Conceptual Climate Models

Minitutorial Part I

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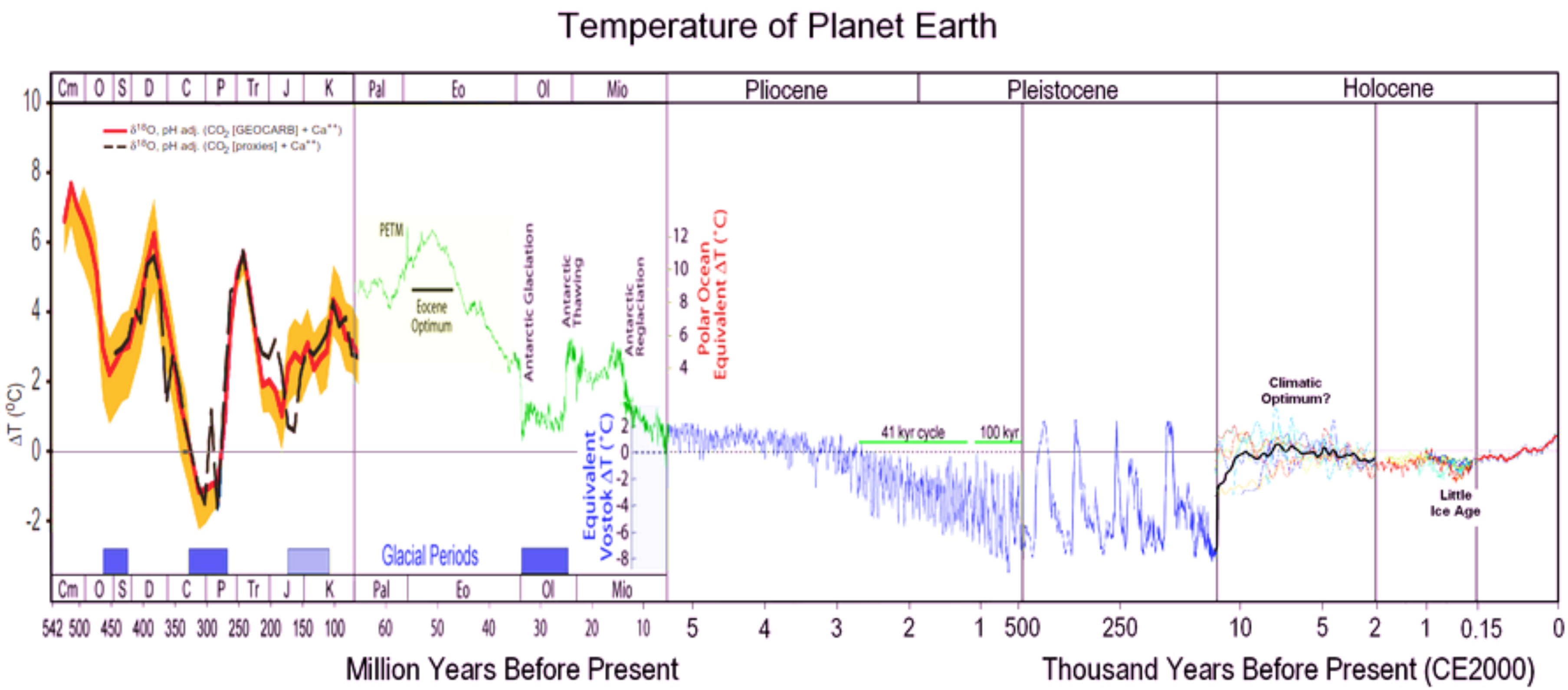
Summary

- A brief history of Earth's climate
- What is climate?
- Some physics background
- 0D Energy Balance Model (EBM)
- Tipping points/ Bifurcations/ Hysteresis
- 1D EBM (infinite dimensional)

A brief history of climate



Major features: glacial cycles with a change in amplitude and frequency, some warm climate in the past, hockey stick at PETM.

Image source: Wikipedia

What is climate?

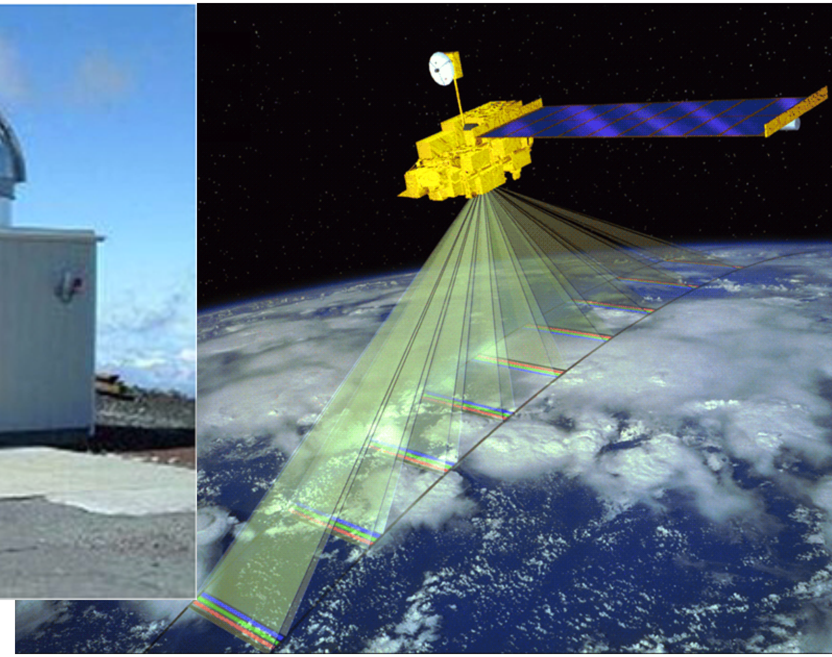
- **Climate = 30 year average of weather**

Weather: Will I need an umbrella tomorrow?

Climate: Should I own an umbrella?

Slide credit: S. Oestreicher, JMM Minitutorial 2013

How do we observe climate?

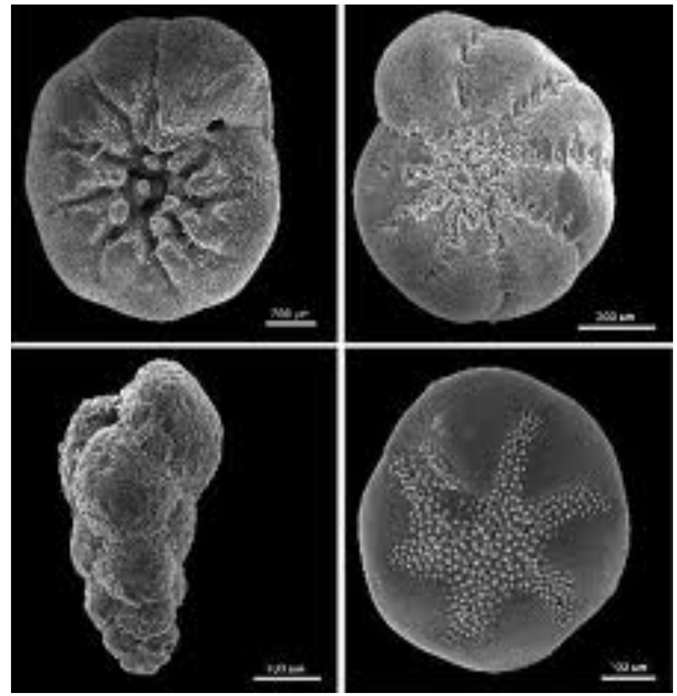


http://spaceplace.nasa.gov/earth-card-game/terra-lrg.en.png

Observing current climate

http://cdiac.ornl.gov/d13_flask_mauna_loa.html
http://www.dartmouth.edu/~mpayres/People/Sharon.7506.web.jpg http://www.whoi.edu/ooi_cgsn/auvs-gliders?tid=1621&cid=137956&article=95673

How do we observe past climate?

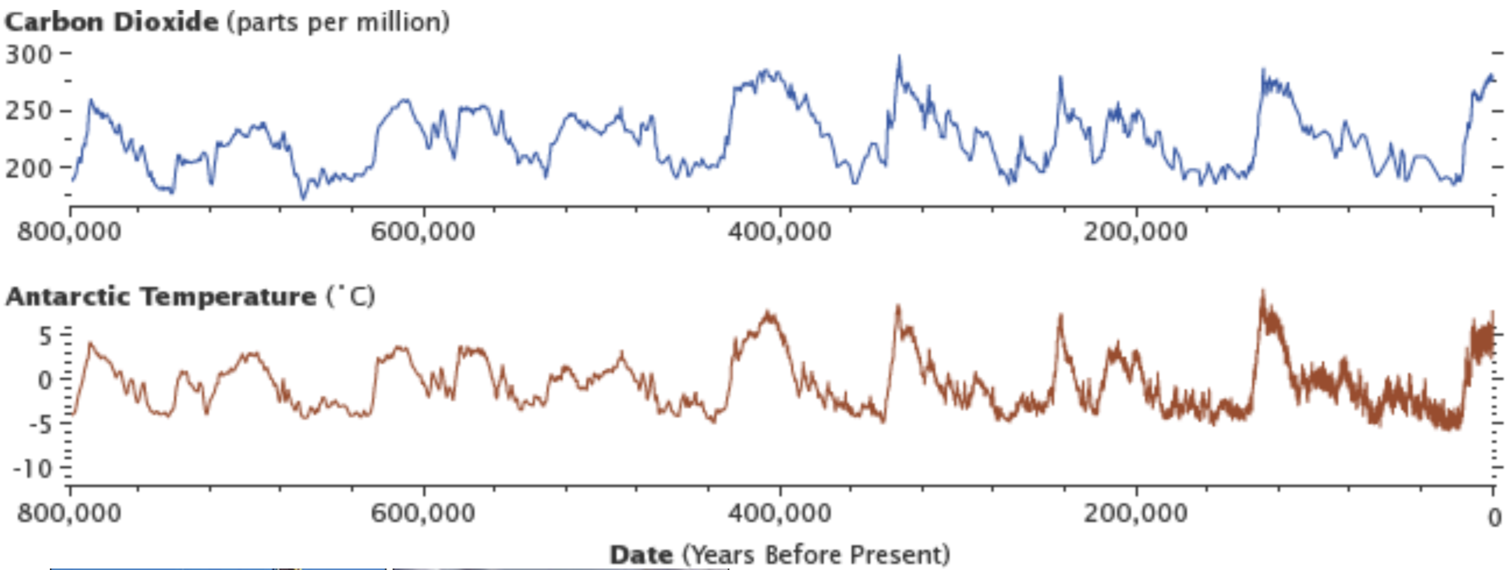


Benthic foraminifera

Hold the long term memory of Earth’s climate

Image source: usgs.gov

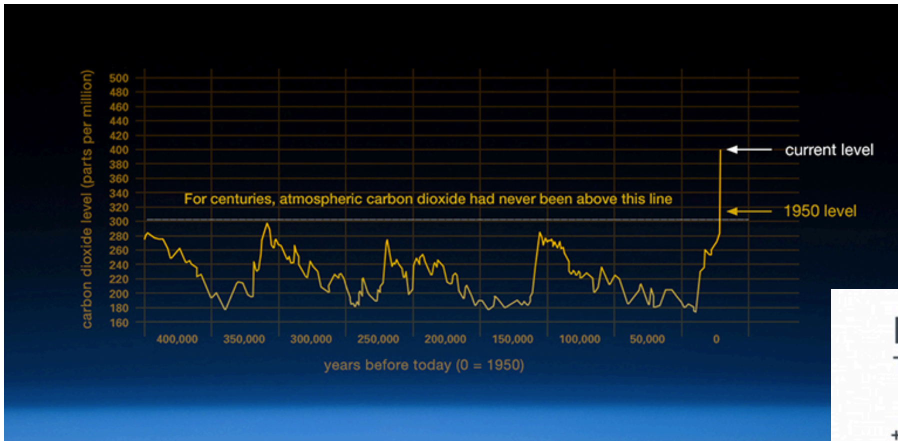
How do we observe past climate?

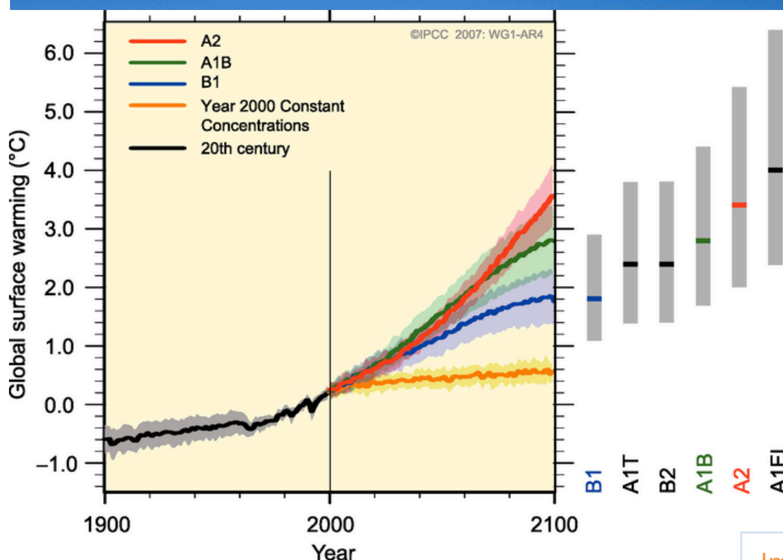


Antartic ice core

Image source:
http://earthobservatory.nasa.gov/Features/CarbonCycle/page4.php

How does CO^2 affect climate?





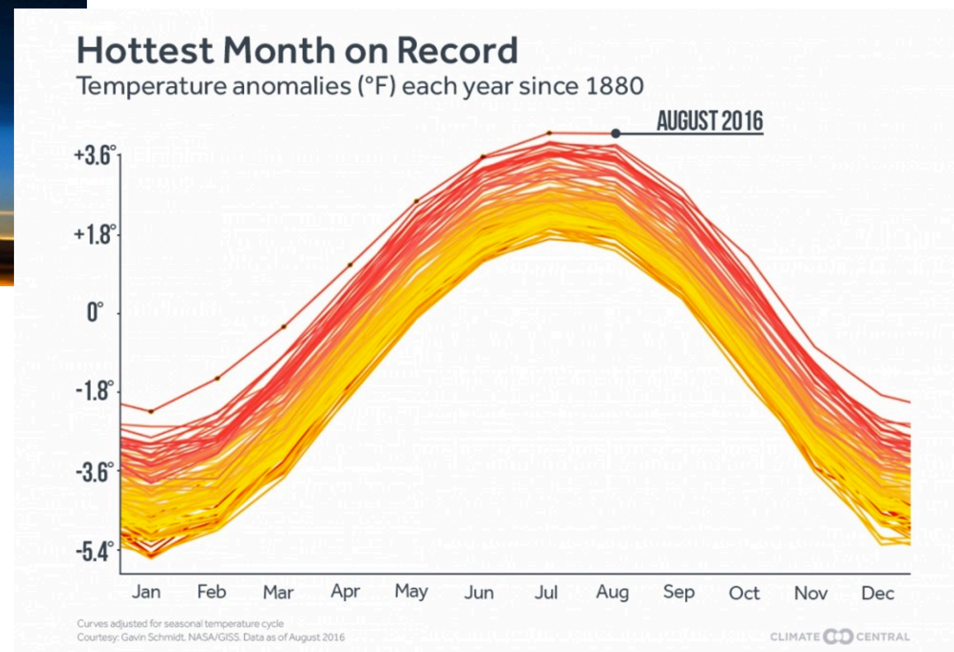
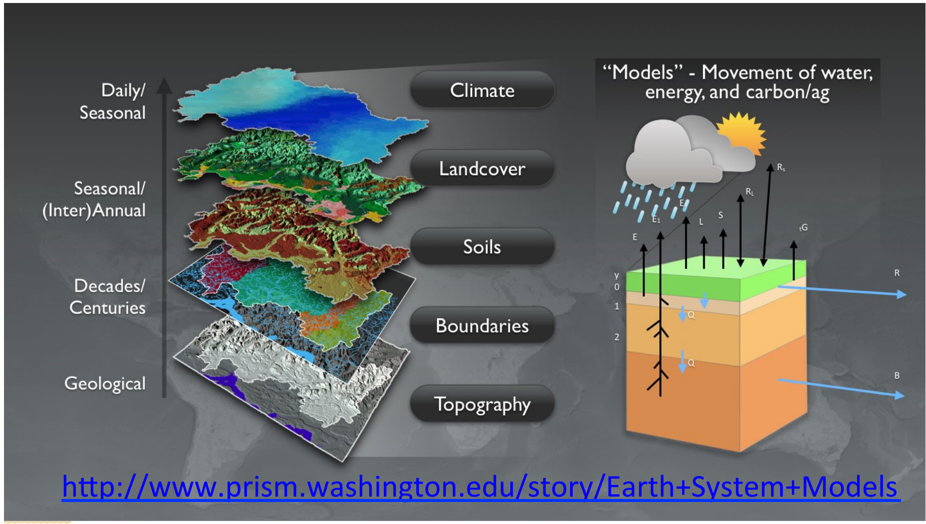


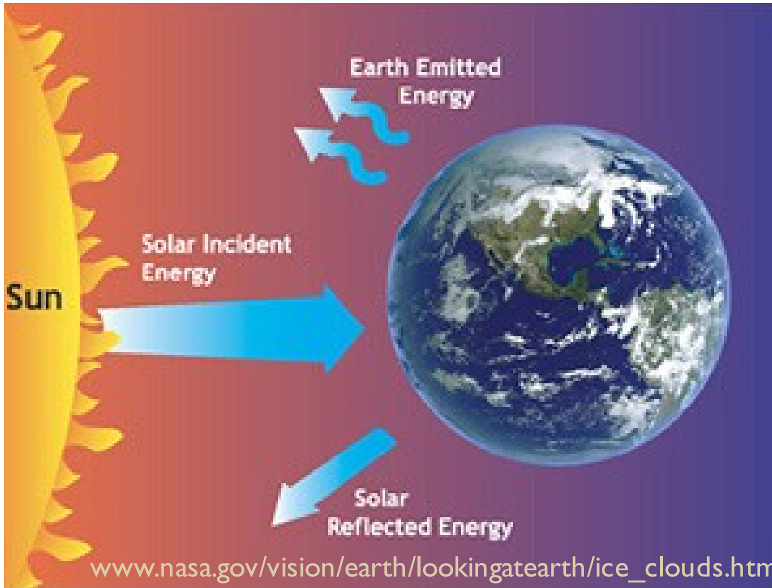
Image source:
http://climate.nasa.gov/climate_resources/24/
https://medium.com/@350/temperature-check-1f076624c55b#.mvvf6oqf8
https://www.ipcc.ch/publications_and_data/ar4/wg1/en/spmsspm-projections-of.html

How to model climate?

- No detail is too small:
  Global Circulation Models
  GCM



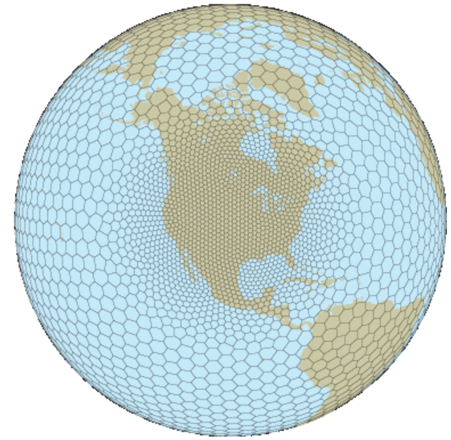
- The rest is details:
  Conceptual Climate Models
  CCM

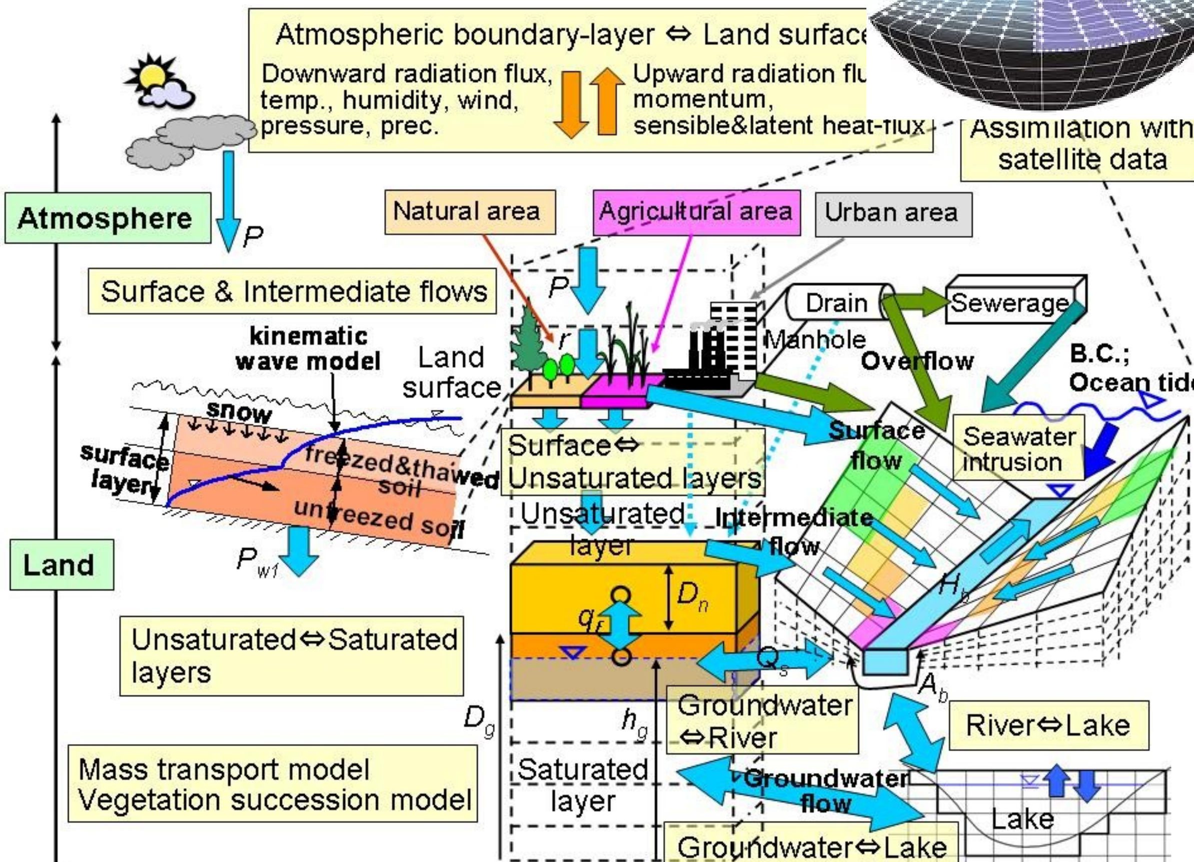


Slide source: S. Oestreicher
JMM Minitutorial 2013.

Global Circulation Models

Complicated choices starting from the grids.



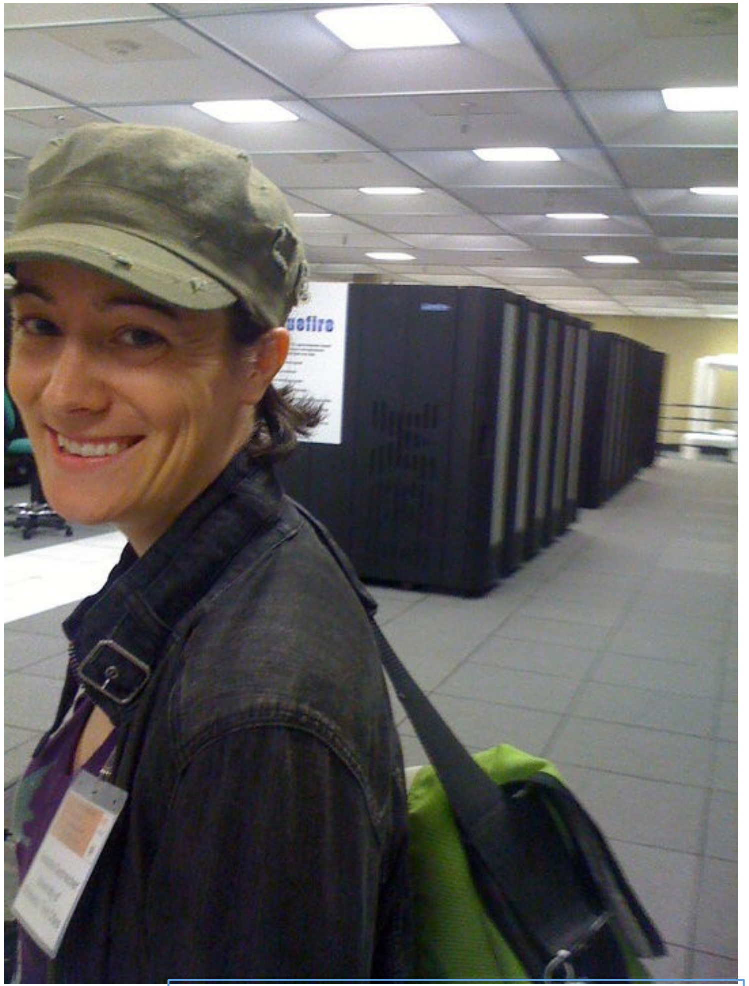


Must decide the the processes of each part and interactions among them.

Slide source: S. Oestreicher JMM Minitutorial 2013.

Global Climate Models

- Processes: physics, biology, chemistry
- Computer Science
  Data mining, coupling non-similar grids, error analysis, parallel processing, time optimization
- Statistics
  Extreme events, trends, and averaging
- Mathematics
  Data assimilation, numerical analysis, PDE



Slide source: S. Oestreicher (now S. Schumacher), JMM 2013

A Simpler Life: Conceptual Climate Modeling

Can one develop a mathematical model that captures Earth's climate, ie through **temperature, ice cover, and CO2 level**?

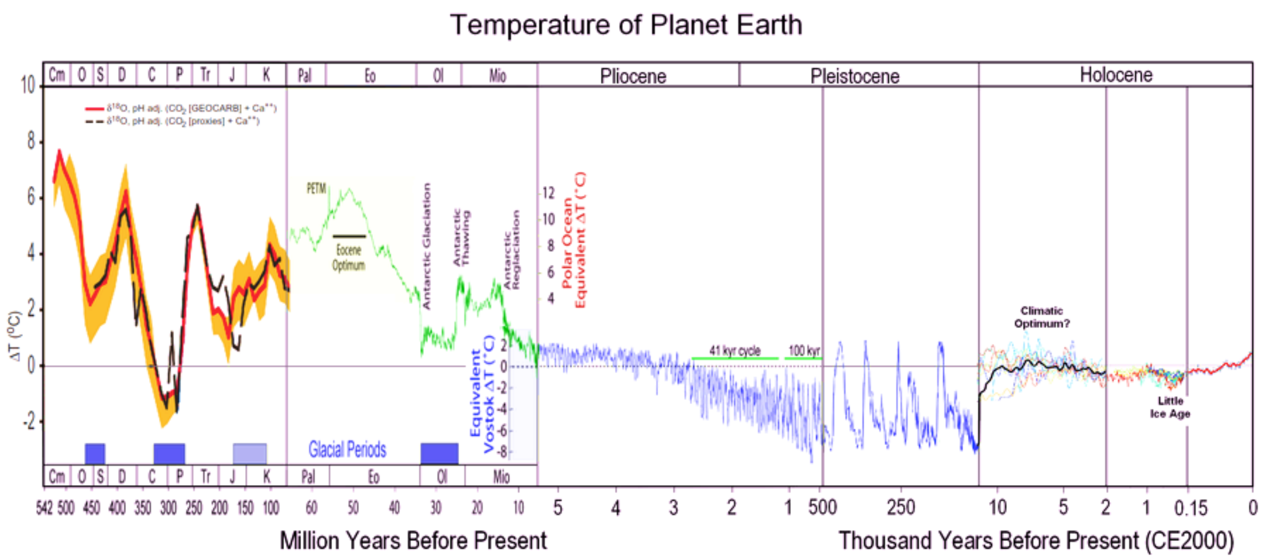


Image source: Wikipedia

The Modeling Cycle

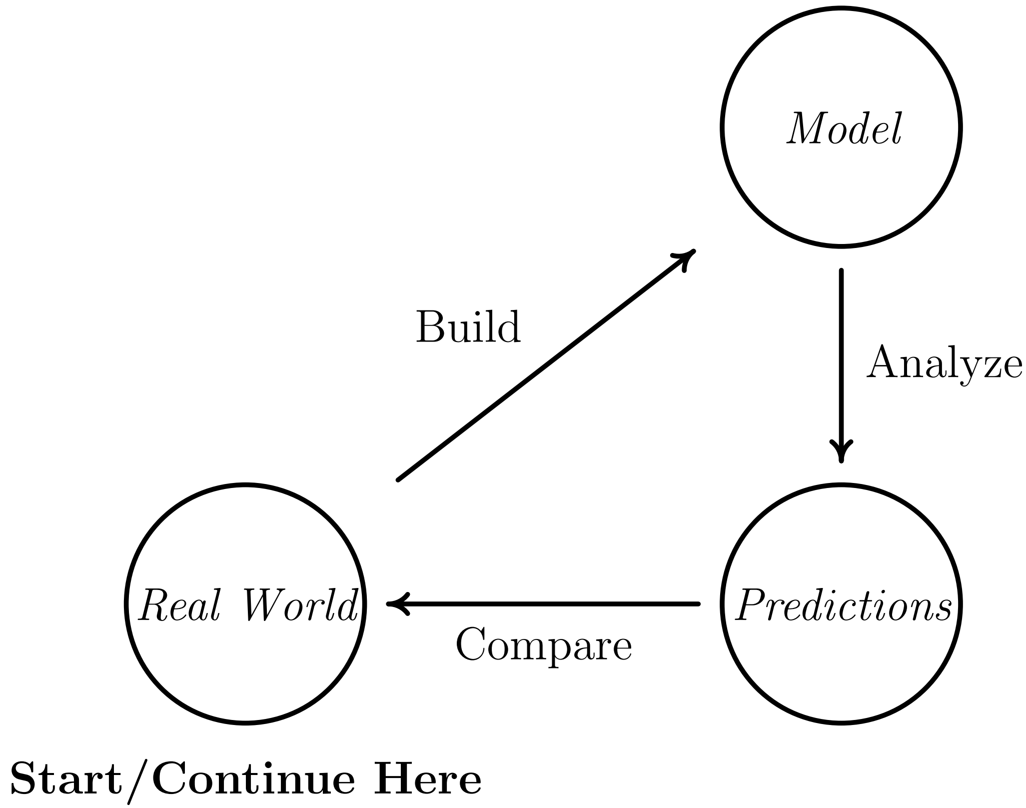


Figure 1: The modeling cycle.

Source: http://dimacs.rutgers.edu/MPE/Energy/DIMACS-EBM.pdf

Some well known laws

1. Conservation of energy: Energy in = Energy out
2. Earth gets its energy from the sun
3. The Stefan Boltzmann black body radiation:

(Incoming solar energy is used to heat up the planet)

“A black body will emit a certain amount of energy *E* depending on it’s temperature *T*.”

$$E = \sigma T^4$$

What does Earth do with all that energy?

Global Energy Flows W m⁻²

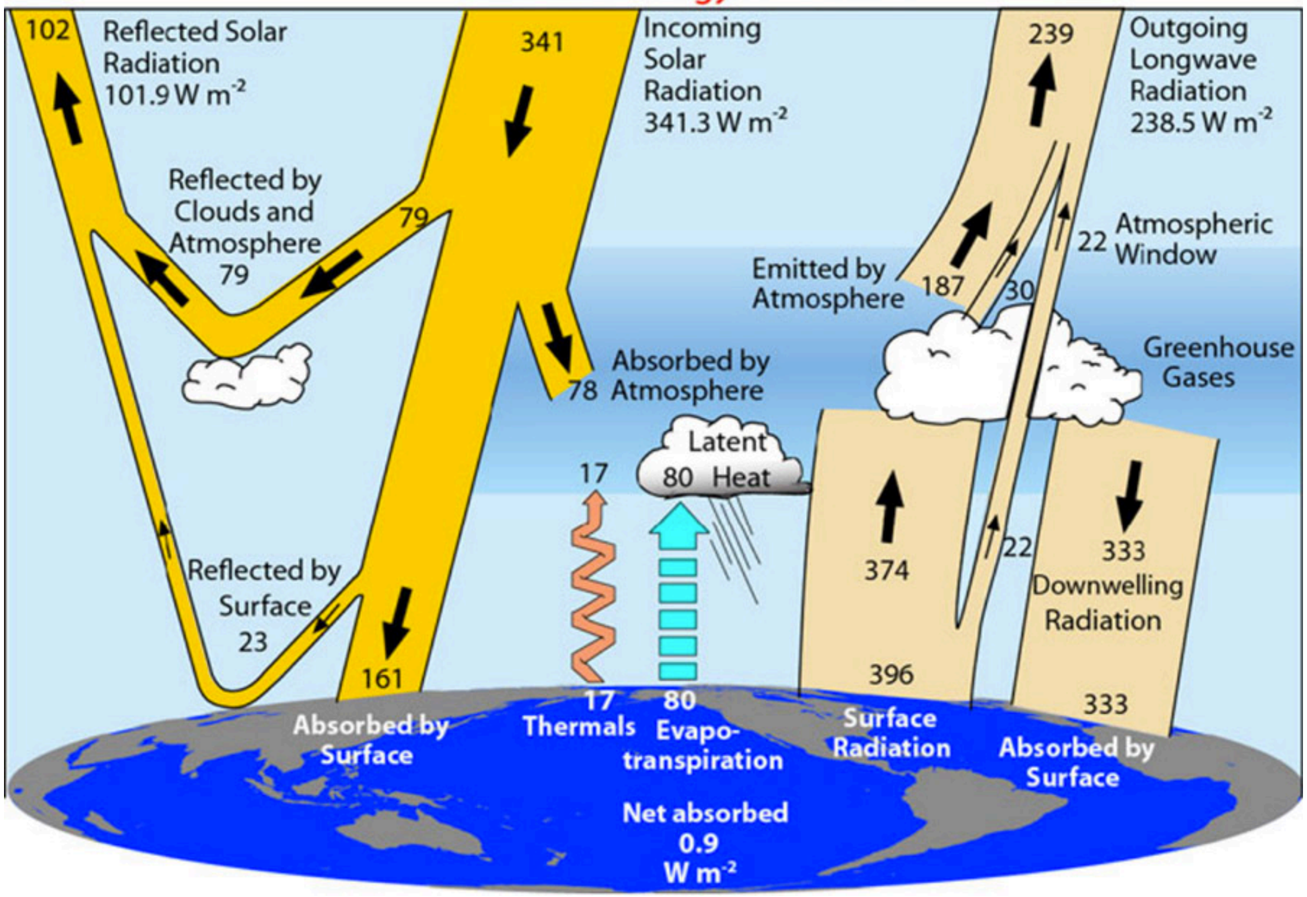


Image source: Trenberth, Kiehl, Fazulo
http://scied.ucar.edu/radiation-budget-diagram-earth-atmosphere

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

The climate of Earth is represented by one point, the global annual average temperature.

ENERGY BALANCE EQUATION

Energy in = Energy out

Any left over energy is used to heat up the planet

Temperature change = Energy in – Energy out

Cycle # 1

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

$$R\frac{dT}{dt} = Q - \sigma T^4$$

$$= Q - \sigma T^4$$

Q = Solar energy received (343 Watt m^{-2})

R = heat capacity (unit W s m^{-2} K^{-1})

σ = Stefan Boltzman constant (5.67×10^{-8} W m^{-2} K^{-4})

T = Temperature (in Kelvin)

Exercise:

1. Verify the units
2. Assuming Earth's climate is in equilibrium,

what is Earth's global annual average temperature?

Cycle # 1

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

$$R\frac{dT}{dt} = Q - \sigma T^4$$

$$= Q - \sigma T^4$$

$T_{equilibrium}$ ~ **279K = 6°C**

Observed Earth's global temperature is about **287K = 14°C**

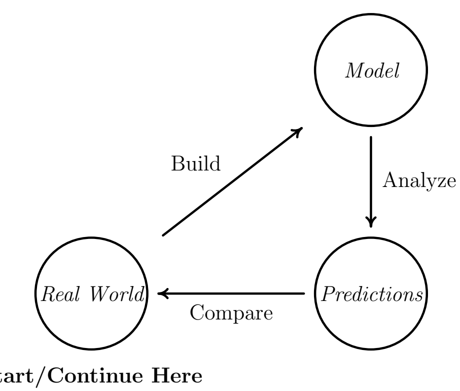


Figure 1: The modeling cycle.

Cycle # 1

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

$$R\frac{dT}{dt} = Q - \sigma T^4$$

$$= Q - \sigma T^4$$

$T_{equilibrium}$ ~ **279K = 6ºC**

Observed Earth's global temperature is about **287K = 14ºC**

Add more process in the model?

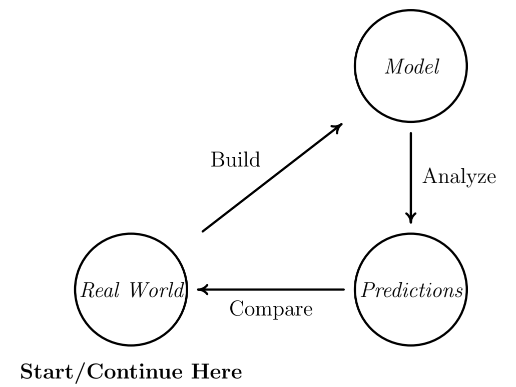
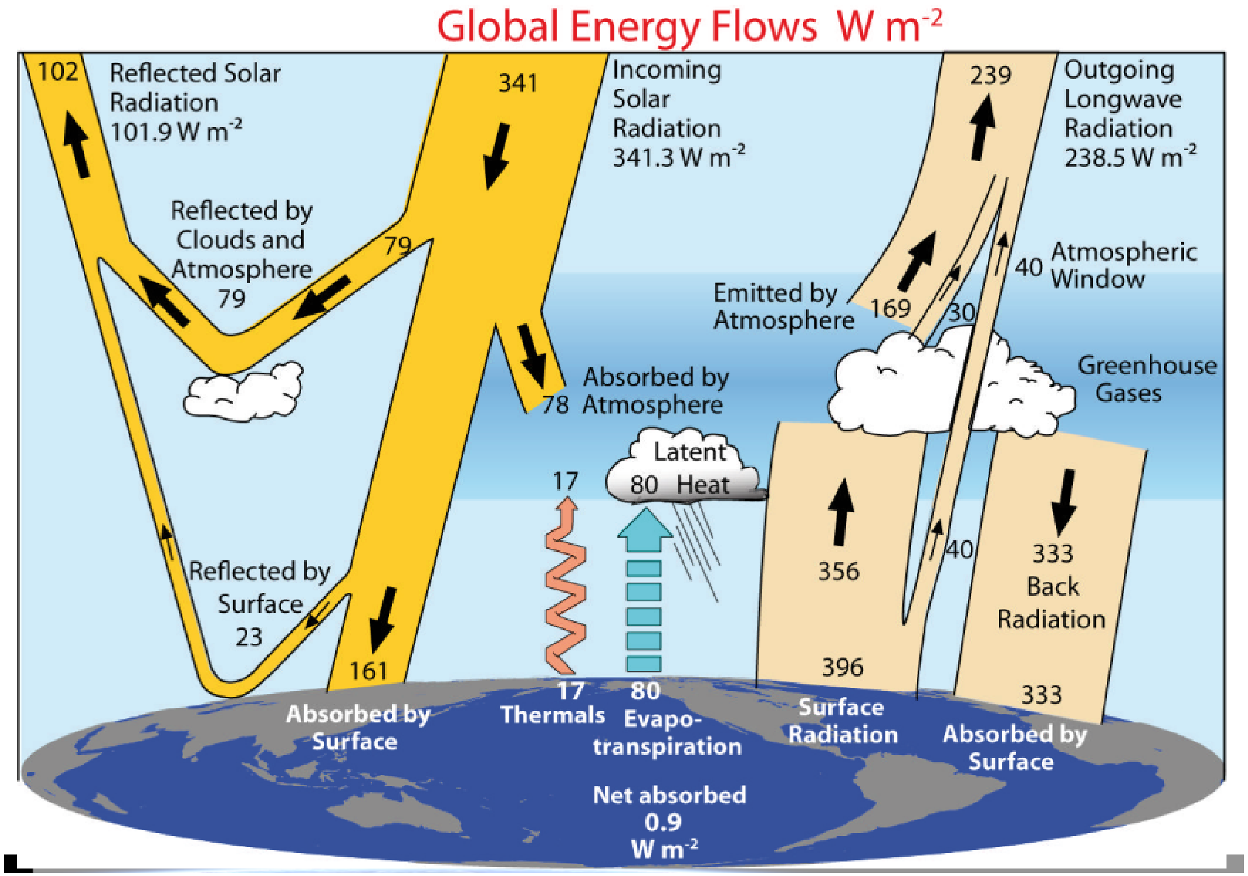


Figure 1: The modeling cycle.

Overview of climate



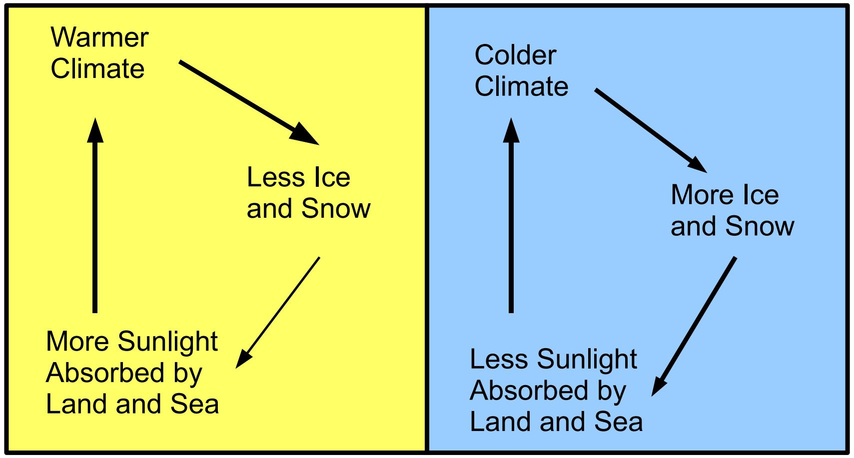
Two types of Earth's reradiation:

Short wave and long wave.

Albedo/ reflectivity affects short wave reradiation.

Some *not* so well known laws

4. Ice albedo feedback:
ice/ snow –lighter color, reflects energy
land/ ocean –darker color, absorbs energy



The Ice Albedo Feedback

Cycle # 2

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

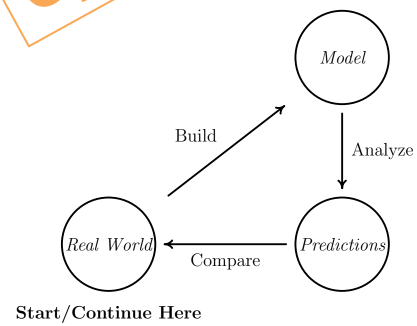


Figure 1: The modeling cycle.

$$R\frac{dT}{dt} = Q - Q\alpha - \sigma T^4$$

$$= Q(1-\alpha) - \sigma T^4$$

Q = 343 Watt m^{-2}

σ = Stefan Boltzman constant (5.67×10^{-8} W m^{-2} K^{-4})

α = planetary albedo ~ 0.3 (non dimensional constant)

Exercise:

1. Verify the units

2. Assuming that Earth's climate is in equilibrium, what is Earth's global annual average temperature?

Cycle # 2

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

$$R\frac{dT}{dt} = Q - Q\alpha - \sigma T^4$$

$$= Q(1-\alpha) - \sigma T^4$$

Exercise 2.

Assuming Earth's climate is in equilibrium, what is Earth's global annual average temperature?

Q = 343 Watt m^{-2}

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Cycle # 2

Modeling Earth's temperature
Zero spatial dimension
Global Energy Balance Model

$$R\frac{dT}{dt} = Q - Q\alpha - \sigma T^4$$

$$= Q(1-\alpha) - \sigma T^4$$

2. Assuming Earth's climate is in equilibrium, what is Earth's global annual average temperature?

T_{eq} = 255 K = -18°C!!!

Far from the observed 14°C.

The model gets worse.

Q = 343 Watt m^{-2}

σ = Stefan Boltzman constant (5.67×10^{-8} W m^{-2} K^{-4})

α = planetary albedo ~ 0.3 (non dimensional constant)

What do we do next?

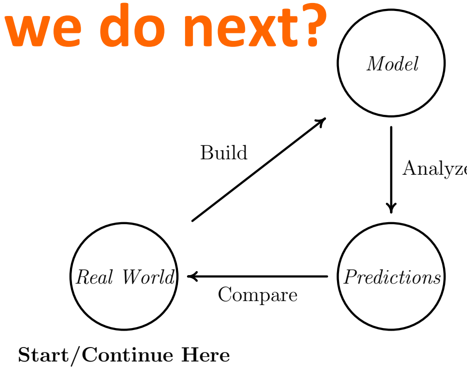
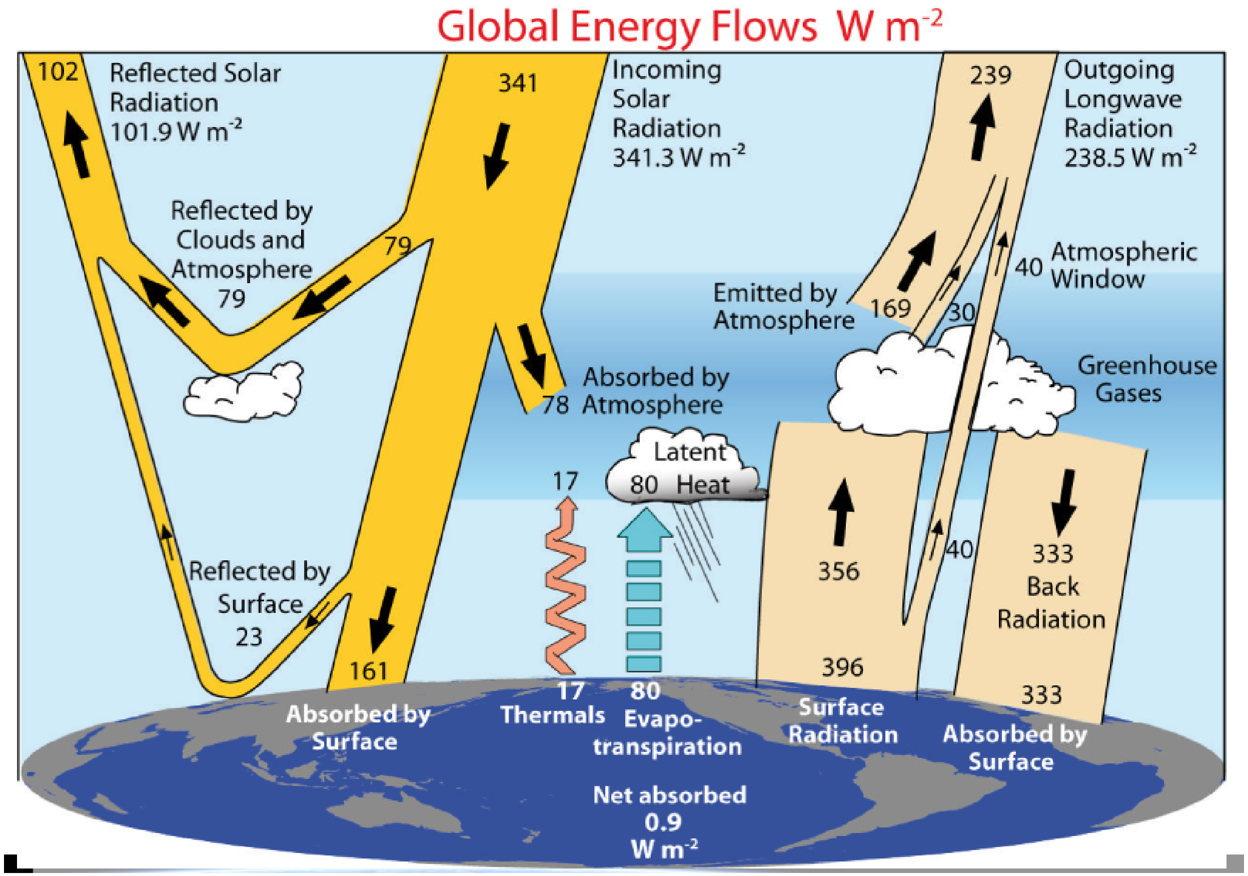


Figure 1: The modeling cycle.

Overview of climate



Two types of Earth's reradiation:

Short wave and long wave.

Long wave radiation is due to eg. green house effect.

Cycle # 3

Modeling Earth's temperature
Global Energy Balance Model

$$R\frac{dT}{dt} = Q(1-\alpha) - \boxed{\varepsilon} \cdot \sigma T^4$$

Stefan Boltzmann Law is radiation law for black body. Since Earth is not black body, must use different approximation of the long wave re-radiation.

Green house gas effect must be included: introduce the factor ε.

Exercise 2b. Find the value of ε that fits the observation.

Cycle # 3

Modeling Earth's temperature
Global Energy Balance Model

$$R\frac{dT}{dt} = Q(1-\alpha) - \boxed{\varepsilon} \cdot \sigma T^4$$

Stefan Boltzmann Law is radiation law for black body. Since Earth is not black body, must use different approximation of the long wave re-radiation.

Green house gas effect must be included: introduce the factor ε.

Exercise 2b. Find the value of ε that fits the observation.
Answer: ε = 0.6 will do it.

Modeling Earth's temperature
Global Energy Balance Model

Cycle # 3
Another approach

$$R\frac{dT}{dt} = Q(1-\alpha) - (A + BT)$$

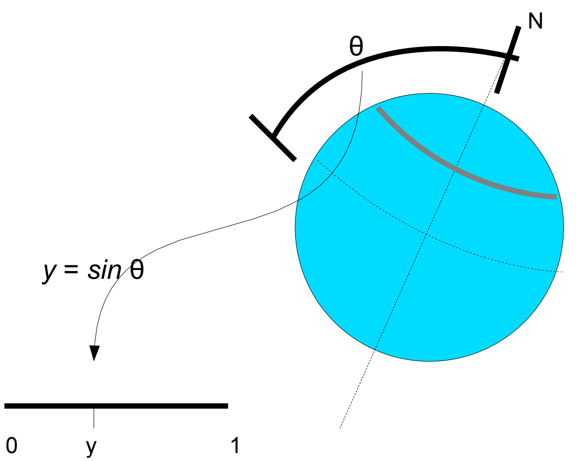
Stefan Boltzmann Law is radiation law for black body.
Since Earth is not black body, must use different approximation of the long wave re-radiation.

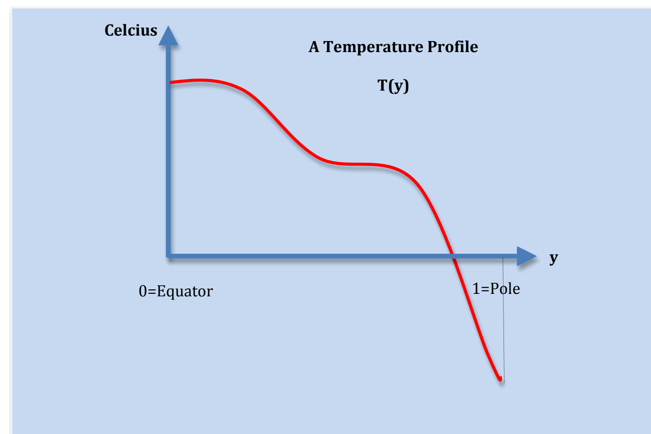
Graves, et al (1993): ~~σT^4~~ instead use **A + B T**
A = 202 Wm^{-2} and B = 1.9 $Wm^{-2}C^{-1}$
are constants obtained from satellite observation.

Modeling Earth's temperature profile
One spatial dimension
Zonal Energy Balance Model

Some assumptions that make the work easier:

1. Symmetry about the equator, so we only look at eg. the northern hemisphere.
2. Annual average temperature along the same latitude, say θ.





Modeling Earth's temperature profile
Zonal Energy Balance Model

$$\frac{\partial T}{\partial t} = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{insolation}} \underbrace{-(A+BT(y))}_{\text{apprx. Stephan-Boltzman}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\right)$$

$R =$ planetary heat capacity

$T = T(y) = T(y,t)$

$s(y) =$ a distribution function

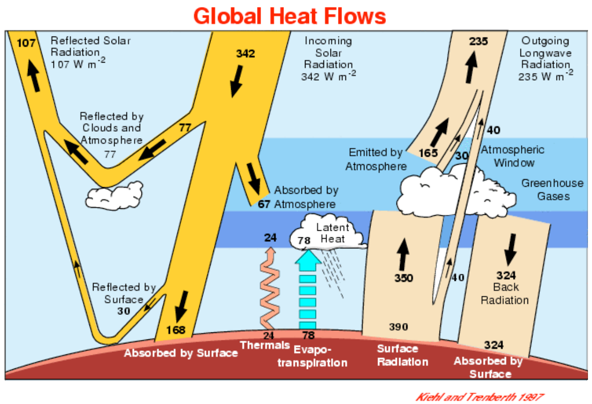
$\alpha(\eta, y) =$ the albedo at y given that the ice line is at η

$\overline{T} = \int_0^1 T(\xi)d\xi$

A = the "Greenhouse Gas Parameter"

B, C are nonnegative parameters

$$\alpha(\eta,y) = \begin{cases} 0.32 & if \quad y < \eta \\ 0.47 & if \quad y = \eta \\ 0.62 & if \quad y > \eta \end{cases}$$



An example of the albedo function.
Another is the smooth approximation of this.

Equilibrium temperature profile

$$\frac{\partial T}{\partial t} = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{insolation}} \underbrace{-(A+BT(y))}_{\text{apprx. Stephan-Boltzman}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\right)$$

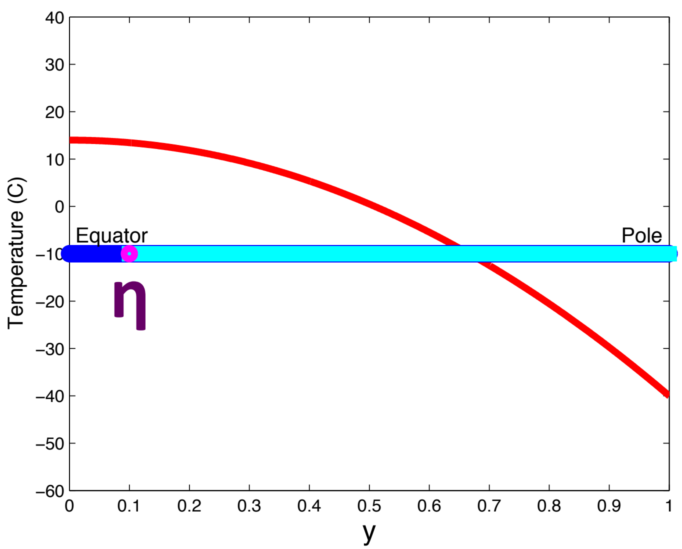
Exercise:

Compute the equilibrium temperature profile for a fix η $T^*_\eta(y)$.

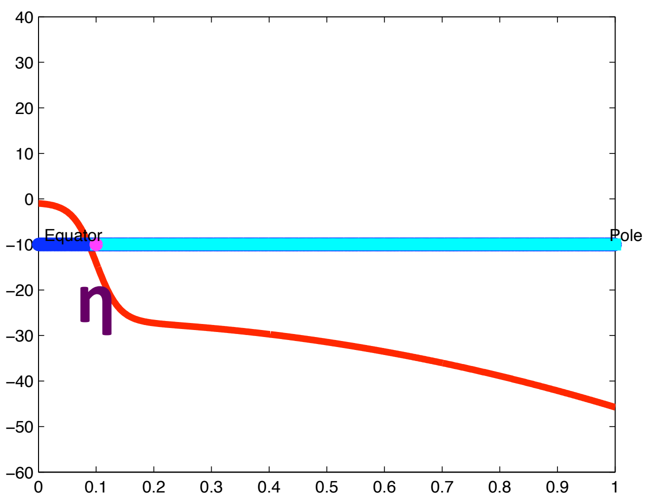
Equilibrium temperature profile

$$\frac{\partial T}{\partial t} = \frac{1}{R}\left(\underbrace{Qs(y)(1-\alpha(\eta,y))}_{\text{insolation}} \underbrace{-(A+BT(y))}_{\text{apprx. Stephan-Boltzman}} - \underbrace{C\left(T(y)-\overline{T}\right)}_{\text{transport}}\right)$$

Initial T(y,0)

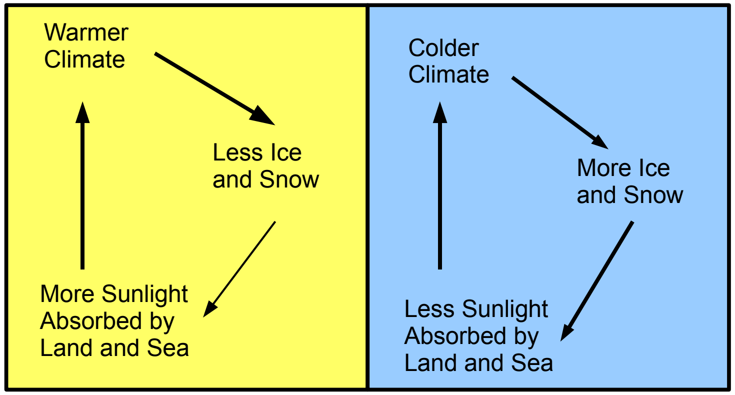


$T^*_\eta(y)$.



ICE LINE EVOLUTION

$$\frac{d\eta}{dt} = \varepsilon(T(\eta,t) - T_C)$$



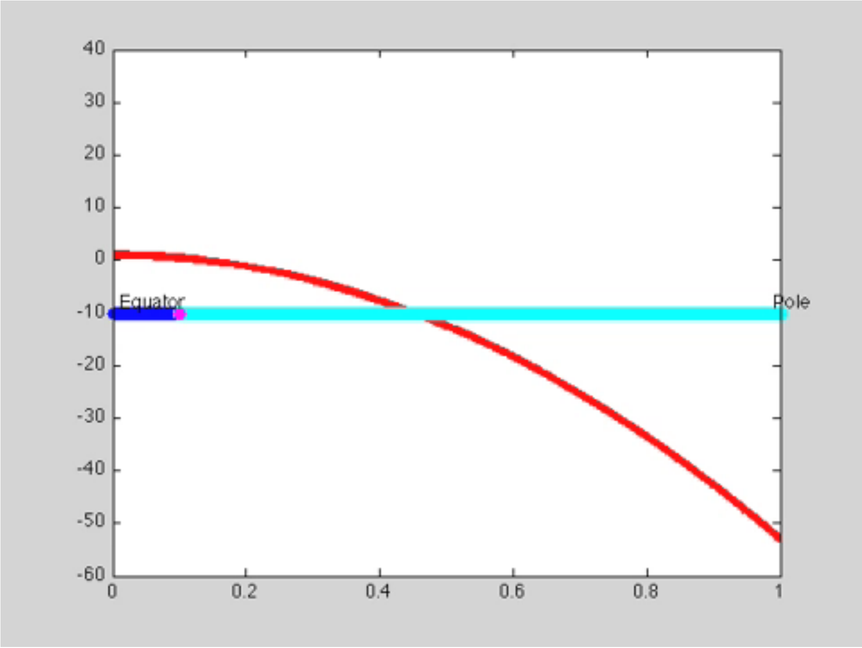
The Ice Albedo Feedback

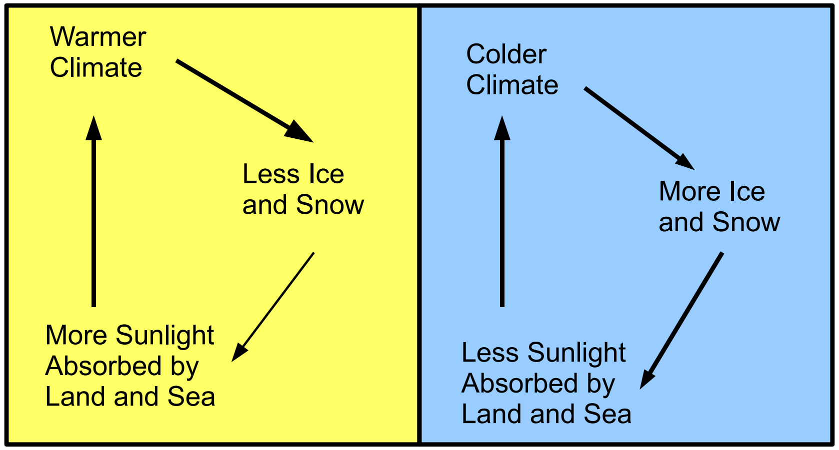
Compare the temperature at the ice line with a critical temperature T_c.

If too warm, ice melts, ice line moves to pole.
If too cold, ice forms, ice line moves to equator.

ICE LINE EVOLUTION

$$\frac{d\eta}{dt} = \varepsilon(T(\eta,t) - T_C)$$

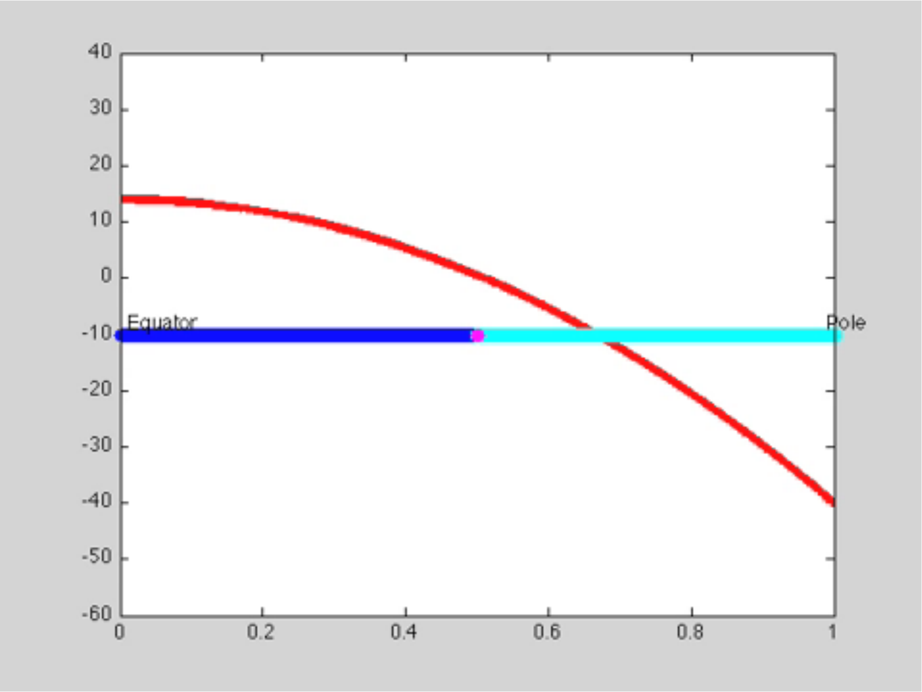


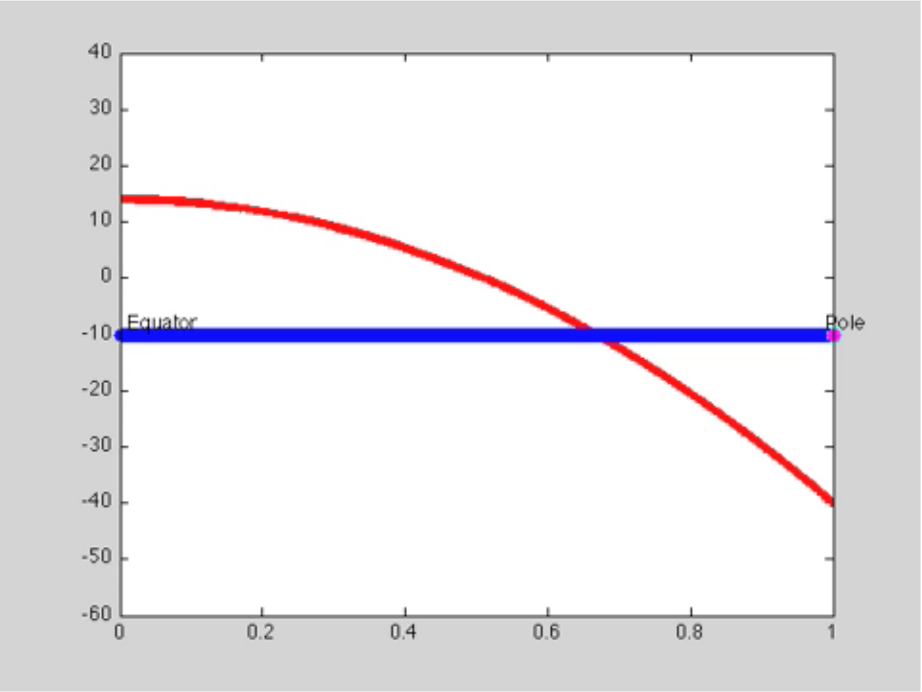


The Ice Albedo Feedback

Low latitude initial ice line

More simulations



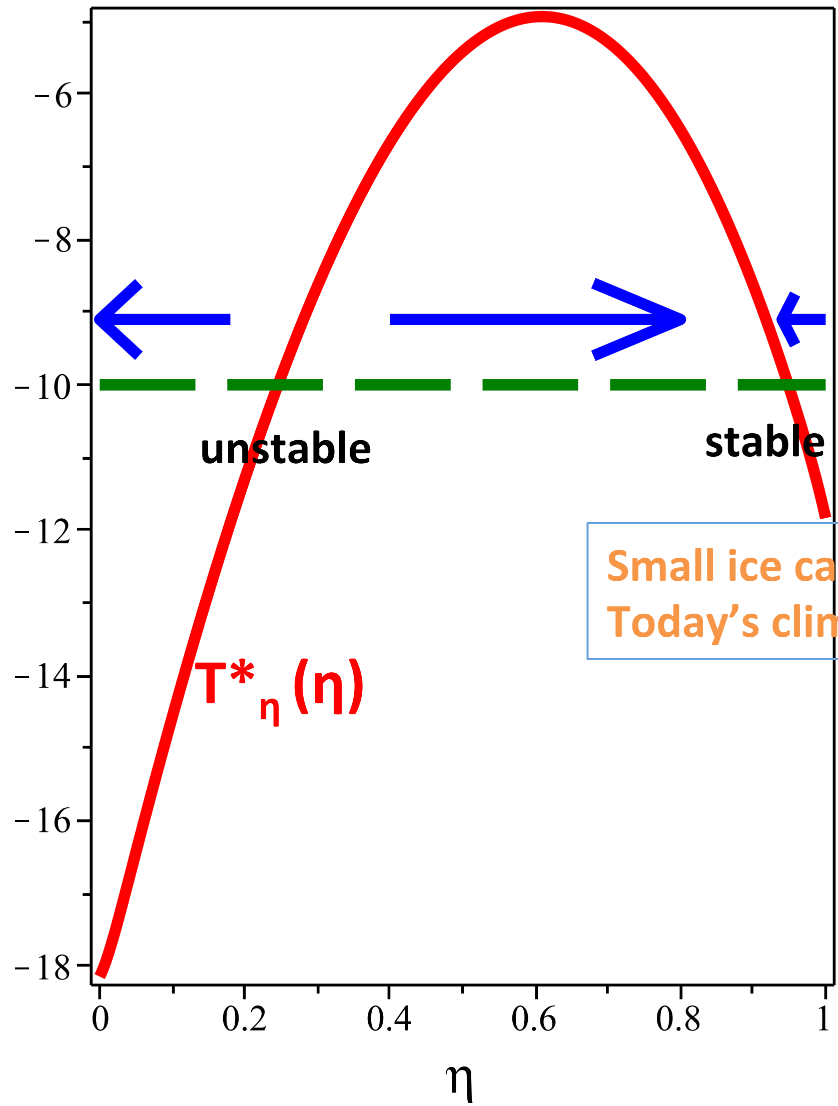


Mid latitude initial ice line

High latitude initial ice line

Ice line stability?

$$\frac{d\eta}{dt} = \varepsilon(T(\eta,t) - T_C)$$



Small ice cap
Today's climate.

NEXT?

- Reduction of the infinite dimensional system to finite dimensional system.
- Bifurcation analysis

Resources:

Modeling module
http://dimacs.rutgers.edu/MPE/Energy/DIMACS-EBM.pdf

Conceptual Climate Module
https://mcrn.hubzero.org/resources/523/supportingdocs. Material developed for the MAA-NCS Summer Seminar Conceptual Climate Models, held in Minneapolis, July 2013. Contributors: A. Barry, R. McGehee, S. Oestreicher, J.A. Walsh, E. Widiasih.

What little bugs in the ocean **don't** tell us: Snowball Earth

Snowball Earth
http://www.wwnorton.com/college/geo/animations/snowball_earth.htm

Image source: Wikipedia

What the model says about Snowball Earth

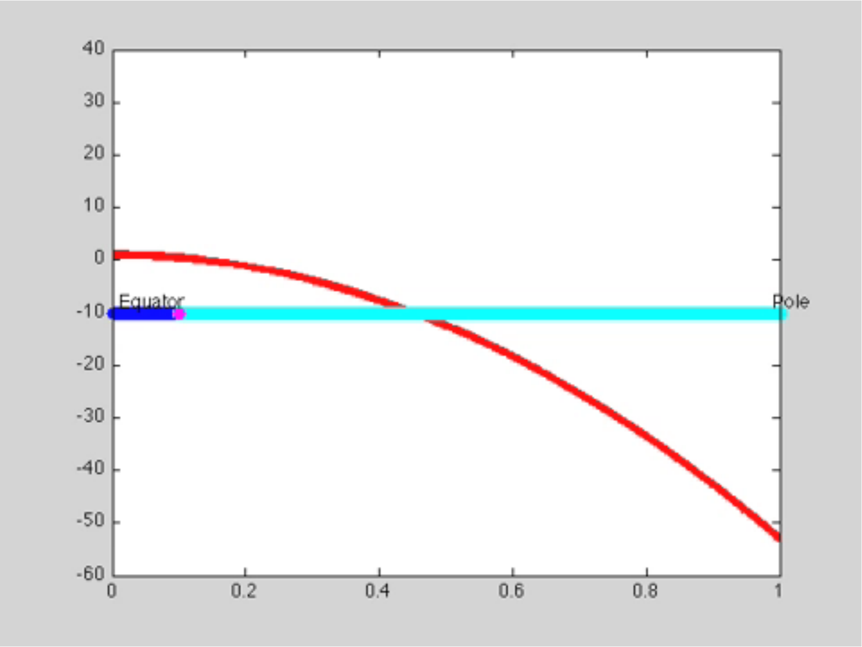


Image source: Wikipedia

Did the snowball earth ever happen?

Some evidence:
Dropstone deposited by glacier into a marine sediment (Namibia)

Image source:
Terra Nova, 2002