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From Categorical to Numerical:
Multiple Transitive Distance Learning and Embedding

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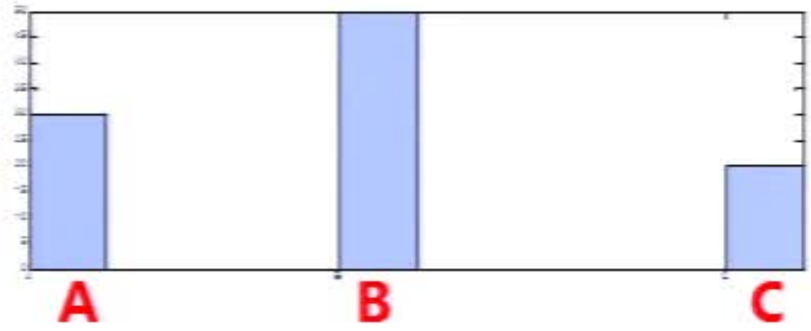
Outline

- Categorical Data and Challenges
- Why from Categorical to Numerical?
- Multiple Transitive Distance Learning
- Experimental Results

Categorical Data

Definition

- only takes limited number of values
  - levels, categories, groups, etc.
- nominal (no order) or ordinal (order)

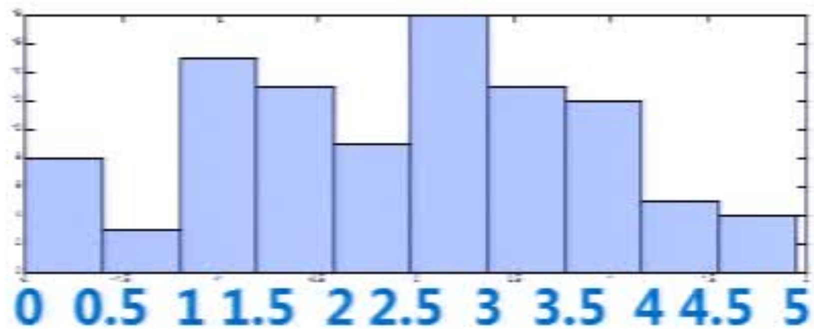


distribution of a categorical variable

Examples

- social, biology, psychology, …

attributes	symbols
Severity (symptom)	mild, moderate, severe
Weather	windy, cloudy, sunny
Blood type	A, B, AB, O



distribution of a numerical variable

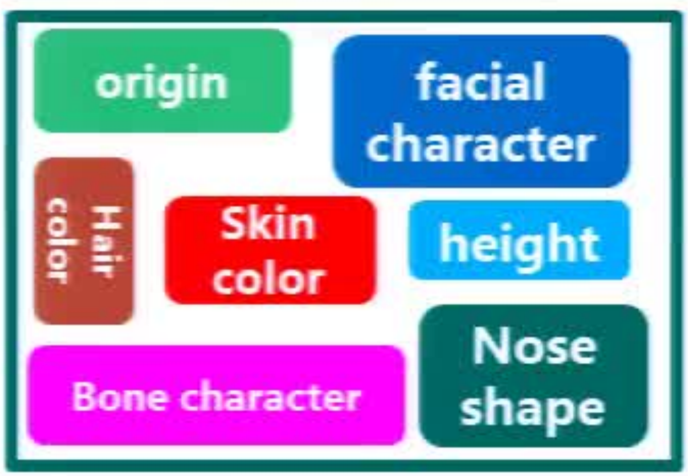
Categorical Data are Difficult

Curse of cardinality
- joint relation among symbols (and response)

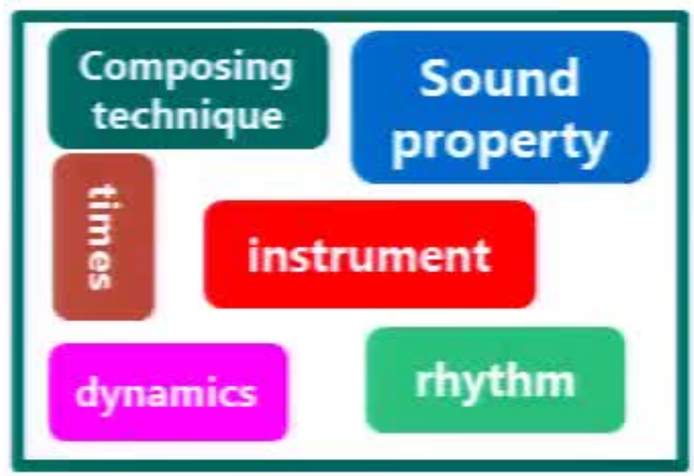
Lack of distance measures
- distance between symbols undefined

Richness of information
- one symbol can be a composite status

RACE



MUSIC



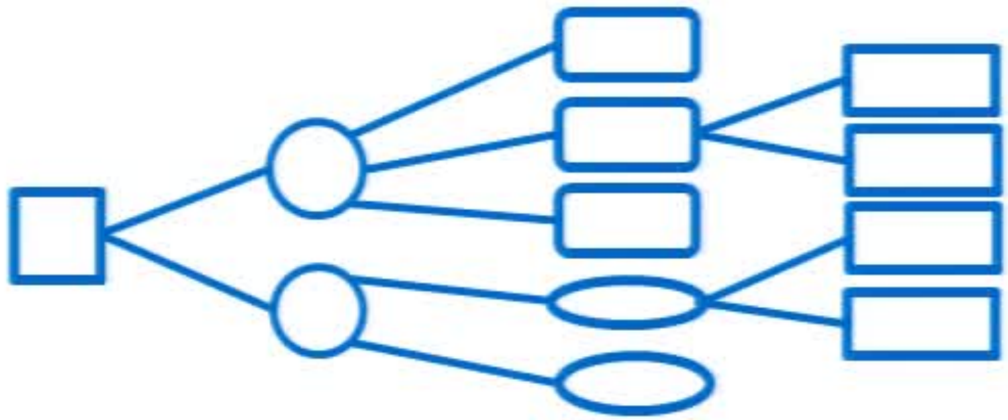
Existing Methods

Statistical modelling

- Binomial, multinomial, poisson, etc.
- Generalized Linear Models
- Logistic regression

Rule Analysis

- Decision Tree



Variable Coding

- Contrast coding
- Regression coding

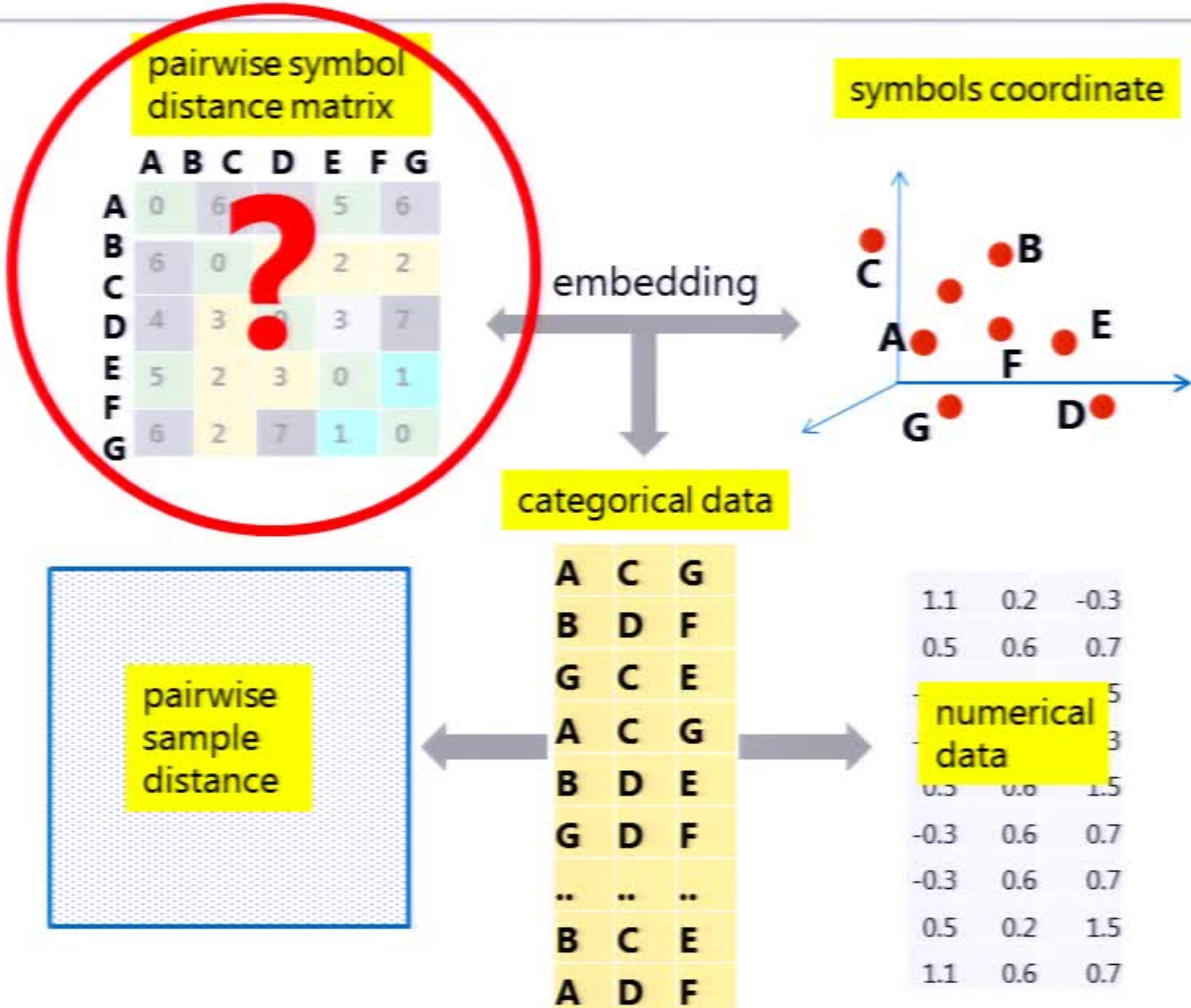
Level of race	New variable 1 (c1)	New variable 2 (c2)	New variable 3 (c3)
1 (Hispanic)	1	0	0
2 (Asian)	0	1	0
3 (African American)	0	0	1
4 (white)	-1	-1	-1

From categorical to Numerical

Why?

- continuous data sometimes easier to handle
  - distribution estimation
  - visualization
  - correlation analysis
  - ....
- abundant algorithms designed for numerical data

Key Idea



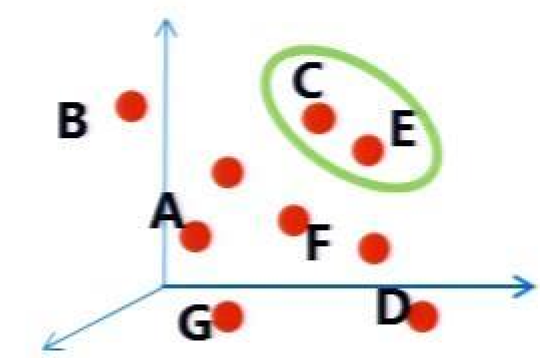
pairwise symbol distance matrix
A B C D E F G
?
embedding
symbols coordinate
categorical data
A C G
B D F
G C E
A C G
B D E
G D F
.. .. ..
B C E
A D F
pairwise sample distance
numerical data
1.1 0.2 -0.3
0.5 0.6 0.7
-0.3 0.6 0.7
-0.3 0.6 0.7
0.5 0.2 1.5
1.1 0.6 0.7

Distance and Embedding

Multidimensional Scaling [Cox and Cox 2001]

- pairwise distances of n objects $S \in R^{n \times n}$
- eigen-decomposition $-\frac{1}{2}HSH = U\Delta U$
- embedding $U\Delta^{-1/2}$ recovers distances in S

pairwise distance



low-dimensional embedding

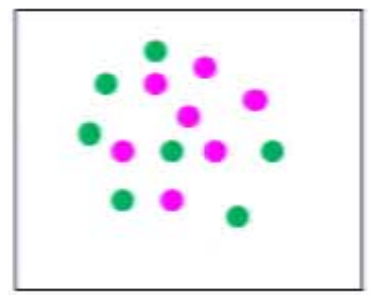
Basic Criterion

What are good symbol distances?

	A	B	C
A	0	2	3
B	2	0	4
C	3	4	0

	A	B	C
A	0	2	3
B	2	0	10
C	3	10	0

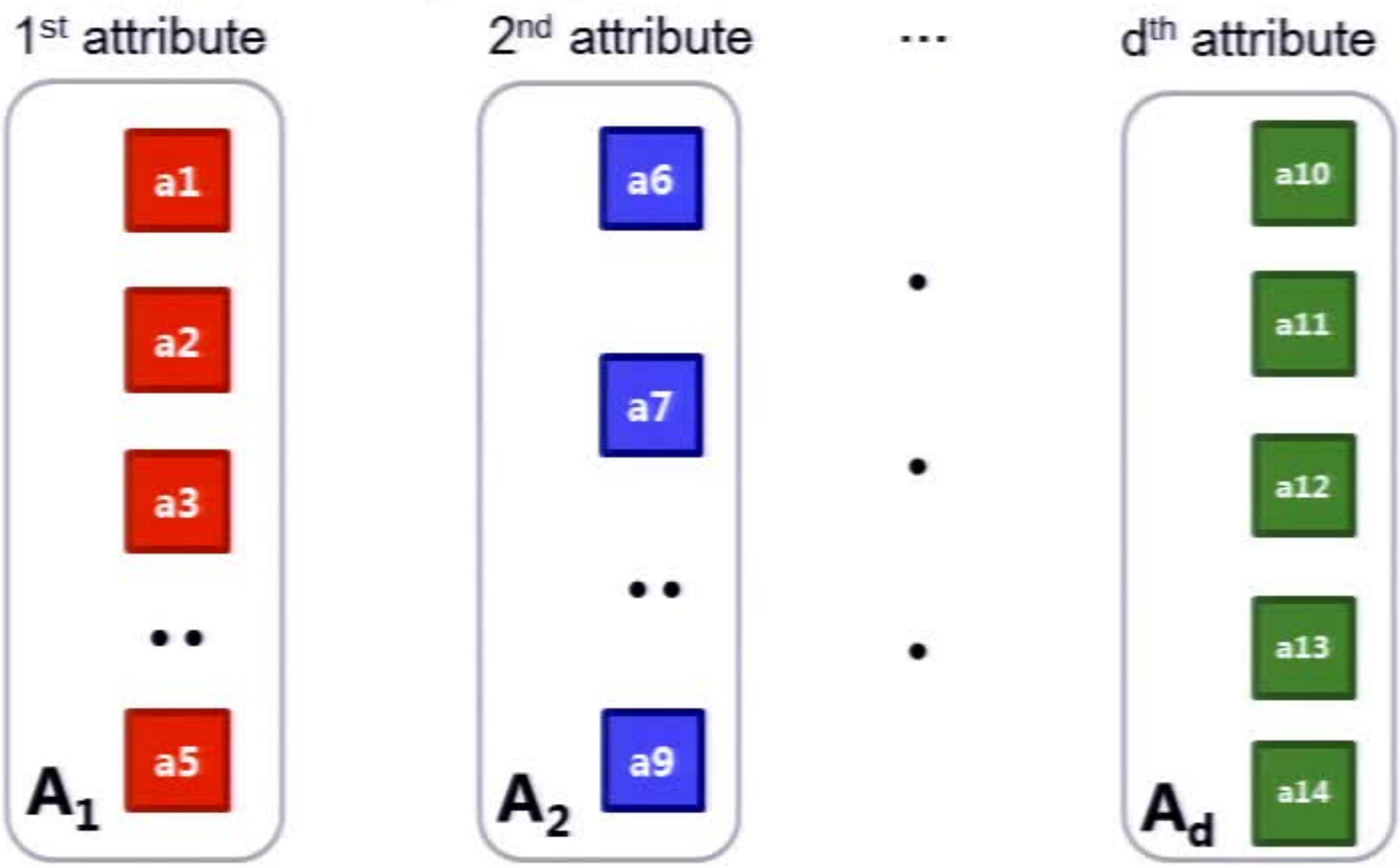
- **Transitivity**
  - if A ↔ B, and A ↔ C, then B ↔ C
  - co-occurrence statistics may fail
  - measuring proximity in categorical data [Gibson et al. 1998]



- **Consistency** (with target)
  - symbol distances determine *sample geometry*
  - induced geometry should be predictive of learning target

Multiple Transitive Distance Learning

- Notations: given n-by-d symbol matrix X



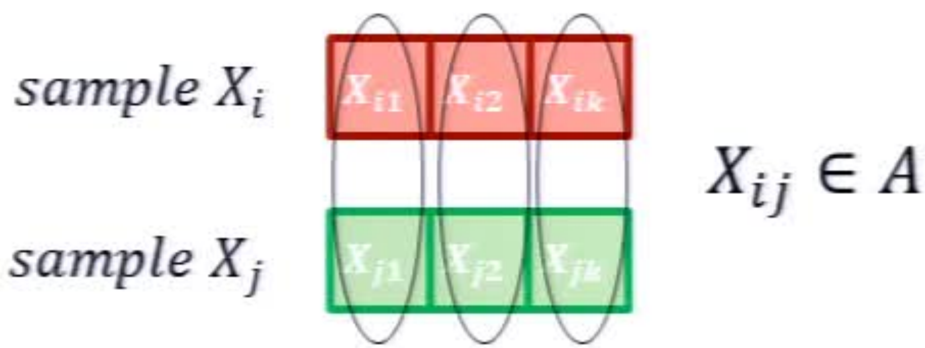
$$A = \{a_1, a_2, \dots, \quad a_6, a_7, \dots, \quad \dots, \quad a_{10}, a_{11}, \dots\}$$

Basic Assumptions

L-θ distance computation

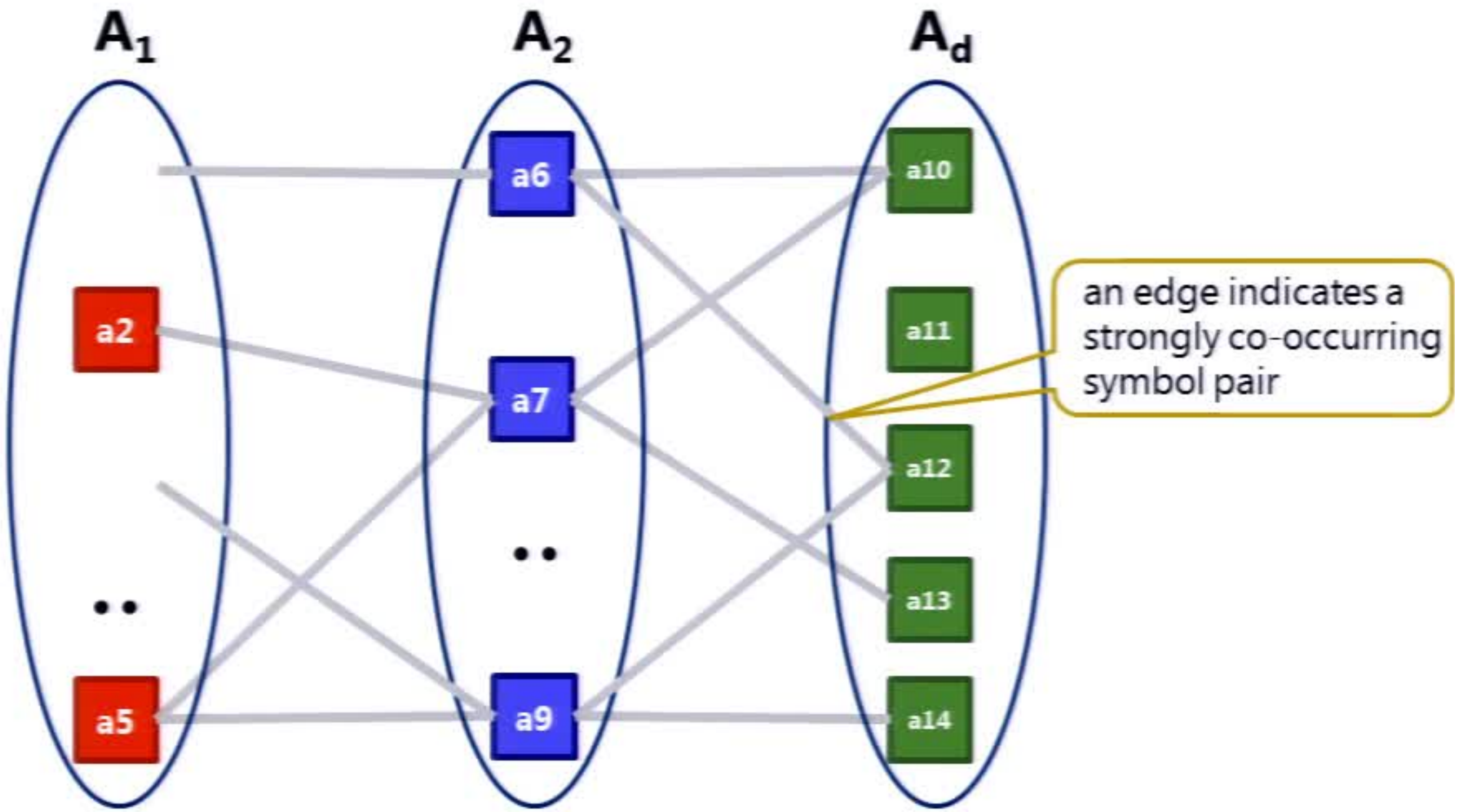
- distance between two samples is the sum of distances over each attribute

$$\mathrm{dis}(X_i, X_j)^\theta = \sum_{k=1}^{d} \mathrm{dis}(X_{ik}, X_{jk})^\theta$$



Multiple Transitive Distance Learning

- Transitive distances: shortest path



Initial d-partite graph of symbols

Basic Assumptions

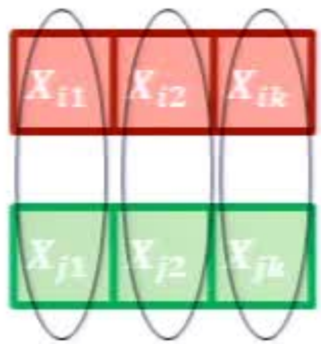
L-θ distance computation

- distance between two samples is the sum of distances over each attribute

$$\mathrm{dis}(X_i, X_j)^\theta = \sum_{k=1}^{d} \mathrm{dis}(X_{ik}, X_{jk})^\theta$$

sample X_i

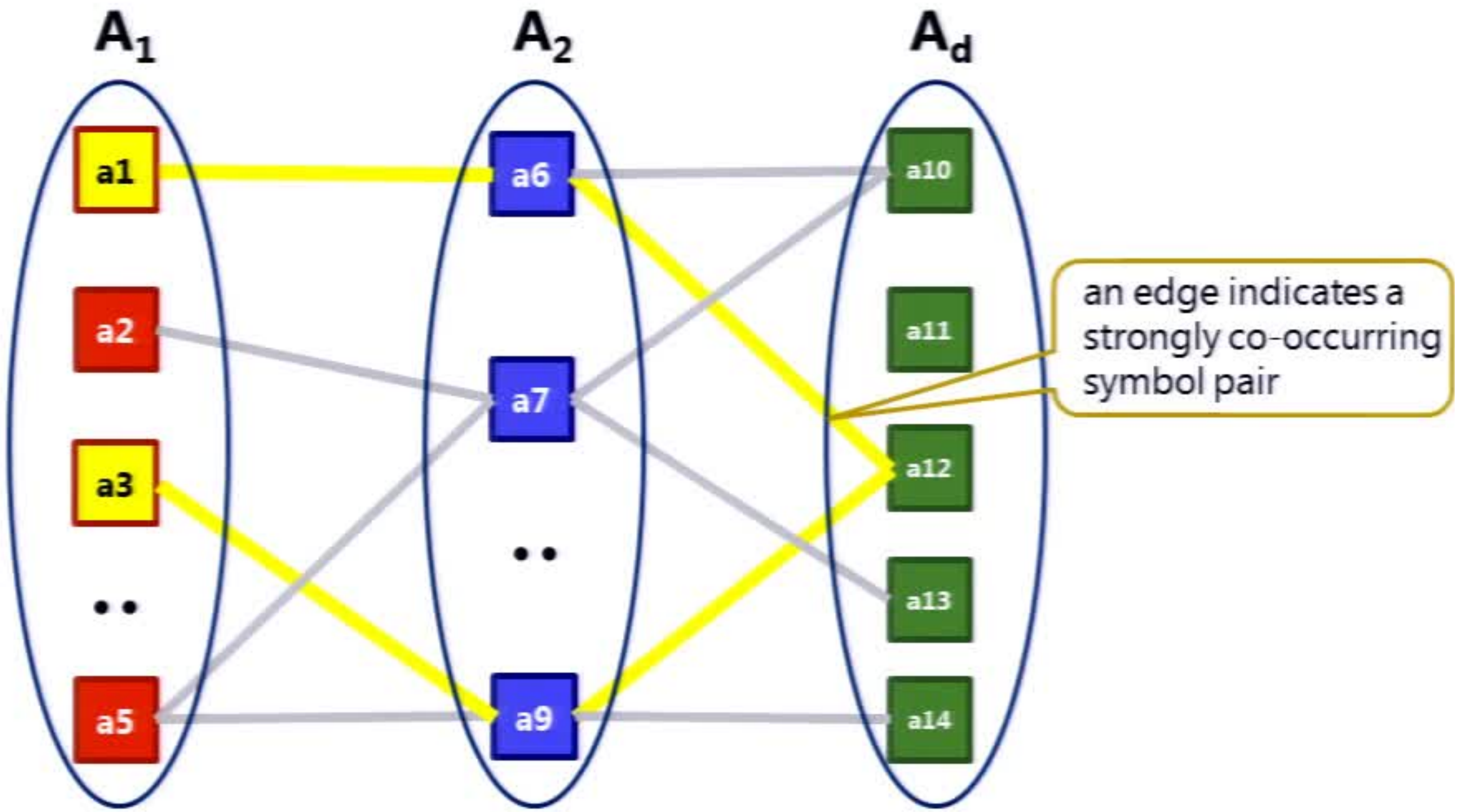
sample X_j



$$X_{ij} \in A$$

Multiple Transitive Distance Learning

- Transitive distances: shortest path



Initial d-partite graph of symbols

Choice of d-partite graphs

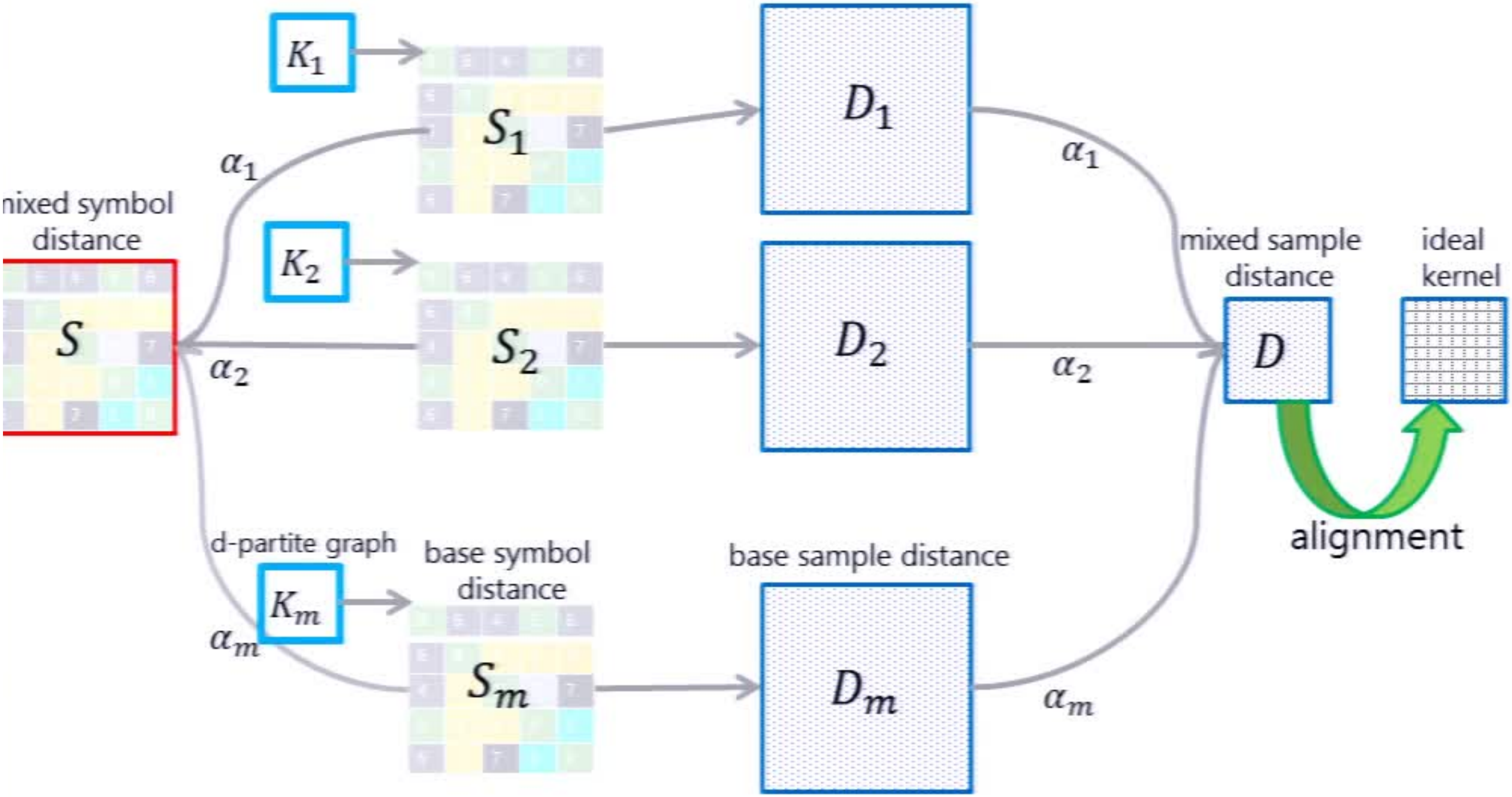
k-partite graph ***K*** using different proximities

- step 1: sparsification
- step 2: turn similarity to distance ($\frac{1}{x}, -\log(x)$)

	Proximity	Connected edge	Non-conn.
Similarities	co-occurrence	$\lvert a_p \cap a_q \rvert$	0
	norm. co-occr.	$\lvert a_p \cap a_q \rvert / \lvert a_p \cup a_q \rvert$	
	mutual info.	$\sum p(a_p = e, a_q = \hat{e}) \log\left(\frac{p(a_p = e, a_q = \hat{e})}{p(a_p = e) p(a_q = \hat{e})}\right)$	
Distances	hamming dist.	$\lvert a_p - a_q \rvert$	∞
	Euclidian dist.	$\lVert a_p - a_q \rVert_2$	
	cosine dist.	$arccos\left(a_p' a_q / \sqrt{\lVert a_p \rVert \lVert a_q \rVert} \right)$	

Multiple Distance Learning

- Align multiple base distances to class labels



Multiple distance learning

Given constraints: must-link $\boldsymbol{S}$; cannot-link $\boldsymbol{D}$

- sum of within-class distance pairs: $\mu_{ij} = [D_{1(i,j)}, D_{2(i,j)}, \ldots, D_{m(i,j)}]$

$$\boldsymbol{J}_S^{\epsilon} = \sum_{(i,j)\in S} \left(\sum_{m=1}^{L} \alpha_m D_{m(i,j)} \right)^{\epsilon} = \sum_{(i,j)\in S} (\langle \mu_{ij}, \boldsymbol{\alpha} \rangle)^{\epsilon}$$

- sum of inter-class distance pairs $\boldsymbol{J}_D = \sum_{(i,j)\in D} (\langle \mu_{ij}, \boldsymbol{\alpha} \rangle)^{\epsilon}$

Discriminative embedding

- linear program

$$\max_{\boldsymbol{\alpha}} \boldsymbol{J}_D^1 - \boldsymbol{J}_S^1 = \left(\sum_{(i,j)\in D} \mu_{ij} - \sum_{(i,j)\in S} \mu_{ij} \right)^T \boldsymbol{\alpha}$$

$$s.t. \ \ \boldsymbol{\alpha} \geq 0, \boldsymbol{\alpha}^T 1 = 1$$

- quadratic program

$$\min_{\boldsymbol{\alpha}} \frac{\boldsymbol{J}_S^2}{\boldsymbol{J}_D^2} = \frac{\boldsymbol{\alpha}^T \left(\sum_{(i,j)\in S} \mu_{ij} {\mu_{ij}}^T \right) \boldsymbol{\alpha}}{\boldsymbol{\alpha}^T \left(\sum_{(i,j)\in D} \mu_{ij} {\mu_{ij}}^T \right) \boldsymbol{\alpha}}$$

Experimental Results

Competing Methods

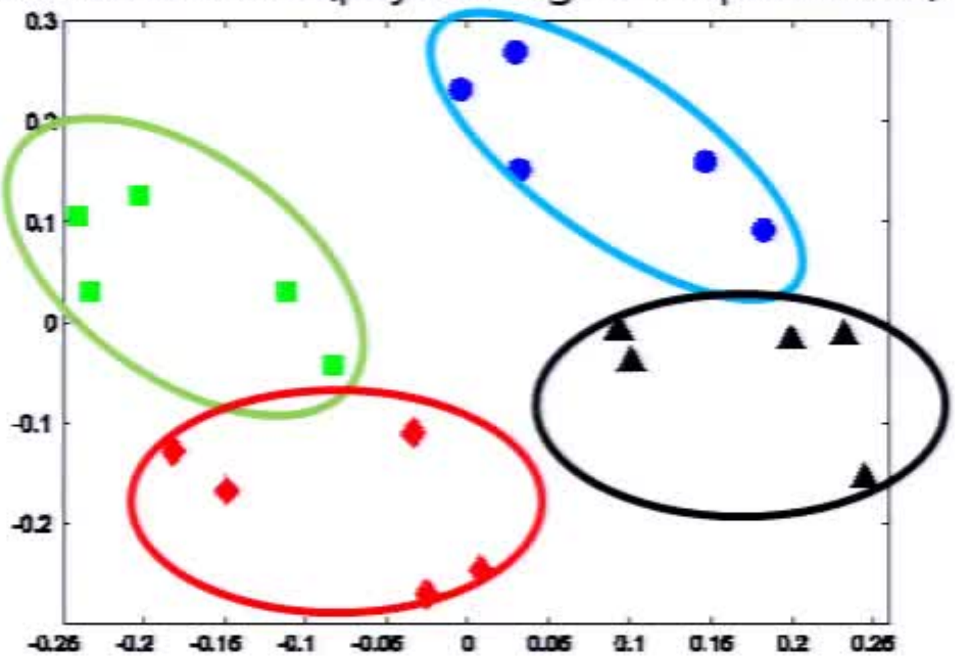
1. decision tree
2. dummy coding
3. density-based logistic regression [Chen et al 2013]
4. our method

Mean and variance of the classification accuracy on UCI datasets.
(random splits 50/50 as training/test data repeated 30 times)

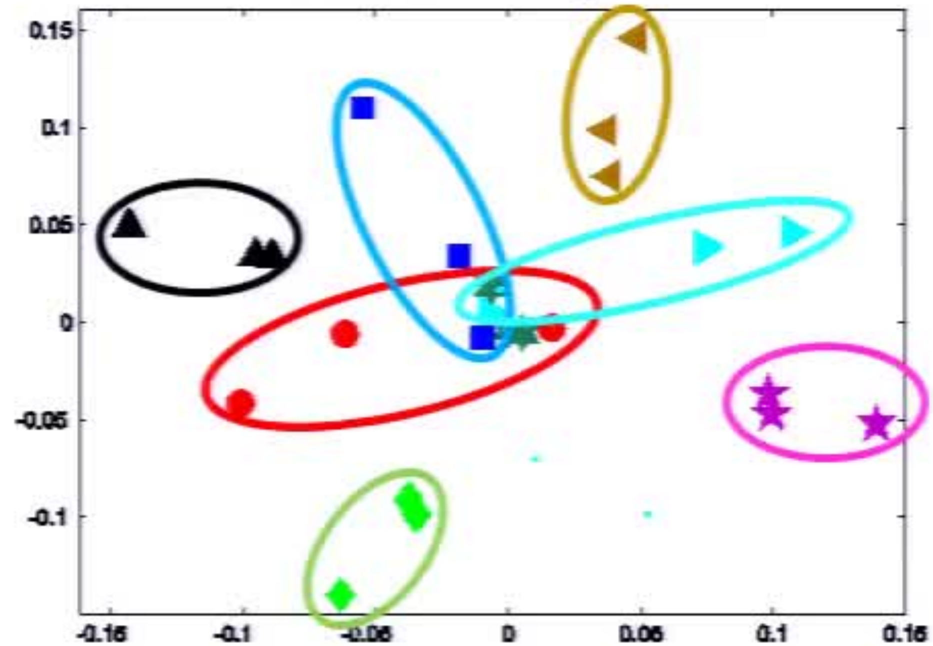
	Balance	Mushroom	Tic-Tac-Toe	Splice	Cancer	Hayes-Roth	Monk
Dummy	98.50±0.75	99.04±0.73	95.25±0.42	91.18±0.94	95.81±1.23	77.42±6.82	96.24±1.13
Density	96.59±1.58	98.43±0.57	70.60±2.34	**91.29±0.98**	**96.7±0.76**	57.7±8.94	96.37±0.85
Dec. Tree	88.49±1.63	**99.40±0.54**	87.48±2.18	88.68±1.73	94.25±1.38	73.93±7.74	97.13±0.72
Ours	**99.42±0.58**	**99.54±0.46**	**98.25±0.44**	**91.51±1.20**	95.94±0.93	**83.91±3.64**	**97.81±1.23**

Embedding Case Study

Balance-scale (psychological experiment)



Tic-Tac-Toe Endgame



1. Left-Weight: (1, 2, 3, 4, 5)
2. Left-Distance: (1, 2, 3, 4, 5)
3. Right-Weight: (1, 2, 3, 4, 5)
4. Right-Distance: (1, 2, 3, 4, 5)

1. top-left-square: {x,o,b}
2. top-middle-square: {x,o,b}
3. top-right-square: {x,o,b}
4. middle-left-square: {x,o,b}
5. middle-middle-square: {x,o,b}
6. middle-right-square: {x,o,b}
7. bottom-left-square: {x,o,b}
8. bottom-middle-square: {x,o,b}
9. bottom-right-square: {x,o,b}