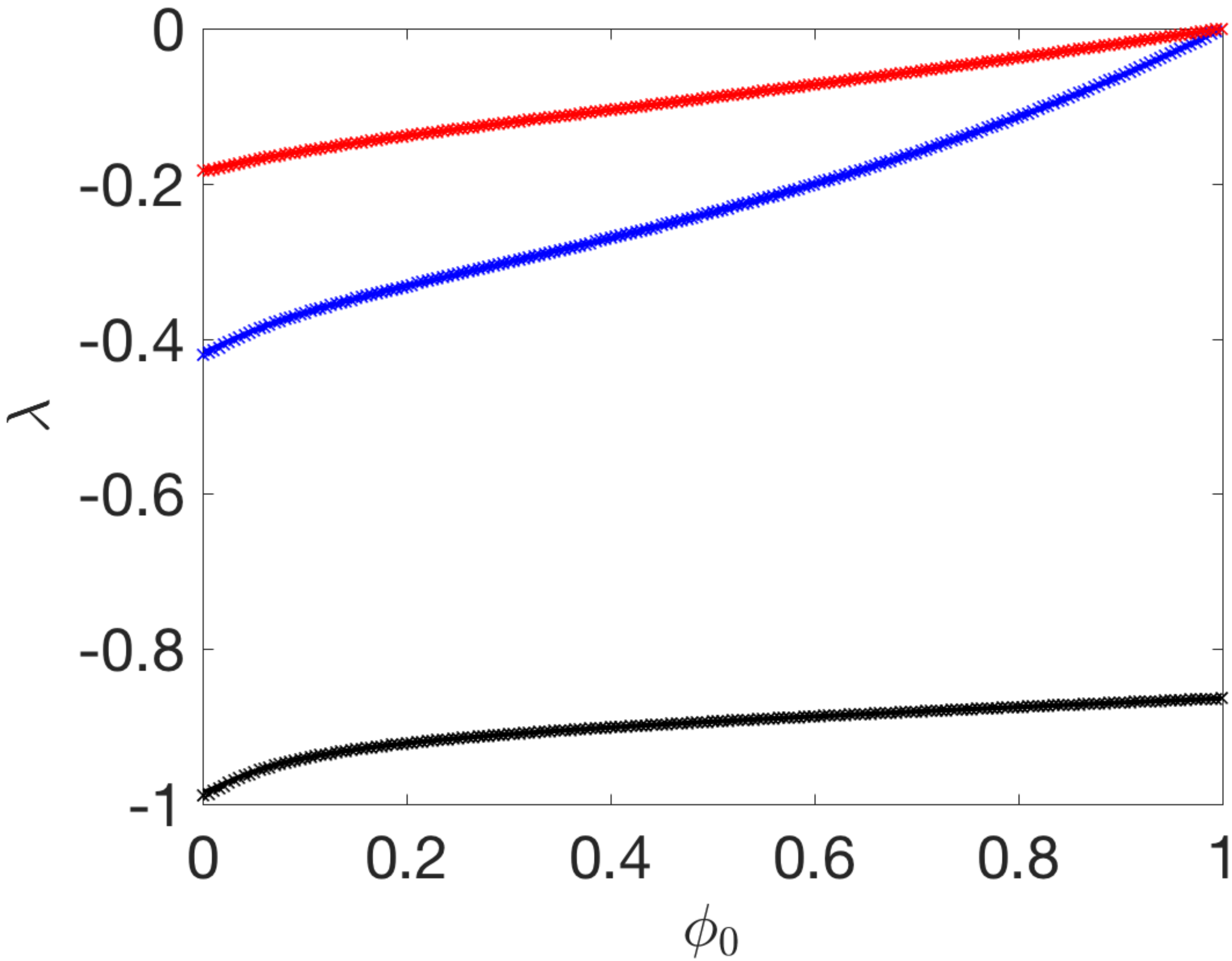
$$\mathbf{A}\begin{pmatrix}\alpha_1\\ \alpha_3\\ \alpha_5\end{pmatrix} = \lambda\mathbf{B_0}\begin{pmatrix}\alpha_1\\ \alpha_3\\ \alpha_5\end{pmatrix}$$

$$\frac{\xi\lambda}{(1+\mu_s\lambda)\phi_0} \to \frac{\xi}{\mu_s\phi_0} \text{ as } \mu_s|\lambda| \gg 1$$

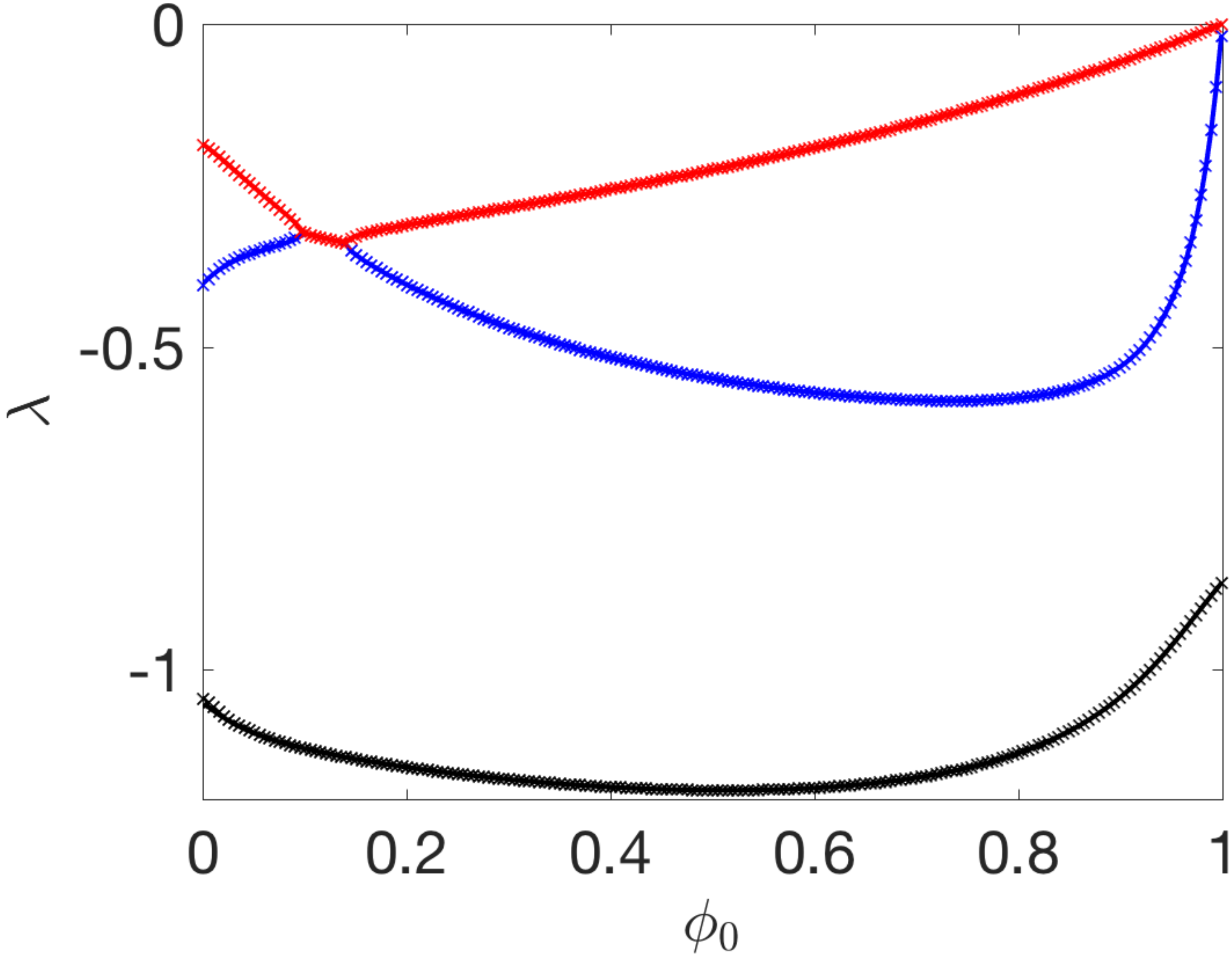
# Linear dynamics of small-deformation of a Darcy drop: $\beta=0,\eta_1=0$



# Linear dynamics of small-deformation of a Darcy drop: $\beta$=1,$\eta_1$=0



# Linear dynamics of small-deformation of a Darcy drop: $\eta_1$=20



# Conclusion and ongoing work

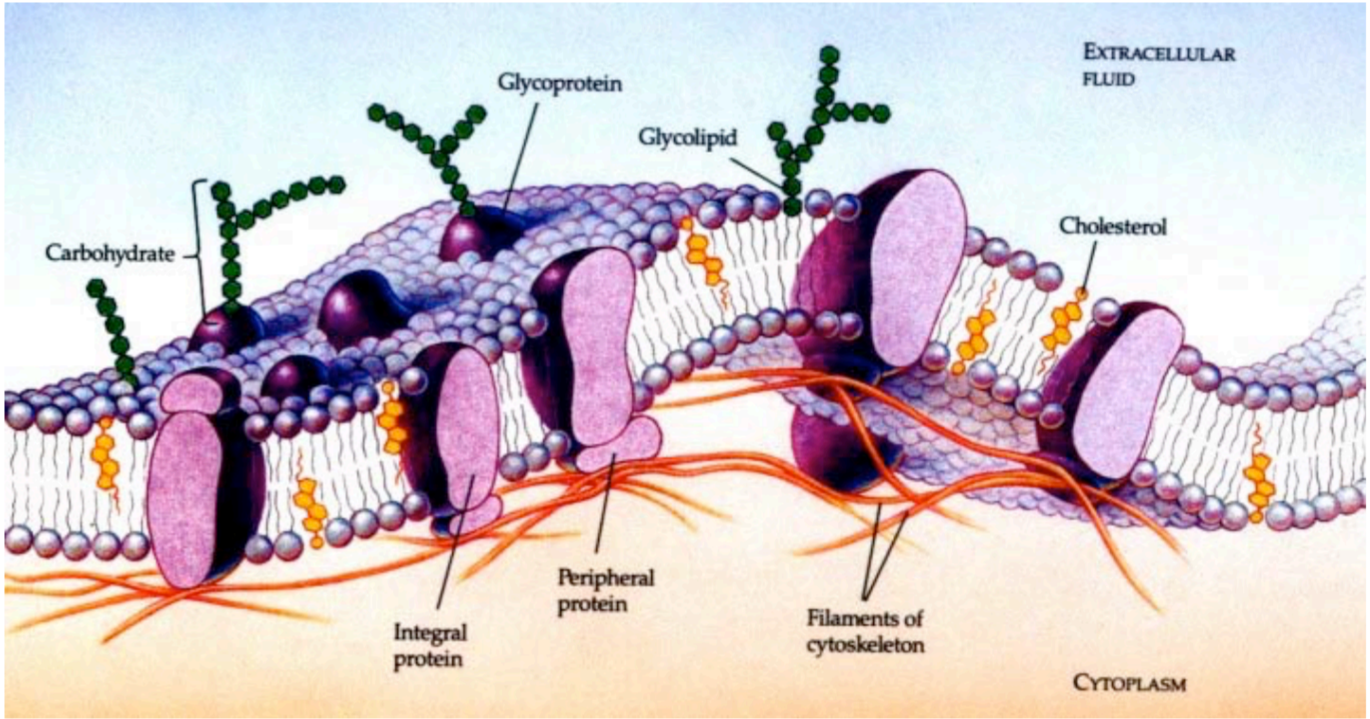
- Both permeability and slip change the flow around the soft drop even in the small-deformation limit.

- Soft poroelastic drop in extensional flow and shear flow. Brinkman drop is also considered (in a separate paper).

- Large deformation of soft poroelastic drop in Stokes flow? (Strychalski *et al*, JCP, 2014, Wrobel *et al*., JFM, JCP, 2016)

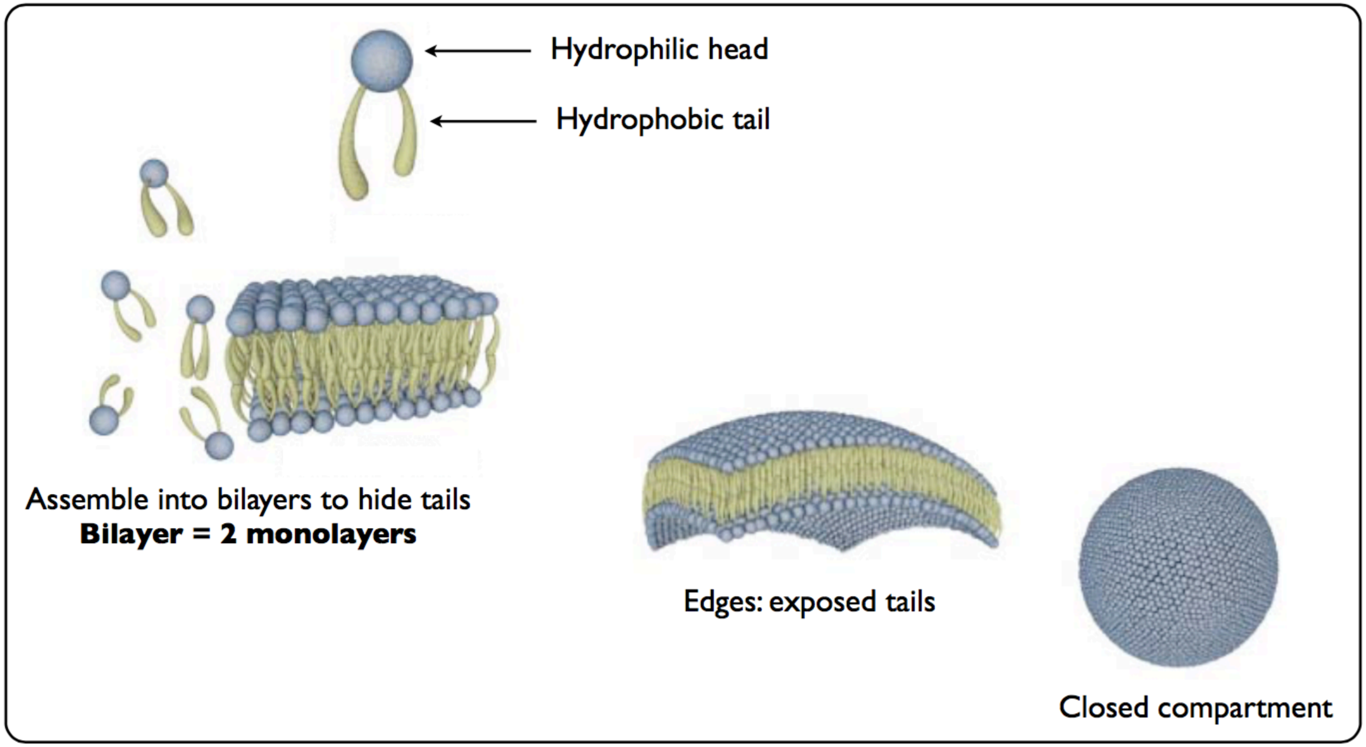
- Swelling and charge transport and electrohydrodynamics of polyelectrolytic solution?

# Cell membrane




- A drawing of a cellular cytoplasmic membrane.

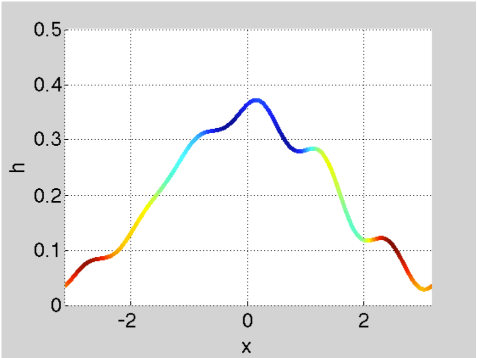
# Structures of lipid bilayer membranes



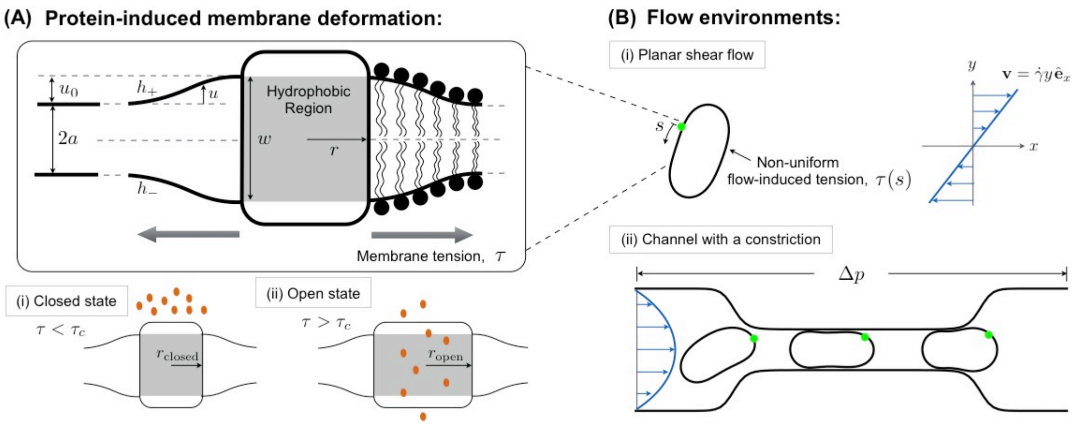

- Bilayer of thickness $2a$ ~ 5 nm with a bending modulus $\kappa$, area stretching modulus $K$, membrane conductance $\sigma_m$ and permittivity $\varepsilon_m$ (membrane capacitance $c_m = \varepsilon_m / 2a$)

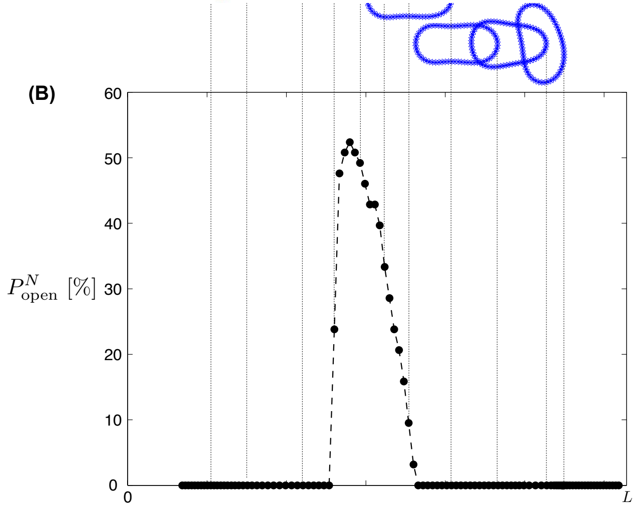
# Hydrodynamics of lipid bilayer membranes

1. Electrohydrodynamics of a planar lipid bilayer membrane (JFM 2014, PoF 2015)

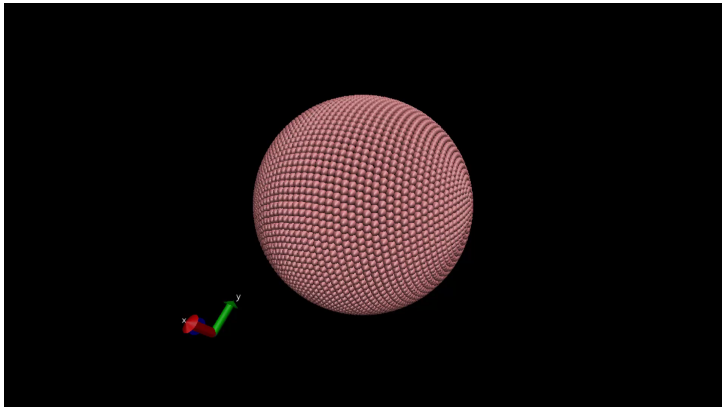



2. Protein-lipid interaction (PNAS 2015, Acta Mechanica Sinica 2016)



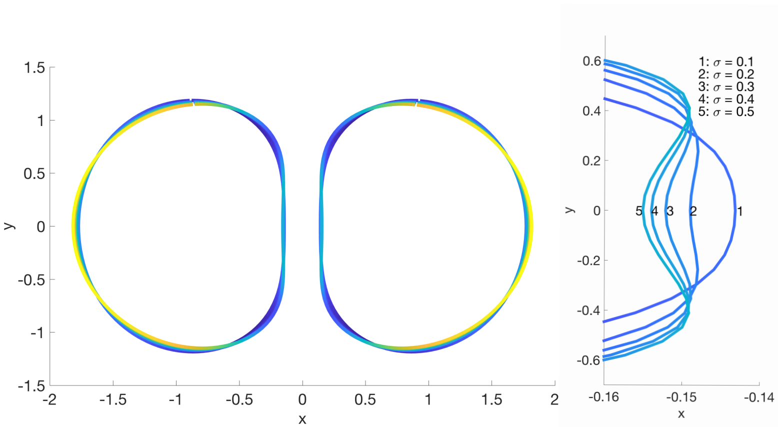




3. Poration of a vesicle (CPC 2017)

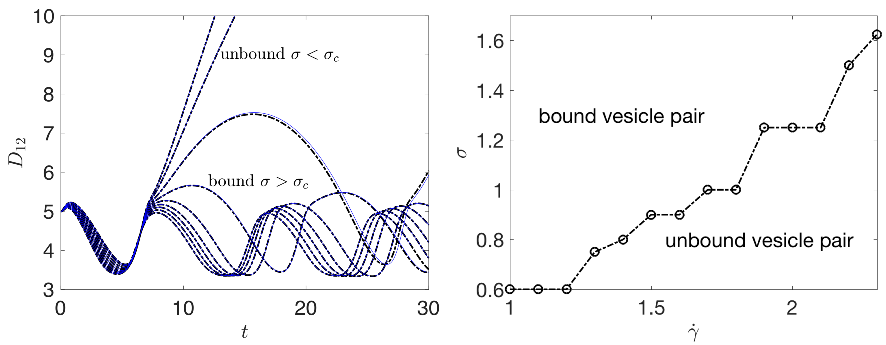


# vdW interactions between two vesicles

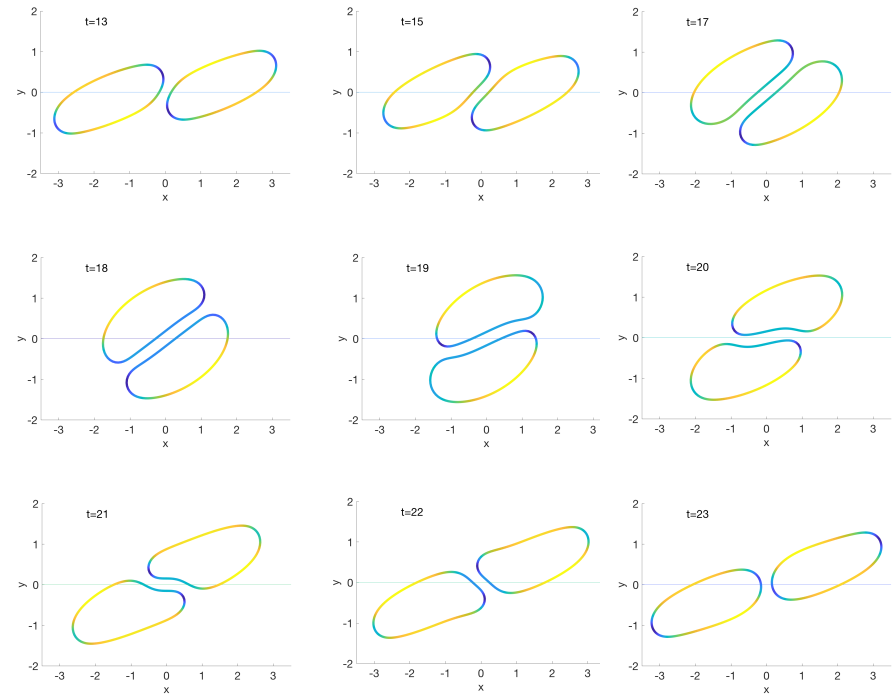
- Two vesicles interacting with each other via a vdW potential

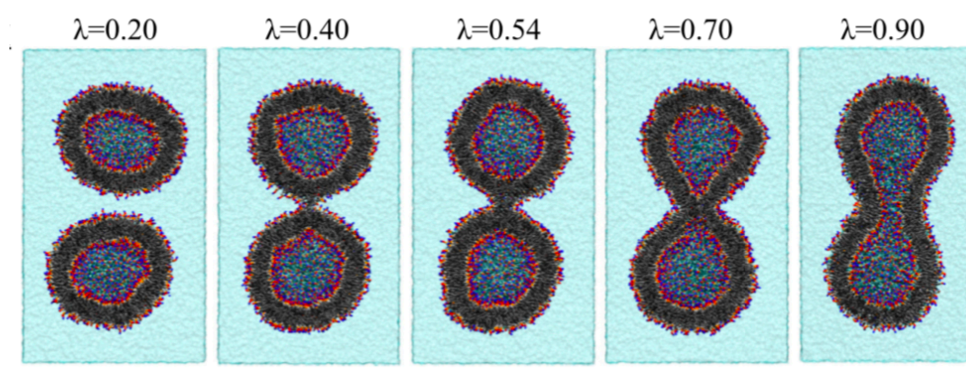



- Two vesicles interacting with each other via a vdW potential under a shear flow




- Hydrodynamics of a vesicle doublet in a shear flow

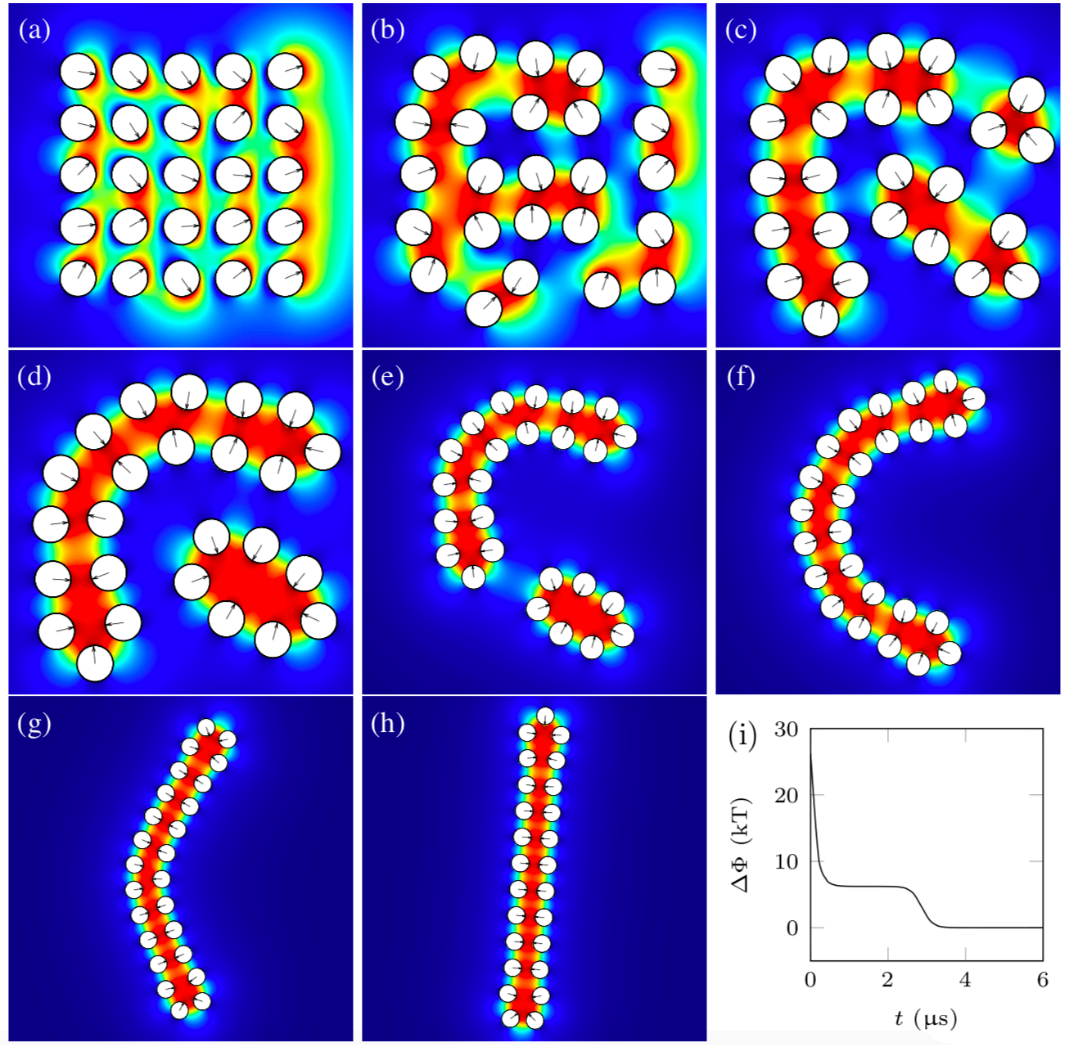


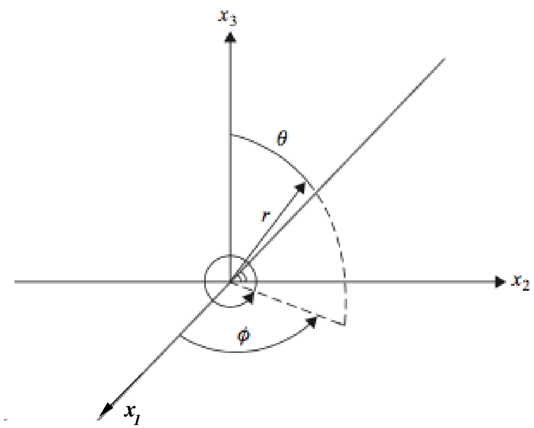

Kawamoto, J. Chem. Phys., 2015.

# Hydrophobic interaction as a dominant mechanism

- Splay, saddle splay, tilt and stretching deformations from classic continuum membrane mechanics arise as a consequence of large-scale hydrophobic attraction minimization on membrane-like configurations.
- "Long-range" hydrophobic interaction sufficient to replace Helfrich free energy?
- Fu *et al.*, submitted for publication in SIAM Multiscale Modeling.



# A viscous drop in a uniform streaming flow




- Spherical coordinate system
- Interior viscosity increases from top to bottom
- Streamlines around an osmophoretic drop (Anderson, 1981, 1983)