Predicting Monsoon Intraseasonal Precipitation using a Low-Order Nonlinear Stochastic Model

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Introduction

Monsoon Intraseasonal Oscillation (MISO)

- the prominent mode of tropical intraseasonal variability in boreal summer
- propagating northeastward
- strongly associated with the boreal summer monsoon rainfall over south Asia
- interactions with El Niño ...

Prediction of the MISO

1. operational/dynamical models
   - capturing more refined structures
   - computationally expensive
2. low-order statistical models
   - capturing only large-scale features
   - cheaper and more accurate

Procedure of prediction with low-order models

1. developing effective MISO indices (e.g, a few PCs of the high dim raw data)
2. designing low-order models to predict the MISO indices
3. spatiotemporal reconstruction (indices + the associated spatial bases)

Some existing indices for real-time monitoring and forecast of the MISO

- EEOF on longitudinal averaged rainfall anomalies (Suhas et al 2013; Sahai et al. 2013)
- multivariate EOF on surface zonal winds and outgoing longwave radiation (Lee et al. 2013)
- other EOF and EEOF methods ... (Kikuchi et al. 2012; Goswami et al. 1999)

Common features of these covariance-based approaches

- ✓ in general capturing the spatiotemporal MISO patterns reasonably well
- ✓ recovering the northeastward propagating intraseasonal periodicity
- × ad hoc seasonal extraction and longtitudinal averaging leading to loss of predictive information
- × sometimes mixing with other modes due to the nonlinear nature
- × potential inadequacy in capturing the rare/extreme events

EEOF: extended empirical orthogonal function

A new MISO index based on NLSA

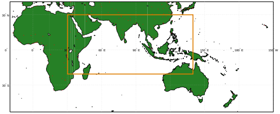
Novel time series technique: **Nonlinear Laplacian Spectrum Analysis (NLSA).**

- combining lagged embedding, machine learning, adaptive weights, spectral entropy criteria (Giannakis & Majda, PNAS 2012). applying to data with huge dimensions.

Advantages of NLSA over classical covariance-based techniques

- objective – by design no ad hoc detrending or spatiotemporal filtering of the full data set
- capturing both intermittency and low frequency variability
- higher memory and predictability in the NLSA MISO modes

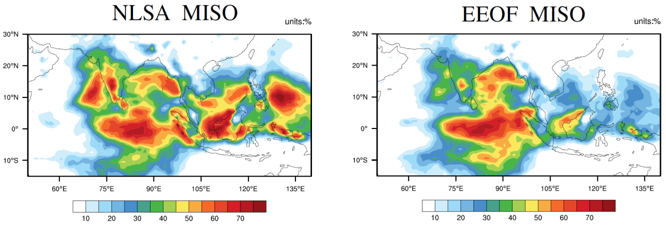
Dataset: the daily Global Precipitation Climatology Project (GPCP) rainfall data (Huffman et al. 2001) over the Asian summer monsoon region for period 1997-2014 (with spatial resolution $1^o \times 1^o$).



Apply NLSA to GPCP dataset with a lagged embedding of $q = 64$ days.
(Sabeerali et al, Climate Dynamics, 2016)

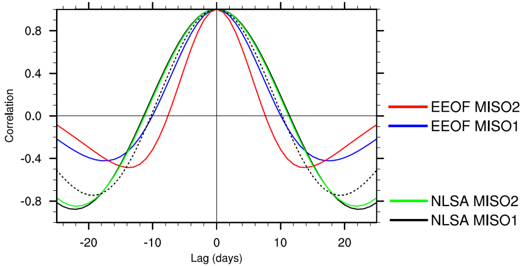
Comparison of MISO modes associated with the NLSA and EEOF

- fractional variance of rainfall anomalies



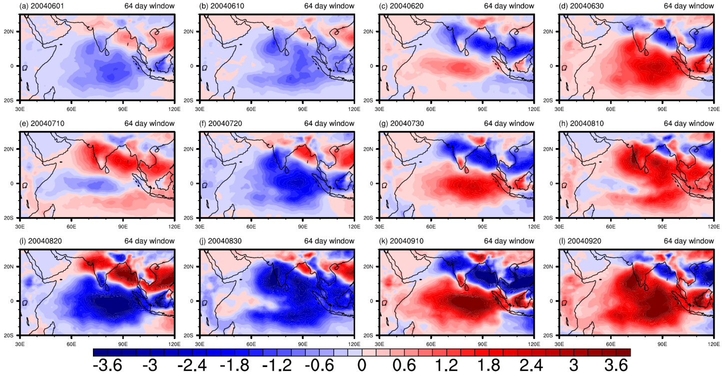
Capturing the variability over Indo-West Pacific is extremely important in determining the propagation features of MISO (Pillai & Sahai 2015).

- autocorrelations of the NLSA and EEOF indices



The NLSA MISO modes

- Reconstruction of MISO evolution from June 2004 to September 2004.



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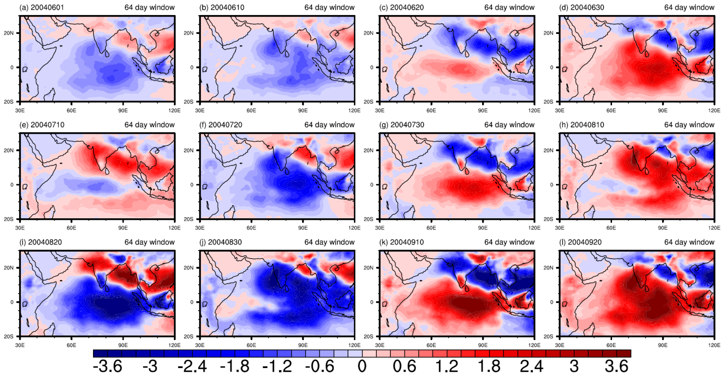
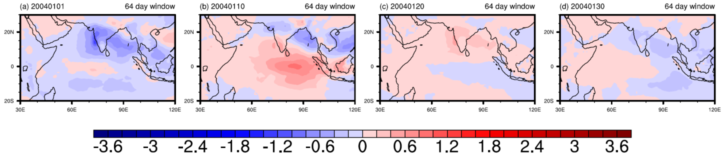
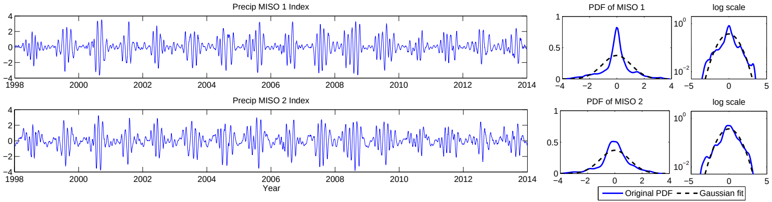


- Reconstruction of MISO evolution for January 2004.



Predicting the MISO indices via a low-order stochastic model



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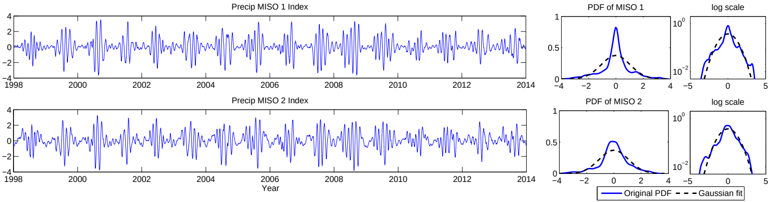


Low-Order Stochastic Model

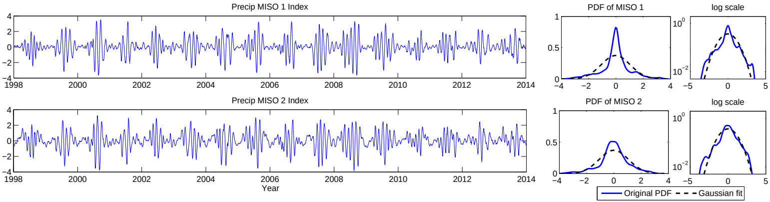
$$du_1 = (-d_u(t)\, u_1 \qquad - \hat{\omega}\, u_2)\, dt + \sigma_u\, dW_{u_1},$$
$$du_2 = (-d_u(t)\, u_2 \qquad + \hat{\omega}\, u_1)\, dt + \sigma_u\, dW_{u_2},$$

with

$$d_u(t) = d_{u0} + d_{u1} \sin(\omega_f t + \phi).$$

- Observed variables u_1, u_2: MISO 1 and MISO 2 indices from NLSA.

Predicting the MISO indices via a low-order stochastic model



Low-Order Stochastic Model

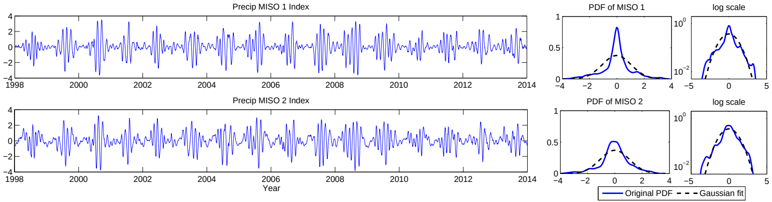
$$
\begin{aligned}
du_1 &= (-d_u(t)\, u_1 + \gamma\, v\, u_1 - \omega\, u_2)\, dt + \sigma_u\, dW_{u_1},\\
du_2 &= (-d_u(t)\, u_2 + \gamma\, v\, u_2 + \omega\, u_1)\, dt + \sigma_u\, dW_{u_2},\\
dv &= (-d_v\, v)\, dt + \sigma_v\, dW_v,\\
d\omega &= (-d_\omega \omega + \hat{\omega})\, dt + \sigma_\omega\, dW_\omega,
\end{aligned}
$$

with

$$d_u(t) = d_{u0} + d_{u1} \sin(\omega_f t + \phi).$$

- Observed variables u_1, u_2: MISO 1 and MISO 2 indices from NLSA.
- Hidden variables v, ω: stochastic damping and stochastic phase.

Predicting the MISO indices via a low-order stochastic model



Physics-Constrained Low-Order Stochastic Model

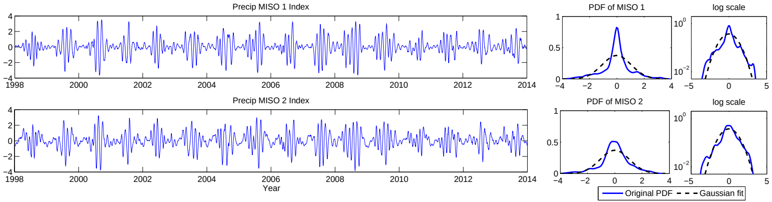
$$du_1 = (-d_u(t)\, u_1 + \gamma\, v\, u_1 - \omega\, u_2)\, dt + \sigma_u\, dW_{u_1},$$
$$du_2 = (-d_u(t)\, u_2 + \gamma\, v\, u_2 + \omega\, u_1)\, dt + \sigma_u\, dW_{u_2},$$
$$dv = (-d_v\, v - \gamma\, (u_1^2 + u_2^2))\, dt + \sigma_v\, dW_v,$$
$$d\omega = (-d_\omega \omega + \hat{\omega})\, dt + \sigma_\omega\, dW_\omega,$$

with

$$d_u(t) = d_{u0} + d_{u1}\sin(\omega_f t + \phi).$$

- Observed variables u_1, u_2: MISO 1 and MISO 2 indices from NLSA.
- Hidden variables v, ω: stochastic damping and stochastic phase.
- Energy-conserving nonlinear interactions between (u_1, u_2) and (v, ω).

Predicting the MISO indices via a low-order stochastic model



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d\omega &= (-d_\omega \omega + \hat{\omega})\, dt + \sigma_\omega\, dW_\omega,
\end{aligned}
$$

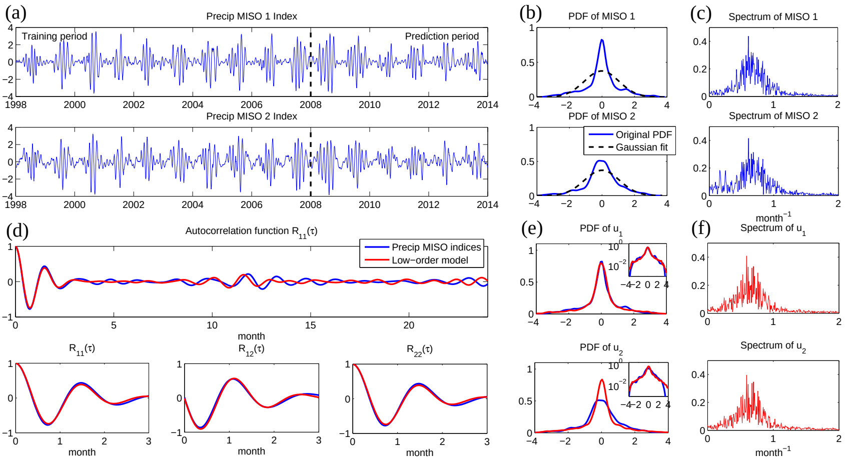
with

$$
d_u(t) = d_{u0} + d_{u1} \sin(\omega_f t + \phi).
$$

- Observed variables u_1, u_2: MISO 1 and MISO 2 indices from NLSA.
- Hidden variables v, ω: stochastic damping and stochastic phase.
- Energy-conserving nonlinear interactions between (u_1, u_2) and (v, ω).
- Effective data assimilation algorithm to determine the initial values of (v, ω) that facilitate the ensemble prediction scheme (Liptser & Shiryaev 2001).

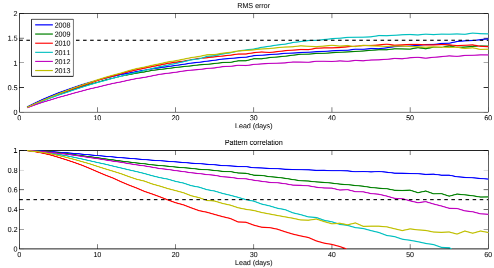
Model Calibration

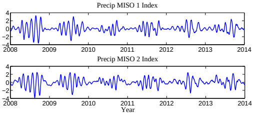
Calibration of parameters using *Information Theory* (Robust parameters)
Model vs. Observations: Non-Gaussian statistics match

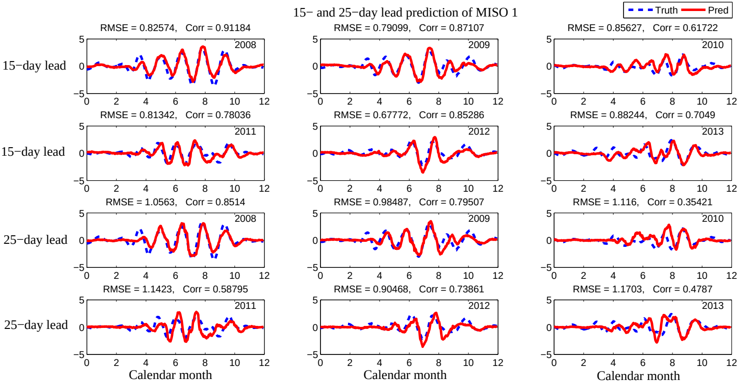


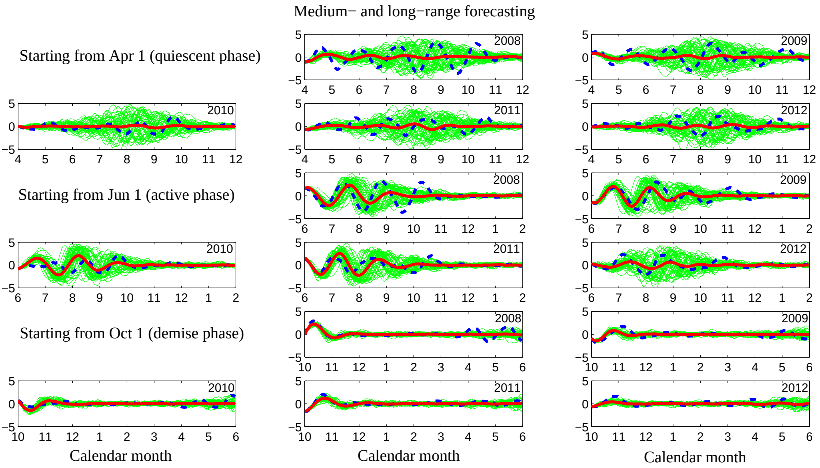
(Linear models fail to accurately match the statistics.)

Prediction of the MISO indices





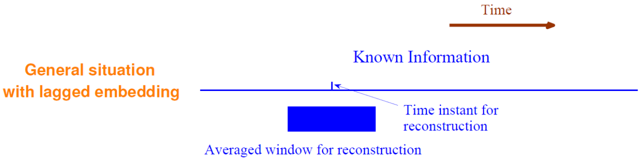




ensemble spread ⇔ forecast uncertainty

Spatiotemporal reconstruction

1. spatial basis × time series
2. averaging over the lagged embedding window



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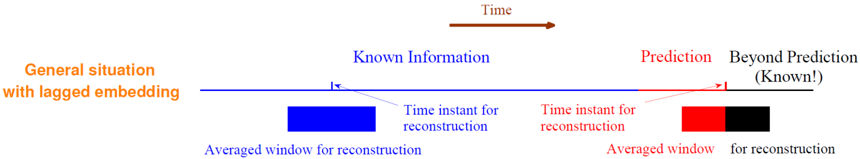

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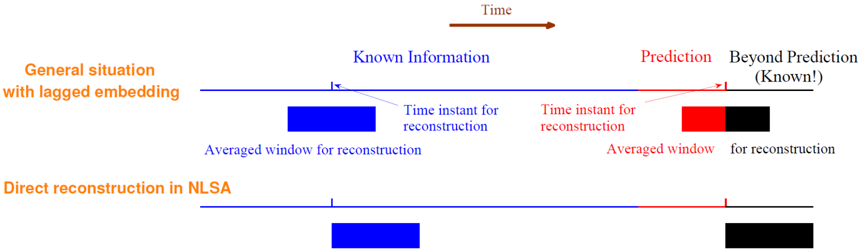
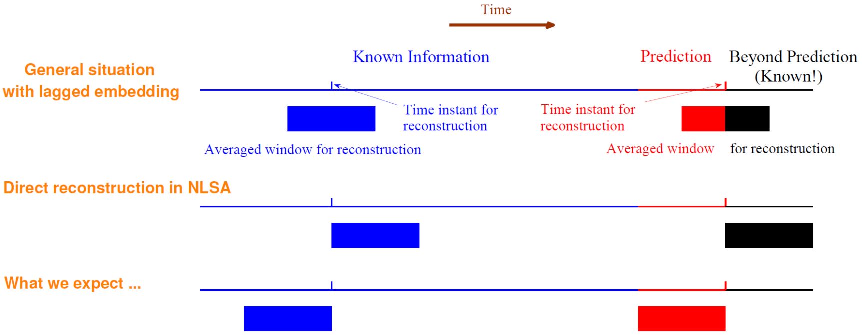


$$
\begin{pmatrix}
z_1 & z_2 & \cdots & z_{N-q+1} & z_{N-q+2} & \cdots & z_{N-1} & \boxed{z_N} \\
z_2 & z_3 & \cdots & z_{N-q+2} & z_{N-q+3} & \cdots & \boxed{z_N} & z_{N+1} \\
\vdots & \vdots & \ddots & \vdots & \cdots & \cdots & \cdots & \vdots \\
z_{q-1} & z_q & \cdots & z_{N-1} & \boxed{z_N} & \cdots & z_{N+q-3} & z_{N+q-2} \\
z_q & z_{q+1} & \cdots & \boxed{z_N} & z_{N+1} & \cdots & z_{N+q-2} & z_{N+q-1}
\end{pmatrix}
$$

Spatiotemporal reconstruction

1. spatial basis × time series
2. averaging over the lagged embedding window



$$
\begin{pmatrix}
z_1 & z_2 & \cdots & z_{N-q+1} & z_{N-q+2} & \cdots & z_{N-1} & \boxed{z_N} \\
z_2 & z_3 & \cdots & z_{N-q+2} & z_{N-q+3} & \cdots & \boxed{z_N} & z_{N+1} \\
\vdots & \vdots & \ddots & \vdots & \cdots & \cdots & \cdots & \vdots \\
z_{q-1} & z_q & \cdots & z_{N-1} & \boxed{z_N} & \cdots & z_{N+q-3} & z_{N+q-2} \\
z_q & z_{q+1} & \cdots & \boxed{z_N} & z_{N+1} & \cdots & z_{N+q-2} & z_{N+q-1}
\end{pmatrix}
$$

More details of the direct method ...

$$A \cdot [\Phi; \Phi^f] = [X; X^f].$$

A: spatial basis Φ: time series X: spatiotemporal modes $\cdot^f$: forecast

$$A\cdot\left[\Phi;(\Phi^f)\right] = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_q \end{pmatrix} \cdot \begin{pmatrix} \Phi_1, \cdots, & \Phi_N, & \Phi_1^f, & \Phi_2^f, & \cdots, & \Phi_q^f \end{pmatrix}$$

$$= \left(\begin{array}{cccc|ccccc} A_1\Phi_1 & A_1\Phi_2 & \cdots & A_1\Phi_N & A_1\Phi_1^f & A_1\Phi_2^f & \cdots & A_1\Phi_{q-1}^f & \boxed{A_1\Phi_{\mathbf{q}}^f} \\ A_2\Phi_1 & A_2\Phi_2 & \cdots & A_2\Phi_N & A_2\Phi_1^f & A_2\Phi_2^f & \cdots & \boxed{A_2\Phi_{\mathbf{q-1}}^f} & A_2\Phi_q^f \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{q-1}\Phi_1 & A_{q-1}\Phi_2 & \cdots & A_{q-1}\Phi_N & A_{q-1}\Phi_1^f & \boxed{A_{q-1}\Phi_{\mathbf{2}}^f} & \cdots & A_{q-1}\Phi_{q-1}^f & A_{q-1}\Phi_q^f \\ A_q\Phi_1 & A_q\Phi_2 & \cdots & A_q\Phi_N & \boxed{A_q\Phi_{\mathbf{1}}^f} & A_q\Phi_2^f & \cdots & A_q\Phi_{q-1}^f & A_q\Phi_q^f \end{array}\right),$$

$$[X; X^f]: \left(\begin{array}{cccc|ccccc} z_1 & z_2 & \cdots & z_{n-2q+1} & z_{n-2q+2} & z_{n-2q+3} & \cdots & z_{n-q} & \boxed{z_1^f} \\ z_2 & z_3 & \cdots & z_{n-2q+2} & z_{n-2q+3} & z_{n-2q+4} & \cdots & \boxed{z_1^f} & z_2^f \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ z_{q-1} & z_q & \cdots & z_{n-q+1} & z_{n-q} & \boxed{z_1^f} & \cdots & z_{q-1}^f & z_{q-1}^f \\ z_q & z_{q+1} & \cdots & z_{n-q} & \boxed{z_1^f} & z_2^f & \cdots & z_q^f & z_q^f \end{array}\right),$$

Predicting $z_{\mathbf{1}}^f$ requires the information up to $\phi_{\mathbf{q}}^f$!

Direct Reconstruction:

Training period

$$X = A\Phi^T \quad \Longrightarrow \quad A = cX\Phi$$

Prediction period

$$A \cdot [\Phi; \Phi^f] = [X; X^f]$$

$$X = \begin{pmatrix} z_1 & z_2 & \cdots & z_{n-2q+1} \\ z_2 & z_3 & \cdots & z_{n-2q+2} \\ \vdots & \vdots & \ddots & \vdots \\ z_{q-1} & z_q & \cdots & z_{n-q-1} \\ z_q & z_{q+1} & \cdots & z_{n-q} \end{pmatrix}$$

An improved method:

Training period

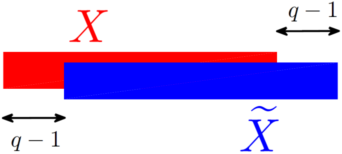
$$\tilde{X} = \tilde{A}\Phi^T \quad \Longrightarrow \quad \tilde{A} = c\tilde{X}\Phi$$

Prediction period

$$\tilde{A} \cdot [\Phi; \Phi^f] = [\tilde{X}, \tilde{X}^f]$$

$$\widetilde{X} = \begin{pmatrix} z_q & z_{q+1} & \cdots & z_{n-q} \\ z_{q+1} & z_{q+2} & \cdots & z_{n-q+1} \\ \vdots & \vdots & \ddots & \vdots \\ z_{2q-2} & z_{2q-1} & \cdots & z_{n-2} \\ z_{2q-1} & z_{2q} & \cdots & z_{n-1} \end{pmatrix}$$

$\tilde{A}$ is computed using only the training data.



More details of the improved method ...

$$\tilde{A}\cdot[\Phi;\Phi^f] = [\tilde{X},\tilde{X}^f].$$

$\tilde{A}$: spatial basis Φ: time series $\tilde{X}$: spatiotemporal modes $\cdot^f$: forecast

$$\widetilde{A}\cdot[\Phi;\Phi^f] = \begin{pmatrix}\widetilde{A}_1\\ \widetilde{A}_2\\ \vdots\\ \widetilde{A}_q\end{pmatrix}\cdot\left(\Phi_1,\quad \cdots,\quad \Phi_{N-q},\ \middle|\ \Phi_{N-q+1},\quad \cdots,\quad \Phi_N,\ \middle|\ \Phi^f_1,\quad \Phi^f_2,\quad \cdots,\quad \Phi^f_q\right)$$

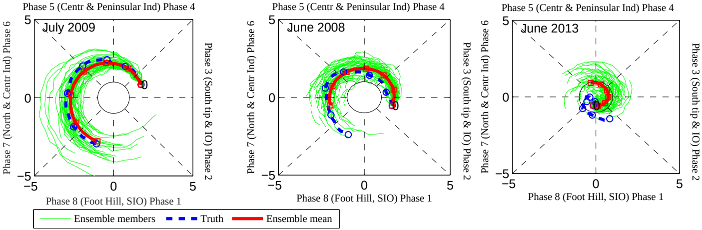
$$= \left(\begin{array}{ccc|cccc|ccc}
\widetilde{A}_1\Phi_1 & \cdots & \widetilde{A}_1\Phi_{N-q} & \widetilde{A}_1\Phi_{N-q+1} & \widetilde{A}_1\Phi_{N-q+2} & \cdots & \widetilde{A}_1\Phi_N & \boxed{\widetilde{A}_1\Phi^f_1} & \cdots & \widetilde{A}_1\Phi^f_q \\
\widetilde{A}_2\Phi_1 & \cdots & \widetilde{A}_2\Phi_{N-q} & \widetilde{A}_2\Phi_{N-q+1} & \widetilde{A}_2\Phi_{N-q+2} & \cdots & \boxed{\widetilde{A}_2\Phi_N} & \widetilde{A}_2\Phi^f_1 & \cdots & \widetilde{A}_2\Phi^f_q \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \\
\widetilde{A}_{q-1}\Phi_1 & \cdots & \widetilde{A}_{q-1}\Phi_{N-q} & \widetilde{A}_{q-1}\Phi_{N-q+1} & \widetilde{A}_{q-1}\Phi_{N-q+2} & \cdots & \widetilde{A}_{q-1}\Phi_N & \widetilde{A}_{q-1}\Phi^f_1 & \cdots & \widetilde{A}_{q-1}\Phi^f_q \\
\widetilde{A}_q\Phi_1 & \cdots & \widetilde{A}_q\Phi_{N-q} & \widetilde{A}_q\Phi_{N-q+1} & \boxed{\widetilde{A}_q\Phi_{N-q+2}} & \cdots & \widetilde{A}_q\Phi_N & \widetilde{A}_q\Phi^f_1 & \cdots & \widetilde{A}_q\Phi^f_q
\end{array}\right),$$

$$[\widetilde{X};\widetilde{\widetilde{X}}]: \left(\begin{array}{ccc|cccc|ccc}
z_q & \cdots & z_{n-2q} & z_{n-2q+1} & z_{n-2q+2} & \cdots & z_{n-q} & \boxed{z^f_1} & \cdots & z^f_{q-1} \\
z_{q+1} & \cdots & z_{n-2q+1} & z_{n-2q+2} & z_{n-2q+3} & \cdots & \boxed{z^f_1} & z^f_2 & \cdots & z^f_q \\
\vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
z_{2q-2} & \cdots & z_{n-q} & z_{n-q+1} & z_{n-q} & \cdots & z^f_{q-1} & z^f_{q-1} & \cdots & z^f_{2q-2} \\
z_{2q-1} & \cdots & z_{n-q-1} & z_{n-q} & \boxed{z^f_1} & \cdots & z^f_q & z^f_q & \cdots & z^f_{2q-1}
\end{array}\right).$$

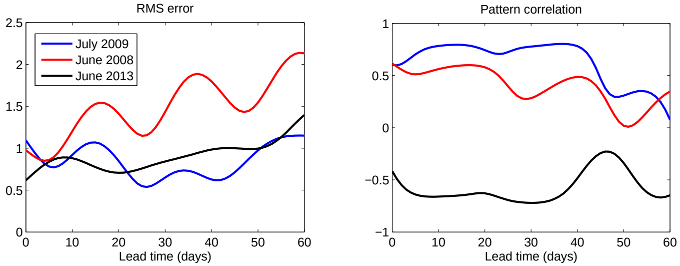
Predicting $z^f_{\mathbf{1}}$ only requires the information only up to $\phi^f_{\mathbf{1}}$.

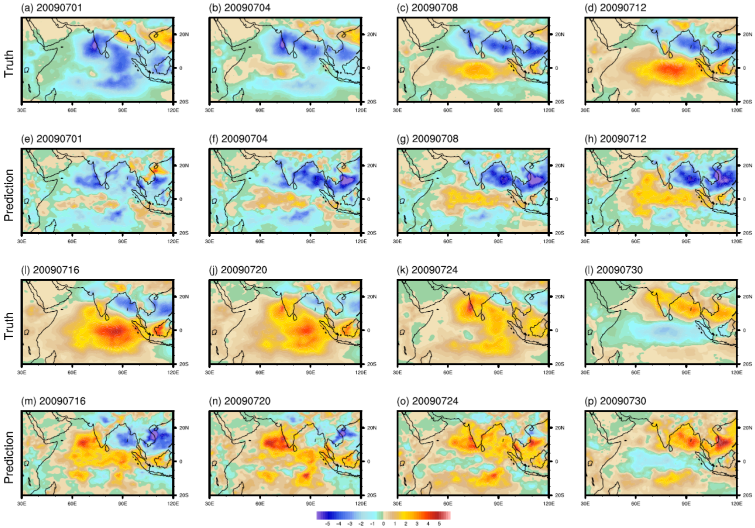
Results: Spatiotemporal reconstruction

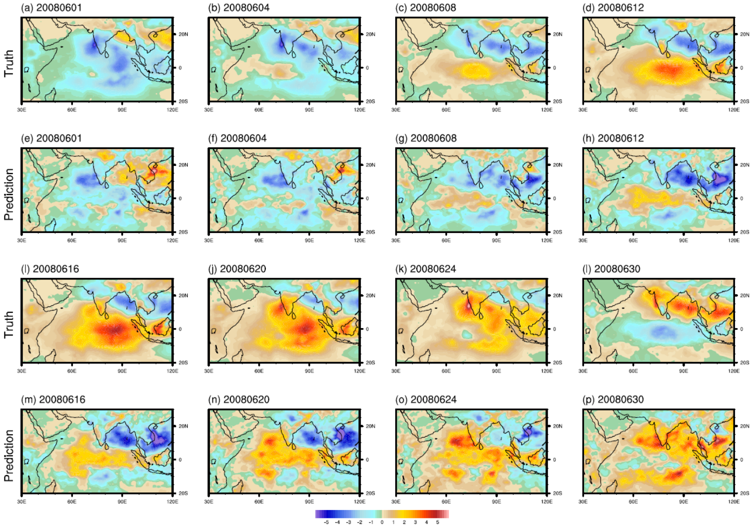
(a) Phase diagrams of MISO indices and predictions



(b) Skill scores for predicting the reconstructed spatiotempheral patterns

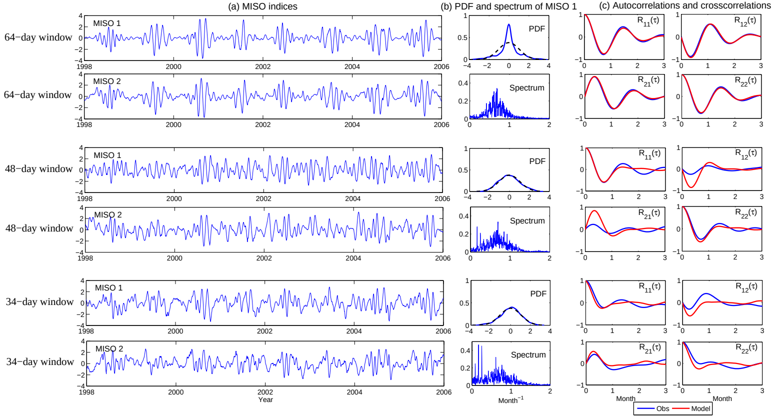


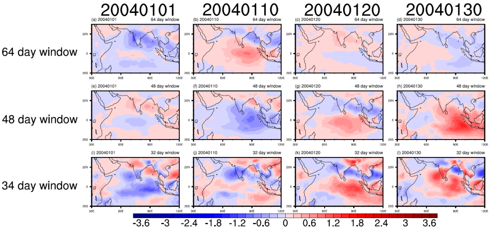

(a) 20090701
(b) 20090704
(c) 20090708
(d) 20090712
Truth
(e) 20090701
(f) 20090704
(g) 20090708
(h) 20090712
Prediction
(i) 20090716
(j) 20090720
(k) 20090724
(l) 20090730
Truth
(m) 20090716
(n) 20090720
(o) 20090724
(p) 20090730
Prediction
30E 60E 90E 120E
20N 0 20S
-5 -4 -3 -2 -1 0 1 2 3 4 5


(a) 20080601
(b) 20080604
(c) 20080608
(d) 20080612
Truth
(e) 20080601
(f) 20080604
(g) 20080608
(h) 20080612
Prediction
(i) 20080616
(j) 20080620
(k) 20080624
(l) 20080630
Truth
(m) 20080616
(n) 20080620
(o) 20080624
(p) 20080630
Prediction
30E
60E
90E
120E
20N
0
20S
-5 -4 -3 -2 -1 0 1 2 3 4 5

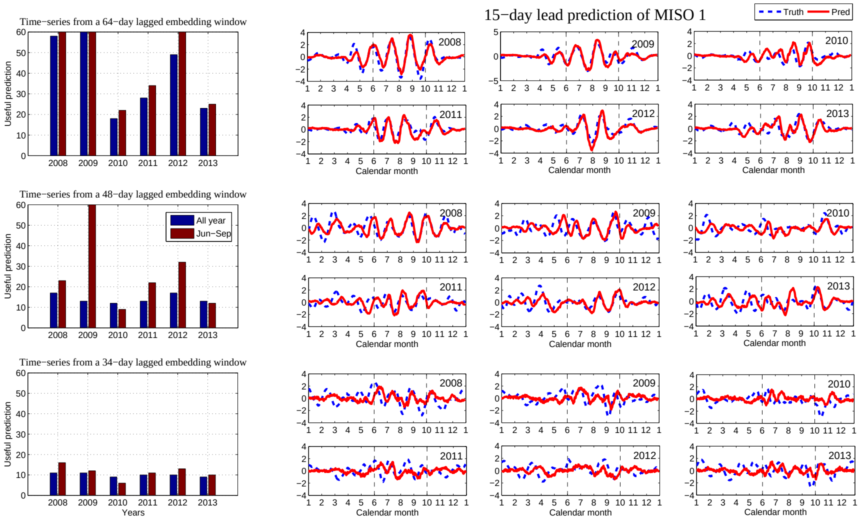
- Qualitative similar skill is found in the spatiotemporal reconstruction as in the predicted time series.
- Error in the spatiotemporal reconstruction does not vanishing at very short terms. This is due to the error in the approximation of the “predicted” spatial basis $\tilde{A}$.
- The “predicted” spatial basis $\tilde{A}$ here is stationary.
  Clustering methods are potentially promising techniques for recovering more detailed features of spatial basis conditioned on different phases.

MISO modes with different lagged embedding window sizes





Prediction of the MISO indices with different lagged embedding window sizes



Summary

- NLSA is applied to the precipitation dataset to obtain the MISO modes.
- A physics-constrained nonlinear low-order stochastic model is applied to predict the MISO indices.
- An improved spatiotemporal reconstruction method is develop that leads to practical predictions.
- A lagged embedding window with intraseasonal time length in NLSA is important in capturing the key features of MISO.

Nan Chen*, Andrew J. Majda, C. T. Sabeer, R. S. Ajayamohan, Predicting Monsoon Intraseasonal Precipitation using a Low-Order Nonlinear Stochastic Model, *Climate Dynamics*, 2017

Thank you

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