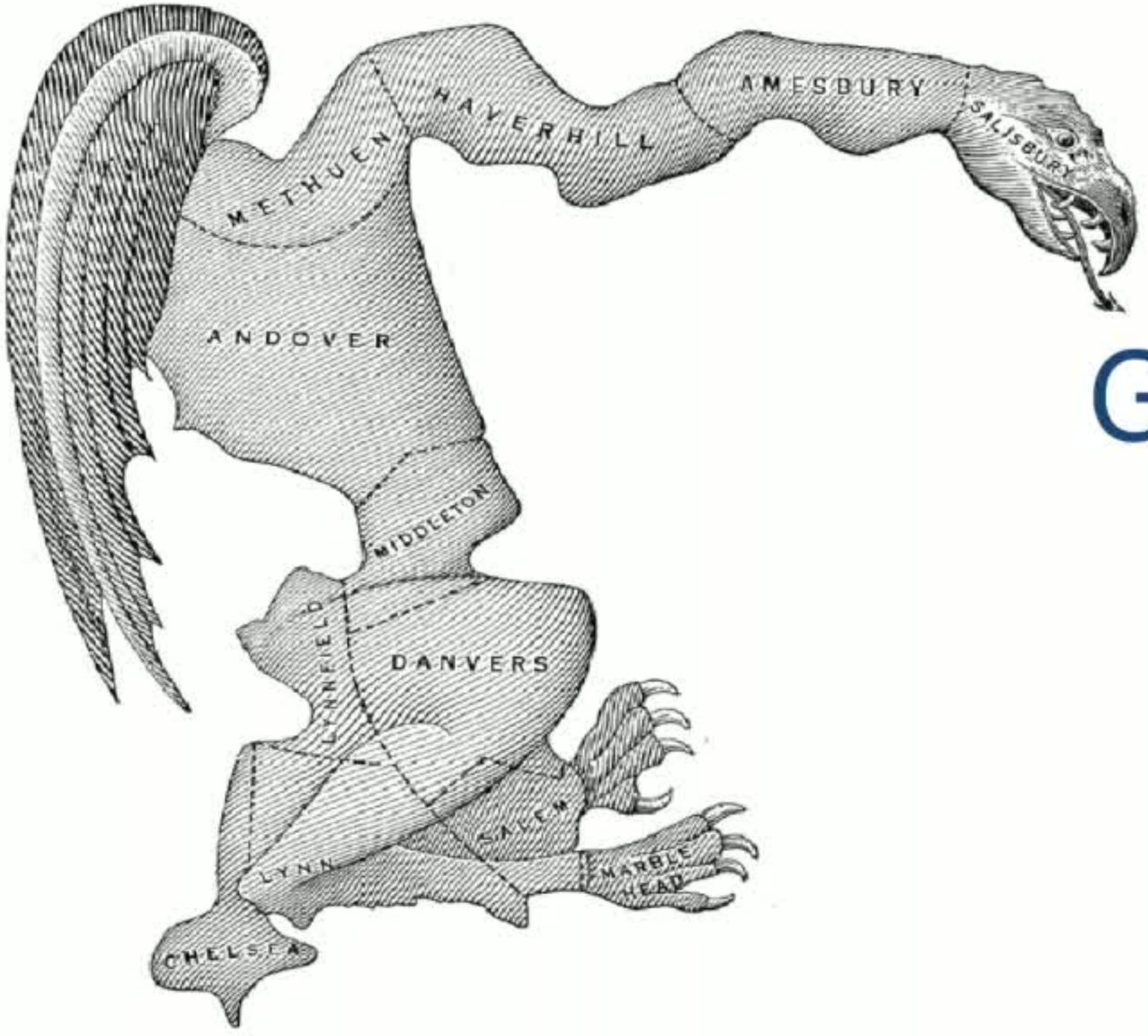
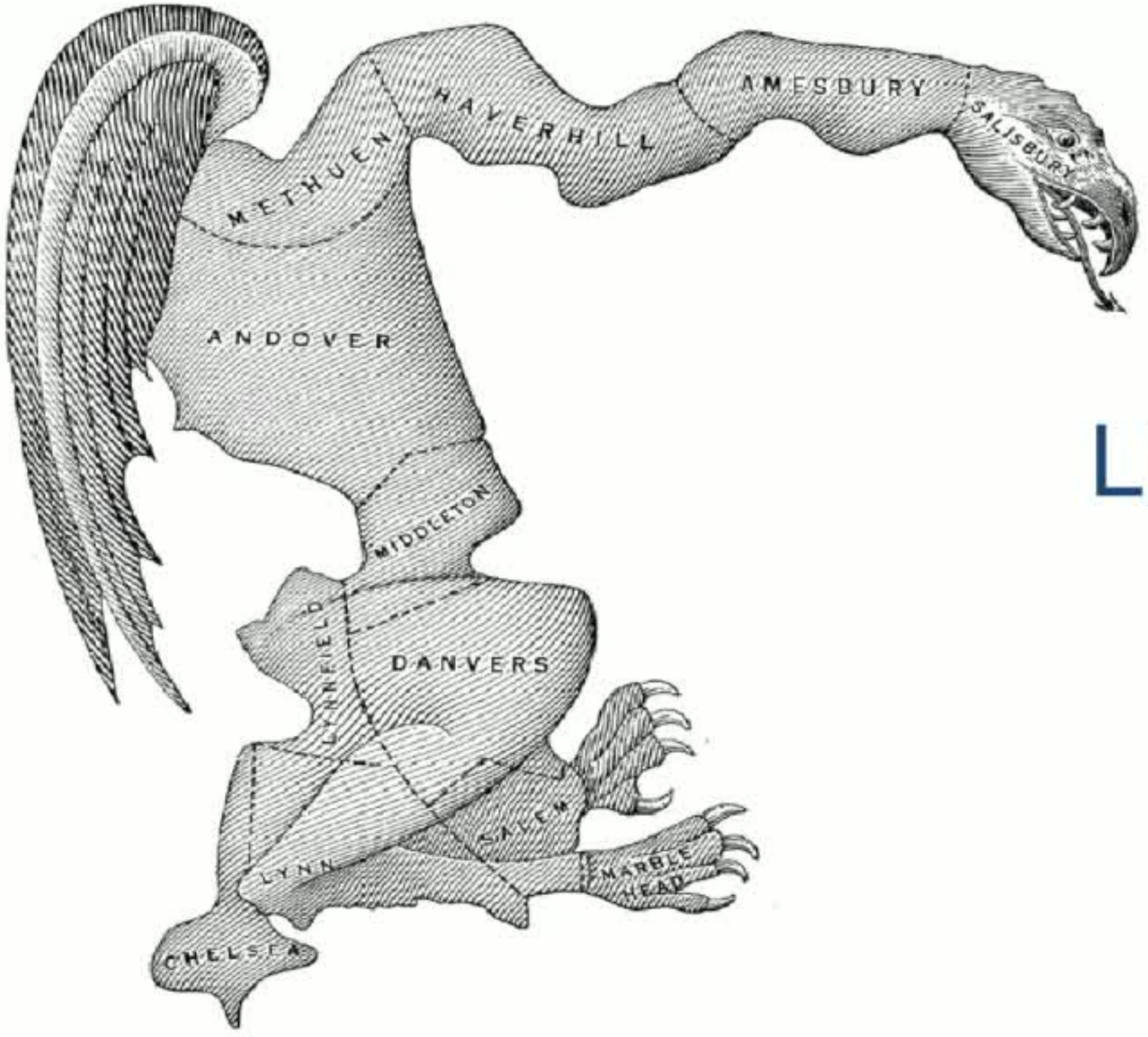


QUANTIFYING GERRYMANDERING

SAMPLING THE GEOPOLITICAL GEOMETRY OF A STATE

DS19
SNOWBIRD
MAY, 2019

JONATHAN MATTINGLY ,GREGORY HERSCHLAG, LISA LEBOVICH
+THE TEAM AT DUKE (AND BEYOND)



How a nice mathematician like me ended up in court

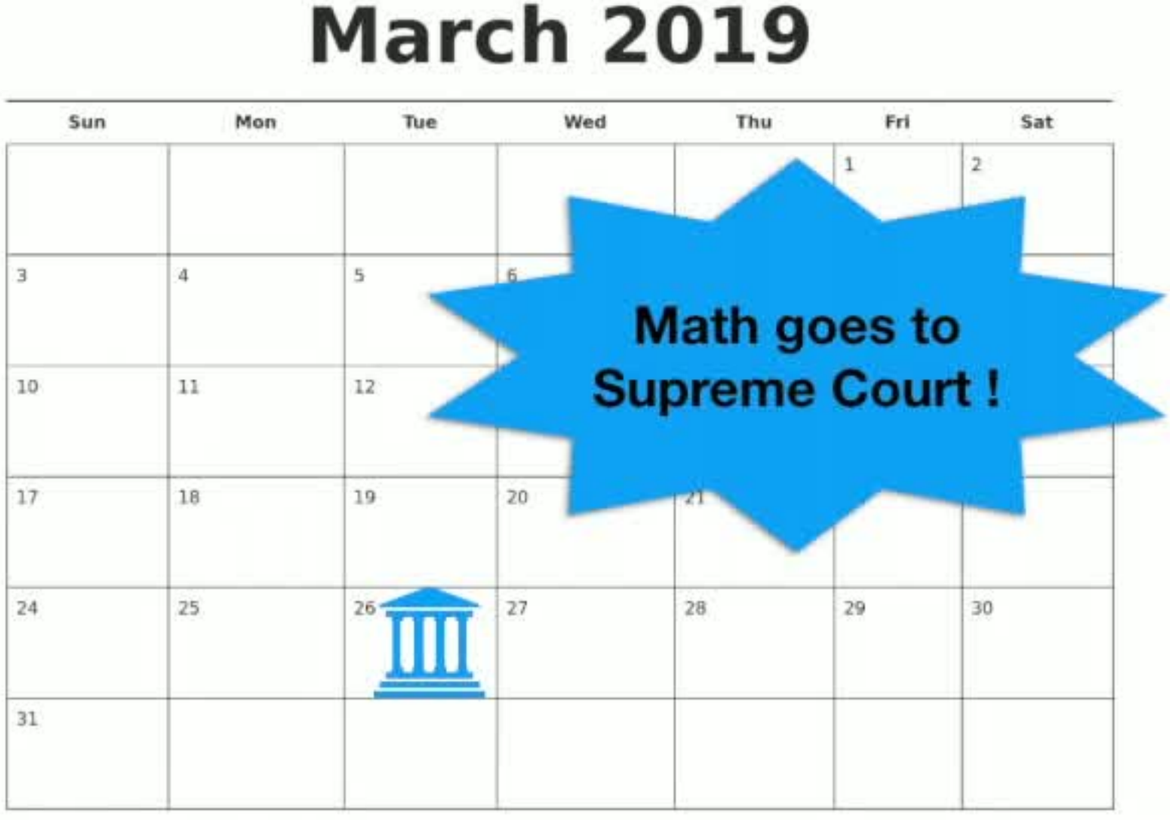
DS19
Snowbird
May, 2019

Jonathan Mattingly ,Gregory Herschlag, Lisa Lebovich
+the Team at Duke (and beyond)

Common Cause/LWV v. Rucho

N.C. Congressional Partisan Gerrymandering Case

- **3 judge panel ruled unconstitutional Gerrymander in summer 2018**
- Provided expert testimony and report in lawsuit
- Heavily cited in court **280p.** judgment
- SCOTUS March 26th along with Maryland case.



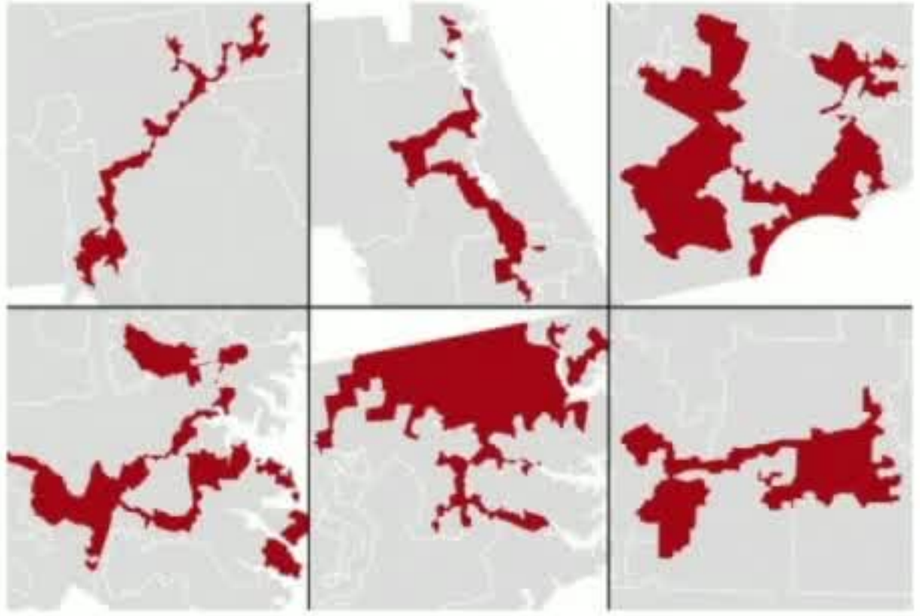
More info

- read Amicus Briefs (www.brennancenter.org/legal-work/common-cause-v-rucho) esp. by Lander, Chin, Pegden, Duchin
- https://www.commoncause.org/page/our-lawsuit-could-end-gerrymandering-for-good/

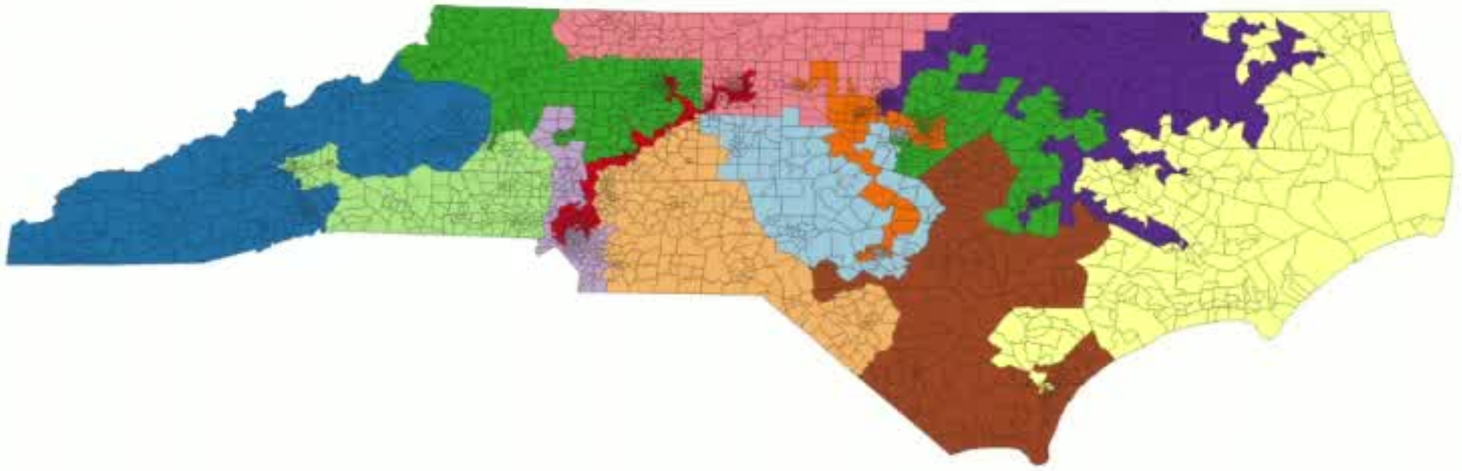
What is Gerrymandering ?

What is Gerrymandering ?

Gerrymandering as oddly shaped districts



"America's most gerrymandered congressional districts." Washington Post



NC 2012 Congressional Districts

Gerrymandering as "Startling" Election Results

NC : US House 2012

	Vote	Seats
Democratic	50.65%	4 (31%)
Republican	48.80%	9 (69%)

WI : Gen Assembly 2014

	Vote	Seats
Democratic	51.28%	36 (36%)
Republican	48.72%	63 (64%)

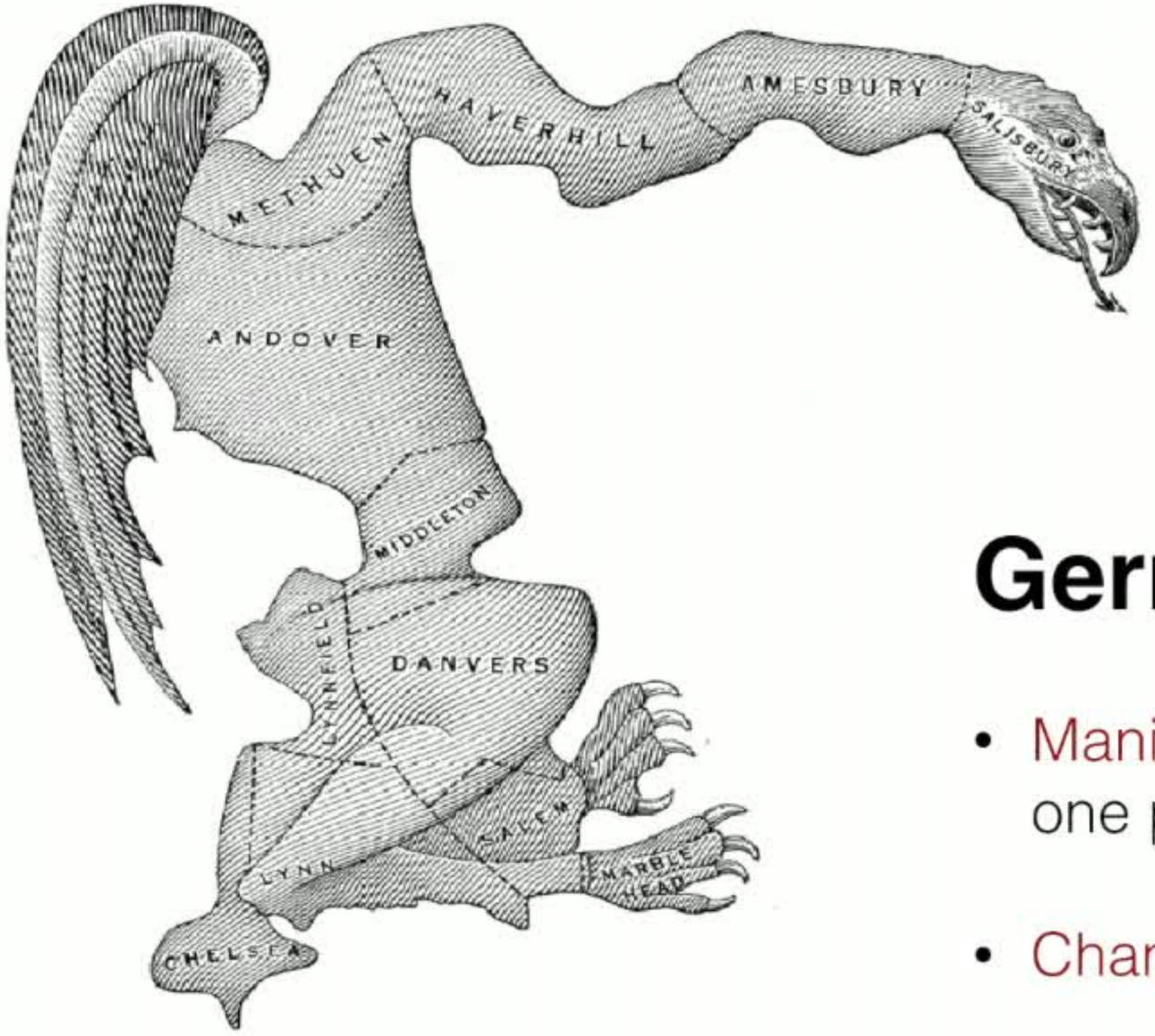
MD : US House 2012

	Vote	Seats
Democratic	63%	7 (87.5%)
Republican	33%	1 (12.5%)

USA : US House 2012

	Vote	Seats
Democratic	50.65%	4 (31%)
Republican	48.80%	9 (69%)

- The most Democratic district had 79.63% Democratic votes
- The most Republican district had 63.11% Republican votes.



Boston Gazette — 26 March, 1812

What would have happen if no political agenda had been inserted in the process ?

Gerrymander

- Manipulate district boundaries to favor one party (partisan) or class (racial)
- Change the outcome of an election

"gerrymander the results"

What if we drew the districts randomly?

with no regard for party registration or most demographics

Look for the *expected* behavior of a collection of non-partisan district plans

create a null-hypothesis without partisan bias

The Recipe

1. Determine what it means for a map to be compliant.

2. Sample a collection of compliant random redistricting plans

3. For each plan count number of Democratic and Republican votes in each of the new districts using actual votes. Use the vote counts to determine the winner.

Use Markov Chain Monte Carlo to sample
a distribution on redistricting plans (will return to this)

Example of compliance:
NC Congressional Districts - HB92

1. Districts have equal population
2. The districts are contiguous and compact,
3. Splitting counties is minimized, and
4. Black voters are sufficiently concentrated in 2 districts to affect the winner.

Sampling

Markov Chain Monte Carlo

$$\text{Density of plan} \quad \propto \quad e^{-\beta(\text{score of plan})}$$

$$P(\xi) = \frac{1}{\mathcal{Z}} e^{-\beta J(\xi)}$$

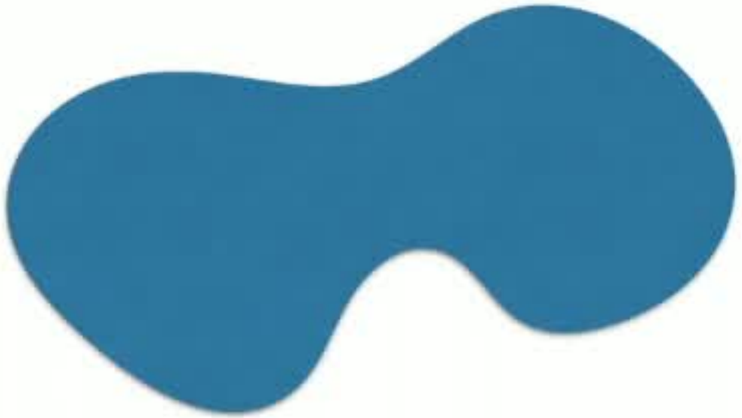
$$\xi : \{Precincts\} \mapsto \{D_1, D_2, D_3, ..., D_n\}$$

$$J(\xi) = w_{population} J_{population}(\xi) + w_{compact} J_{compact}(\xi) + w_{VRA} J_{VRA}(\xi) + w_{county} J_{county}(\xi)$$

$$J_{population}(\xi) = \sum_{d=1}^{n} (\text{Ideal}_{\text{population}} - \text{Population of district d})^2$$

(a 13 color Potts Model with an unusual energy)

Compactness score

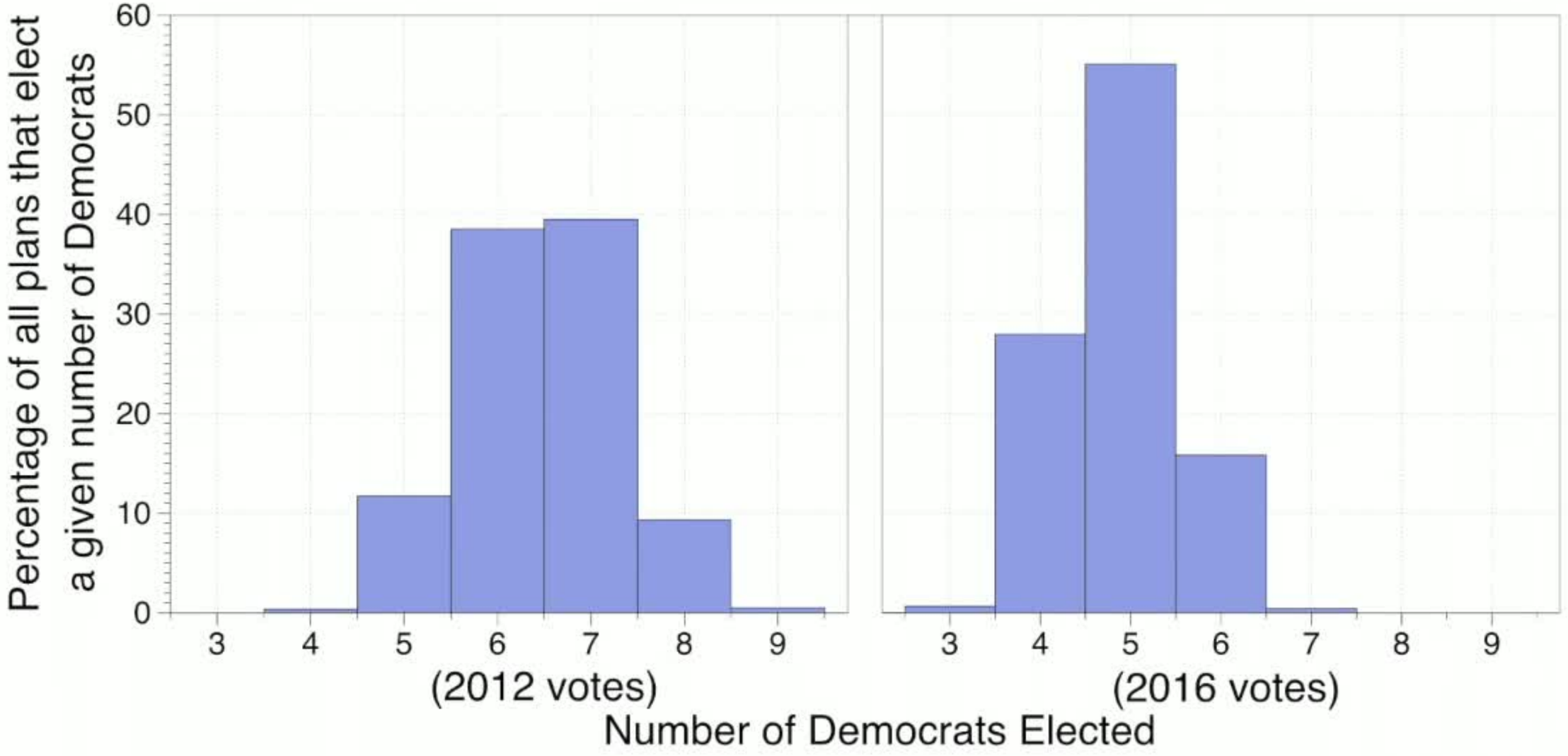


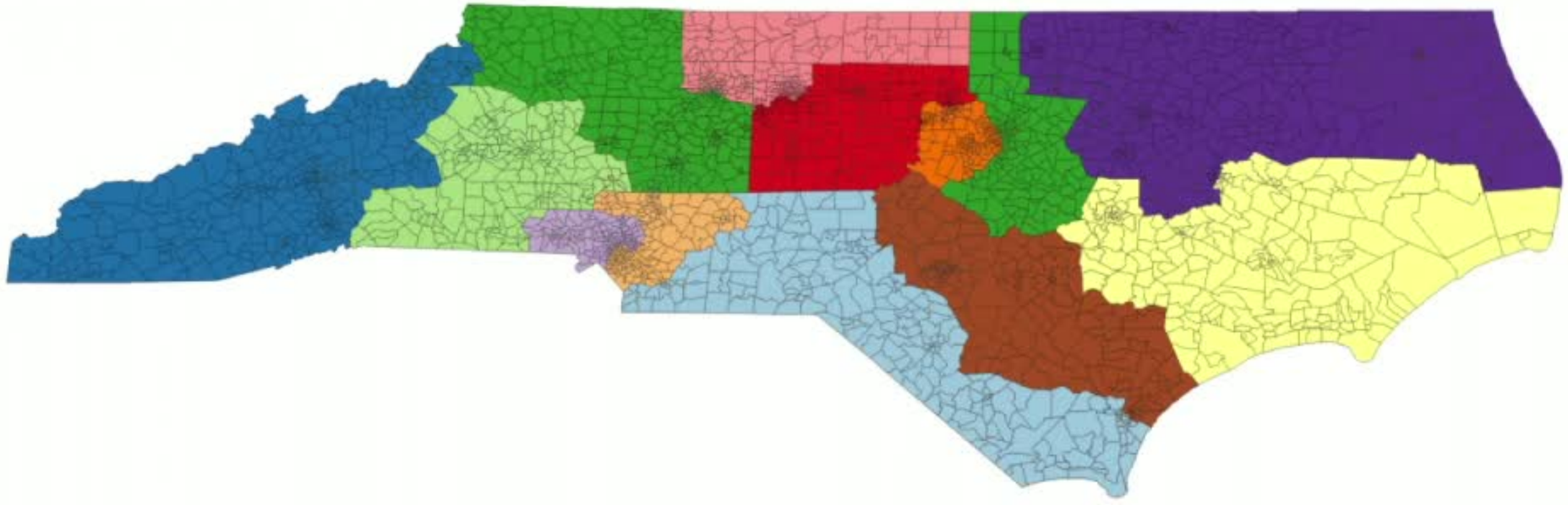
$$\frac{(\text{Perimeter})^2}{\text{Area}} \geq 4\pi \approx 12.5$$

Minimized for a circle

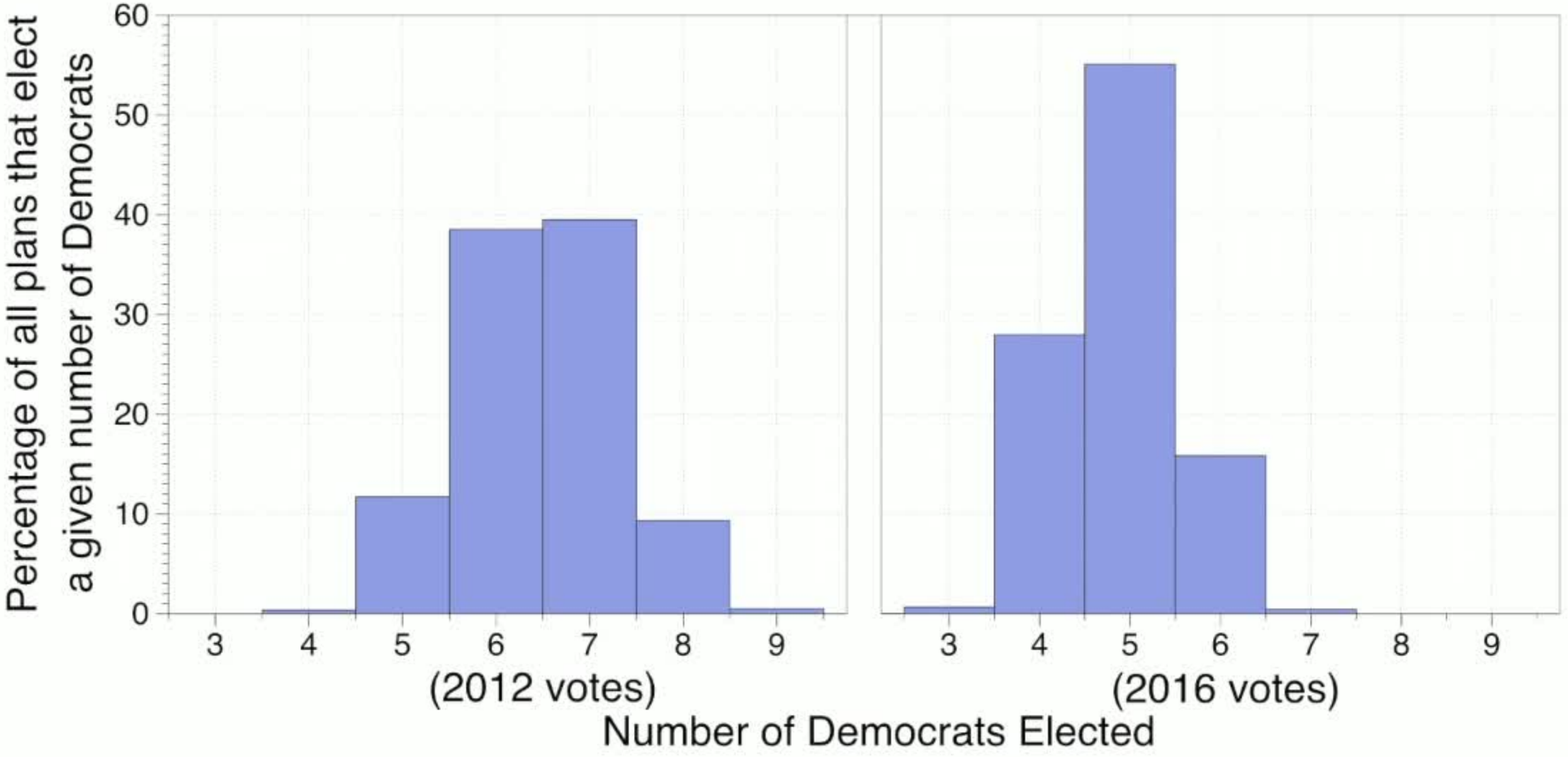
Also considered the ratio of district's area to the smallest circumscribing rectangle

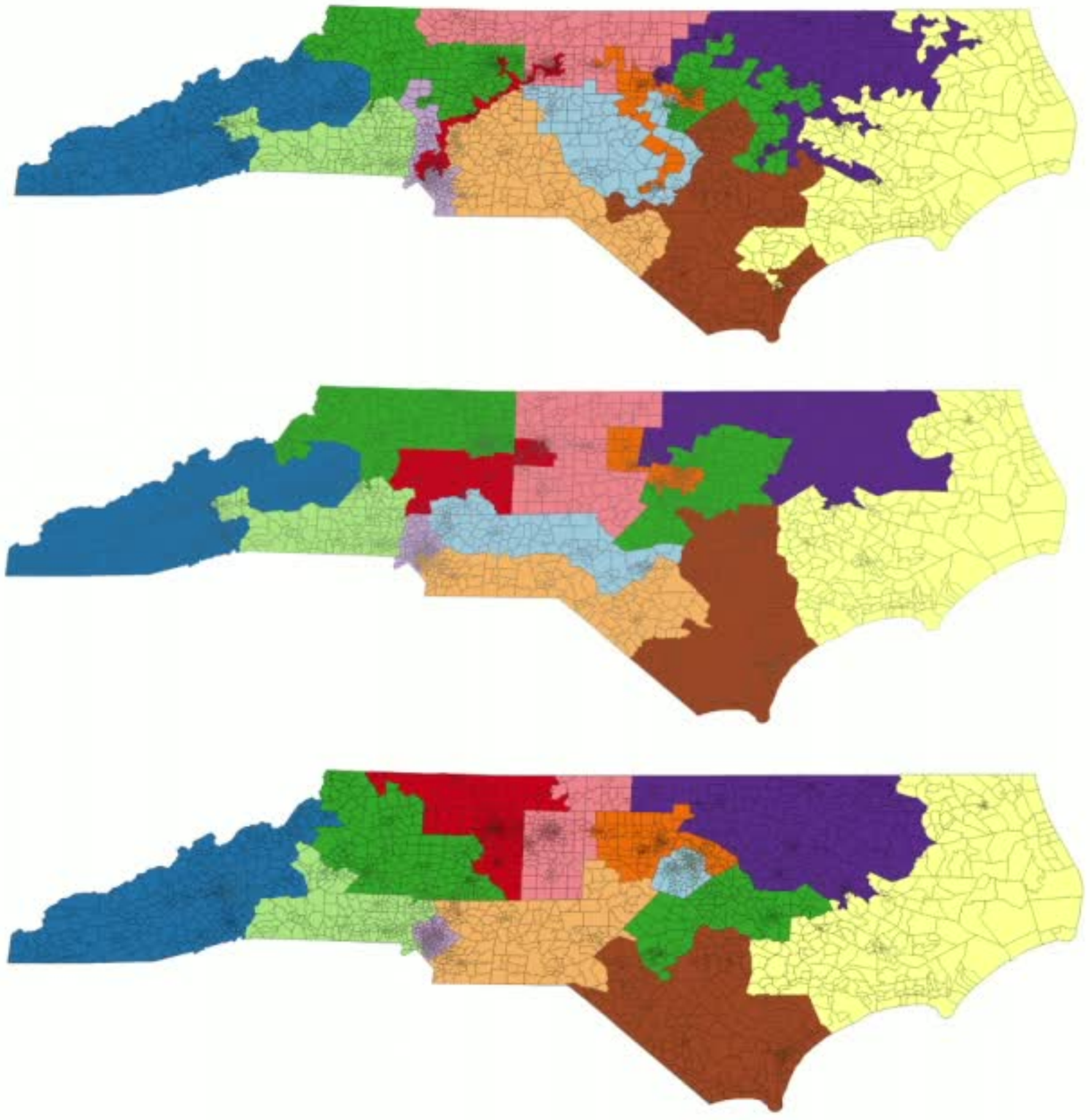
The number of Democrats elected over 24,000 district plans



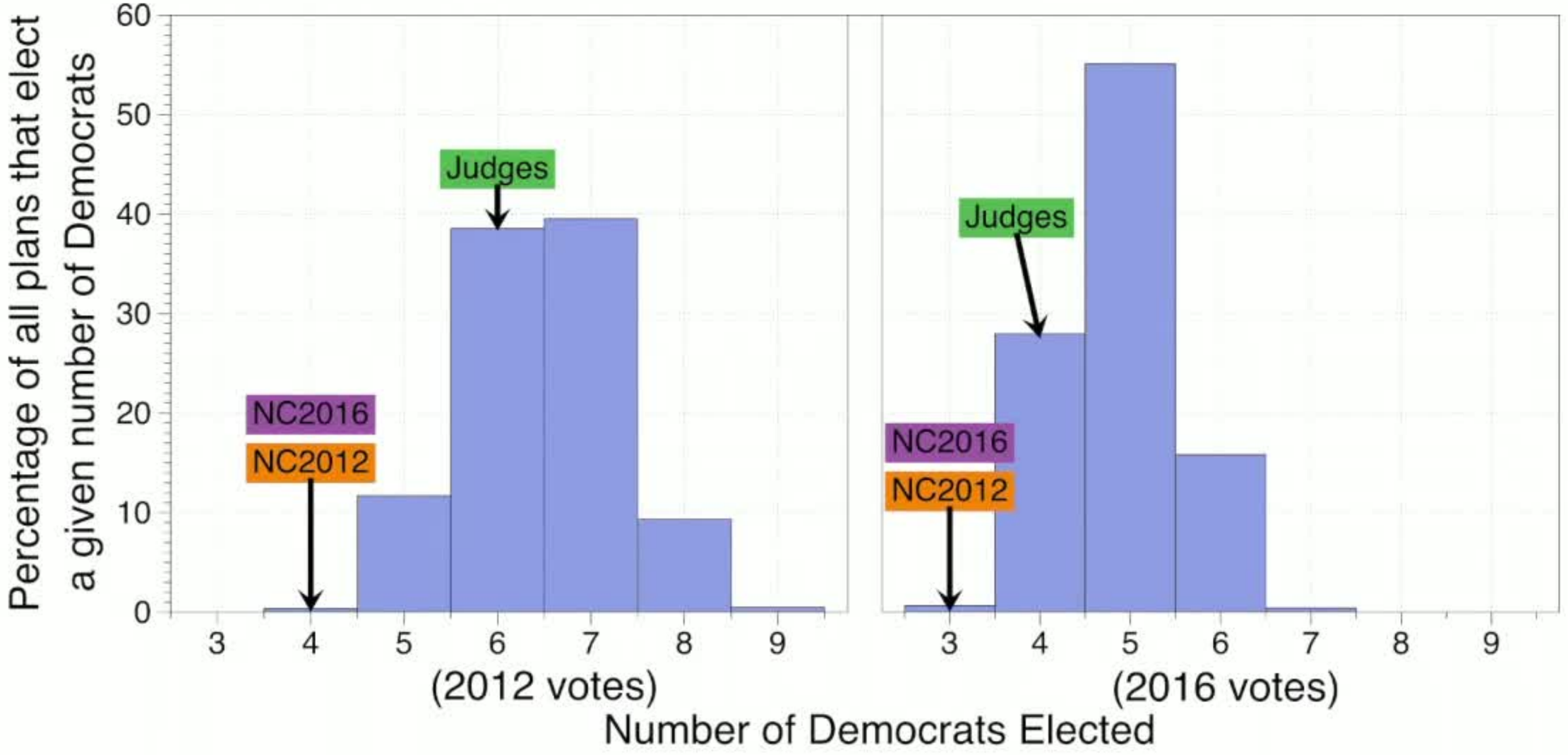


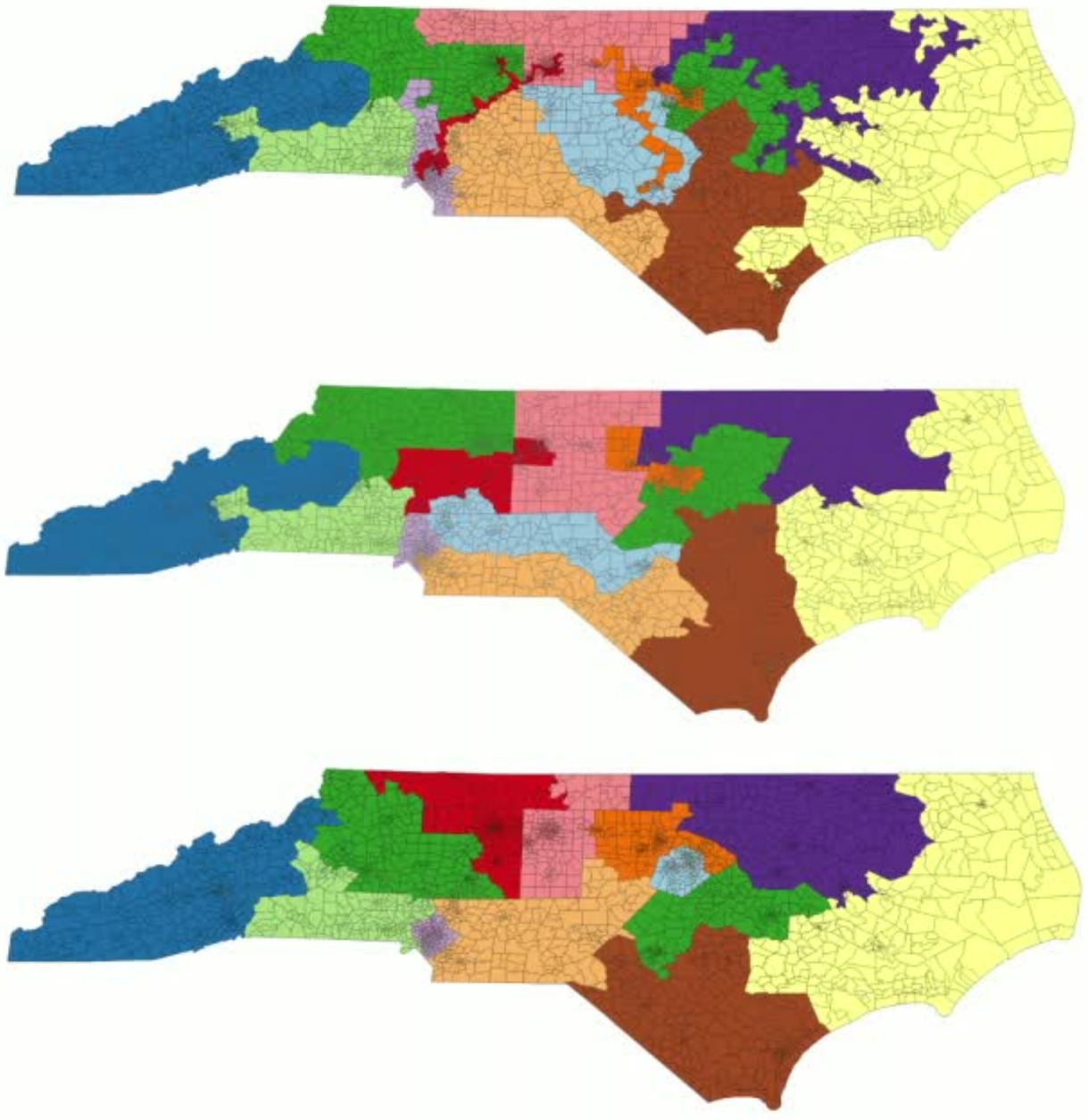
The number of Democrats elected over 24,000 district plans



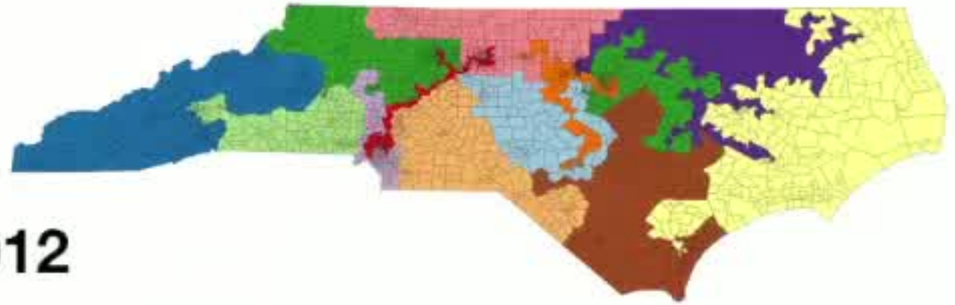


Compare particular maps with the 24,000 district plans





Gerrymandering can occur in the absence of oddly shaped districts



NC 2012

Atypical

NC 2016

Atypical

Panel of Judges

Typical

Gerrymandering can occur in the absence of oddly shaped districts

NC 2012

Atypical

NC 2016

Atypical

Panel of Judges

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Gerrymandering can occur in the absence of oddly shaped districts

NC 2012

Atypical

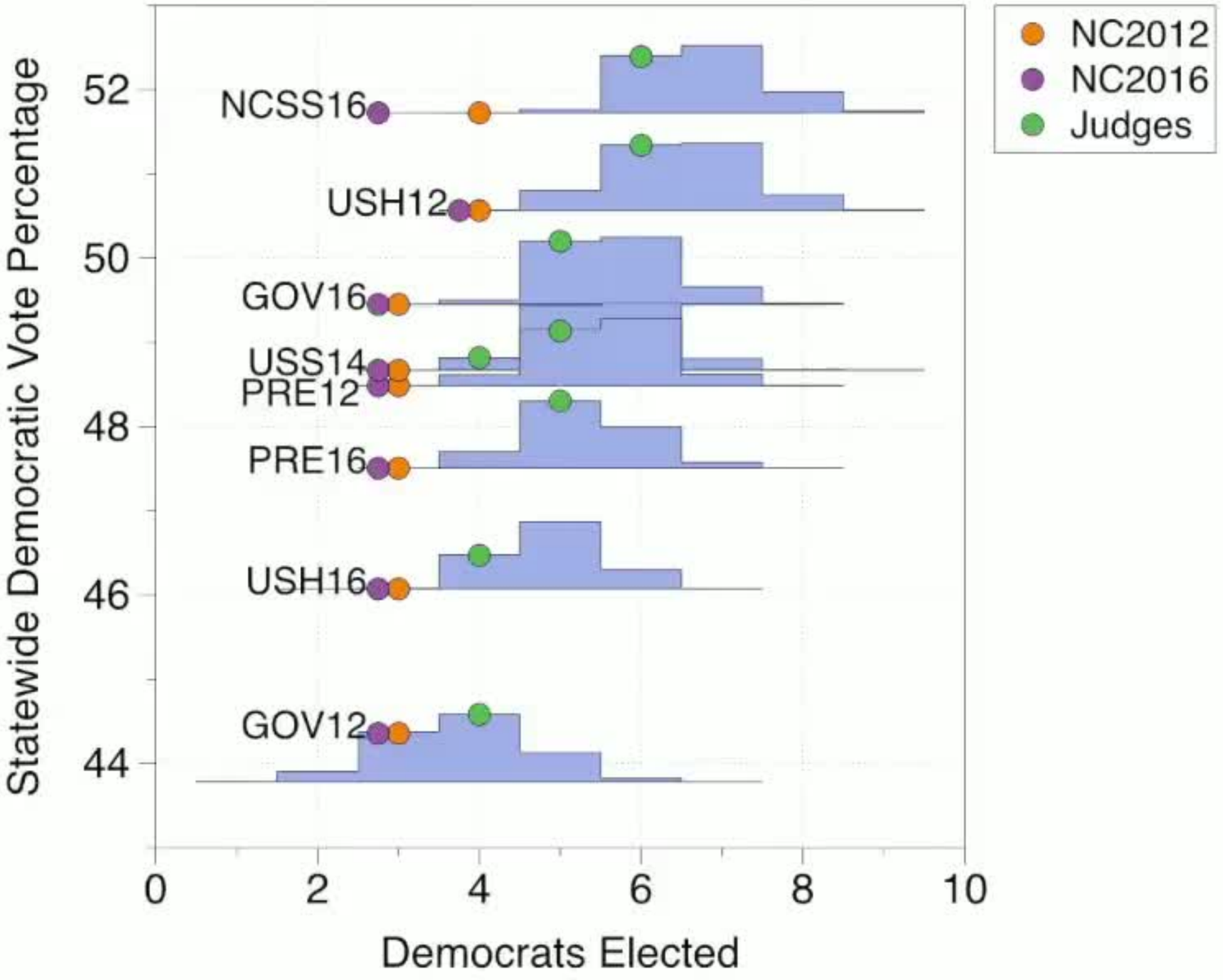
NC 2016

Atypical

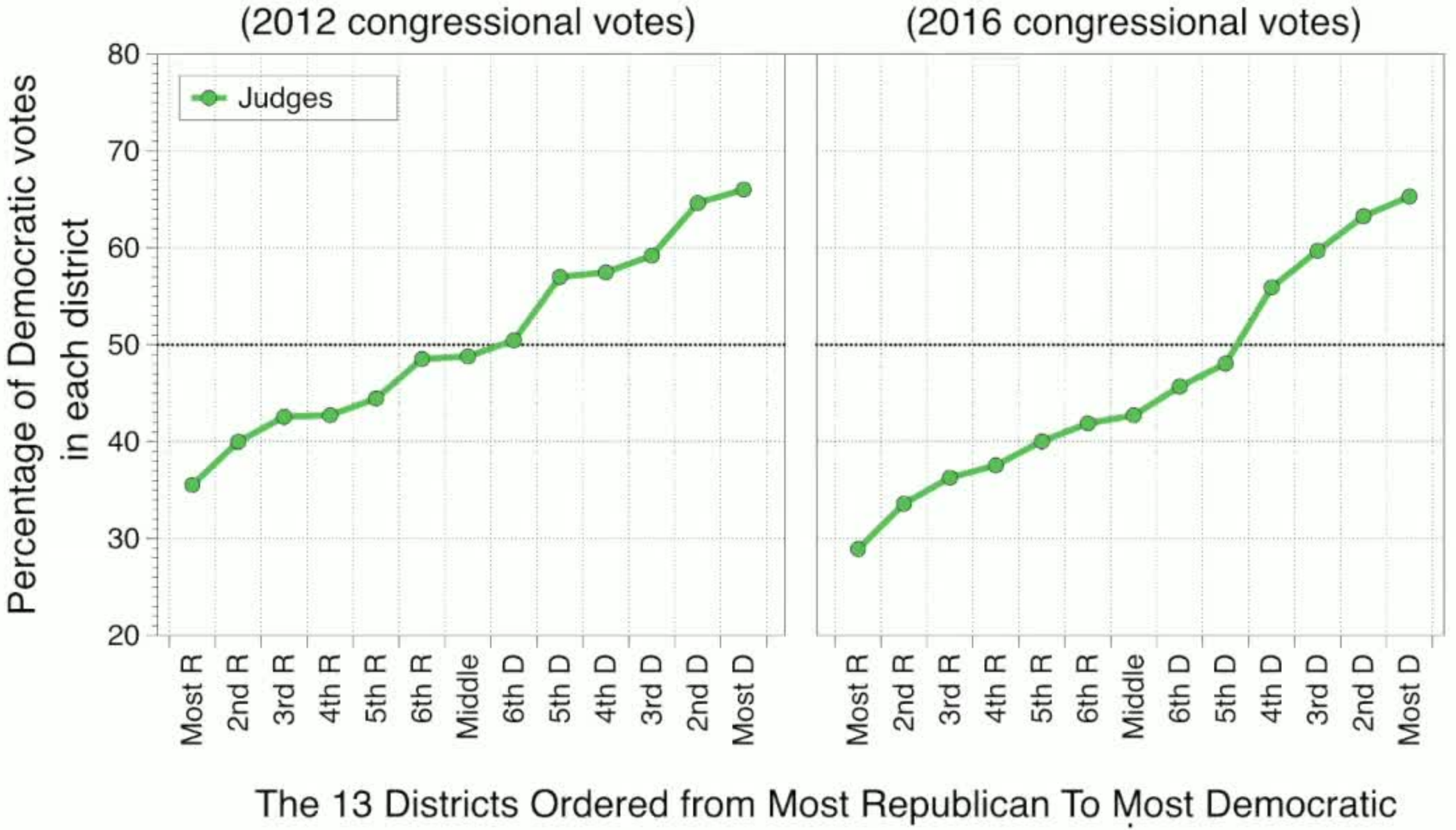
Panel of Judges

Typical

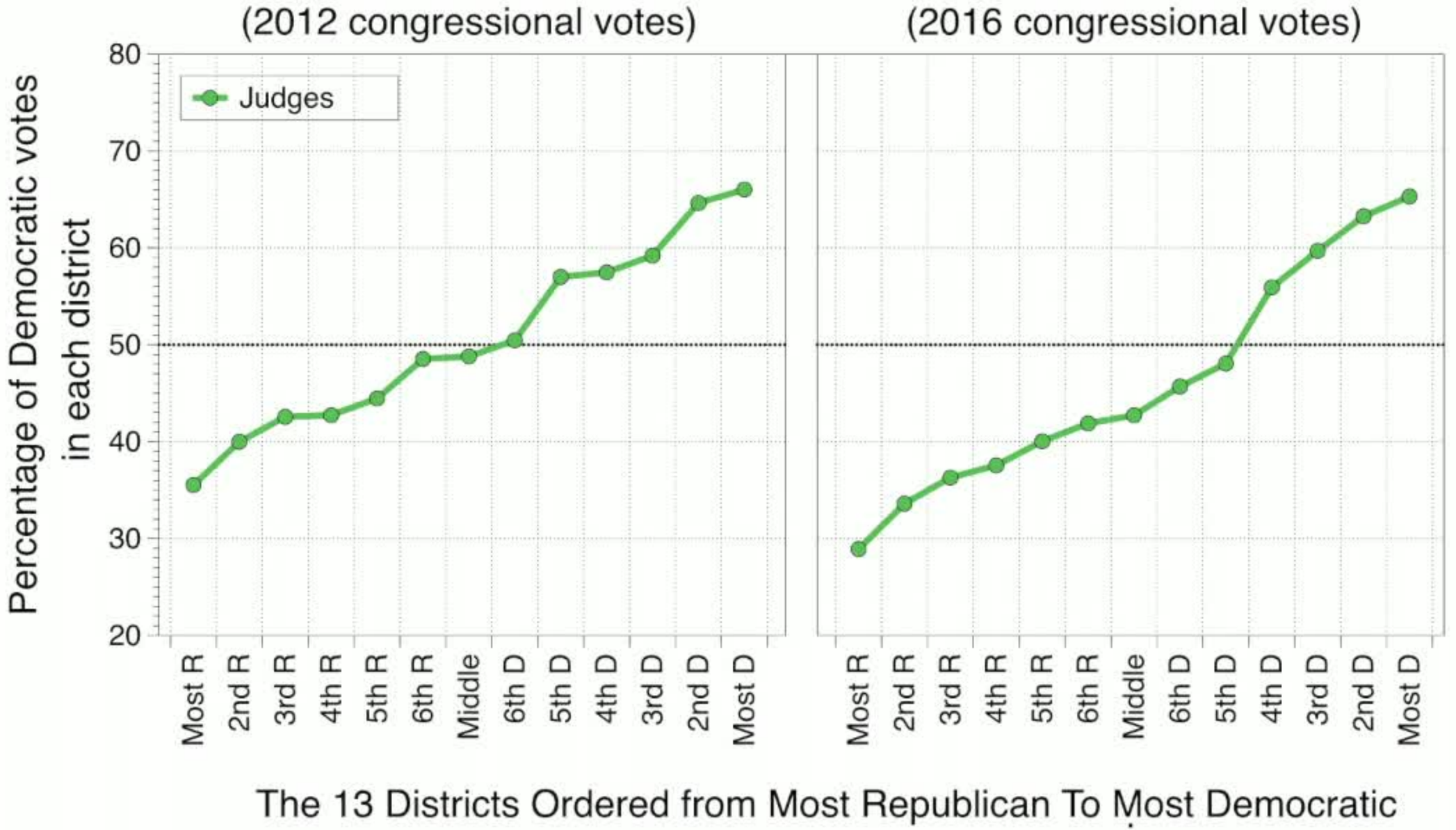
Across many elections
Does incumbency matter?



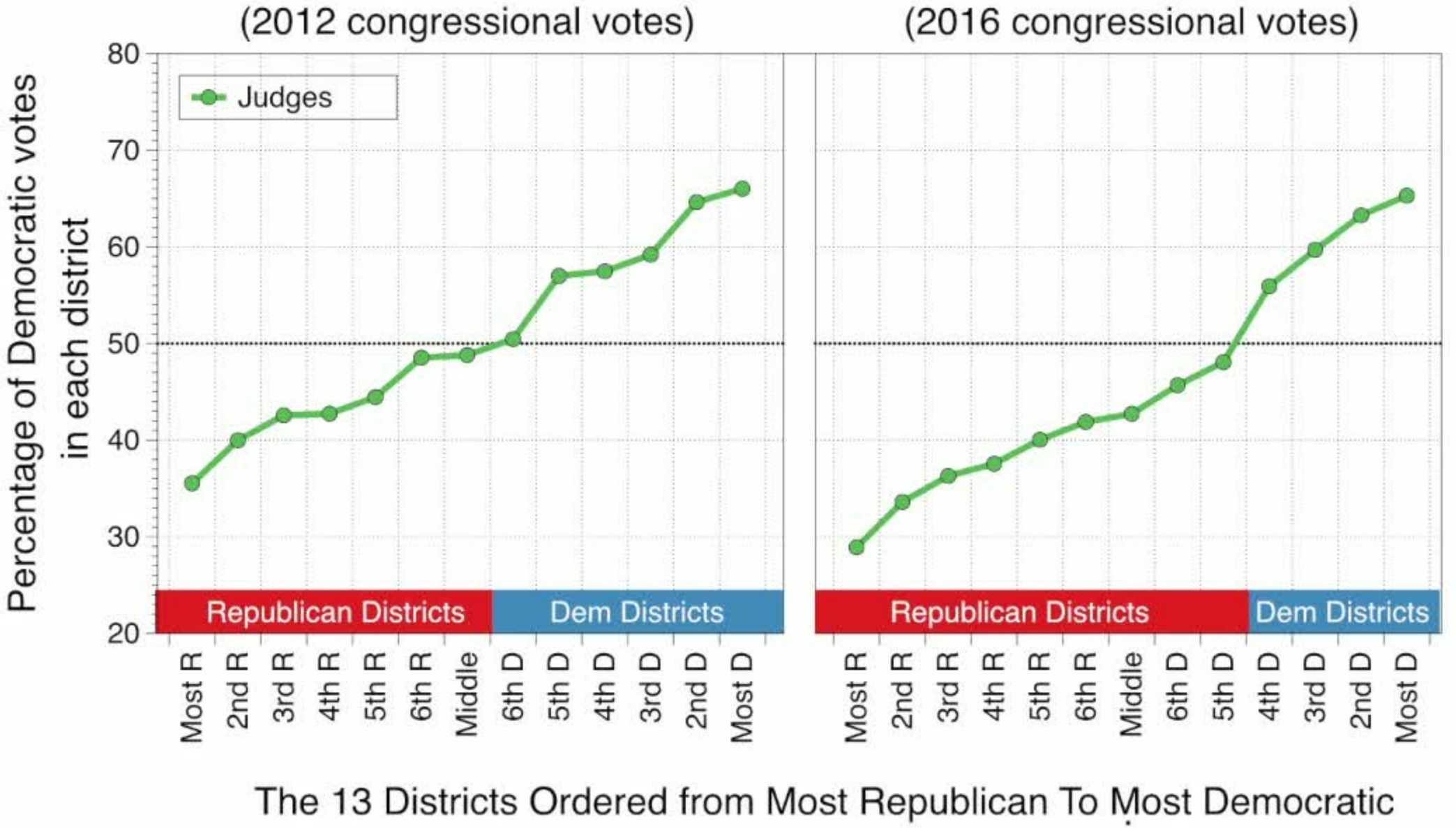
Visualizing election details on the Judges' Map



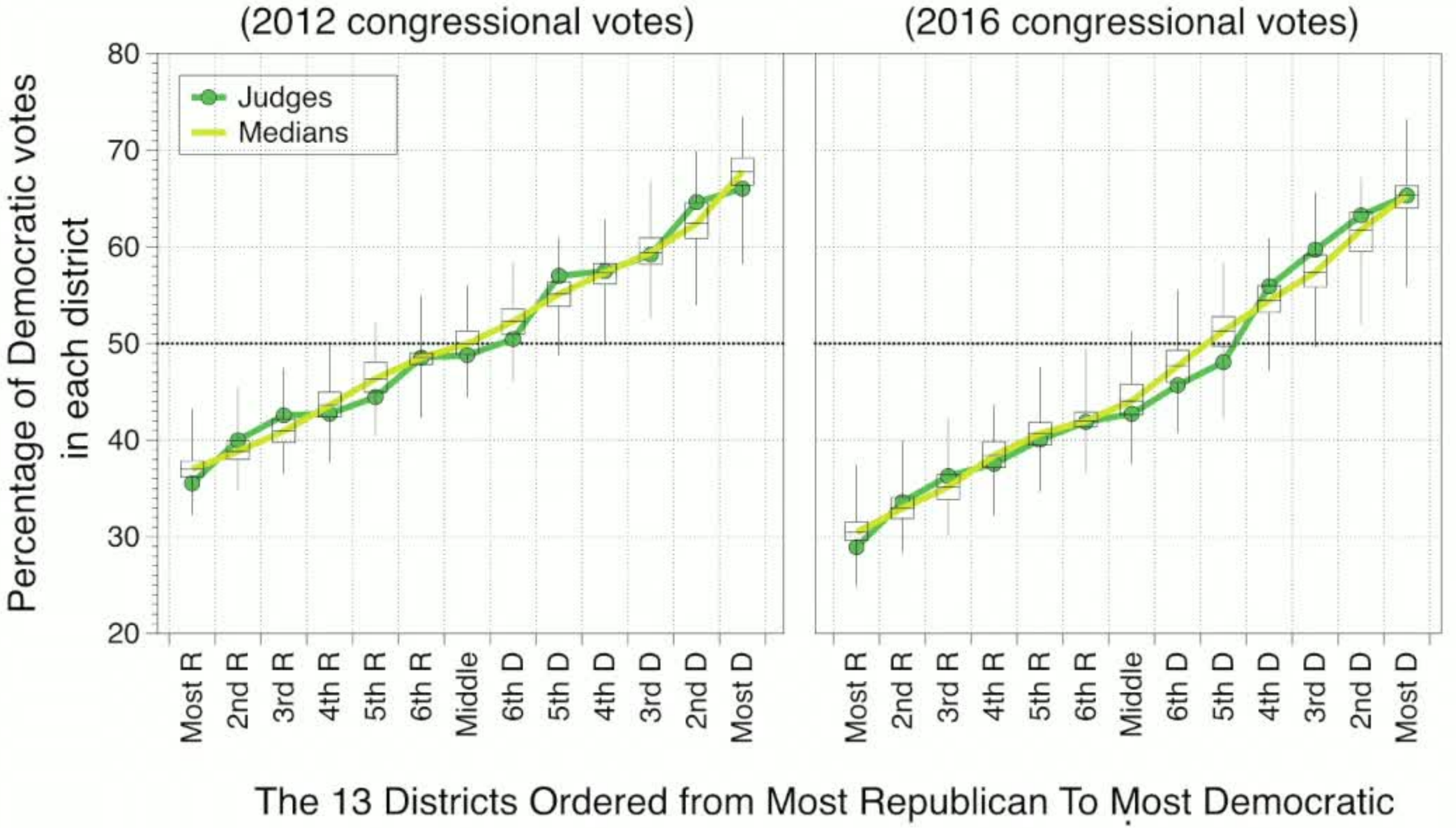
Visualizing election details on the Judges' Map



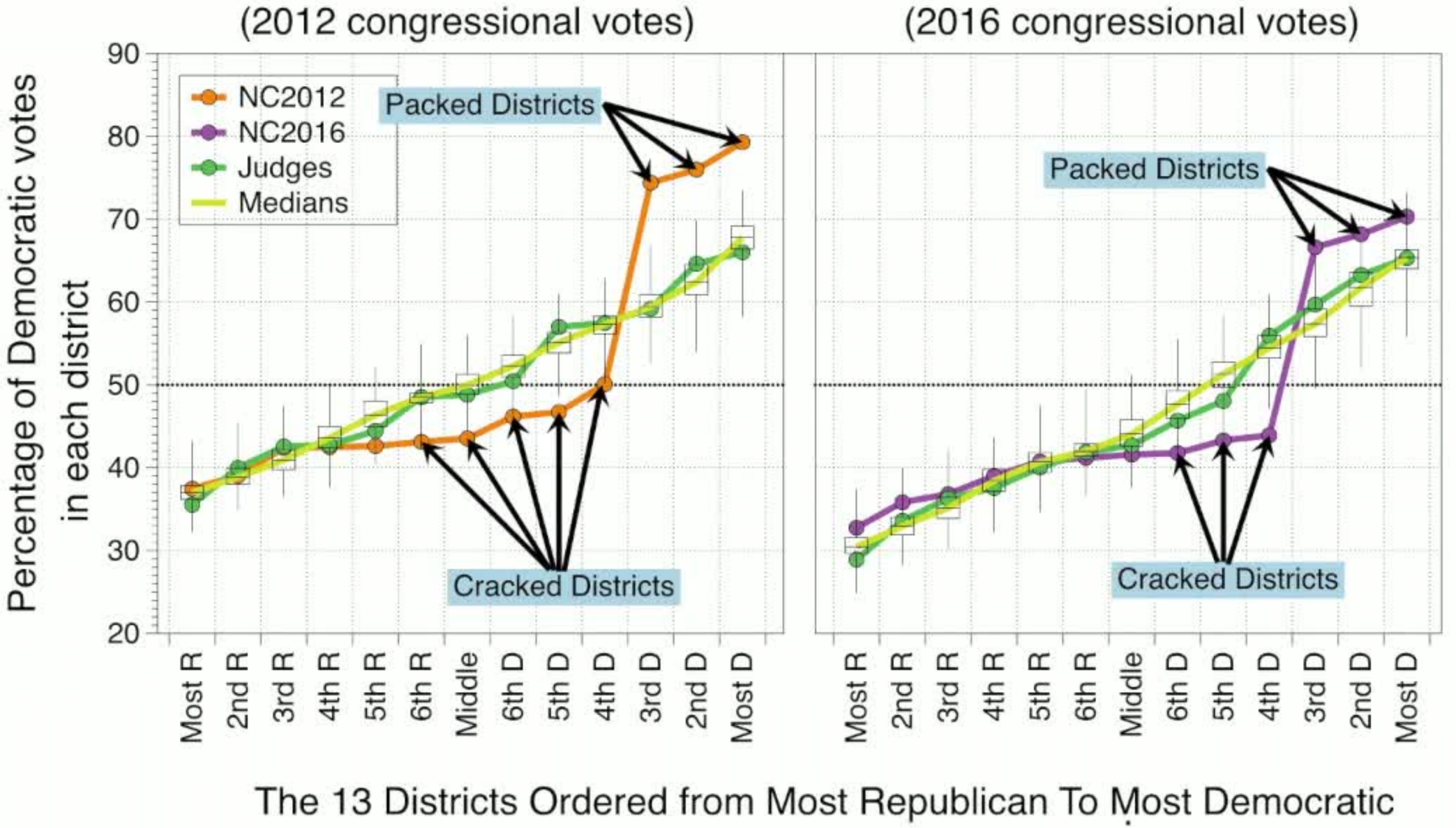
Visualizing election details on the Judges' Map



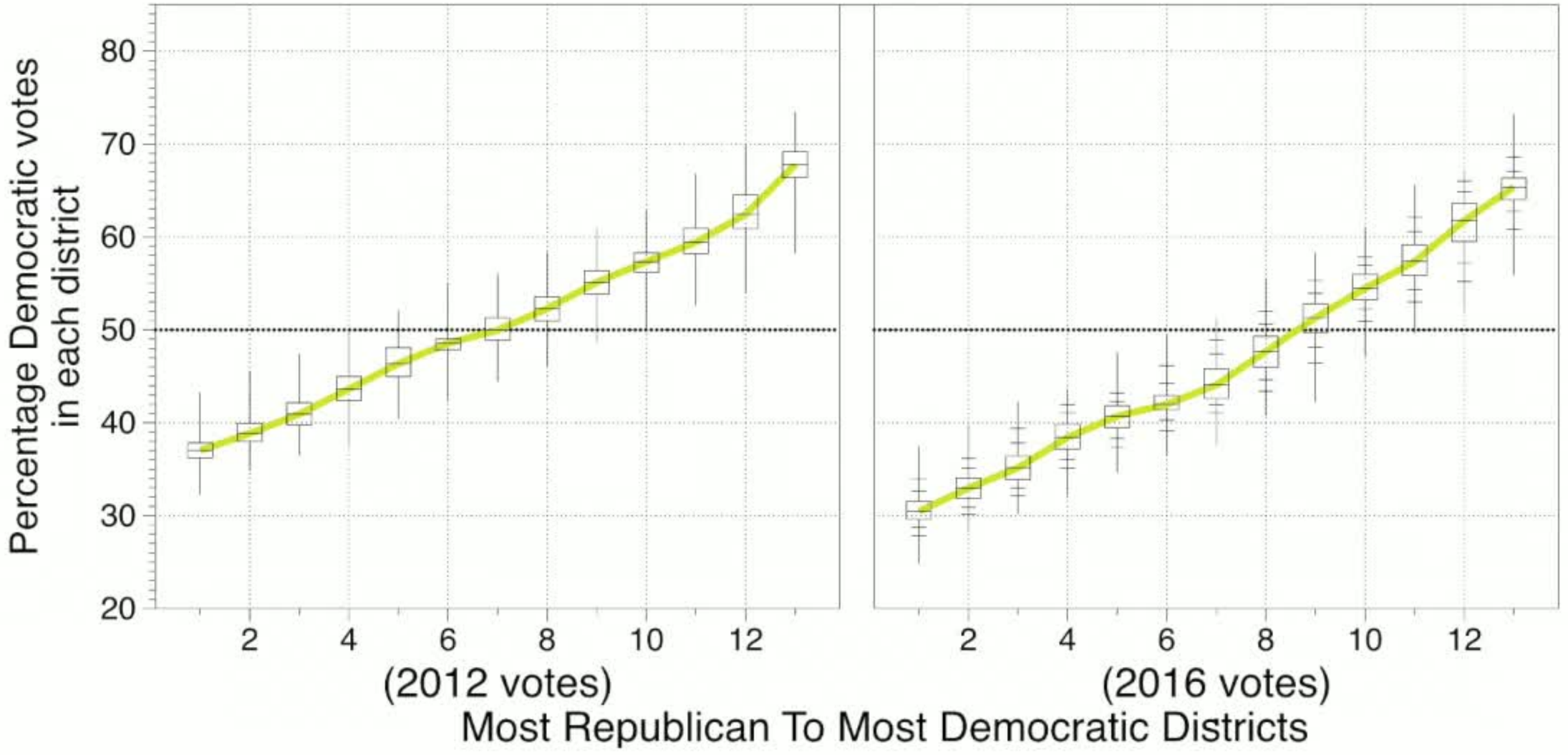
Visualizing details on the collection of maps



Visualizing details on the enacted plans



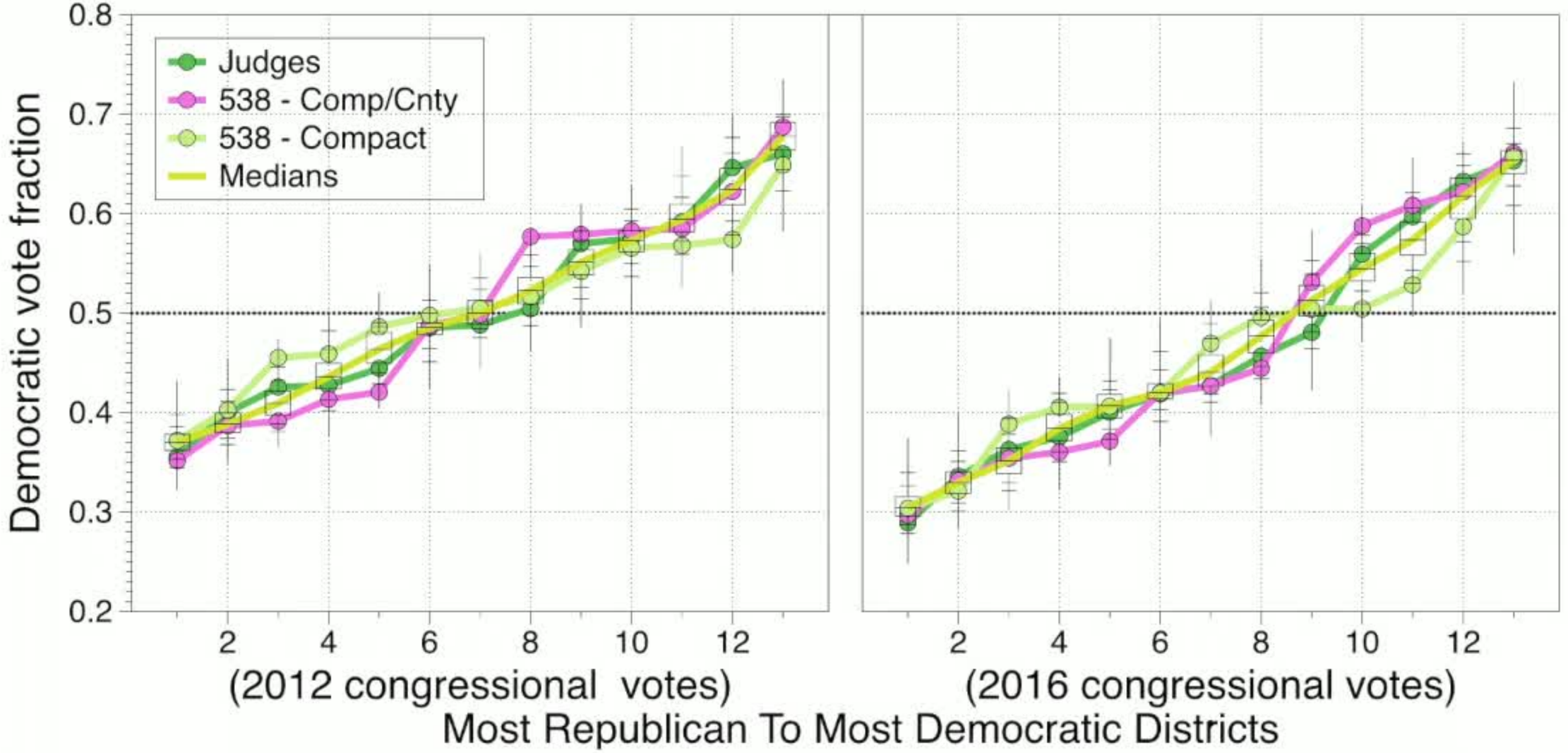
NC Congressional Delegation



Rep (538)

Dem (538)

Are we sampling the space in a reasonable way?



Judges

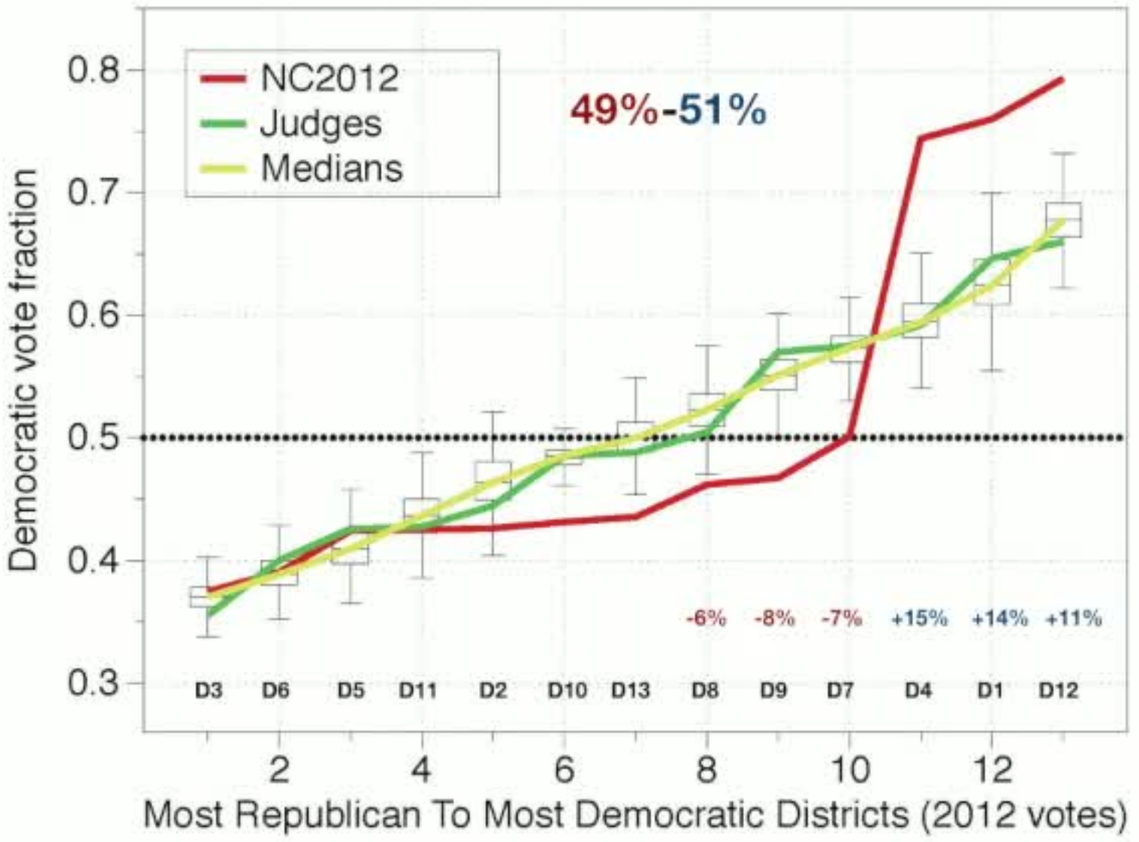
Comp/County

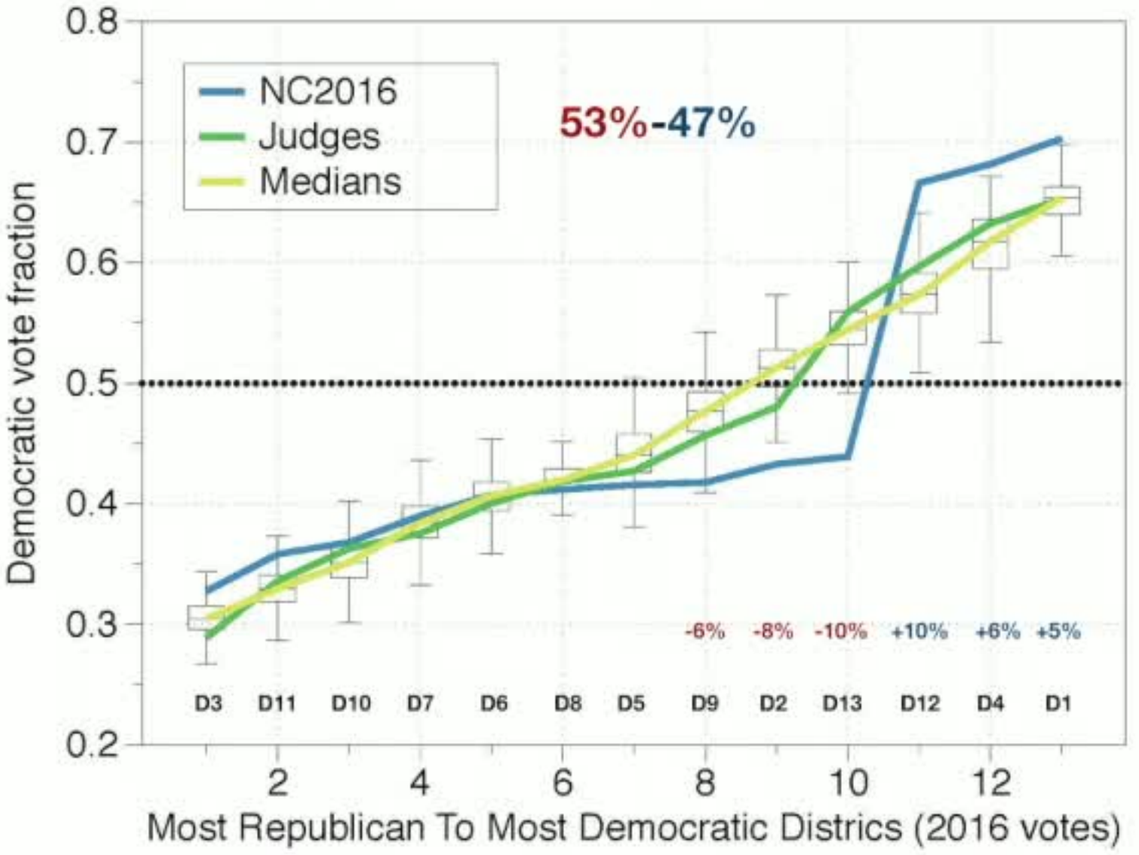
a 538 plan

Compact

a 538 plan

Signature of Gerrymandering

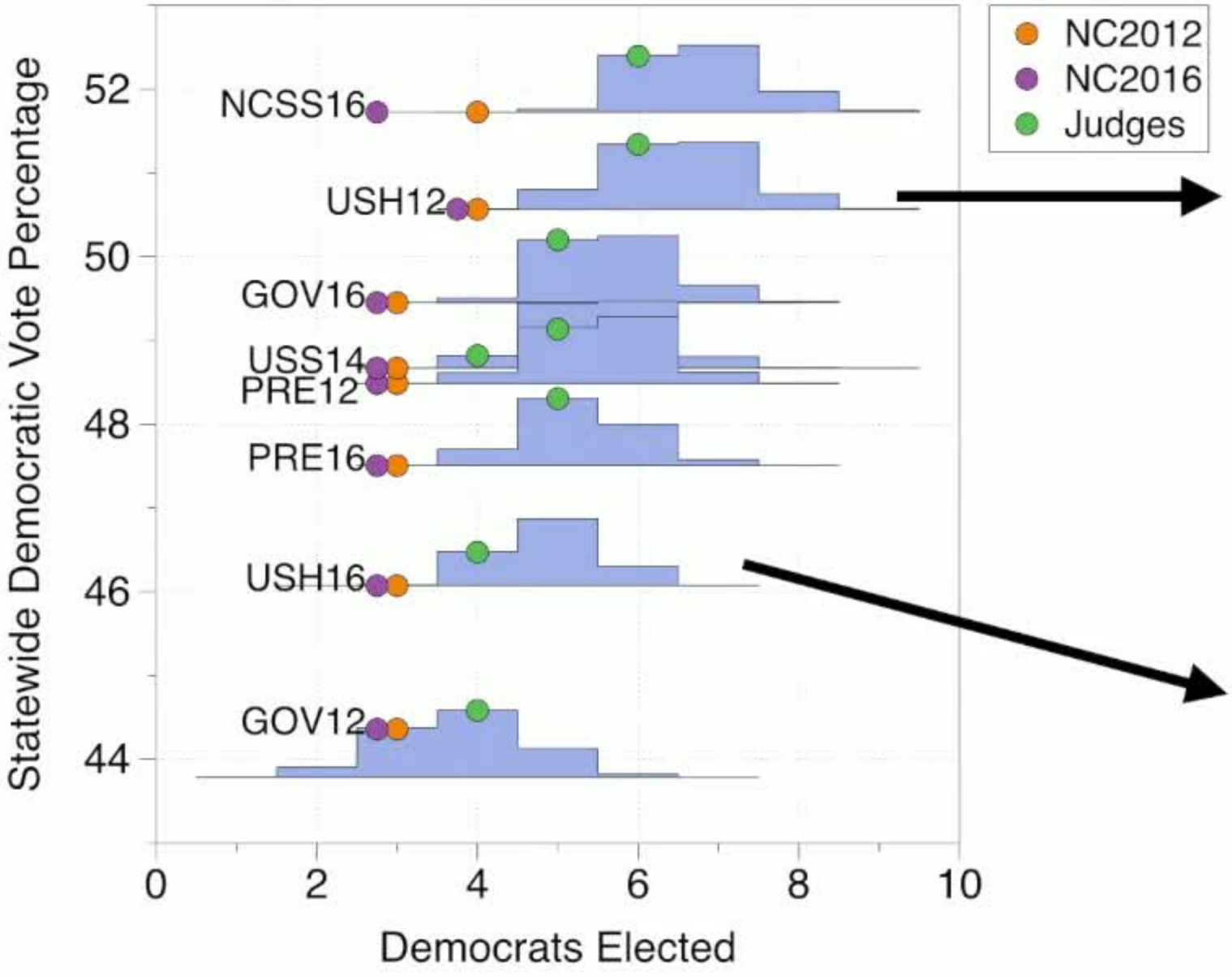


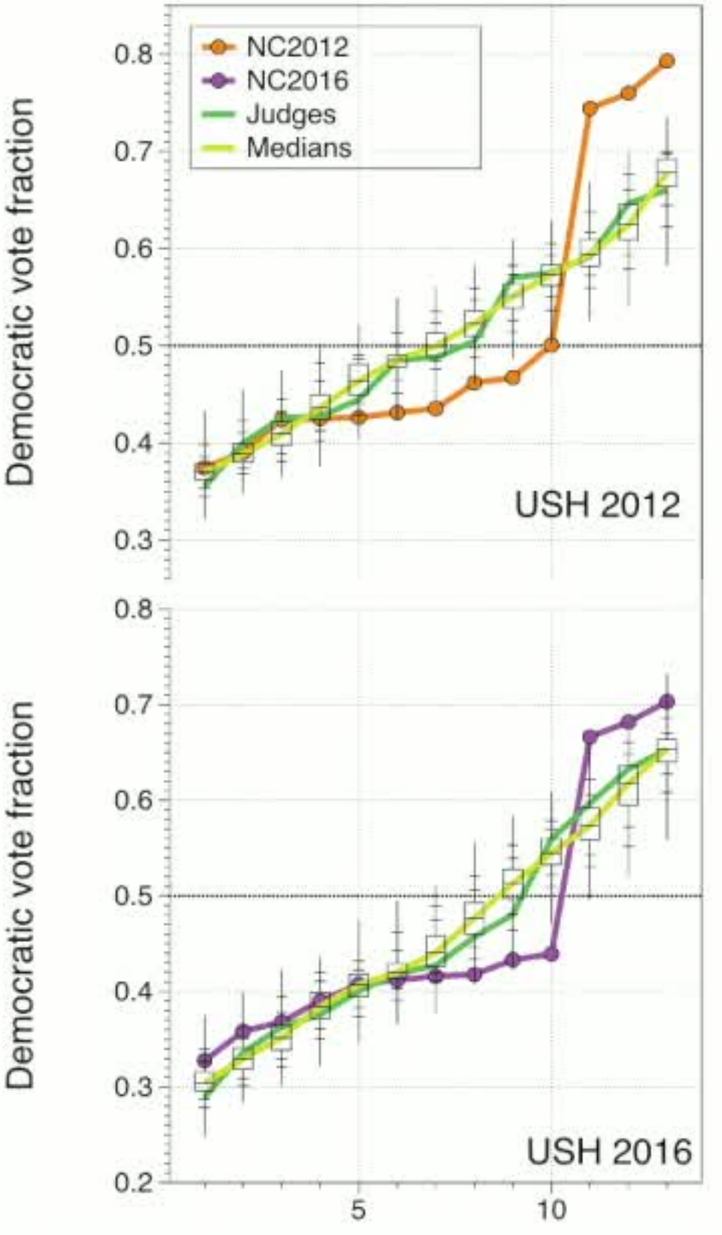


Two principle plots presented in Common Cause v. Rucho

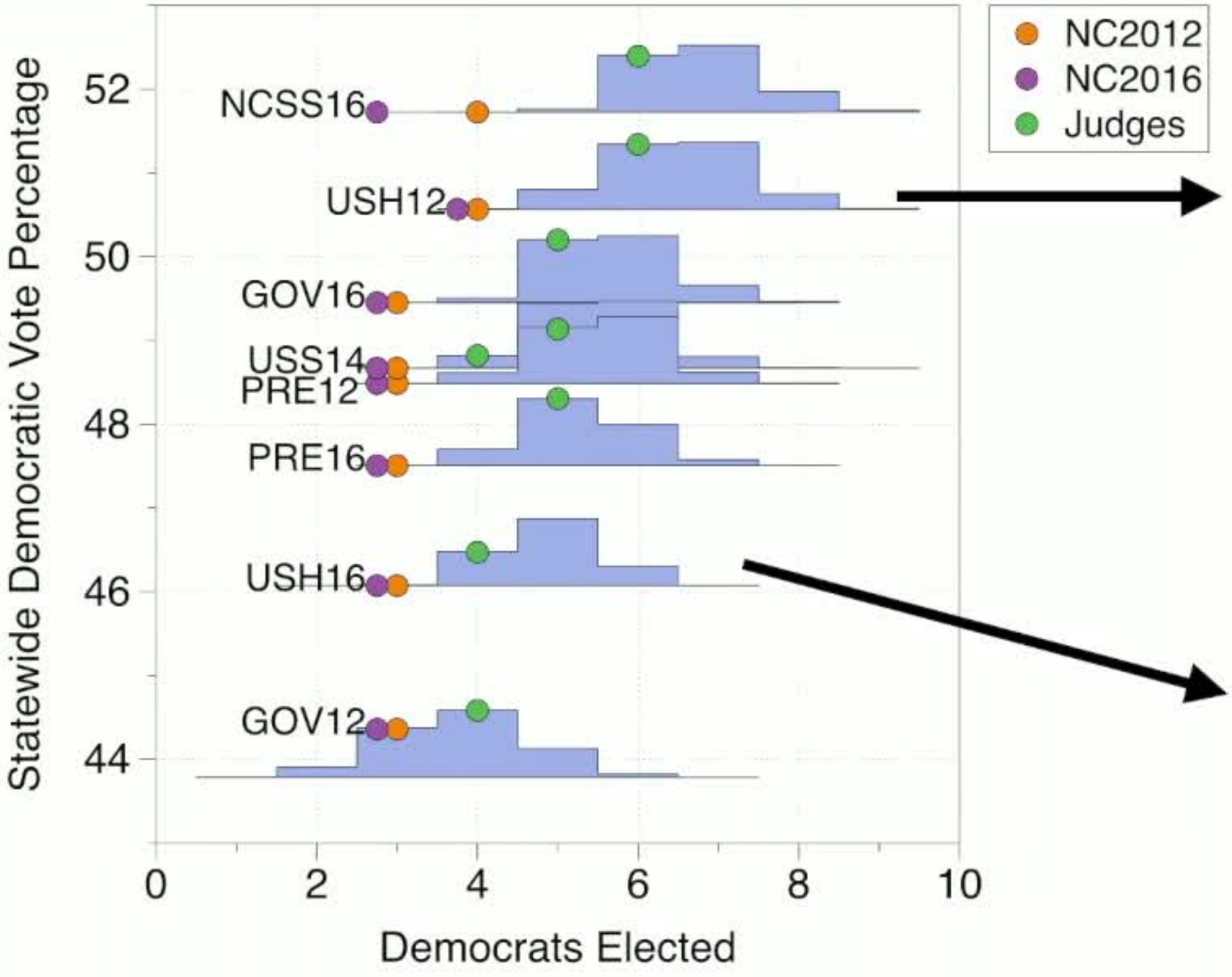
Identify Cracked and Packed districts

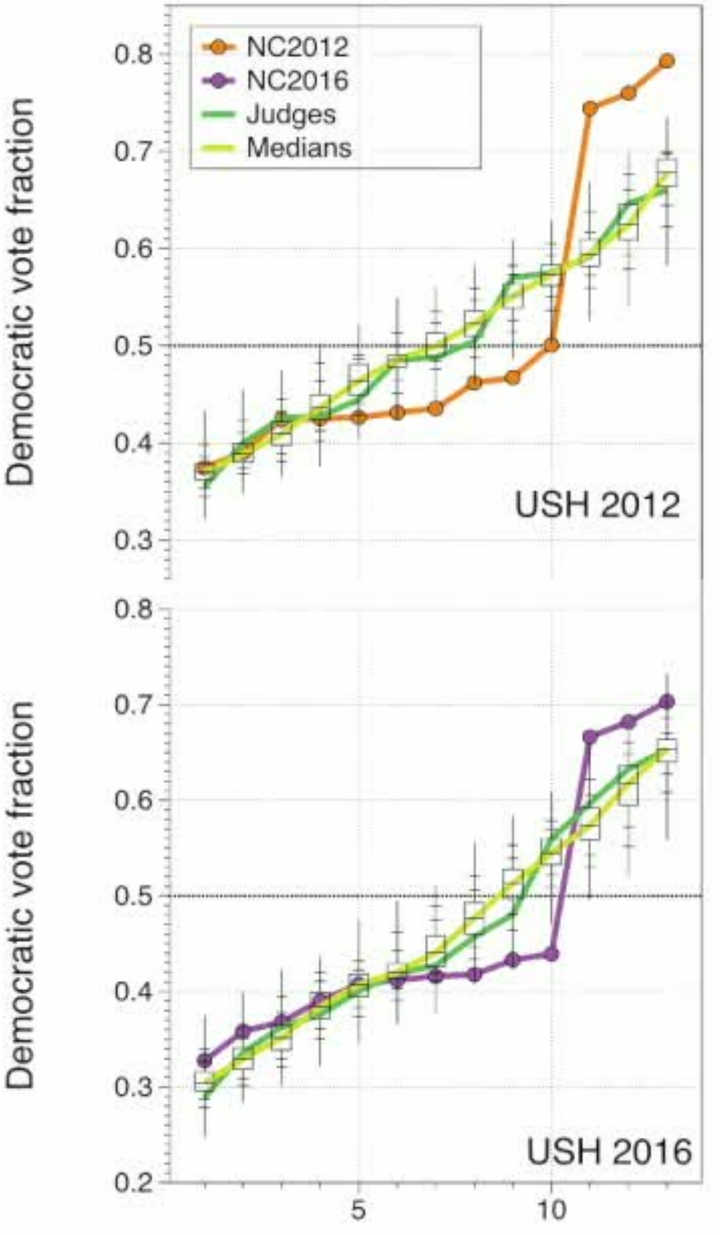
Stagnating election results





Stagnating election results





Gerrymandering as strange results

NC : US House 2012

	Vote	Seats
Democratic	50.65%	4 (31%)
Republican	48.80%	9 (69%)

WI : Gen Assembly 2014

	Vote	Seats
Democratic	51.28%	36 (36%)
Republican	48.72%	63 (64%)

Back to “startling” results

Gerrymandering as strange results

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	Vote	Seats
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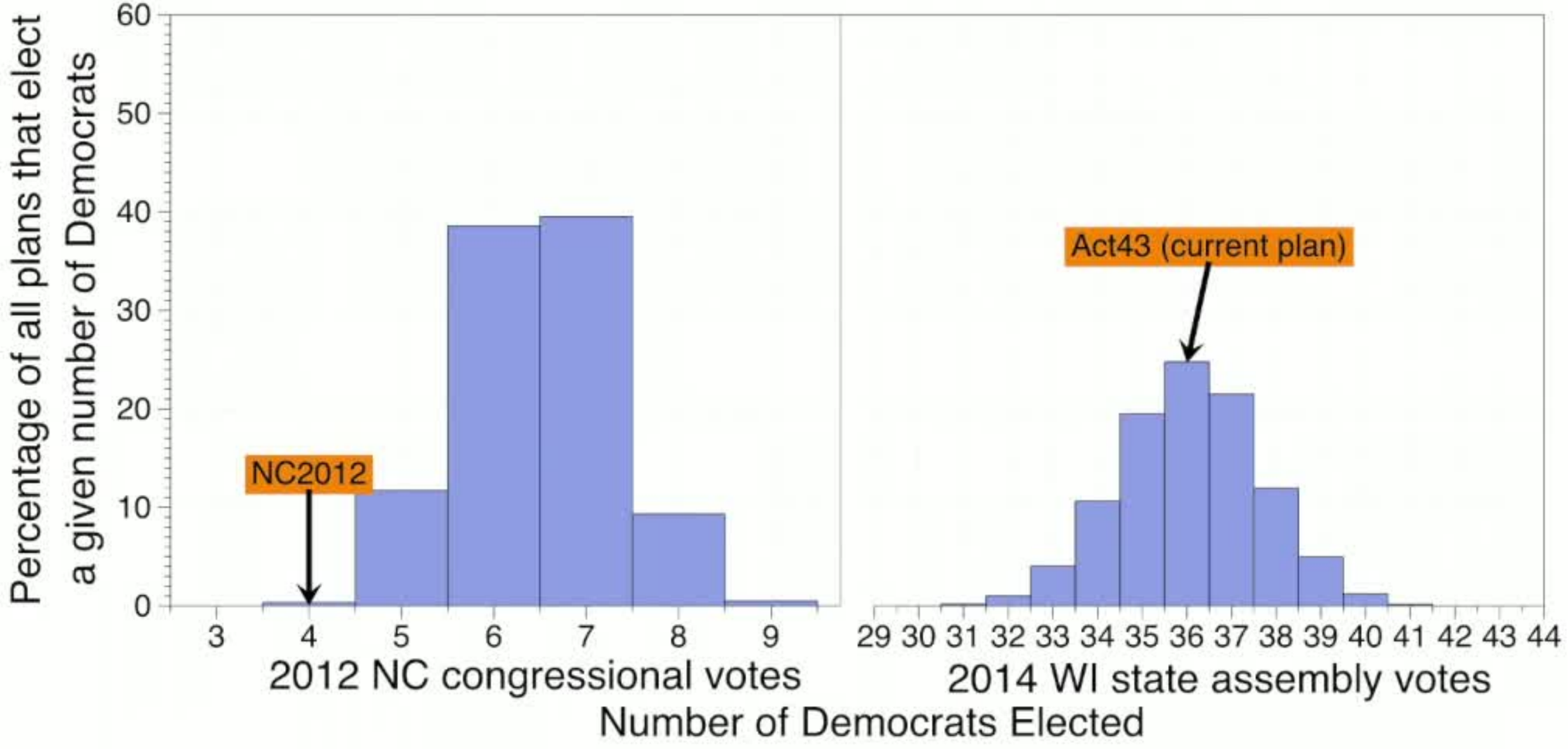
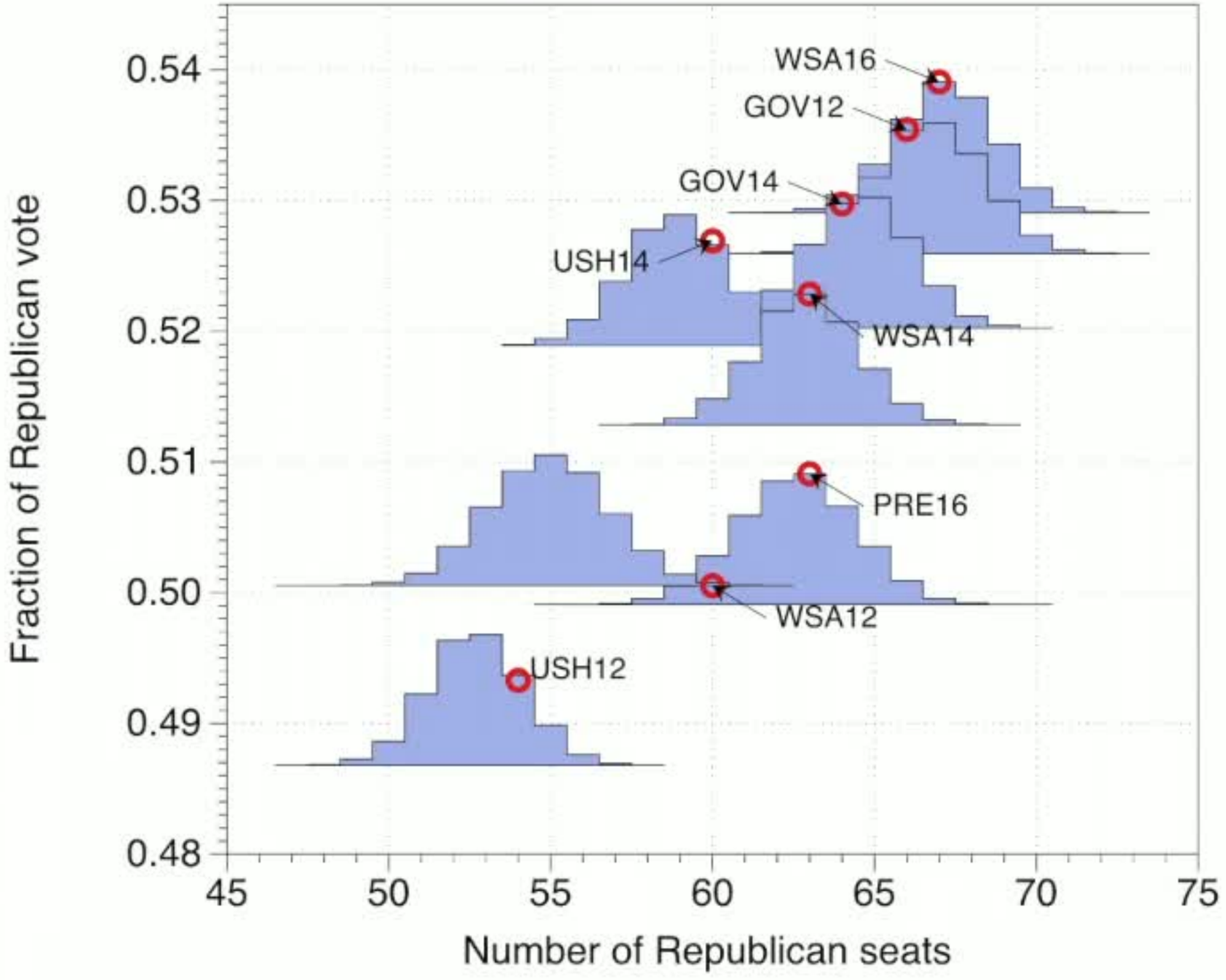
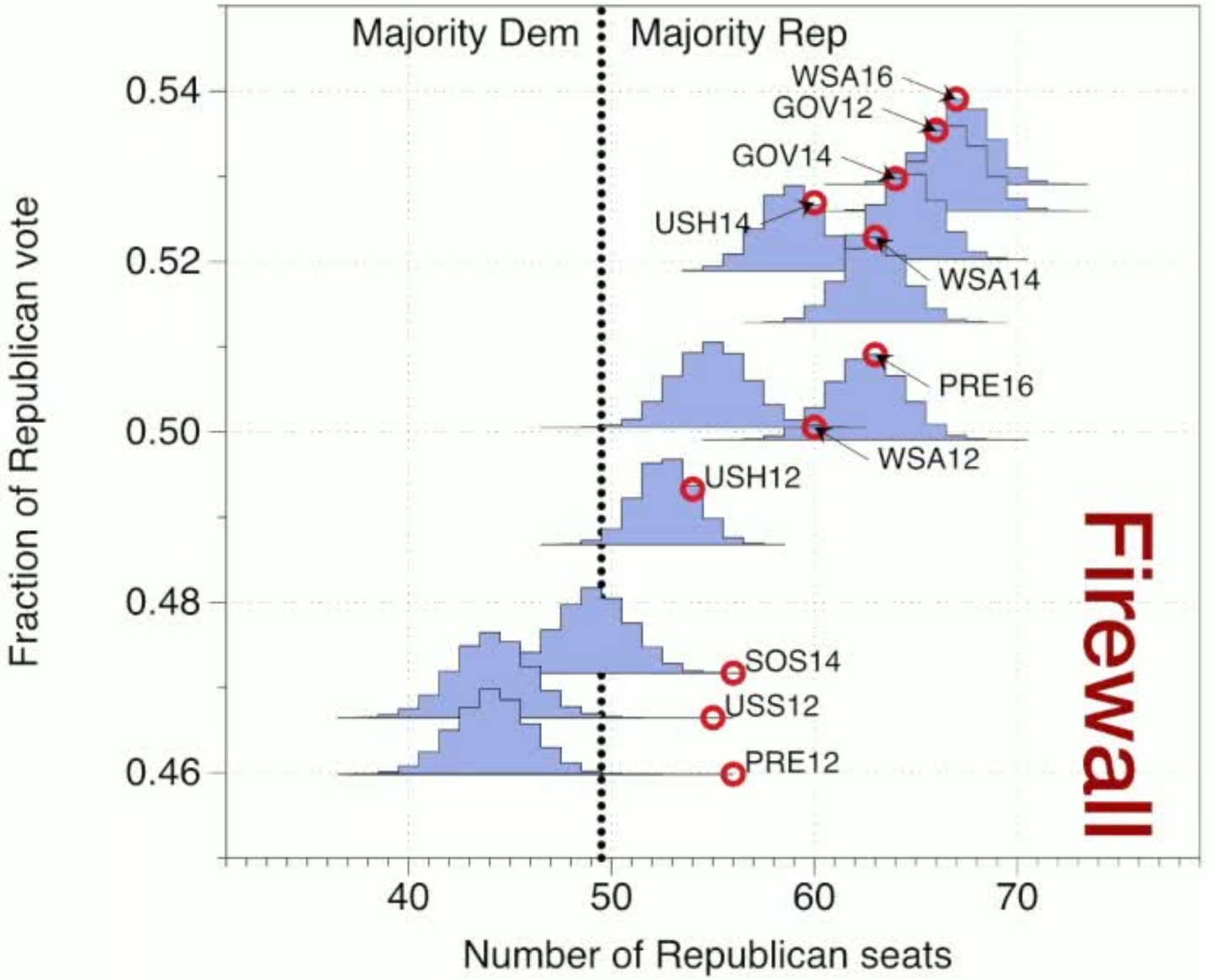
Surprising election results do not necessarily imply Gerrymandering



Wisconsin result is robust over many elections

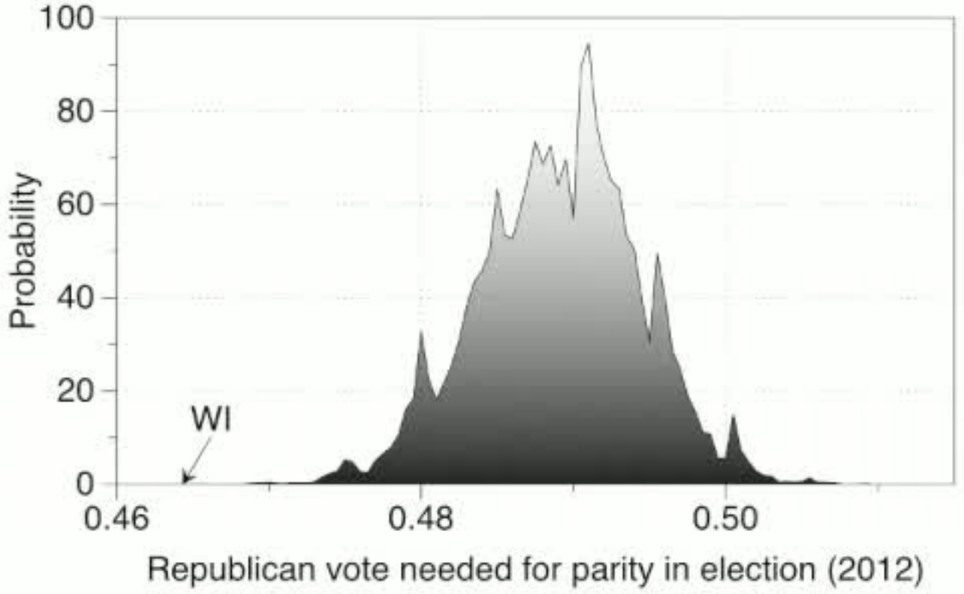


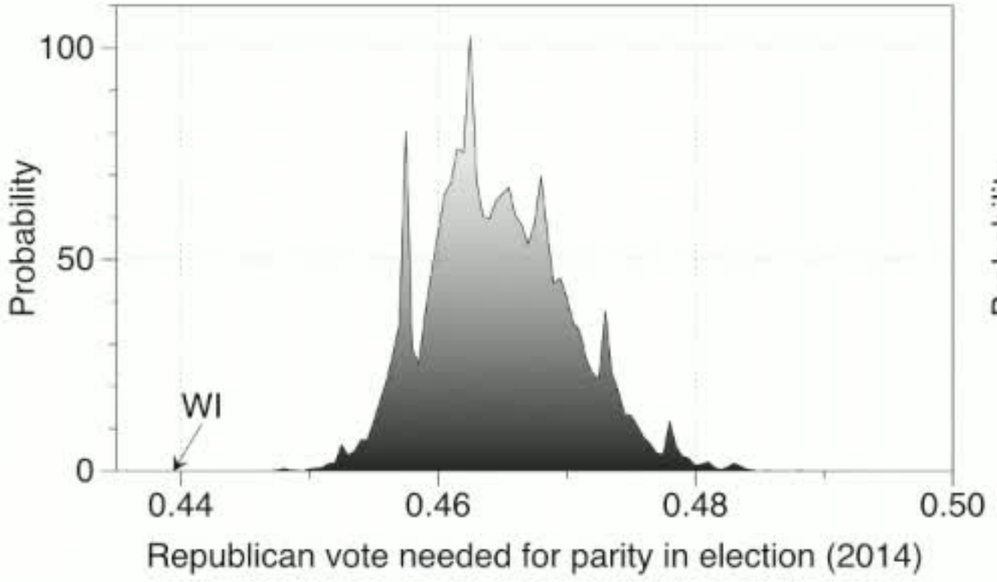
Wisconsin historical elections

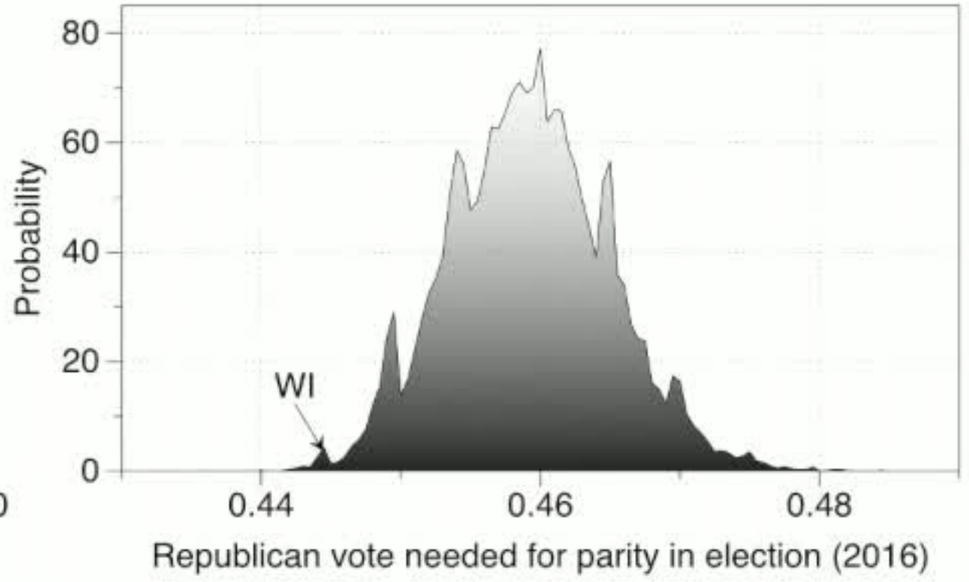


Probe Structural Bias

Structural advantage exists; sampling decouples geopolitical effects from Gerrymandered effects

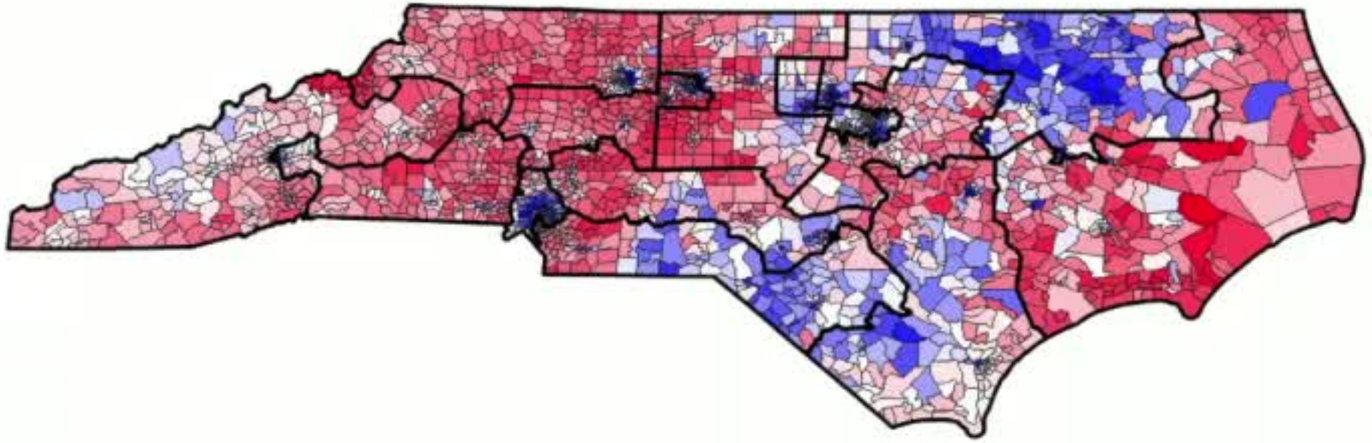




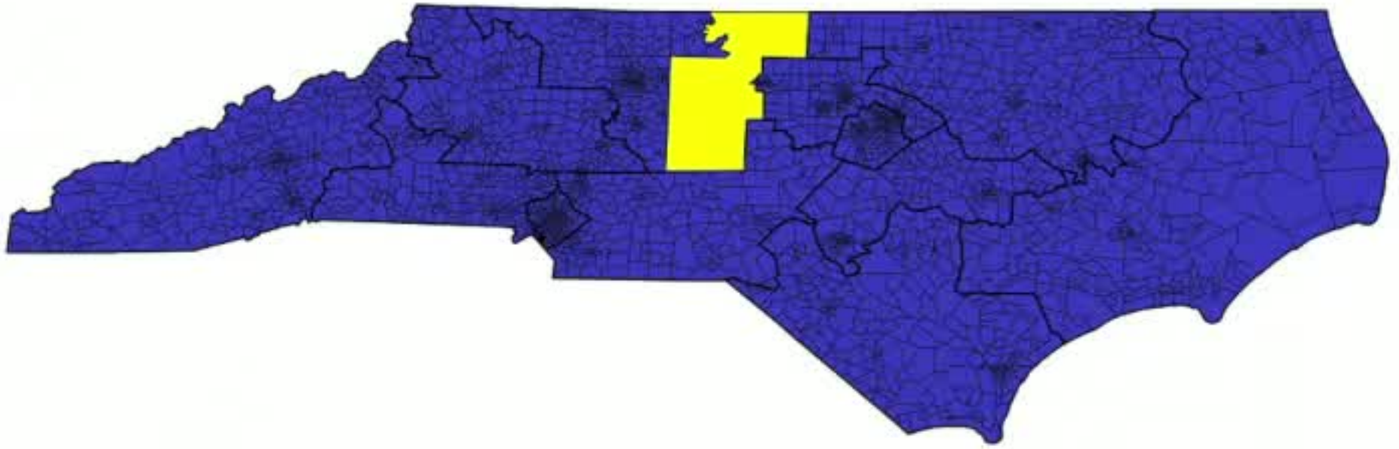


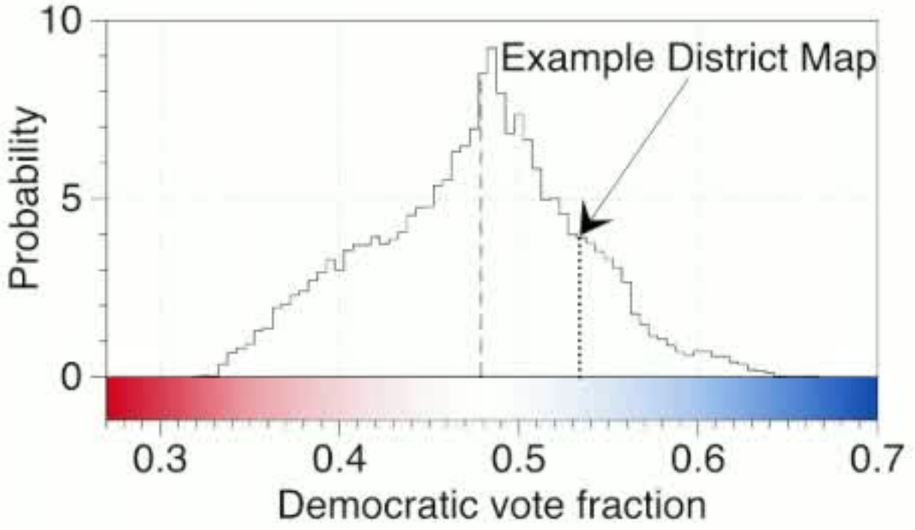
Measuring Localized Harm

Precinct Level Analysis

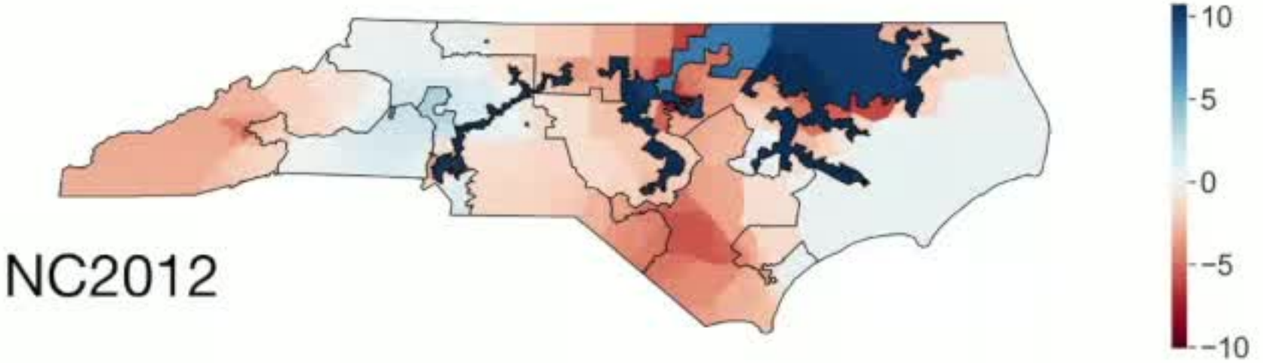


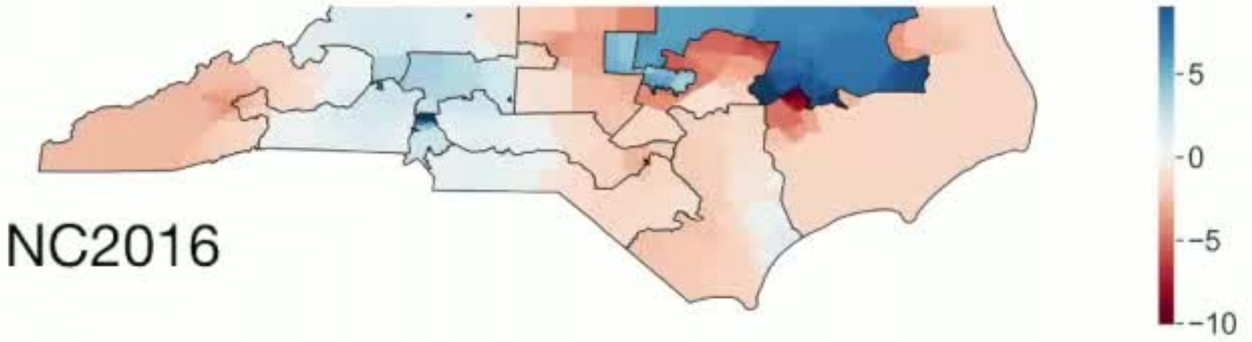
vote fraction at predict level

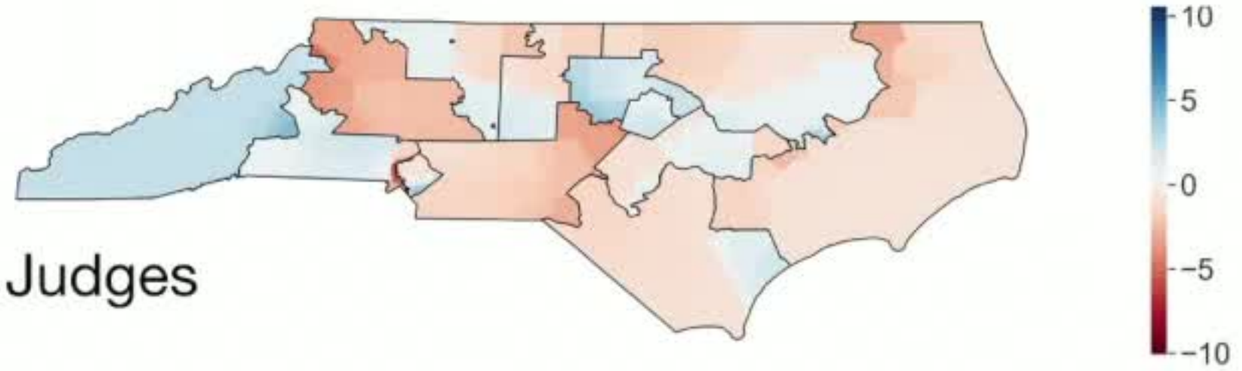




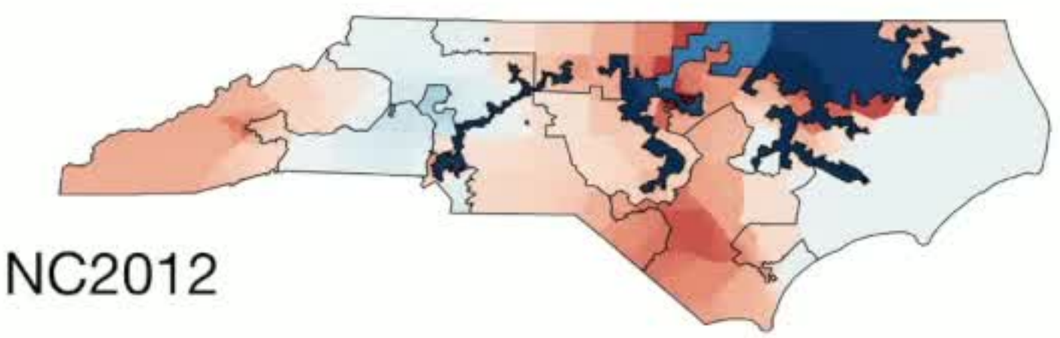
Which Precincts are Outliers?





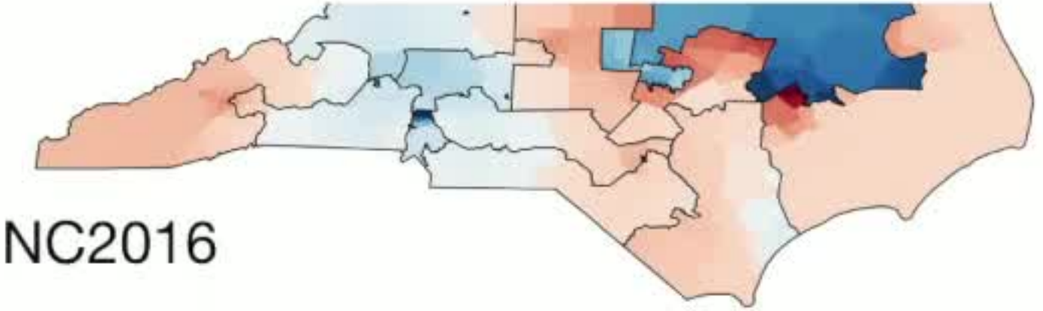


Which Precincts are Outliers?



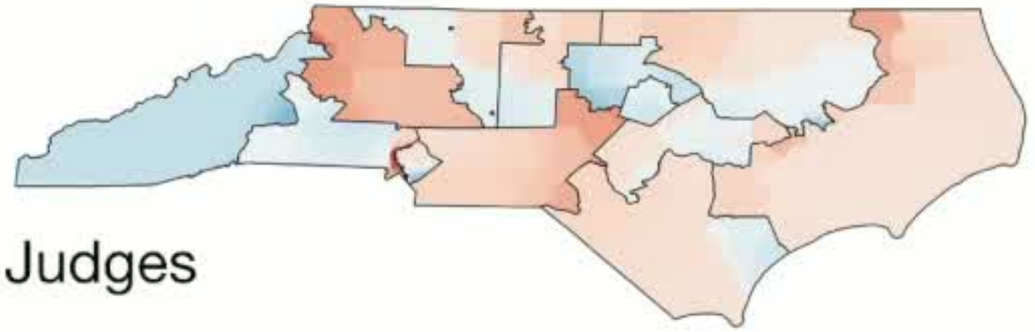
NC2012

NC2012



NC2016

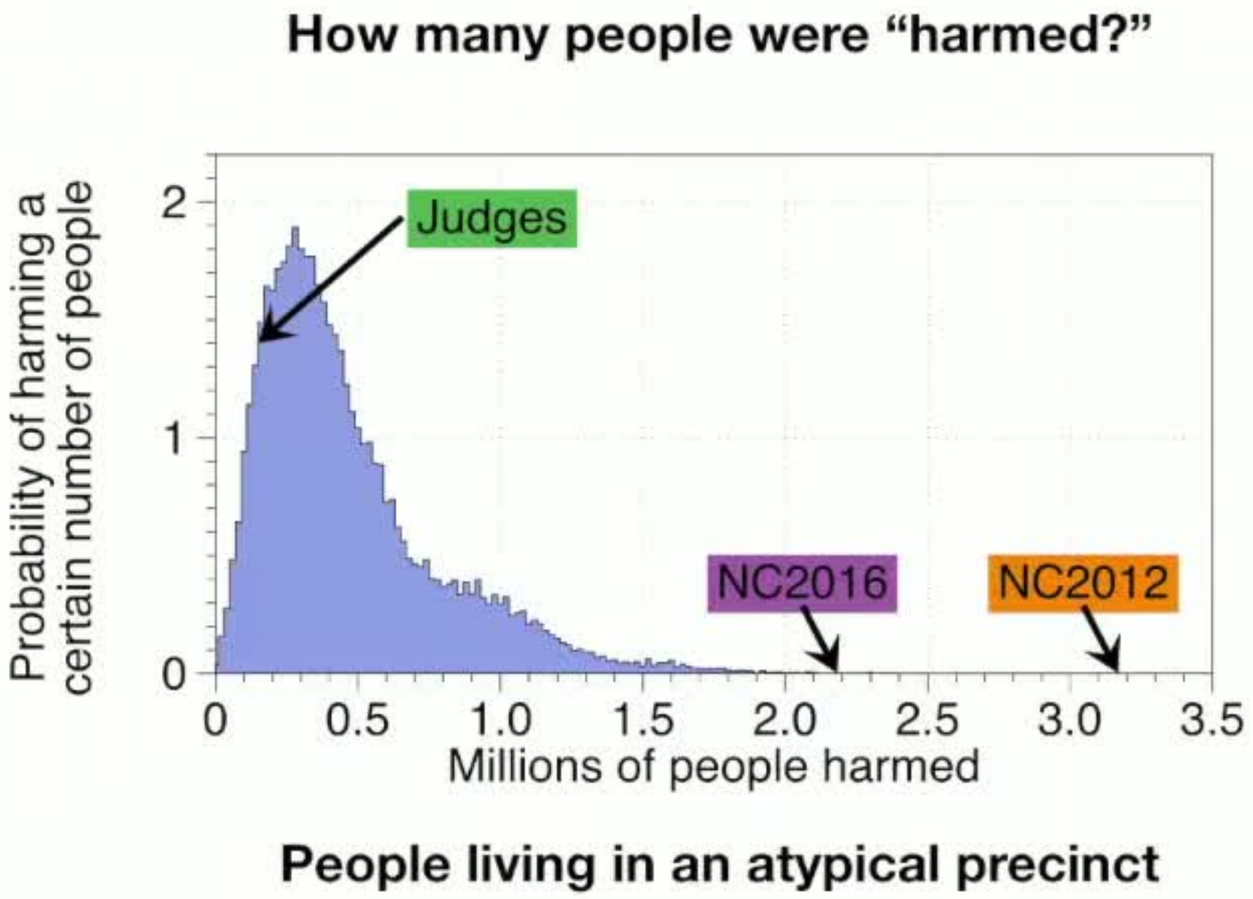
NC2016

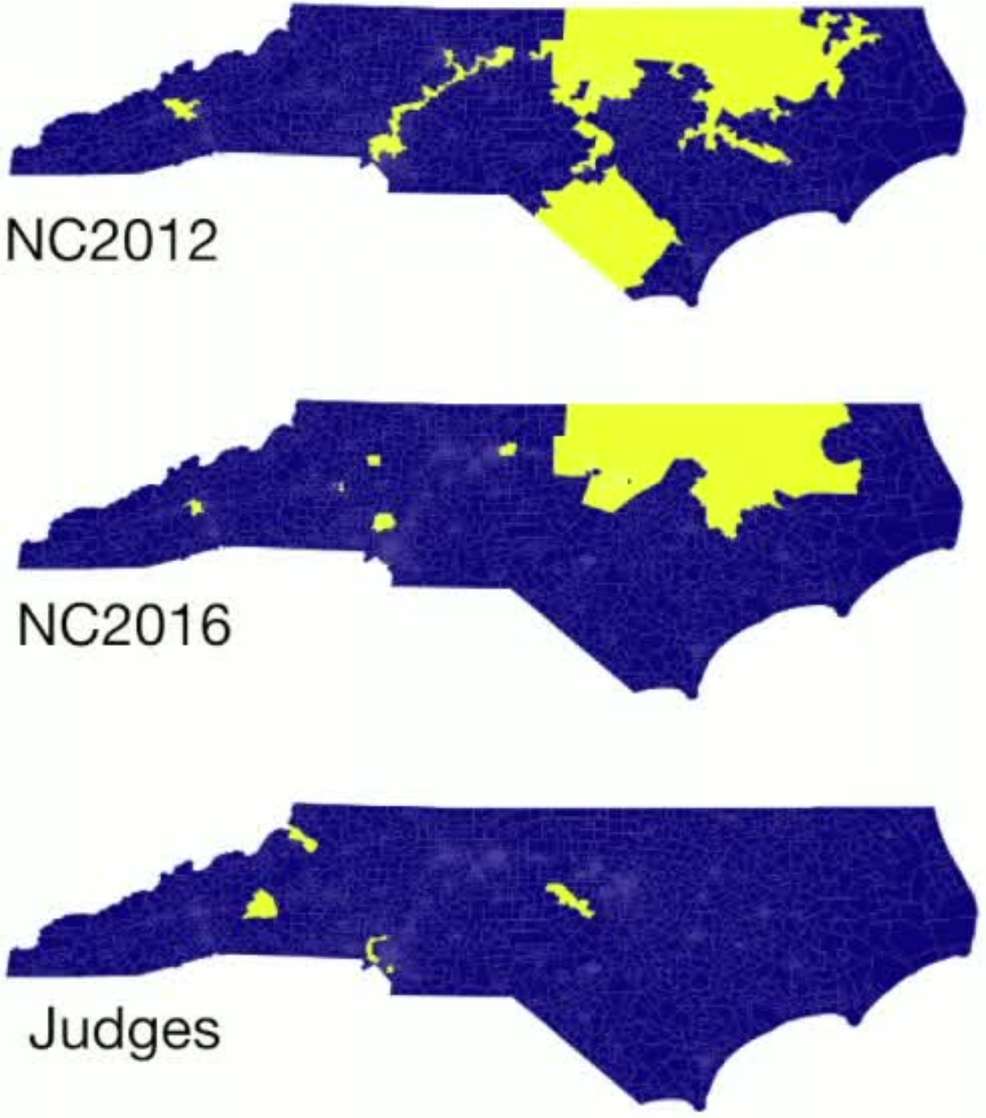


Judges

Judges

How to discuss the overall amount of harm





Groups using algorithmic generated maps to benchmark

Institution/Group	(Some of) the Members	Methodology
Duke	Greg Herschlog, JCM	MCMC; Simulated annealing on large spaces
Princeton	Kosuke Imai, Benjamin Fifield, Sam Wang, Will Alder	MCMC; Simulated/parallel tempering on small spaces
Carnegie Mellon/Pitt	Wes Pegden, Alan Frieze, Maria Chikina	MCMC; localized analysis at constant temp
Tufts	Daryl Dedford, Moon Duchin	MCMC; Promising work to study a uniform measure on compliant plans (split and merge)
Michigan/Stanford	Jowei Chen, Jonathan Rodden	Constructive algorithms (flood/conquer)
SUNY Binghamton	Daniel Magleby, Robin Best, Michael McDonald	Constructive algorithms (graph partitioning)
University of Illinois at Urbana-Champaign	Wendy Tam Cho, Yan Y. Liu	Constructive algorithms (conquer with genetic mixing)

On going work

work on samplers

Implementing tempered heat bath proposals (GH, MB)
More efficient proposals, parallel, simulated tempering

Split and Merge Proposals (See Duchin et al)
(with NCSSM)

Implementing Non reversible samplers
(GH, Matthias Sachs)

- a generalized skew-detailed balance structure (Diaconis , Holmes , Neal)
- A sense of momentum allows more systematic progress

theoretical models

Models of Natural Packing (City/Rural)

Generative models for box-plot structure

National Effect of Gerrymandering

Non-Partisan Team of Students

Christy Vaughn Graves (UG; 2013-2016)
Sachet Bangia (UG; 2016-2017)
Sophie Guo (UG; 2016)
Bridget Duo (UG; 2016)
Hansung Kang (UG; 2016-2017)
Justin Luo (UG; 2016-2017)
Michael Kepler (MS; 2018)
Sam Eure (UG; 2018-Present)
Rahul Ramesh (UG; 2018-Present)
Lisa Lebovich (MS; 2018-Present)
Mike Bell (GS; 2017-Present)
Robert Ravier (GS; 2016-Present)
Andrew Chin (2018-Present)
Gregory Herschlag (2016-Present)
Jonathan Mattingly (2013-Present)

Plus 16 new members in a research based undergraduate course

Collaborators

Guy-Uriel Charles
Janice McCarthy
Lydia Kwee
Colin Rundel
Adam Graham-Squire
Stephen Schecter

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Blog: https://sites.duke.edu/quantifyinggerrymandering/

Possible ideas to improve sampling

Many existing MCMC methods used in a variety of applications
From glass tempering, to molecular dynamics, to drug design, to image processing, and more

- Parallel tempering
- Hierarchical sampling
- Cluster proposals
- Modifications/variations on any of the above
- And much more…

Constitutional Arguments

- **14th Amendment** : Equal protection under the law
- **1st Amendment** : Protection against retaliation due to political speech (ie voting)
- No Question of intent in NC case.

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