Defining equations of probability tree models

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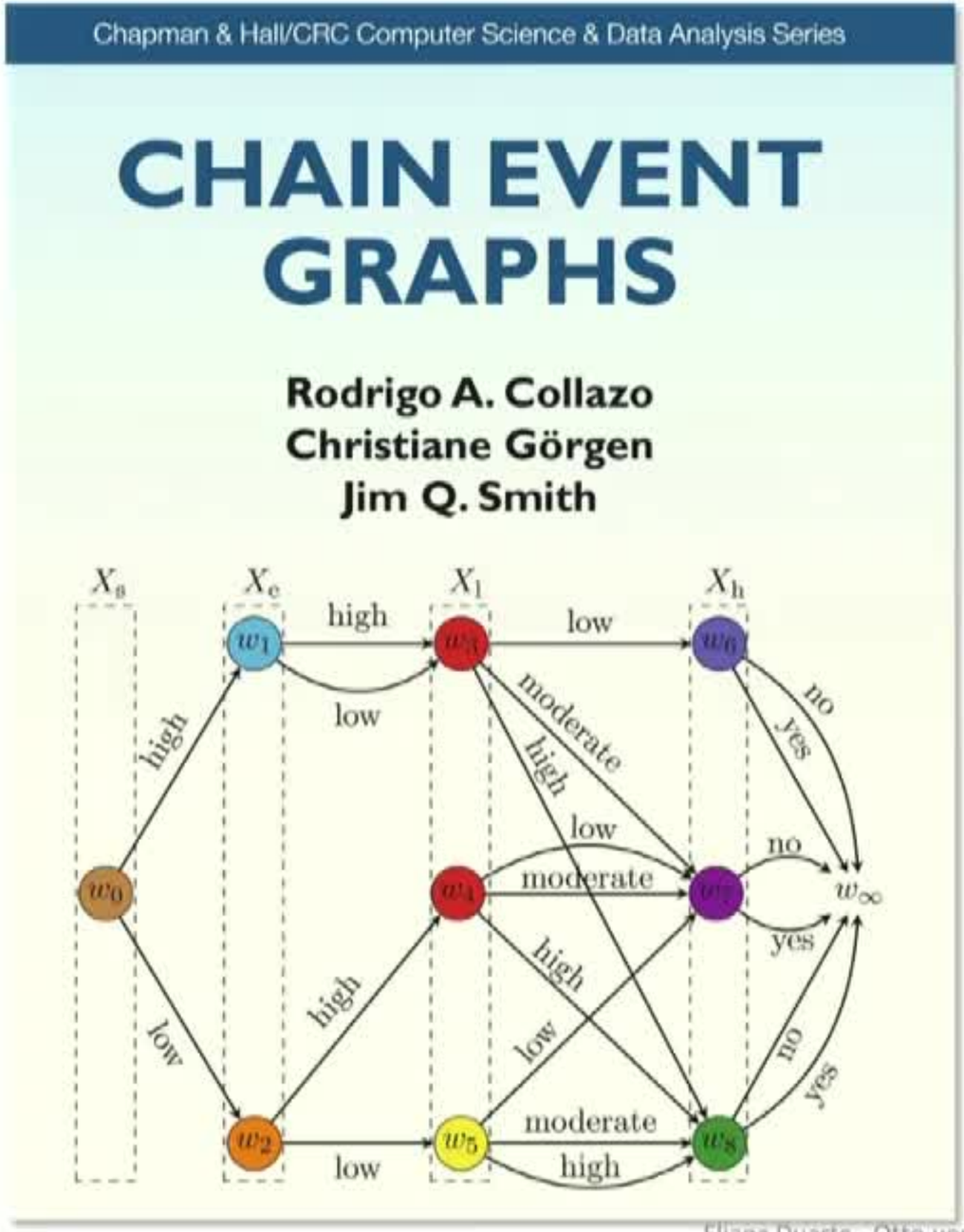
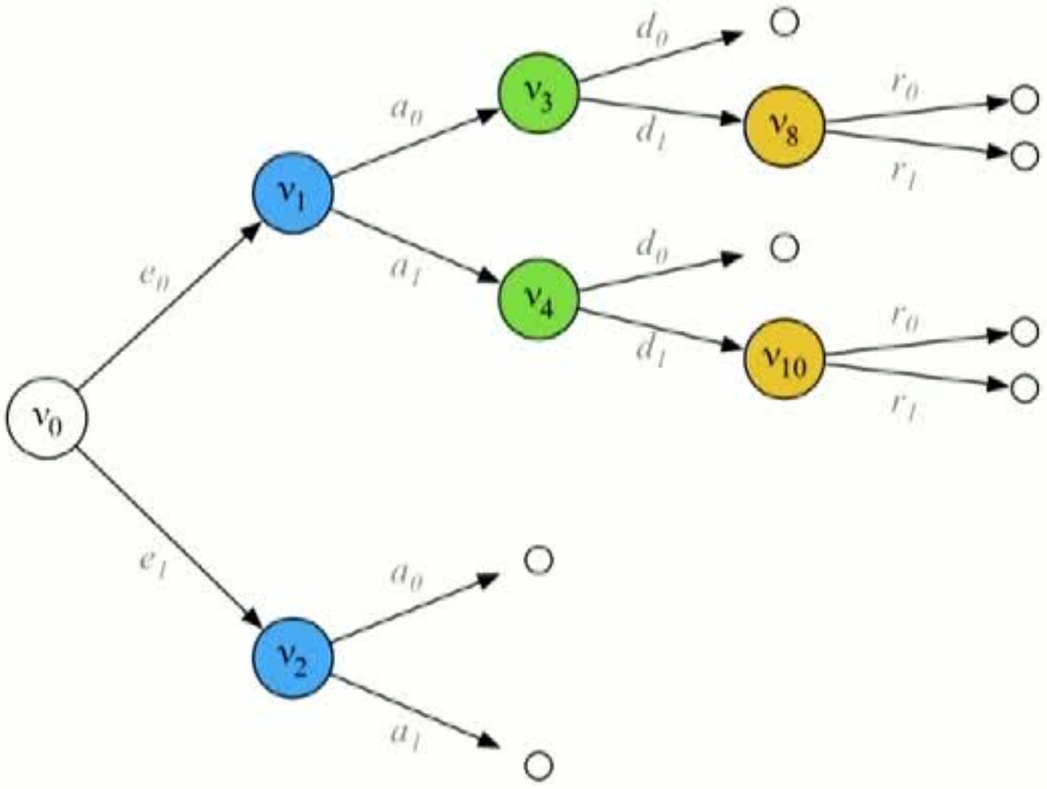


- Chain event graphs provide a way to encode relationships in terms of unfolding of events.
- **Chain Event Graphs** gives these models a solid mathematical and statistical framework.

Unfolding of events in a cell culture



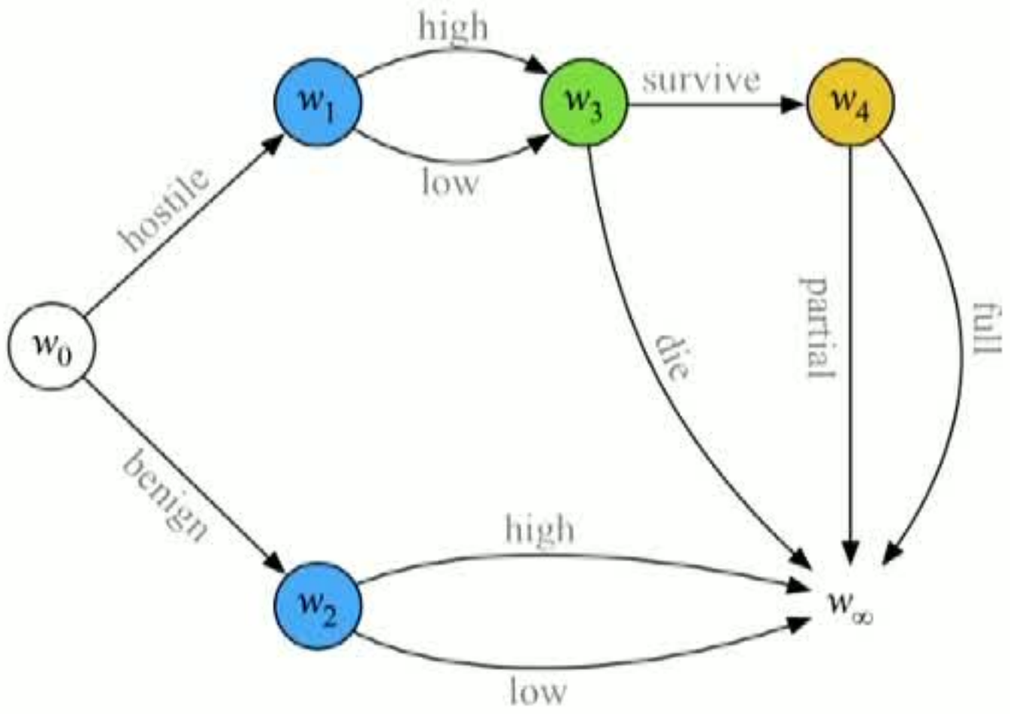
The statistical model $\mathcal{M}$ associated to $(\mathcal{T}, \boldsymbol{\theta}_{\mathcal{T}})$ is the image of

$$\psi : [0,1]^4 \longrightarrow \mathbb{R}^8$$

$$(e_0, a_0, d_0, r_0) \mapsto (e_0 a_0 d_0, \ldots, e_1 a_1).$$

$$\begin{cases} & P_0 + \cdots + P_7 - 1, & \\ P_5P_6 - P_2P_7, & P_3P_6 - P_0P_7, & P_2P_3 - P_0P_5, \\ P_4P_6 - P_1P_7, & P_2P_4 - P_1P_5, & P_1P_3 - P_0P_4 \end{cases}$$

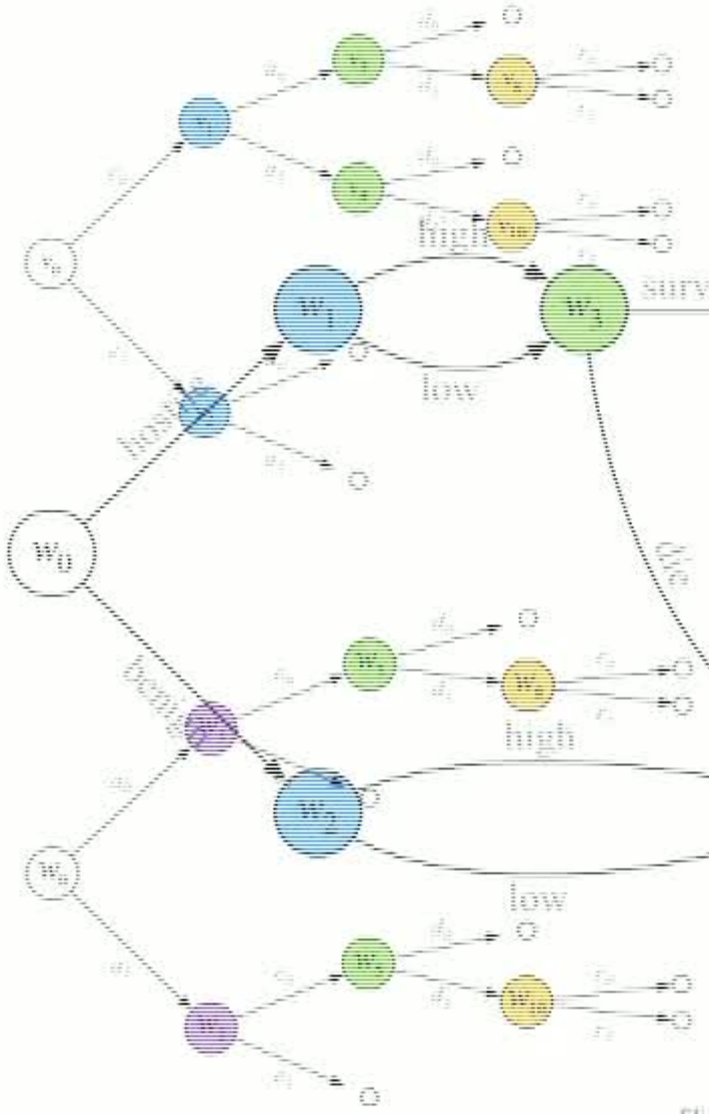
Unfolding of events in a cell culture



- Level of activity is independent of the environment.
- If the environment is hostile then a cell gets damaged and might either survive or die.
- Whether a cell dies or survives does not depend on its activity.
- If a cell survives the recovery is not affected by the history.

Chain event graph for the **staged tree** $(\mathcal{T}, \boldsymbol{\theta}_{\mathcal{T}})$.

Statistical equivalence of staged trees



$(\mathcal{T}_1, \theta_{\mathcal{T}_1}), (\mathcal{T}_2, \theta_{\mathcal{T}_2})$ are **statistically equivalent** if the image of their parameterization coincides inside the probability simplex.

Interpolating polynomial:

Two staged trees $(\mathcal{T}_1, \theta_{\mathcal{T}_1}), (\mathcal{T}_2, \theta_{\mathcal{T}_2})$ with the same label set are called **polynomially equivalent** if their interpolating polynomials are the same.

Polynomial equivalence

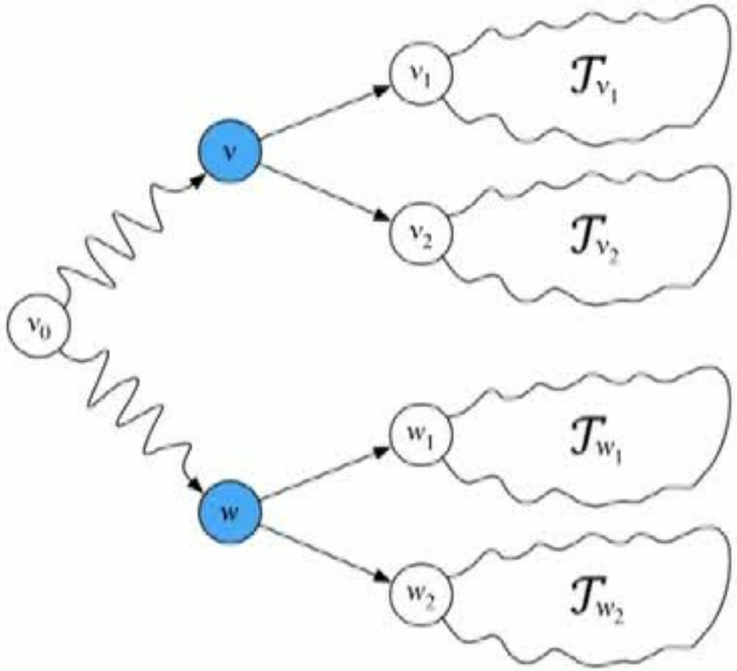
- The activity is independent of the environment.
- If the environment is hostile then a cell gets damaged and might either survive or die.
- Whether a cell dies or survives does not depend on its activity.
- If a cell survives the recovery is

Defining equations of staged tree models

When are staged tree models toric varieties?

Theorem (D., Görgen, arXiv:1802.04511v1):

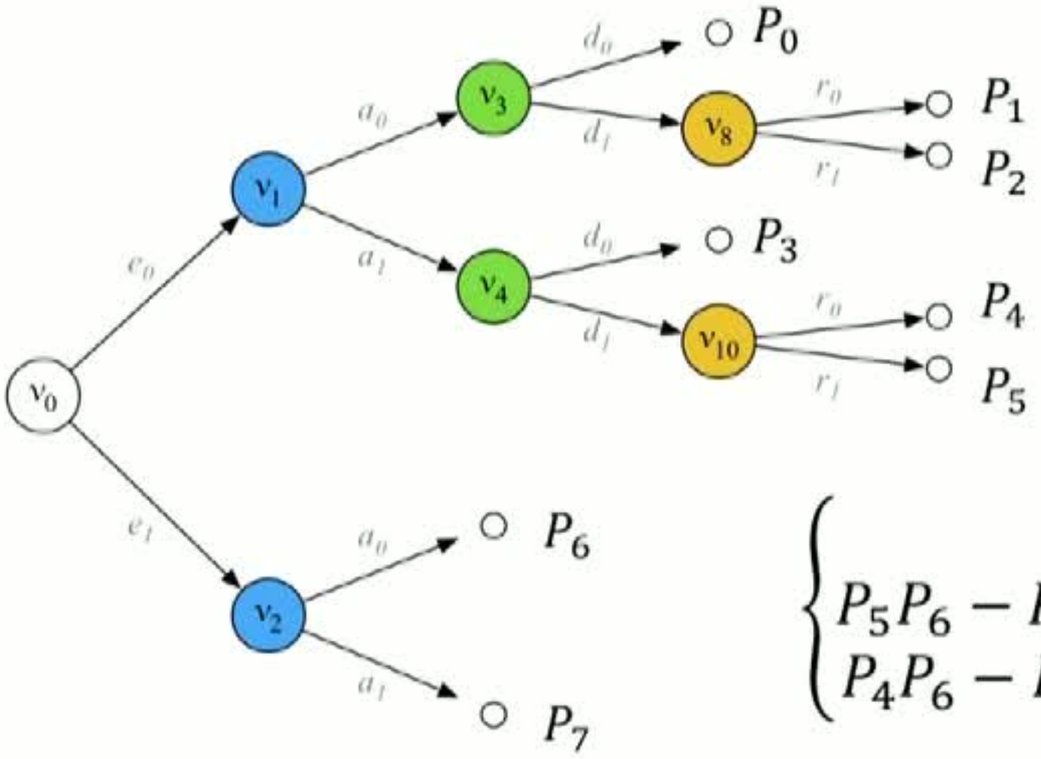
A binary staged tree model is a toric variety if and only if all vertices in the same stage satisfy



$$t(\mathcal{T}_{v_1})t(\mathcal{T}_{w_2}) = t(\mathcal{T}_{w_1})t(\mathcal{T}_{v_2}).$$

Reading the defining equations from the tree

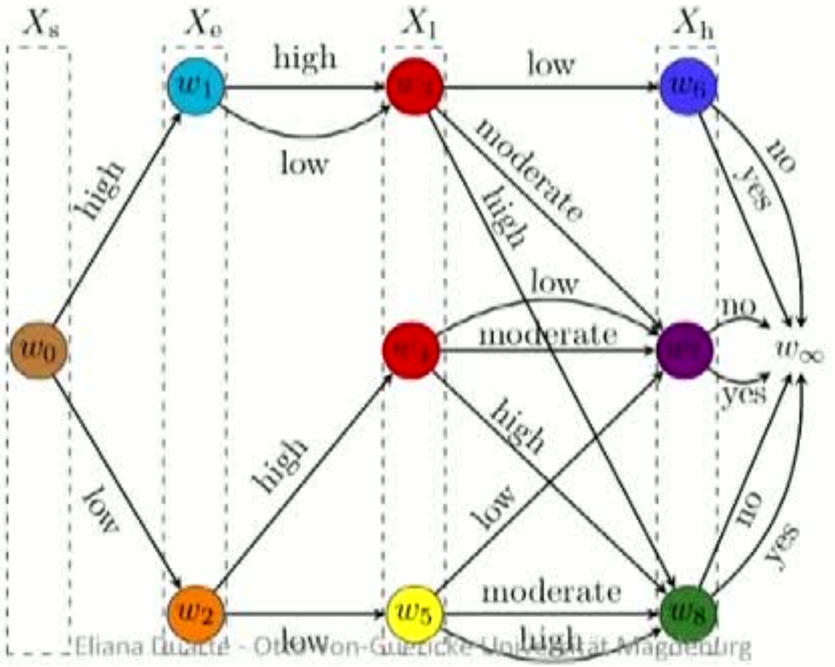
When are staged tree models toric varieties? How to obtain the defining equations of staged tree models?



$$\begin{cases} P_0 + \cdots + P_7 - 1, & & \\ P_5P_6 - P_2P_7, & P_3P_6 - P_0P_7, & P_2P_3 - P_0P_5, \\ P_4P_6 - P_1P_7, & P_2P_4 - P_1P_5, & P_1P_3 - P_0P_4 \end{cases}$$

Outlook

- Staged trees provide a intuitive way to understand sum-to-one conditions of the parameters space of Bayesian Networks.
- Using this framework we would like to give a complete description of the defining equations of non-toric staged tree models.



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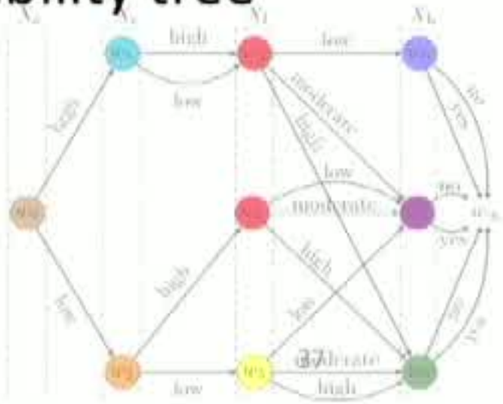

Complexity Reduction in Algebraic Statistics

- Monday November 26th to Tuesday Nov 27th, Otto-Von-Guericke Universität Magdeburg https://www.math.ovgu.de/AlgStat.html

- **Paul Breiding, MPI MIS Leipzig**
- **Mathias Drton (TBC)University of Washington**
- **Emil Horobet, Sapientia Hungarian University of Transylvania**
- **Kaie Kubjas, MIT**
- **Hugo Maruri Aguilar, Queen Mary University**
- **Fatemeh Mohammadi, University of Bristol**
- **Nina Otter MPI MIS Leipzig**
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- **Eva Riccomagno, University of Genoa**
- **Henry Wynn, London School of Economics**