



Source: National Geographic, 2011. The World's Coral Reefs

# Spatiotemporal Model of Coral Reefs

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# Why Study Coral ?

Coral reef systems are **critical and complicated**

25% of marine life began in a reef

Billion dollar industry (UNEP, 2018).

Projected temperature increases -> more coral mortality

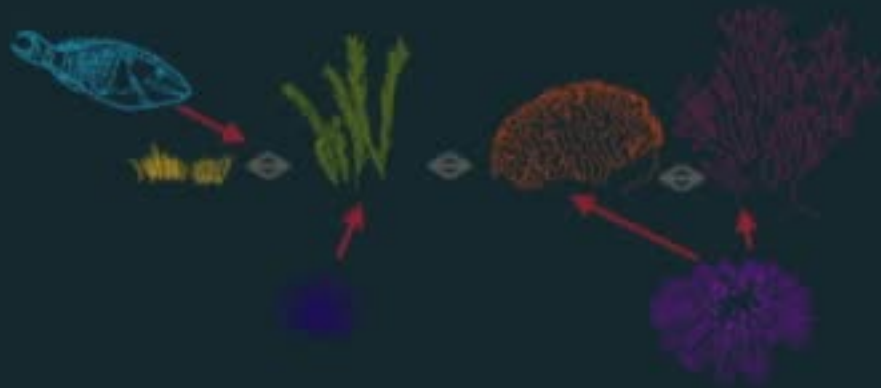
Intriguing nonlinearities



# Overview

Coral Reef Introduction

Underlying ODEs



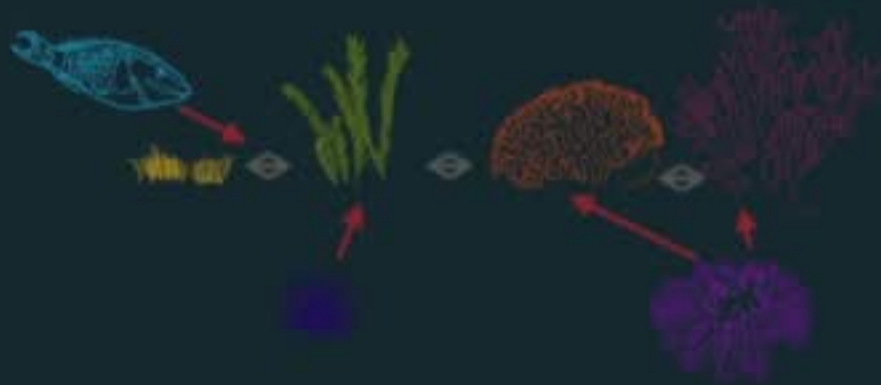
$$\frac{dM}{dt} = aMC - \frac{gM}{M+T} + \gamma MT$$

$$\frac{dC}{dt} = rTC - dC - aMC$$

$$\frac{dT}{dt} = \frac{gM}{M+T} - \gamma MT - rTC + dC$$

# Overview

## Coral Reef Introduction



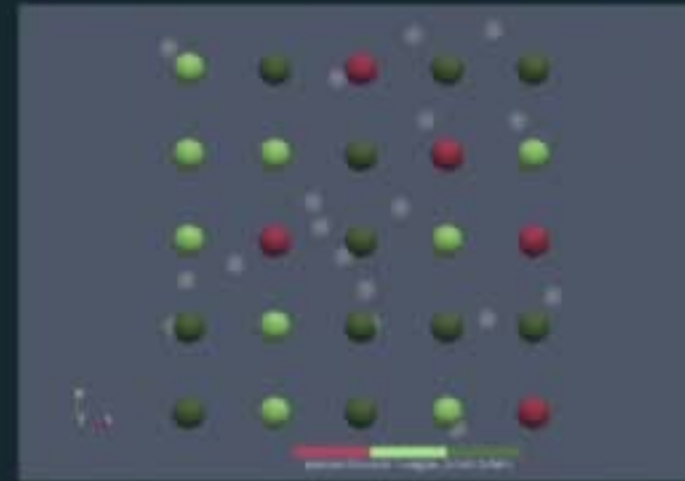
## Underlying ODEs

$$\frac{dM}{dt} = aMC - \frac{gM}{M+T} + \gamma MT$$

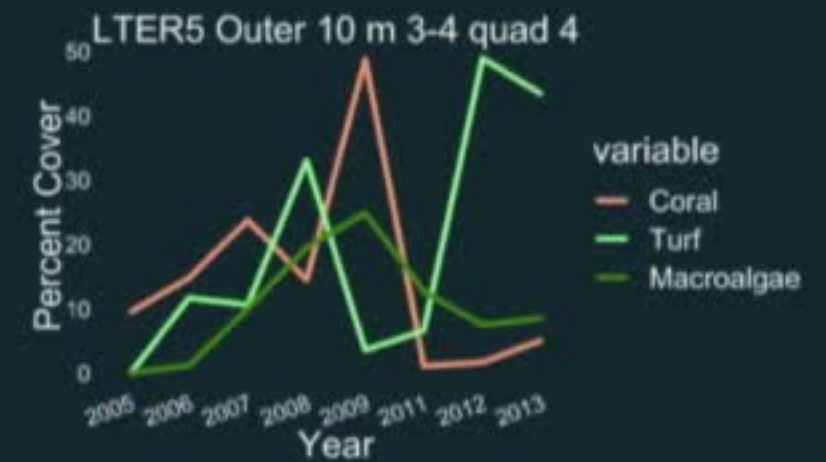
$$\frac{dC}{dt} = rTC - dC - aMC$$

$$\frac{dT}{dt} = \frac{gM}{M+T} - \gamma MT - rTC + dC$$

## Space and Stochasticity



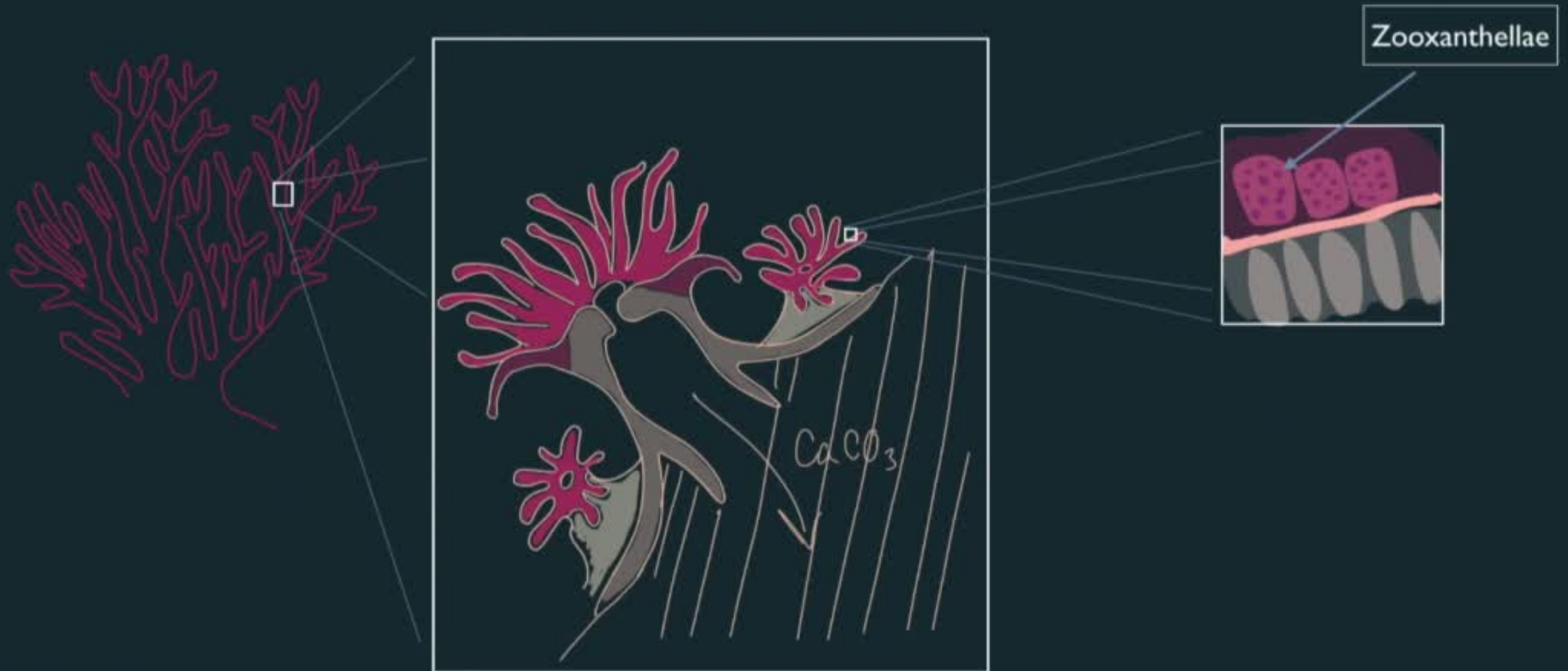
## Reef Representation



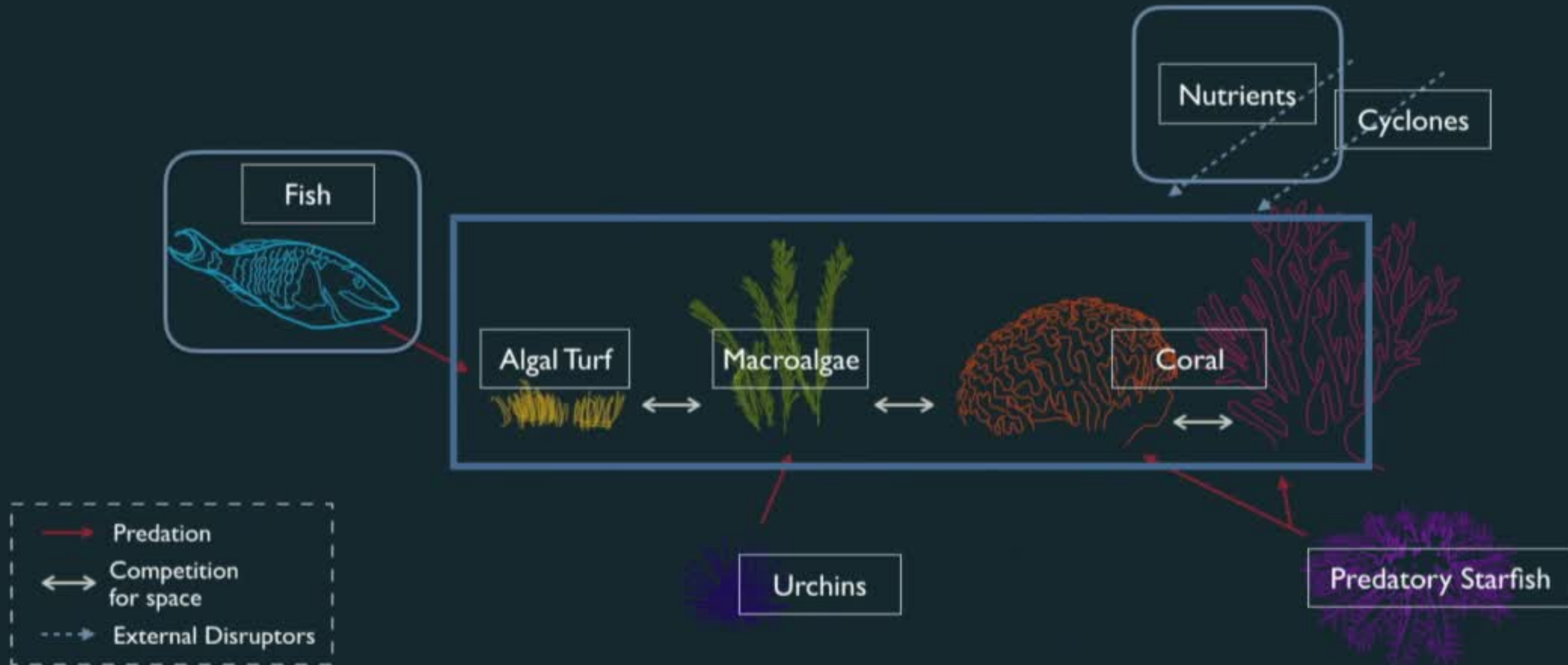
# The Coral Organism and its symbiont



# The Coral Organism and its symbiont



# Coral Reef Predation and Competition



# Coral Reef Predation and Competition Equations

Mumby, Hastings, Edwards 2007

$$\frac{dM}{dt} = aMC - \frac{gM}{M+T} + \gamma MT$$

$M$  Macroalgae

$r$  Growth rate of coral over turf

$d$  Death rate of coral

$$\frac{dC}{dt} = rTC - dC - aMC$$

$C$  Coral

$a$  Growth rate of macroalgae over coral

$g$  Rate of grazing

$\gamma$  Growth rate of macroalgae over turf

$$\frac{dT}{dt} = \frac{gM}{M+T} - \gamma MT - rTC + dC$$

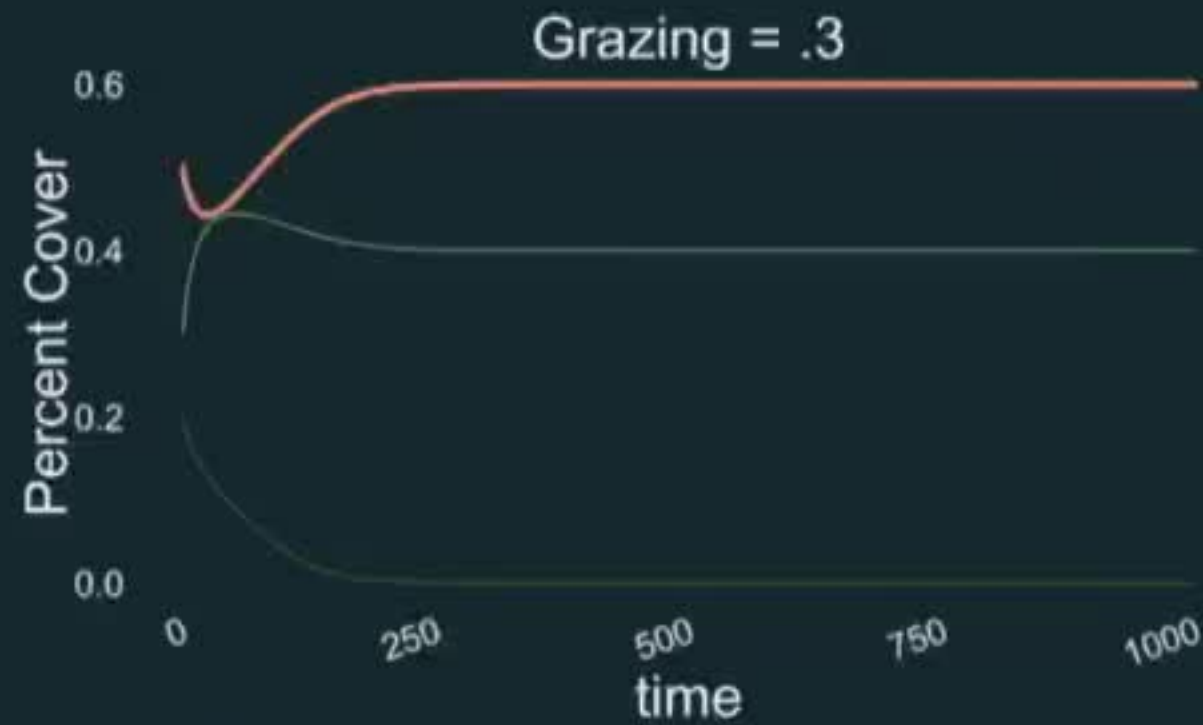
$T$  Turf



# Equations are Deterministic

## Show Bifurcation through grazing parameter changes

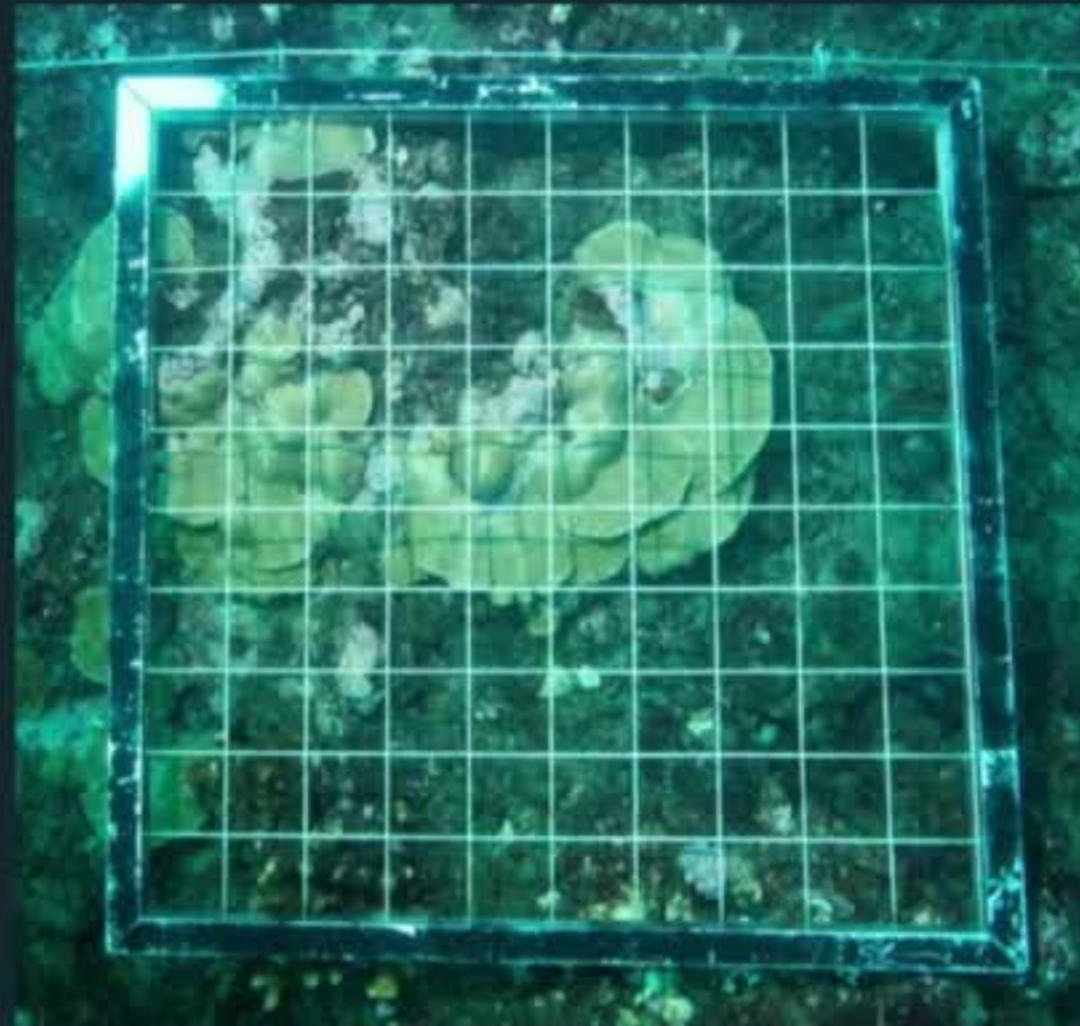
Turf Macroalgae Coral



# Coral Surveying : Moorea, French Polynesia



Source: "Biological and Physical Interactions on a Tropical Island Coral Reef: Transport and Retention Processes Around Moorea, French Polynesia" J.J. Leichter et al.



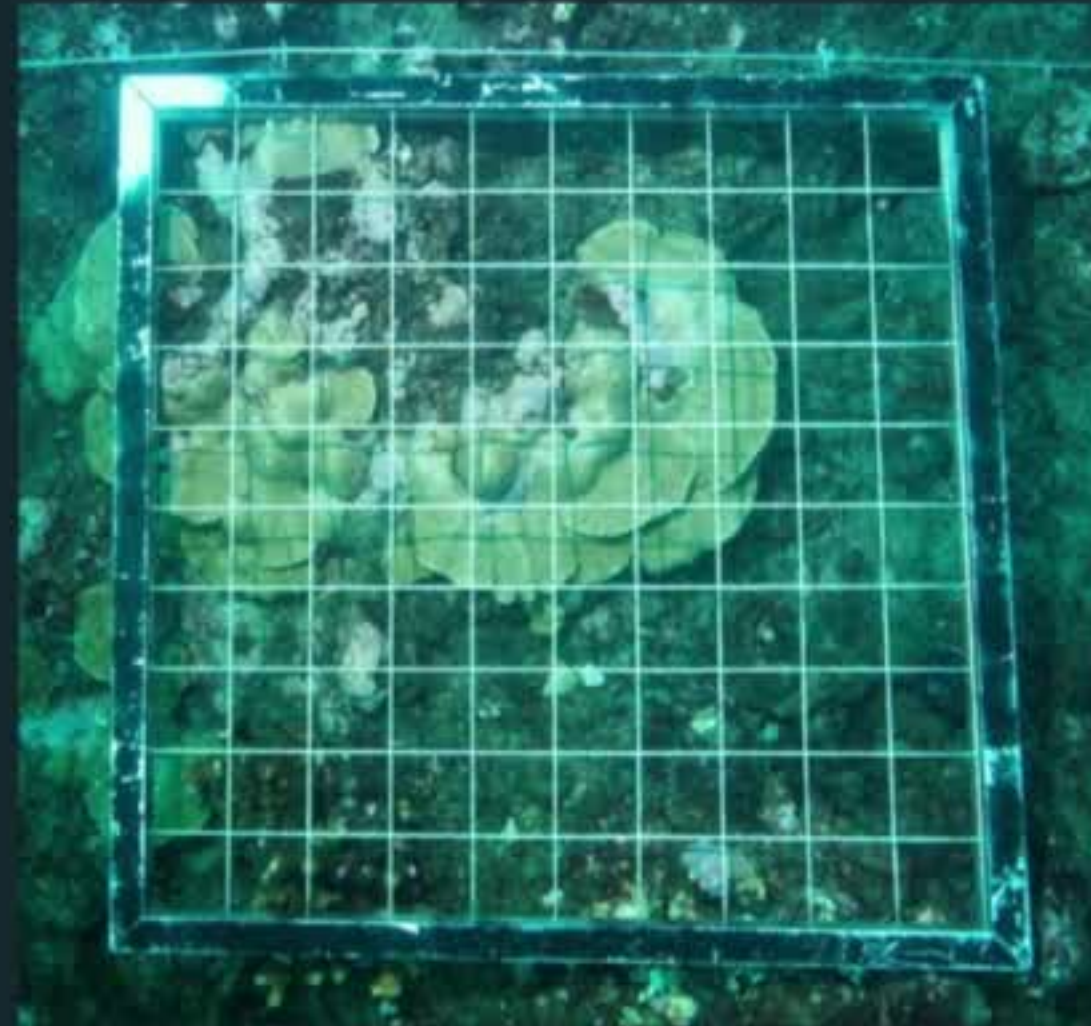
Source: Criobe station, SO CORAIL. <http://observatoire.criobe.pf>

# Toy Model Development: Does spatial discretization allow for how to model meta-stability between ecological regime shifts

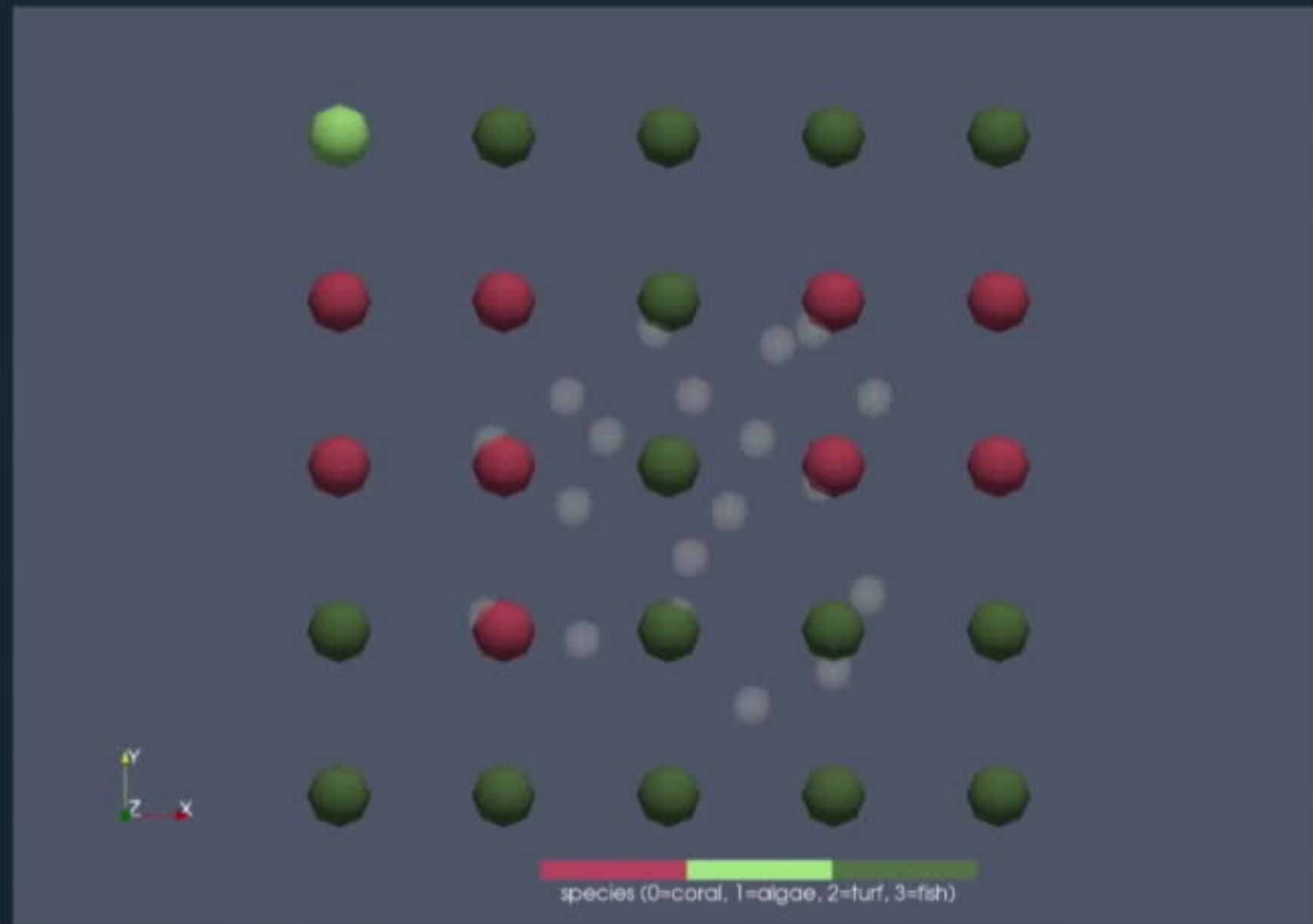
$$\frac{dM}{dt} = aMC - \frac{gM}{M+T} + \gamma MT$$

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$$\frac{dT}{dt} = \frac{gM}{M+T} - \gamma MT - rTC + dC$$



# Our Model



# Grid Update

```
U = random.uniform(0,1)

if Coral
    if U < d * grid_dt:
        Change to Turf
    if U < d * grid_dt + a * density_macroalgae * grid_dt:
        Change to Macroalgae
if Turf
    if U < gamma * grid_dt * density_macroalgae :
        Change to Macroalgae
    if U < (gamma * grid_dt * density_macroalgae] +
        r * grid_dt * density_coral:
        Change to Coral
if Macroalgae
    if U < g * density_grazers * grid_dt:
        Change to Turf
```

$r$  Growth rate of coral over turf

$d$  Death rate of coral

$a$  Growth rate of macroalgae  
over coral

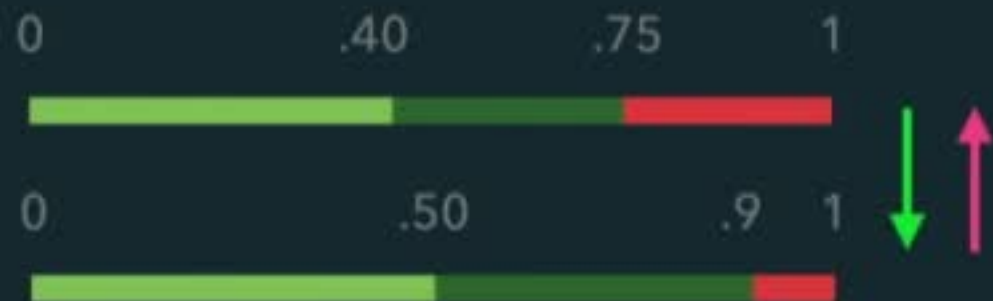
$g$  Rate of grazing

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# Introduction of Spatial Variability and Stochasticity

Model Objects

Coral, Macroalgae, and Turf are all the same object with the following attributes: Type (C, M, T, F), Velocity, Neighboring types

Random variable U

For initial grid setup, initial fish velocity, and grid update

Morse force

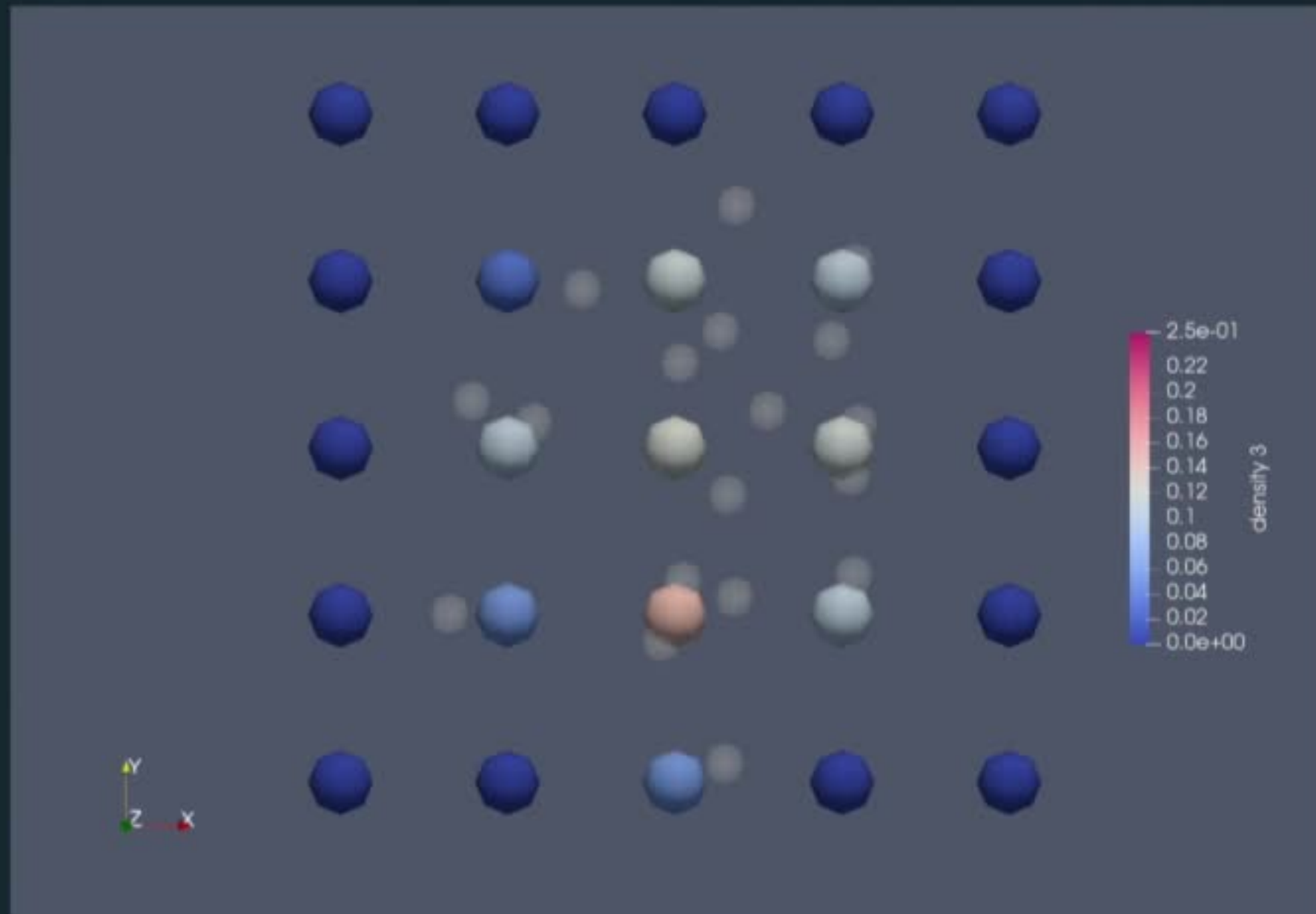
$$= C_r * e^{(-r/l_r)} - C_a * e^{(-r/l_a)}$$

For fish movement (attraction to coral and macroalgae, repulsion from turf, and attraction/repulsion to each other for "flocking" movement)

Neighborhood search

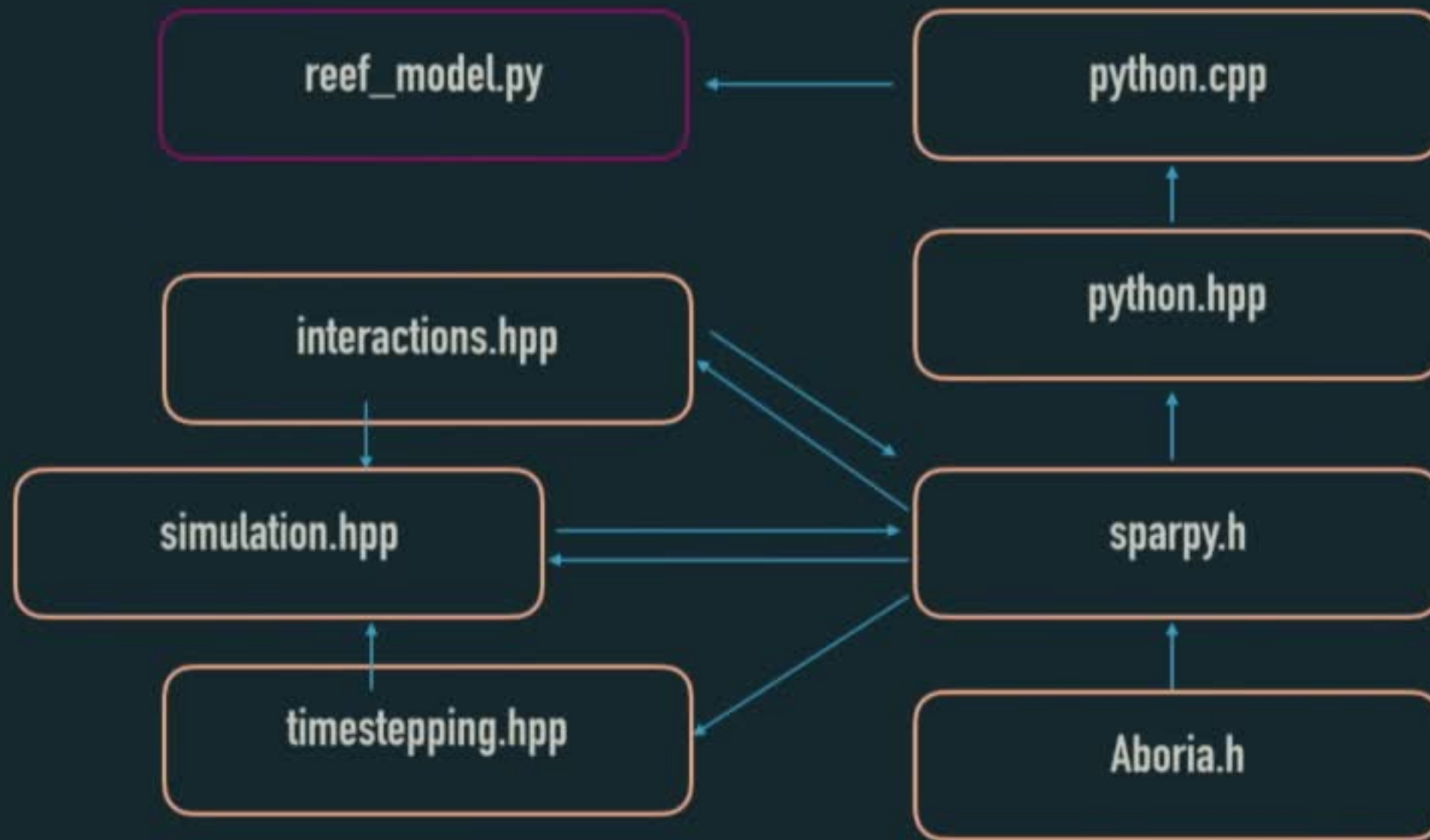
For particle interactions and weights in grid update

# Density Knowledge





# C++ Backend

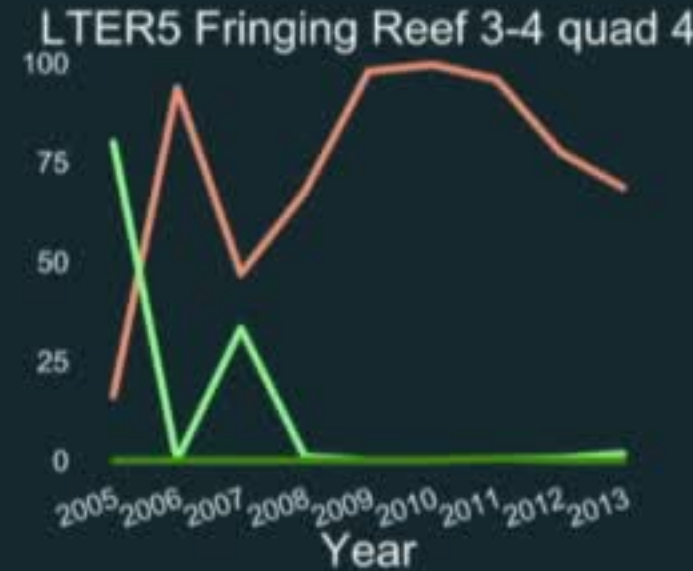
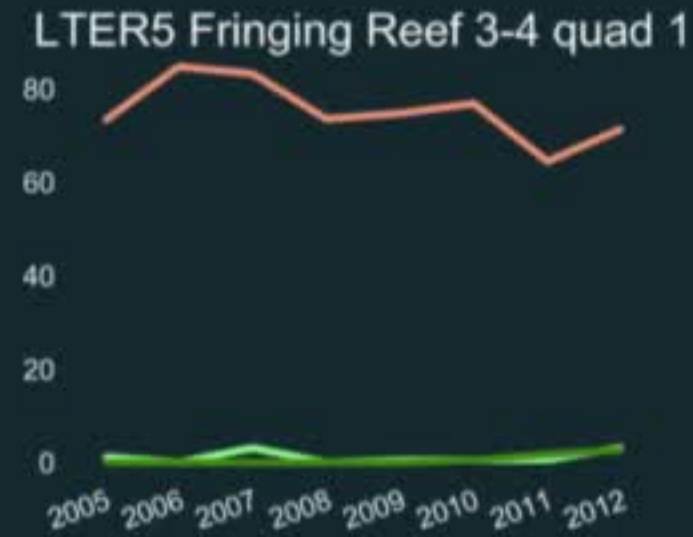
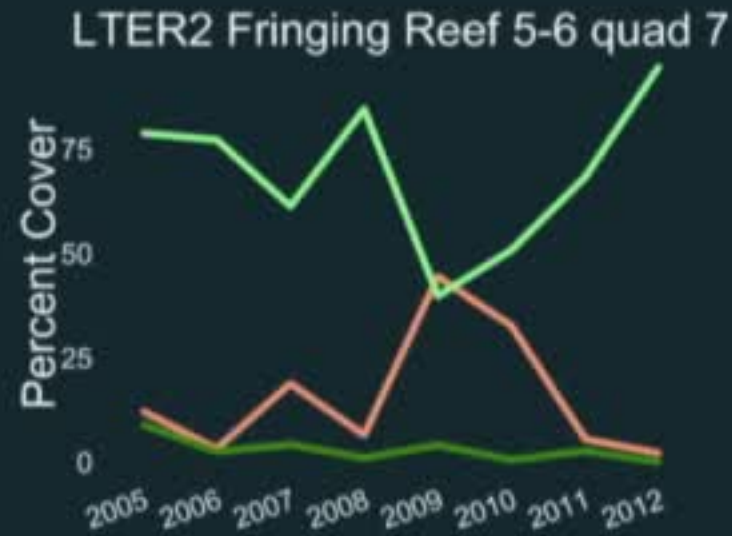


# Variety in Reef “Types”

Turf Macroalgae Coral

## Algae Favoring

## Coral Favoring

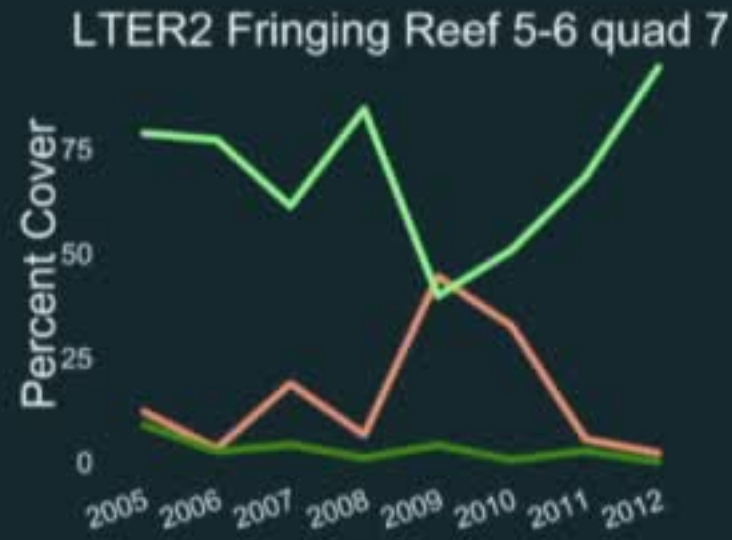




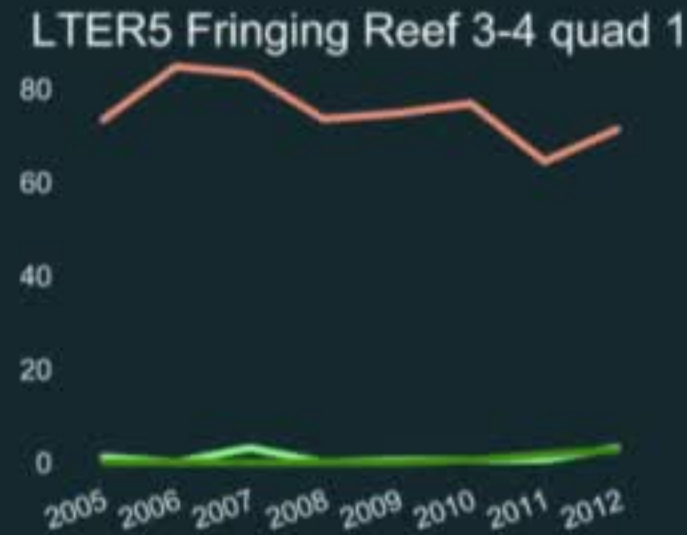
# Variety in Reef "Types"

— Turf    — Macroalgae    — Coral

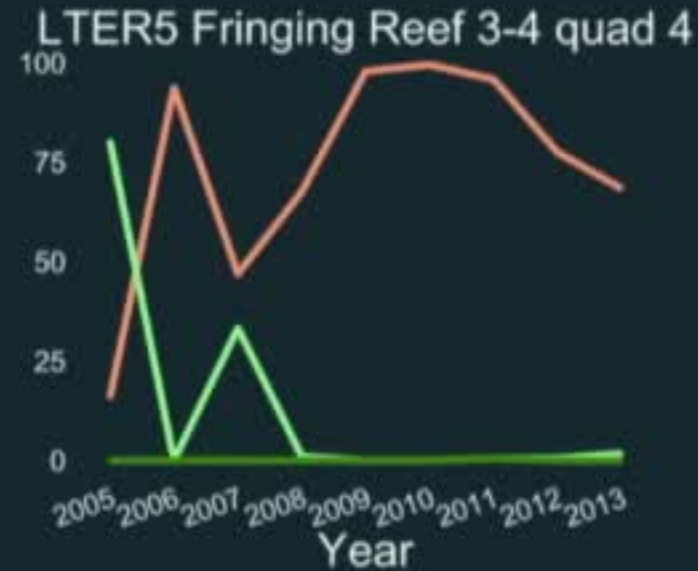
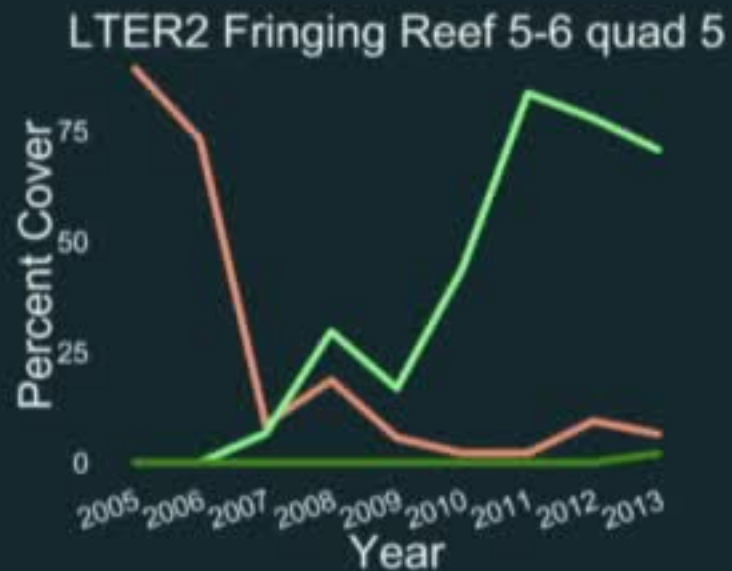
## Algae Favoring



## Coral Favoring



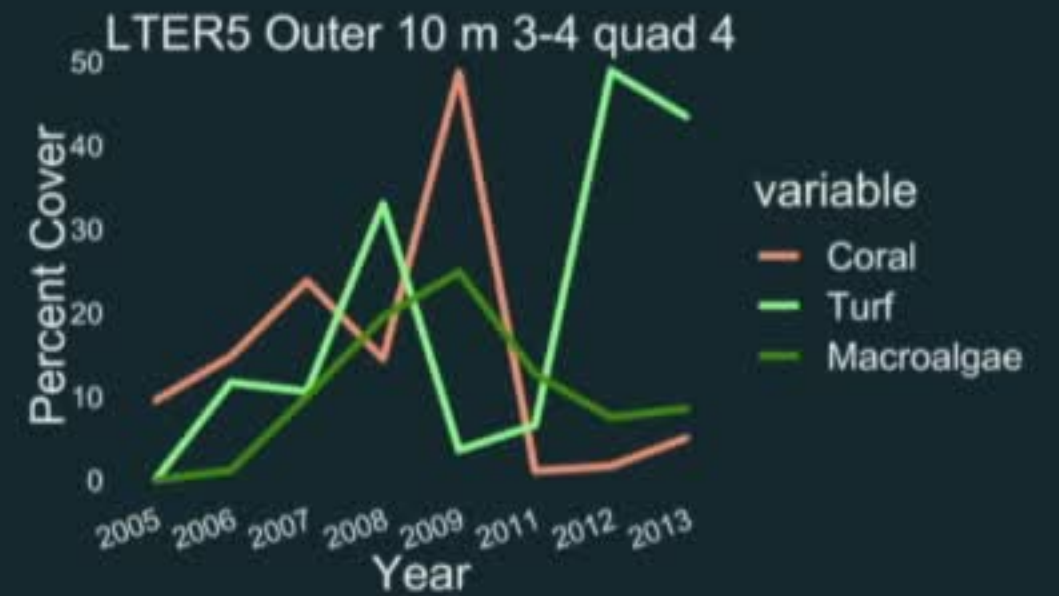
## Variable Outcomes



# Further Examples of Switching Behavior in Patches of the Same Reef

## Variable Outcomes

Turf Macroalgae Coral



# Original Outputs



# Mimicking Switching Behavior

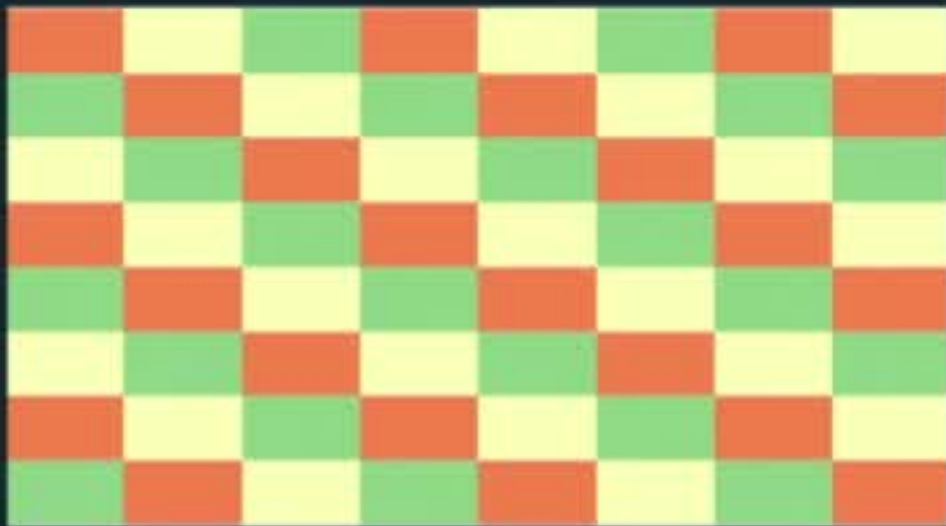
Through weighing stochasticity more heavily by neighboring benthic coverage, including the **same species**, and **increasing number of nodes**



# Mimicking Switching Behavior

Through weighing stochasticity more heavily by neighboring benthic coverage, including the same species, and increasing number of nodes

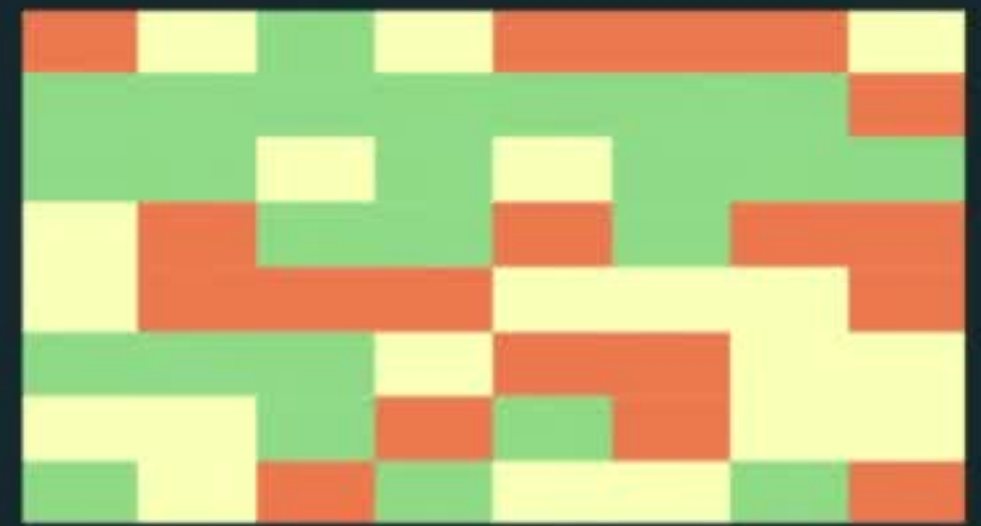
Step 0



Step 50

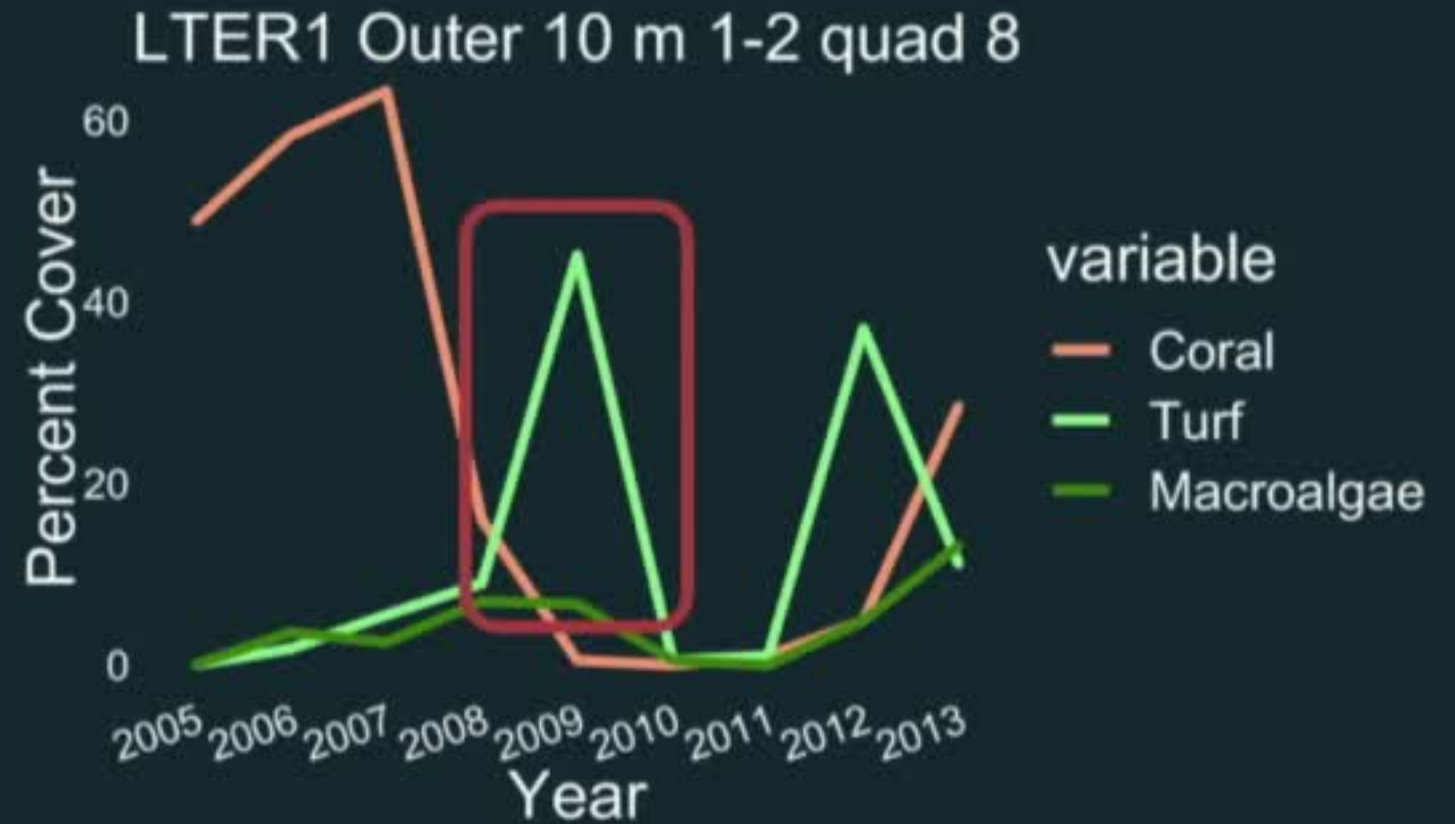
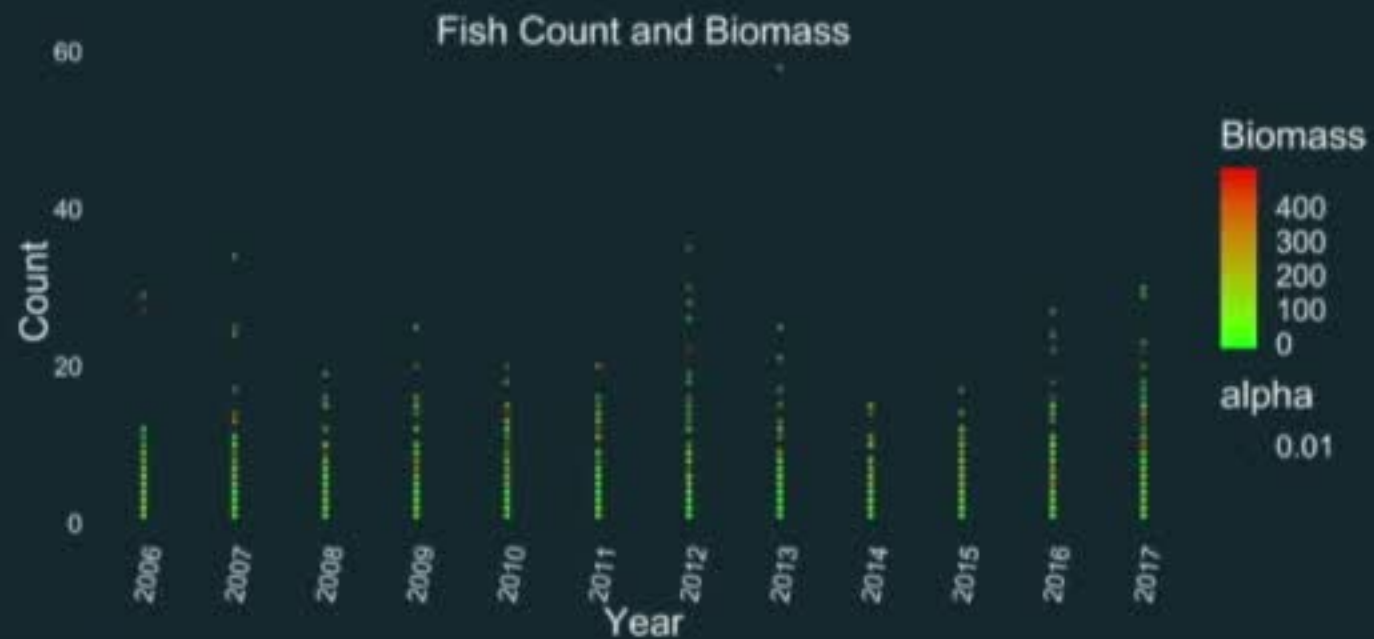
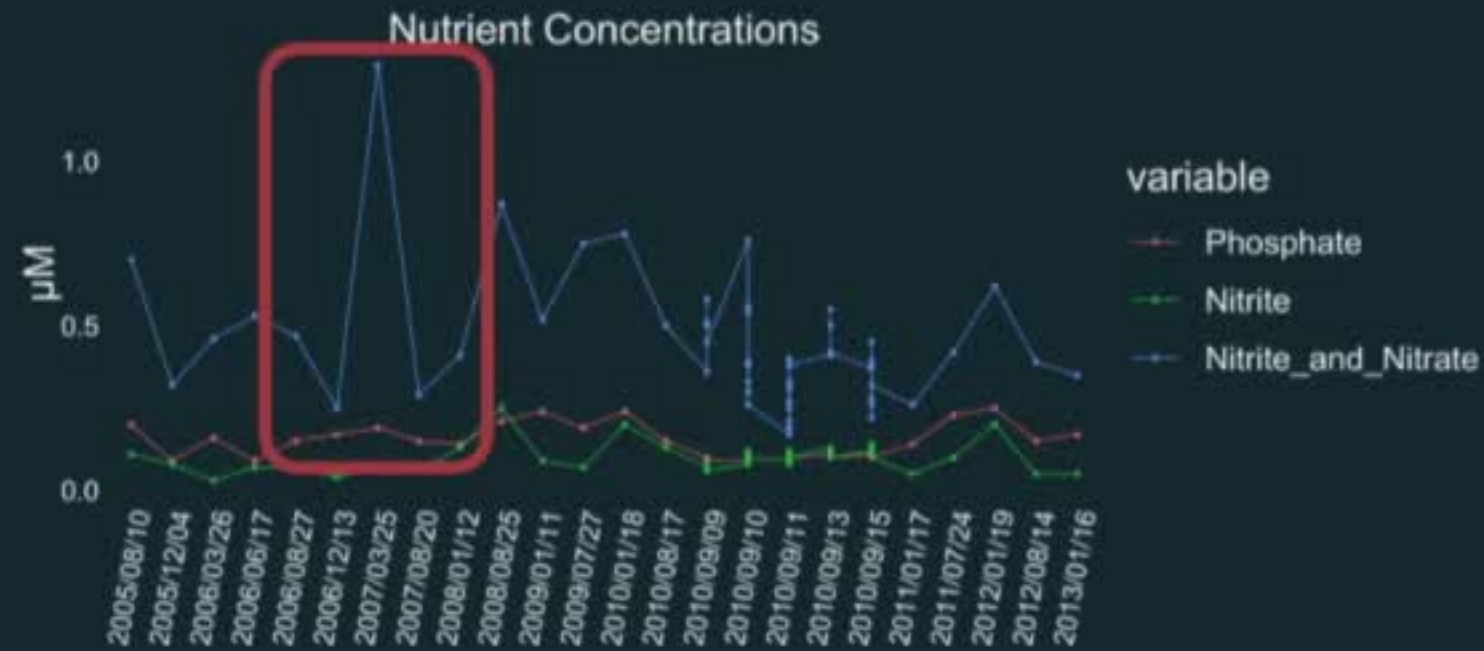


Step 100



Turf Macroalgae Coral

# Transfer Entropy (Dreams)



Brooks, A of Moorea Coral Reef LTER. 2016. MCR LTER: Coral Reef Community Dynamics: Abundance and Species Richness of Fishes Associated with the Coral *Porites rus* 2000 thru 2011. [knb-lter-mcr.1.22 doi:10.6073/pasta/286f844cc890fbbd19f5adcb689502d](https://doi.org/10.6073/pasta/286f844cc890fbbd19f5adcb689502d)

Allredge, A of Moorea Coral Reef LTER. 2019. MCR LTER: Coral Reef Water Column Nutrients, ongoing since 2005. [knb-lter-mcr.1034.9 doi:10.6073/pasta/9328a024f2bf16eccc66024f07dbcc574](https://doi.org/10.6073/pasta/9328a024f2bf16eccc66024f07dbcc574)



# Information Flow Examples in Hydrology

Ruddell, B. L., & Kumar, P. (2009). Ecohydrologic process networks: 1. Identification. *Water Resources Research*, 45(3), doi:10.1029/2008WR007279

Ma, H., Larsen, L. G., & Wagner, R. W. (2018). Ecogeomorphic Feedbacks that Grow Deltas. *Journal of Geophysical Research: Earth Surface*. <https://doi.org/10.1029/2018jf004706>

Rinderer, M., Ali, G., & Larsen, L. G. (2018). Assessing structural, functional and effective hydrologic connectivity with brain neuroscience methods: State-of-the-art and research directions. *Earth-Science Reviews*, 178, 29–47. <https://doi.org/10.1016/j.earscirev.2018.01.009>

# Preliminary Results and Takeaways

Stochasticity coupled with (self-inflicting) neighborhood knowledge can keep the system from converging to either coral or turf dominating states

Different models for different parts of the reef

Transfer Entropy still a possibility, need to look at lags and various binning

Stochastic Spatiotemporal model has a lot of exploration potential