2017 SIAM CSE
Atlanta, Georgia

Towards Efficient Multiscale Numerical Methods for Kinetic Models in Plasma Physics

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Joint work with: Erica Johnson (ISU), Lopamudra Palodhi (ISU)

Partially funded by: NSF Grant DMS–1620128

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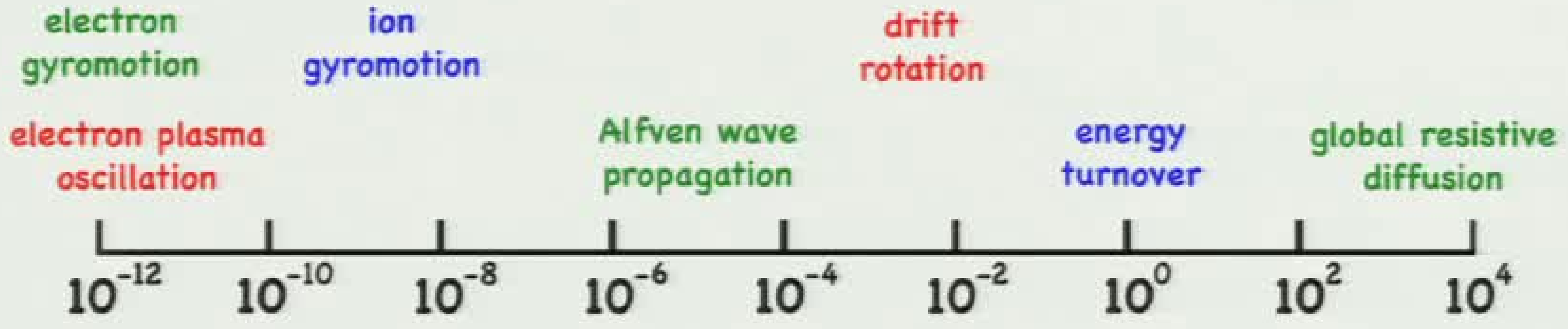
Outline

Real-world problems are multiscale

Example: Magnetically-confined fusion



electron gyromotion

ion gyromotion

drift rotation

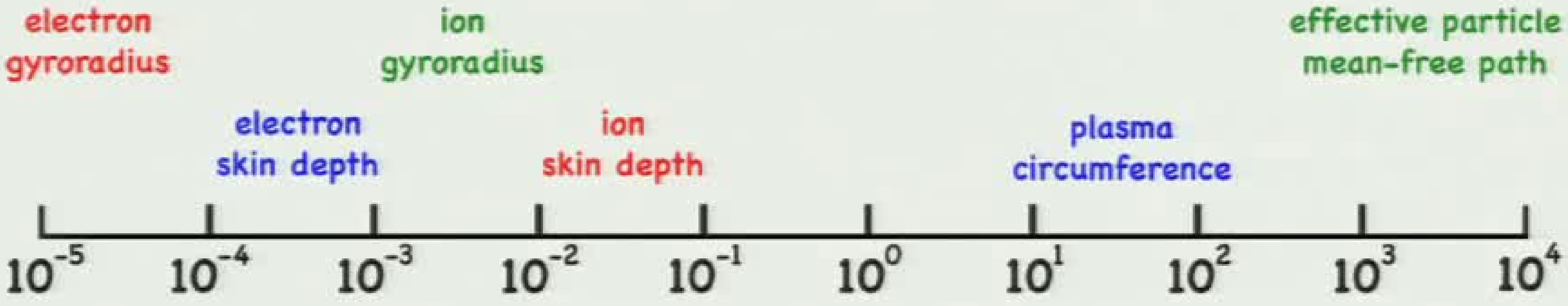
electron plasma oscillation

Alfven wave propagation

energy turnover

global resistive diffusion

Characteristic Time Scales in ITER (seconds)



electron gyroradius

ion gyroradius

effective particle mean-free path

electron skin depth

ion skin depth

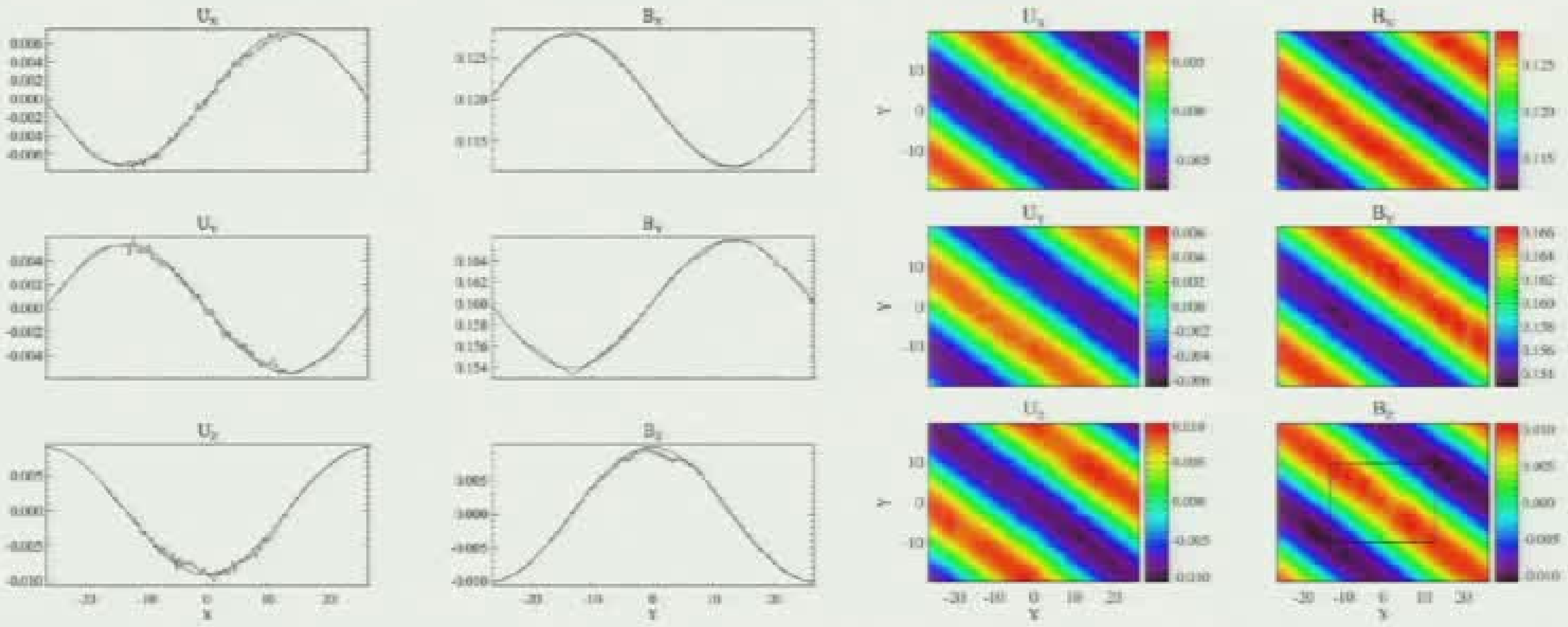
plasma circumference

Characteristic Length Scales in ITER (meters)

- ITER: effort to develop largest tokamak for magnetic confined fusion
- Nonlinear effects couple many of the space and time scales, very stiff PDEs

State-of-the-art I: Hall MHD + PIC

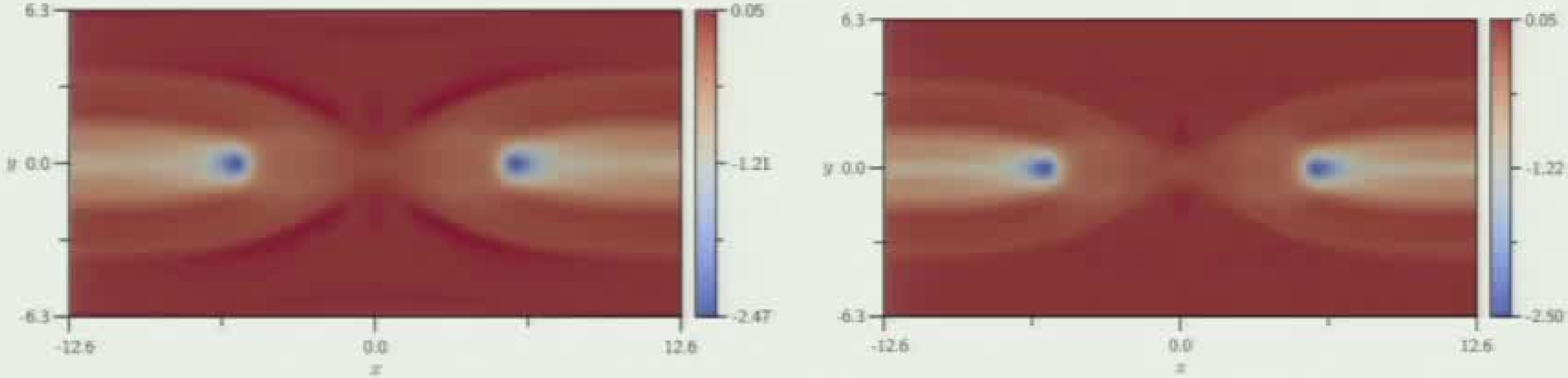
[Daldorff, Tóth, Gombosi, Lapenta, Amaya, Markidis, & Brackbill 2014]

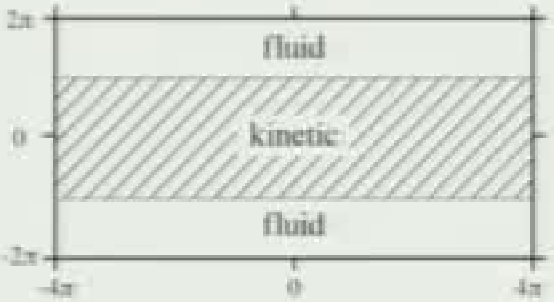


- PIC code region embedded in Hall MHD model
- Restriction (PIC → Hall MHD): modified Ohm's law
- Prolongation (Hall MHD → PIC): BCs of PIC region
- **Difficulty #1:** consistency problems between models (quasineutrality)
- **Difficulty #2:** PIC introduces statistical noise
- **Difficulty #3:** dynamically deciding fluid/kinetic boundary

State-of-the-art II: Vlasov + 2-Fluid

[Rieke & Grauer, 2015]





- Fluid region: 2-fluid 5-moment (Euler-Maxwell) using finite differences
- Kinetic region: operator split finite differences
- Restriction (Kinetic → Fluid): compute moments of kinetic PDF (+ smoothing)
- Prolongation (Fluid → Kinetic): construct PDF from moments
- **Difficulty:** dynamically deciding fluid/kinetic boundary

Paradigms for multiscale simulations

Fluid and kinetic regions that couple at boundaries

- How do the regions couple?
- How do we determine dynamically where to place regions?
- (Fluid everywhere/kinetic in some regions) vs. (non-overlapping regions)

Global kinetic solvers that are asymptotic-preserving to fluid regime

- No coupling, but inefficient in fluid regimes
- Try to achieve efficiency via AMR techniques (not trivial in phase space)

Fluid solvers that try to extend to kinetic regime by including high moments

- How many moments do we need?
- How to close the moment expansion? (physical accuracy, well-posedness)

Micro-macro decomposition

- Write the solution as (fluid) + (kinetic corrections) and split the two
- Solve both everywhere, but kinetic can be coarsely resolved when not important
- How to design good schemes for (kinetic corrections)?

Asymptotic-preserving [Jin, 1999], [Jin, 2012]

Commutative diagram

$$\begin{tikzcd}
\boxed{\begin{array}{c}\text{Method for kinetic:}\\ f_{,t}+\mathbf{v}\cdot\nabla_{\mathbf{x}}f+\mathbf{a}\cdot\nabla_{\mathbf{v}}f=\frac{1}{\varepsilon}(\mathcal{M}-f)\end{array}} \arrow[r, "\begin{bmatrix}\Delta t\\ \Delta x\\ \Delta v\end{bmatrix}\to 0^{+}"] \arrow[d, "\varepsilon\to 0^{+}"] & \boxed{\begin{array}{c}\text{Exact solution of kinetic:}\\ f_{,t}+\mathbf{v}\cdot\nabla_{\mathbf{x}}f+\mathbf{a}\cdot\nabla_{\mathbf{v}}f=\frac{1}{\varepsilon}(\mathcal{M}-f)\end{array}} \arrow[d, "\varepsilon\to 0^{+}"'] \\
\boxed{\begin{array}{c}\text{Method for fluid:}\\ q_{,t}+\nabla\cdot\mathbf{F}(q)=\psi(q)\end{array}} \arrow[r, "\begin{bmatrix}\Delta t\\ \Delta x\end{bmatrix}\to 0^{+}"] & \boxed{\begin{array}{c}\text{Exact solution of fluid:}\\ q_{,t}+\nabla\cdot\mathbf{F}(q)=\psi(q)\end{array}}
\end{tikzcd}$$

- AP applies to a large class of singularly-perturbed ODEs/PDEs
- AP: Numerical method limits to a convergent method when $\varepsilon\to 0^{+}$
- AP is a necessary condition for efficient numerical methods in the limit $\varepsilon\to 0^{+}$
- AP is not sufficient, since limiting method could be convergent but not efficient

Gaussian-based moment closure

[Groth, Gombosi, Roe, & Brown, 1994, 2003]

- Knock-out missing moments by pretending they come from a Gaussian

$$\textbf{20 - moment}: \quad \mathbb{R} \leftarrow 3\mathbb{P}\mathbb{P}/\rho \quad (\text{Kurtosis: } \mathbb{K} = \mathbb{R} - 3\mathbb{P}\mathbb{P}/\rho \equiv 0)$$

$$\textbf{35 - moment}: \quad \mathbb{S} \leftarrow 10\rho\mathbb{P}\mathbb{Q}$$

- Example: 20-moments in 1D (reduces to only 4 moments):

$$q = \begin{bmatrix} \rho \\ \rho u \\ p + \rho u^2 \\ q + 3pu + \rho u^3 \end{bmatrix} \quad \text{and} \quad f(q) = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ q + 3pu + \rho u^3 \\ \frac{3p^2}{\rho} + 4qu + 6pu^2 + \rho u^4 + (\mathbb{K} = 0) \end{bmatrix}$$

- Four eigenvalues of flux Jacobian:

$$\lambda = u + s\sqrt{\frac{p}{\rho}}, \qquad s^4 - 6s^2 - 4sh + 3 = 0, \qquad h := \frac{q}{p}\sqrt{\frac{\rho}{p}}$$

- **Advantage:** no direct moment inversion
- **Disadvantage:** limited hyperbolicity: $|h| < \sqrt{\sqrt{8} - 2} \approx 0.9102$