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# Using Fast Forward Solvers to enable Uncertainty Quantification in Seismic Imaging

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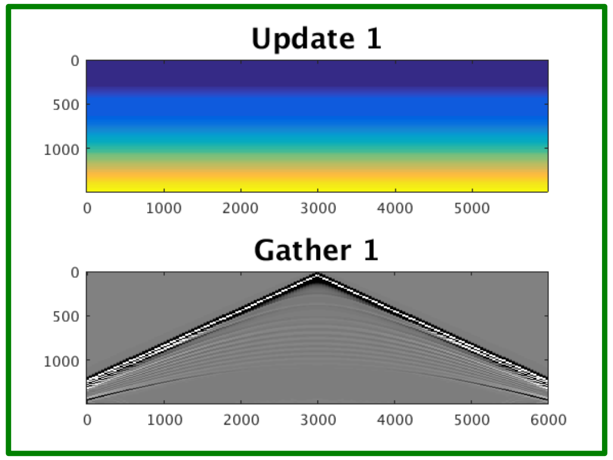
# Outline

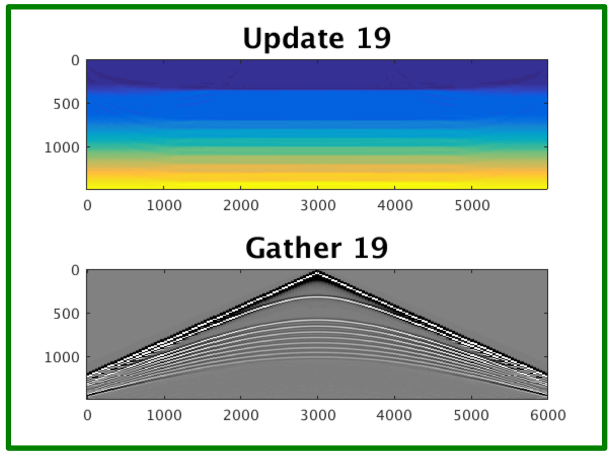
- Bayesian Seismic Inversion
- Metropolis Hastings algorithm
- Field Expansion Method
  - ☑ Theory
  - ☑ Example
- Local Acoustic Solver
  - ☑ Theory
  - ☑ Example
- Conclusions

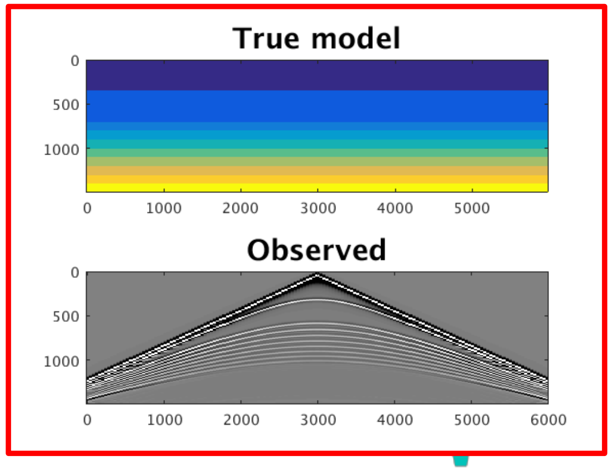

https://evaluationrevisited.wordpress.com/cartoons/cartoons-day-1/

# Full Waveform Inversion:

$$J(m) = \frac{1}{2} || \boxed{G(m)} - \boxed{d} ||_2^2$$



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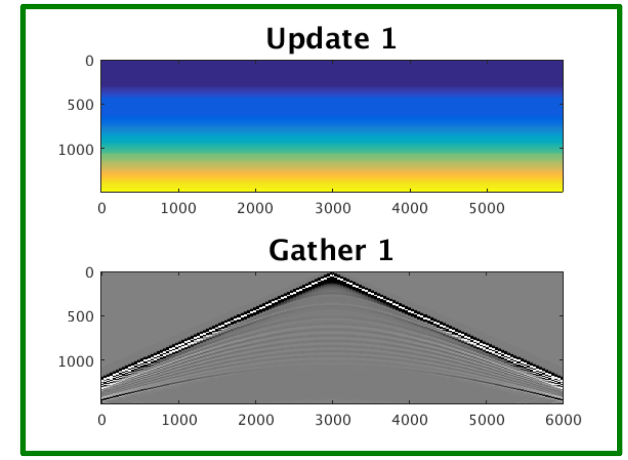


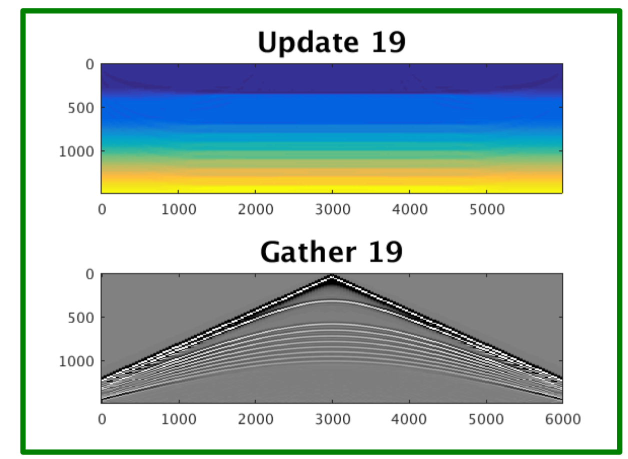


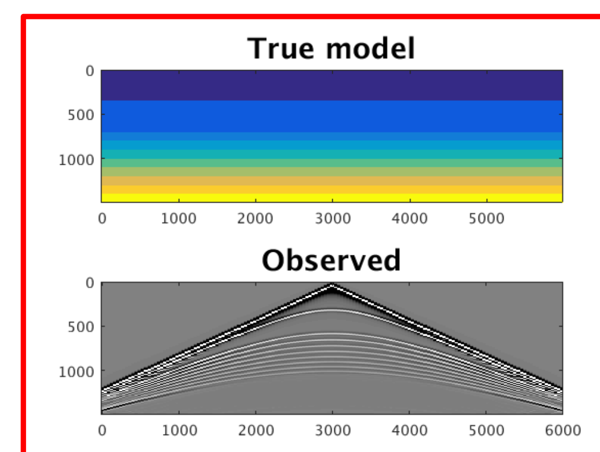


<u>Challenges:</u>

- Sensitive to initial model (non-convex, non-linear problem)
- Expensive forward solves
  (finite difference, finite element)
- Single image & no uncertainty quantification
  “How wrong are we?”

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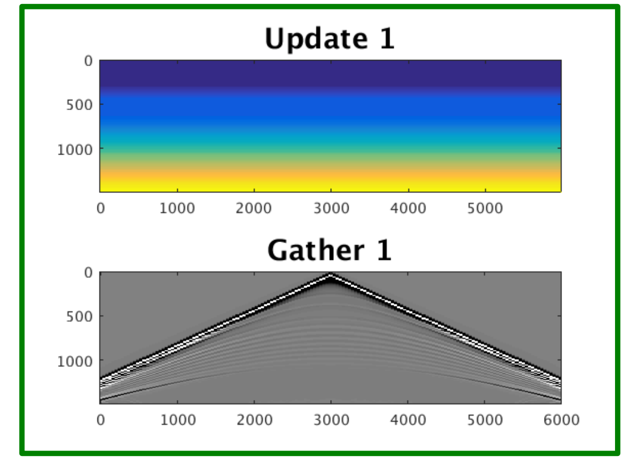


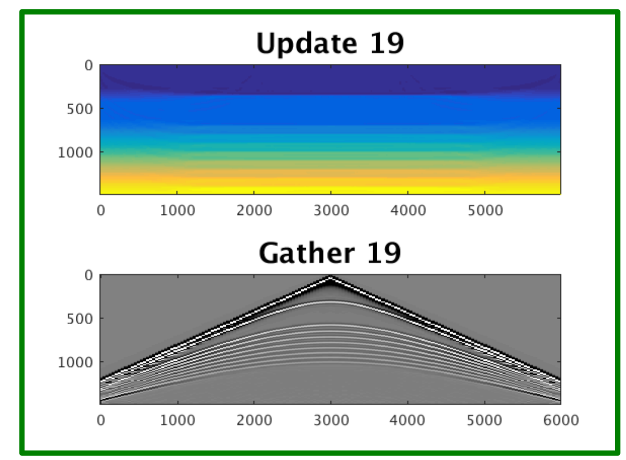


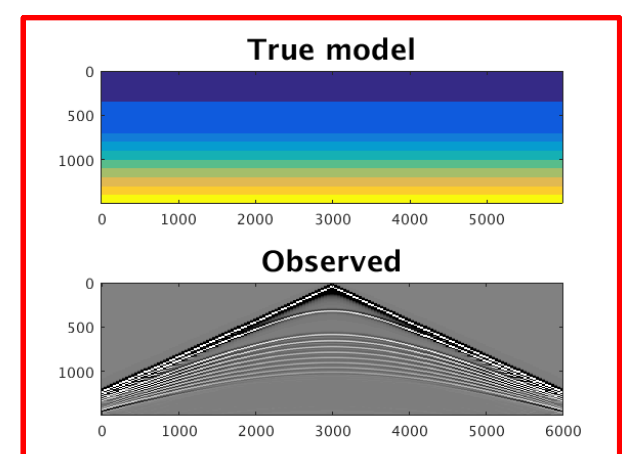


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- Solution: Markov-Chain Monte Carlo & fast forward solver

# Bayesian Seismic Inversion:

**Synthetic seismic data:**

$$\mathbf{d} = F(\mathbf{m}) + n$$

$\mathbf{d}$ : Simulated Wavefield

$\mathbf{m}$ : Model

$F$ : Forward Solver

$n$ : Gaussian noise

**-Tarantola A., 2005**: Inverse problem theory and methods for model parameter estimation: *SIAM*

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**Likelihood function:**

$$L(\mathbf{m}) \equiv p(\mathbf{d}|\mathbf{m}) \propto exp\left[-\frac{1}{2}(f(\mathbf{m}) - \mathbf{d})^T \Sigma^{-1} (f(\mathbf{m}) - \mathbf{d})\right]$$

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**Posterior calculation:**

$$p(\mathbf{m}|\mathbf{d}) = \frac{p(\mathbf{d}|\mathbf{m})p(\mathbf{m})}{p(\mathbf{d})}$$

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# Metropolis Hastings Overview:

**Require:** $m_0, L(m_0)$ ← *Initial model & likelihood*

-**Haario H., E. Saksman and J. Tamminen, 2001**: An adaptive metropolis algorithm: *Bernouli*
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$u \in [0,1]$

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**If** $u < \alpha_i$

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$u \in [0,1]$

**If** $u < \alpha_i$

$m_i = m_*$ — *Accept*

**else**

$m_i = m_{i-1}$ — *Reject*

**End if**

**End for**

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- Generate samples directly from your posterior (m0,m1…)
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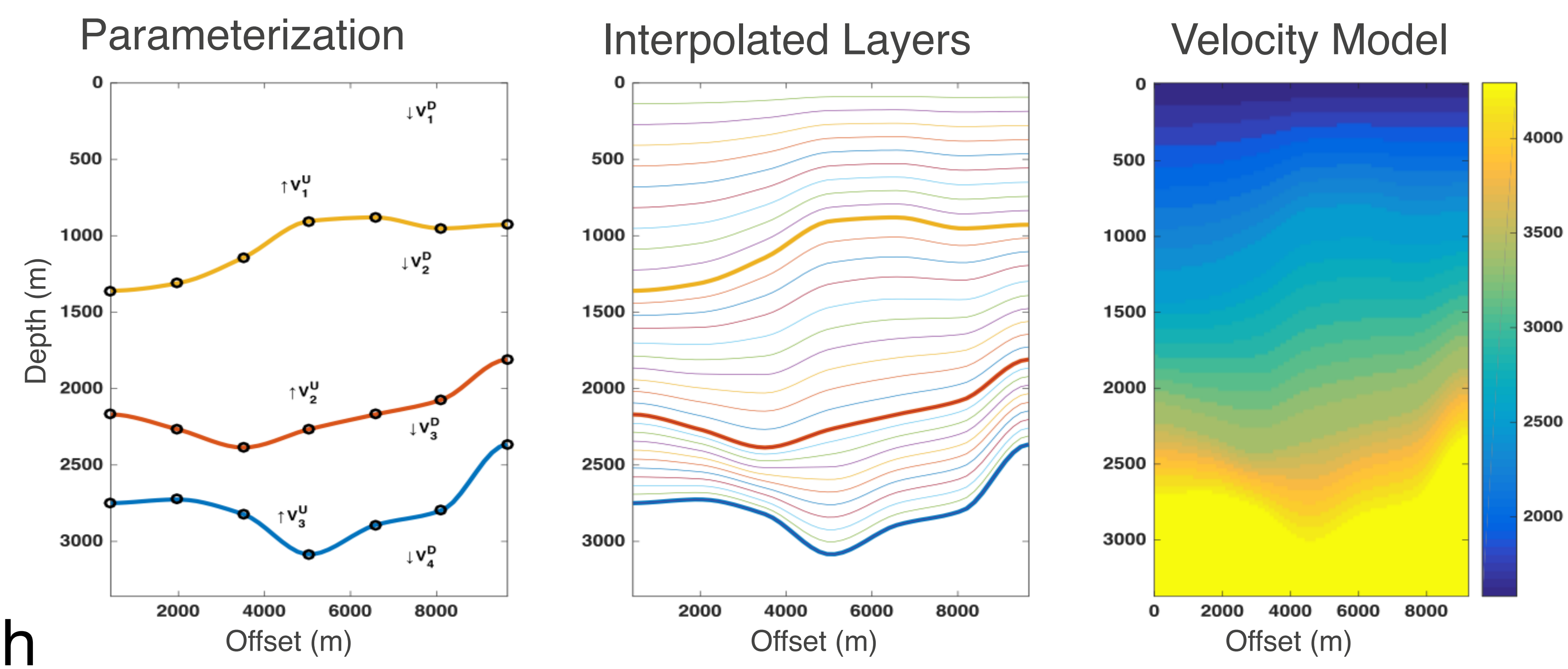
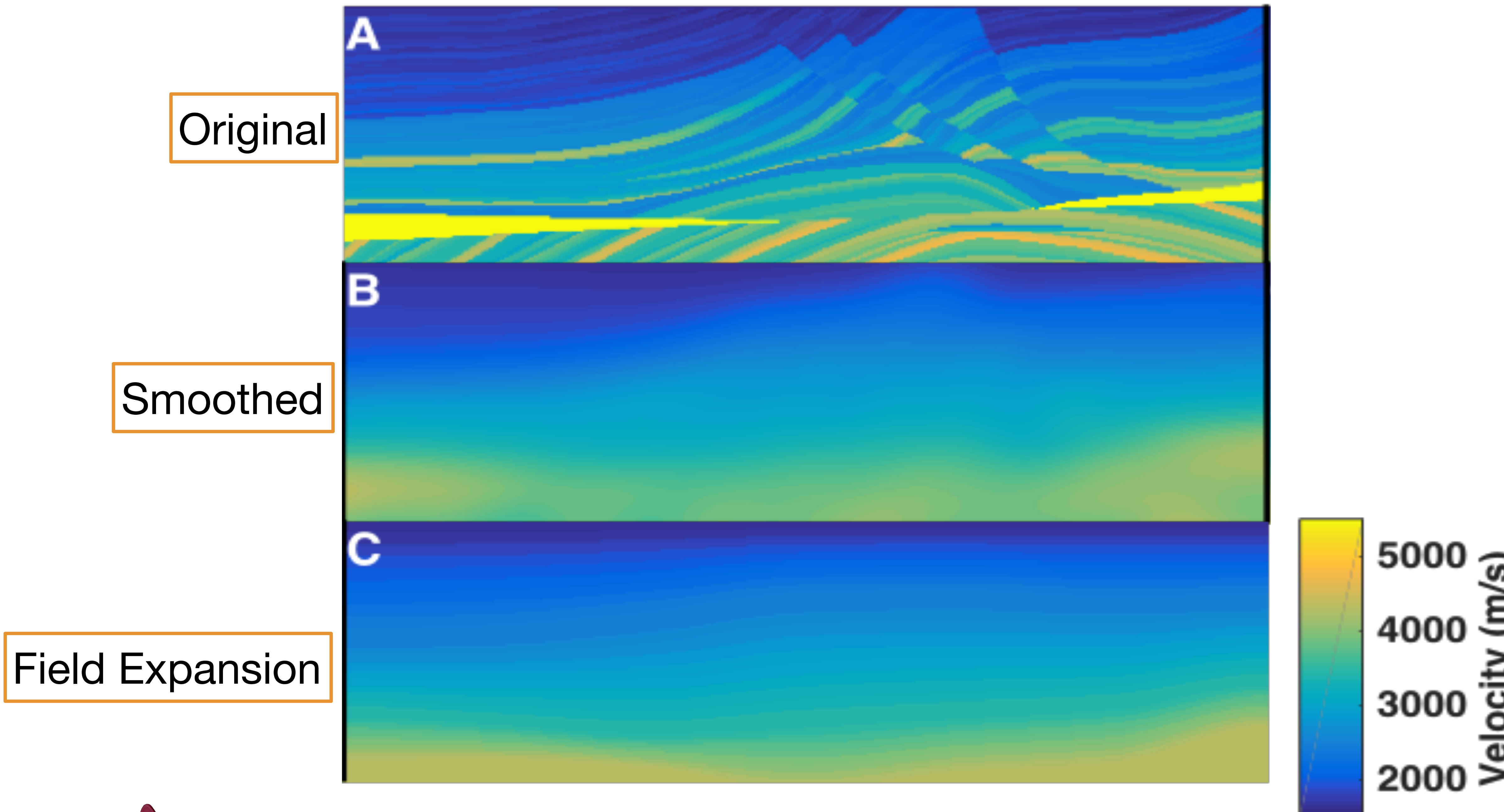
# Field Expansion Method in 1 minute:

- Parameterize model to master layers (M) with $N_q$ control points
- Linear gradient $V_i^{up}$ and $V_i^{down}$
- Layers with FEM
- Migrate reflector after with 0-offset time migration




-**Zelt C. and R. Smith, 1992**: Seismic travel time inversion for 2D crustal velocity structure: *GJI*
-**Malcolm A. and D. Nicholls, 2011**: A field expansion method for scattering by periodic multilayered media: *The Journal of Acoustic Society of America*
-**Ely G, A. Malcolm and D. Nicholls, 2015**: Combing global optimization and boundary integral methods to robustly estimate subsurface velocity models: *SEG Expanded Abstracts*
-**Datta D. and M. K, Sen, 2016**: Estimating a starting model for full-wave- form inversion using a global optimization method: *Geophysics*

# UQ with Field Expansion Method:



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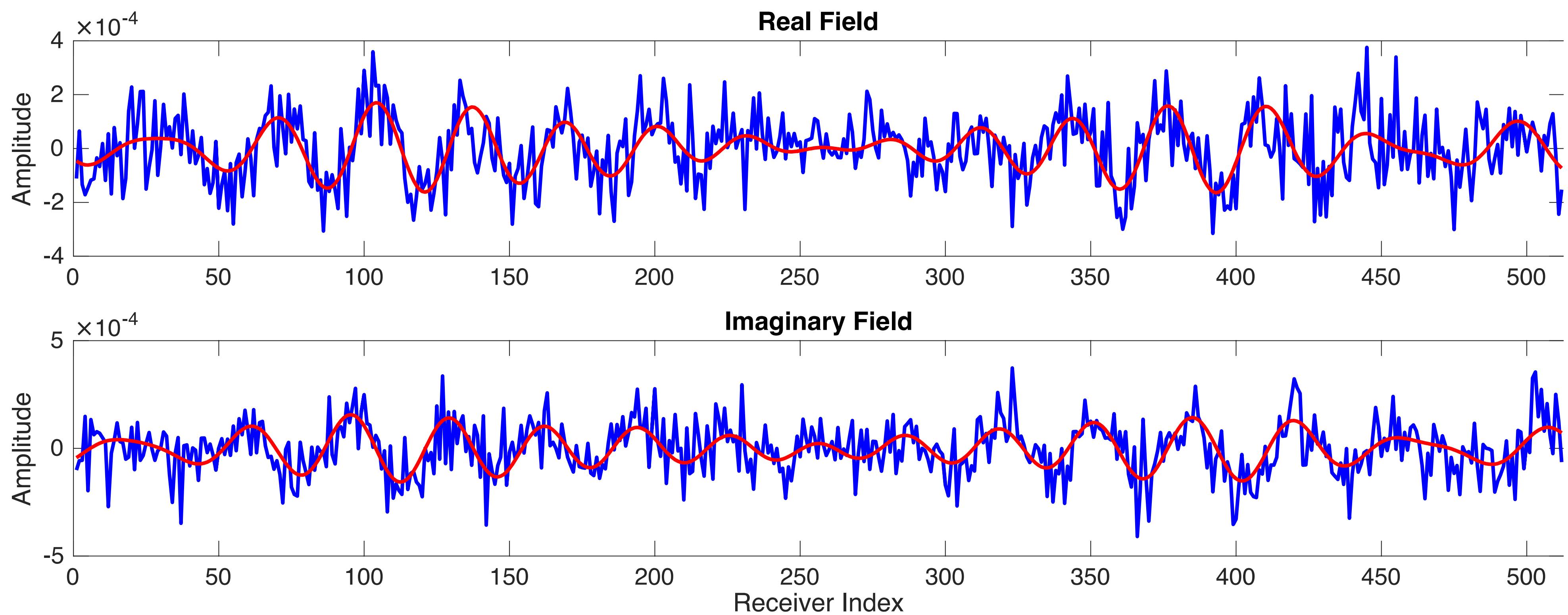


- 3Hz single shot, 256 receivers, SNR 0.75, 31 velocity model parameters
- MCMC Run: 500,000 iterations, discard first 250,000
- Focus on stability of images (reflectors of interest)

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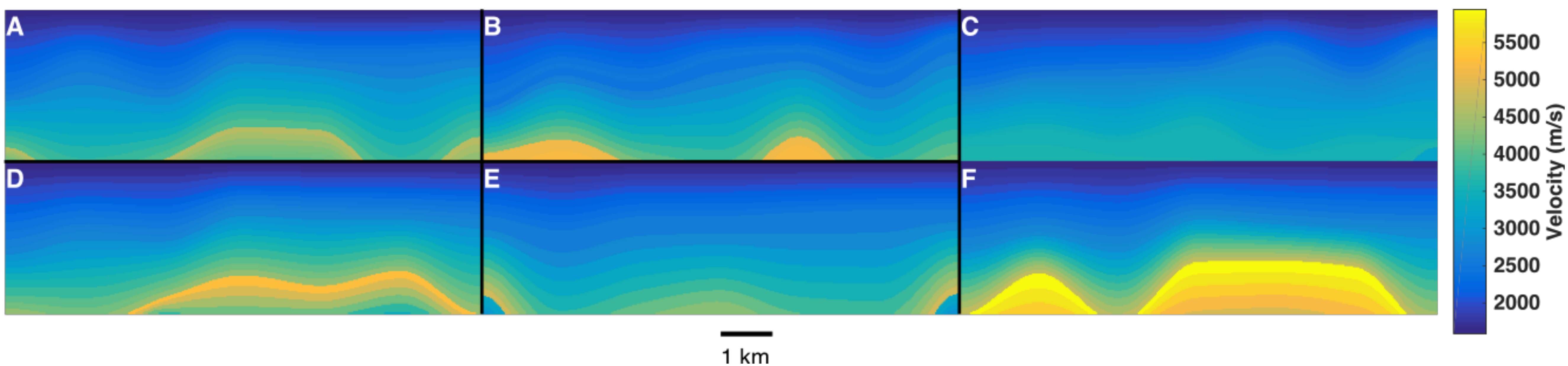
# UQ with Field Expansion Method:

## Posterior Distribution: Velocity Models




- 250,000 models of 500,000 discarded, 6 models randomly selected
- Shallow velocity structure better constrained than deeper
- Too complex to show error bars

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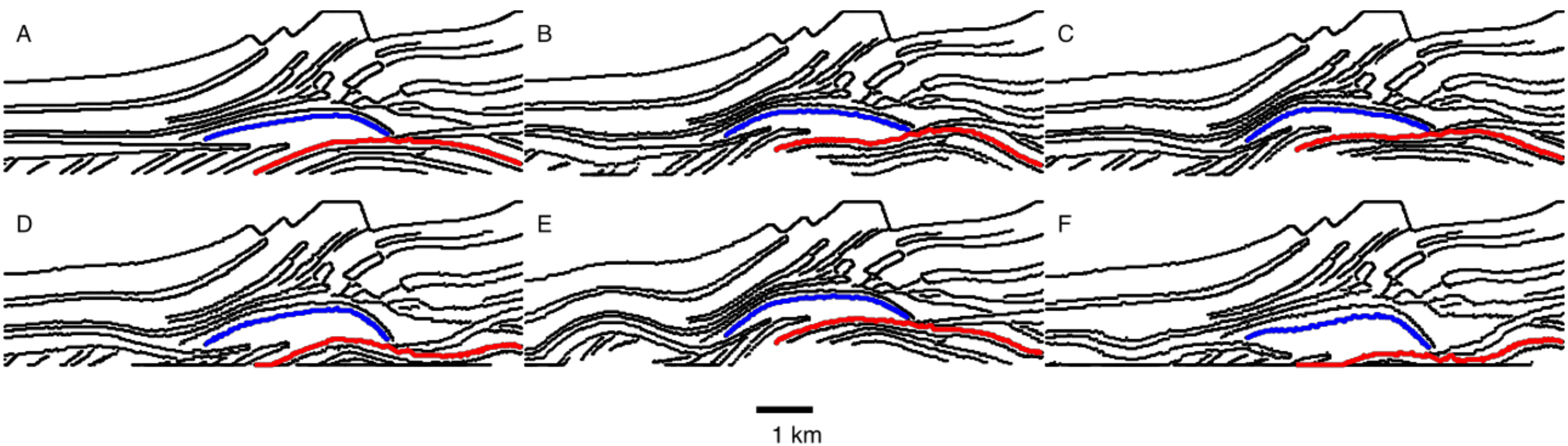
# UQ with Field Expansion Method:

## Posterior Distribution: Migrated images




- Generated reflectivity model -> travel times
- Zero offset migrate travel times with each velocity model
- Upper structure more stable than lower

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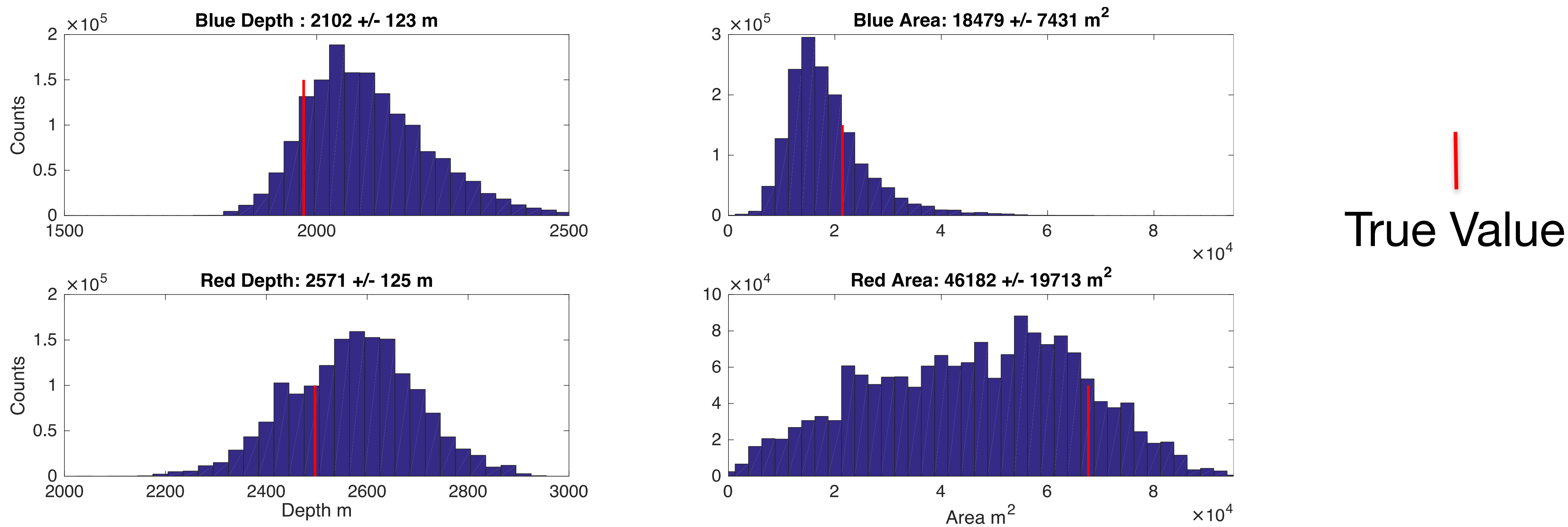
# UQ with Field Expansion Method:




- Ran MCMC 8 times -> 2 million posterior samples
- Deeper red anticline area, poorly constrained
  - Non-Gaussian distribution
  - Significant samples near zero area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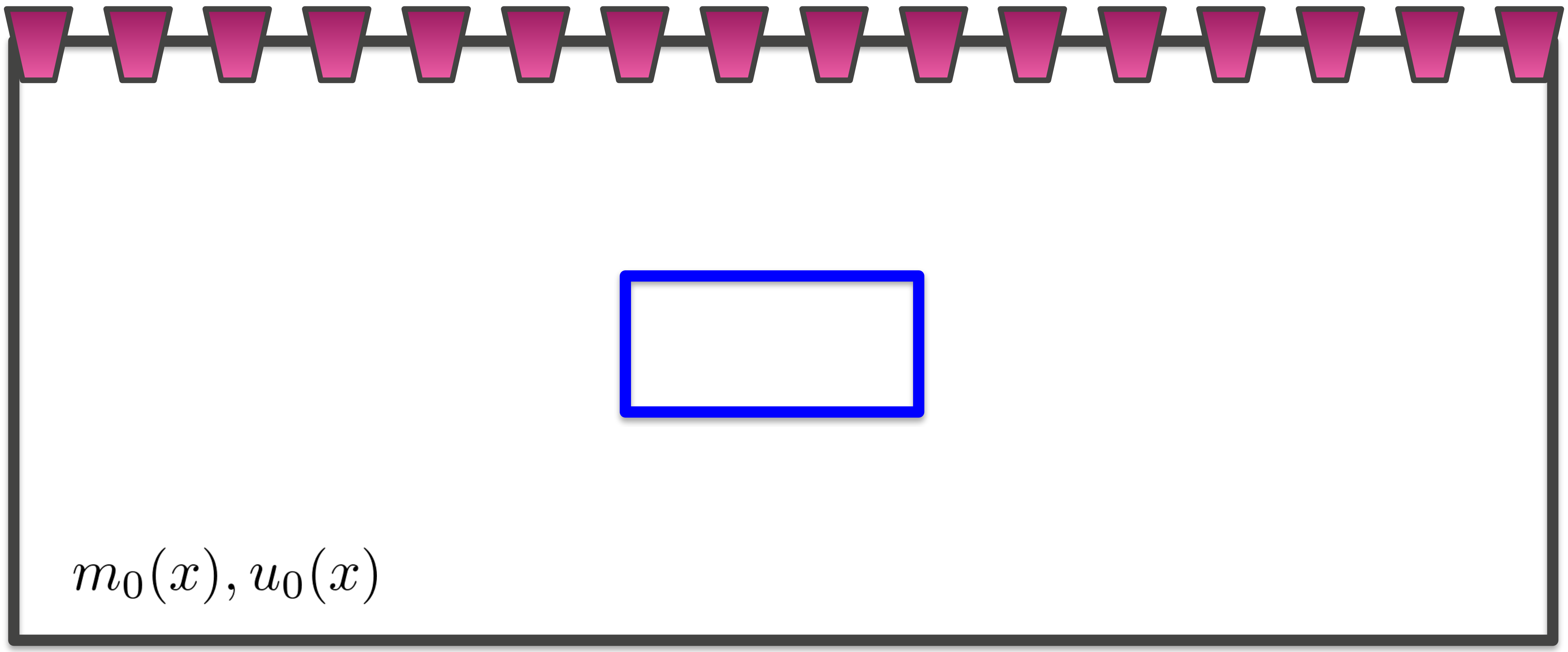
# Local Acoustic Solver in 1 minute:

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$$m(x) = m_0(x) + \delta m(x)$$

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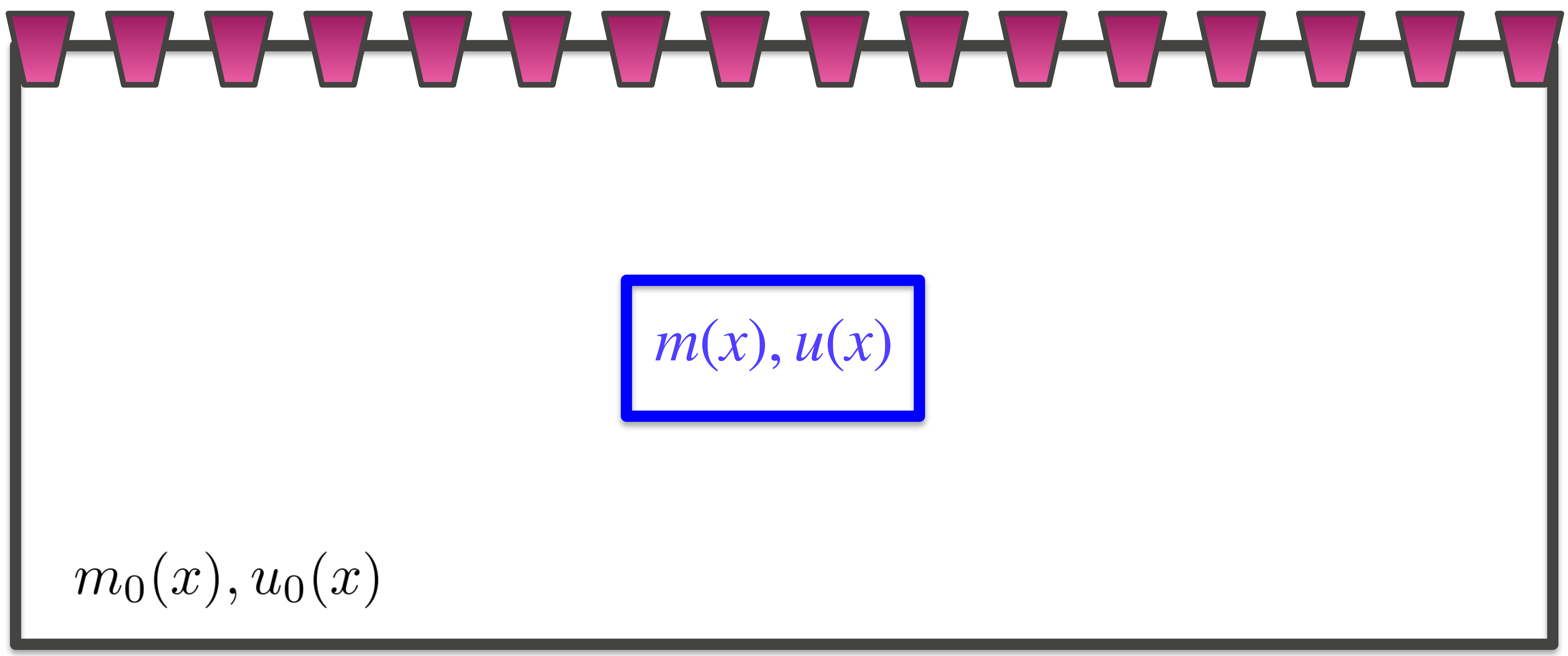


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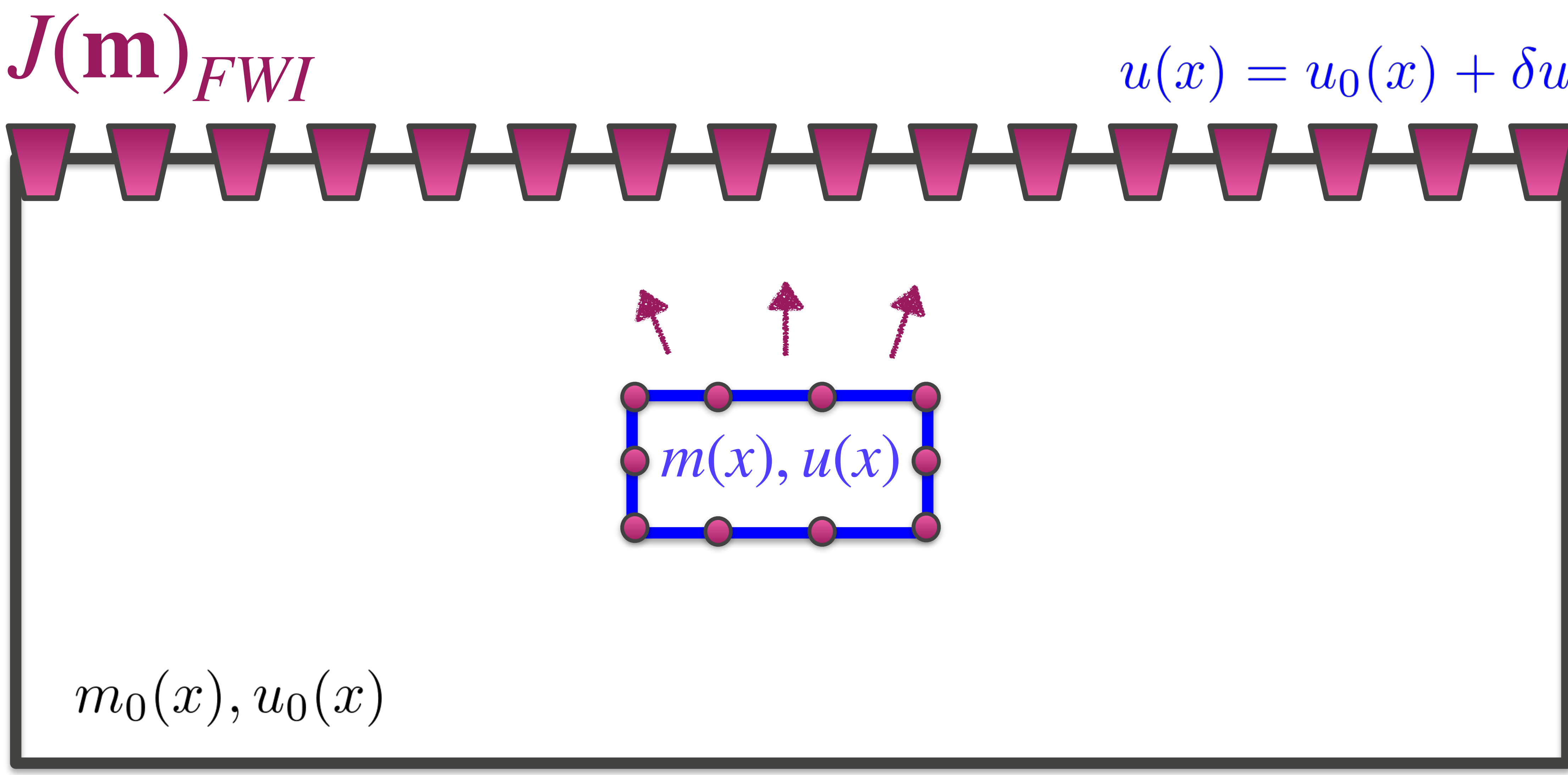



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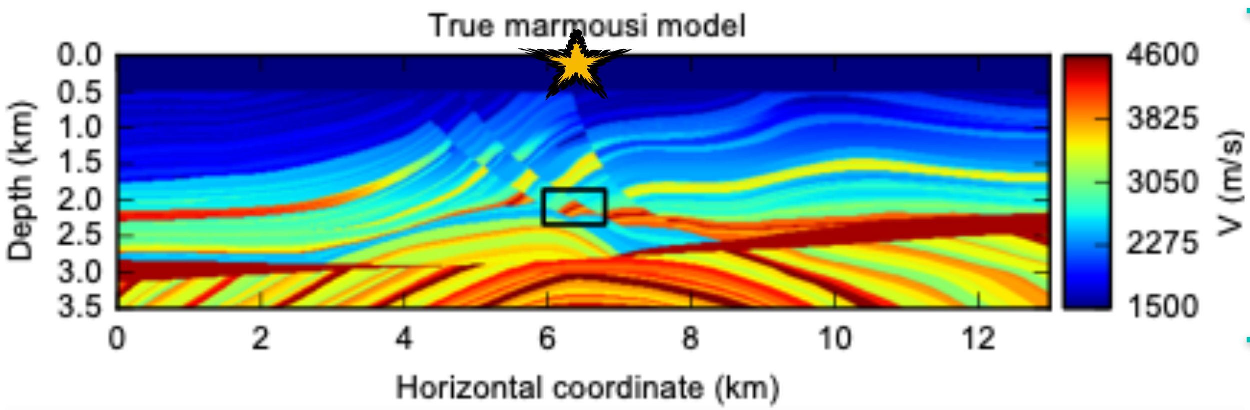
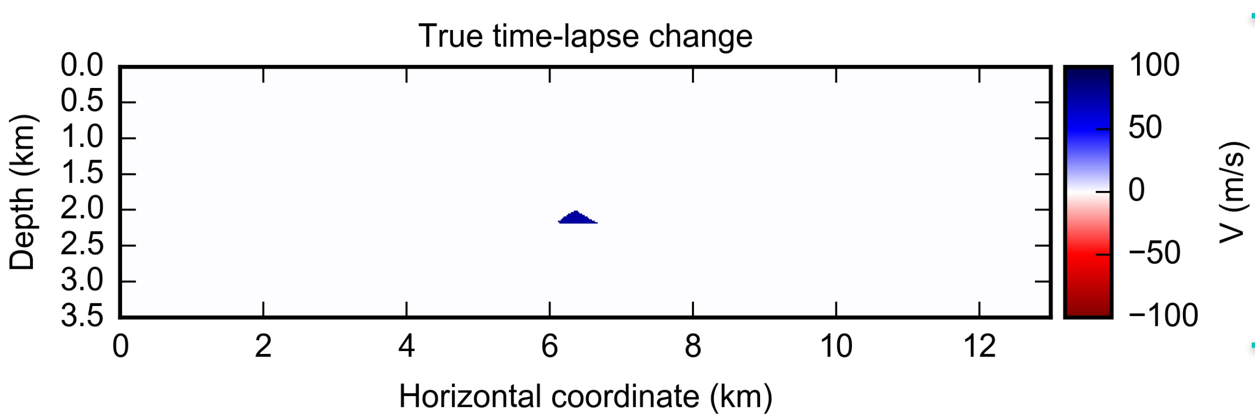



$m_0, u_0$




$\delta m$

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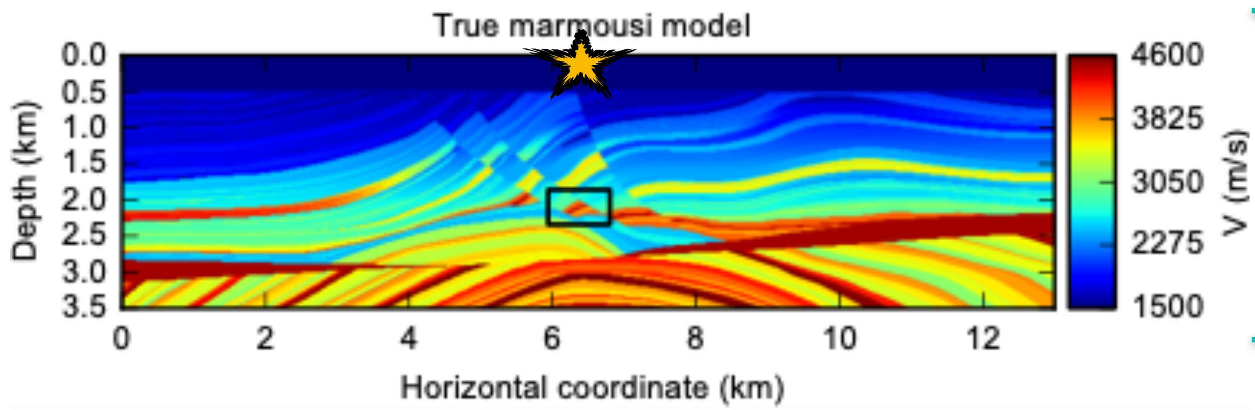


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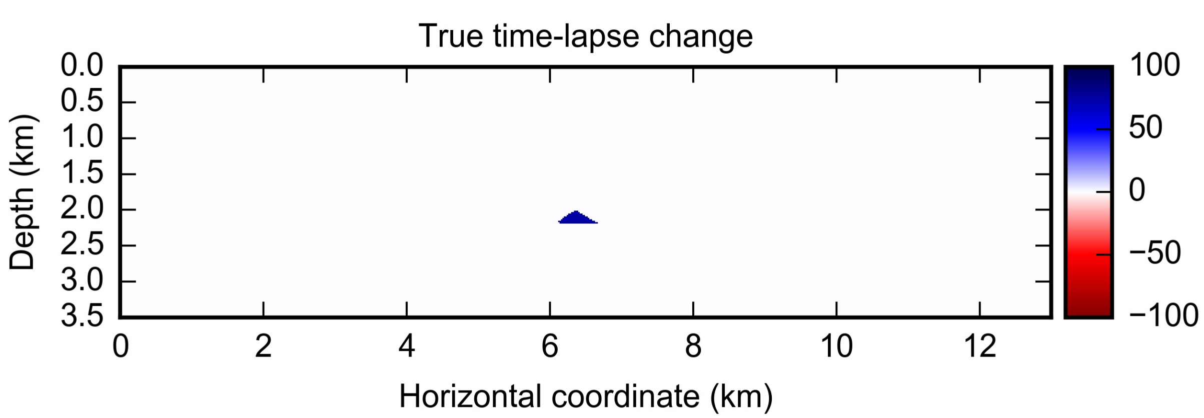


$\delta \mathbf{m}$

**Simulation parameters:**

- Single source: Ricker wavelet with a peak frequency of 6 Hz
- 651 Receivers
- Uniform spacing grid
- Bayesian Inversion for a single frequency of 8 Hz

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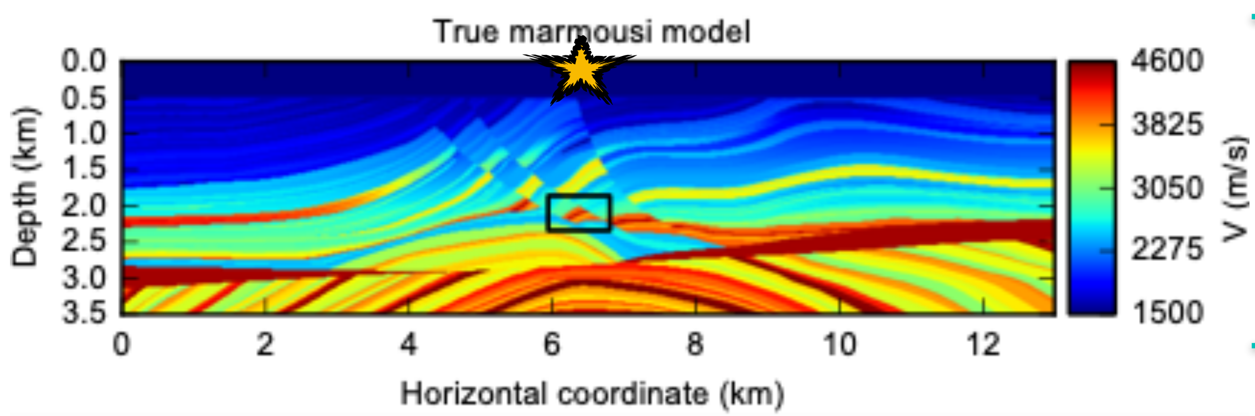


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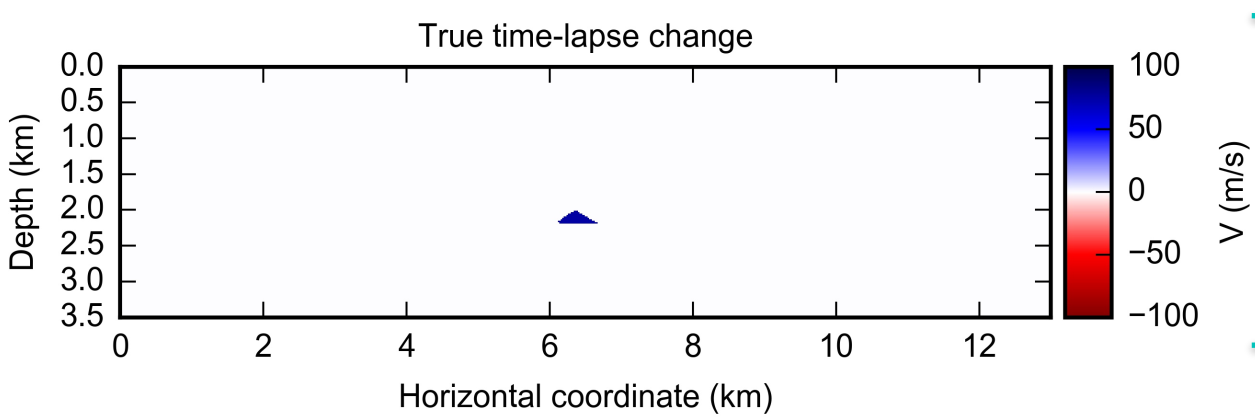


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**Assumptions:**

- Random noise in the data and want to recover the distribution of δm.
- Shape of δm constant -> 1 DOF

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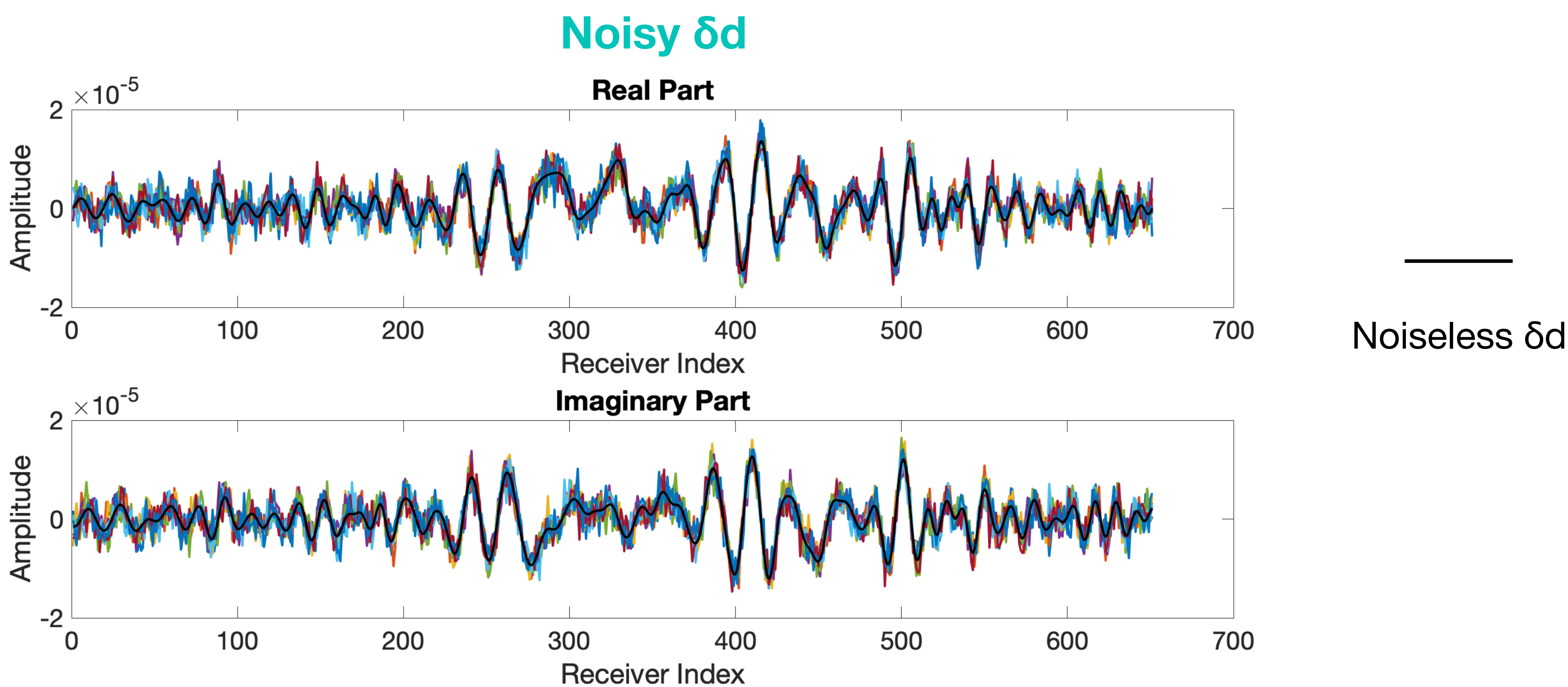
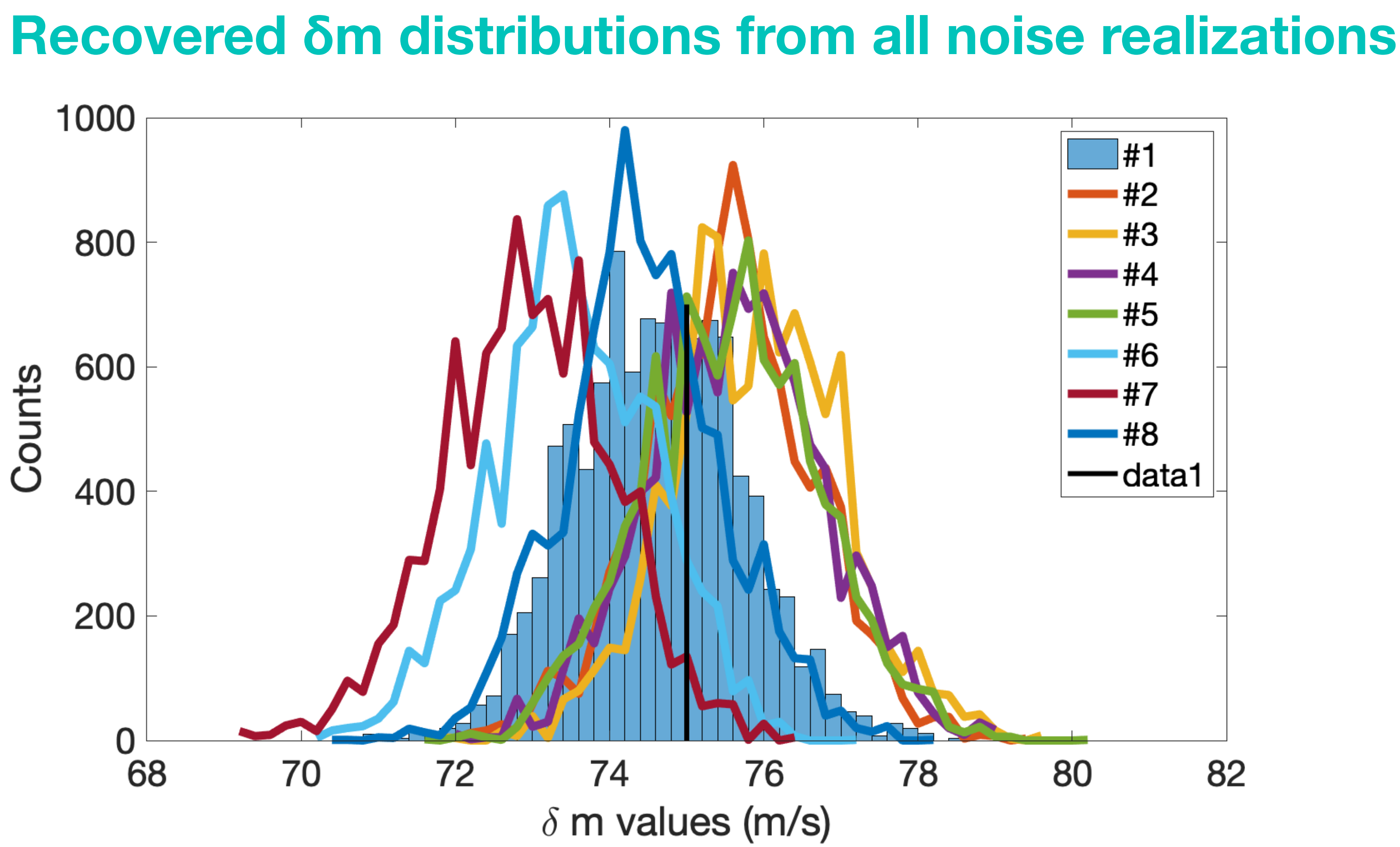



- 8 different noise realizations described by the same covariance matrix Σ
- Noise-to-signal ratio is 0.5

# UQ with Local Acoustic Solver:




- Run MCMC 20,000 & discard 1st half to drop dependancy on the starting model

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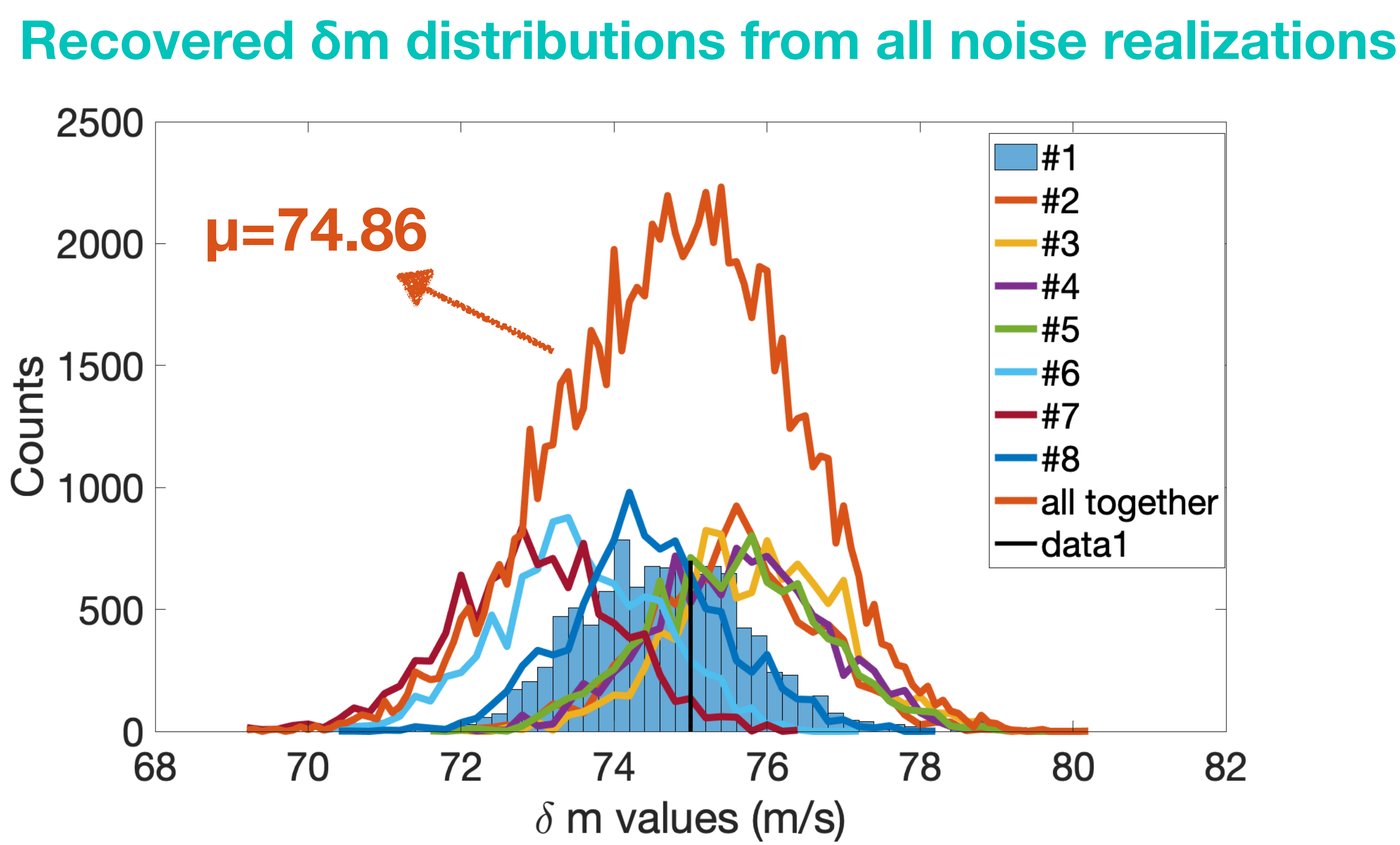


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# Summary

* It is important to have an error bar in our measurement

* Need of a computationally feasible frameworks
    - Need a lot of runs to converge
    - Doing this with a normal solver will be expensive

* What questions you ask vs. what forward solver you use

* Future work: incorporate more refined UQ techniques as well as increase the DOF

"I only believe in statistics that I doctored myself"
**— Winston S. Churchill**

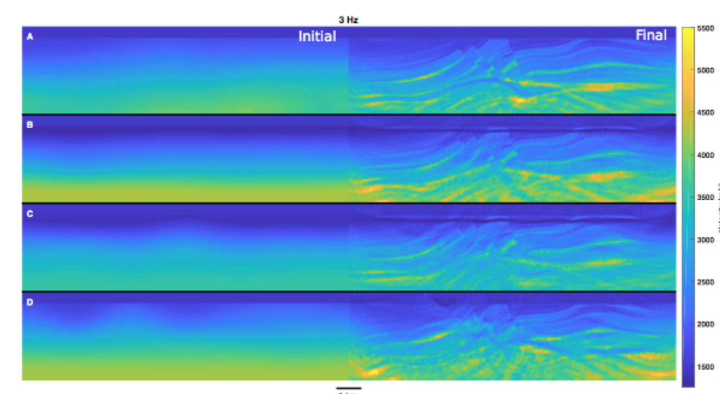
**Research Funding:**

Chevron
Hibernia
NSERC CRSNG
Newfoundland Labrador
InnovateNL

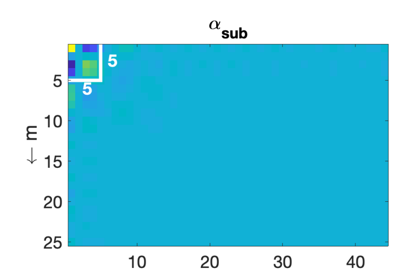
# Thank you for your attention!
# Ευχαριστώ για την προσοχή σας!

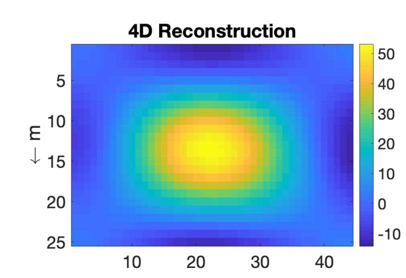
## Work in progress:

1. "**Global Optimization for Full Waveform Inversion: Understanding Trade-offs and Parameter Choices**" Gregory Ely, Alison Malcolm, and David Nicholls; *Geophysics (Submitted)*




2. "**4D Multi-parameter Metropolis Hastings Inversion** " Maria Kotsi, Alison Malcolm, and Gregory Ely, *in preparation for SEG 2019*



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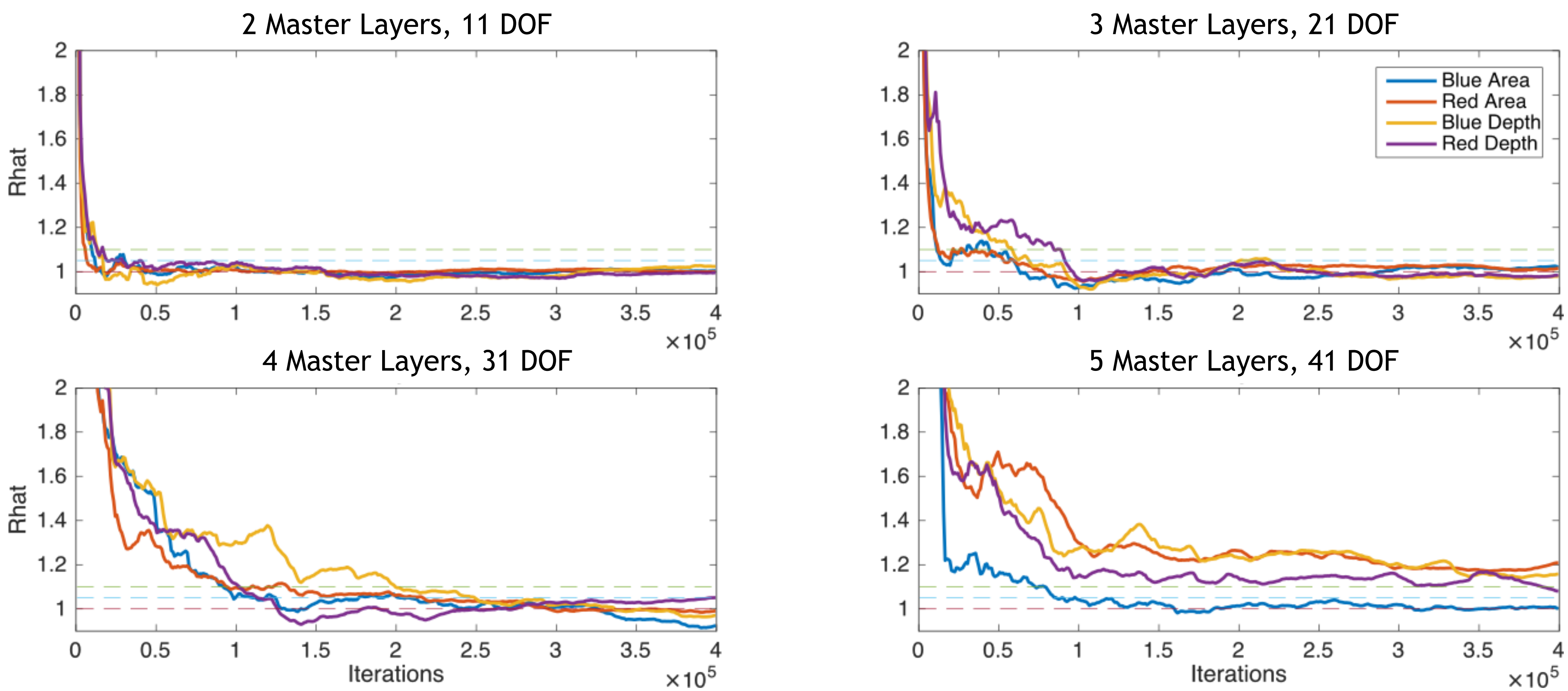
# Convergence: Degrees of Freedom




- 2-5 Master layers with 11 to 41 degrees of freedom (DOF)
  - 14 chains discard 6 lowest acceptance 400,000 iterations, 200,000 discarded
- Convergence rate dependent on number of DOF
  - Failed to converge for 41 DOF

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