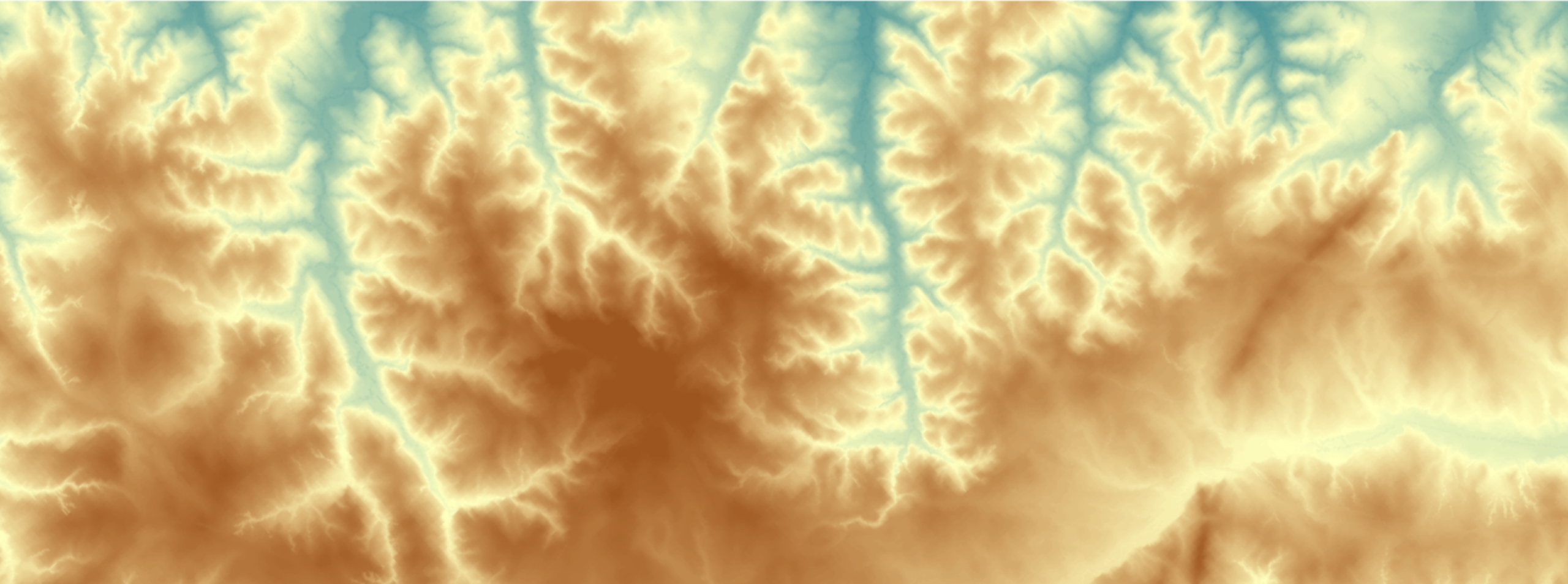


# Ecohydrological Drivers of Vegetation Patterns and Landscape Evolution

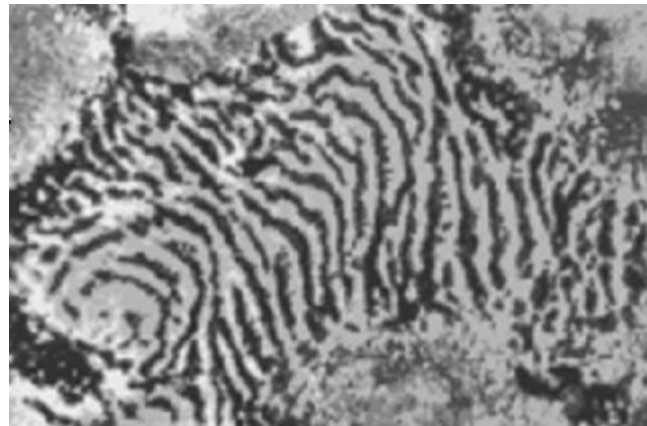
---

Amilcare Porporato, Sara Bonetti, and Milad Hooshyar  
CEE & PEI, Princeton University





A labyrinth of green biomass observed in the northern **Negev** (200 mm annual rainfall). Distance between green patches is ~ 15 cm.



Vegetation bands on hillslopes near Niamey, **Niger**





# Outline

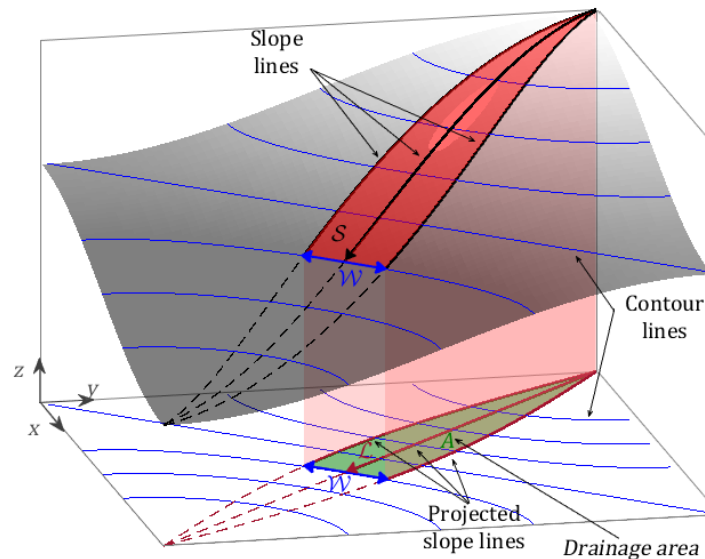
- Stochastic dynamics of plant water stress



- Erosion thresholds and land degradation

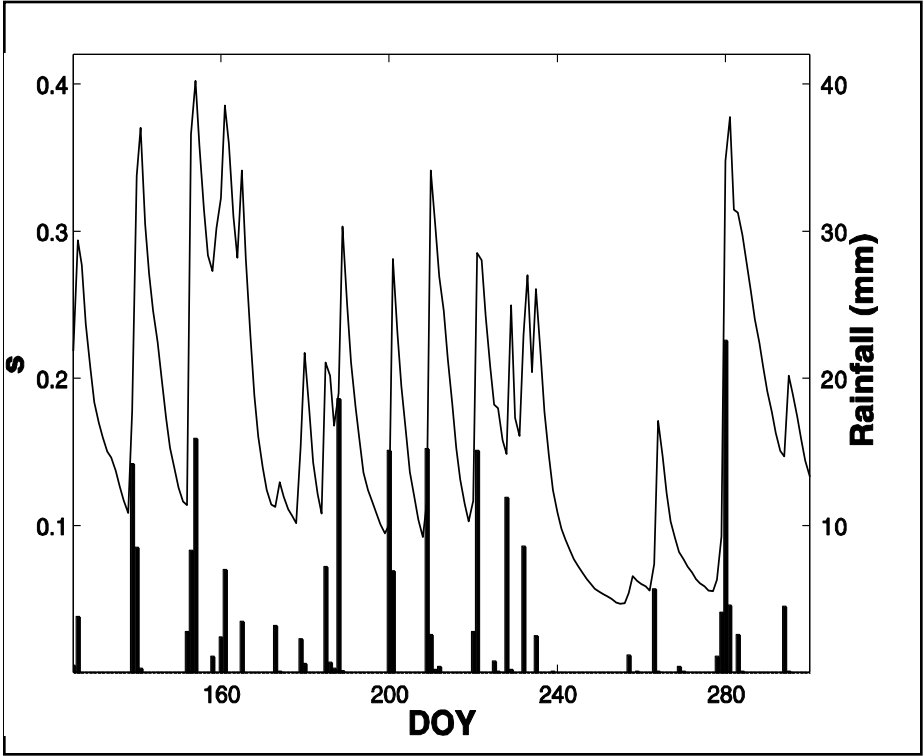


- Landscape evolution  
(preliminary)



# Stochastic soil moisture dynamics

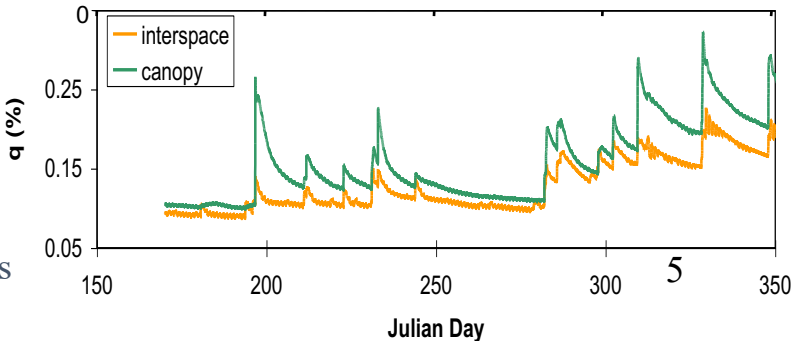
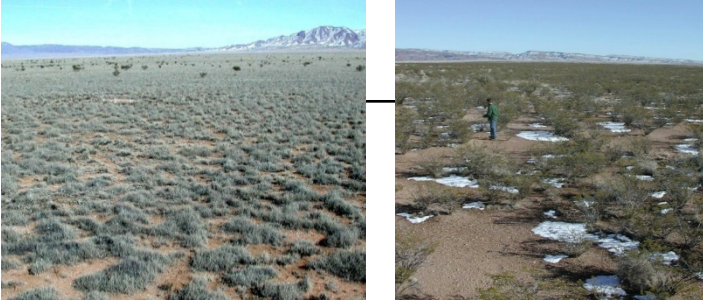
Daly et al. WRR 2006



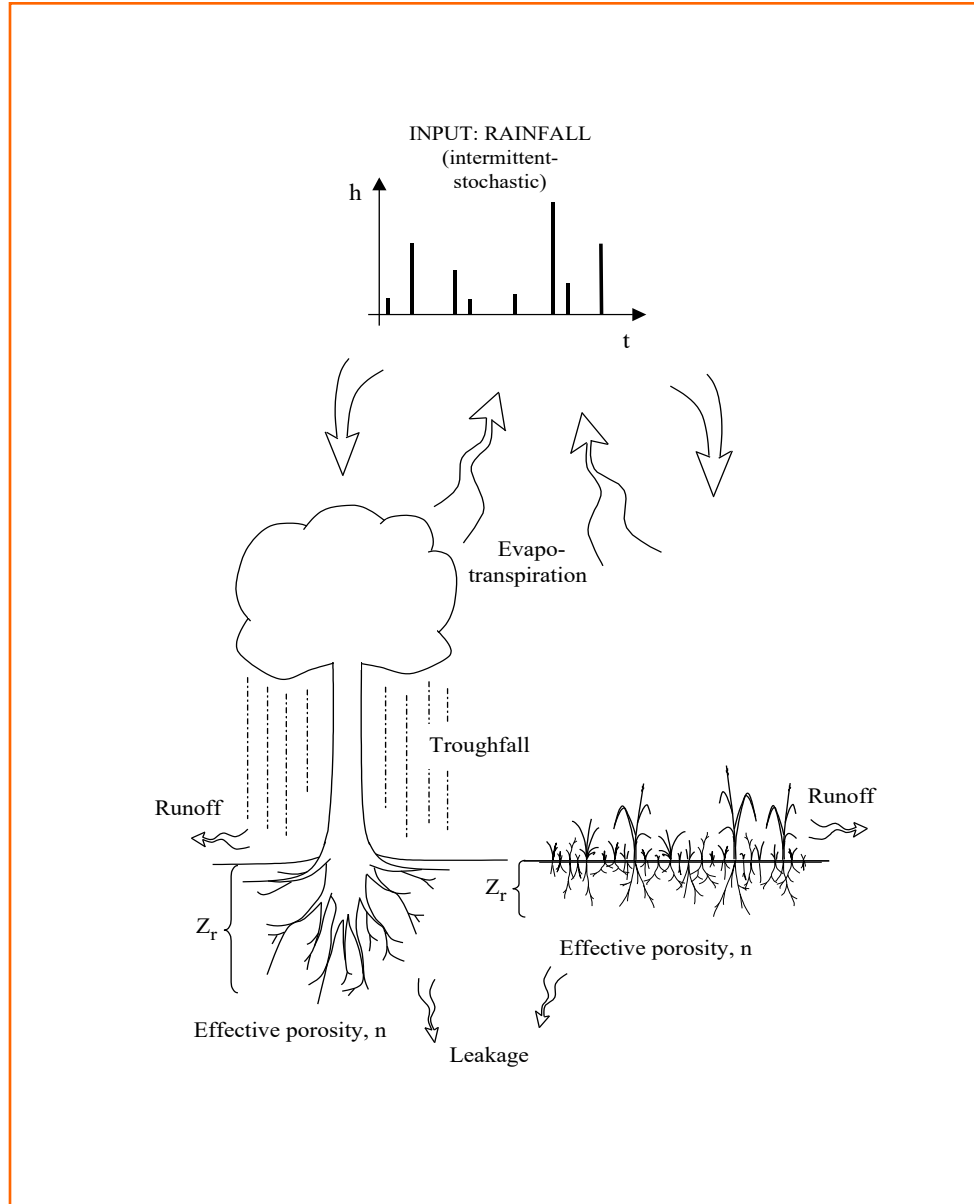
Daily Rainfall and Soil moisture in the Duke forest site (soil moisture is the average of the measurements from 4 TDR probes located over 30 cm



Loblolly pine (*Pinus taeda*) and surrounding hardwood forest at the CO<sub>2</sub>-enriched atmosphere (FACE) experiment) at Duke



# The Soil Water Balance



secondary role of vegetation

primary role of vegetation

$$nZ_r \frac{ds(t)}{dt} = \underbrace{R(t) - I(t) - Q[s(t),t]}_{Y[s(t),t]} - E[s(t)] - L[s(t)]$$

$n$  = porosity

$Z_r$  = active soil depth

$s(t)$  = relative soil moisture

$R(t)$  = rainfall rate

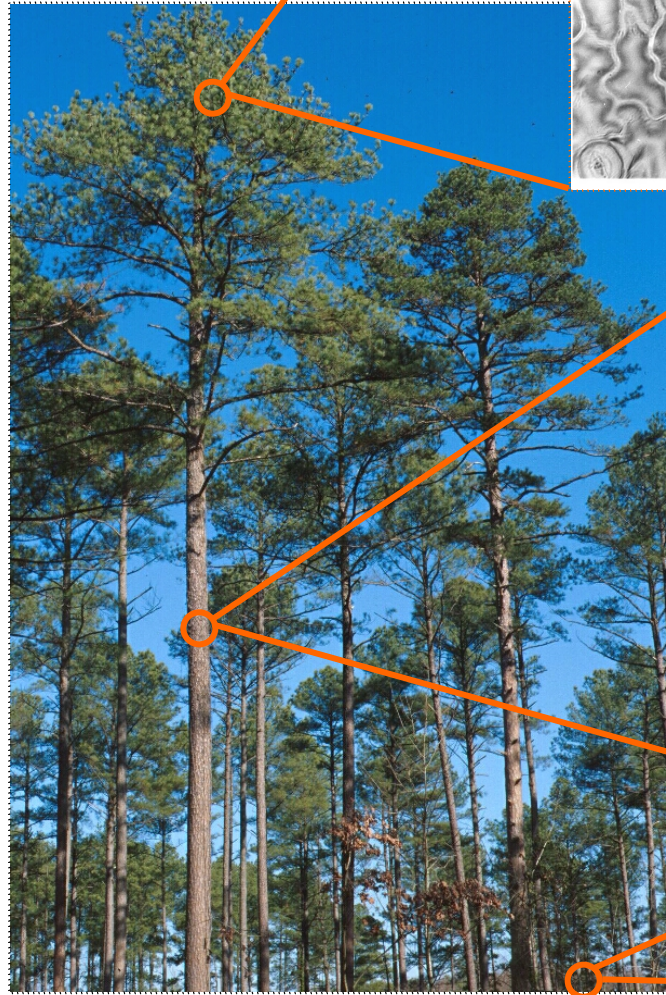
$I(t)$  = canopy interception

$Q[s(t),t]$  = runoff

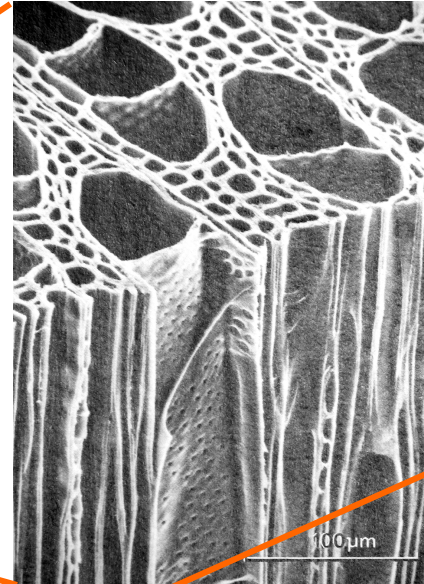
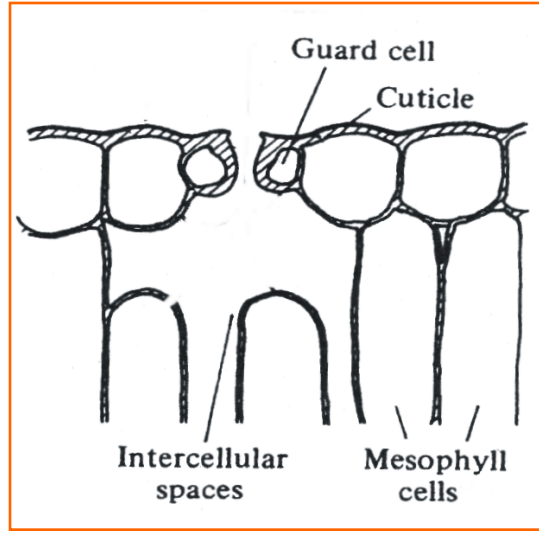
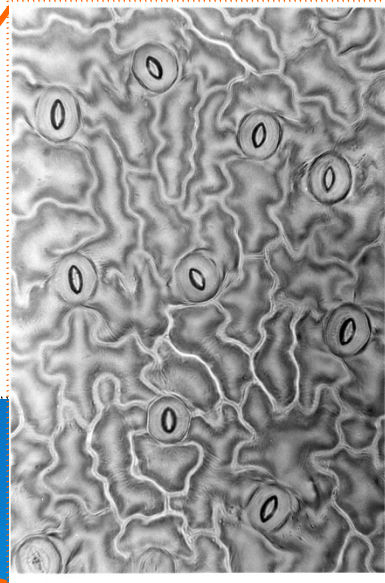
$E[s(t),t]$  = evapotranspiration

$L[s(t),t]$  = leakage

- Daily time scale
- Vertically averaged over the root zone
- Negligible topographic effects

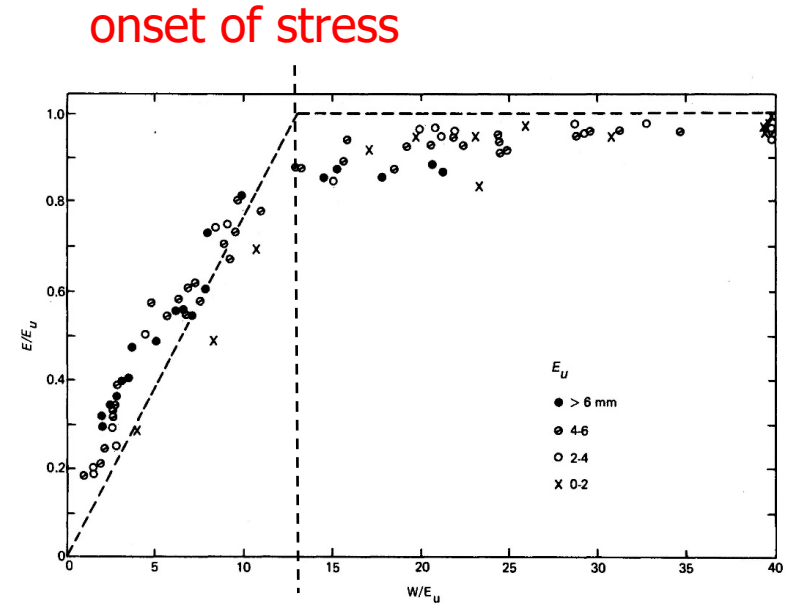
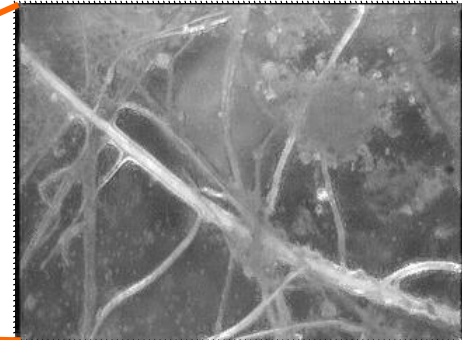


stomata



xylem vessels

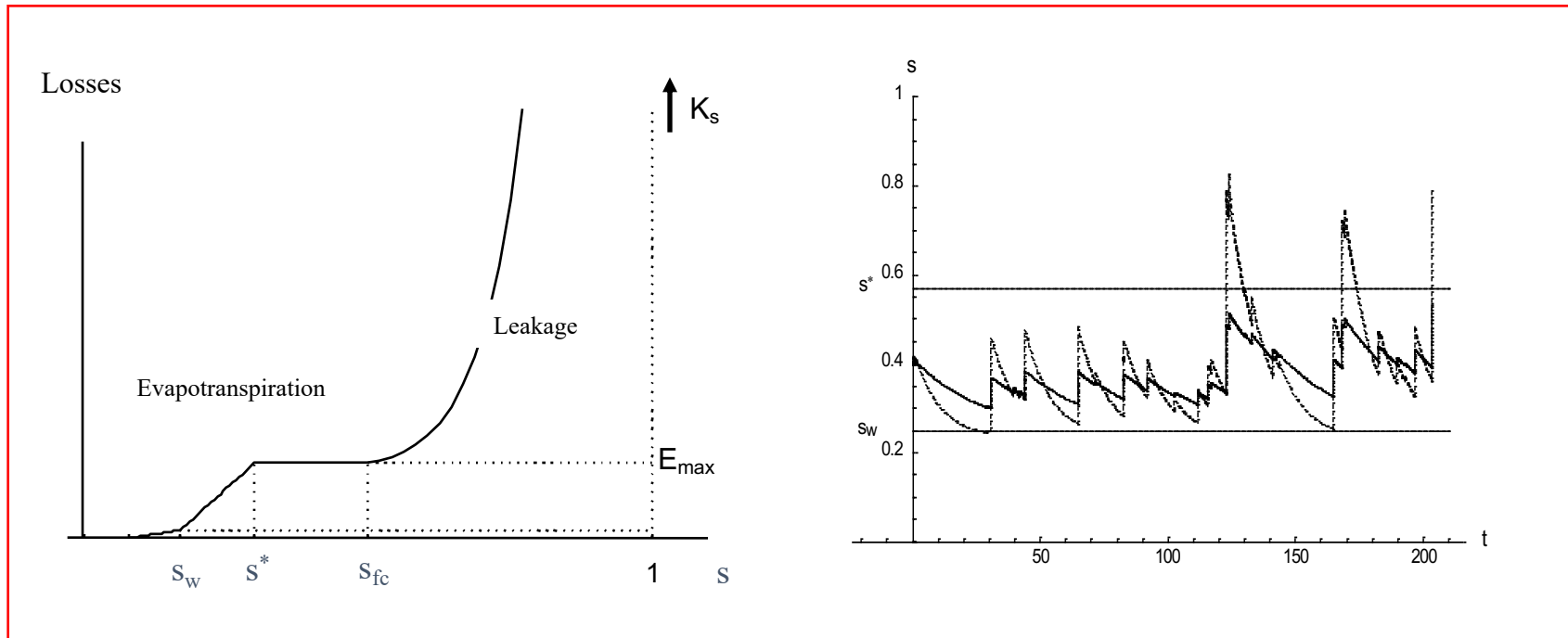
soil-root interface



After: Federer, *Water Resour. Res.*, 15 (3), 555-561, 1979.

# 1-D Non-linear (stochastic) differential equation driven by a state-dependent Poisson noise

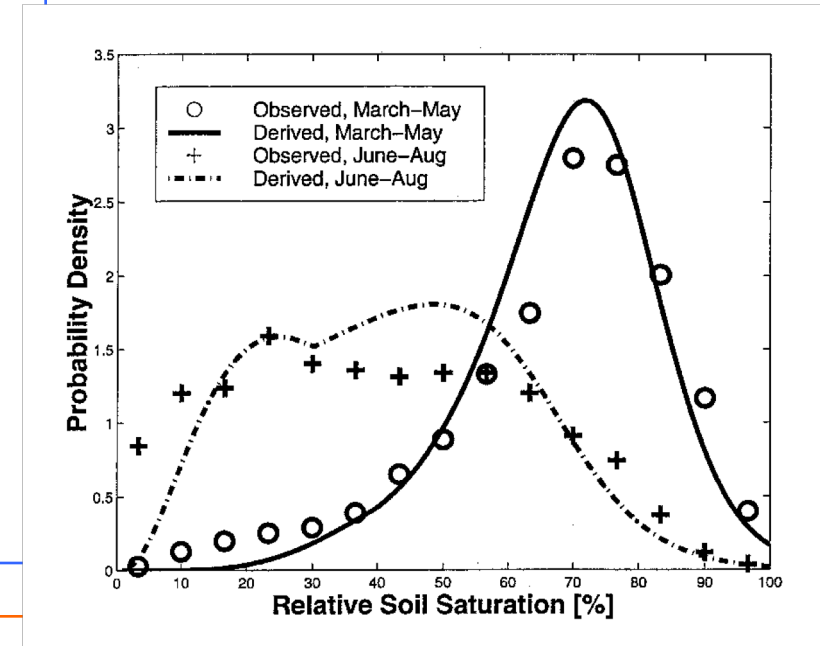
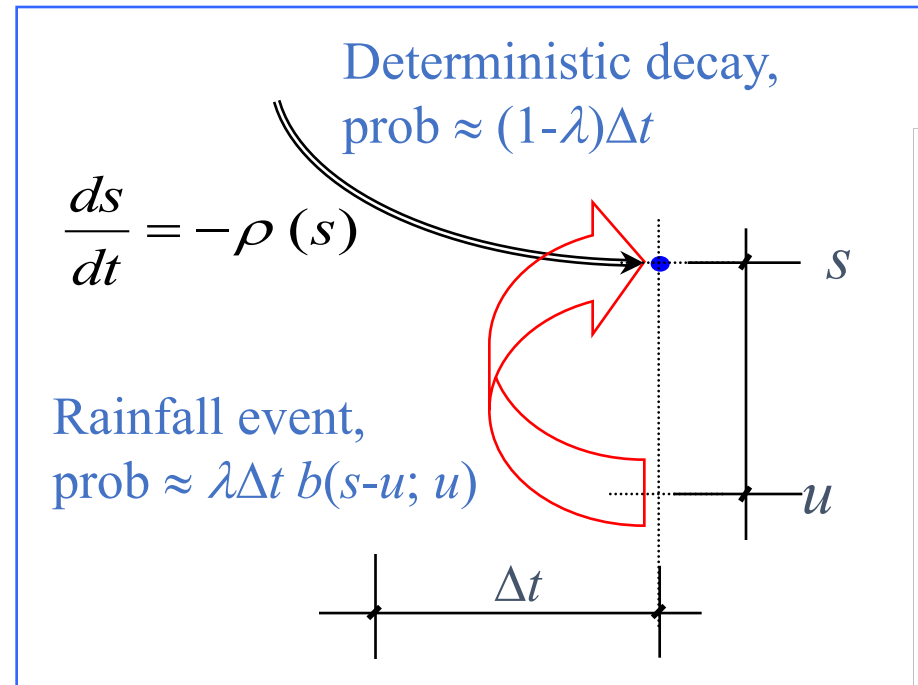
$$\frac{ds}{dt} = -\rho(s) + F(t, s)$$





# Chapman-Kolmogorov eq. (Bartlett & Porporato PRE 2018)

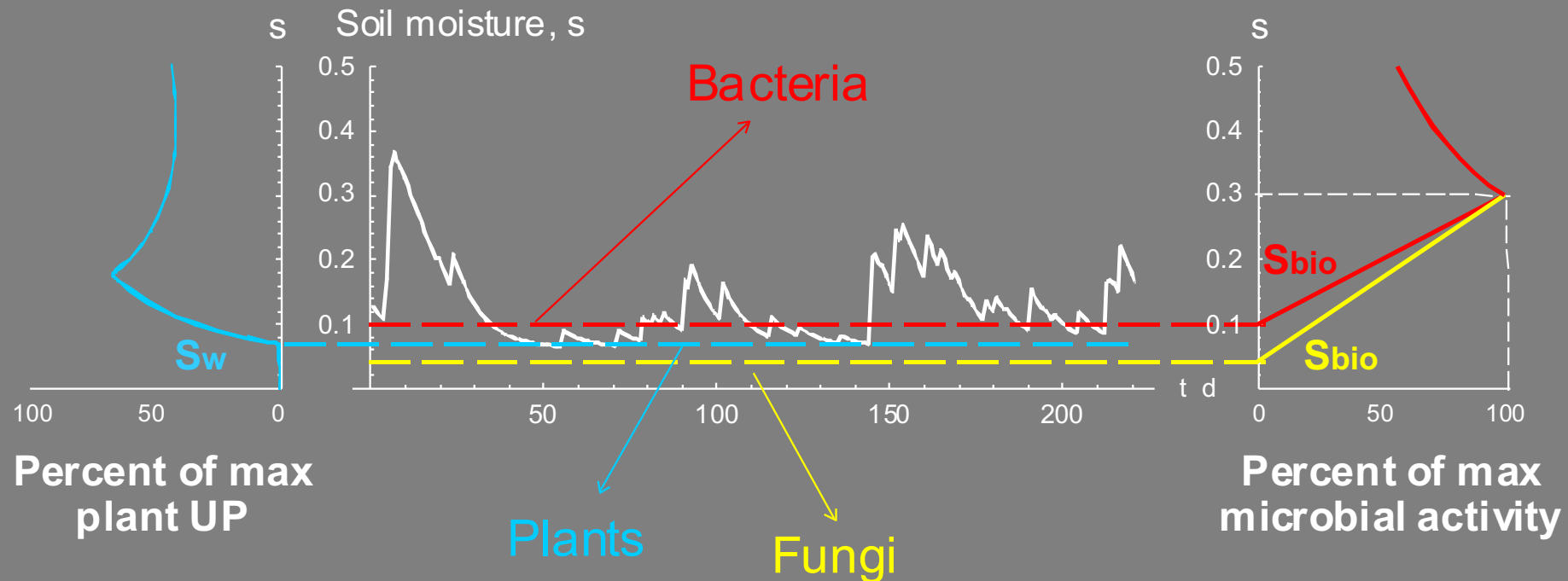
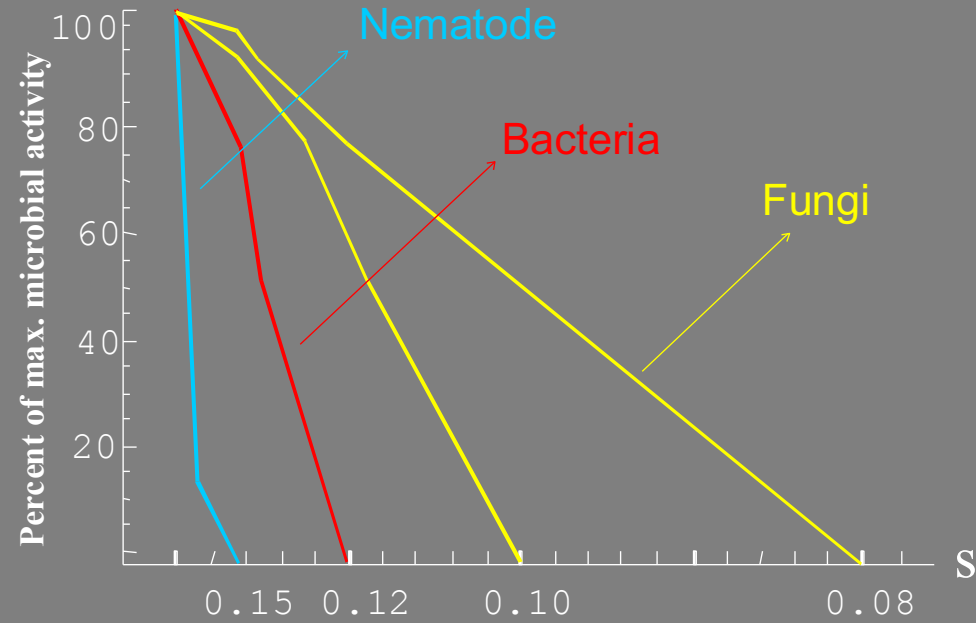
$$p(s, t + \Delta t) = \underbrace{(1 - \lambda \Delta t) p(s + \Delta s, t) d(s + \Delta s)}_{\text{det. decay}} + \underbrace{\lambda \Delta t \int_0^s p(u + \Delta u, t) b(s - u; u) d(u + \Delta u) ds}_{\text{jumps}}$$



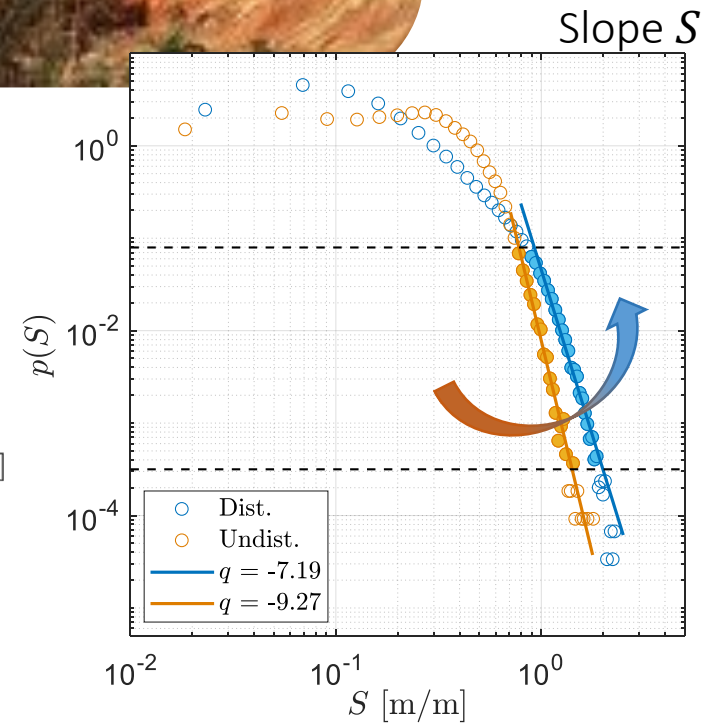
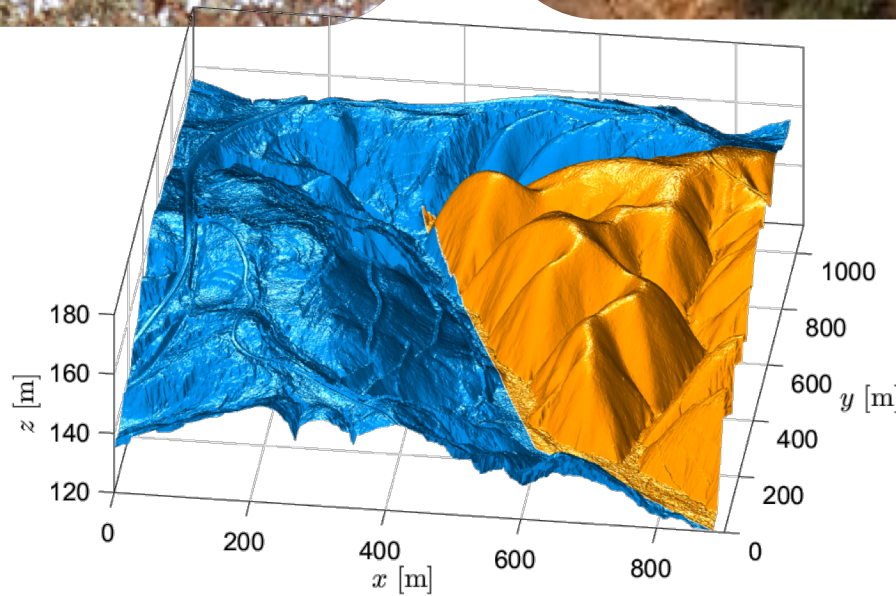
frequency of crossing

$$\frac{\partial}{\partial t} p(s, t) = \frac{\partial}{\partial s} \left( p(s, t) \rho(s) \right) - \lambda p(s, t) + \lambda \int_0^s p(u, t) \gamma e^{-\gamma(s-u)} du$$

# Crossing of stress points



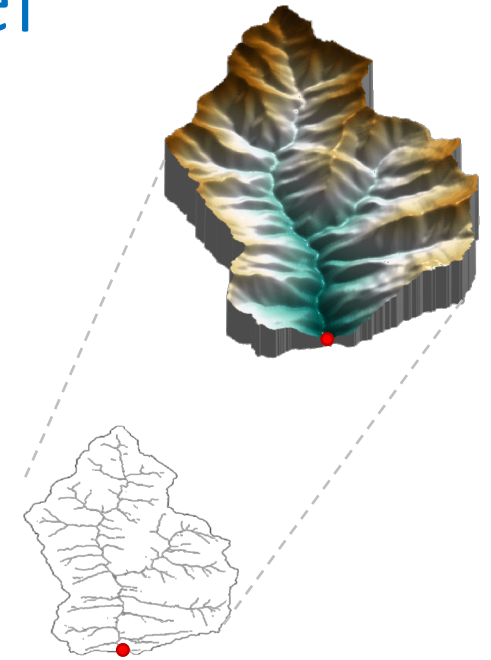
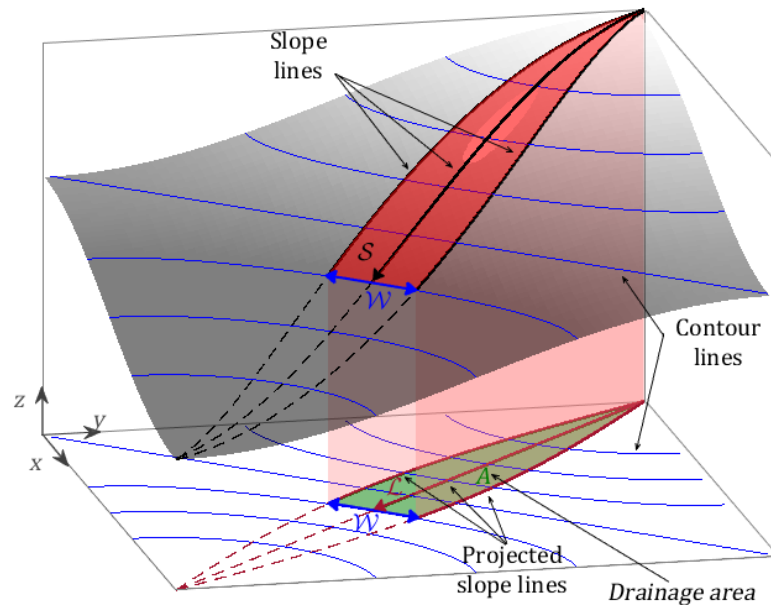
# Vegetation 'disturbances' and landscape morphology



# Minimalist Landscape-Evolution Model

$$\frac{\partial z}{\partial t} = D\nabla^2 z - Ka^m |\nabla z|^n + U$$

Theoretical definition of **drainage area**



- Contour lines
- Slope lines (direction given by  $-\nabla z$ )
- Drainage area  $A$  [ $L^2$ ]
- Specific drainage area  $a$  [ $L$ ]

$$a = \lim_{w \rightarrow 0} \frac{A}{w}$$

- Local slope  $|\nabla z|$

# Mathematical model

$$\frac{\partial h}{\partial t} = R - \nabla \cdot (q\mathbf{n})$$

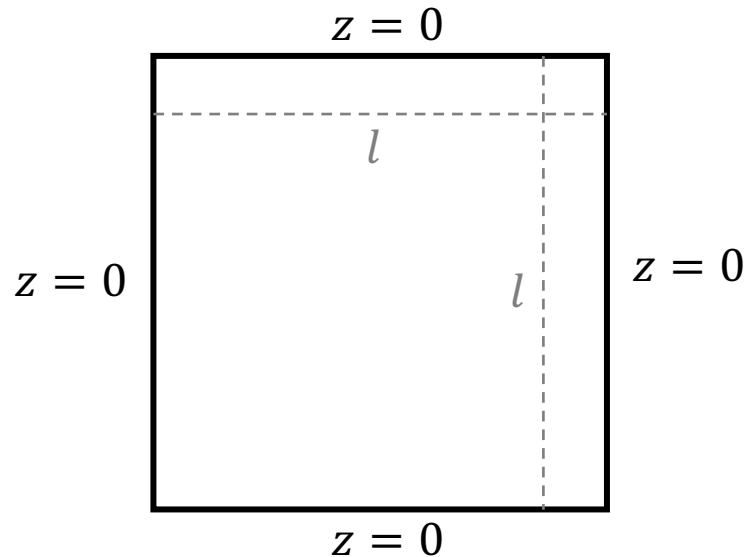
$$\frac{\partial z}{\partial t} = D\nabla^2 z - Kq^m |\nabla z|^n + U$$

$$\mathbf{n} = \frac{\nabla z}{|\nabla z|} \quad q = hv_0$$



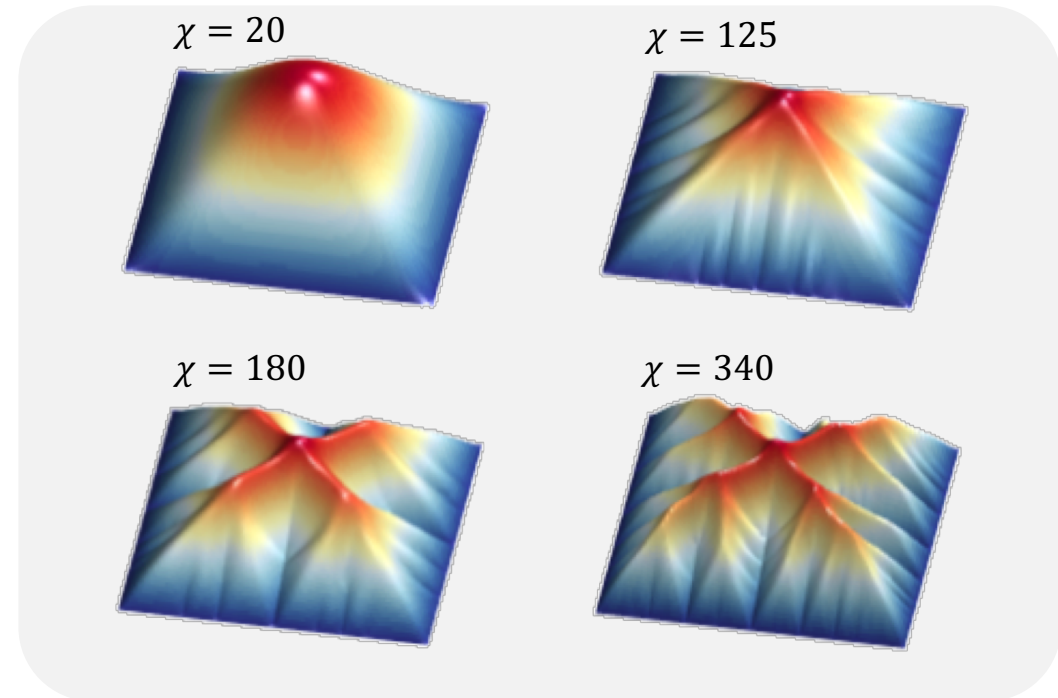
$$-\nabla \cdot \left( a \frac{\nabla z}{|\nabla z|} \right) = 1$$

$$\frac{\partial z}{\partial t} = D\nabla^2 z - Ka^m |\nabla z|^n + U$$

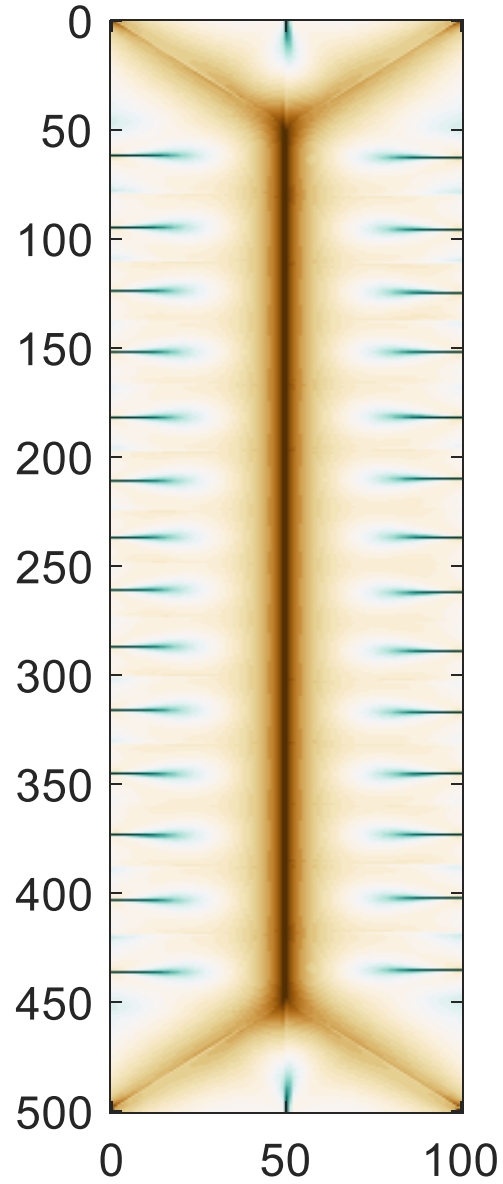


$$\chi = \frac{K l^{m+n}}{D^n U^{1-n}}$$

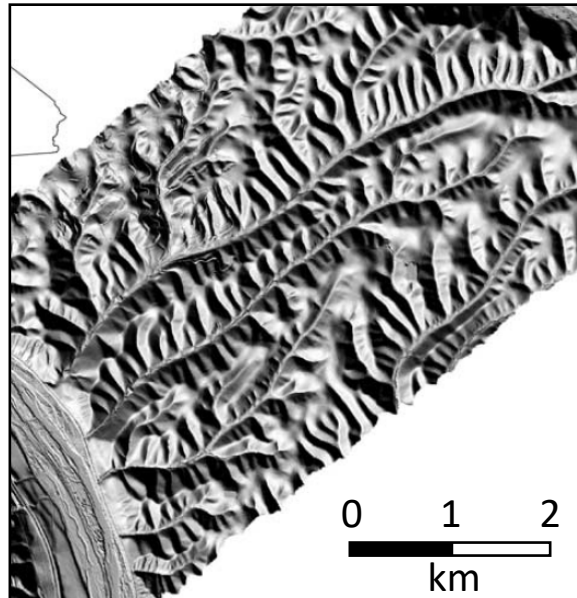
role of vegetation?



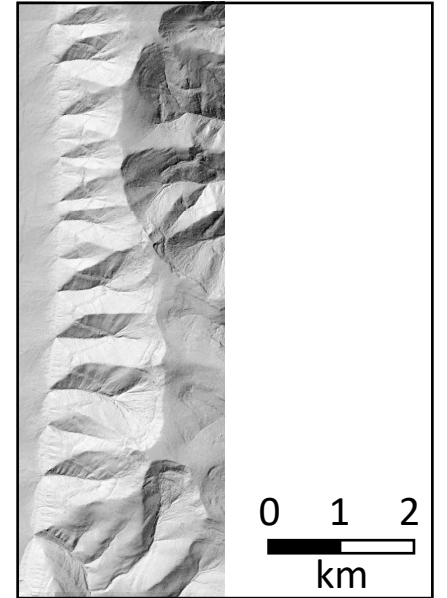
# Evenly-spaced valleys



Orland, CA (Perron et al., 2008)

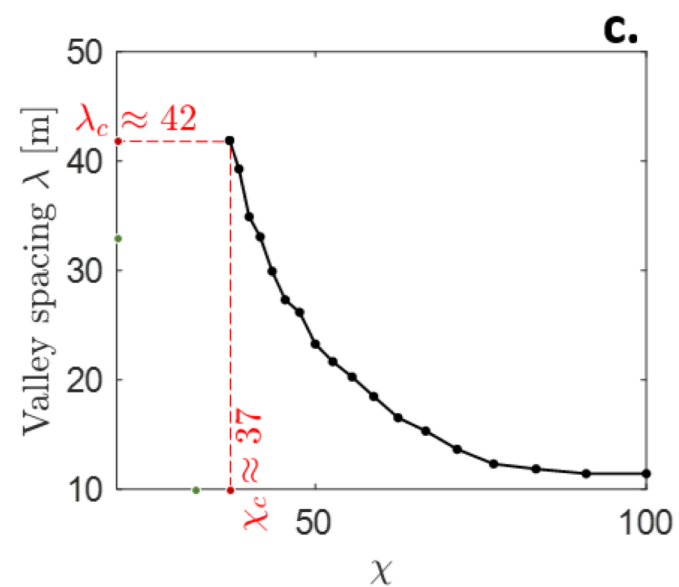
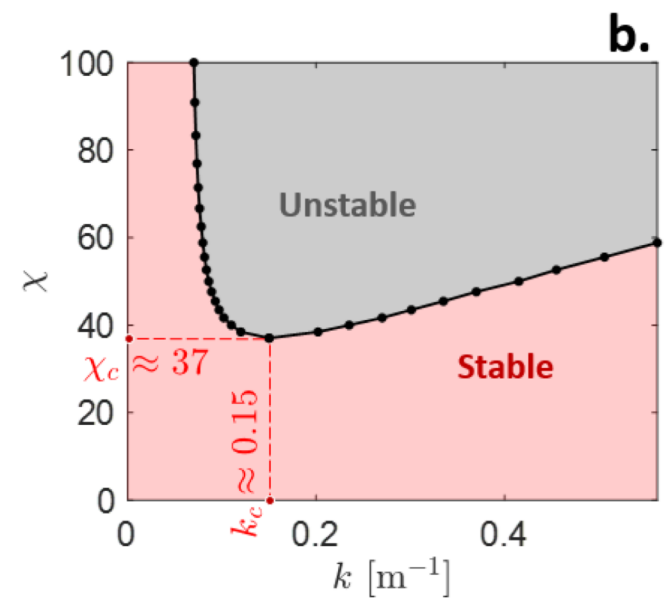
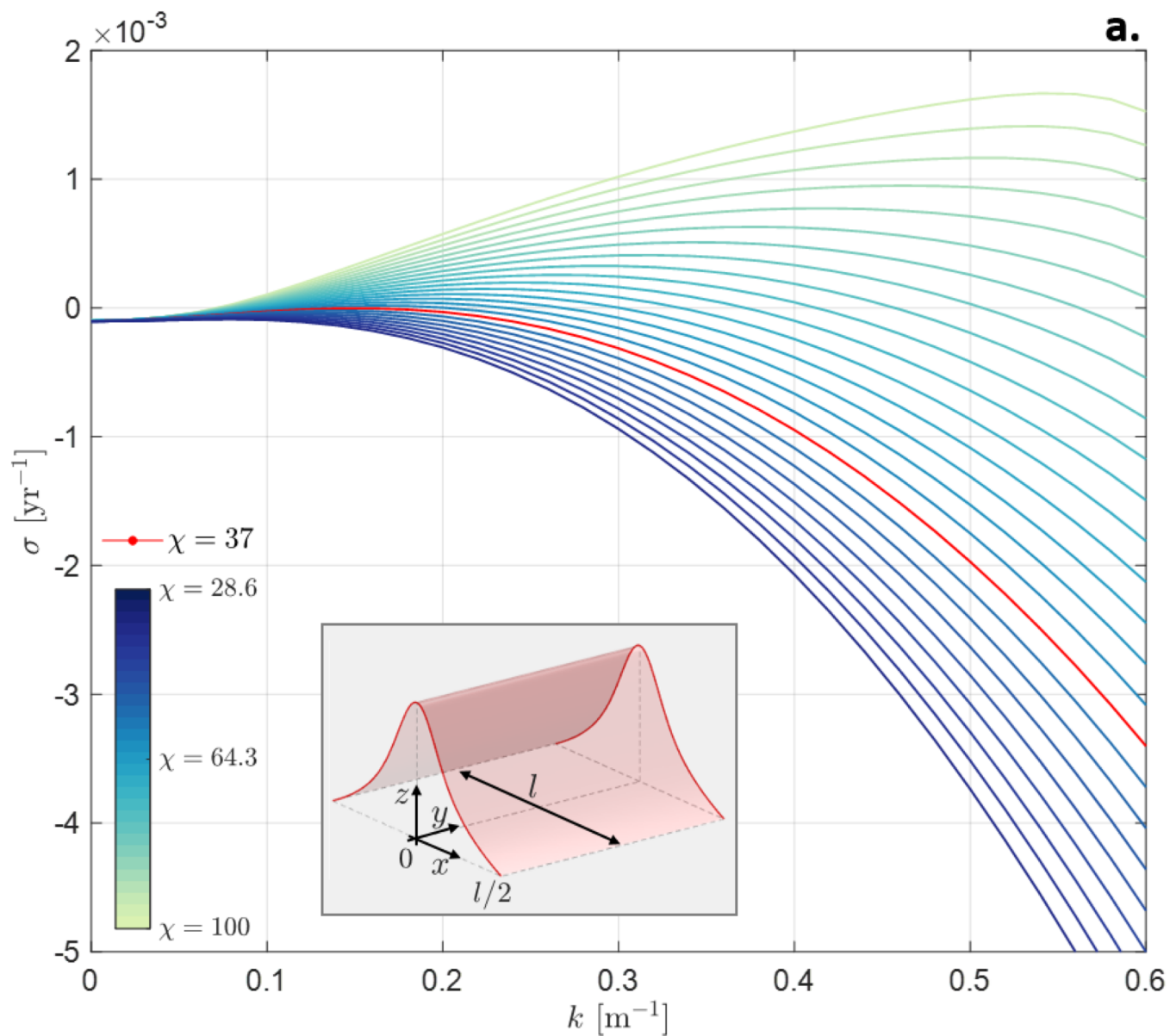


Gabilan Mesa, CA (Perron et al., 2008)

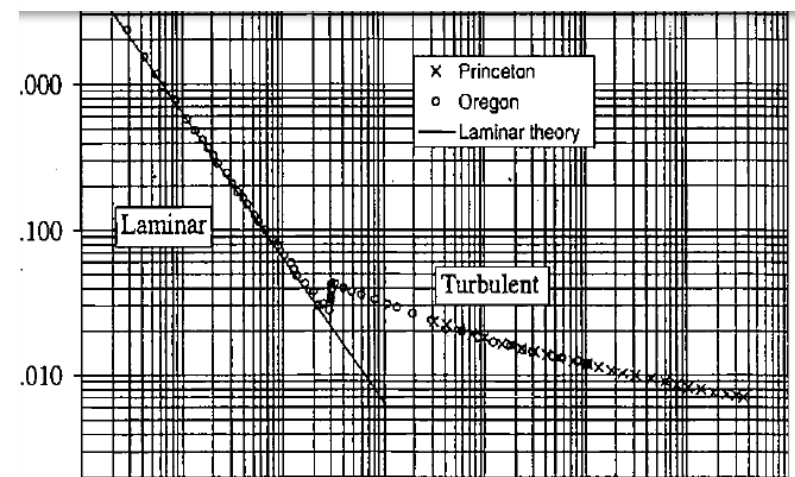
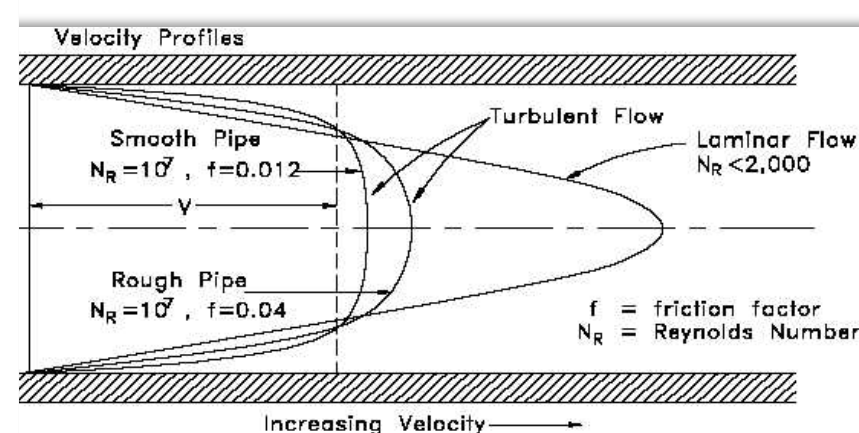
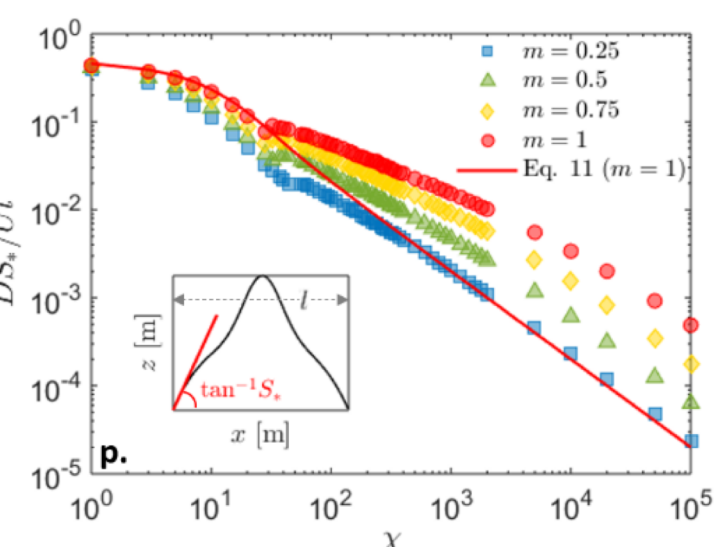
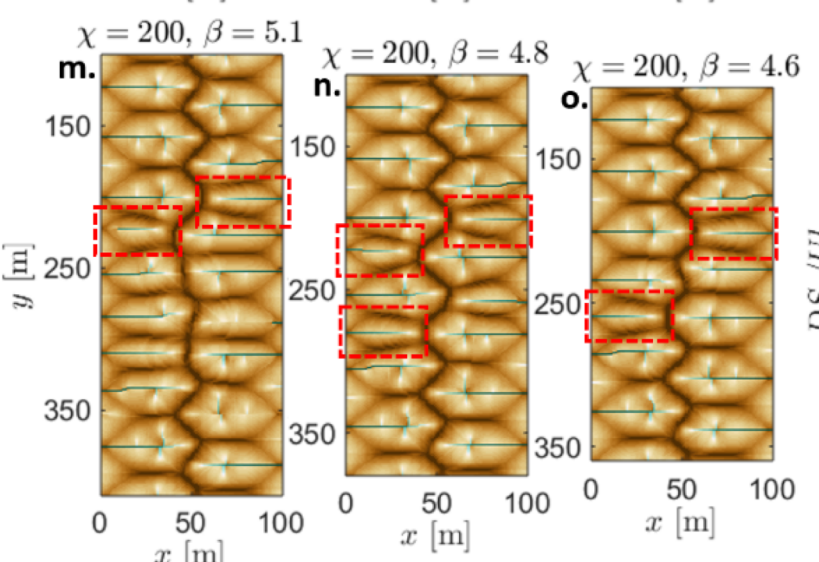
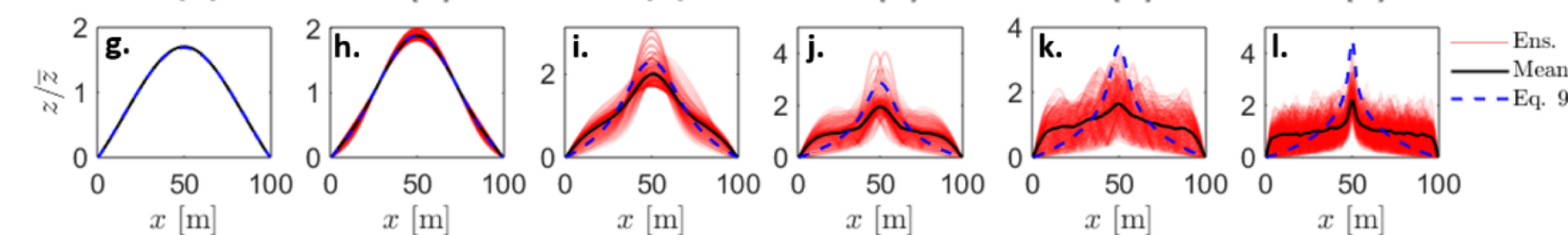
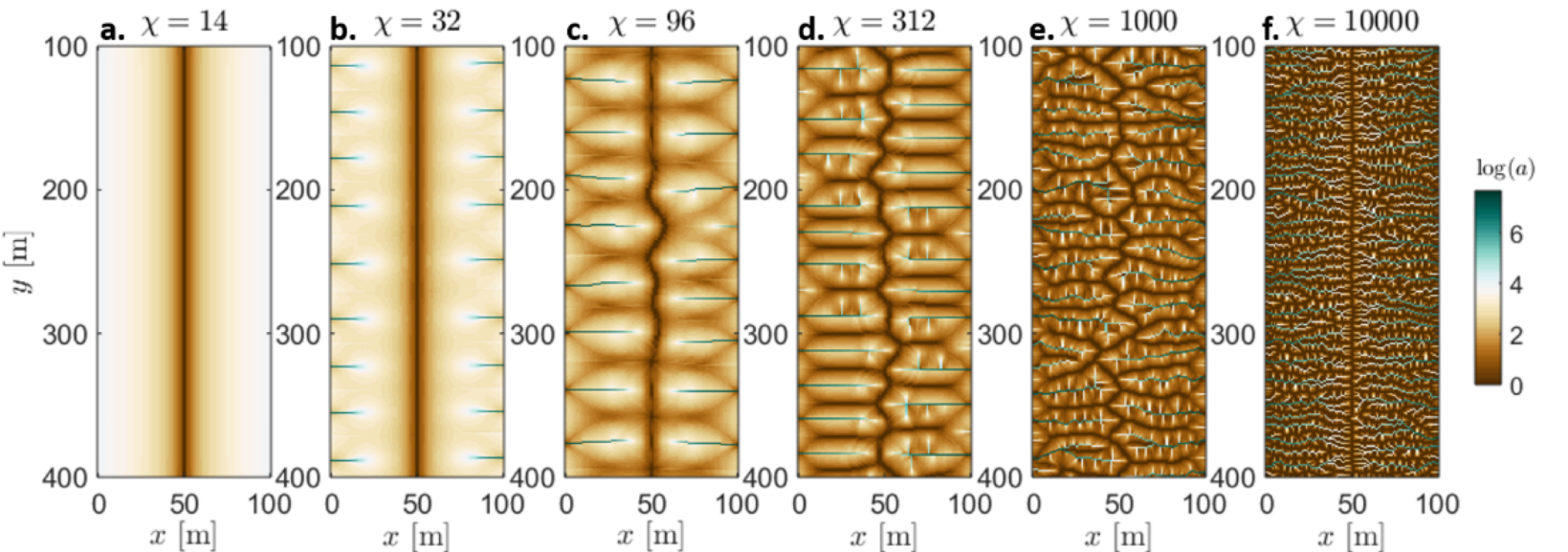


Manti-La Sal National Forest, UT

# Linear Stability Analysis



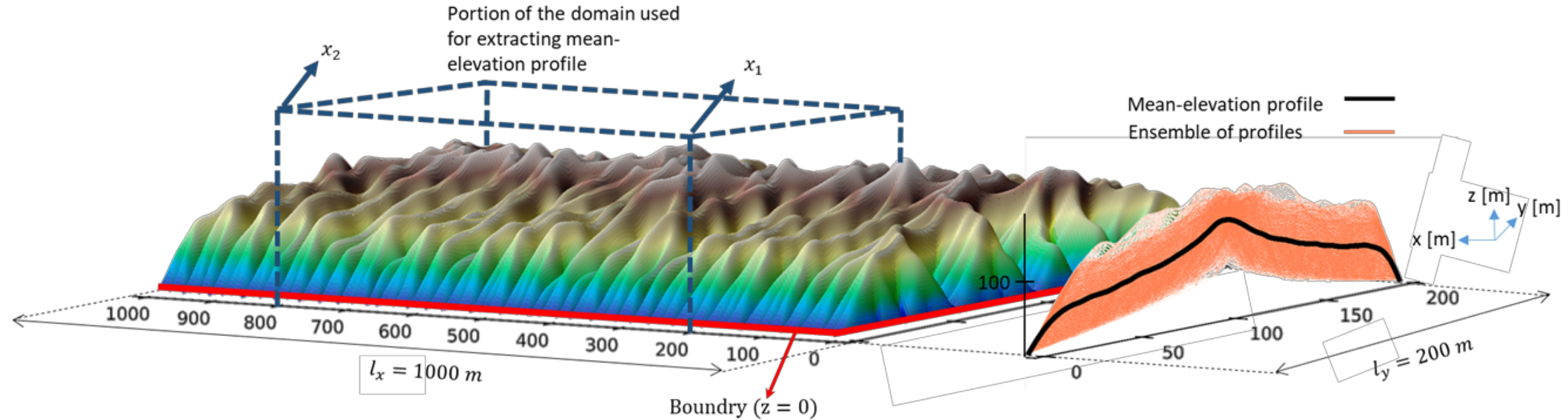
$\chi$  increases complexity





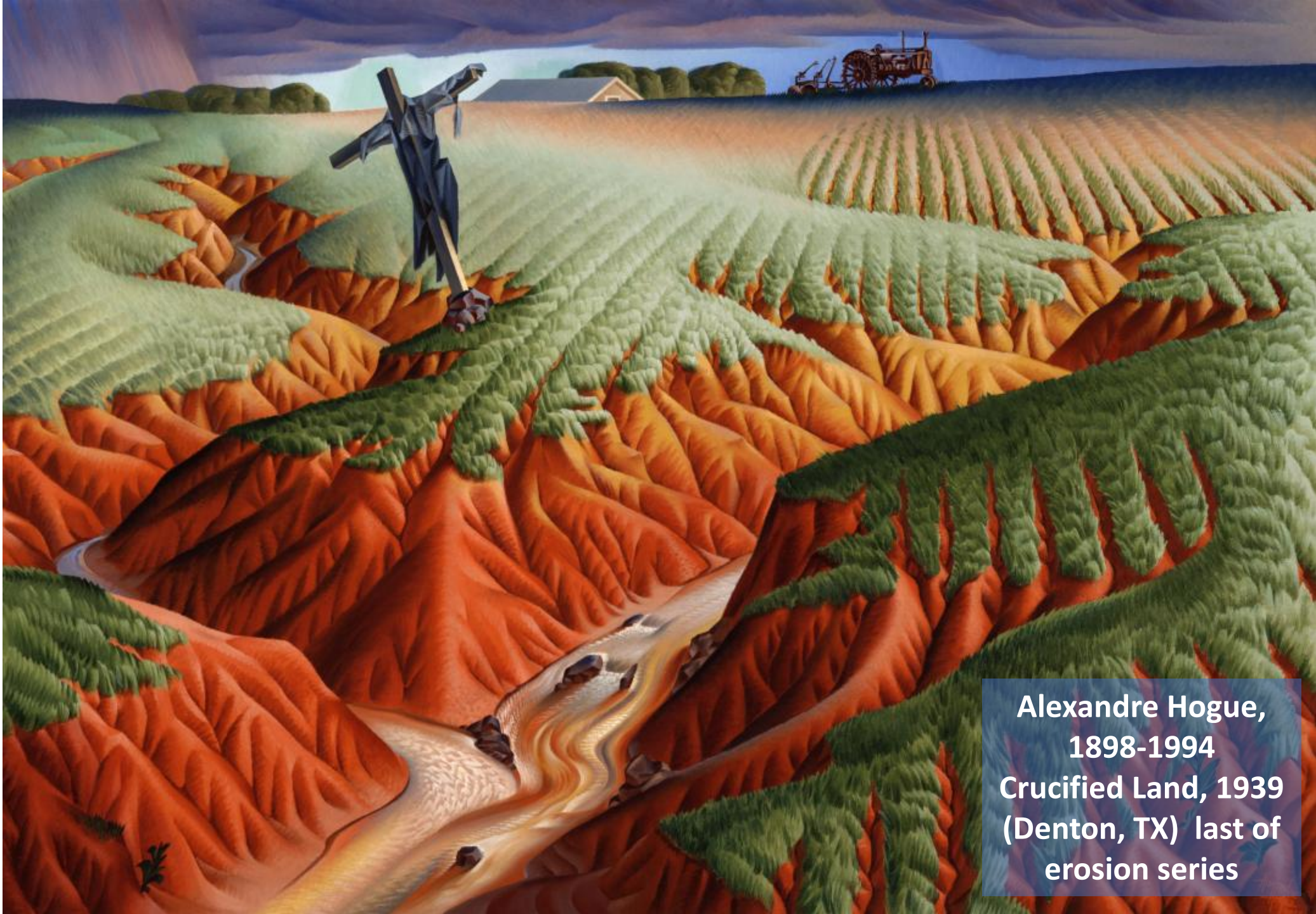
# Logarithmic elevation profile

Hooshyar et al. (poster Wed evening)





Thank you!



Alexandre Hogue,  
1898-1994  
Crucified Land, 1939  
(Denton, TX) last of  
erosion series