Assessing Educational Interventions: Moving from ``Does It Work?" to ``What Do They Know?"

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  - I told them about the great grades the students earned
  - I told them about the fact that of the 32 students that started the class, 32 finished the class and 31 passed.
  - I told them of how great the conversations were and how my students wrote good proofs, definitions, and even developed conjectures

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  The common question, *did your intervention work*, suddenly became much more complex.

Underlying terms

- Evaluation vs. Assessment

- Evaluation: final description of the quality/efficacy of the effort—e.g., Grades!

- Assessment: provides information for improving learning and teaching. Assessment is an interactive process between students and faculty that informs faculty how well their students are learning what they are teaching.

Underlying terms

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- Evaluation: final description of the quality/efficacy of the effort

Exams, homework, projects! … [we often call this assessment, and project rubrics are your friend! –want an example?]

- Assessment: provides information for improving learning and teaching. Assessment is an interactive process between students and faculty that informs faculty how well their students are learning what they are teaching.

--Way more than just exams and homework! Listening to students talk and writing down what they say, recording them, journaling or informal writing, taking pictures of stuff, talking to them, …

Different types of assessment

- Exploratory: How are students thinking about these ideas? How do they interact with the materials? What changes in how students think about the ideas?
- Standard- or objective-based: What proficiency do students demonstrate with X algorithm? Can students …?

Let me try for a relatively simple example…

- This is a micro-level assessment
  - A case study of the meaning that one professor and six of his students saw with one proof presentation
  - Goal is to generate hypotheses that can account for the phenomenon that students do not learn from a clear lecture

This study was done with *Kristen Lew, Keith Weber, and Juan Pablo Mejia-Ramos…*

Naturalistic case study of a real analysis lecture

- Case study– One professor (Dr. A) with 30 years experience and an excellent reputation as a real analysis instructor
- One 11-minute proof that a sequence $\{x_n\}$ with the property that $|x_n - x_{n+1}| < r^n$ for some $0 < r < 1$ is convergent
- Our aim:
  - Find out what Dr. A was trying to convey
  - See the extent to which six of Dr. A's students recognized that the content that Dr. A was trying to convey in his presentation

Methods- Lecture analysis by professor

- Instructor was audiotaped in an interview on lecture.
  - First asked to describe why he presented this proof to students
  - Then asked to stop the video recording at every point he thought he was trying to convey mathematical content

Methods- Lecture analysis by students

- Three student pairs were interviewed where we made four passes through the data.
- *Pass 1:* Students were asked to refer to their notes and state what they thought were the main ideas of the proof.
- *Pass 2:* Students watched the lecture again in its entirety, taking notes, and were asked the same question.
- *Pass 3:* Students were shown individual clips of the video and asked what they thought the professor was trying to convey.
- *Pass 4:* Students were told one thing that you might get from some proofs of this theorem was the content that Dr. A highlighted and asked if they got that from this proof.

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Results- Summary

Content conveyed By professor	Group #1	Group #2	Group #3
To show sequence is convergent without a limit candidate, show it is Cauchy	Pass 3	Pass 3	Never
Triangle inequality is important for proofs in real analysis	Pass 2	Pass 3	Pass 3
Geometric series in one’s “toolbox” for working with bounds and keeping quantities small	Never	Never	Never
How to set up a proof to show a sequence is Cauchy	Pass 4	Pass 2	Pass 4
Cauchy sequences can be thought of as “bunching up”	Pass 3	Pass 3	Pass 3

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Results: Cauchy heuristic

- At three points in the proof, Dr. A emphasized the Cauchy heuristic: if you want to show a sequence is convergent when you do not have a limit candidate, you can show it is Cauchy.

Dr. A: How can we proceed to show that this is a convergent sequence? Anybody have a guess?

Student: [Incomprehensible utterance]

Dr. A: Well that’s not quite the right term. What kind of sequences do we know converge even if we don’t know what their limits are? [pause] It begins in ‘c’.

Student: Cauchy.

Dr. A: Cauchy! We’ll show it’s a Cauchy sequence.

Different ways to ask questions

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This question is hard to answer without a direct comparison of pre- and post-intervention. You need a ‘control’ group with matched characteristics (e.g., pretest knowledge) and reasonably similar study habits (*unless you think your intervention will change work habits*)

- Instead, you might ask the following kinds of questions:
  - What is it that they’re supposed to learn?
  - How well have you specified it?
  - How can you ‘capture’ habits and processes?

The key to standards-based questions:

The more detail with which you specify the intended outcomes, the better you can argue that your intervention is successful *(or not)*

Action verbs are key! Watch out for words that imply you know what’s in a student’s head…

Let's operationalize objectives

Some questions and goals in David's and Elizabeth's talks:

- If we rotate 45 degrees, how does the angle change?
- If you change from z = x^2 - y^2 to something else, what changes?
- Realize thaf f is constant along lines y=kx
- Understand the two different models of (2x^2y)/(x^4+y2) illustrate approaching via parabolas and rays and the relationship and...?
- Understand a geometric interpretation of a mixed partial derivative...
- The space between y = x and y = x^2 is not what ***you*** think! And, not what we draw... [that other set of things] it's not what ***you*** think! Or, what you draw!
- What does it look like when you slice a torus?
- We ask that students become familiar with either Maple or Mathematica.

fundamental claim: you think differently

- If we want to write standards, what would we write?

A idea from TJ's talk

- TJ: here are some questions structures I've found helpful. The implication is that 'these might work for you too'
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  - What 'productive' means
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- How can we test whether these questions structures lead to productive conversation? What do we need to agree on?
  - What the 'question structures' are (Do we all agree what 'Generalize known content' means?)
  - What 'productive' means
  - What 'conversation' means (maybe? Maybe we mean something else)?
    - One idea seems to be, "can students X" where X might be 'develop conjectures' or 'develop definitions' but even then, we need to agree on what counts as a definition!

But, what would the 'exploratory' assessment look like?

What did the students say they learned in Pass 2?

- Cauchy sequences will be on the mid-term (Pair 1)
- The triangle inequality is important in real analysis (Pair 1)
- There is a consistent format to follow when writing a proof that a sequence is convergent (Pair 2)
- Dr. A expanded the toolbox for simplifying expressions (Pair 2)
- You can use prior knowledge from courses like calculus when writing proofs in real analysis (Pairs 1, 2, and 3)

Big ideas—when asking, “does it work?” you’ll want to think about:

- What’s your actual intervention?
  - What are the components?
  - If someone else was going to take it up, what would you ask that they do?
  - What freedom does the other person have?
- What do you mean by ‘work’?
  - What are the intended outcomes? [How can you capture habits?]
  - What are your measures? Have other people used them? Does the field agree on them? [e.g., The Calc Concept Inventory?*]
- When you ‘test’ it, how much do you change the other parts of your teaching practice?

A larger-scale example

- With colleagues I developed a *Real Analysis for Teachers* course. Our course goals included:
  - Using actual classroom situations to motivate the real instruction
    - This implies that we can prompt significant mathematical and pedagogical conversations
  - Changing teacher's perception of the utility of real analysis (they think it useless)
  - Helping students to learn more about secondary content and real analysis, and to be able to describe connections between them.

A slide about the intervention

- We developed detailed lesson plans, including statements of learning goals for each lesson, as well as anticipated student responses to prompts (we'll share!). From the cover page

Lesson 1

REAL ANALYSIS Content

Theorem 1.2.6. Two real numbers *a* and *b* are equal iff for every $\varepsilon > 0$ it follows that $|a-b| < \varepsilon$.

SECONDARY Mathematics

State and explain the structure of real numbers represented as infinite decimals – i.e., is simply a statement of equivalence like $^1/_2 = {}^2/_4$.

PEDAGOGICAL AIM – Secondary teaching practice

Principle of Good Teaching 4. Select examples that exemplify nuances within and boundaries around a mathematical idea – in this case, around real numbers expressed as decimals.

Principle of Good Teaching 2. Clarify implicit assumptions and mathematical limitations in *students'* mathematical statements or arguments – in this case, about real numbers expressed as decimals.

What did we learn?

- We engaged in standards-based assessment. For example, this summer I read transcripts from every initial pedagogical conversation and looked for instances where the students did or did not engage in mathematical and pedagogical conversations.

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- We engaged in standards-based assessment. For example, this summer I read transcripts from every initial pedagogical conversation and looked for instances where the students did or did not engage in mathematical and pedagogical conversations.
  - But, what if we don't agree on what *significant* is?
  - How do we evaluate whether we *changed* their beliefs about real analysis?

What else did we learn?

We also engaged in exploratory data analysis:

- *How did the students engage with the pedagogical situations*? How did the students reason about mathematical questions and ideas like quantification (e.g., looking at instances of negating quantified statements)?
  - They spent a lot of time criticizing aspects of teaching practice that we did not anticipate. They critiqued expressions, they critiqued phrasings that didn't change the mathematical meaning, and, most importantly, we overestimated their knowledge of the school mathematics content and so they couldn't *mathematically evaluate* some of the things that we portrayed.

What else did we learn?

We also engaged in exploratory data analysis:

- How did the students engage with the pedagogical situations? *How did the students reason about mathematical questions and ideas like quantification (e.g., looking at instances of negating quantified statements)*?
  - We just didn't know what they'd do! So, we had to give them tasks and wait to see what they did well and what they struggled with.

How could we tell

- If we changed their beliefs about the utility of real analysis for teaching practice?
- If we’ve changed their teaching practices?
- If we’ve changed their knowledge of secondary content?
- If they can discuss connections between secondary content and real analysis?

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We can't!

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So we revised our questions, for example, instead we asked, "what are their beliefs" and compared with prior groups.

We found out that they considered *everything* we did during the course real analysis, even the secondary content and pedagogical situations!

As a reminder, when asking, "does it work?" you'll want to think about:

- What's your actual intervention?
  - What are the components?
  - If someone else was going to take it up, what would you ask that they do?
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- What do you mean by 'work'?
  - What are the intended outcomes? [How can you capture habits?]
  - What are your measures? Have other people used them? Does the field agree on them? [e.g., The Calc Concept Inventory?*]

- How do your students engage? Reason? Think/talk about the ideas that you're trying to convey?

Thank you

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Some papers discussed in this talk can be found at:
pcrg.gse.rutgers.edu

Some can be had in draft form by emailing me…

Results:
Cauchy heuristic

- At three points in the proof, Dr. A emphasized the Cauchy heuristic: if you want to show a sequence is convergent when you do not have a limit candidate, you can show it is Cauchy.

Dr. A: We will show that this sequence converges by showing that it is a Cauchy sequence [writes this sentence on the board as he says it aloud, then turns around to face class]. A Cauchy sequence is defined without any mention of limit.

Results: Cauchy heuristic

- At three points in the proof, Dr. A emphasized the Cauchy heuristic: if you want to show a sequence is convergent when you do not have a limit candidate, you can show it is Cauchy.

And now we'll state what it is we have to show. We will show that there is an N-epsilon for which x_n minus x_m would be less than epsilon when m and n are greater than this number N-epsilon. [Dr. A writes this sentence on the board as he says it aloud] This is how we prove it is a Cauchy sequence. [Turns around and faces class]. See there is no mention of how the terms of the sequence are defined. There is no way in which we would be able to propose a limit L. So we have no way of proceeding except for showing that it is a Cauchy sequence or a contractive sequence. So let's look and see how we proceed.

Results: Cauchy heuristic

- Our research team highlighted the Cauchy heuristic as the main point of presenting this proof.
- The other real analysis instructor described this as “the main objective”.
- Dr. A highlighted these three excerpts where he was trying to convey important content
- No student mentioned this in Pass 1 or Pass 2.

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