Why Do Math?

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Three Interpretations of Whydomath

- Personal: Why do we do math?

  SIAM provides infrastructure — through publications and conferences — to carry out research projects

- Societal: Government supports math sciences: Why?

  SIAM has an extensive Washington advocacy role

- Educational: Why should students study math?

  SIAM runs math competitions for high school students
  supports student chapters
  has a careers section
  has an Education Committee

Personal

- Adrenaline rush of the light bulb effect

  It's addictive

- Unexpected relationship of abstract with applied

  - Symmetric Chaos

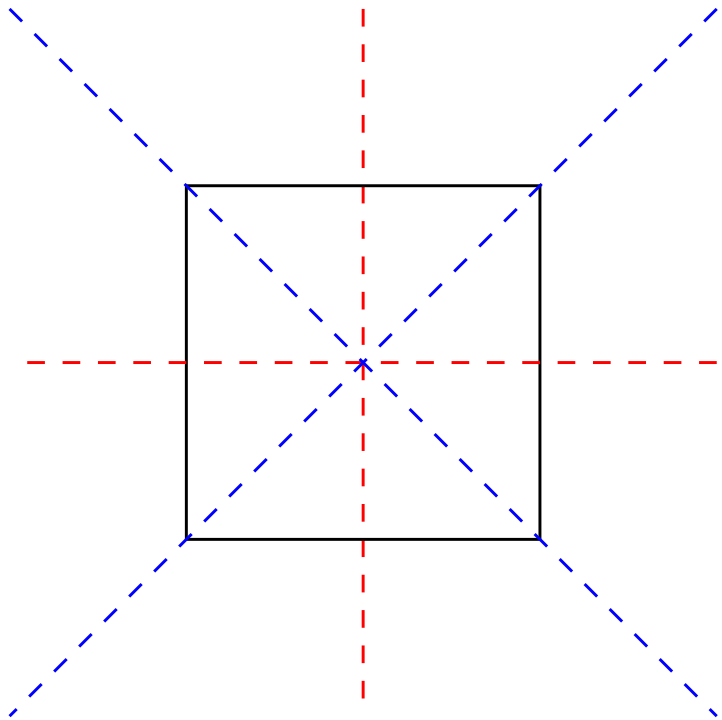
    Pascal Chossat (Luminy)
    Michael Dellnitz (Paderborn)
    Mike Field (Houston)
    Ian Melbourne (Surrey)

Planar Dynamics; Symmetric Chaos

- Let $f : \mathbf{R}^2 \to \mathbf{R}^2$
- Choose a point z_0 in the plane
- Let $z_1 = f(z_0)$, $z_2 = f(z_1)$, $z_3 = f(z_2)$ ...
- View attractors by throwing away transients
- What kinds of attractors appear for symmetric f?

Finite Symmetries on the Plane

- $\mathrm{D}_3 =$ symmetries of an equilateral triangle
  $\mathrm{D}_4 =$ symmetries of a square
  $\mathrm{D}_5 =$ symmetries of a regular pentagon



Four Rotations Four Reflections

Symmetric Maps

- A planar map has $\mathbf{D}_m$ symmetry if for every $g \in \mathbf{D}_m$

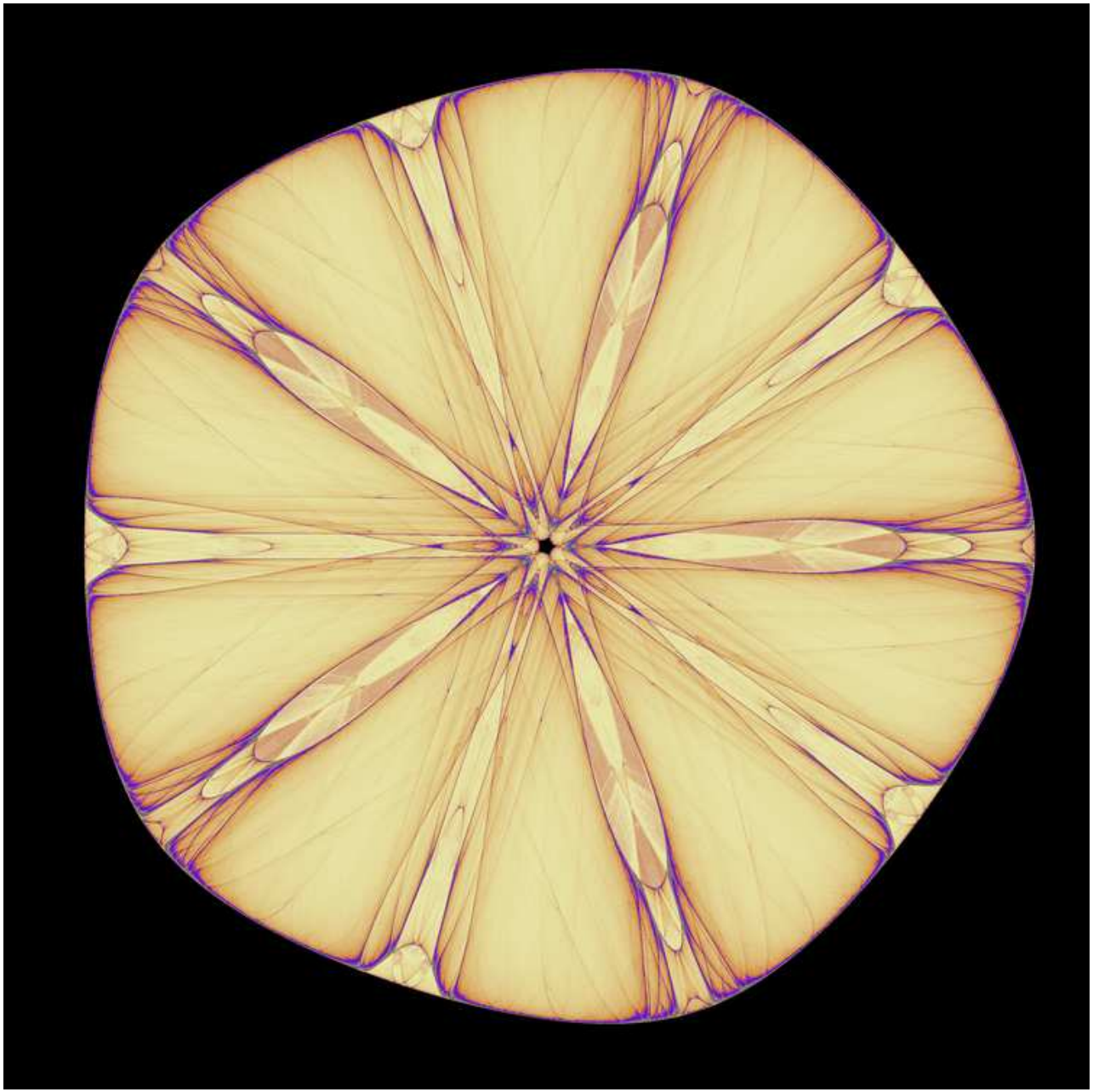
$$f(gz) = gf(z) \qquad z \in \mathbf{C} \cong \mathbf{R}^2$$

- Example of map with $\mathbf{D}_m$-symmetry is

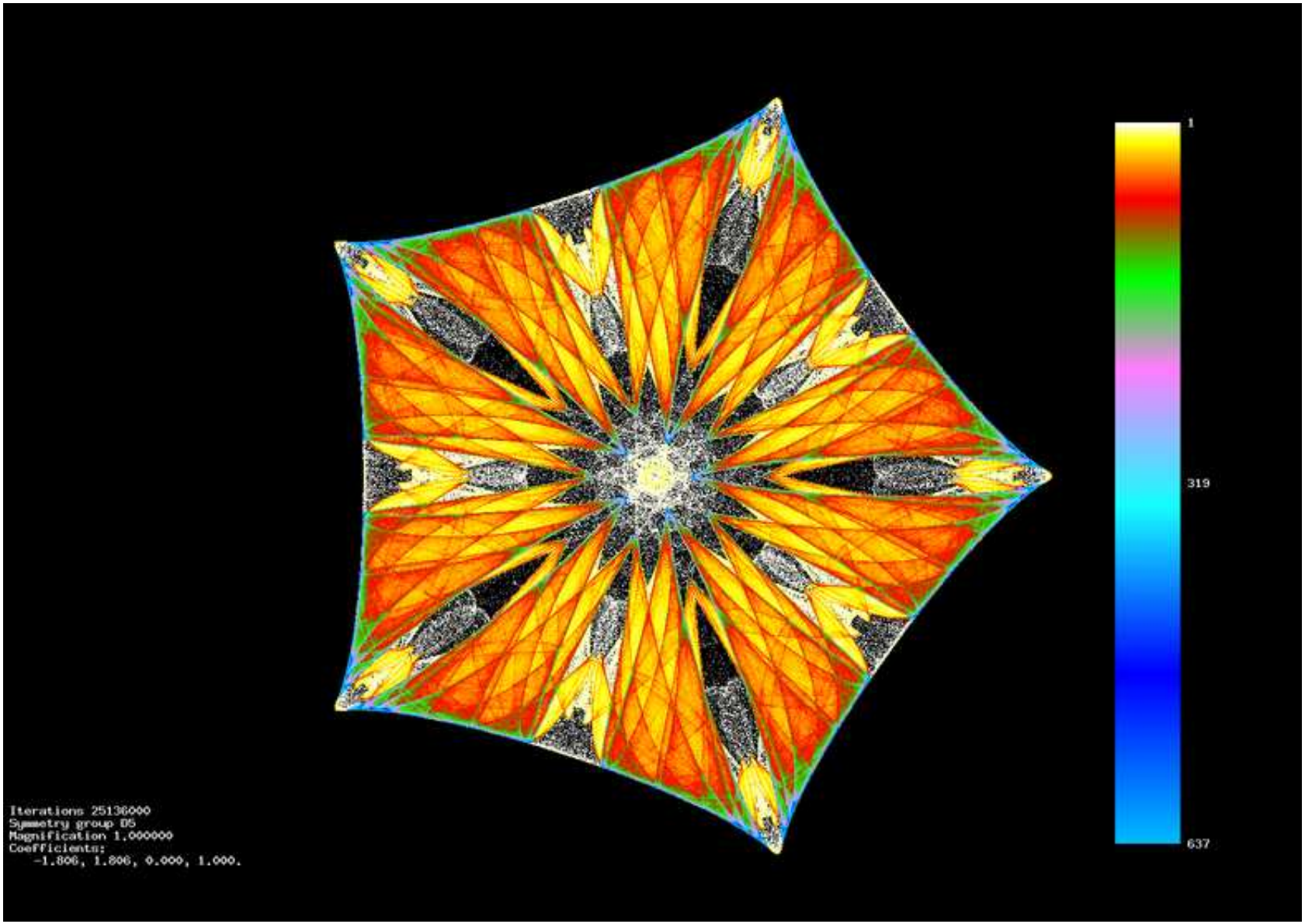
$$\mathbf{f(z)} = (\lambda + \alpha \mathbf{z}\bar{\mathbf{z}} + \beta \mathrm{Re}(\mathbf{z}^{\mathbf{m}}))\,\mathbf{z} + \gamma \bar{\mathbf{z}}^{\mathbf{m-1}}$$

m	λ	α	β	γ
5	-2.34	2	0.2	0.1
5	-1.806	1.806	0	1
5	2.6	-2	0	-0.5

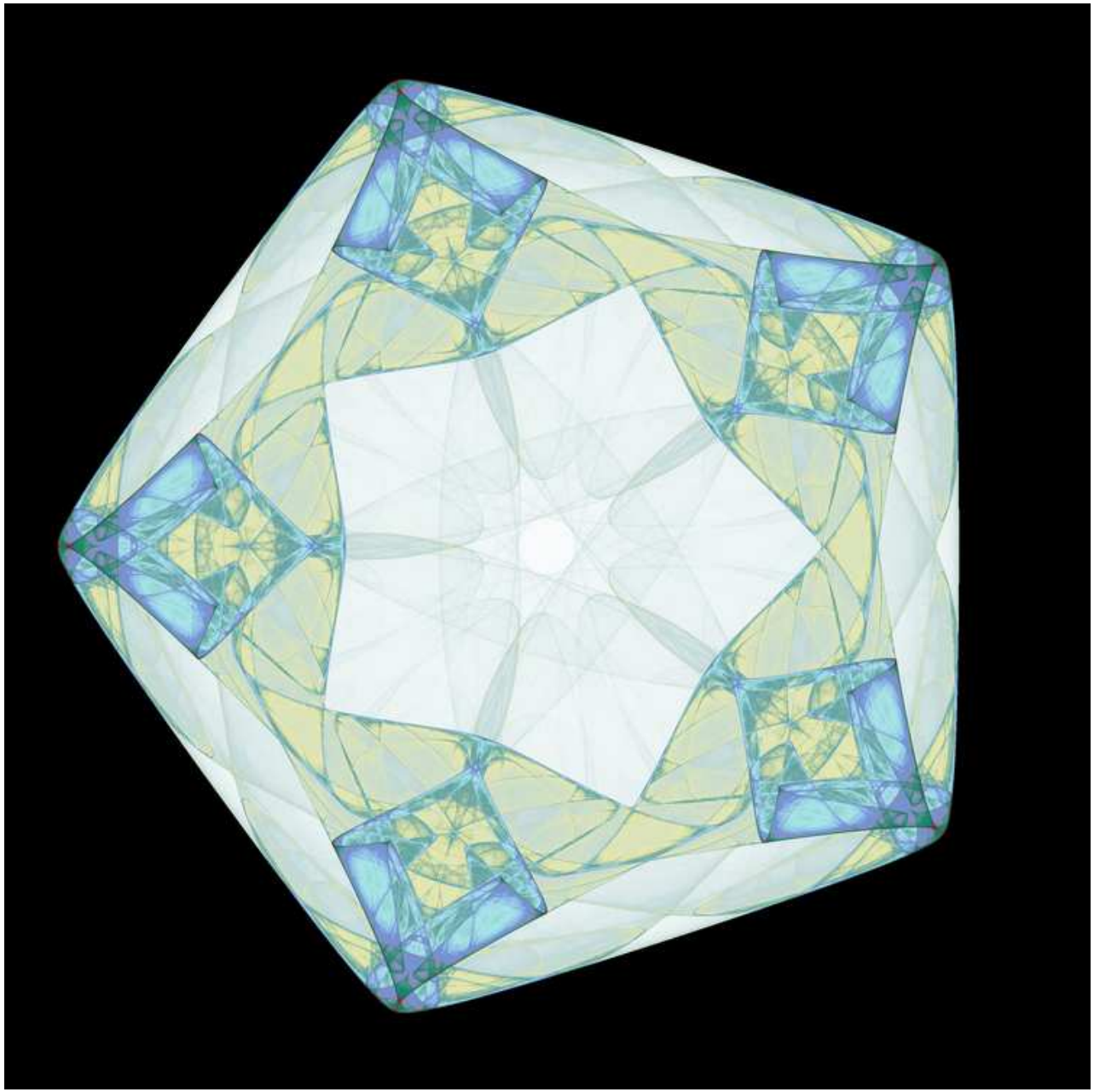
Sanddollar



Emperor’s Cloak



Pentagon Attractor



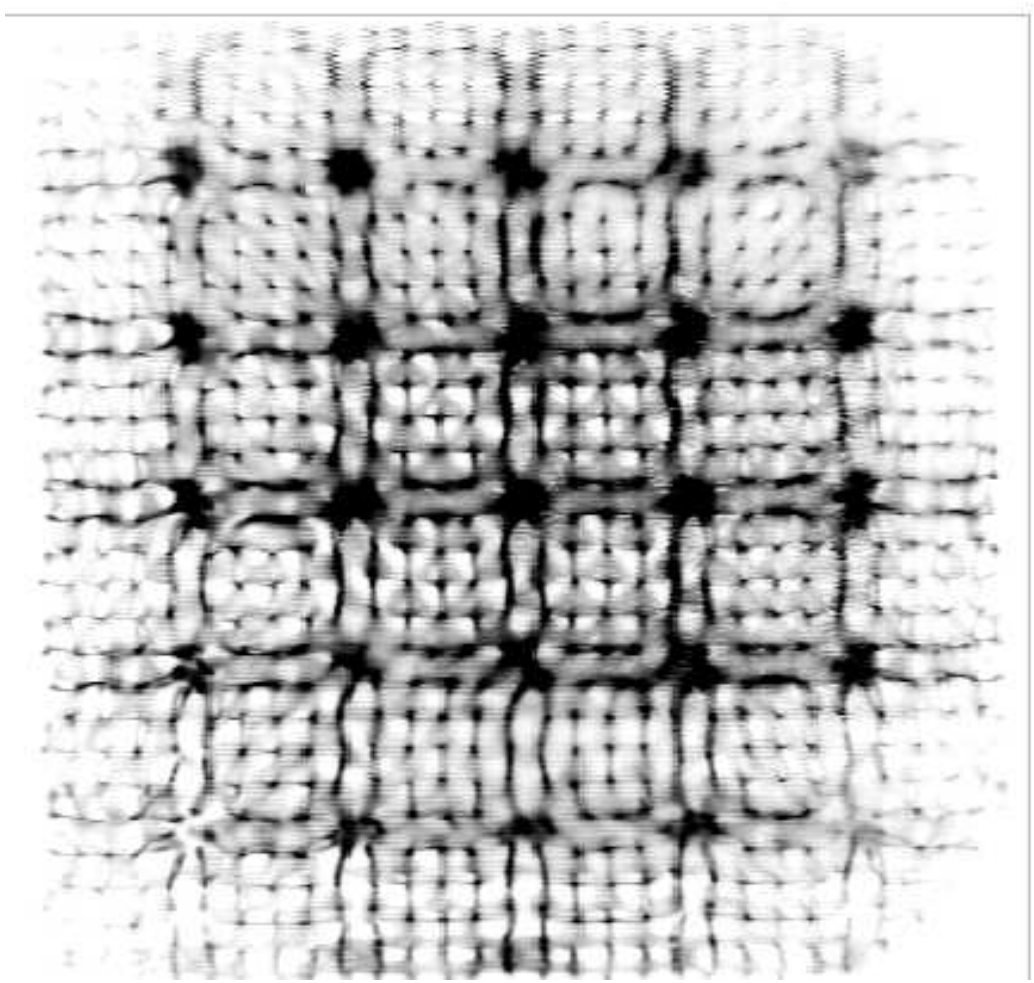
Consequence of Attractor Symmetry

- Let $u(x,t)$ be a time series
- Ergodic Theorem: Time Average = Space Average
- Time average has same symmetries as attractor
  - Let $U(x) =$ average of $u(x,t)$ over t
  - Then $U(\sigma x) = U(x)$ for every symmetry σ

Faraday Surface Wave Experiment

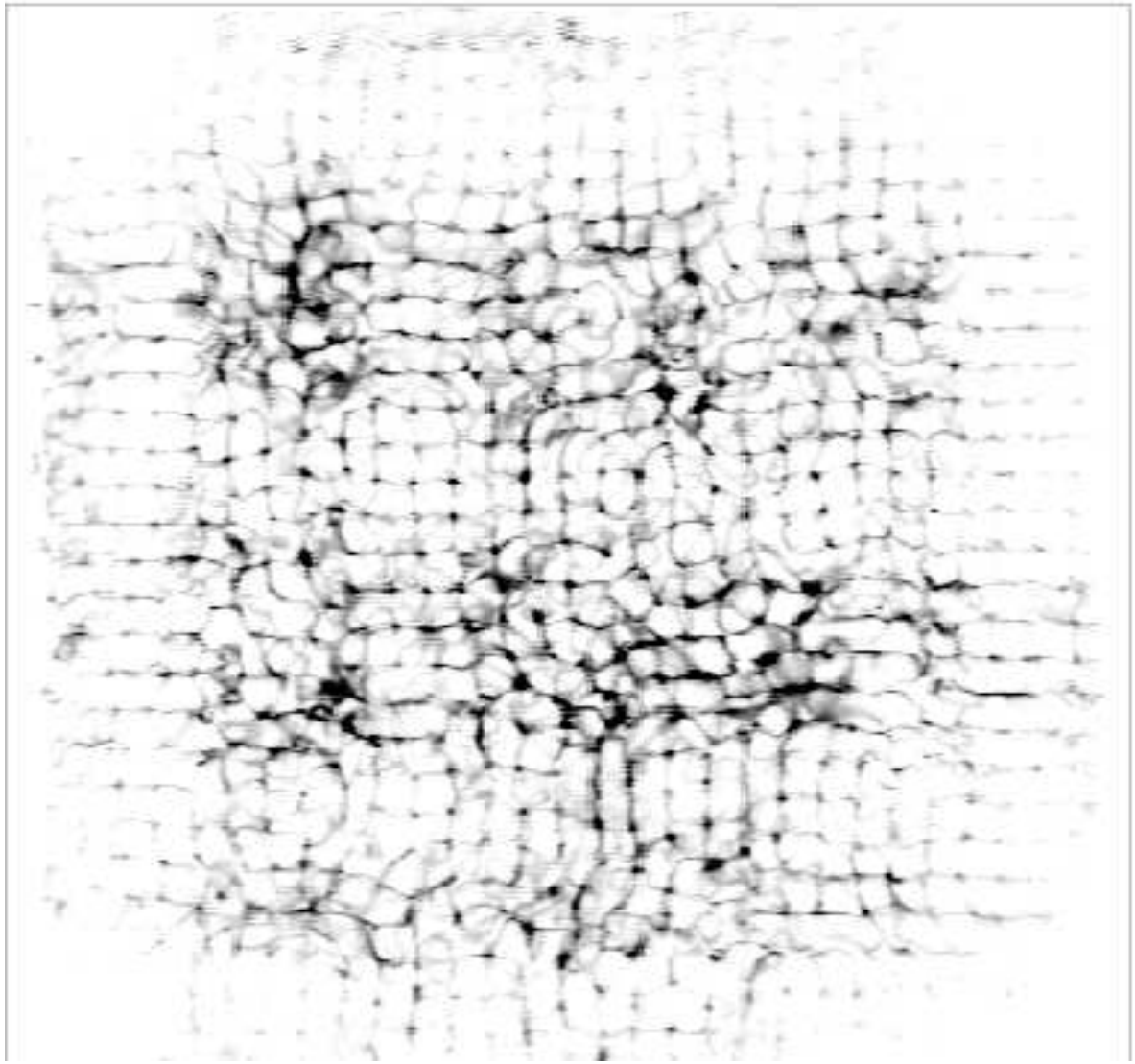
- Vibrate a fluid layer at fixed frequency and amplitude
- At small amplitude — surface is flat
- At large amplitudes — surface deforms
- Take picture at each period of forcing
- Light is transmitted through the fluid
  - **Dark areas**: surface is concave up
  - **Bright areas**: surface is concave down

The Faraday Experiment (2)



Gollub, Gluckman, Marcq, & Bridger (1993)

The Faraday Experiment (3)



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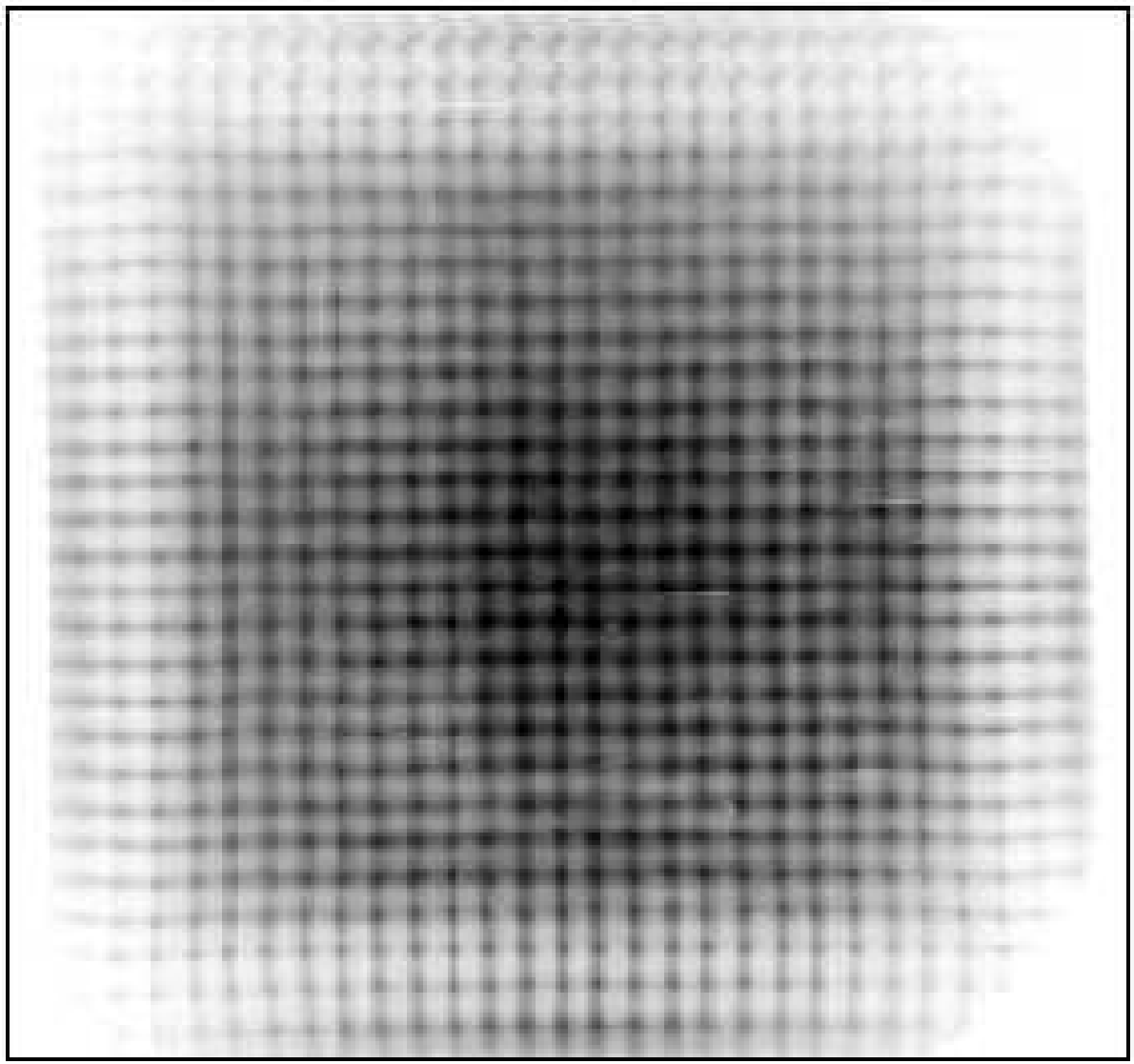
The Faraday Experiment (4)



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Societal: Why Fund Math?

- Education: Need for a technically trained work force
- Research: Has proved useful

NSF Mission Statement

. . . advance the national health, prosperity, and welfare, and secure the national defense by funding

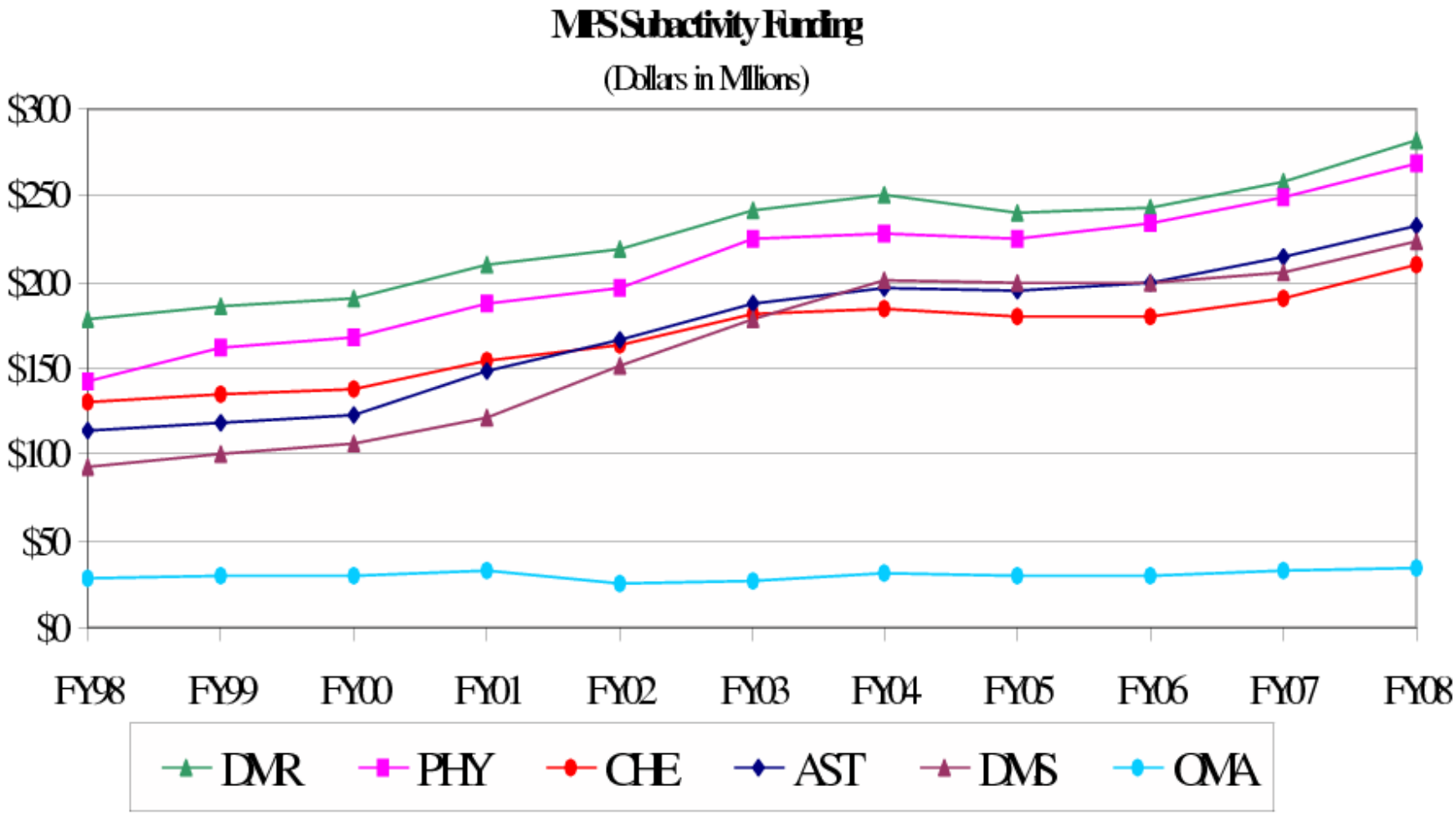
- basic and applied scientific research and research fundamental to the engineering process,
  . . .
- science and engineering education programs,
  . . .

Math Funding Data: FY2006

DoD			76.2
	AFOSR	32.1	
	ARO	10.0	
	ONR	13.6	
	DARPA	16.5	
	NSA	4.0	
DOE			29.4
NSF			199.3
NIH			79.1
	NIGMS	38.0	
	NIBIB	41.1	
TOTAL			$384M

NSF Funding

Ten-Year Funding History



Rising Above the Gathering Storm-2007

- As percentage of GDP federal funding of research in physical sciences was 45% less in 2004 than in 1976.
- Increase funding of long-term science and math basic research by 10% each year over the next 7 years.
- Increase number of US citizens who earn bachelor's degrees in science, engineering, and math.

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- American Competitiveness Initiative and America Competes Act respond to Gathering Storm
- The role of the mathematical sciences in competitiveness must be explained

Why Study Math?

- Number of U.S. citizens receiving degrees in math has decreased.

  Need for a technically trained workforce

- Why should students who do not want to teach math take math?

  How do we transmit the excitement & utility of math?

- Can most teachers in first year courses answer the question *Why study math?*

  Can we describe the breadth of careers that benefit from math?

http://dev.whydomath.org/

- A website of many nodes — each node a success story of mathematics or computational science
- Aimed at freshman/sophomore/popular science level
- Provide a community resource to help answer the questions “Why study math?” and “Why fund math?”
- Highlight the exciting and varied possibilities of mathematics and computational science

Features of Whydomath

- Nodes
  - Mathematics constructs interplanetary superhighway
  - Mathematics helps win America's Cup
  - Mathematics revolutionizes theoretical neuroscience
  - Mathematics helps save lives
  - Mathematics helps enable the internet
  - And much much more . . .
- Reading Room
  - Ian Stewart's math columns from *Scientific American*
  - Articles from *SIAM News*
- Links to Other Math Websites
- Links to Career Sites

New Nodes

- Nodes in preparation
  - JPEG and wavelets
  - Option pricing
  - Search engines, pagerank, and Google
- Nodes wish list
  - global positioning systems
  - human genome and the shotgun method
  - motion picture special effects
  - cryptography
  - crowd dynamics
  - car bodywork, design and manufacture
  - oil recovery

People Involved

Katherine Socha; Jessica Stephenson; Donna Witzleben

Chris Budd; John Burns; Rob Ghrist; Peter Turner

Mary Huang; Shannon Slaughter; Jonathan Holm

Dave Marshall; Michelle Montgomery; Ted Kull; Jim Crowley

Shane Ross; Brent Doiron; Eric Shea-Brown

Cathryn Mitchell; Mike Jones; Joe Skufca; Pat Van Fleet

Roger Lee; Ronnie Sircar; Gil Strang; Justin Court

Alfio Quarteroni; Nick Hieb; Ian Stewart; Barry Cipra

Whydomath is a Community Effort

- Do you have a success story to tell? Volunteer
- Are you an excellent writer? Volunteer
- Do you have excellent web skills? Volunteer
- Do you know good articles for Reading Room? Tell us
- CAVEAT: Whydomath nodes are history lessons—not grant proposals

Email: whydomath@siam.org