

A data-dependent weighted LASSO under Poisson noise

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Weighted
LASSO



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The LASSO for sparse inverse problems

The diagram illustrates the LASSO model for sparse inverse problems. It shows the equation $y = Ax + \varepsilon$. The vector y is $n \times 1$, the matrix A is $n \times p$, the vector x^* is $p \times 1$, and the error vector ε is $n \times 1$. The matrix A and vector x^* are shown with colored blocks representing non-zero elements, indicating sparsity.

The LASSO estimator:

$$\min_x \frac{1}{2} \|y - Ax\|^2 + \gamma \|x\|_1$$

Results

- ▶ If we have tight bounds on the backprojected residuals of the form

$$|\tilde{A}^\top (\tilde{y} - \tilde{A}x^*)|_k \leq d_k,$$

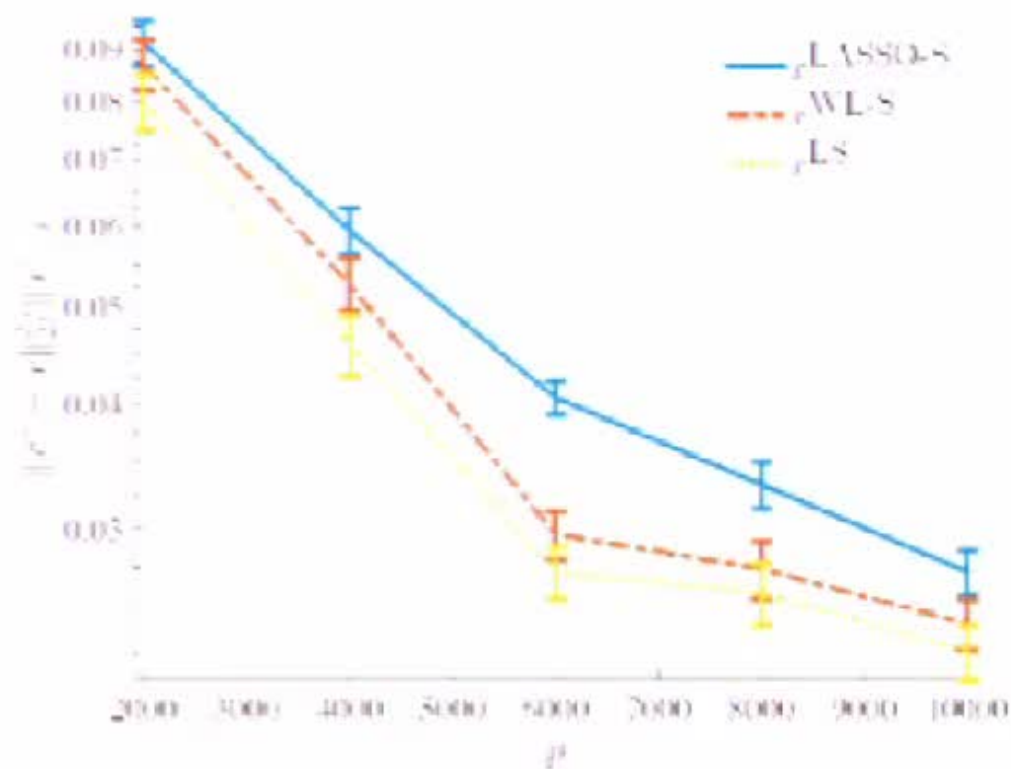
then using these weights within the weighted LASSO can perform nearly as well as a support oracle.



- ▶ When there is significant variation among the d_k s, the weighted LASSO can perform significantly better than the typical unweighted LASSO.
- ▶ We have examined two example systems:
 - ▶ **Bernoulli ensembles for compressive imaging:** we see similar rates as in earlier minimax analyses.
 - ▶ **Random convolution in genetic motif analysis:** earlier analyses could not address this!

Motif rate results

	Small m	Large m
LASSO	$\frac{s \ x^*\ _1 \log^3 p}{m}$	$\frac{s \ x^*\ _1 \log^2 p}{p}$
Weighted LASSO	$\frac{\ x^*\ _1 \log p}{m}$	$\frac{s \ x^*\ _1 \log^2 p}{p}$



m is the number of parent events (blue dots)

$$m \propto \sqrt{p} \log p$$