

Biofluids of reproduction: oscillators, viscoelastic networks and sticky situations.

AWM-SIAM
July 11, 2016

Lisa J. Fauci
Tulane University, Math Dept.
New Orleans, Louisiana, USA

Collaborators:

Susan Suarez

Cornell University

Robert Dillon

Washington State Univ.

Sarah Olson

Worcester Polytechnic

Julie Simons

Cal State Maritime

Ricardo Cortez

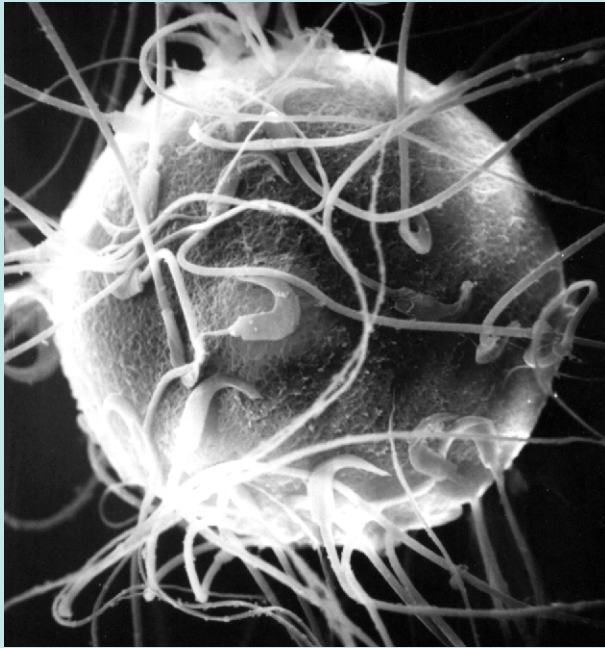
Tulane University

Jacek Wrobel

Raytheon



Reproduction – No better illustration of complex fluid-structure interactions



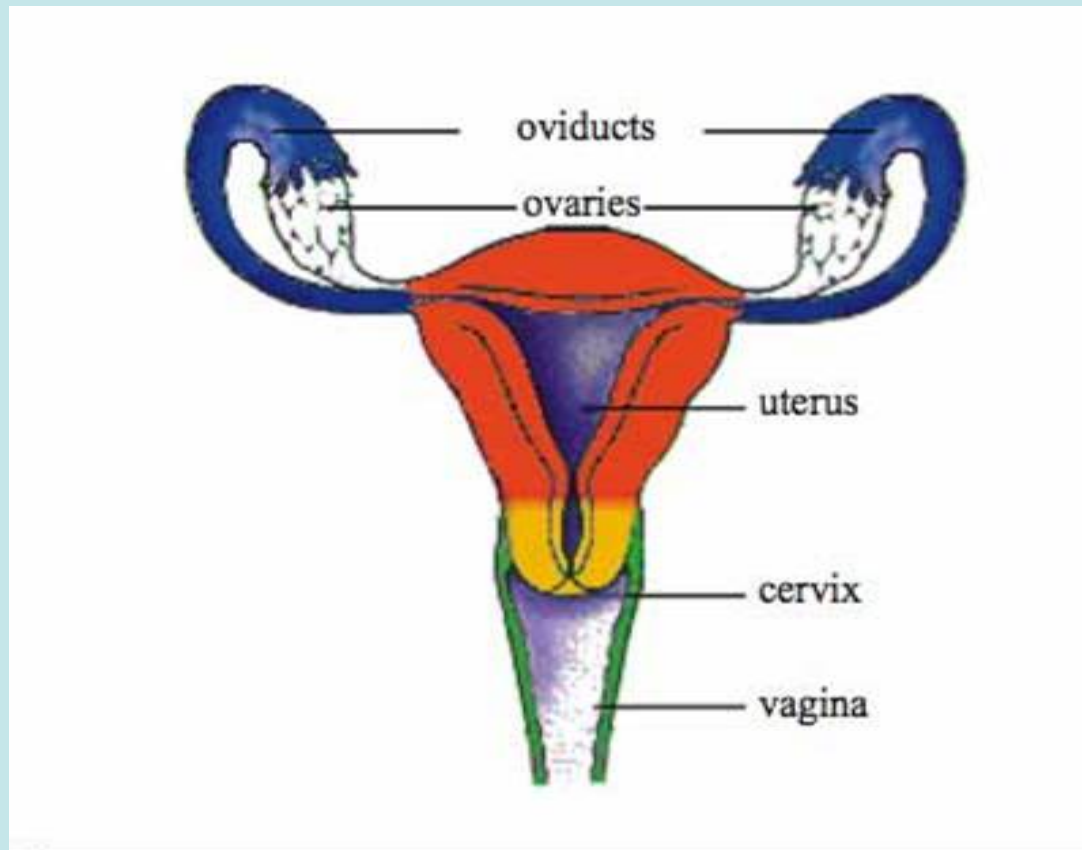
ZP –glycoprotein layer
surrounding oocyte

Fauci and Dillon,
Ann. Rev. Fluid Mech.,
Vol. 38, 2006

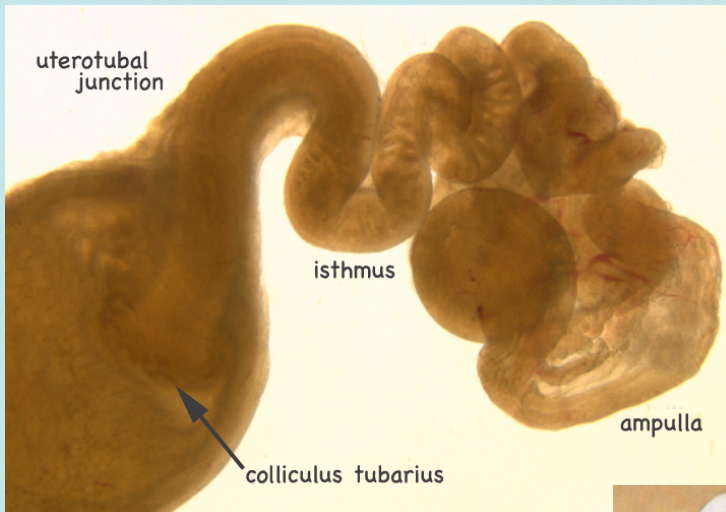
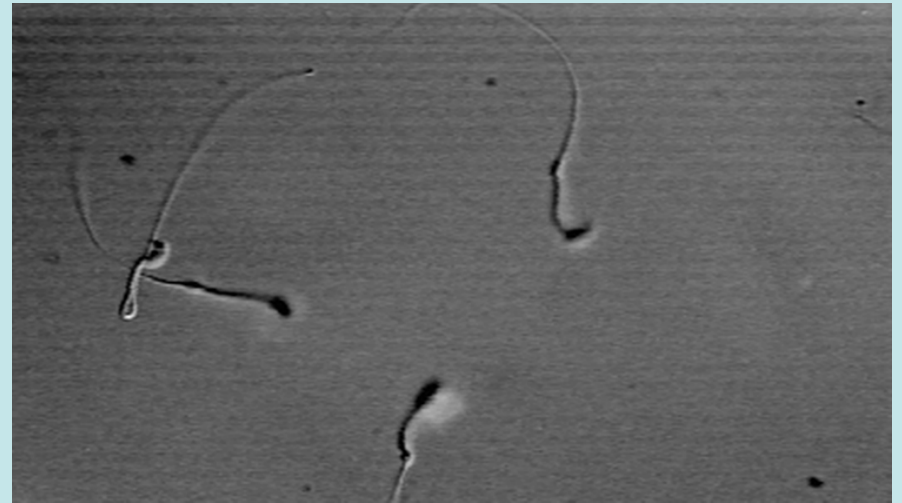
A scanning electron
micrograph of hamster
sperm bound to a
zona pellucida.

courtesy of
P. Talbot, Cell Biol.
UC Riverside

- Transport of sperm to site of fertilization
- Transport of oocyte cumulus complex (OCC) to oviduct
- Transport and implantation of embryo in uterus

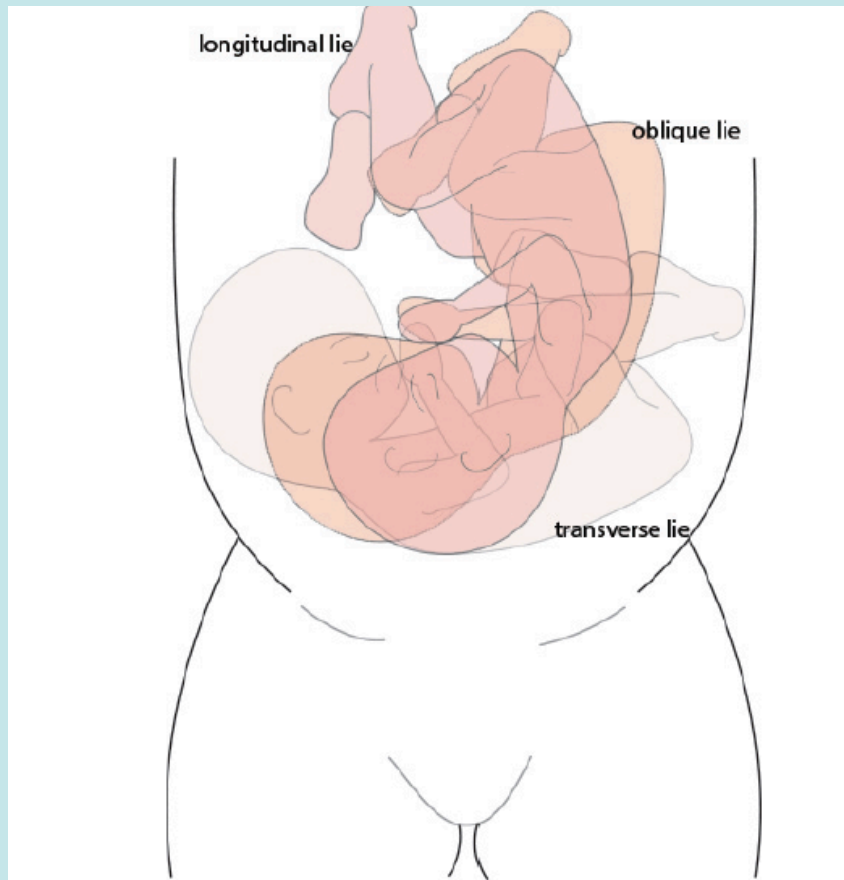


- Motile spermatozoa
- Muscular contractions
- Ciliary beating



Courtesy: Susan Suarez,
Cornell U.

Human birth: What forces are experienced by the fetus?
How does the force depend upon fetal lie (angle)?
How does the force depend upon fluid properties?
What are the fluid properties?



Megan Leftwich, GWU
Alexa Baumer, GWU
Roseanna Pealatore, Tulane

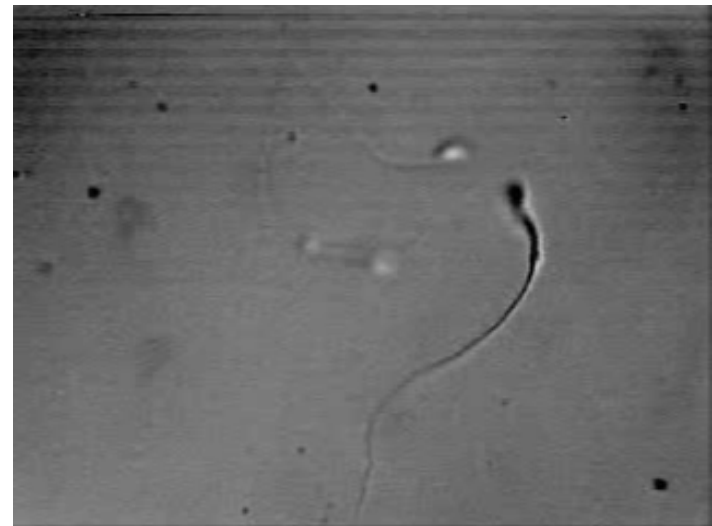
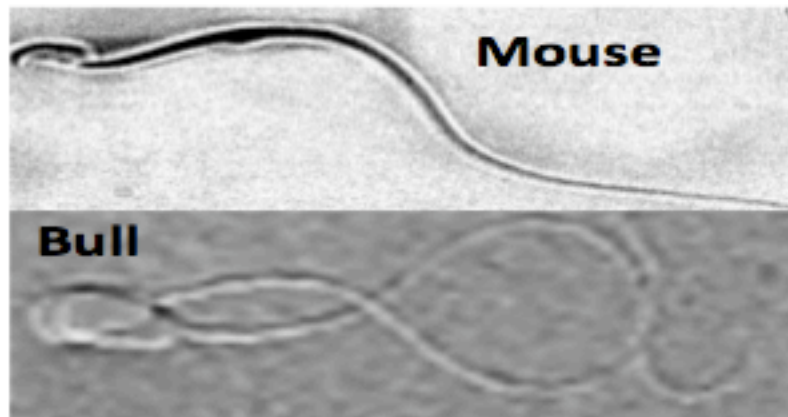
Today's tour:

- **Sperm motility/hyperactivation**
- Viscoelastic networks
- Human birth

Types of Sperm Motility

Activated motility

- Symmetrical flagellar bending
- Linear trajectories
- Resting level of Ca^{2+}



Movie courtesy of S. Suarez

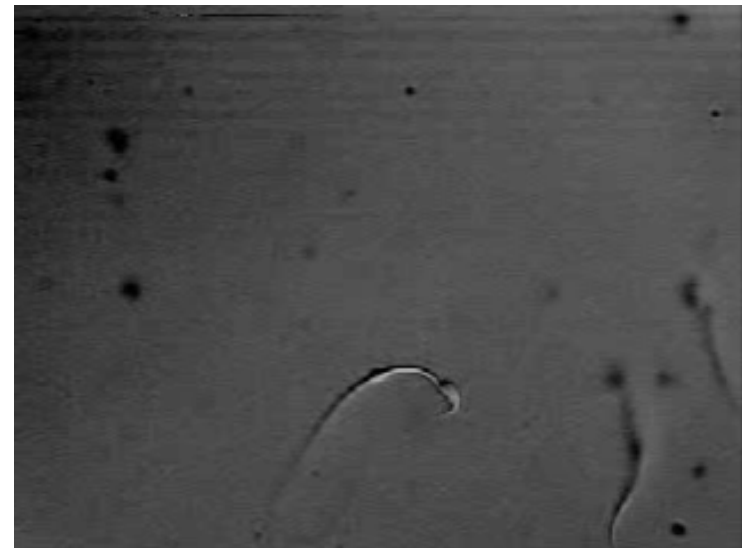
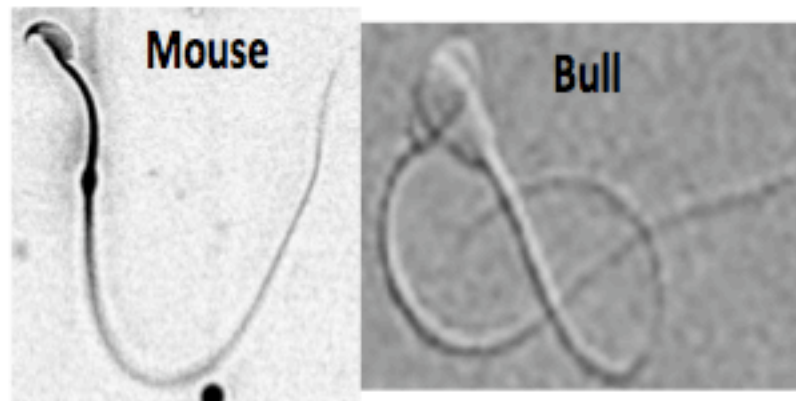
Marquez and Suarez. Different signaling pathways in bovine sperm regulate capacitation and hyperactivation, *Biol Reprod* 70 (2004) 1626–1633.

Chang and Suarez. Rethinking the relationship between hyperactivation and chemotaxis in mammalian sperm, *Biol Reprod* 83 (2010) 507–513.

Types of Sperm Motility

Hyperactivated motility

- Highly asymmetrical flagellar bending
- Increased curvature, amplitude
- Increased Ca^{2+}



Movie courtesy of S. Suarez

Marquez and Suarez. Different signaling pathways in bovine sperm regulate capacitation and hyperactivation, *Biol Reprod* 70 (2004) 1626–1633.

Chang and Suarez. Rethinking the relationship between hyperactivation and chemotaxis in mammalian sperm, *Biol Reprod* 83 (2010) 507–513.

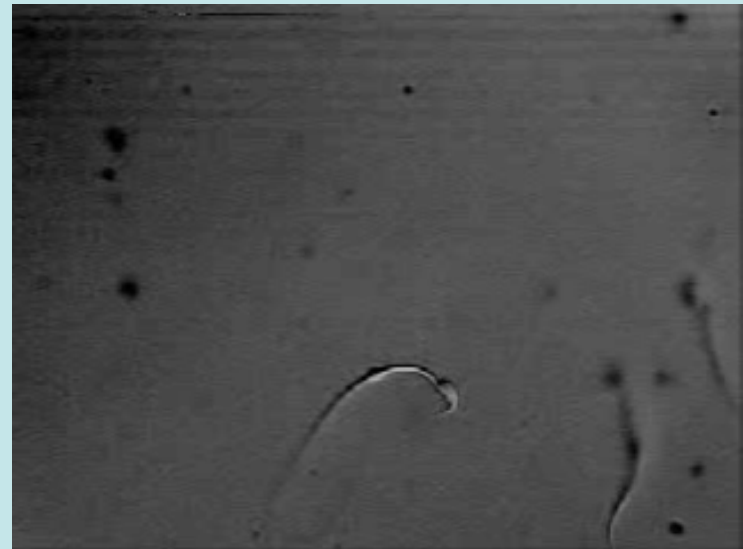
Computational fluid dynamics

What do we want to learn?

What are the complications?

What choices do we make?

- What are biochemical pathways that initiate hyperactivation?
- How do these biochemical signals change the internal force-generating mechanisms?
- What are the functional implications of a hyperactivated waveform?



What are the complications?

What are the complications?

- Moving interfaces!

What are the complications?

- Moving interfaces!
- Complicated geometries!

What are the complications?

- Moving interfaces!
- Complicated geometries!
- Many interfaces!

What are the complications?

- Moving interfaces!
- Complicated geometries!
- Many interfaces!
- Flexible, elastic, actuated interfaces!

What are the complications?

- Moving interfaces!
- Complicated geometries!
- Many interfaces!
- Flexible, elastic, actuated interfaces!
- Non-Newtonian fluid!!

What are the complications?

- Moving interfaces!
- Complicated geometries!
- Many interfaces!
- Flexible, elastic, actuated interfaces!
- Non-Newtonian fluid!!
- Biochemical signaling!!

What are the choices?

- 2D versus 3D?

What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?

What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?
- Start with one interface (cilium or sperm)?

What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?
- Start with one interface (cilium or sperm)?
- Prescribe kinematics?

What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?
- Start with one interface (cilium or sperm)?
- Prescribe kinematics?
- Idealized fluid model ?

What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?
- Start with one interface (cilium or sperm)?
- Prescribe kinematics?
- Specify preferred curvature?
- Ignore biochemical signaling or include a simple model?

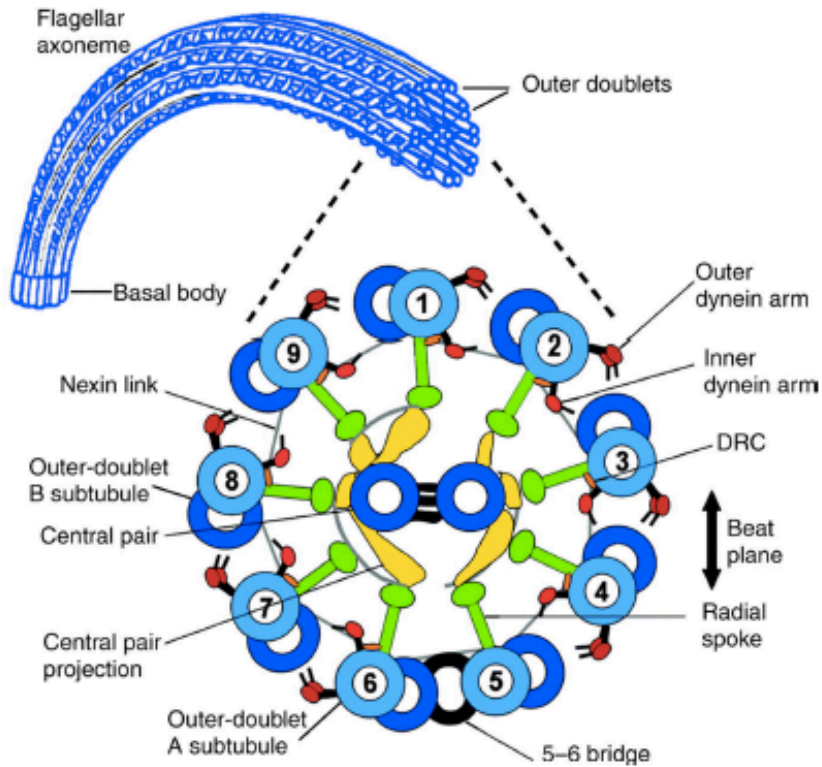
What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?
- Start with one interface (cilium or sperm)?
- Prescribe kinematics?
- Specify preferred curvature?
- Ignore biochemical signaling or include a simple model?
- Simple elastic model (Kirchoff rod?)

What are the choices?

- 2D versus 3D?
- Simpler domain geometries (a box)?
- Start with one interface (cilium or sperm)?
- Prescribe kinematics?
- Specify preferred curvature?
- Ignore biochemical signaling or include a simple model?
- Simple elastic model (Kirchoff rod?)
- Viscoelastic model?

NEARLY PLANAR SWIMMING



Lindemann C.B. and K.A. Lesich. *J. Cell Sci*, 2010.

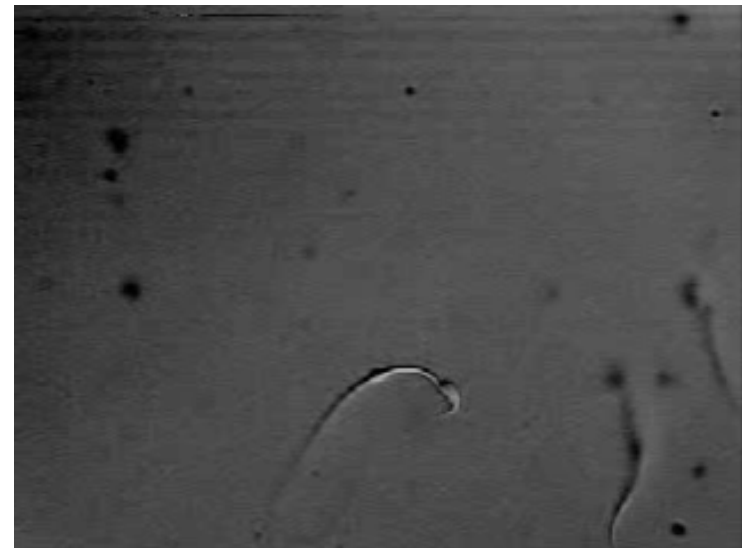
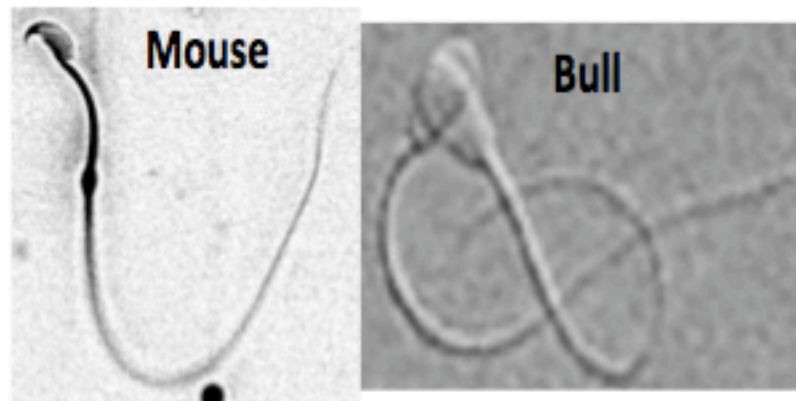


Kantsler, V. et al. *eLife*, 3, 2014.

Types of Sperm Motility

Hyperactivated motility

- Highly asymmetrical flagellar bending
- Increased curvature, amplitude
- Increased Ca^{2+}



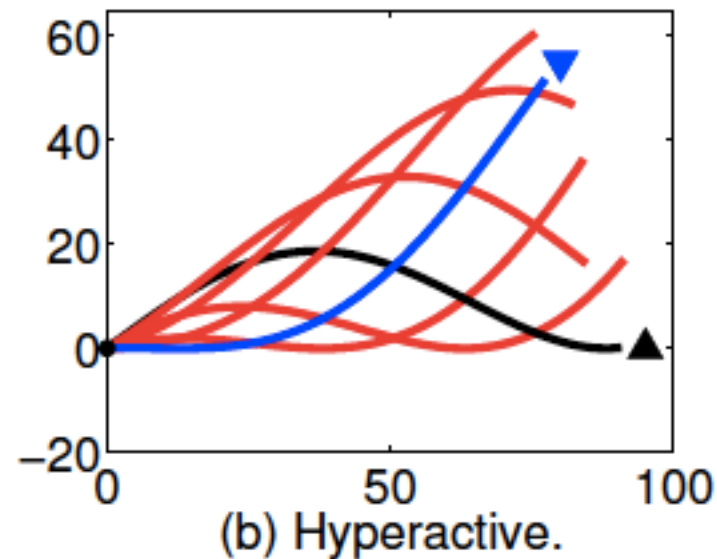
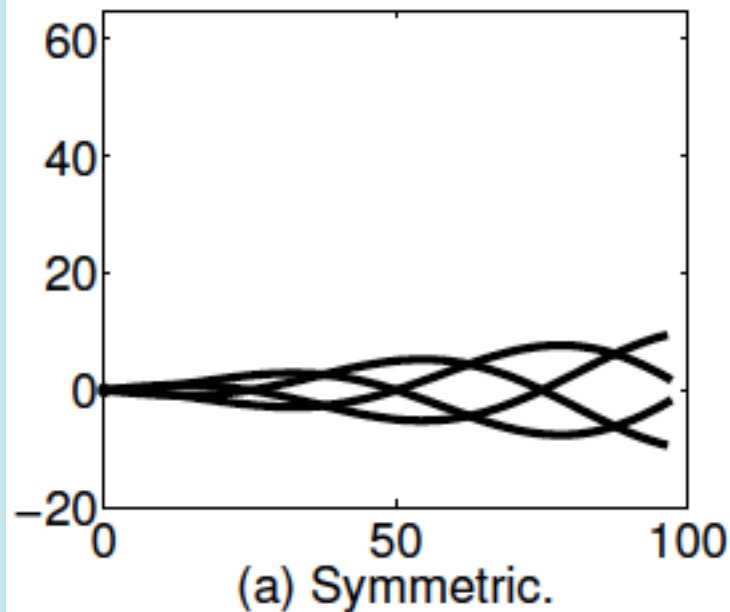
Movie courtesy of S. Suarez

Marquez and Suarez. Different signaling pathways in bovine sperm regulate capacitation and hyperactivation, *Biol Reprod* 70 (2004) 1626–1633.

Chang and Suarez. Rethinking the relationship between hyperactivation and chemotaxis in mammalian sperm, *Biol Reprod* 83 (2010) 507–513.

- Recent work by *Curtis, M. P. , Kirkman-Brown, J. C., Connolly, T. J. and E. A. Gaffney, 2012, J. Theor. Biol.*

showed that for wave of **given kinematics**, asymmetry could produce both tugging and thrusting forces at different wave phases.



Forces

- The bending filament, $\mathbf{X}(s, t)$, is taken to be a generalized Euler elastica whose energy, E , is given by:

$$E = \int_{\Gamma} (\varepsilon_{tens} + \varepsilon_{bend}) ds$$
$$\varepsilon_{tens} = S_1 \left[\left\| \frac{d\mathbf{X}}{ds} \right\| - 1 \right]^2, \quad \varepsilon_{bend} = S_2 \left[\frac{\partial \Theta}{\partial s} - \zeta(s, t) \right]^2$$

- $\frac{\partial \Theta}{\partial s}$ = shear angle, $\zeta(s, t)$ = preferred curvature
- Energy function is non-negative and translation and rotation invariant
- Force per unit length, \mathbf{g} , is derived from the energy function:

$$\mathbf{g} = -\frac{d}{d\mathbf{X}} (\varepsilon_{tens} + \varepsilon_{bend}) .$$

L. Fauci, A. McDonald, Sperm motility in the presence of boundaries, *Bull Math Biol* 57 (1995) 679–699.

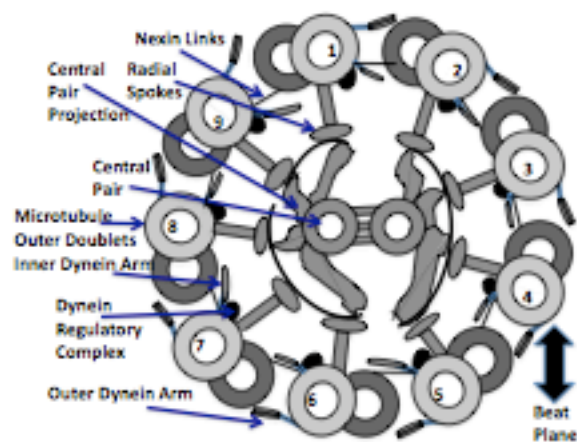
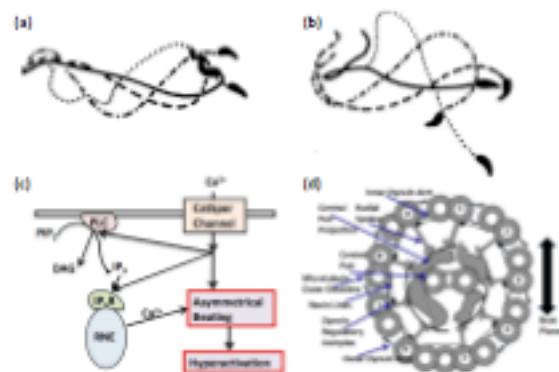
Calcium Dependent Curvature Model

Preferred (signed) curvature function corresponding to a simple sine wave with $x(s, t) = s$ and $y(s, t) = b \sin(\kappa s - \omega t)$ for small amplitude b is:

$$\zeta(s, t) = -\kappa^2 b \sin(\kappa s - \omega t)$$

Dependence of local dynein force generation on calcium:

$$b(s, t) = V_A \frac{Ca(s, t)}{Ca(s, t) + k_A}, \quad k_A = \begin{cases} k_{A,1} & -\kappa^2 \sin(\kappa s - \omega t) > 0 \\ k_{A,2} & -\kappa^2 \sin(\kappa s - \omega t) < 0 \end{cases}$$



Fluid coupled with ‘elastic structure’

Flow is governed by the incompressible Navier Stokes equations:

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right] = -\nabla p + \mu \Delta \mathbf{u} + \sum_{\mathbf{k}} \mathbf{F}^{\mathbf{k}}(\mathbf{x}, t)$$

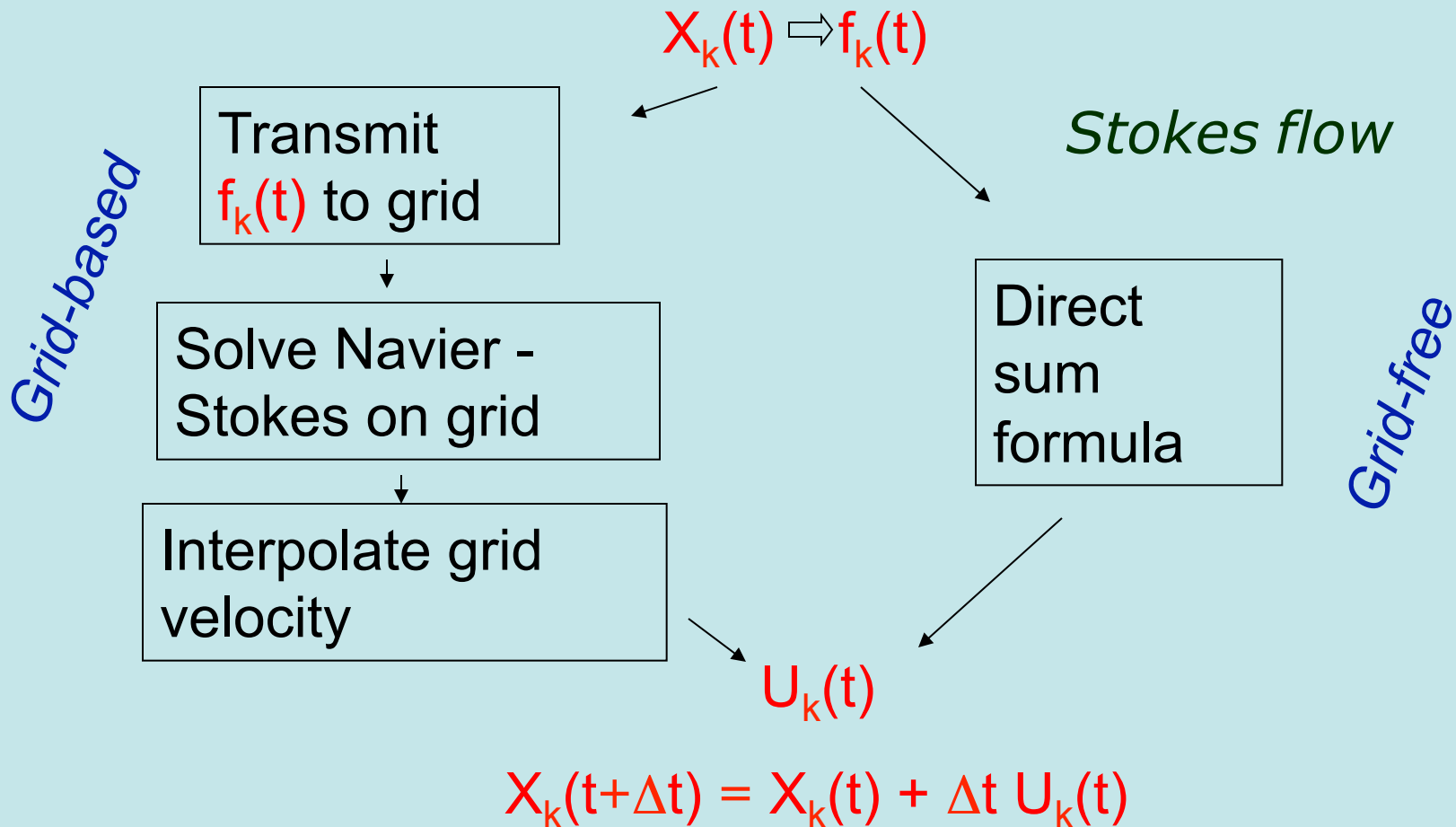
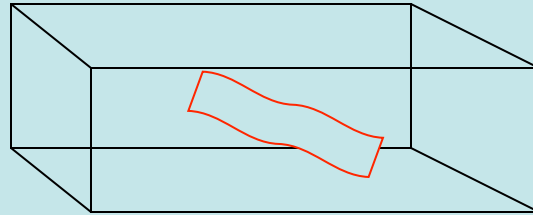
$$\nabla \cdot \mathbf{u} = 0$$

$\mathbf{F}^{\mathbf{k}}$ is a ‘delta function’ layer of force exerted by the \mathbf{k}^{th} filament on the fluid.

$$\mathbf{F}^{\mathbf{k}}(\mathbf{x}, t) = \int_S \mathbf{f}^{\mathbf{k}}(s, t) \delta(\mathbf{x} - \mathbf{X}^{\mathbf{k}}(s, t)) ds$$

$$\frac{\partial \mathbf{X}^{\mathbf{k}}(s, t)}{\partial t} = \mathbf{u}(\mathbf{X}^{\mathbf{k}}(s, t), t) = \int_{\Omega} \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}^{\mathbf{k}}(s, t)) d\mathbf{x}$$

Immersed boundary framework



2D sperm motility



LF and A. McDonald, 1994
Bull. Math. Biol.

Grid – free numerical method for zero Reynolds number

Steady Stokes equations: $\mu\Delta\mathbf{u} = \nabla p - \mathbf{F}$

$$\nabla \cdot \mathbf{u} = 0$$

Method of regularized Stokeslets (R. Cortez, SIAM SISC 2001;
Cortez, Fauci, Medovikov, Phys. Fluids, 2004)

Forces are spread over a small ball -- in the case $\mathbf{x}_k=0$:

$$\mathbf{F}(\mathbf{x}) = \mathbf{f}_0 \phi_\epsilon(\mathbf{x}), \quad \int \phi_\epsilon(\mathbf{x}) d\mathbf{x} = 1$$

For the choice:

$$\phi_\epsilon(\mathbf{x}) = \frac{15\epsilon^4}{8\pi(r^2 + \epsilon^2)^{7/2}}.$$

the resulting velocity field is:

$$\mathbf{u}(\mathbf{x}) = \frac{1}{8\pi\mu} \left\{ \mathbf{f}_0 \frac{2\epsilon^2 + r^2}{(r^2 + \epsilon^2)^{3/2}} + \frac{(\mathbf{f}_0 \cdot \mathbf{x})\mathbf{x}}{(r^2 + \epsilon^2)^{3/2}} \right\}$$

$$\mathbf{u}(\mathbf{x}) = \frac{1}{8\pi\mu} \left\{ \mathbf{f}_0 \frac{2\epsilon^2 + r^2}{(r^2 + \epsilon^2)^{3/2}} + \frac{(\mathbf{f}_0 \cdot \mathbf{x})\mathbf{x}}{(r^2 + \epsilon^2)^{3/2}} \right\}$$

Note:

$\mathbf{u}(\mathbf{x})$ is defined everywhere

$\mathbf{u}(\mathbf{x})$ is an exact solution to the Stokes equations, and is incompressible

$$u_j(\mathbf{x}_0) = \frac{1}{8\pi\mu} \sum_{n=1}^N \sum_{i=1}^3 S_{ij}(\mathbf{x}_n, \mathbf{x}_0) g_{n,i} A_n$$

If regularized forces are exerted at “N” points, the velocities at these points can be computed by superposition of Regularized Stokeslets

or

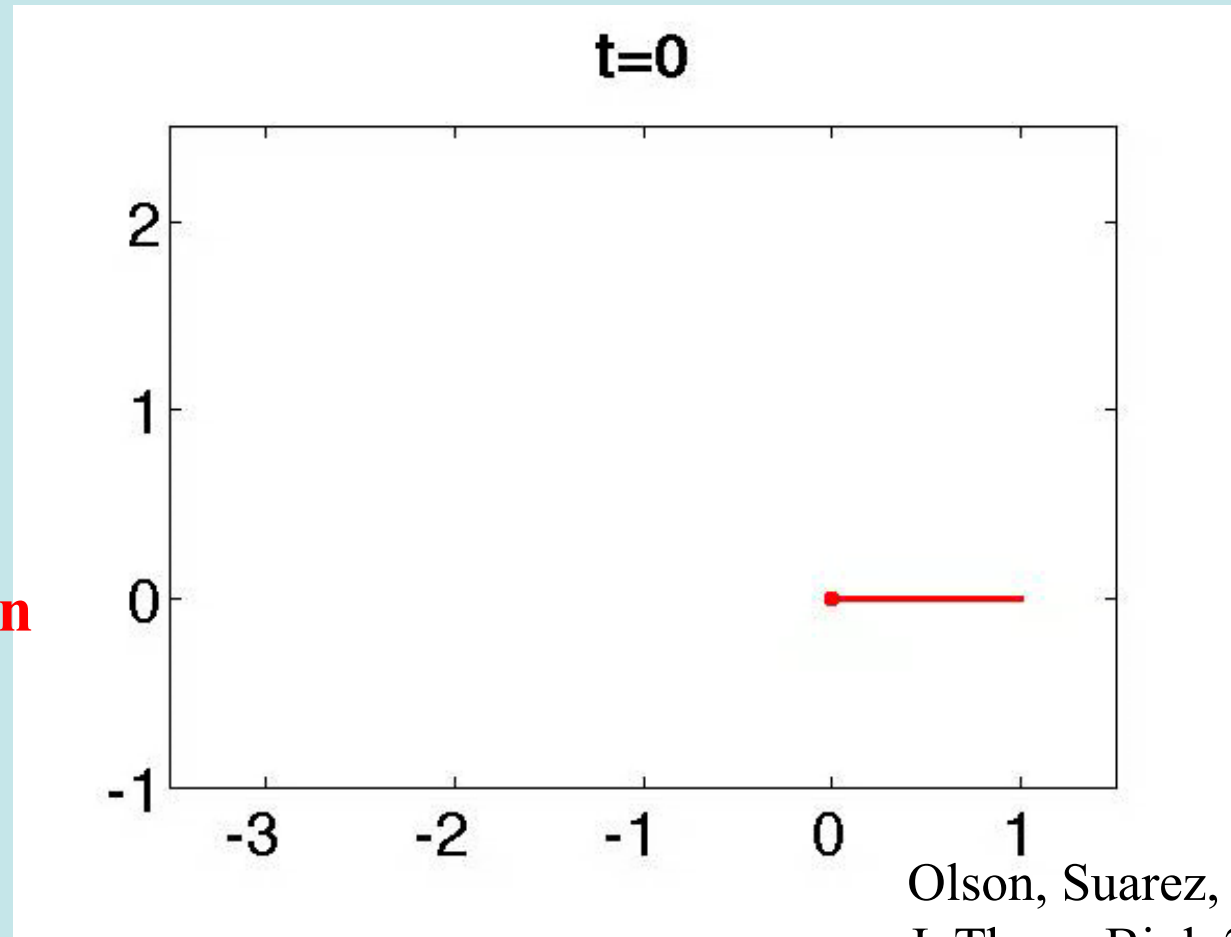
$$u = A g$$

Here A is a $3n$ by $3n$ matrix that depends upon the **geometry**.

Hyperactivated sperm motility

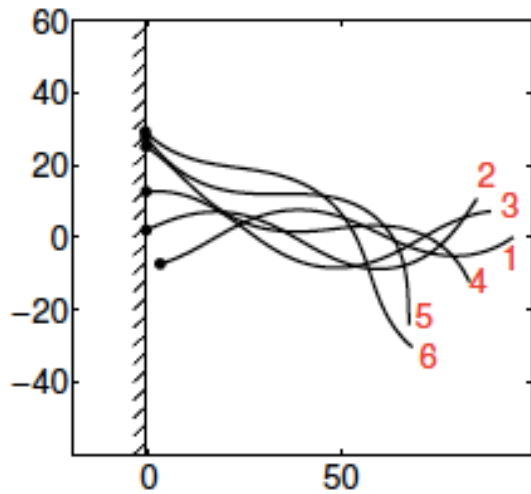
- 3D Stokes
- Planar beat
- *Preferred* kinematics *evolves* from calcium model

- **Symmetric**
- **Asymmetric**
- **Calcium driven**

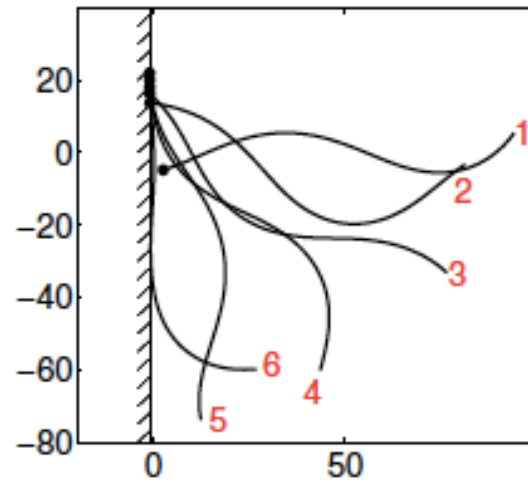


Olson, Suarez, Fauci,
J. Theor. Biol, 2011

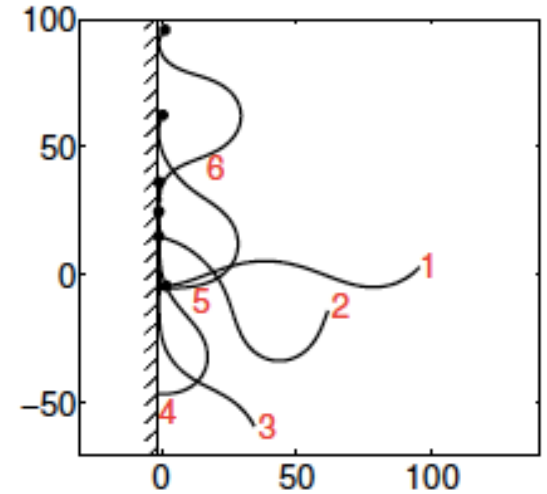
Planar wall



(a) Symmetric.

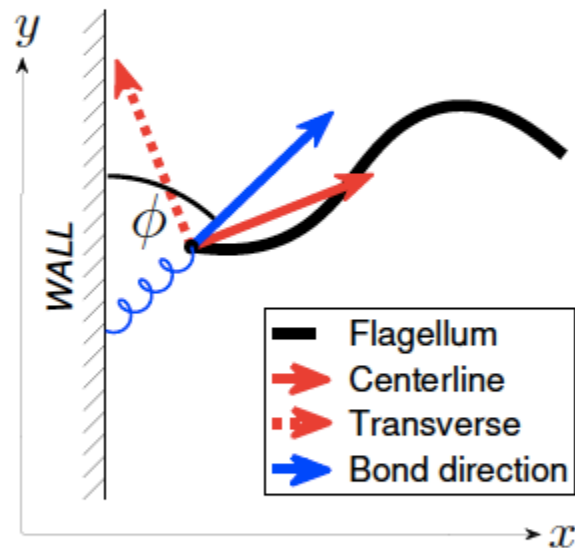


(b) Asymmetric.



(c) Ca^{2+} -dependent.

BOND FORCE



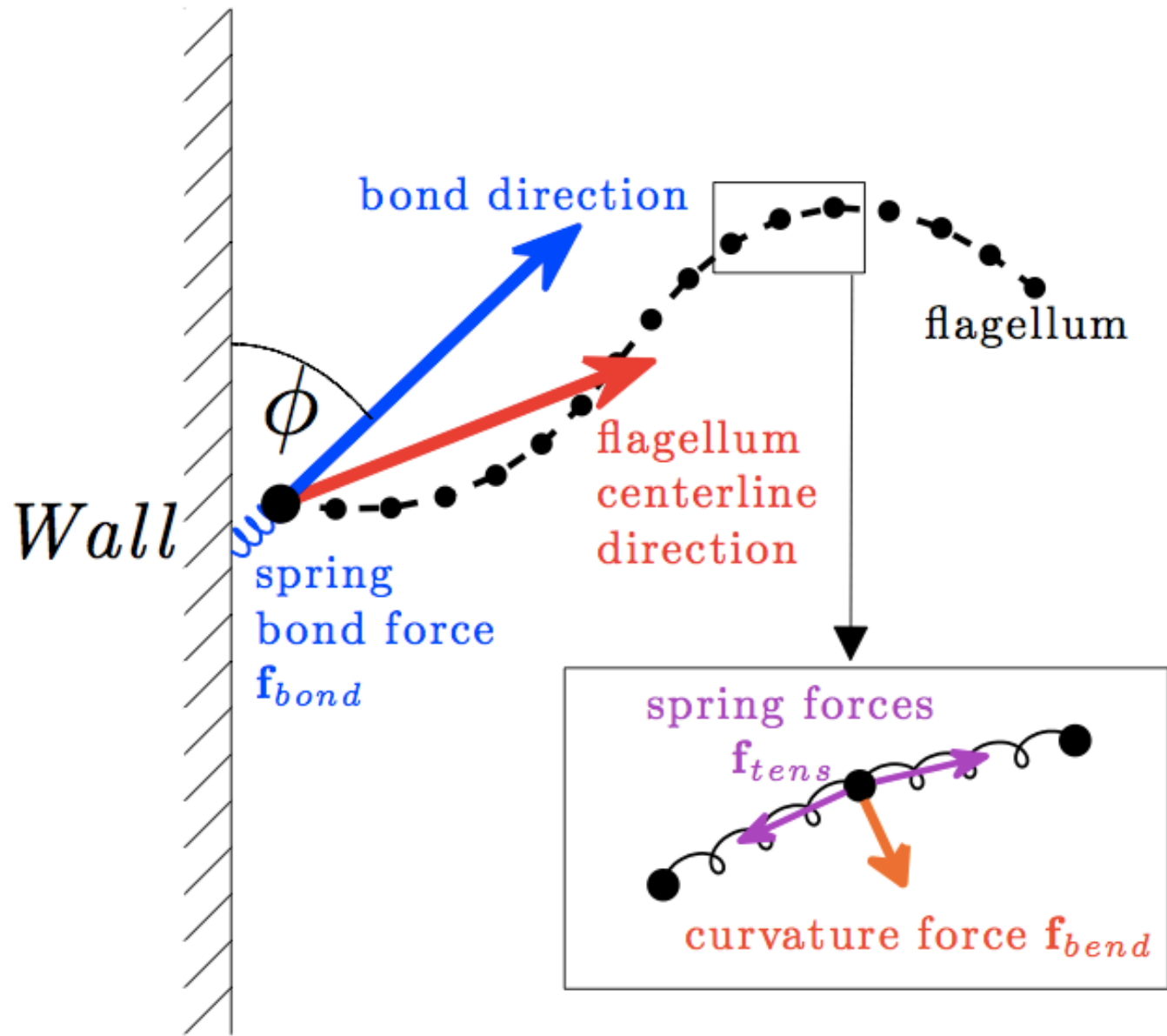
- ▶ **Bond formation:** when head comes within a capture radius L_c of the wall.
- ▶ **Spring bond force** for head of sperm when near surface:

$$f_{bond} = -k_b(L - L_*)$$

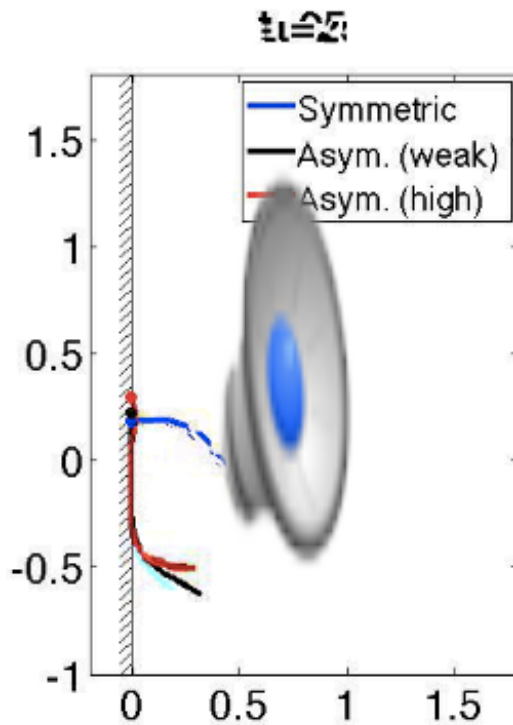
(k_b = ideal spring constant, L = spring length, L_* = resting length.)

- ▶ **Bond breaking:** when spring stretched beyond a breaking distance of

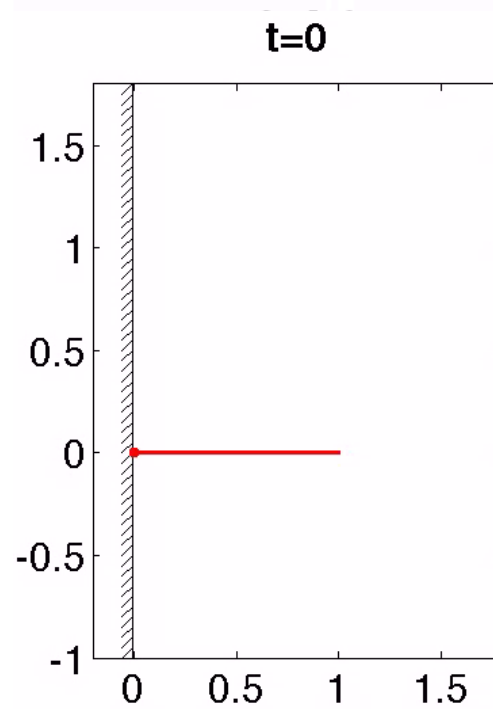
$$[(1 - \alpha) + \alpha \sin(\phi)]L_b$$



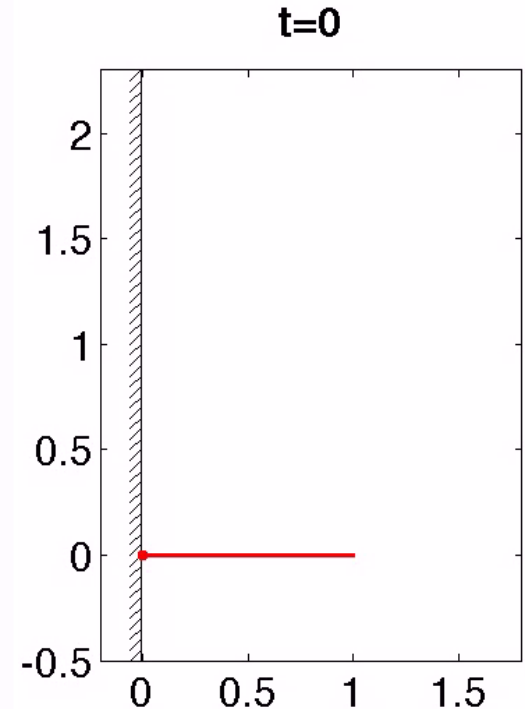
SWIMMING NEAR A WALL



No bond



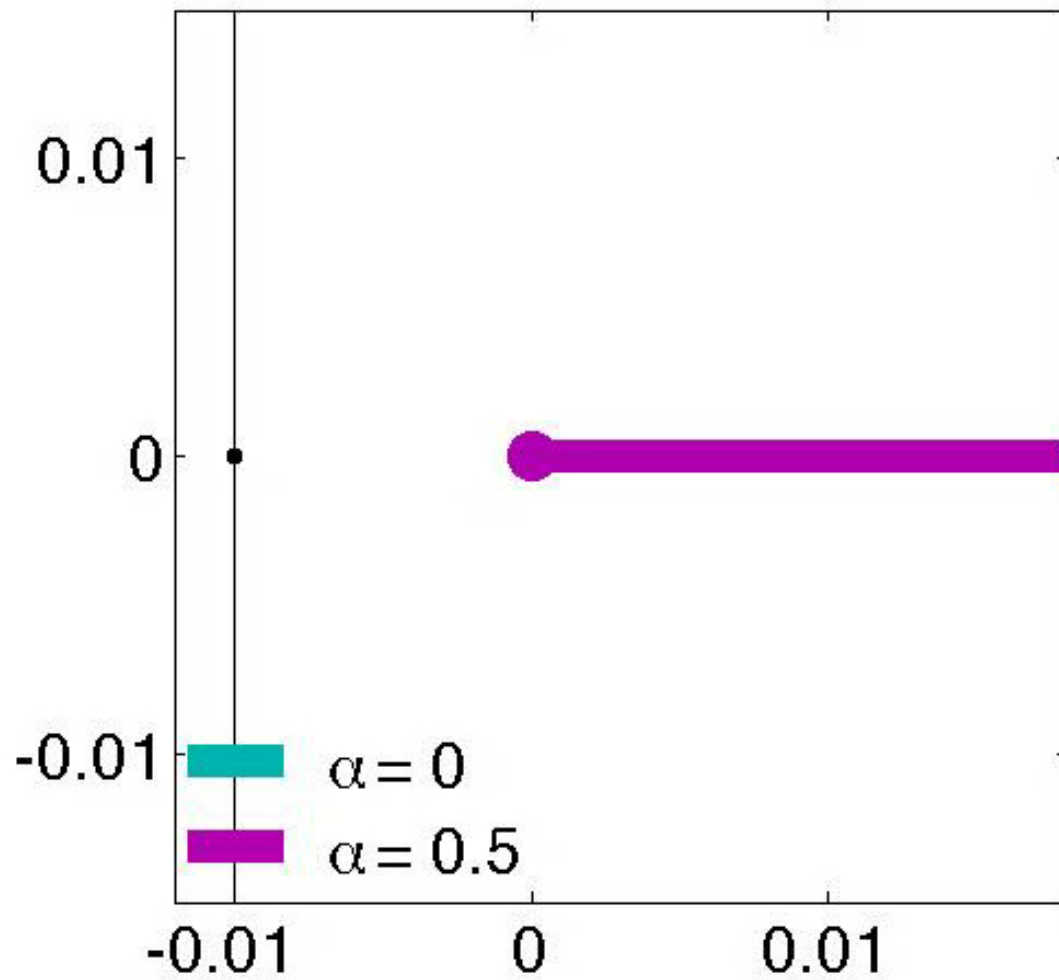
$\alpha = 0$



$\alpha = 0.5$

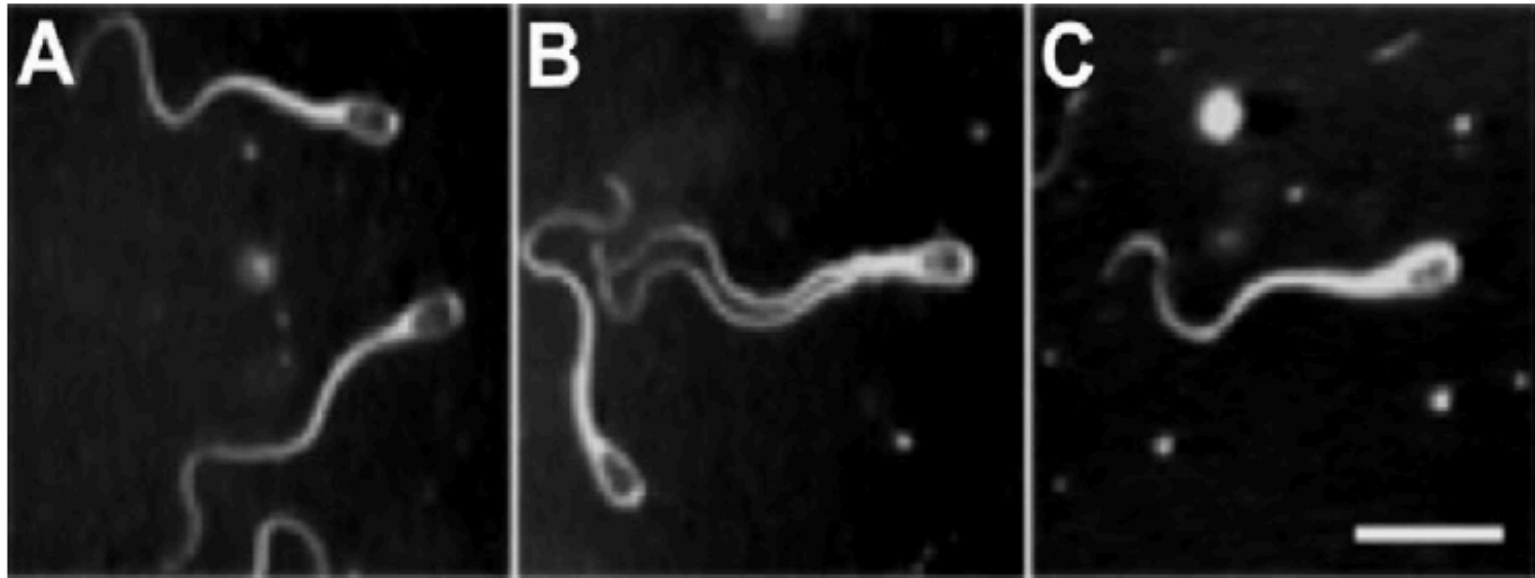
Simons, J., Olson, S.D., Cortez, R. and L. Fauci, J. *Theor. Biol.*, 2014.

t=0



- The frequent binding and attaching of hyperactivated sperm as seen in Chang and Suarez 2012 relies upon both asymmetry and high-amplitude.
- Elastic bond behavior can actually **enable** sperm to move away from epithelium.
- Bonding mechanisms affect detachment dynamics.

ATTRACTING SWIMMERS

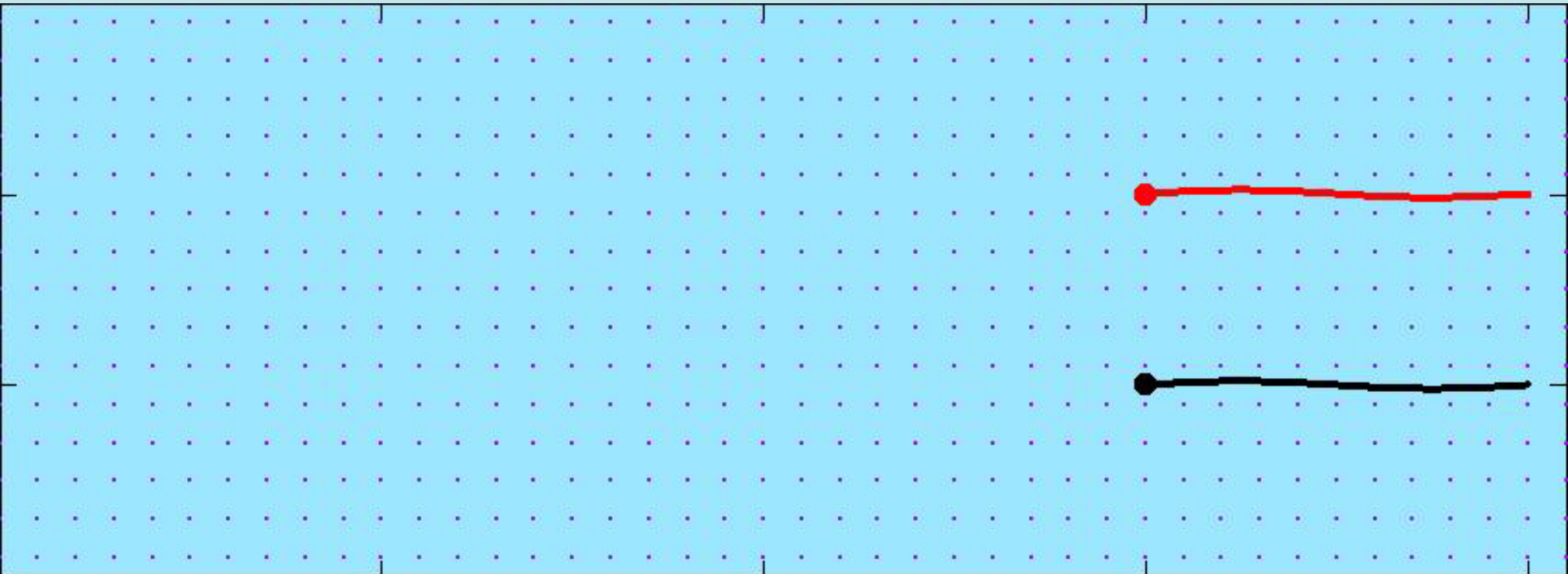


- ▶ Experimental attraction (bull, near surface): Woolley, D. et al., *J. Exp. Biol.*, 2009.
- ▶ Attraction in 2D fluid model: Yang, Y., Elgeti, J., G. Gompper, *Phys. Rev. E.*, 2008.
- ▶ Attraction in 3D fluid with bead-chain model: Llopis, I. et al., *Phys. Rev. E.*, 2013.

3D Stokes

Planar waveform

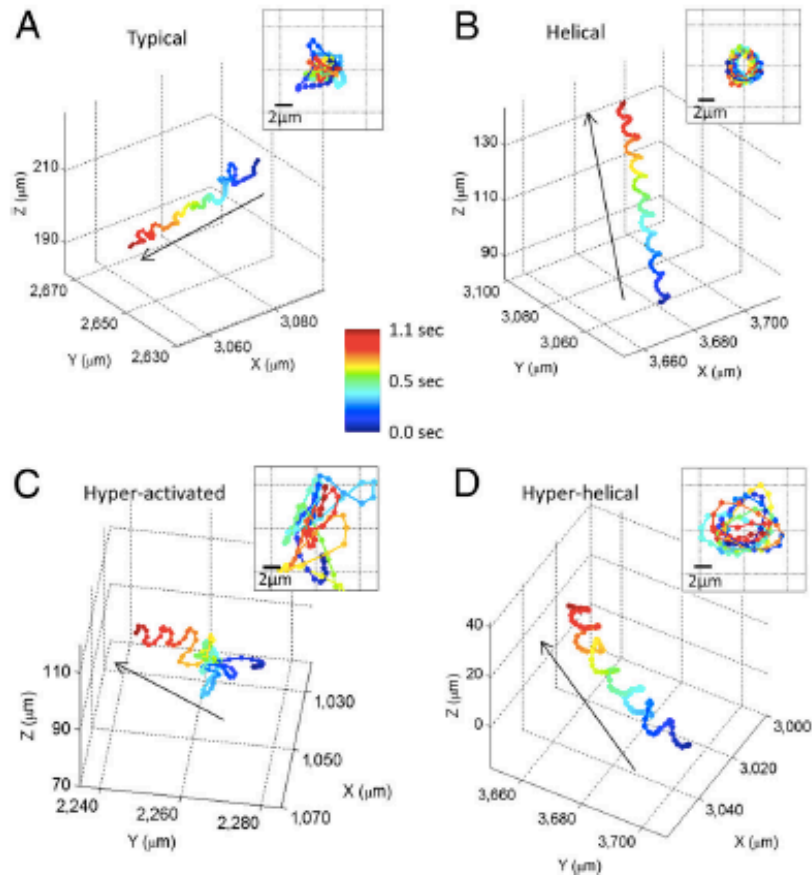
Preferred curvature



Olson, LF, *Phys. Fluids*, 2016

Simons, LF, Cortez, *J. Biomech.*, 2015

3D TRAJECTORIES



Su, T.-W., Xue, L., Ozcan, A., 2012. *High-throughput lensfree 3D tracking of human sperms reveals rare statistics of helical trajectories.* PNAS, 109 (40), 16018–16022.

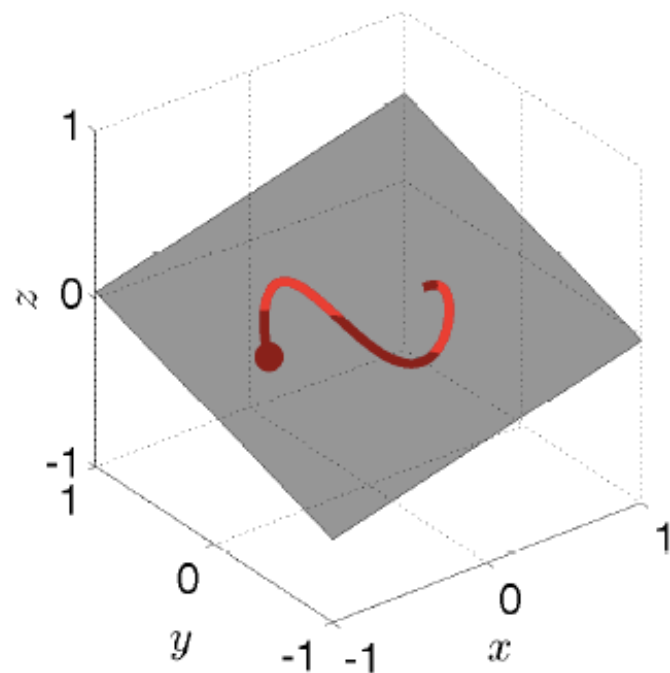
- ▶ Previous (planar) models cannot model the **general** interaction of 2 or more sperm!
- ▶ Can we develop a model that can incorporate 3D effects?
- ▶ How do sperm in populations affect each other's trajectories?
- ▶ Can helical/chiral trajectories arise from a planar motility behavior?

THREE-DIMENSIONAL MODEL EXTENSION

Concept: penalize out-of-plane components.

- ▶ Define the flagellum plane.
- ▶ Define a *new* energy to minimize these out-of-plane deviations:

$$E_{pen} = \frac{1}{2} S_p \int_0^L \left[\frac{\partial \hat{\mathbf{X}}}{\partial s} \cdot \mathbf{e}_3 \right]^2 ds$$

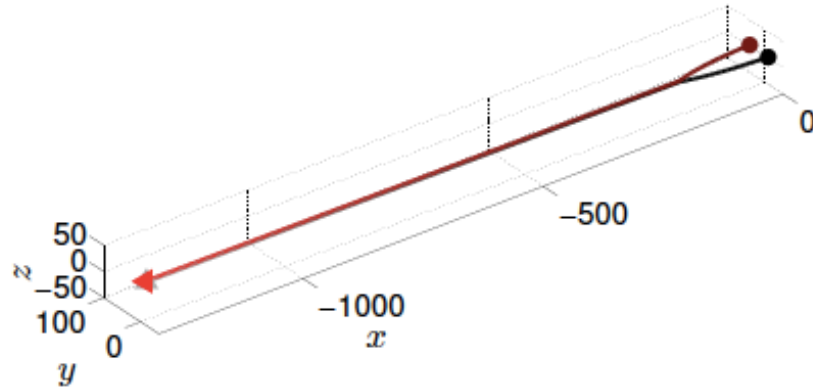


Simons, J., Fauci, L. and R. Cortez, J. *Biomechanics*, 2015.

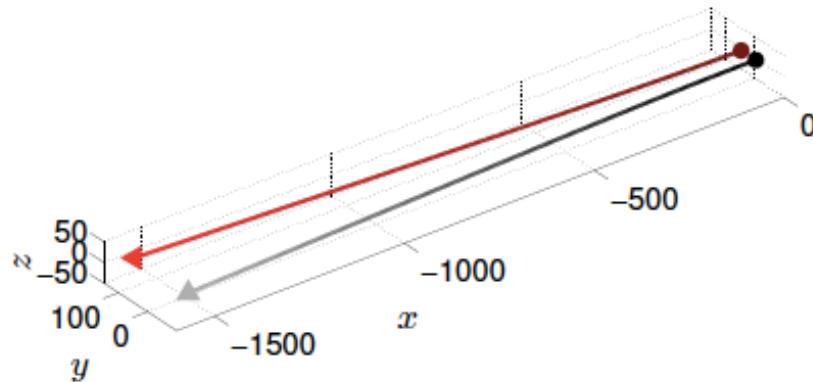
Parallel planar swimmers



TRAJECTORIES



(a) Coplanar.

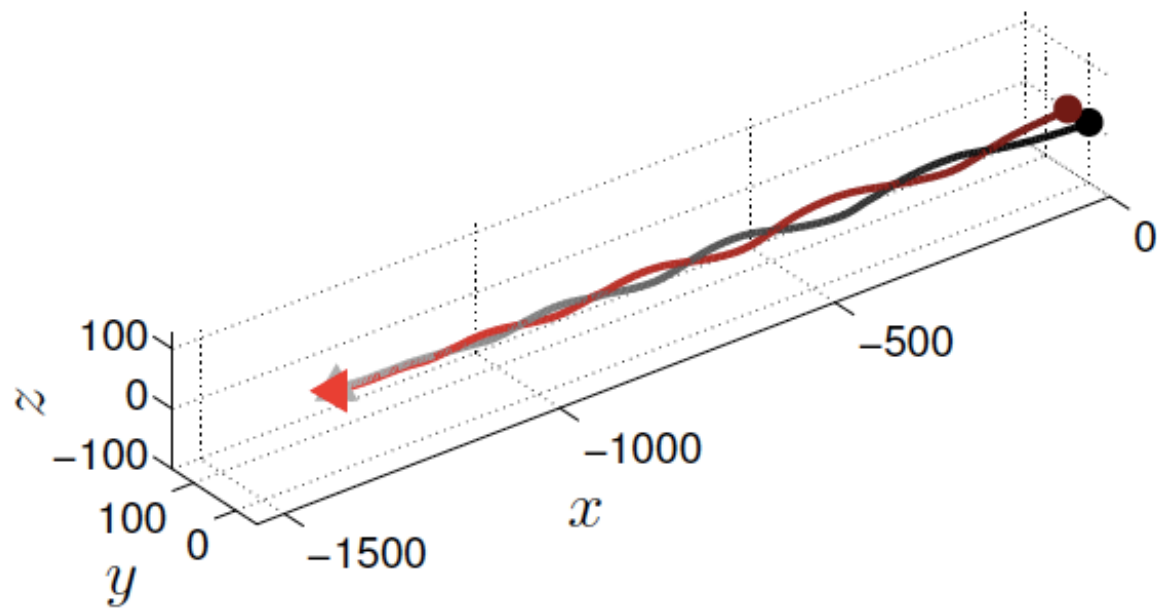


(b) Parallel initial flagellar planes.

Perturbed coplanar swimmers

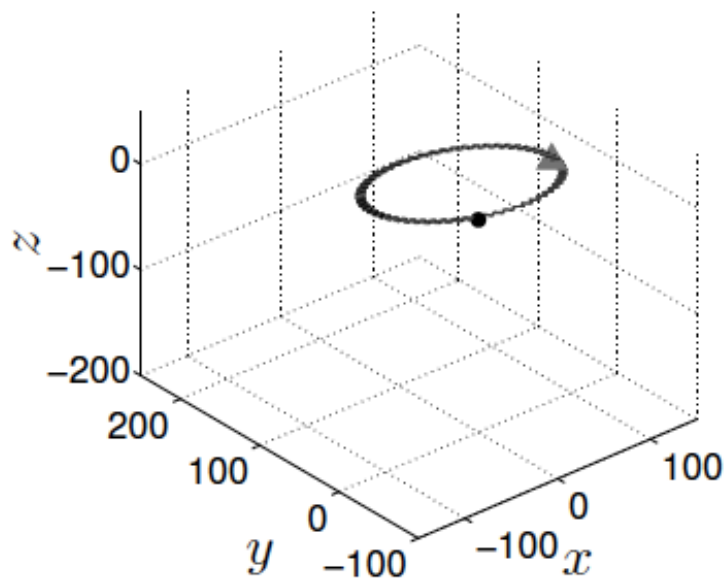


PERTURBED COPLANAR CONFIGURATIONS

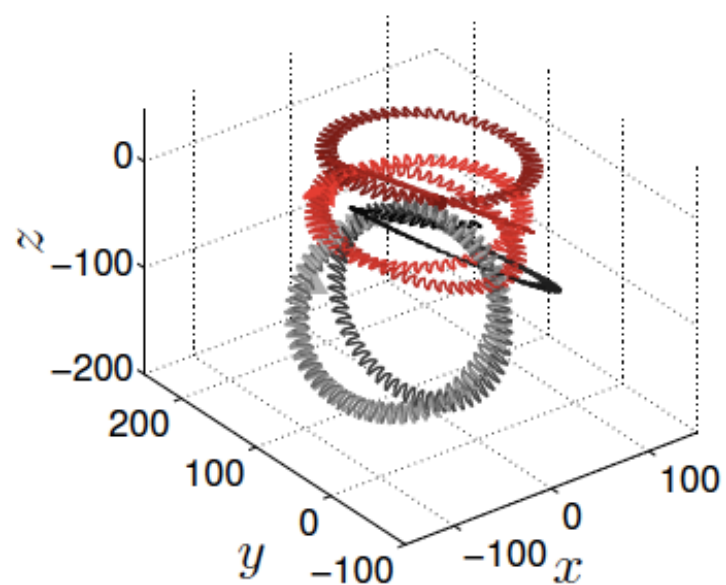




ASYMMETRIC TRAJECTORIES



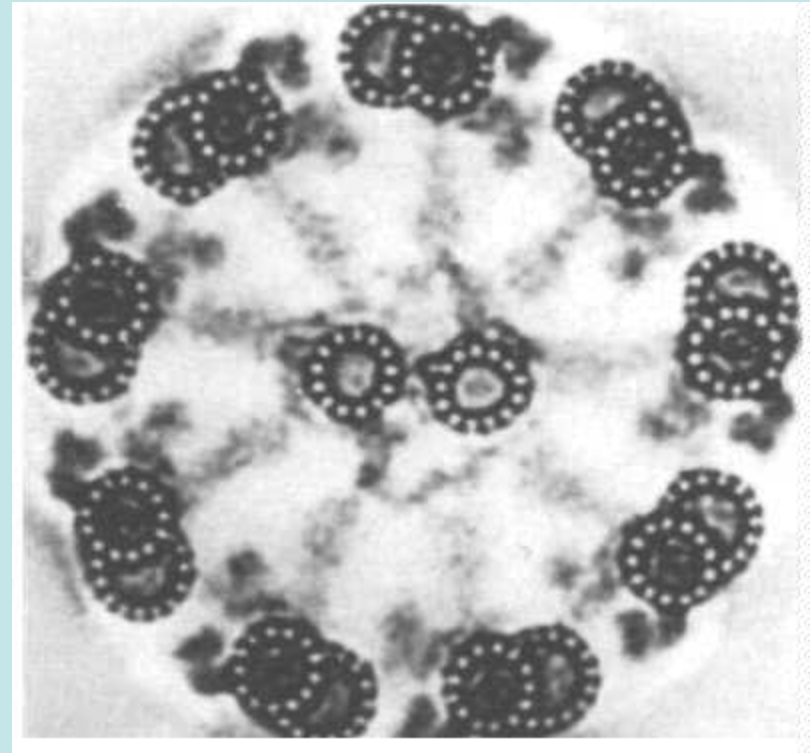
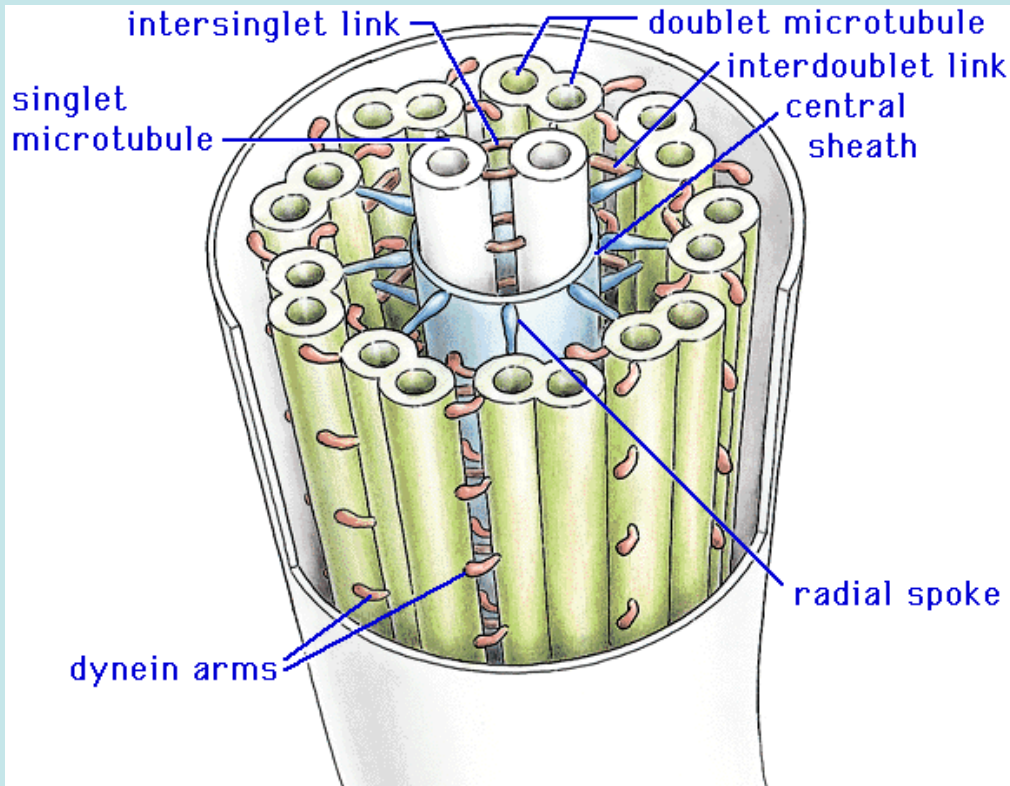
(a) Single sperm.



(b) Two sperm.

- ▶ 3D trajectories can evolve from waveforms that are primarily planar.
- ▶ Intuition from 2D may be misleading for 3D behavior: coplanar cooperative swimming represents an *unstable* state (from a dynamical systems perspective).
- ▶ Complex environments may *enable* sperm to reach the egg more effectively (a fully 3D search).

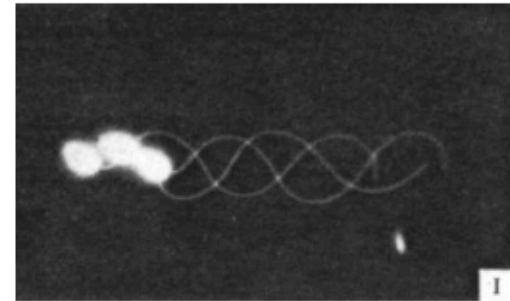
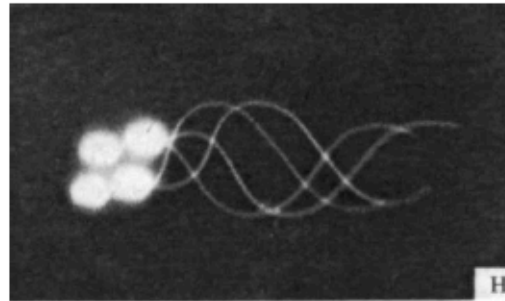
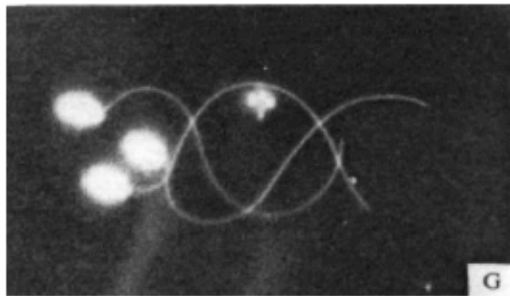
Eucaryotic axoneme



3D schematic

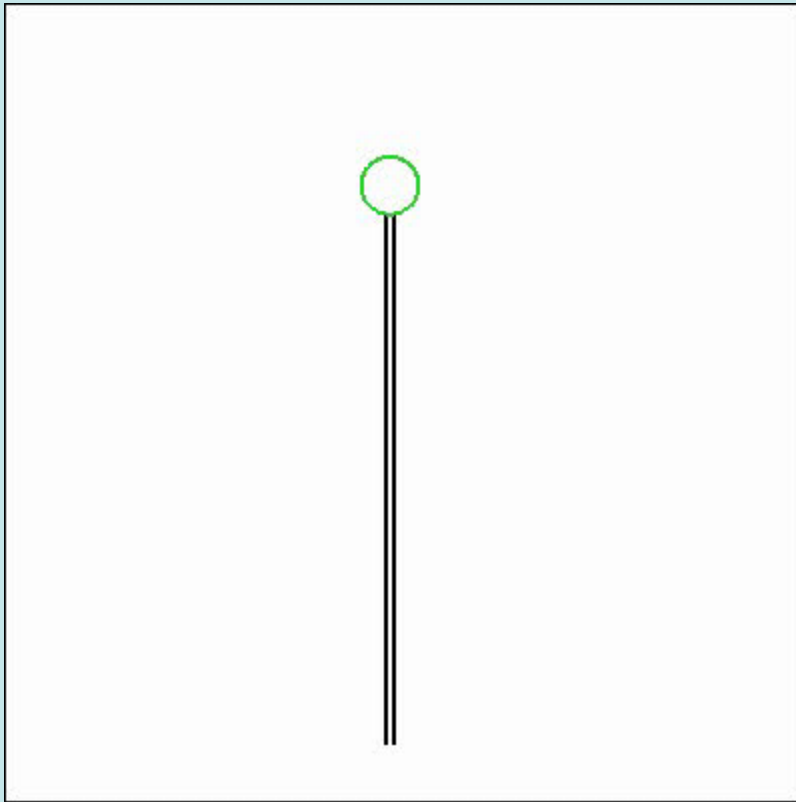
The precise nature of the spatial and temporal control mechanisms regulating various waveforms of cilia and flagella is still unknown.

What are the internal mechanisms that cause flagella (or cilia) to beat?

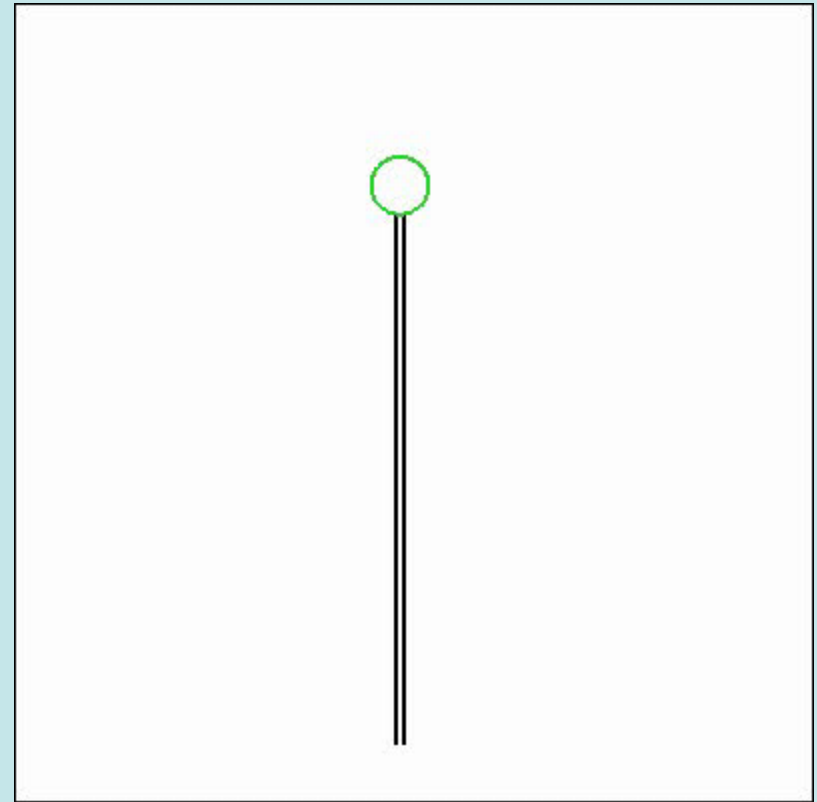


C. Brokaw, sea-urchin sperm swimming in fluids of increasing viscosity....1972

- 2D Navier-Stokes
- Individual dynein motors - activation curvature controlled



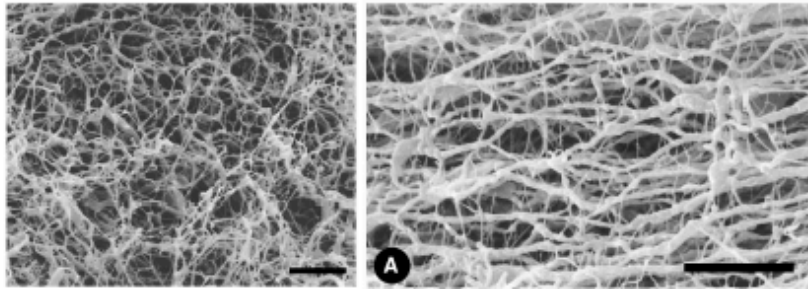
10 centipoise



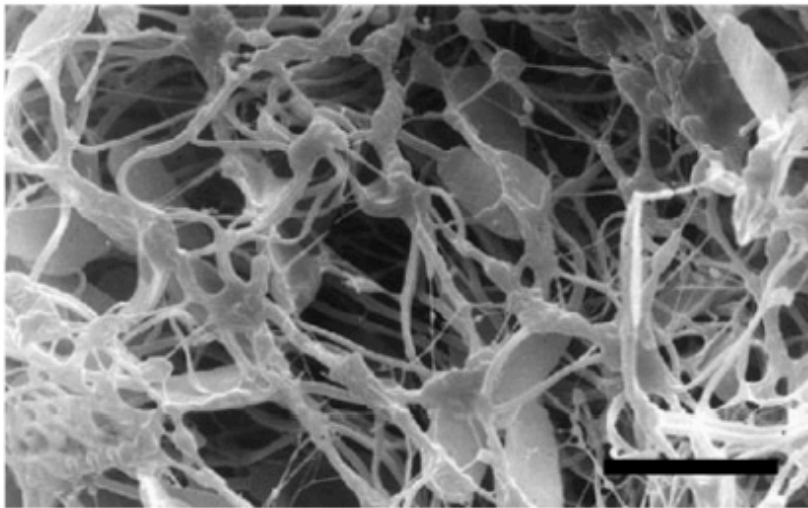
1 centipoise

Highly Heterogeneous, Viscoelastic Media

- Biological fluid is typically highly viscous.
- Biological fluid is composed of networks of branched polymers.
- Goal is to model viscoelastic behavior of medium.
- Swimming through a viscoelastic medium depends on its properties.
- Study of swimming in heterogeneous fluid.



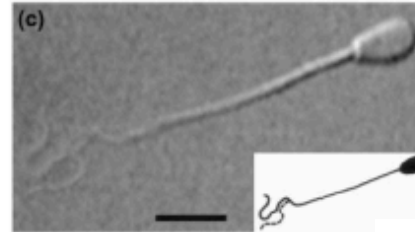
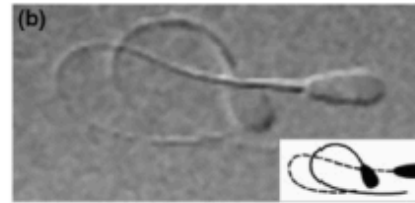
Rutland et al. *J. Anat.* 201, 2002



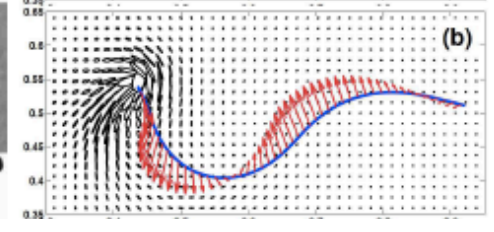
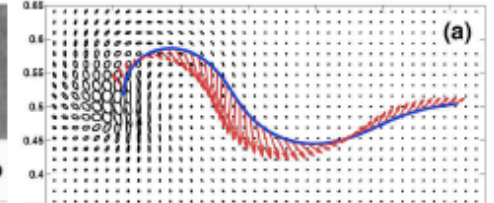
Rutland et al. *Reprod. Dom. Anim.* 40, 2005

Gap size: $0.5 - 25 \mu m$

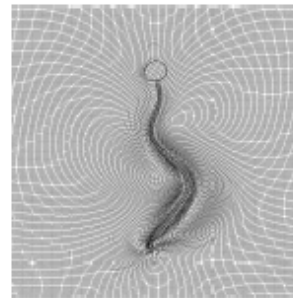
Swimmer's head: $5 - 10 \mu m$ long, $3 \mu m$ wide



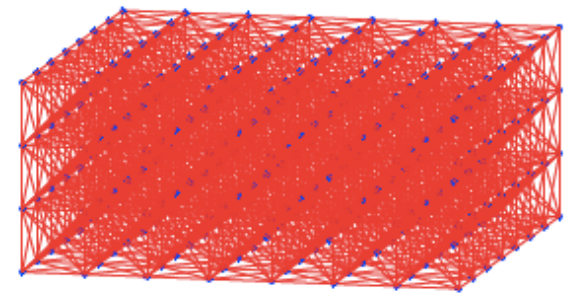
Ho, Suarez, *Reprod.* 122, 2001



Teran, Fauci, Shelley, *PRL* 104 2010



Dillon, Zhuo, *Disc.Cont.Dyn.S* 12, 2011 Wróbel et al. *Phys.Fluids* 24, 2014



Simple Elements in Stokes Flow

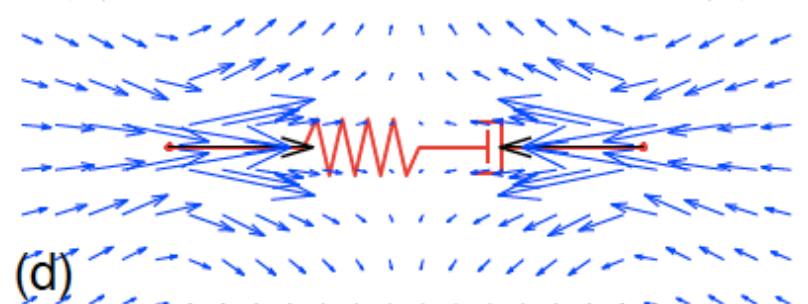
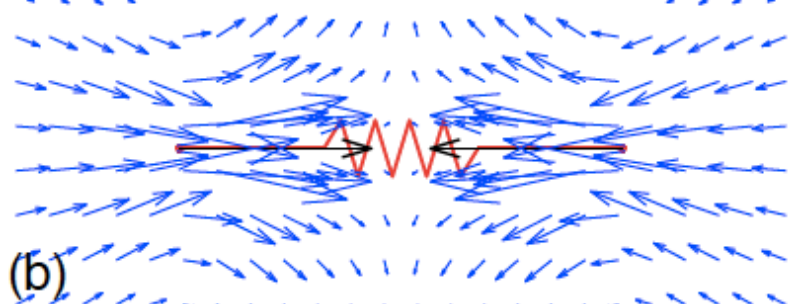
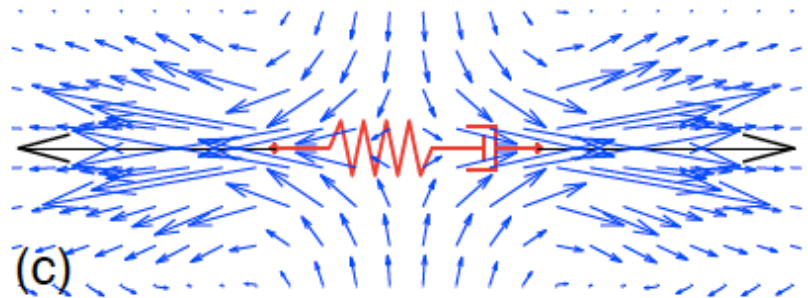
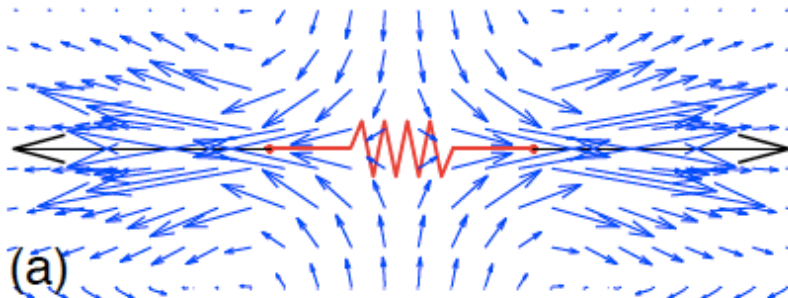
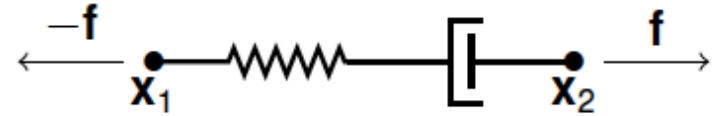
Stokes-Spring element



$$\mathbf{f}_s(\mathbf{x}_1) = -\mathbf{f}_s(\mathbf{x}_2) = E \left(\frac{\|\mathbf{x}_2 - \mathbf{x}_1\|}{\ell} - 1 \right) \frac{\mathbf{x}_2 - \mathbf{x}_1}{\|\mathbf{x}_2 - \mathbf{x}_1\|}$$

$$\frac{d\ell(t)}{dt} = \frac{E\ell_0}{\eta} \left(\frac{\|\mathbf{x}_2 - \mathbf{x}_1\|}{\ell(t)} - 1 \right)$$

Stokes-Maxwell element



Linear Viscoelasticity - Rheology

$\varepsilon(t) = \Delta L/L$ – strain – dimensionless deformation,

$\sigma(t) = \mathbf{f}/A$ – stress – force per area

Creep test – strain response to a step-stress:

$$\sigma(t) = \sigma_0(H(t - t_0) - H(t - t_1)) \longrightarrow \varepsilon(t)$$

Stress Relaxation test – stress response to a constant strain:

$$\varepsilon(t) = \varepsilon_0 H(t - t_0) \longrightarrow G(t) = \frac{\sigma(t)}{\varepsilon_0}$$

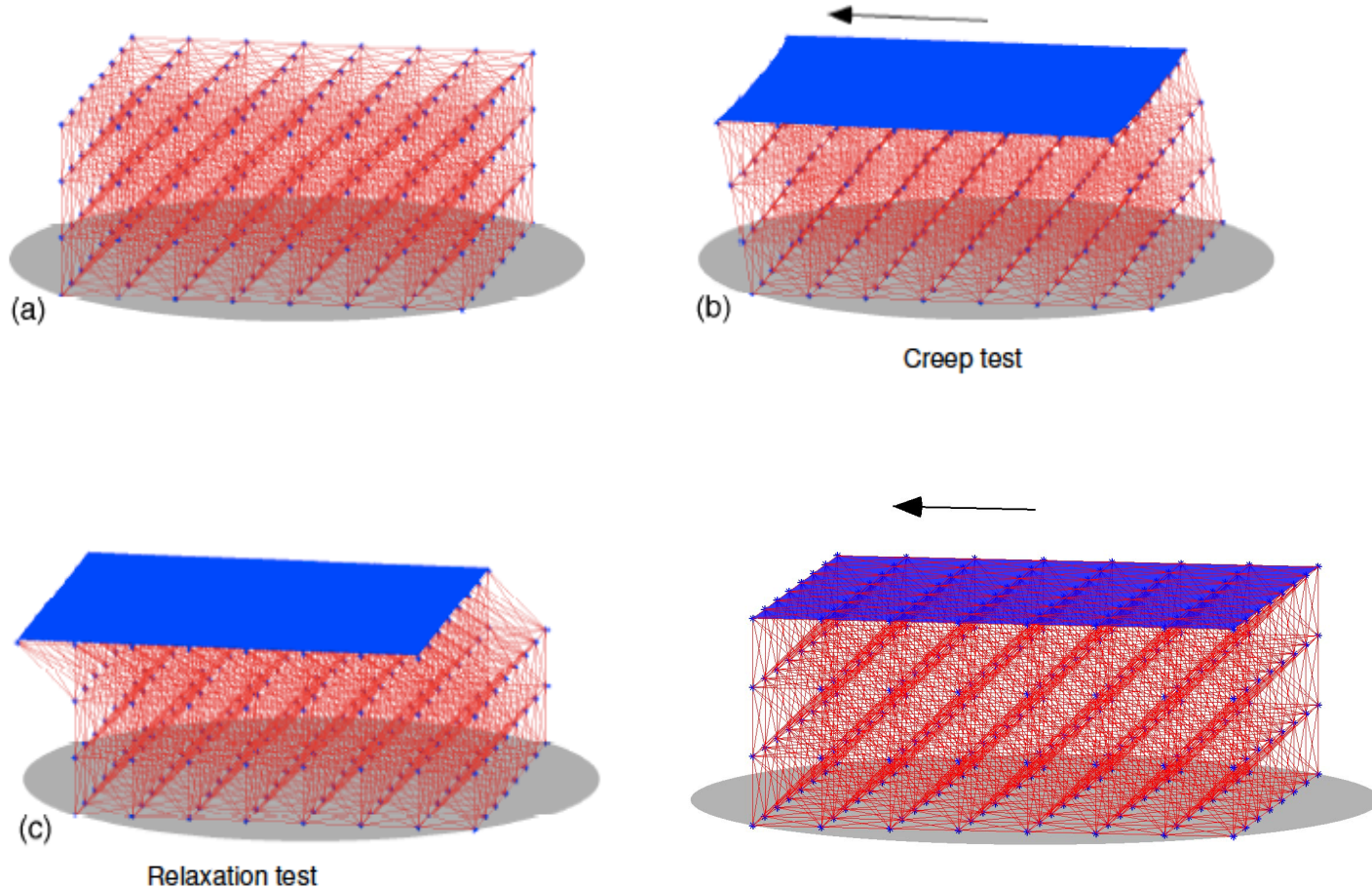
Small Amplitude Oscillatory Shear test – stress response to an oscillatory strain:

$$\varepsilon(t) = \varepsilon_0 \sin(\omega t) \longrightarrow \sigma(t) = \varepsilon_0 [G'(\omega) \sin(\omega t) + G''(\omega) \cos(\omega t)]$$

additionally

$$\eta' = \frac{G''(\omega)}{\omega} \quad \text{– dynamic viscosity,} \quad \eta'' = \frac{G'(\omega)}{\omega} \quad \text{– elastic viscosity}$$

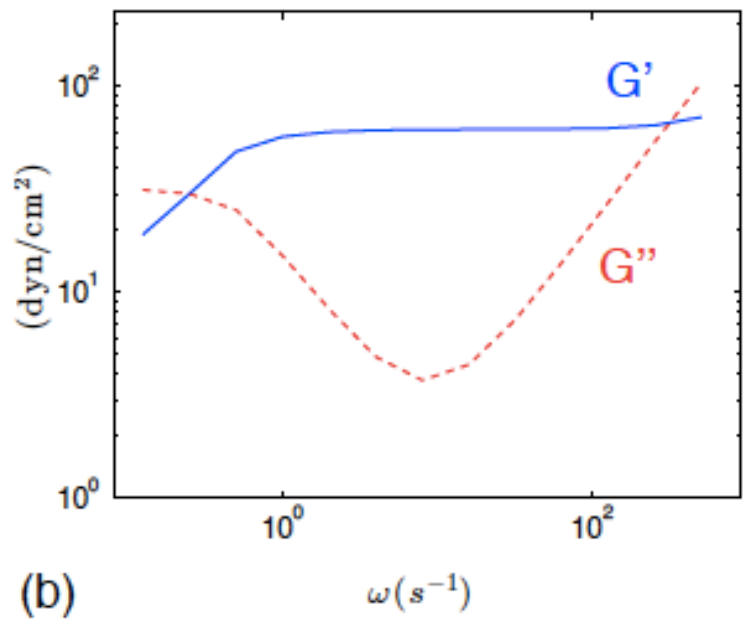
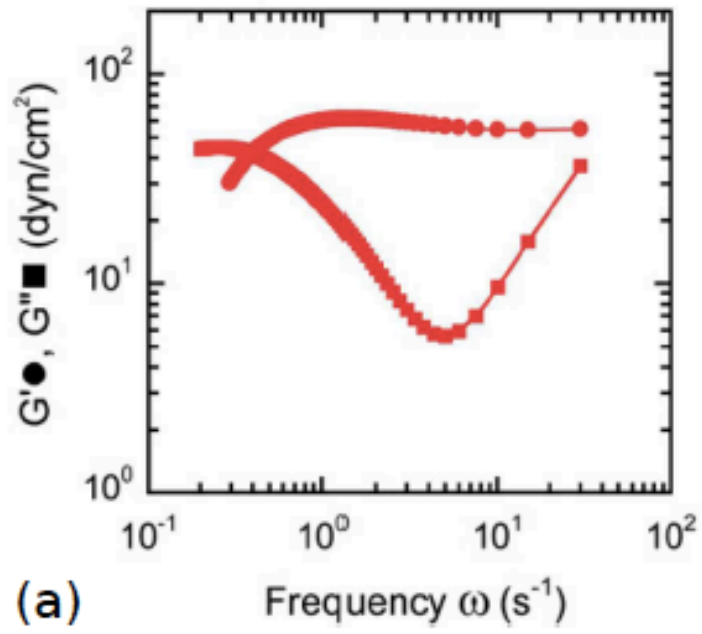
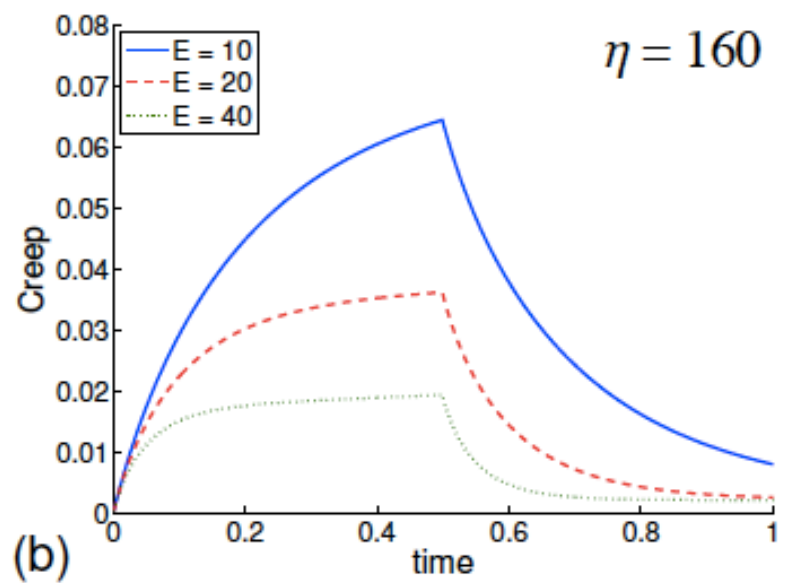
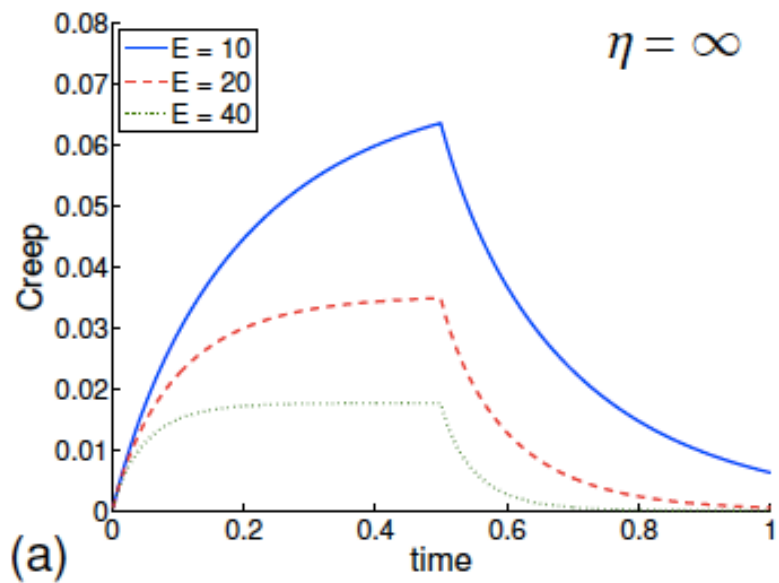
Structure in Stokes Fluid – Sliding Plate Rheometer



The total force at the node \mathbf{x}_i is

$$\mathbf{f}_i = \sum_j \mathbf{f}_j^s = \sum_j \ell_{ij}^2(0) E_{ij} \left(\frac{\|\mathbf{x}_j - \mathbf{x}_i\|}{\ell_{ij}(t)} - 1 \right) \frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|},$$

where E_{ij} is a stiffness and $\ell_{ij}(t)$ is the resting length of the link between nodes \mathbf{x}_i and \mathbf{x}_j .



Free Swimmer Model I – Prescribed Motion

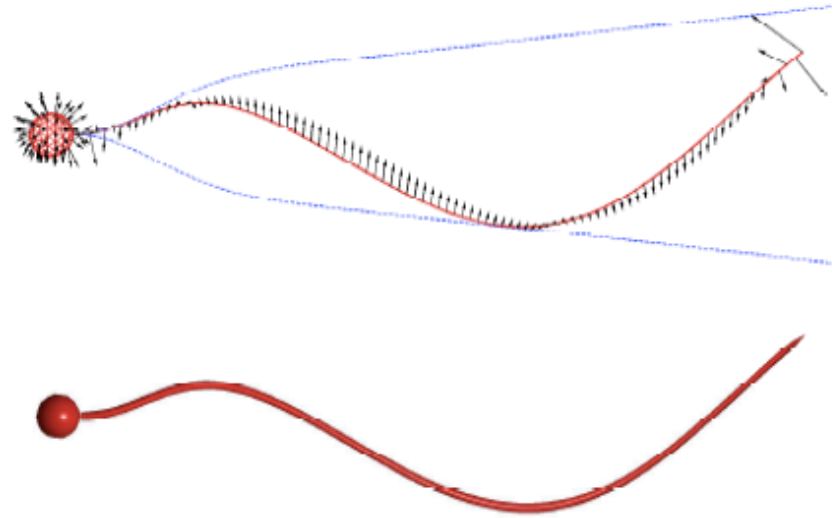
Motility problem:

$$\mathbf{u}_d + \mathbf{U} + \Omega \times (\mathbf{x} - \mathbf{x}_c) = \frac{1}{\mu} \int_{\partial B} \mathbf{S}^\epsilon(\mathbf{x}, \mathbf{x}_0) \mathbf{f} dS$$

Free swimmer:

$$\mathbf{u}_d = \frac{1}{\mu} \int_{\partial B} \mathbf{S}^\epsilon(\mathbf{x}, \mathbf{x}_0) \mathbf{f} dS - \mathbf{U} - \Omega \times (\mathbf{x} - \mathbf{x}_c)$$

$$0 = \int_{\partial B} \mathbf{f} dS \quad 0 = \int_{\partial B} (\mathbf{x} - \mathbf{x}_c) \times \mathbf{f} dS$$

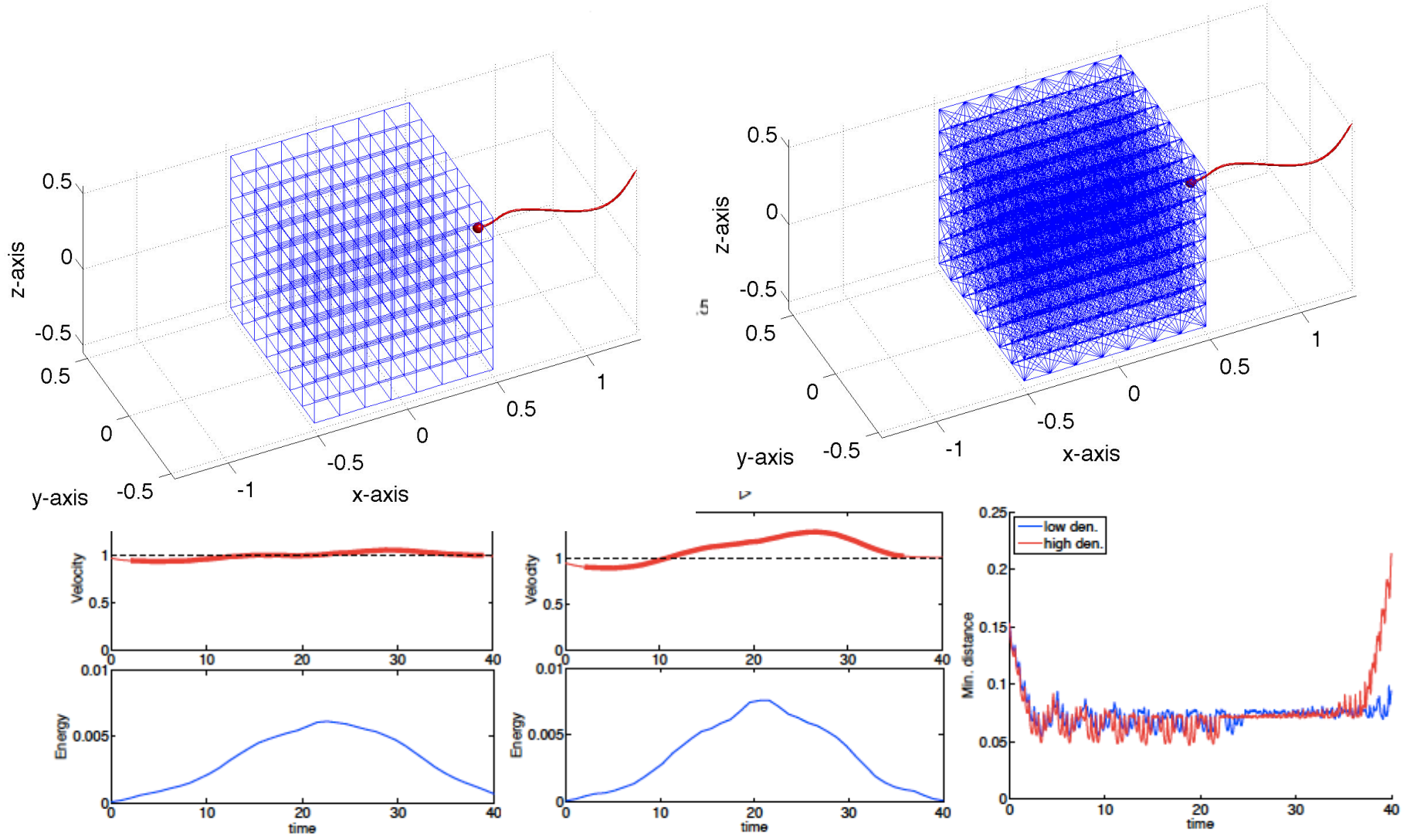


- Wave velocity boundary condition at tail points.
- Zero velocity (no-slip) boundary condition at head points.
- Head points updated using rotation and translational velocities, i.e.

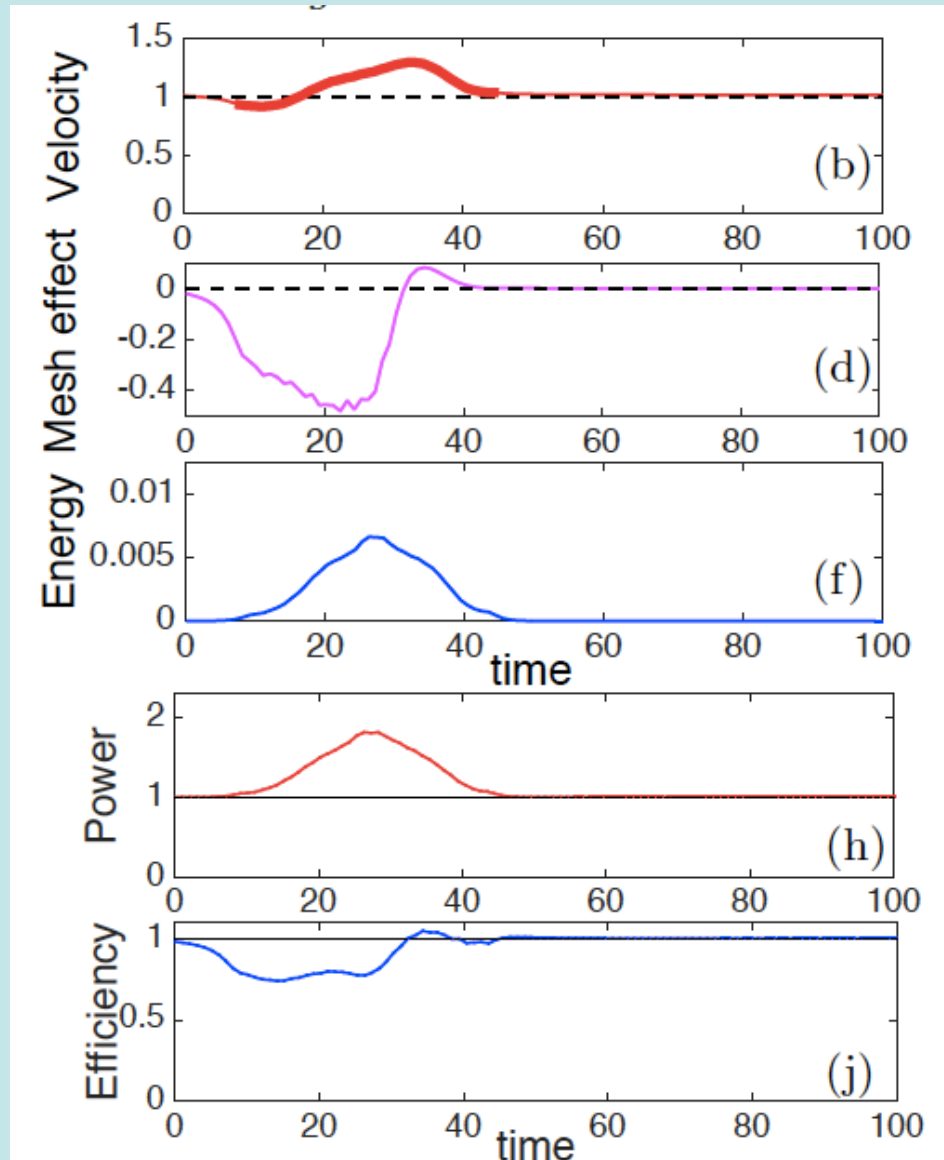
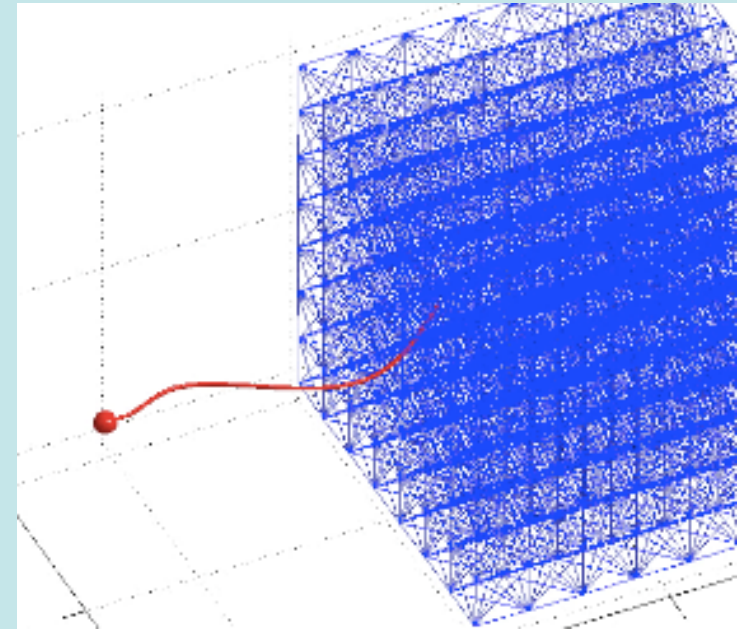
$$\frac{d\mathbf{x}}{dt} = \mathbf{U} + \Omega \times (\mathbf{x} - \mathbf{x}_c)$$

Swimming through Viscoelastic Structures

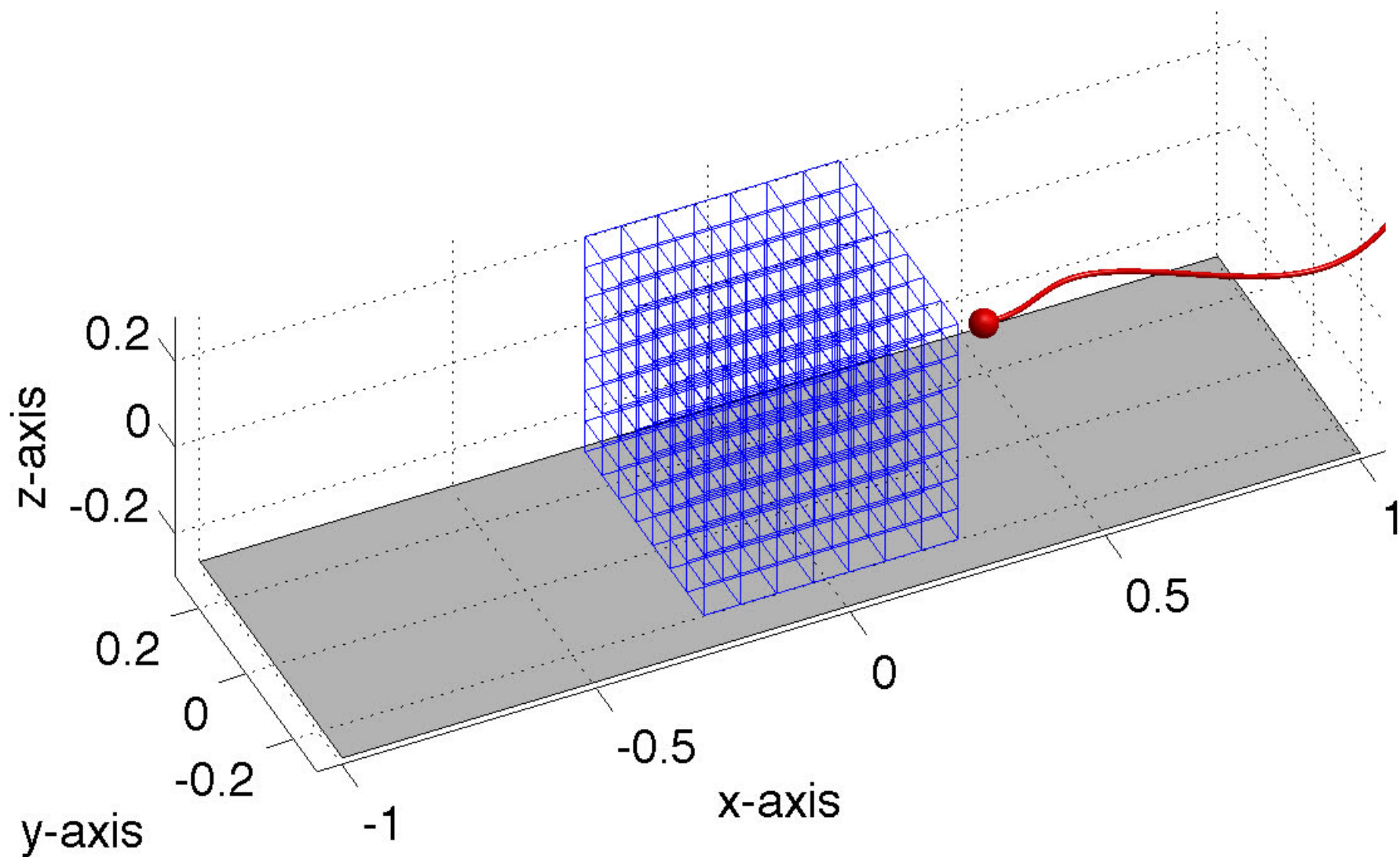
Wrobel, Lynch, Barrett, Fauci, Cortez, J. Fluid Mech. (2016)

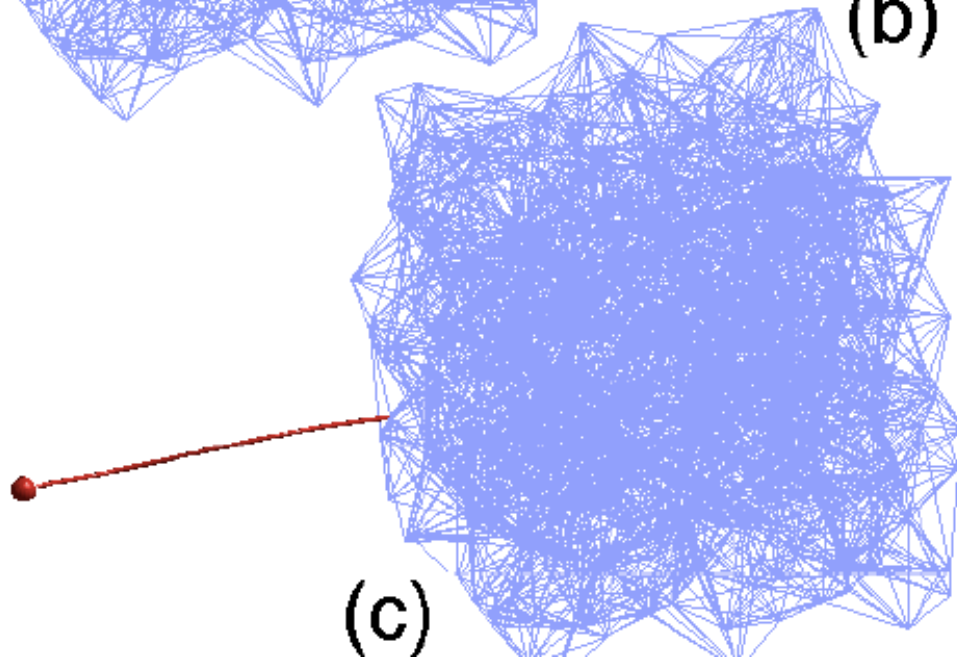
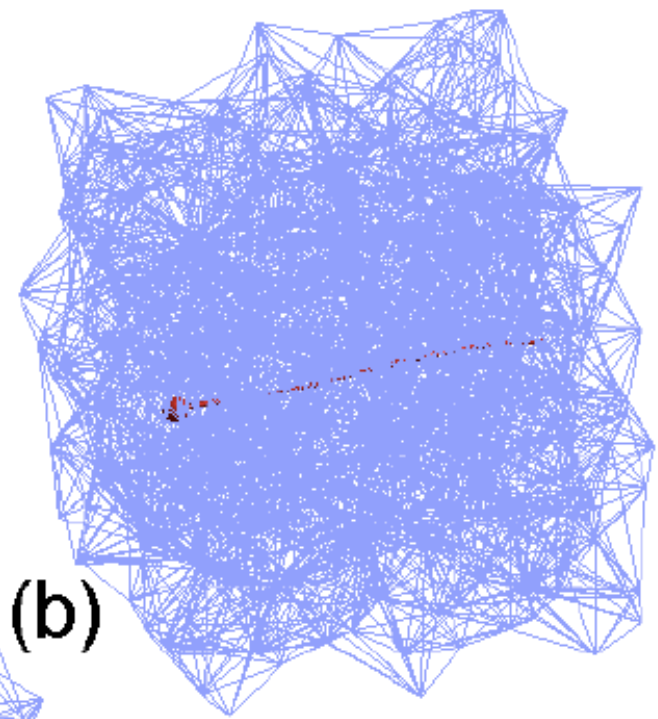
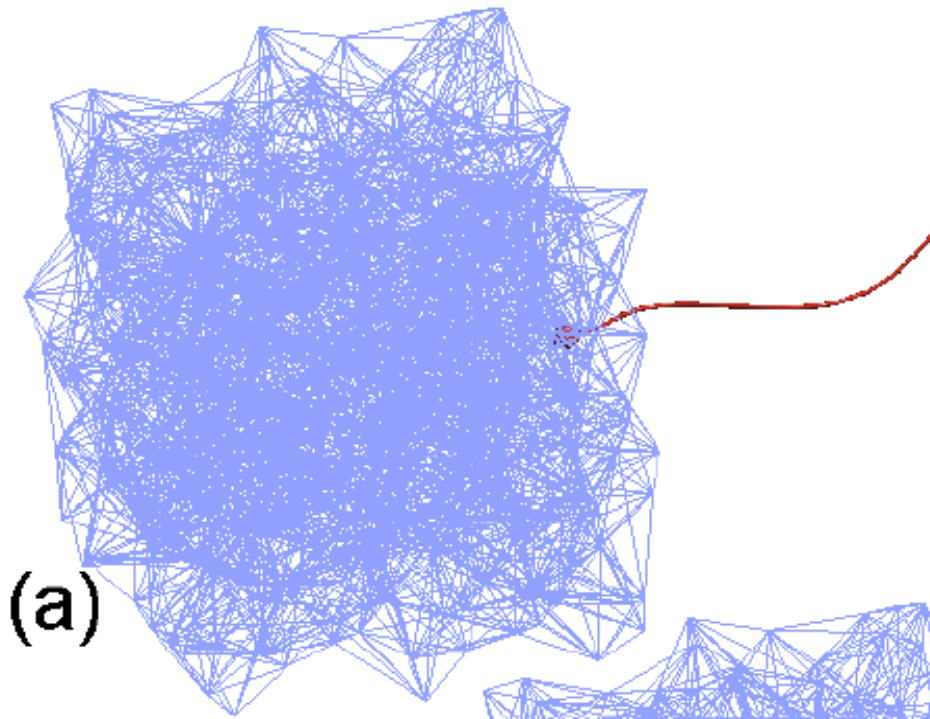


Swimmer gets a boost in velocity within network, but also requires more power to maintain specified waveform.



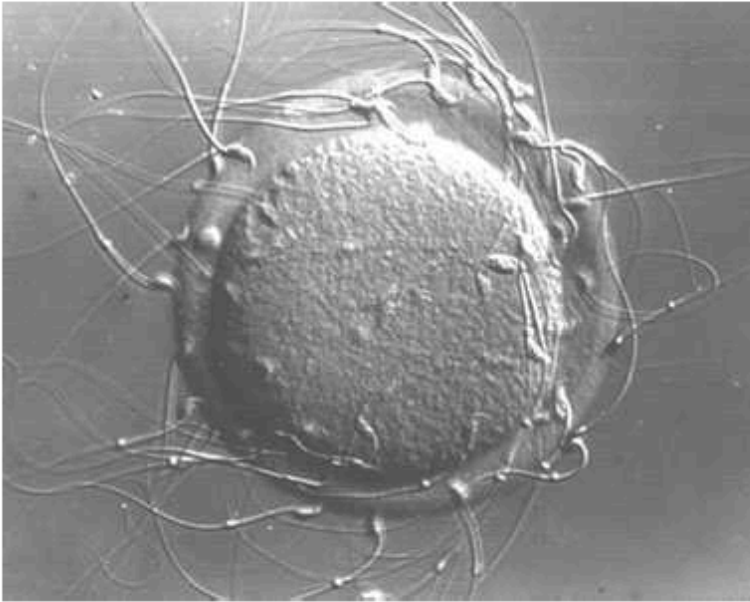
Network attached to wall.



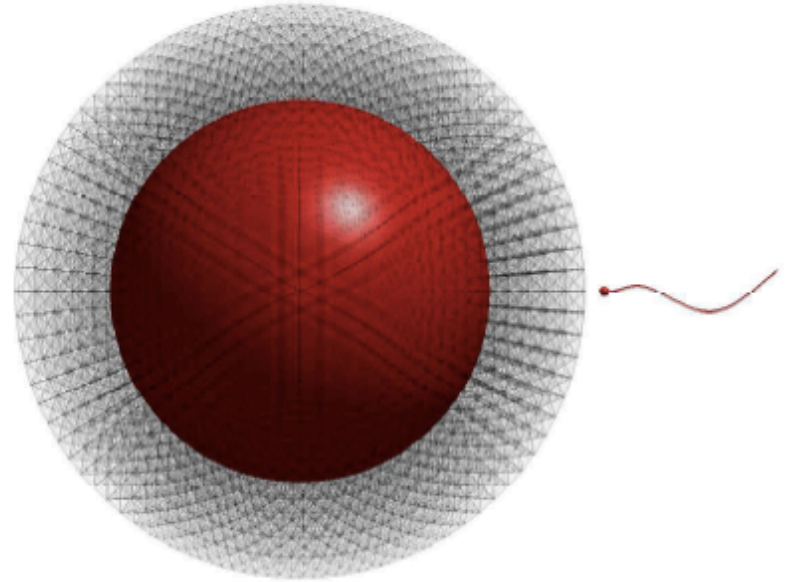


Network changes
orientation of swimmer

Ovum with its outer protective layer

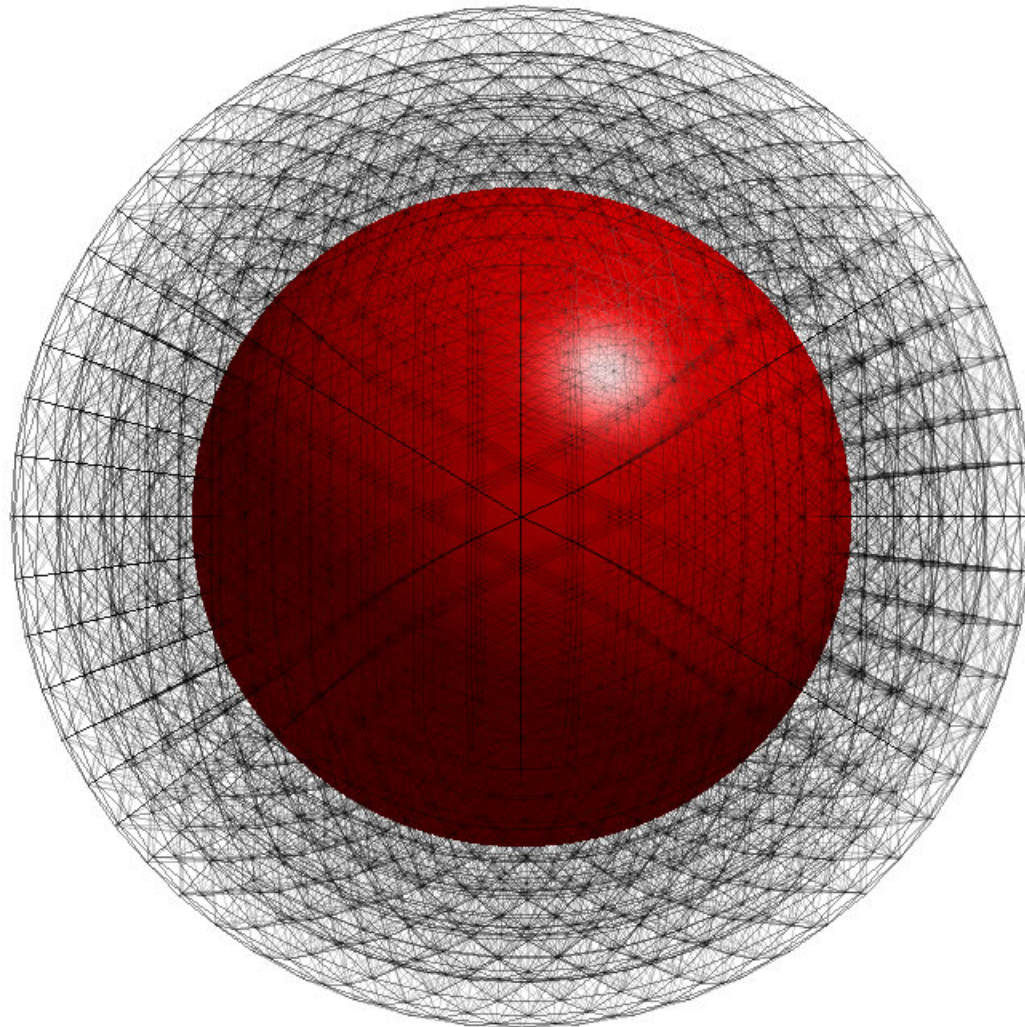


Wassarman, J. Biol. Chem. 283, 2008

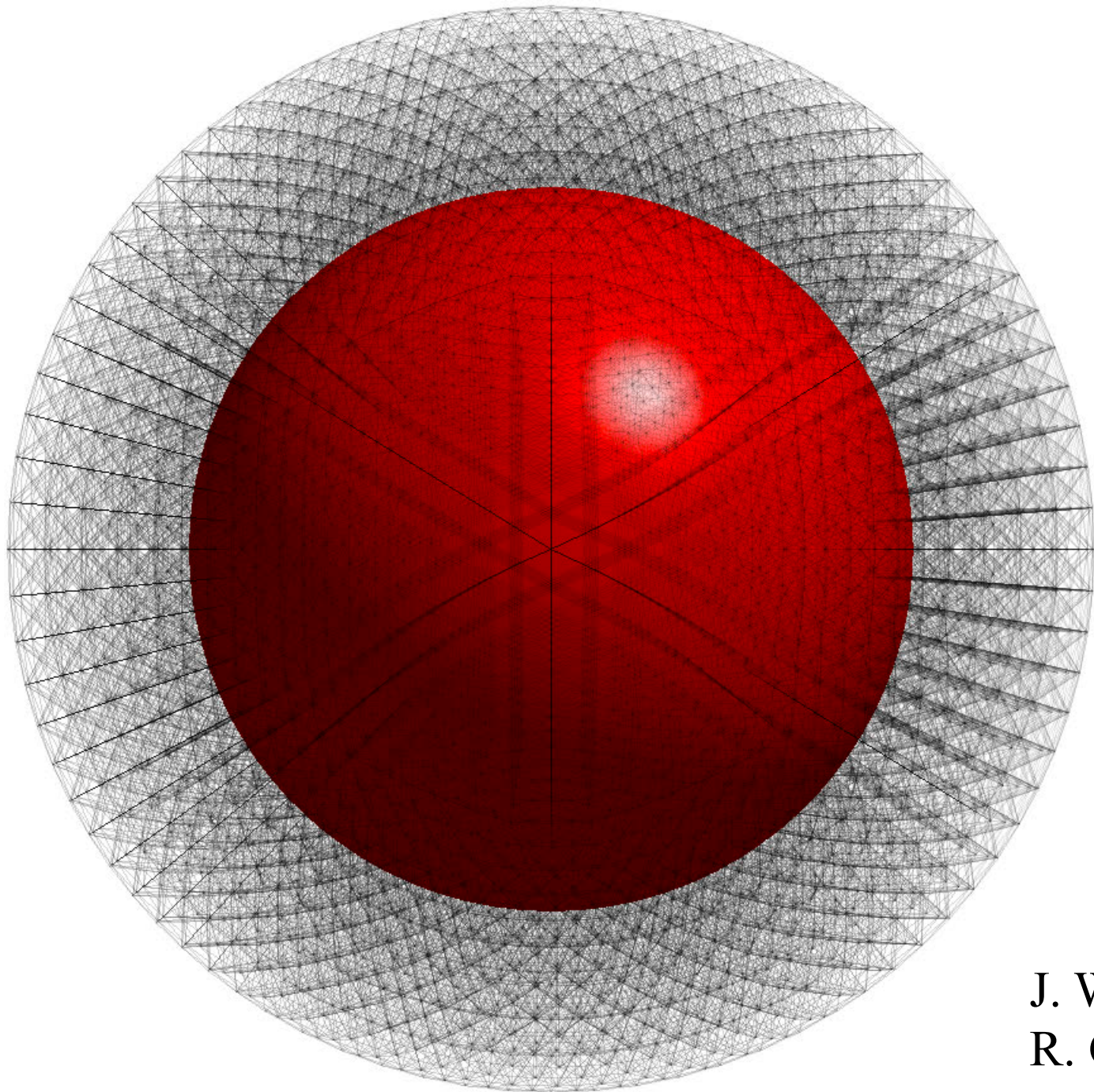


▷

Regularized images for sphere



Wrobel, Cortez,
Varela, Fauci,
J. Comp. Phys. 2016



J. Wrobel, J. Simons,
R. Cortez, LF 2016

Motivation

Vaginal delivery is linked to

- ▶ shorter post-birth hospital stays
- ▶ lower likelihood of intensive care stays
- ▶ lower mortality rates [1]

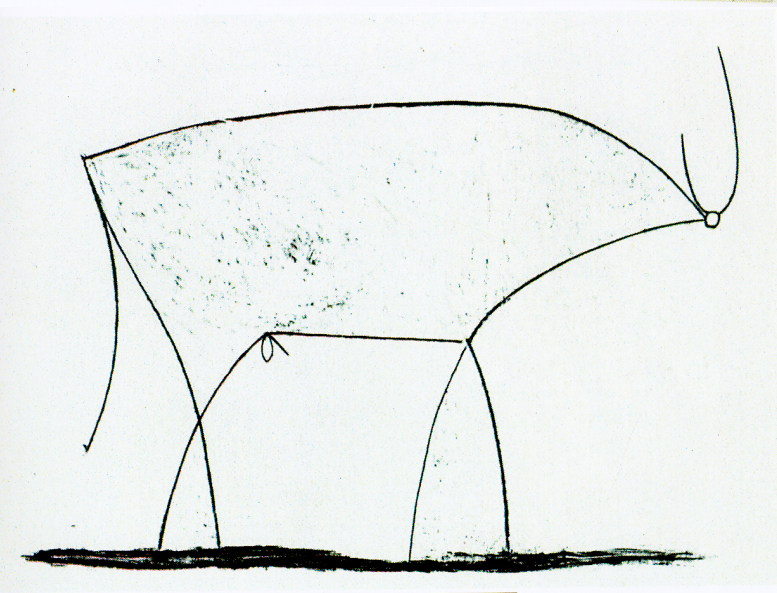
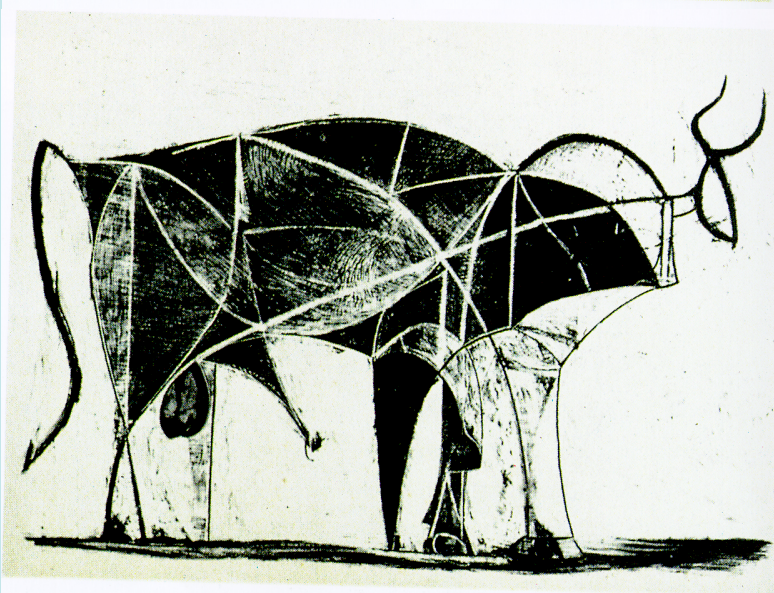
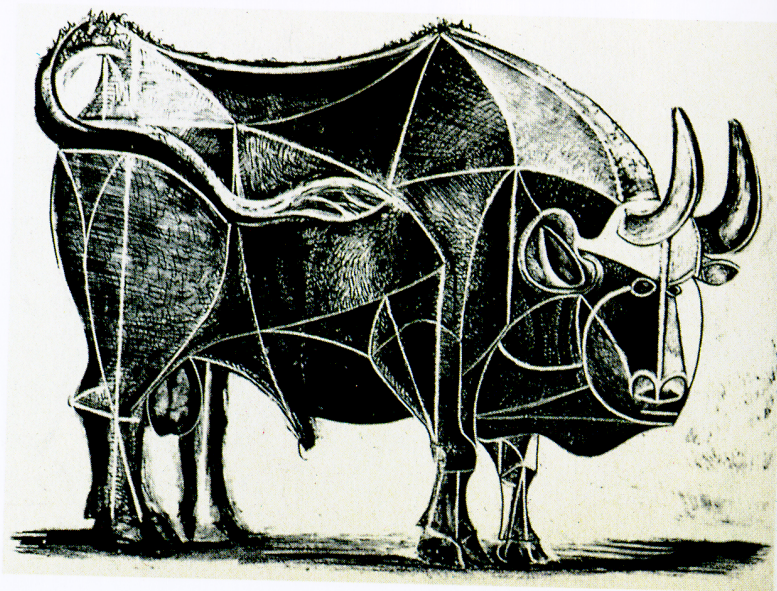
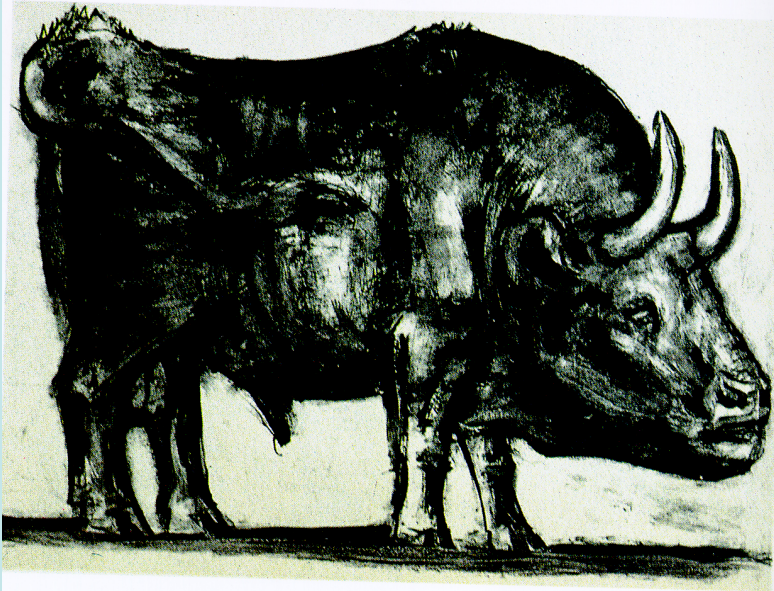
Fluid mechanics greatly informs the total mechanics of birth.

- ▶ vernix caseosa
- ▶ amniotic fluid



[1] C. S. Buhimschi, I. A. Buhimschi (2006). *Advantages of vaginal delivery*, Clinical obstetrics and gynecology.
Fig. 1: "HumanNewborn" by Ernest F - Own work. Licensed under CC BY-SA 3.0 via Commons - <https://commons.wikimedia.org/wiki/File:HumanNewborn.JPG#/media/File:HumanNewborn.JPG>
Fig. 2: "Postpartum baby2" by Tom Adriaenssen - <http://www.flickr.com/photos/inferis/110652572/>.
Licensed under CC BY-SA 2.0 via Commons - https://commons.wikimedia.org/wiki/File:Postpartum_baby2.jpg#/media/File:Postpartum_baby2.jpg

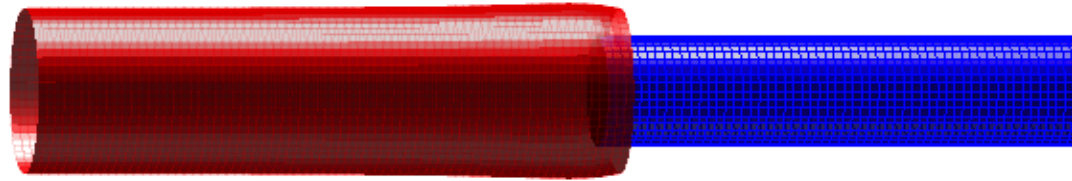
Abstraction - Picasso



Chris Johnson, U. Utah

Baby: rigid cylinder
Uterus: flexible tube





Elastic Tube

- ▶ Tube modeled by network of Hookean springs.
- ▶ Force at \mathbf{x}_l due to spring from

\mathbf{x}_m :

$$\mathbf{f}(\mathbf{x}_l) = \tau \left(\frac{\|\mathbf{x}_m - \mathbf{x}_l\|}{\Delta_{lm}} - 1 \right) \frac{(\mathbf{x}_m - \mathbf{x}_l)}{\|\mathbf{x}_m - \mathbf{x}_l\|}$$

- ▶ τ chosen to match elastic properties to physical experiment.

Rigid Inner Rod

- ▶ A constant velocity \mathbf{u} is specified in the z -direction.

R. Pealater, Tulane



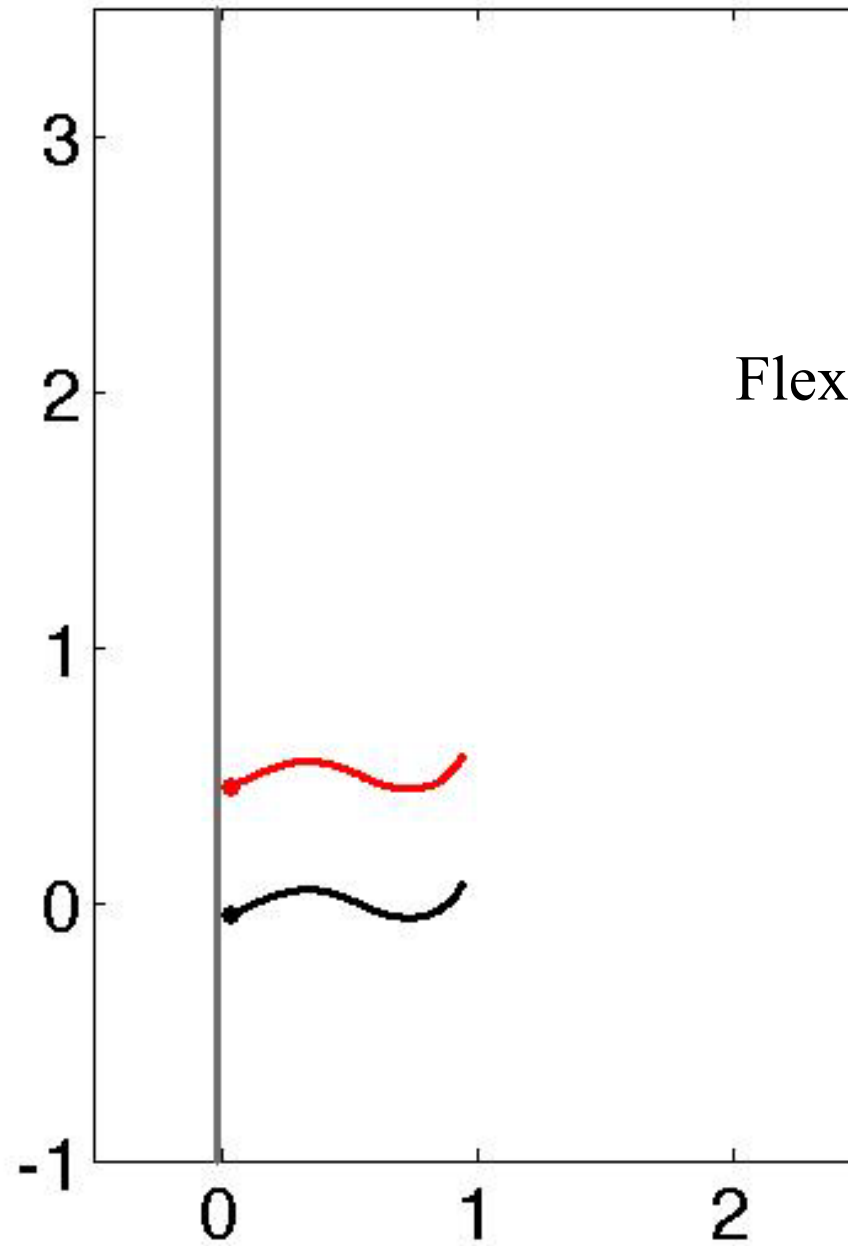


Can we characterize
tube buckling?

Nevermind the baby... this
is interesting elasto/
hydrodynamics...

Thank you so much!!!!!!!!!!!!

$t=0.1$



Flexible wall!!