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Mathematical Challenges in Control of Large-Scale Complex Systems

Be Curious[SM]

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Overview/Outline

Large-scale, interconnected systems present unique mathematical challenges in control

Context of talk

Attributes of complex/complicated systems

Challenges for control

Tools/techniques to address some of the challenges

Complex vs. Complicated Systems

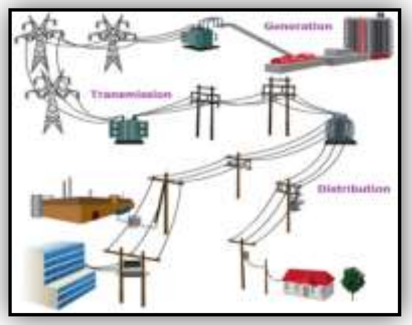
- Complex systems demonstrate adaptation and self organization, emergent behavior
- Complicated systems are not necessarily complex
- Engineering is about assembling components that work in specific ways

essay concepts

Engineering complex systems *

The emergent properties of complex systems are far removed from the traditional preoccupation of engineers with design and purpose.

Complex



Electric smart grid

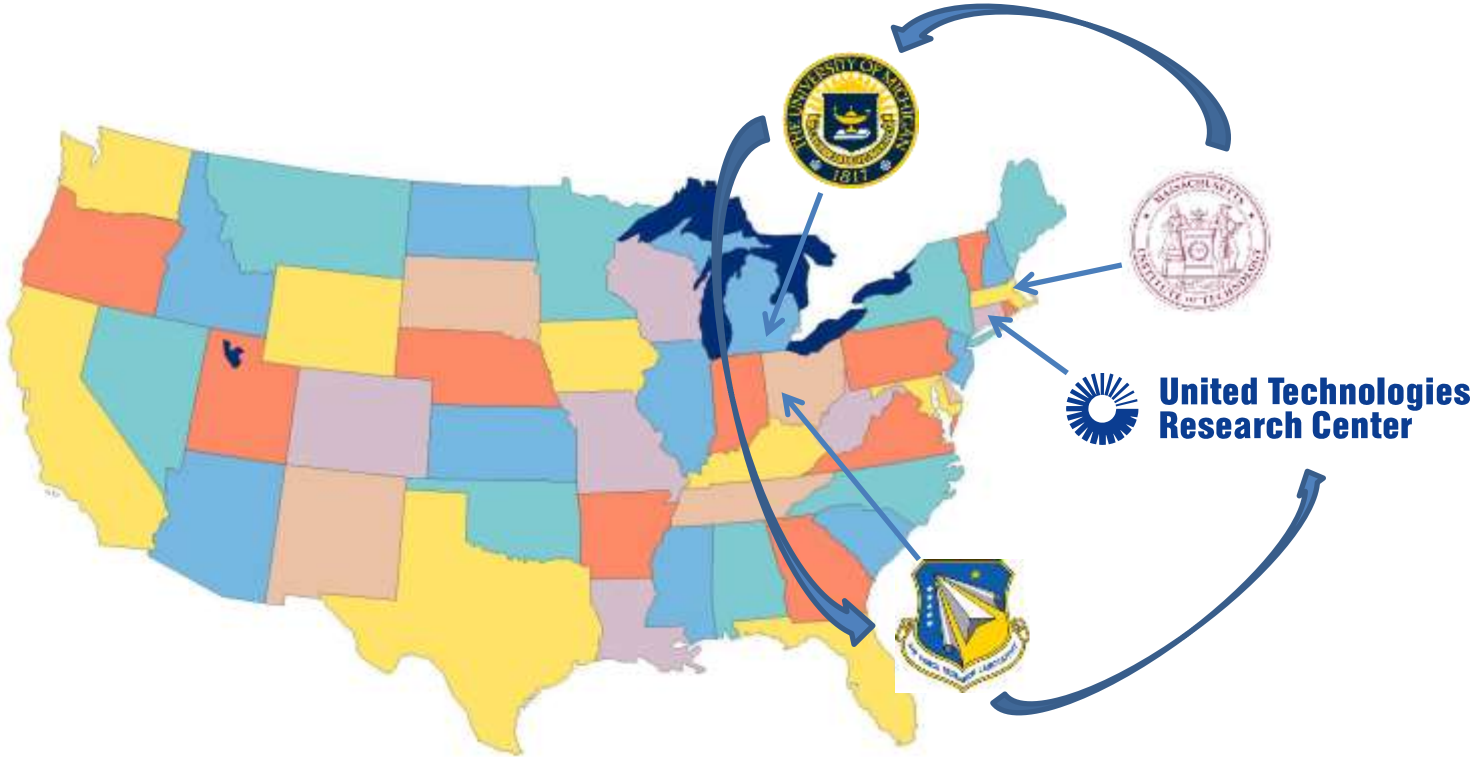
Complicated

Highly Integrated Modern Aircraft System

Future Building Systems

*Engineering Complex Systems, J. M. Ottino, Nature, Vol. 427, 29 January 2004

Personal Background



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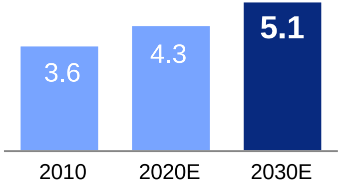
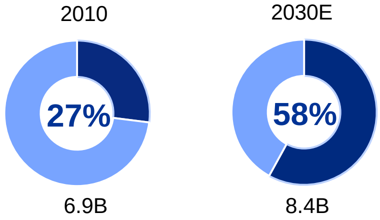
UTC Aerospace Systems | Pratt & Whitney (A United Technologies Company) | United Technologies (Climate | Controls | Security) | Otis (A United Technologies Company)

Large-scale, interconnected systems are everywhere in modern life

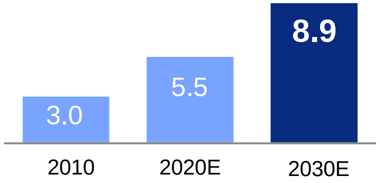
Growing urban population
(billions)



Expanding global middle class
(share of population)



Increasing demand for air travel
(revenue passenger miles in trillions)



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Global megatrends will continue to drive the need for these systems

UTC Aerospace

Massive complexity, challenges for estimation and control

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UTC Buildings

Massive complexity, challenges for design, implementation, modeling, and control

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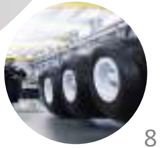
The Systems Department at UTRC develops innovative technology solutions and concepts in the area of complex adaptive systems. We focus on solving problems related to designing, controlling and managing systems that are characterized by complex interactions between a large number of independent and heterogeneous components and subsystems.

Control Systems

- Dynamic system modeling
- Control analysis and design
- Estimation, filtering, perception and navigation
- Systems and control architecture
- Optimization
- Verification and validation



Attributes of Large-Scale, Complicated Systems

High dimension
- Large numbers of components
- Subsystems or components may have dynamic coupling

Composition in multiple domains
- Mechanical, fluid, electrical, thermal, etc.

Model uncertainty

Safety criticality

Operational limits

Challenges for Control

Estimation

Fault detection and diagnostics

Computation

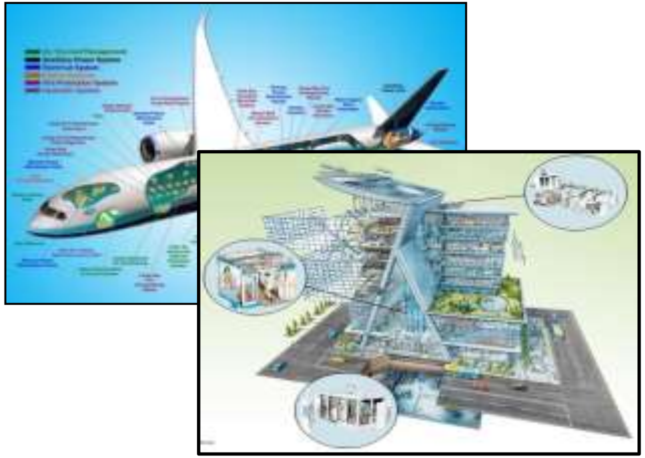
Communication

Cyber physical security

Verification and validation

Control architecture selection

Control gain selection/control design

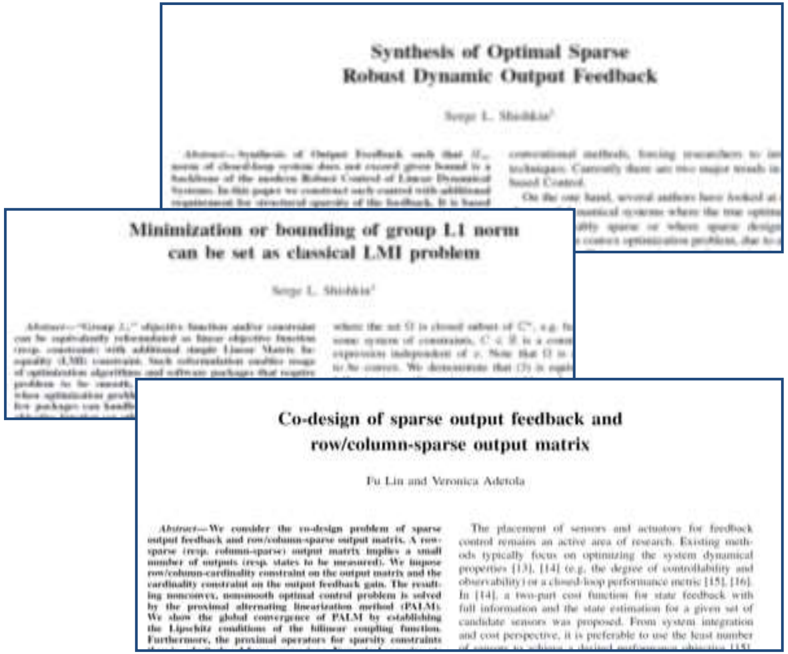


Sparse Control: Efficient, scalable, robust control architecture

Addressing the Challenges

Optimal sparse output feedback control design

Co-design of sparse feedback and outputs



Synthesis of Optimal Sparse
Robust Dynamic Output Feedback

Serge L. Shishkin

Minimization or bounding of group L1 norm
can be set as classical LMI problem

Serge L. Shishkin

Co-design of sparse output feedback and
row/column-sparse output matrix

Fu Lin and Veronica Adetola

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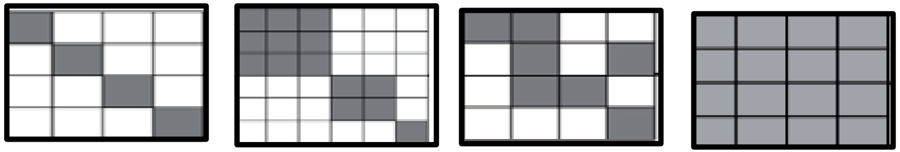
Addressing the Challenges

Possible Control Configurations

Diagonal (SISO) | Block diagonal | Sparse | Full MIMO



APPROACHES

System level decomposition

Hierarchical – information flow
Graph theoretic

Sub-system level decomposition and control

Control architecture selection based on interaction measures

Independent of control design

Distributed control design with constraint on admissible communication links and/or control architecture

Structure + control design

Sparse Output Feedback Control

Given an LTI system with state *x*, inputs *u*, outputs *y*, disturbances *d*

$$\begin{aligned} \dot{x} &= Ax + B_2 u + B_1 d, \\ y &= Cx, \\ z &= \begin{bmatrix} Q^{1/2} x \\ R^{1/2} u \end{bmatrix} \end{aligned}$$

Find the stabilizing linear output-feedback control policy $u = Fy = FCx$ that

1. Guarantees (robust) stability of the closed-loop system
   Minimizes H_2 norm of the transfer function from *d* to *z*

2. Promotes feedback sparsity
   F has minimum number of nonzero entries

F. Lin, M. Fardad and M. Jovanovic : Design of optimal sparse gains via the alternating direction method of multipliers. IEEE TAC 58(9) 2013

Sparse Output Feedback Control

Optimization for Control Sparsity

$$\underset{\text{closed-loop objective}}{minimize \quad J(F)} \quad + \quad \underset{\text{sparsity promoting term}}{\gamma \; g(F)}$$

$$g(F) = card(F) \text{ i.e. number of nonzero elements of matrix } F$$
$$\gamma > 0 \; - \text{performance vs sparsity trade} - \text{off}$$

Augmented Lagrangian Approach

$$\min_{P>0,F,G} \; \frac{1}{2}\text{tr}\left(B_1^T P(F) B_1\right) + \gamma g(G)$$
$$F - G = 0$$

where *P(F)* is defined by the Lyapunov equation:

$$A_F^T P(F) + P(F) A_F = -(Q + C^T F^T R F C), \qquad P(F) > 0 \qquad A_F = A - B_2 F C$$

Augmented Lagrangian for the problem:

$$\mathcal{L}_\rho(F, G, \Lambda) = \frac{1}{2}\text{tr}\left(B_1^T P(F) B_1\right) + \text{tr}\left(\Lambda^T (F - G)\right) + \frac{\rho}{2}\|F - G\|_F^2$$

Algorithm: Alternating direction method of multipliers (ADMM)

Sparse Output Feedback Control

Alternating Direction of Multipliers Method

At iteration *k*, given (F_k, G_k, Λ_k) such that $A_{F_k} < 0$, (A_{F_k}, B_1) controllable:

1. $F_{k+1} = \arg\min_F \quad \mathcal{L}_\rho(F, G_k, \Lambda_k)$,

2. $G_k = \arg\min_G \quad \mathcal{L}_\rho(F_{k+1}, G, \Lambda_k)$,

3. $\Lambda_{k+1} = \Lambda_k + \rho\left(F_{k+1} - G_{k+1}\right)$.

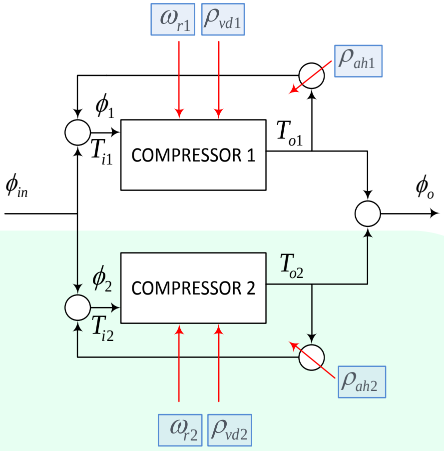
Using results [*,**], we can expect the above procedure to globally converge to a local optimum. The quality of the local optimum is to be seen. The conditions under which it converges to the optimal feedback are to be researched.

[*] Makila, P.M., *Linear Quadratic Design of Structure-Constrained Controllers*, in Proc. of 2nd American Control Conference, San Francisco, CA, 1983, pp.1011--1019

[**] Makila, P.M., *On the Anderson--Moore Method for Solving the Optimal Output Feedback Problem,* IEEE Trans. Automatic Control, AC-29 (9),1984, pp.834--836

Sparse Output Feedback Control

Aircraft Dual Cabin Air Compressor



- Dynamical model with 19 states,
- 3 outputs, 3 control inputs, 1 disturbance input
- Baseline control: SISO PI

- H_2 controller: Static output feedback
- Integrator needed for tracking and disturbance rejection
- Augmented system to include controller integrator states
- Static output feedback controller becomes 3 by 6 matrix.
- Enforce structure on the sparse controller – Link the proportional and integral terms through a change of coordinates

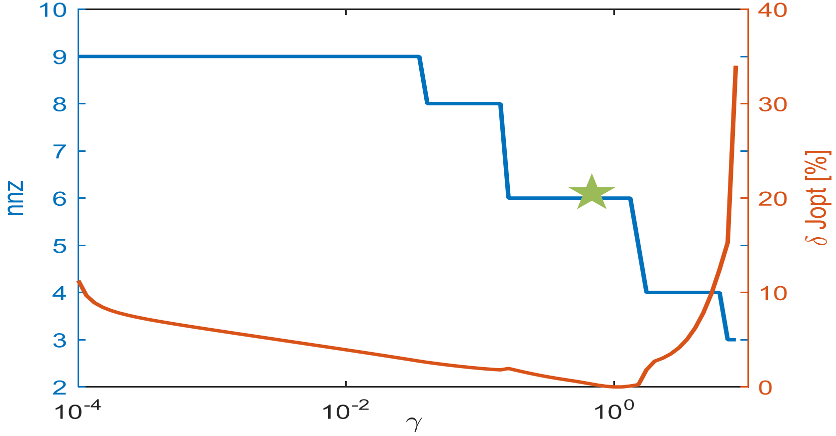
$$F = [F_p\,(3\times 3) : Fi(3\times 3)] \xrightarrow{T} \begin{bmatrix} F_{p11} & F_{i11} & Fp_{12} & F_{i12} & Fp_{13} & F_{i13} \\ F_{p21} & F_{i21} & Fp_{22} & F_{i22} & Fp_{23} & F_{i23} \\ F_{p31} & F_{i31} & Fp_{32} & F_{i32} & F_{p33} & Fi_{33} \end{bmatrix}$$

Block control structure

- Implement block-sparsity with block size [1,2] for each subsystem

Sparse Output Feedback Control
787 Dual Cabin Air Compressor

$$Jopt = tr(B_1{}^T P(F) B_1)$$
$$nnz = \#\ of\ non-zero\ control\ loops$$



Optimal number of non zero loops = 6

Sparsity promoting decentralized control has the same structure as RGA pairing

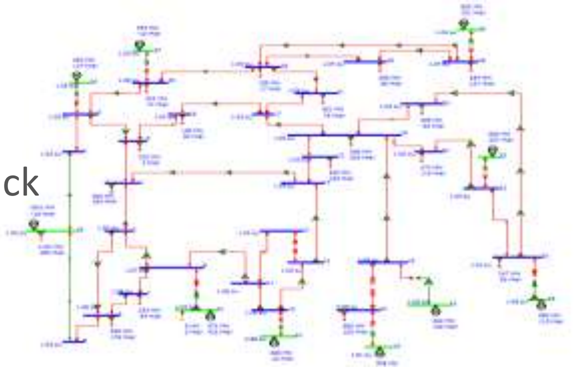
$$RGA = \begin{bmatrix} 1.0223 & -0.0044 & -0.0178 \\ -0.0043 & 0.3162 & 0.6880 \\ -0.0180 & 0.6882 & 0.3298 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

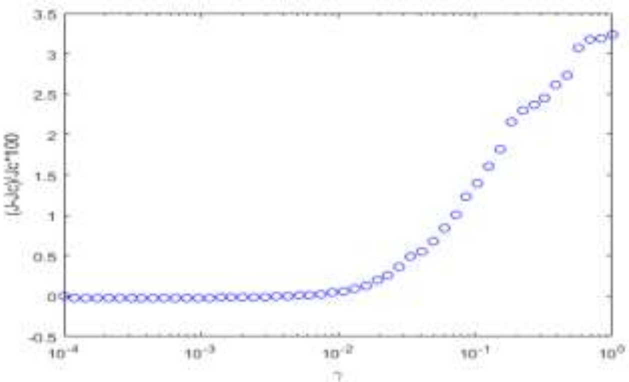
Sparse Output Feedback Control

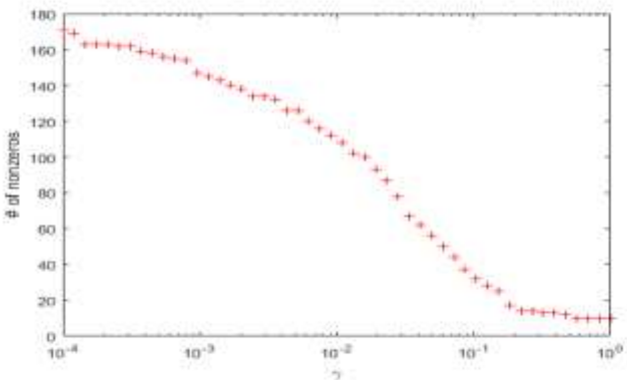
Power System

Demonstrate output feedback examples: New England 39-bus

- 39 buses and 10 generators
- 75 states, 9 inputs, and 19 outputs (phase angles and frequency)
- Objective: suppress wide-area oscillation using sparse output feedback gain: *u = Fy*
- Explicit tradeoff between sparsity and controller performance







Sparse Output Feedback Control – Output Co-Design

Problem Formulation

Given an LTI system with state *x,* inputs *u,* outputs *y,* disturbances *d*

$$\begin{aligned} \dot{x} &= Ax + B_2 u + B_1 d, \\ y &= Cx, \\ z &= \begin{bmatrix} Q^{1/2} x \\ R^{1/2} u \end{bmatrix} \end{aligned}$$

Now promote sparsity of output matrix, zero columns eliminate sensors

Find the stabilizing linear output-feedback control policy $u = Ky = KCx = Fx$ that

1. Guarantees (robust) stability of the closed-loop system

   Minimizes H_2 norm of the transfer function from *d* to *z*

2. Promotes feedback sparsity

   K has minimum number of nonzero entries

Sparse Output Feedback Control – Output Co-Design

Problem Formulation

$$\begin{array}{lll} \text{minimize}_{K,C,F} & J(F) & \text{closed-loop norm } (\mathcal{H}_2, \mathcal{H}_\infty, \ldots) \\ \text{subject to} & F = KC & \text{structured state feedback} \\ & \text{card}(K) \leq s & \text{sparse output feedback} \\ & \text{card}_{\text{row}}(C) \leq r & \text{row/column sparse output} \end{array}$$

$$\begin{array}{ll} \text{minimize}_{K,C,F} & J(F) + \frac{\gamma}{2}\|F - KC\|_F^2 \\ \text{subject to} & \text{card}(K) \leq s, \quad \text{card}_{\text{row}}(C) \leq r \end{array}$$

$$f(K) = \begin{cases} 0, & \text{card}(K) \leq s \\ \infty, & \text{otherwise} \end{cases} \qquad g(C) = \begin{cases} 0, & \text{card}_{\text{row}}(C) \leq r \\ \infty, & \text{otherwise} \end{cases}$$

No convex approximation of the cardinality functions

$$\underset{K,C,F}{\text{minimize}} \; \Phi := f(K) + g(C) + J(F) + h(K,C,F)$$

$$h(K,C,F) := \frac{\gamma}{2}\|F - KC\|_F^2$$

Sparse Output Feedback Control – Output Co-Design

Problem Structure

$$\underset{K,C,F}{\text{minimize}}\ \Phi := f(K) + g(C) + J(F) + h(K,C,F)$$

$$h := \frac{\gamma}{2}\|F - KC\|_F^2$$

Nonconvex nonsmooth

Control objective

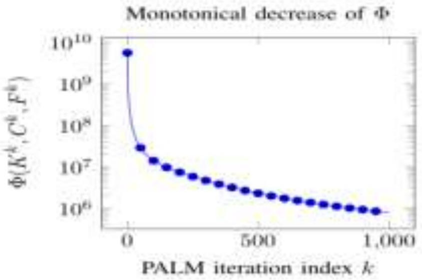
Bilinear coupling

One approach is to use convex relaxation to handle sparsity-promoting terms

Many heuristics to handle bilinear coupling term (but with little guarantee)

We employ the proximal alternating linearization method (PALM)

PALM has convergence guarantee and the objective value is monotonically decreasing throughout the iterates



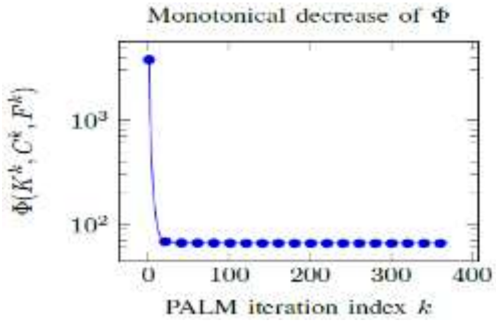
Bolte, Sabach, and Teboulle, Math Prog. ‘14,
Attouch, Bolte, and Svaiter, Math Prog. ‘13.

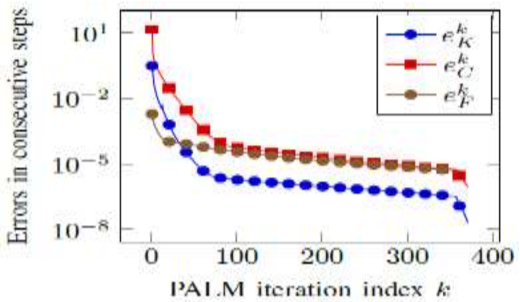
ADMM-based algorithms have appeared for special nonconvex functions:
Wang, Yin, Zheng ‘15, Hong, Luo, Razaviyayn ’16 …

Sparse Output Feedback Control – Output Co-Design

Mass Spring Example

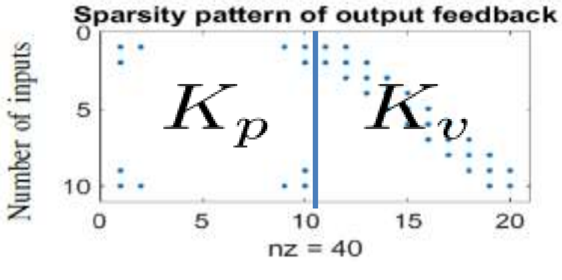
$$A = \begin{bmatrix} O & I \\ T & O \end{bmatrix} \in R^{2N\times 2N}, \quad B_1 = B_2 = \begin{bmatrix} O \\ I \end{bmatrix} \in R^{2N\times N}, \quad x = \begin{bmatrix} p \\ v \end{bmatrix} \in R^{2N}$$

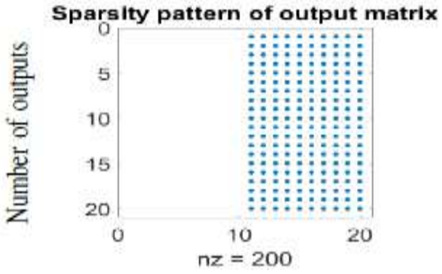




$$e^k_K = \|K^{k+1} - K^k\|_F$$
$$e^k_C = \|C^{k+1} - C^k\|_F$$
$$e^k_F = \|F^{k+1} - F^k\|_F \leq 10^{-5}$$

Impose 20% element sparsity in *K* and 50% column sparsity in *C*

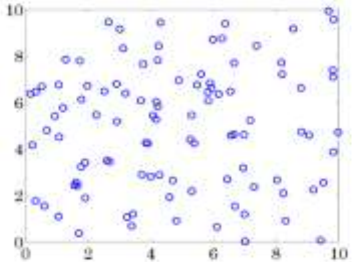




Cardinality functions allow us to control the sparsity levels directly

Sparse Output Feedback Control – Output Co-Design

Numerical Example



A network of 100 unstable coupled systems

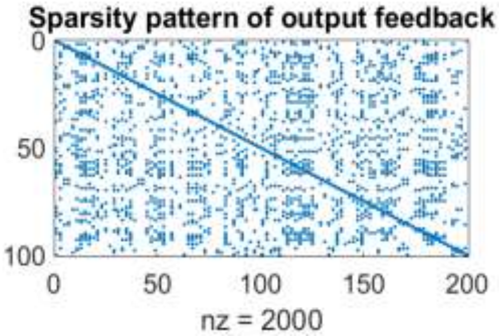
$$\dot{x}_i = A_{ii}x_i + \sum_{i \neq j} A_{ij}x_j + B_i u_i + B_i w_i$$

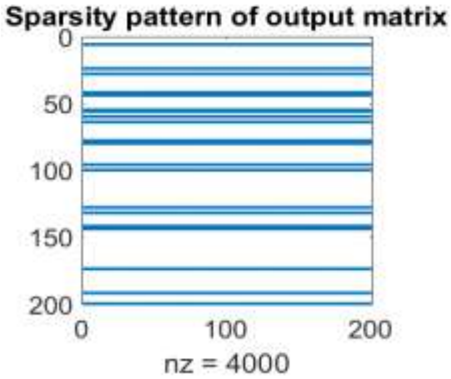
$$A_{ii} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad A_{ij} = \alpha_{ij} I, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_{ij} = e^{-\|p_i - p_j\|_2}, \text{ and } p_i \text{ denotes the position}$$

Design output matrix with 10% nonzero rows

Design output feedback with 10% nonzero elements





Challenges for Control

Estimation

Fault detection and diagnostics

Computation

Communication

Cyber physical security

Verification and validation

Control architecture selection

Control gain selection/control design

Acknowledgements

Wei Kang

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