Effect of Use-Dependent Plasticity on Information Transfer at Hippocampal Synapses

Elham Bayat-Mokhtari, J. Josh Lawrence and Emily F. Stone

University of Montana
elham.bayatmokhtari@umontana.edu

Model Equations

- Concentration of Calcium:

$$C(t) = C_0 e^{-t/\tau_{ca}} + \Delta$$

Model Equations

- Concentration of Calcium:

$$C(t) = C_0 e^{-t/\tau_{ca}} + \Delta$$

Model Equations

- Concentration of Calcium:

$$C(t) = C_0 e^{-t/\tau_{ca}} + \Delta$$

- Fraction of a pool of synapses that release vesicles upon the arrival of spike at the terminal:

$$P_{rel}(t) = P_{max} \frac{C^4(t)}{C^4(t) + K^4}$$

- Fraction of sites that are release-ready:

$$R_{rel}(t) = 1 - (1 - R_0) \left(\frac{C_0 e^{-t/\tau_{ca}} + K_r}{K_r + C_0} \right)^{\Delta k} e^{-k_{min} t}$$

$R_0 = R_{rel}(1 - P_{rel})$

- Inhibitory Postsynaptic Current:

$$IPSC \sim N_{tot} R_{rel} P_{rel}$$

Computational Experiments

- Input: Poisson spike train S with a particular mean firing rate λ.
- Sequence of interspike intervals(ISIs),
  $I_S = \{t_1 - T_1, t_2 - t_1, t_3 - t_2, \cdots, t_n - t_{n-1}\}$.
  - T_1 is the beginning of the record trial.
  - $I_S = \{ISI_1, ISI_2, \cdots, ISI_{n-1}\}$, where $ISIs$ are independent and identically distributed random variables.
  - ISI has exponential distribution, with parameter λ as mean firing rate.
    $$f_{X=ISI}(x; \lambda) = \lambda e^{-\lambda x}$$
- Output: Normalized Postsynaptic Response distribution.

How does the response vary with frequency and synapse type?

Model Equations

- Concentration of Calcium:

$$C(t) = C_0 e^{-t/\tau_{ca}} + \Delta$$

- Fraction of a pool of synapses that release vesicles upon the arrival of spike at the terminal:

$$P_{rel}(t) = P_{max} \frac{C^4(t)}{C^4(t) + K^4}$$

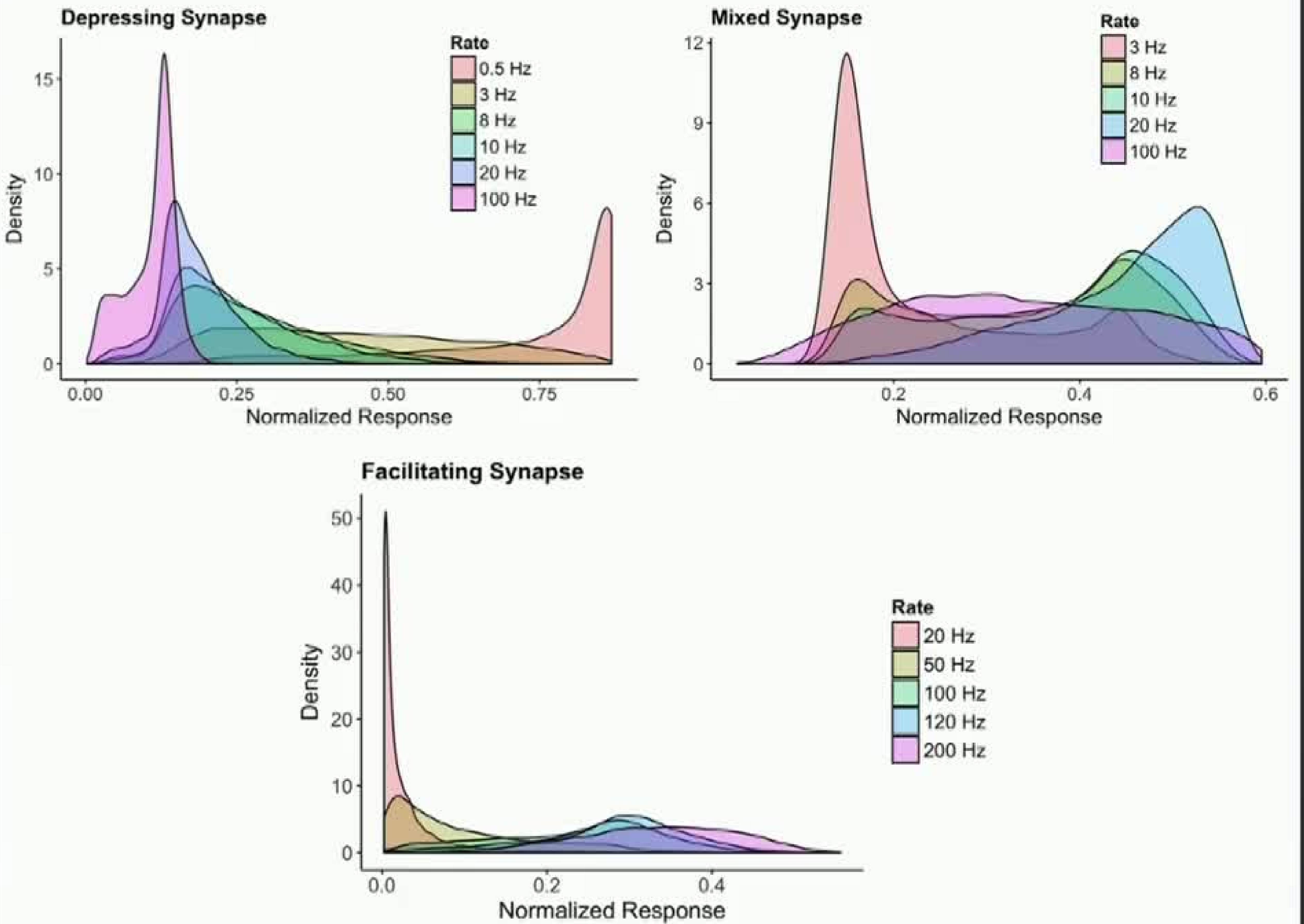
- Fraction of sites that are release-ready:

$$R_{rel}(t) = 1 - (1 - R_0) \left(\frac{C_0 e^{-t/\tau_{ca}} + K_r}{K_r + C_0} \right)^{\Delta k} e^{-k_{min} t}$$

$R_0 = R_{rel}(1 - P_{rel})$

- Inhibitory Postsynaptic Current:

$$IPSC \sim N_{tot} R_{rel} P_{rel}$$



Depressing Synapse
Rate
0.5 Hz
3 Hz
8 Hz
10 Hz
20 Hz
100 Hz
Density
15
10
5
0
0.00
0.25
0.50
0.75
Normalized Response
Mixed Synapse
Rate
3 Hz
8 Hz
10 Hz
20 Hz
100 Hz
Density
12
9
6
3
0
0.2
0.4
0.6
Normalized Response
Facilitating Synapse
Rate
20 Hz
50 Hz
100 Hz
120 Hz
200 Hz
Density
50
40
30
20
10
0
0.0
0.2
0.4
Normalized Response

2-D Map

- Given Interspike interval T, 2-D map (in C and R_{rel}):

$$C_{n+1} = C_n e^{-T/\tau_{ca}} + \Delta$$

$$P_{n+1} = P_{max} \frac{C_{n+1}^4}{C_{n+1}^4 + K^4}$$

$$R_{n+1} = 1 - (1 - (1 - P_n) R_n) \left(\frac{C_n e^{-T/\tau_{ca}} + K_r}{K_r + C_n} \right)^{\Delta k} e^{-k_{min} T}$$

The peak value of IPSC upon the n-th stimulus is $R_n P_n$

Computational Experiments

- Input: Poisson spike train S with a particular mean firing rate λ.
- Sequence of interspike intervals(ISIs), $I_S = \{t_1 - T_1, t_2 - t_1, t_3 - t_2, \cdots, t_n - t_{n-1}\}$.
  - T_1 is the beginning of the record trial.
  - $I_S = \{ISI_1, ISI_2, \cdots, ISI_{n-1}\}$, where $ISIs$ are independent and identically distributed random variables.
  - ISI has exponential distribution, with parameter λ as mean firing rate.

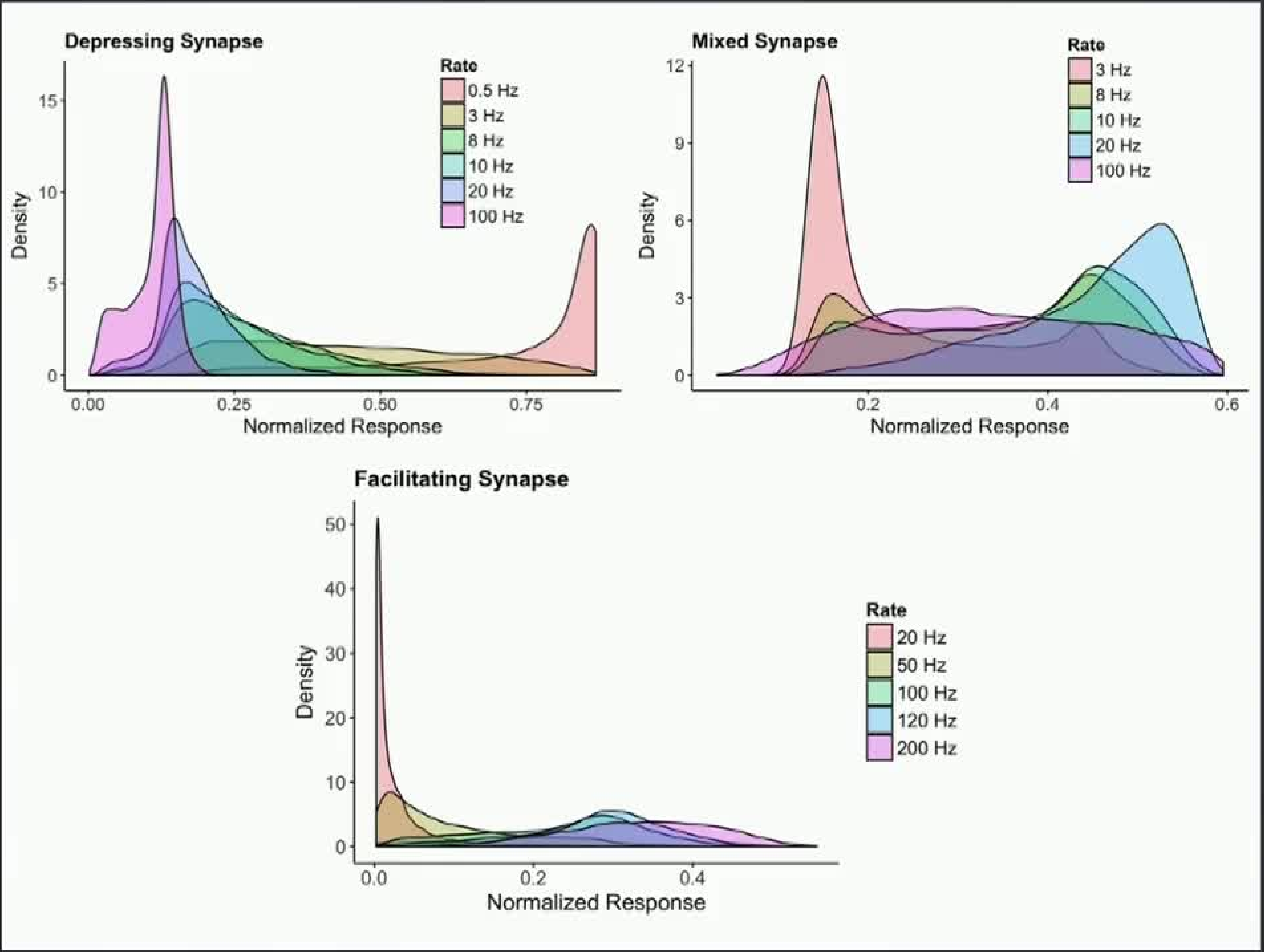
$$f_{X=ISI}(x; \lambda) = \lambda e^{-\lambda x}$$

- Output: Normalized Postsynaptic Response distribution.

How does the response vary with frequency and synapse type?

Some general problems in Neuroscience

- How much information is transmitted between neurons?
    - Information theoretic measures determine the amount of information the neuron could transmit, given the distribution of observed spikes.
    - These measures ignore the order in which the responses occur.
- How to quantify "Memory" in the synapse?
    - Multivariate Information measure estimates the amount of information contained in a response about the sequential number of preceding spike.
    - Computed using Kraskov-Stogbauer-Grassberger (KSG) algorithm (Kraskov et al., 2004).
- We use `Computational Mechanics` to quantify the structure of response with an optimally predictive hidden Markov model or Causal State Model (CSM).


Depressing Synapse
Rate
0.5 Hz
3 Hz
8 Hz
10 Hz
20 Hz
100 Hz
Density
0
5
10
15
0.00
0.25
0.50
0.75
Normalized Response
Mixed Synapse
Rate
3 Hz
8 Hz
10 Hz
20 Hz
100 Hz
Density
0
3
6
9
12
0.2
0.4
0.6
Normalized Response
Facilitating Synapse
Rate
20 Hz
50 Hz
100 Hz
120 Hz
200 Hz
Density
0
10
20
30
40
50
0.0
0.2
0.4
Normalized Response

Causal States

- Chain: $\overleftrightarrow{S} = \overleftarrow{S}_t \overrightarrow{S}_t$
- Past: $\overleftarrow{S}_t = \cdots S_{t-2} S_{t-1} S_t$
- Future : $\overrightarrow{S}_t = S_{t+1} S_{t+2} \cdots$
- Stationary : $P(\overleftrightarrow{S}_t) = P(\overleftrightarrow{S}_0)$
- Two histories $\overleftarrow{s}^L$ and $\overleftarrow{s'}^L$, are equivalent when

$$P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s}^L) = P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s'}^L)$$

- ϵ : function which maps histories to their equivalence classes:

$$\epsilon(\overleftarrow{s}^L) = \{\overleftarrow{s'}^L : P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s}^L) = P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s'}^L)\}$$

- The possible values of ϵ are "Causal States" of the process.

$$\sigma_t = \epsilon(S^L)$$

Some general problems in Neuroscience

- How much information is transmitted between neurons?
  - Information theoretic measures determine the amount of information the neuron could transmit, given the distribution of observed spikes.
  - These measures ignore the order in which the responses occur.
- How to quantify "Memory" in the synapse?
  - Multivariate Information measure estimates the amount of information contained in a response about the sequential number of preceding spike.
  - Computed using Kraskov-Stogbauer-Grassberger (KSG) algorithm (Kraskov et al., 2004).
- We use `Computational Mechanics` to quantify the structure of response with an optimally predictive hidden Markov model or Causal State Model (CSM).

Properties of Causal States

(proof in (Shalizi & Crutchfield, 2001))

- Causal states are minimal sufficient statistics for predicting the process's future.
- Given an initial Causal State and the next symbol from original process, we can define the transition probability

$$T_{ij}^{(s)} \equiv P(\overrightarrow{S}^{1} = s, S' = \sigma_j | S = \sigma_i)$$

- Each causal state has a unique morph, i.e., no two causal states have the same conditional distribution of futures.

Thresholding base on Statistical Complexity

- Input: Poisson Spike train at a certain mean firing rate.
- Output: Continuous response values ranges between $[0, 1]$.
- CSM takes values from a discrete alphabet.
- Partition the output into 0's and 1' based on the `Statistical Complexity Measure`.
- **Statistical Complexity**: average amount of historical information (memory) needed to reproduce the patterns contained in the data set (sequence).
- Statistical Complexity is defined as

$$C_\mu = -\sum_i P(\sigma_i) \log_2 P(\sigma_i).$$

where $P(\sigma_i)$ is the probability of finding a system in the causal state i after the machine has been running infinitely long.

Causal States

- Chain: $\overleftrightarrow{S} = \overleftarrow{S}_t \overrightarrow{S}_t$
- Past: $\overleftarrow{S}_t = \cdots S_{t-2} S_{t-1} S_t$
- Future : $\overrightarrow{S}_t = S_{t+1} S_{t+2} \cdots$
- Stationary : $P(\overleftrightarrow{S}_t) = P(\overleftrightarrow{S}_0)$
- Two histories $\overleftarrow{s}^L$ and $\overleftarrow{s'}^L$, are equivalent when

$$P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s}^L) = P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s'}^L)$$

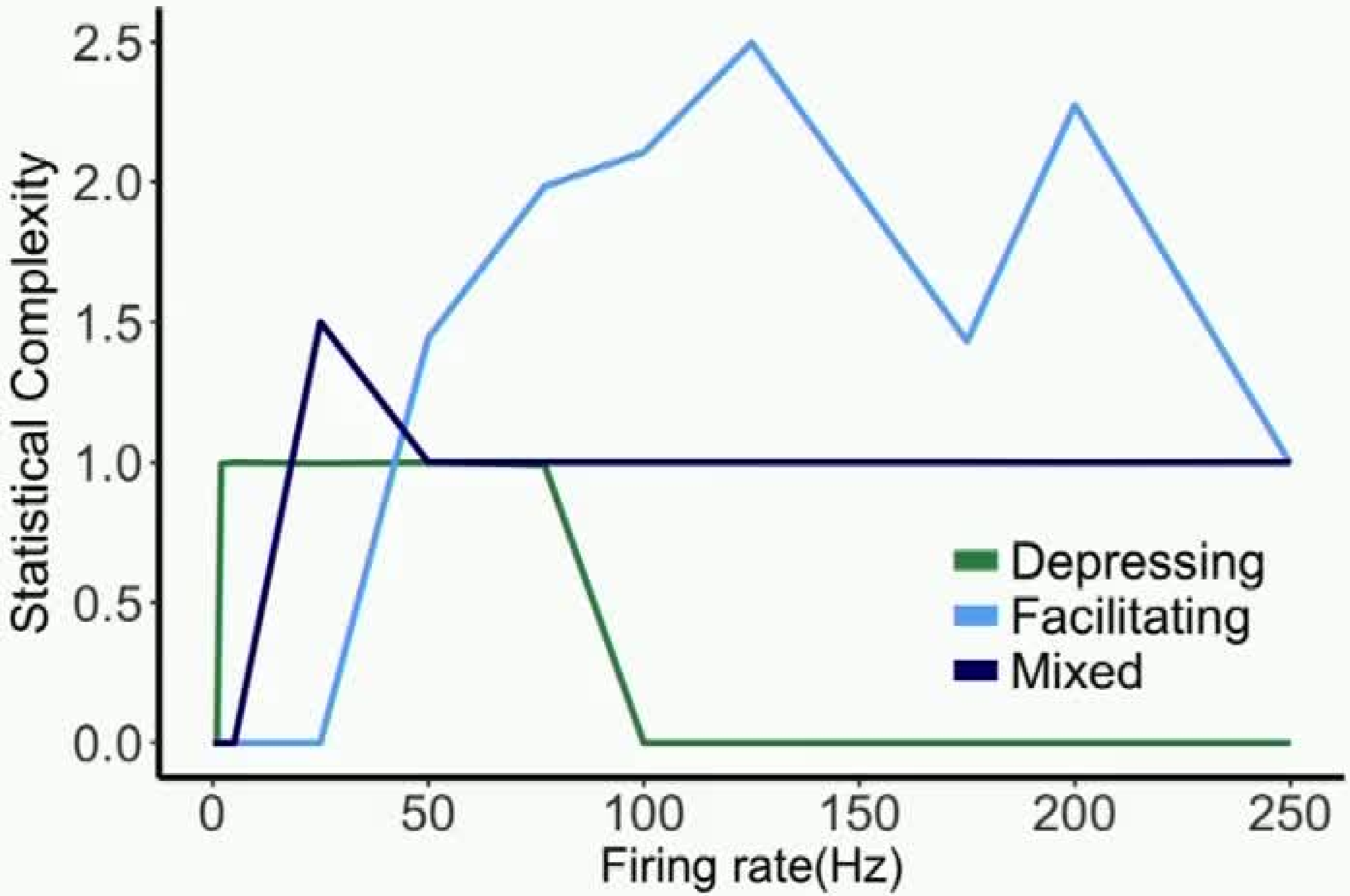
- ϵ : function which maps histories to their equivalence classes:

$$\epsilon(\overleftarrow{s}^L) = \{\overleftarrow{s'}^L : P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s}^L) = P(\overrightarrow{S}^L | \overleftarrow{S}^L = \overleftarrow{s'}^L)\}$$

- The possible values of ϵ are "Causal States" of the process.

$$\sigma_t = \epsilon(S^L)$$

Statistical Complexity for synapse types



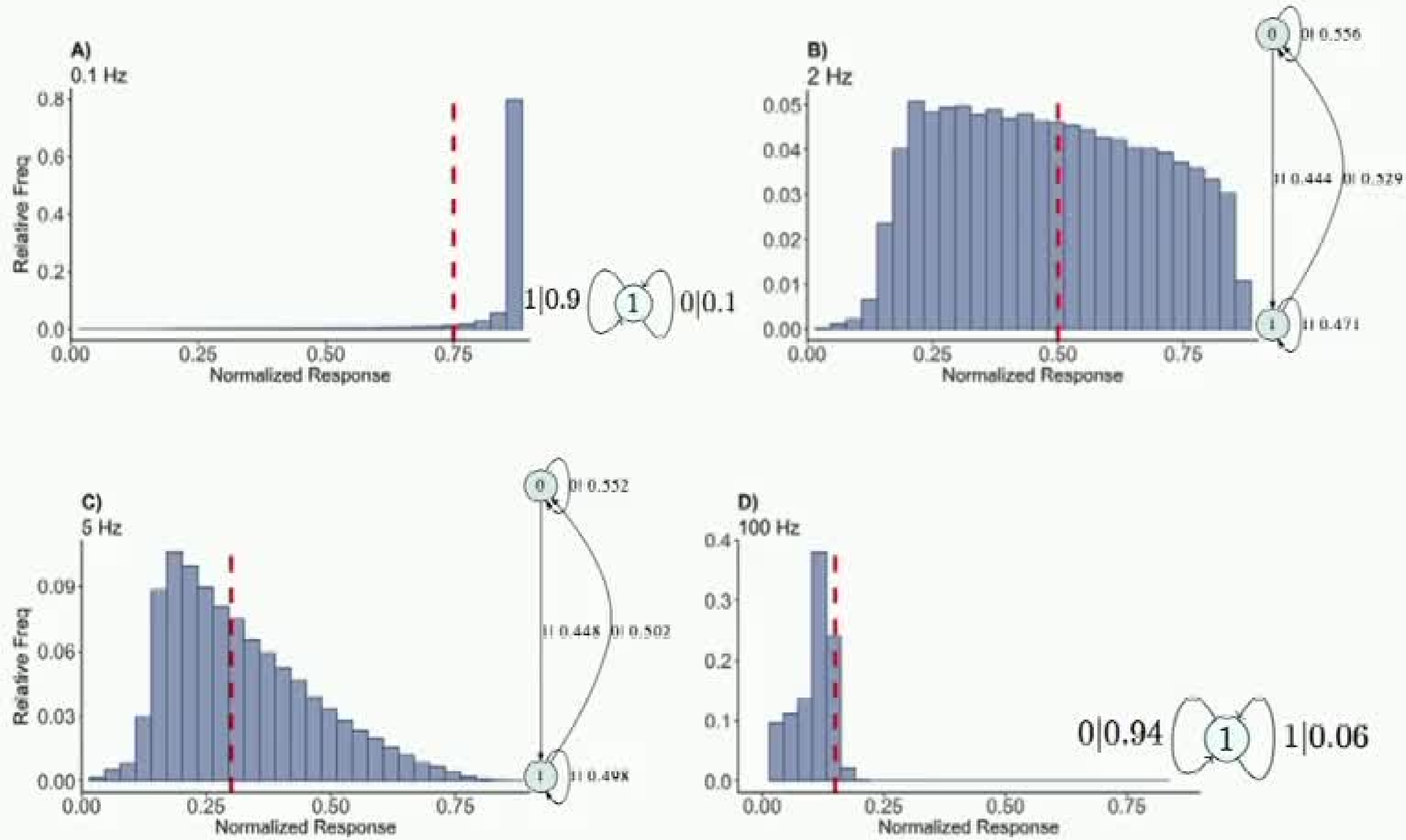
Thresholding base on Statistical Complexity

- Input: Poisson Spike train at a certain mean firing rate.
- Output: Continuous response values ranges between $[0, 1]$.
- CSM takes values from a discrete alphabet.
- Partition the output into 0's and 1' based on the `Statistical Complexity Measure`.
- **Statistical Complexity**: average amount of historical information (memory) needed to reproduce the patterns contained in the data set (sequence).
- Statistical Complexity is defined as

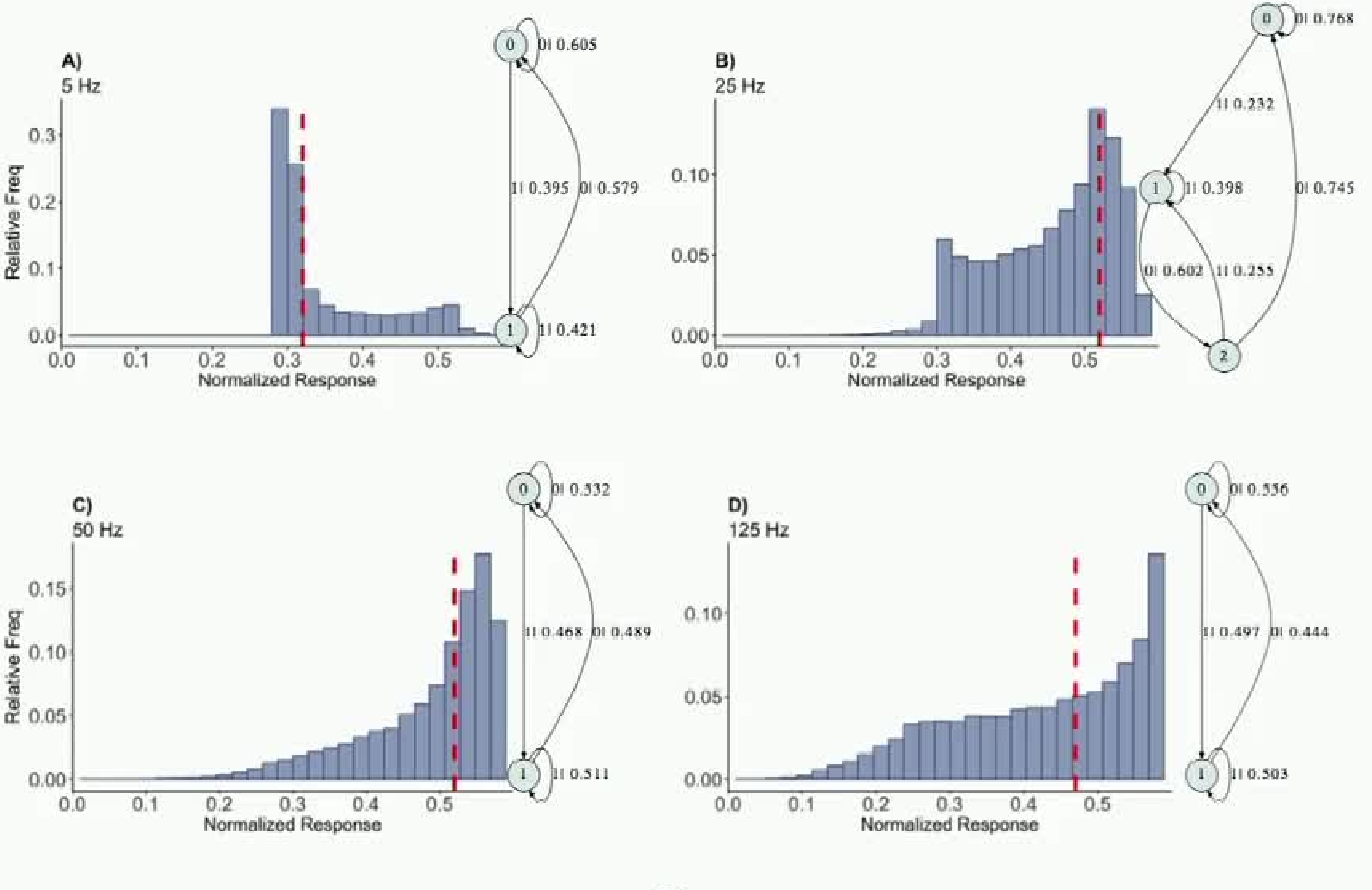
$$C_\mu = -\sum_i P(\sigma_i) \log_2 P(\sigma_i).$$

where $P(\sigma_i)$ is the probability of finding a system in the causal state i after the machine has been running infinitely long.

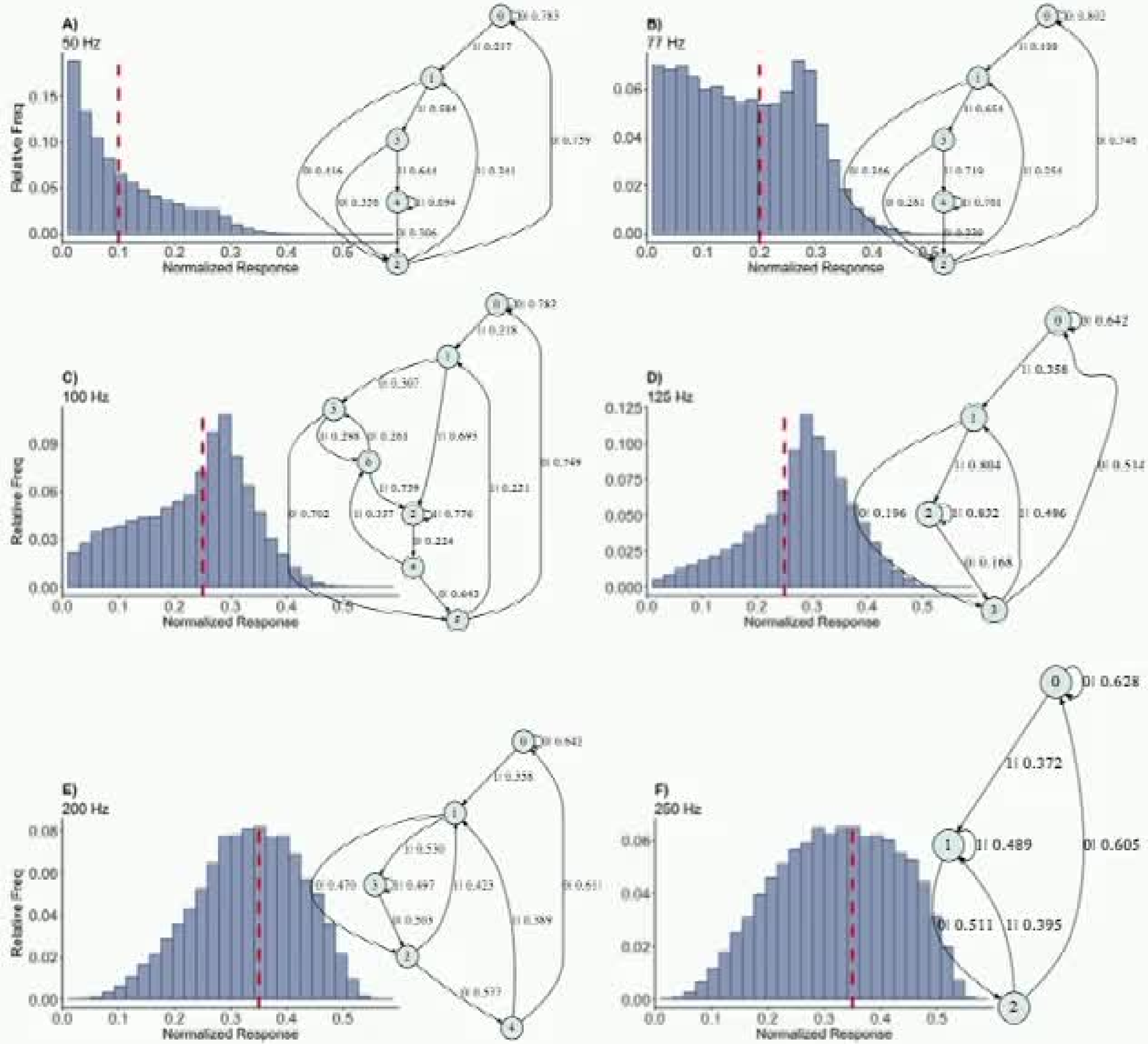
The CSMs reconstructed for depressing synapse



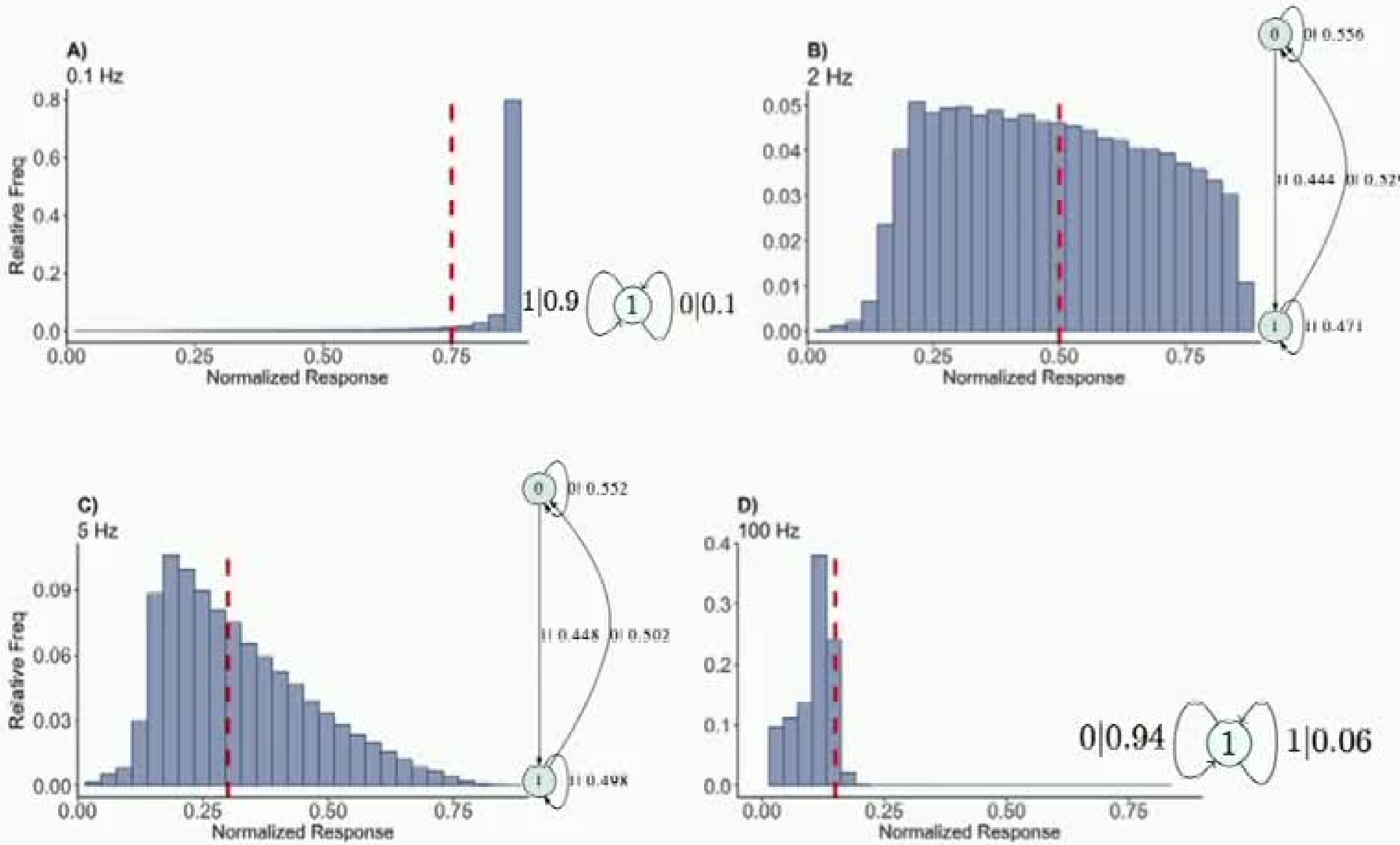
The CSMs reconstructed for mixed synapse



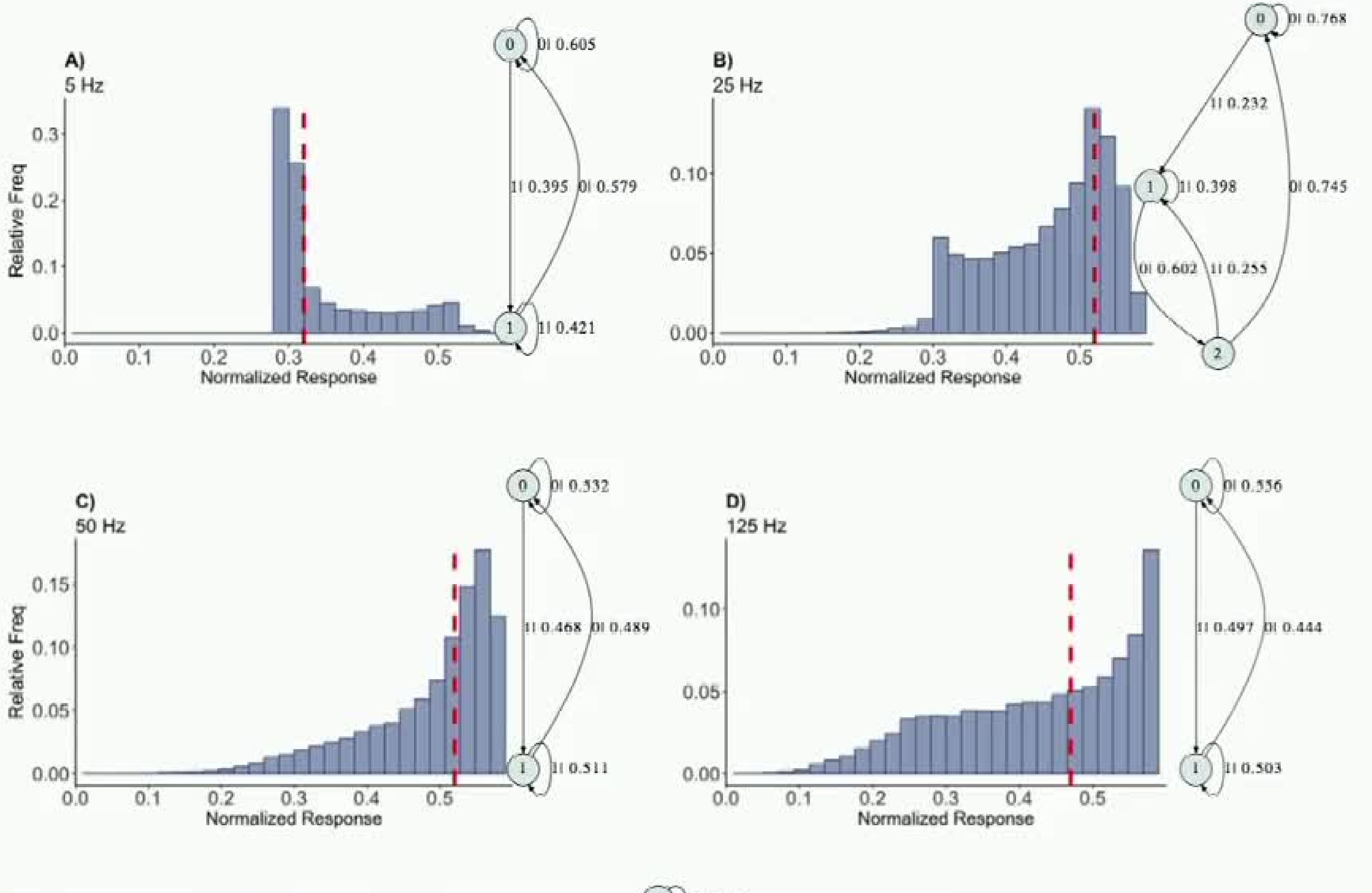
The CSMs reconstructed for facilitating synapse



The CSMs reconstructed for depressing synapse



The CSMs reconstructed for mixed synapse

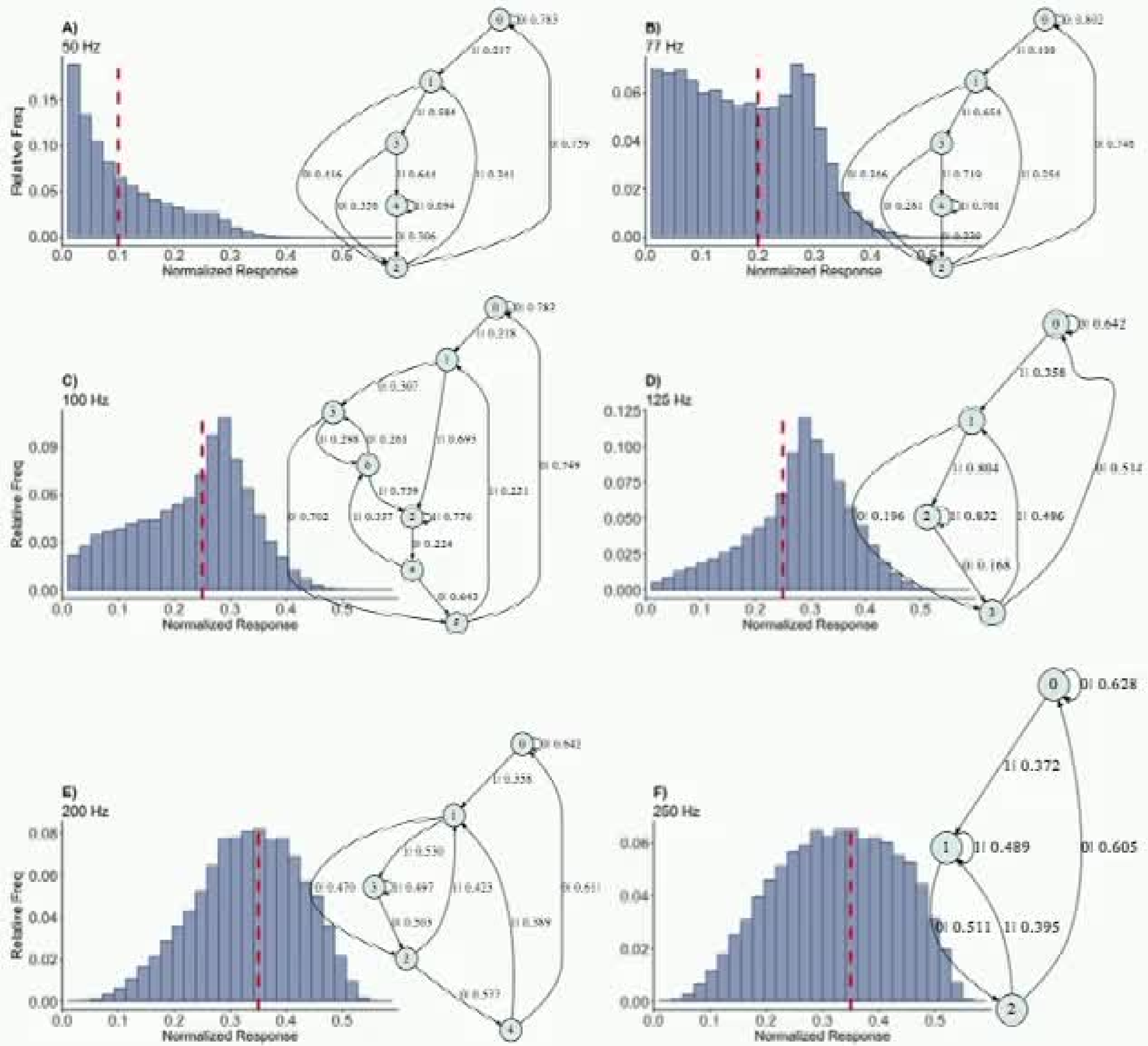


Discussion

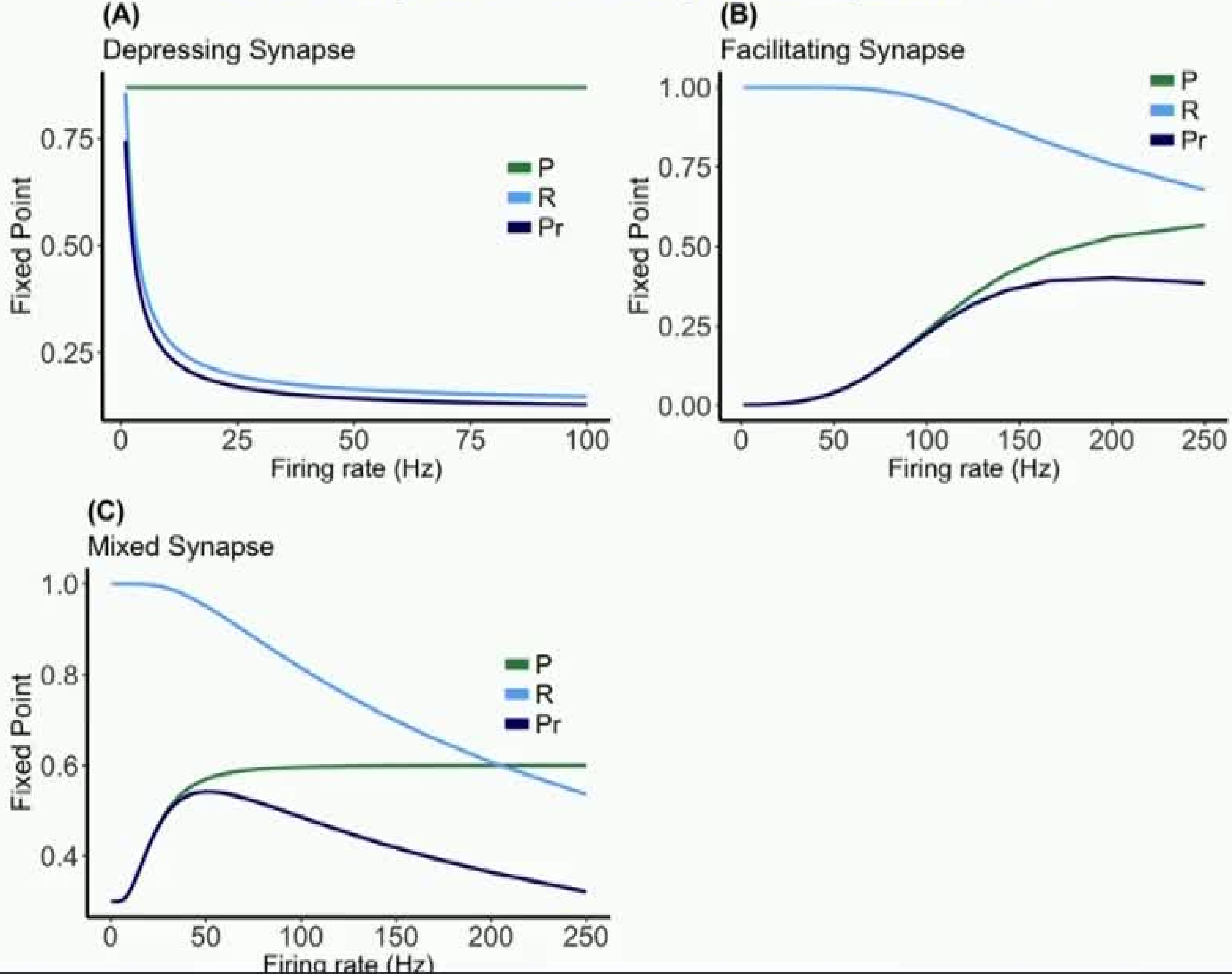
- The machines differ largely between the three types of short-term plasticity, e.g., the dynamics of the depressing type is simple whereas the facilitating type is complicated. These findings are not immediately obvious by looking at the response distributions.

- We can understand this difference by looking at the spectrum of fixed point.

The CSMs reconstructed for facilitating synapse



"Decompose" fixed point spectrum



Discussion

- The machines differ largely between the three types of short-term plasticity, e.g., the dynamics of the depressing type is simple whereas the facilitating type is complicated. These findings are not immediately obvious by looking at the response distributions.

- We can understand this difference by looking at the spectrum of fixed point.

"Decompose" fixed point spectrum