Aggregate Behaviors of Heterogeneous, Delay-Coupled Swarms

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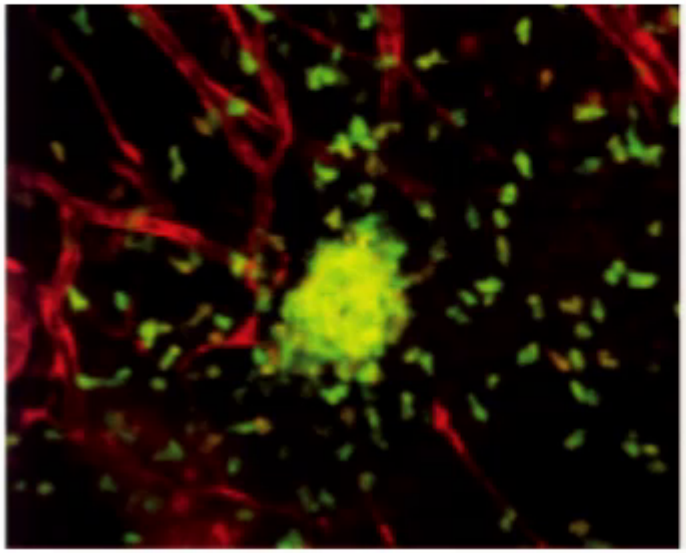
Swarms in Natural and Engineered Systems



Outline

I. Delay-coupled swarms with heterogeneous dynamics

- Dynamical model of the swarming system
- Mean-Field dynamics
- Motion patterns
- Bifurcation diagram

II. Additional extensions

III. Experimental Verification of Results

Model of a globally-coupled swarm with heterogeneous dynamics

- Say $\mathbf{r}_i \in \mathbb{R}^2$ denotes the position of the i^{th} agent in the swarm[1].
- Agents are distinguished by an acceleration factor, κ_i.

$$\ddot{\mathbf{r}}_i = \kappa_i(1 - |\dot{\mathbf{r}}_i|^2)\dot{\mathbf{r}}_i - \frac{a\kappa_i}{N}\sum_{\substack{j=1 \\ j\neq i}}^{N}(\mathbf{r}_i(t) - \mathbf{r}_j(t-\tau)) + \boldsymbol{\eta}_i(t)$$

N - number of particles

a - particle interaction coupling parameter

τ - time delay

D - intensity of noise

$\langle \eta_i^{(\ell)}(t) \rangle = 0$ and $\langle \eta_i^{(\ell)}(t)\eta_j^{(k)}(t') \rangle = 2D\delta(t-t')\delta_{ij}\delta_{\ell k}$ for all $i,j \in \{1,\ldots,N\}$ and $\ell,k \in \{1,2\}$

[1] These results can be easily extended to $\mathbb{R}^3$.

Motion patterns of the swarm

Translating state

agents move in a group with constant velocity

Ring state

agents form counter-rotating rings about a fixed center of mass

Rotating state

agents move as a group about a fixed center of rotation

$N = 100$ number of agents

$a = 1$ coupling parameter

$\kappa_i = 1$ $(\forall i)$ acceleration factor

$D = 0.3$ noise intensity

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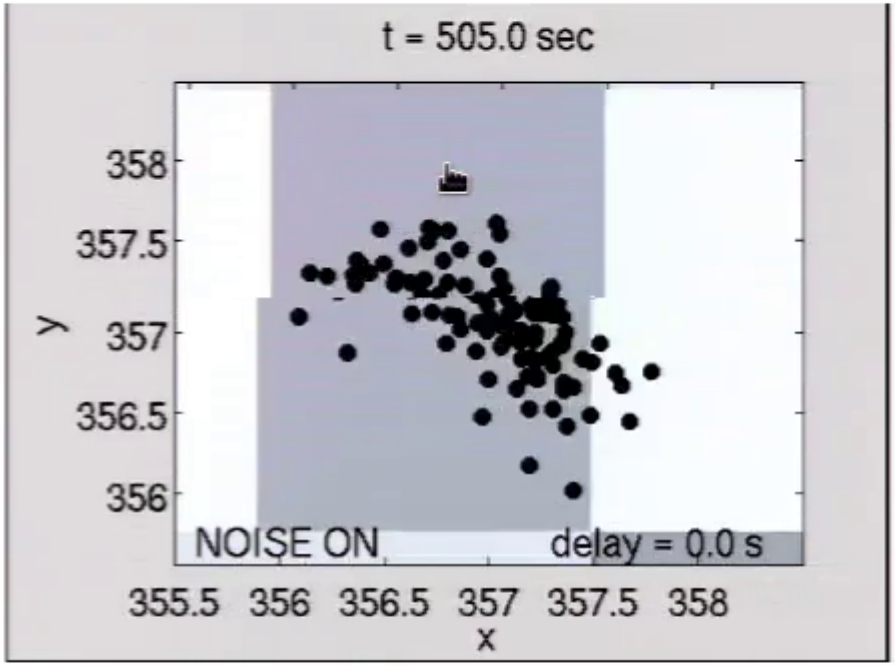

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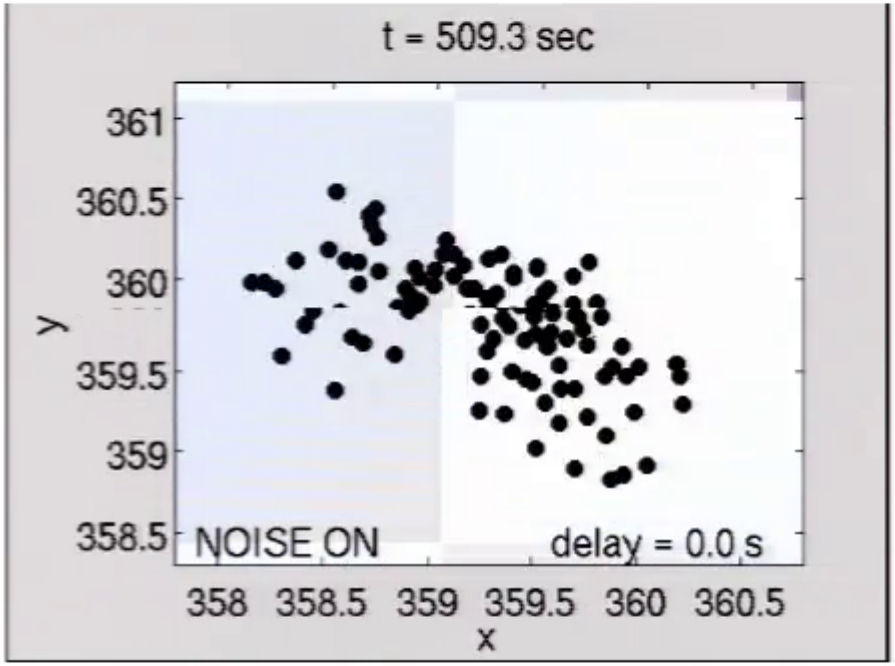

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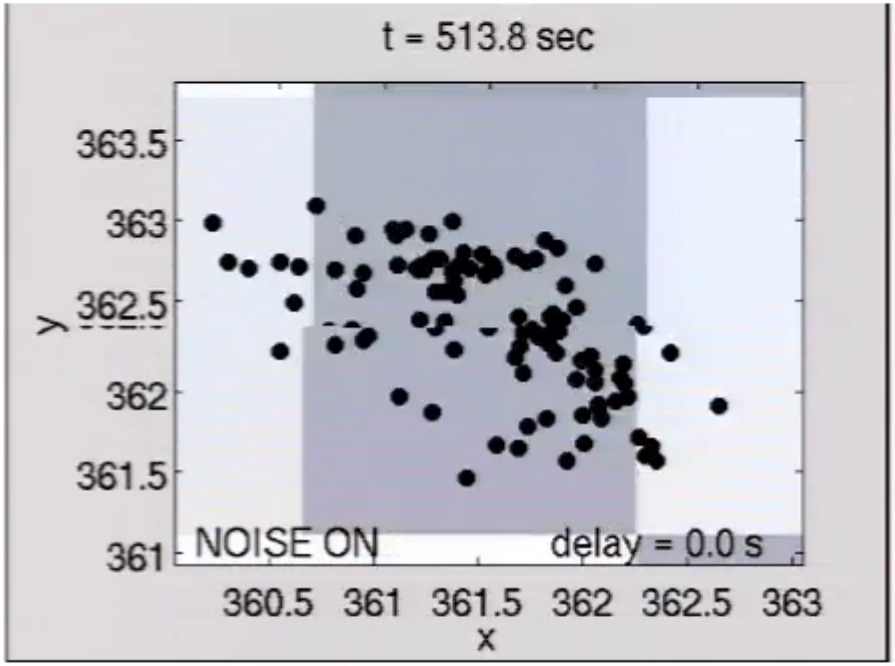

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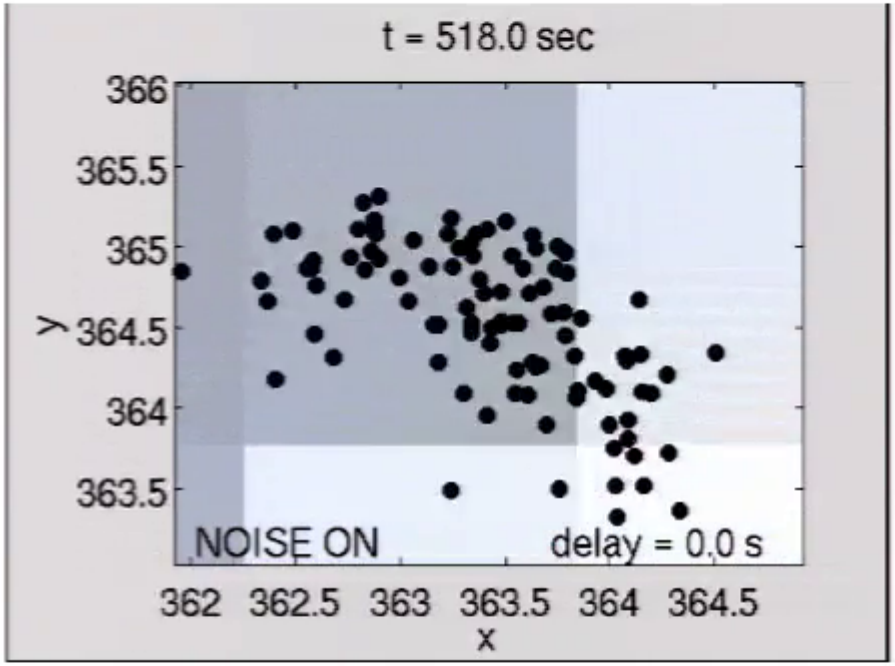

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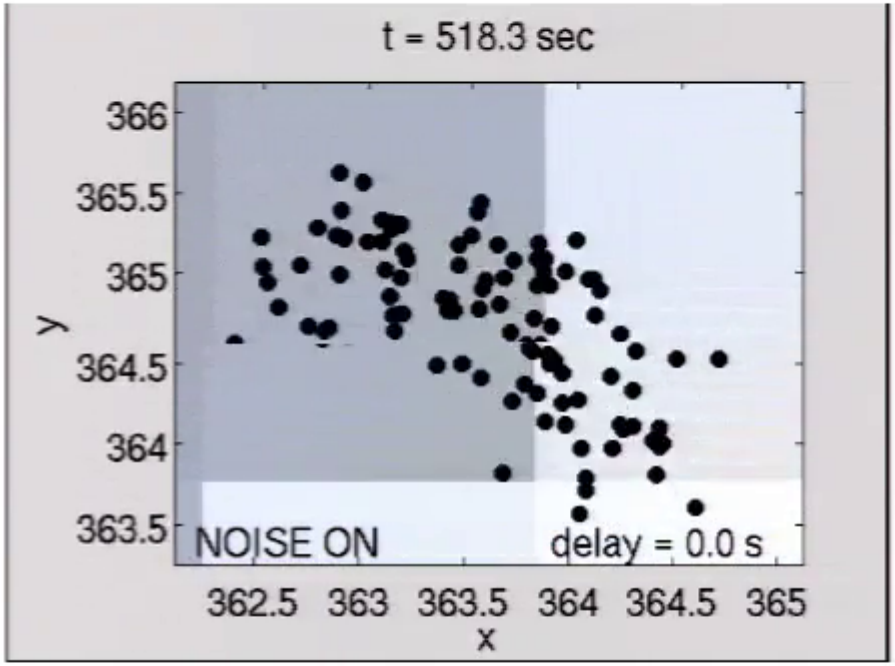

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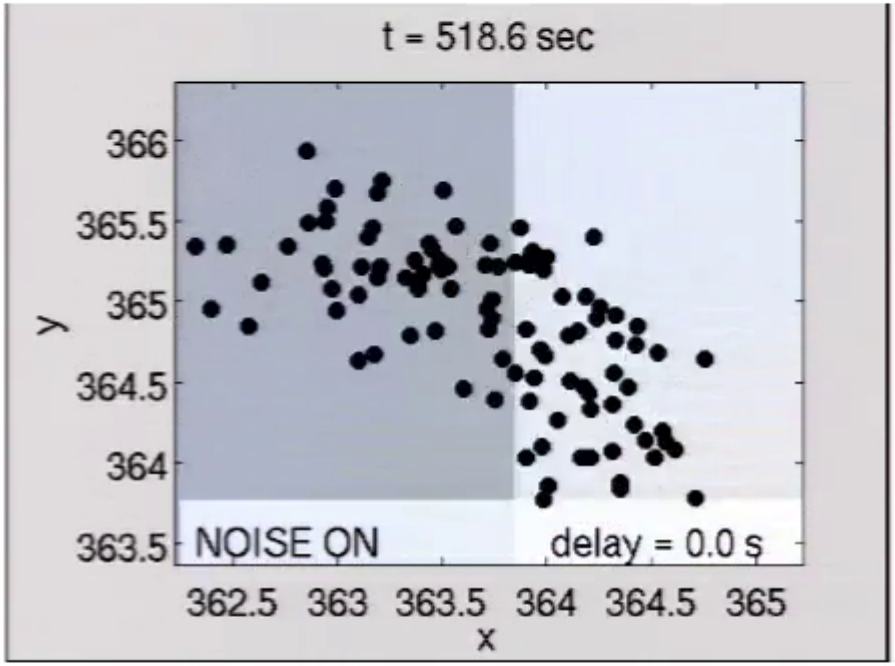

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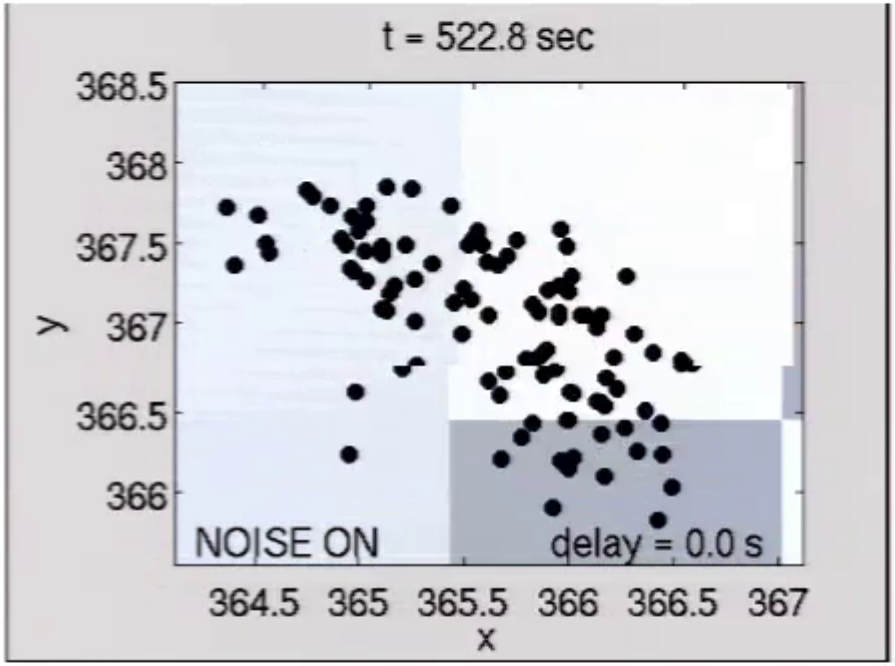

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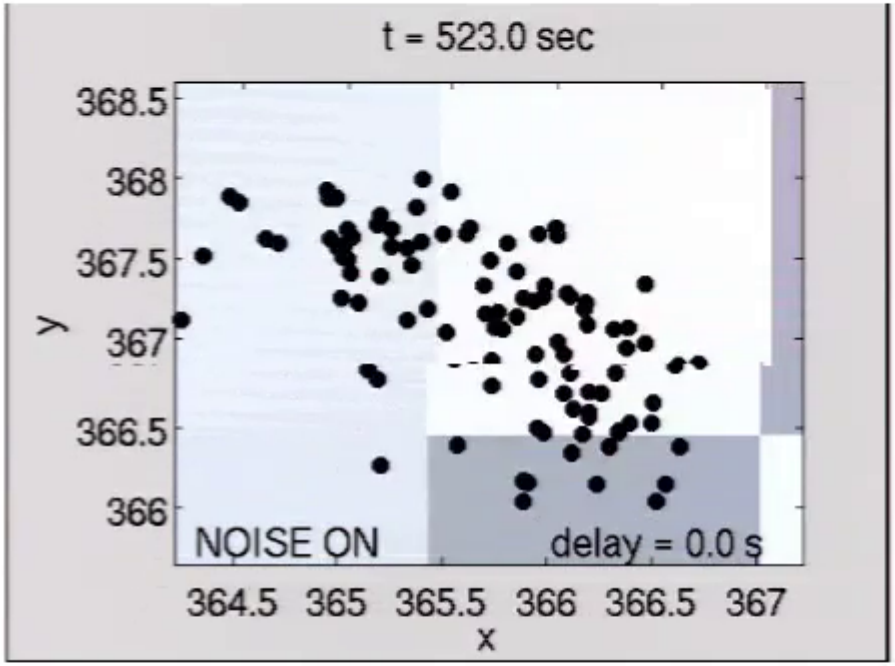

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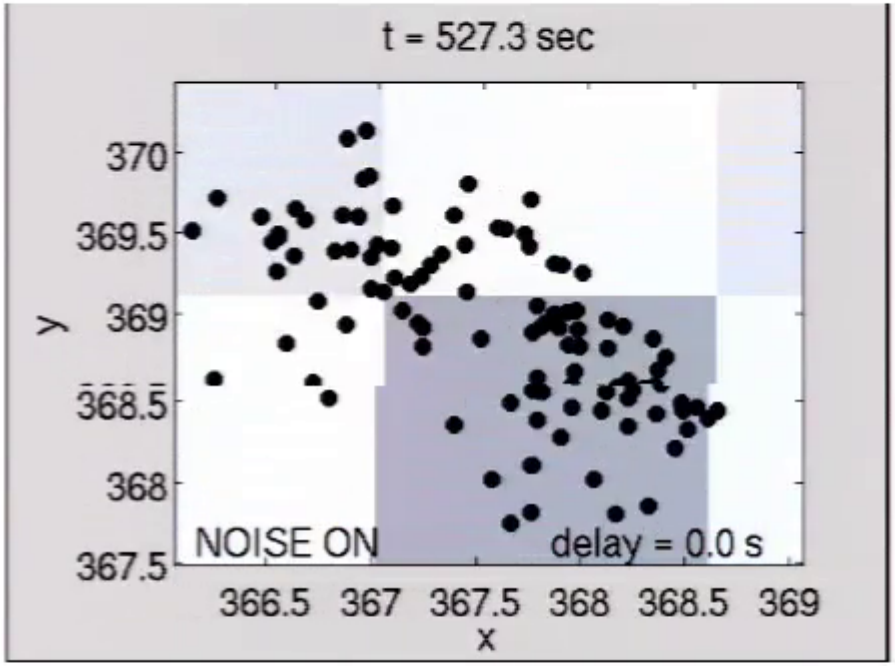

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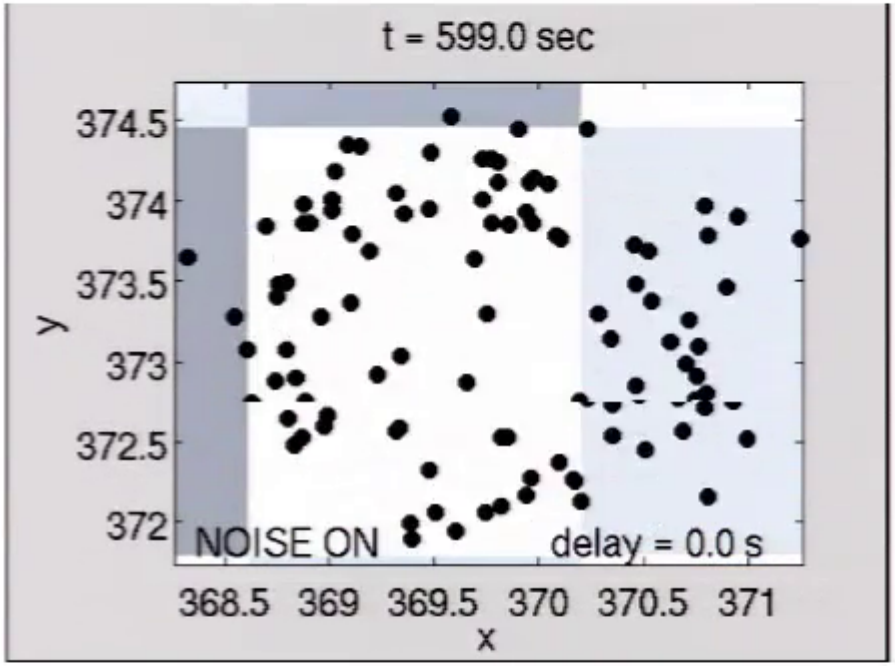

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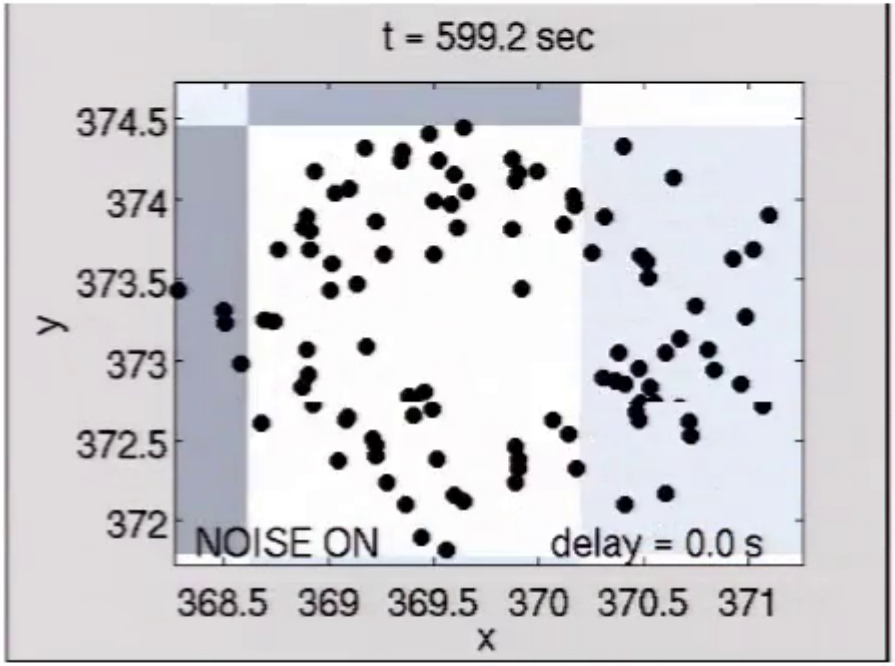

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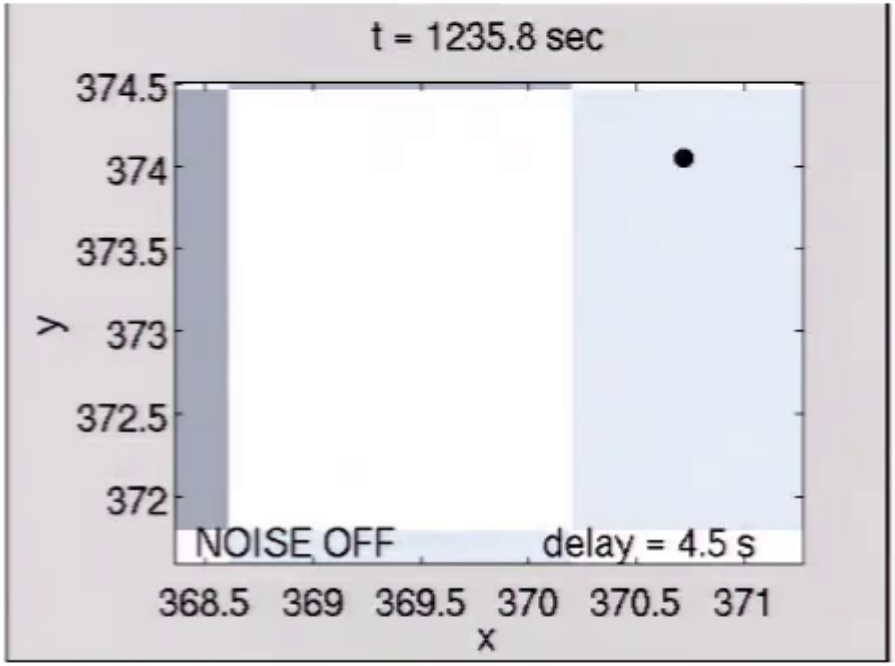

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