Topological Data Analysis of Stochastic Collective Motion

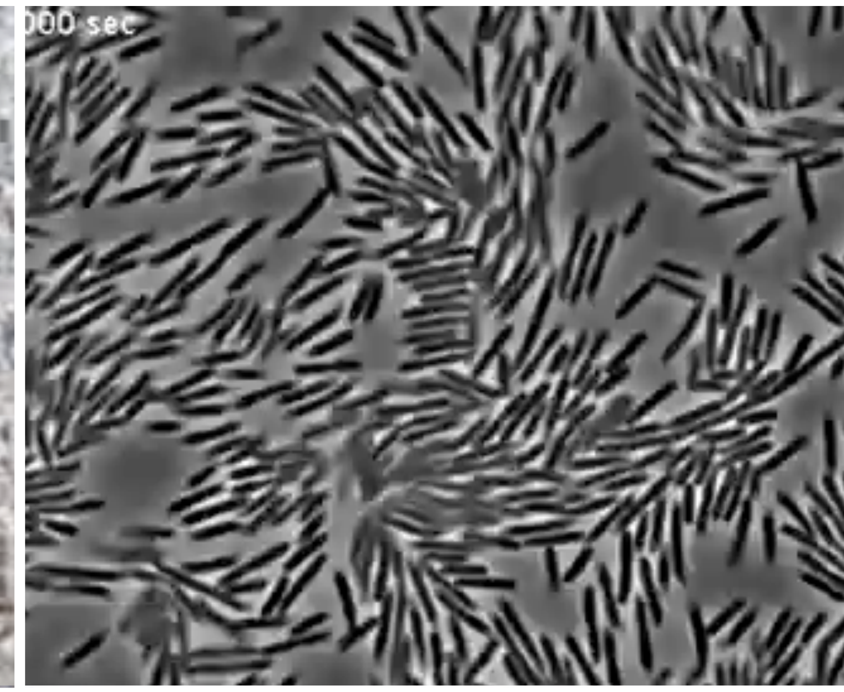
Chad Topaz (Williams College)
Lori Ziegelmeier, Tom Halverson (Macalester College)
NSF DMS-1412674

Main po
this tutorial

- Topological data analysis (for computing and describ
- TDA of time series aids th large data sets arising from collective motion
- Topological time series of collective motion models have a coherent average

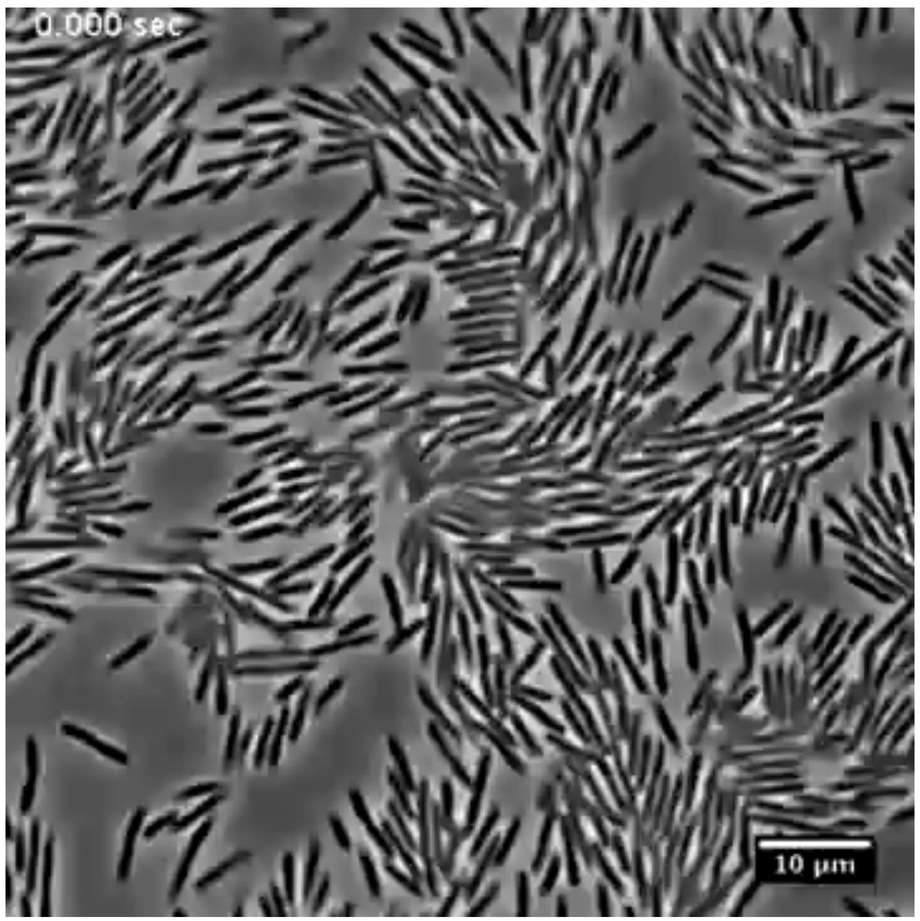
Collective motion occurs across the natural world.






Quantifying group dynamics is a task suited for data science.

https://youtu.be/q27Jn3h4kpE



M. Copeland, University of Wisconsin

300 bacteria
4 pieces info. / (frame x bacteria)
20 frames / second
10 seconds

240,000 pieces of information

MS78
Topological Data Analysis of Time Series from Dynamical Systems

8:30-8:55 Topological Data Analysis of Stochastic Collective Motion
Chad M. Topaz, Macalester College, USA

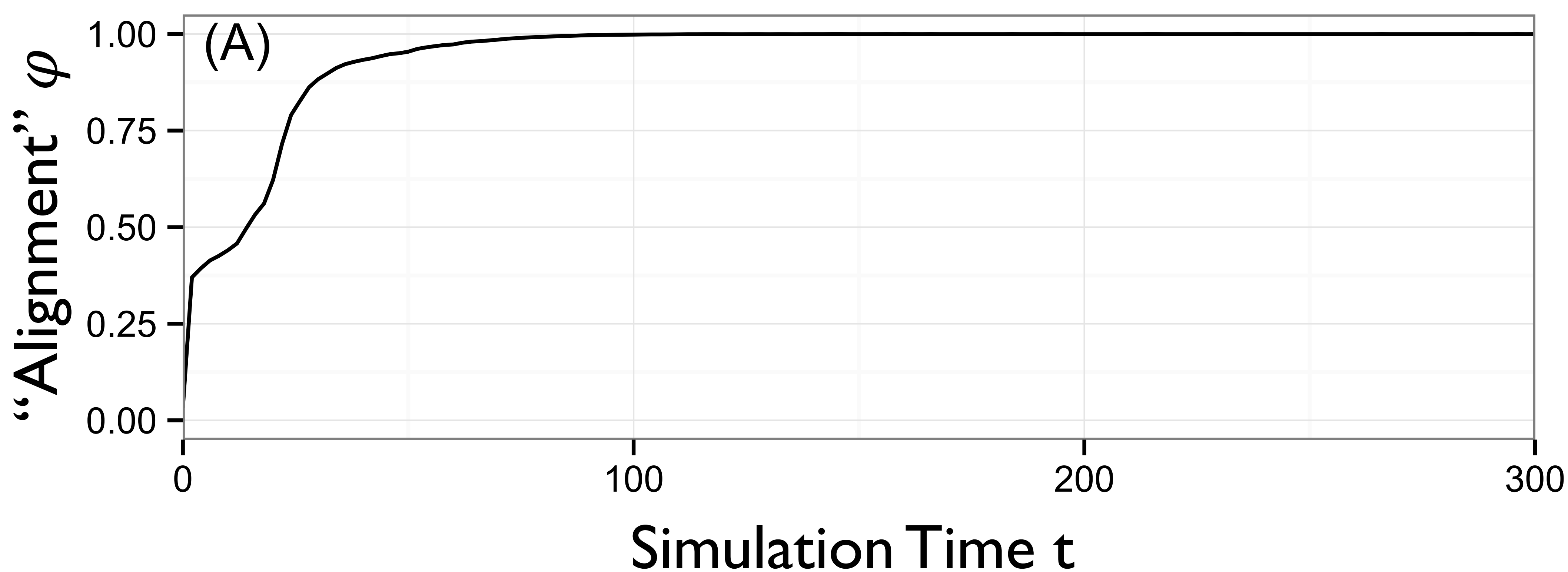
9:00-9:25 Combinatorial Approx. and Discrete-Time Dynamics
***Sarah Day*, College of William & Mary, USA**

9:30-9:55 Classification of Pattern-Forming Systems Using Persistence
Rachel Neville and Patrick Shipman, Colorado State University, USA

10:00-10:25 Witness Complexes for Time Series Analysis
Nicole Sanderson, University of Colorado, USA

Dynamics are often assessed via order parameter time series.

Alignment order parameter: $\varphi(t) = \frac{1}{N v_0} \left| \sum_{i=1}^{N} \mathbf{v}_i(t) \right|$



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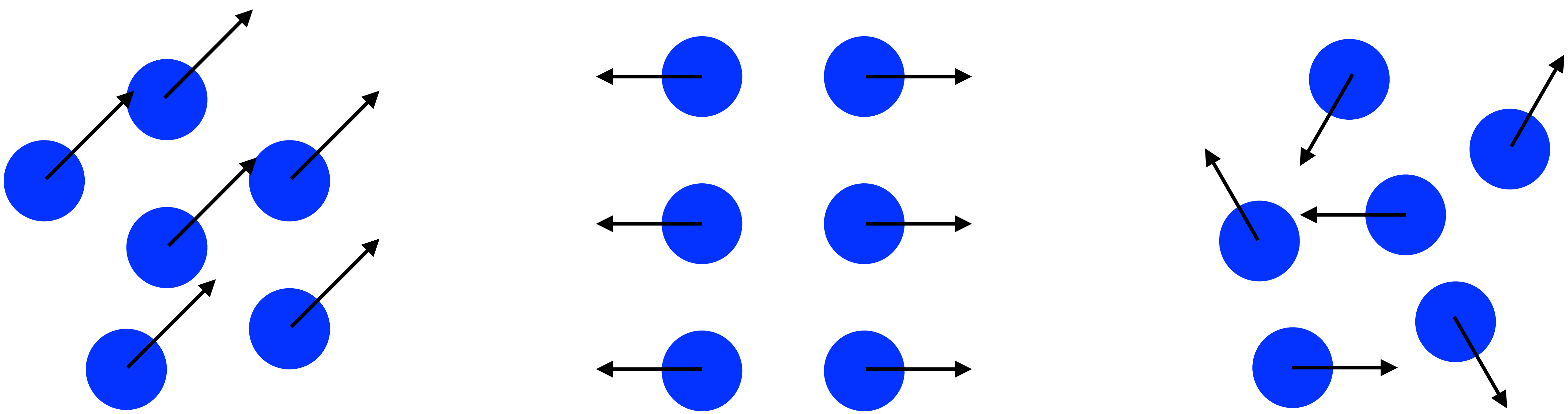


φ = 1 φ = 0 φ = 0

Study data via topology.

1. Computational Homology
   T. Kaczynski, K. Mischaikow, and M. Mrozek. (2004)

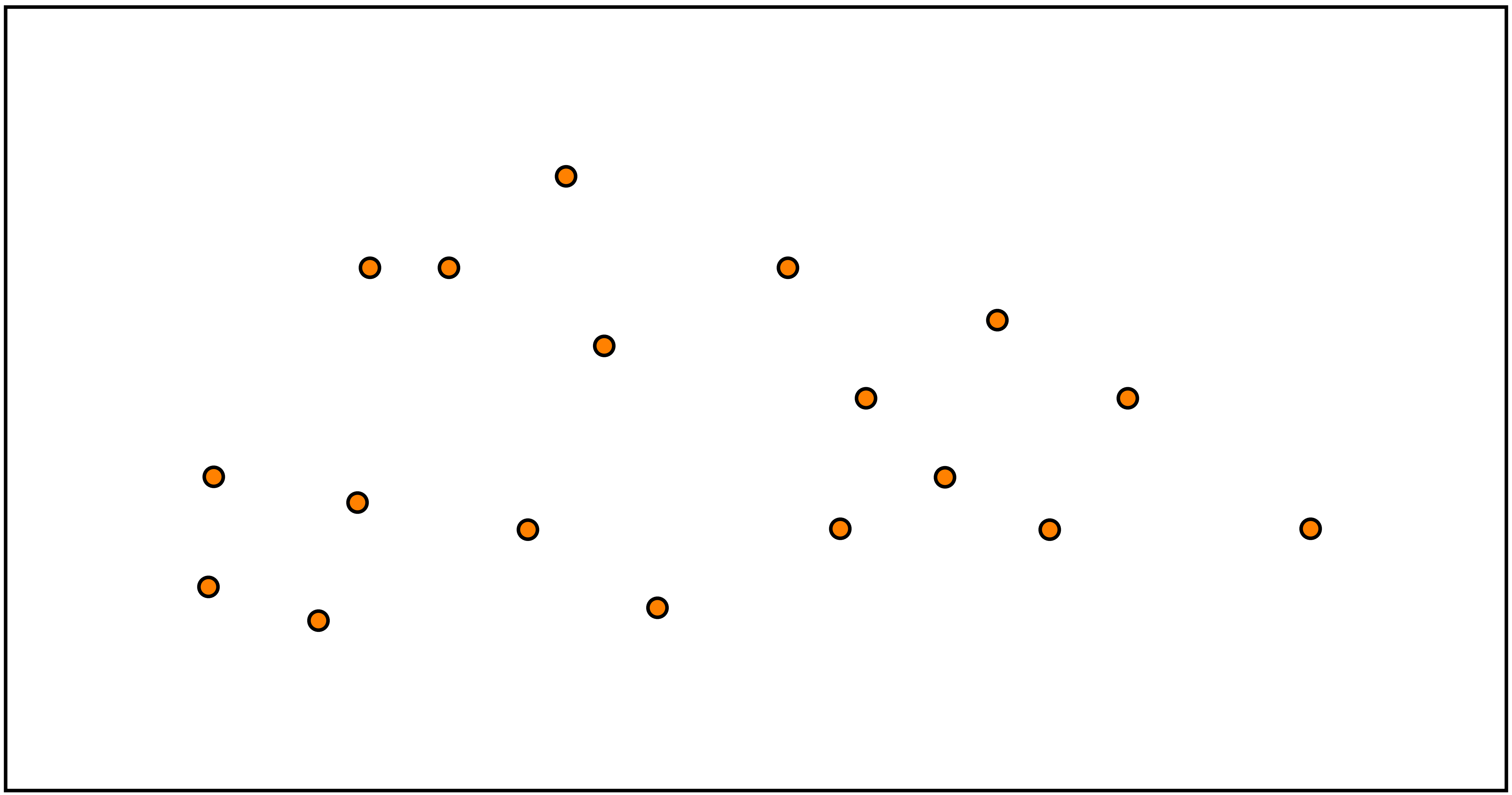
2. Computing persistent homology
   A. Zomorodian, G. Carlsson. Disc. & Comp. Geom. (2005)

3. Barcodes: The persistent topology of data
   R. Ghrist. Bull. Am. Math. Soc. (2008)

4. Persistent homology: A Survey
   H. Edelsbrunner, J. Harer. Contemp. Math. (2008)

5. Topology and Data
   G. Carlsson. Bull. Am. Math. Soc. (2009)

Step 1:
Envision data as point cloud



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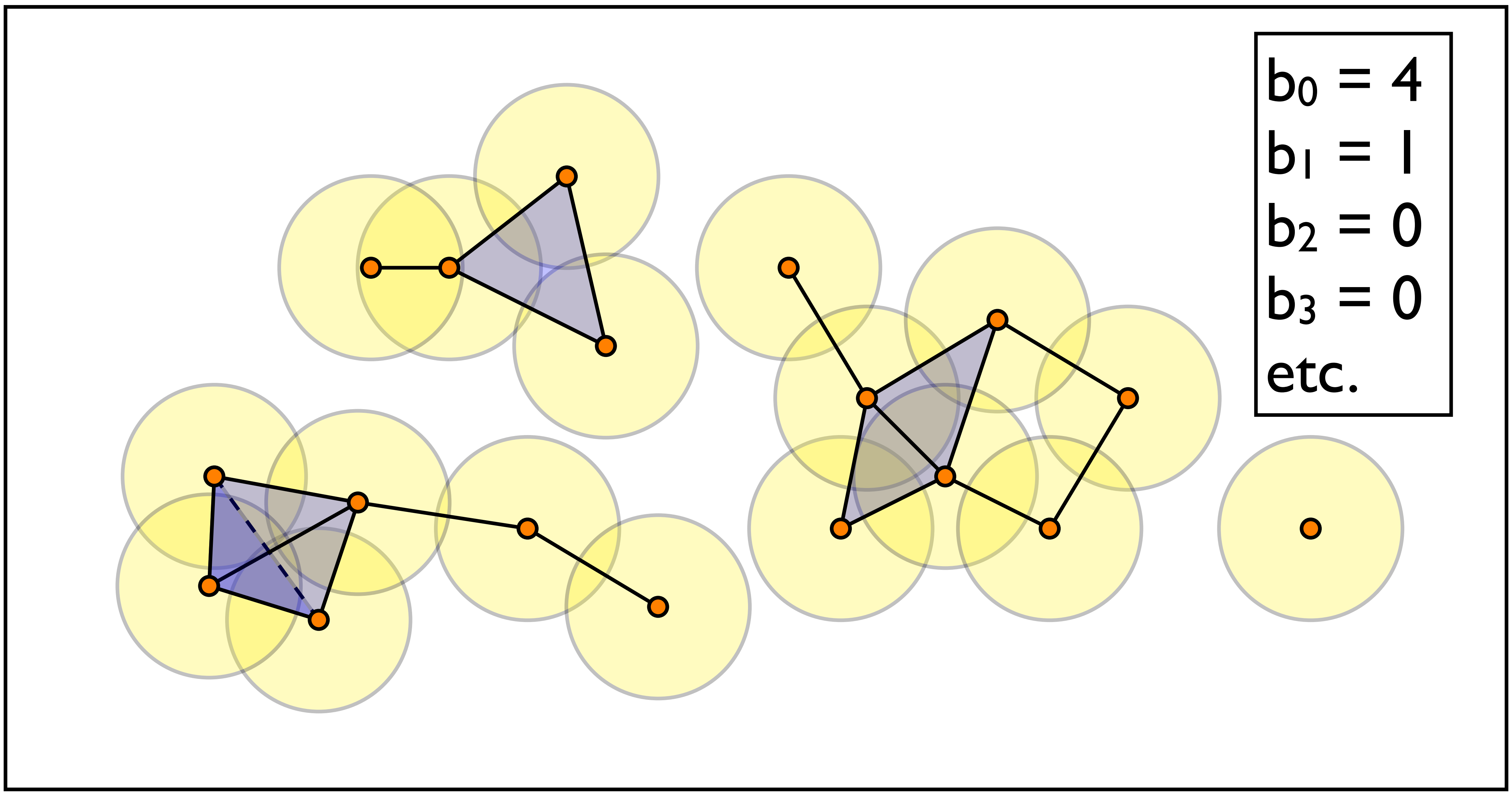
Step 3:
Calculate Betti numbers

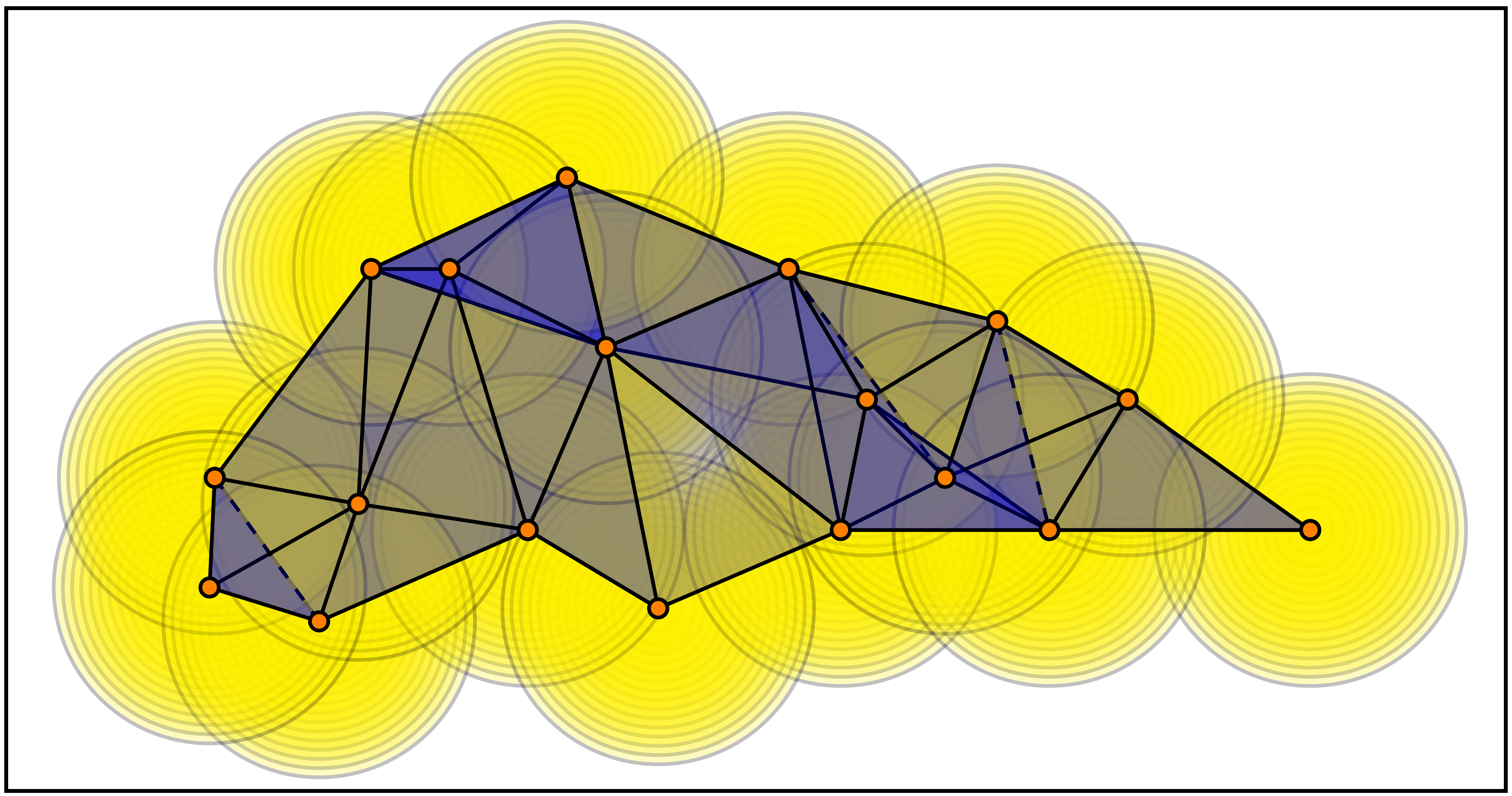


Chad's Self-Help Homology Tutorial For The Simple(x)-Minded

A full-color Extravaganza

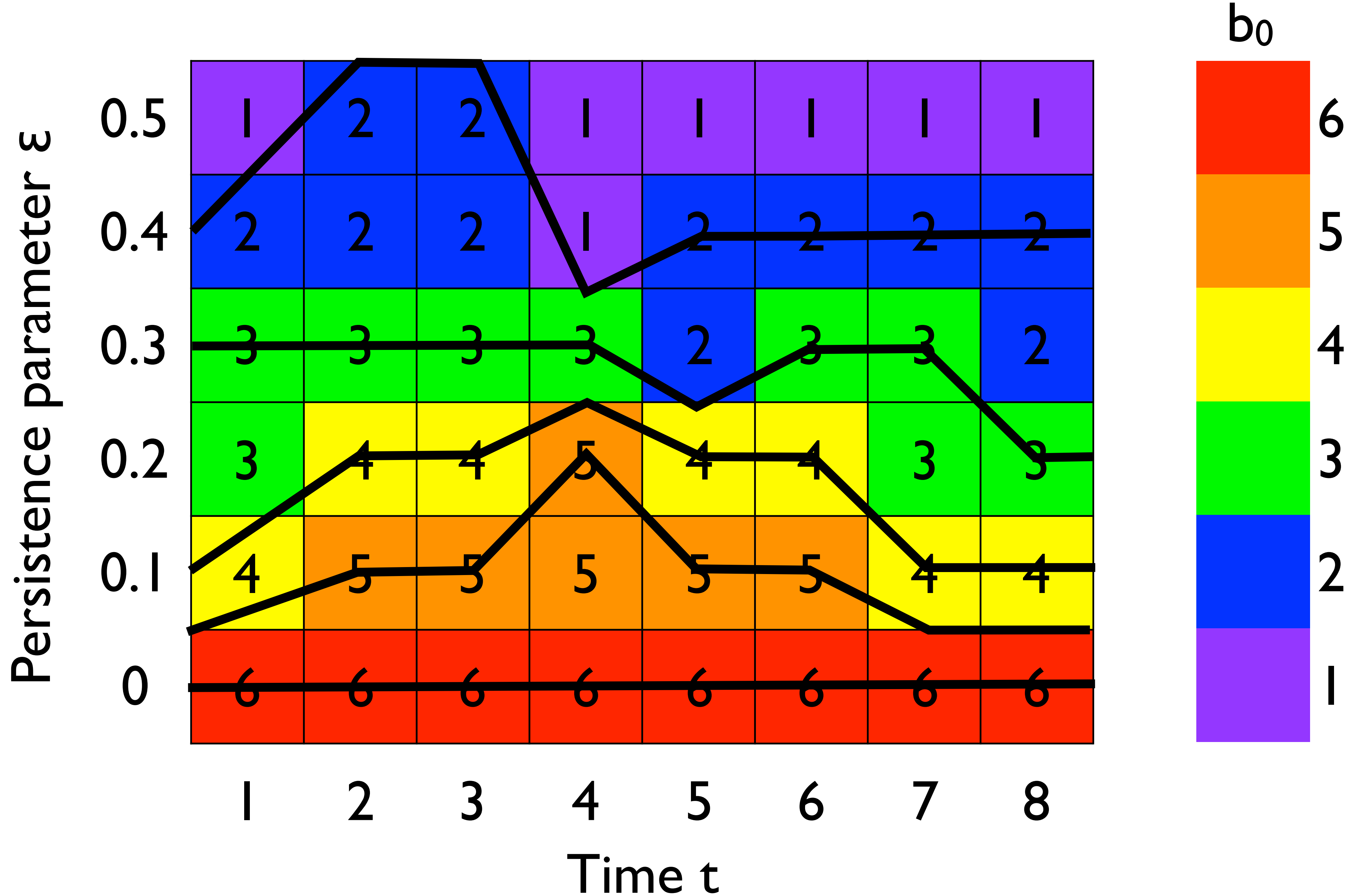
With very sincere thanks and apologies to Lori Ziegelmeier and Tom Halverson, who actually know topology and tried to explain it to me.

Step 4:
Find persistent homology

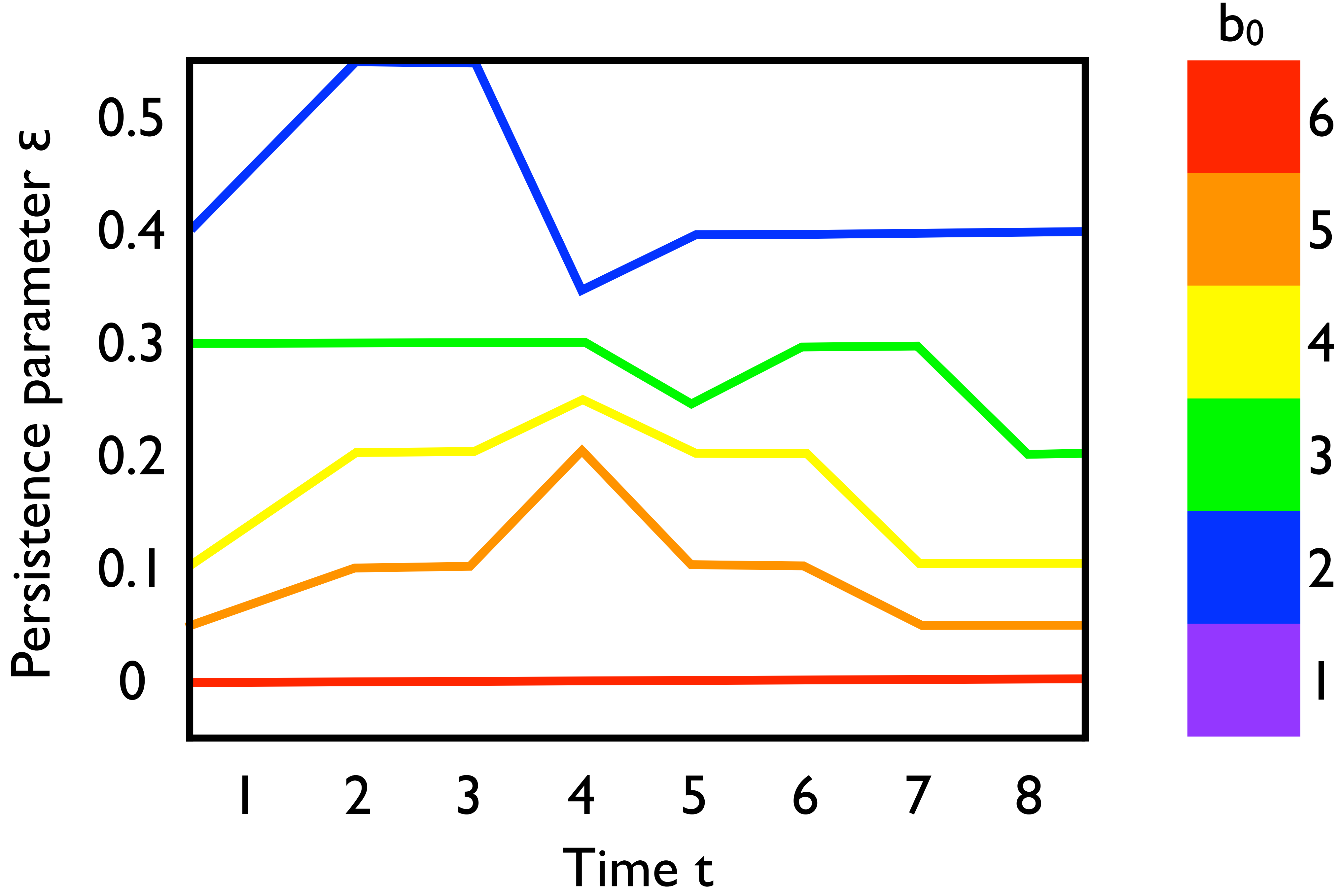


Step 4:
Find persistent homology

Step 5: Evolve in time



Step 5:
Evolve in time



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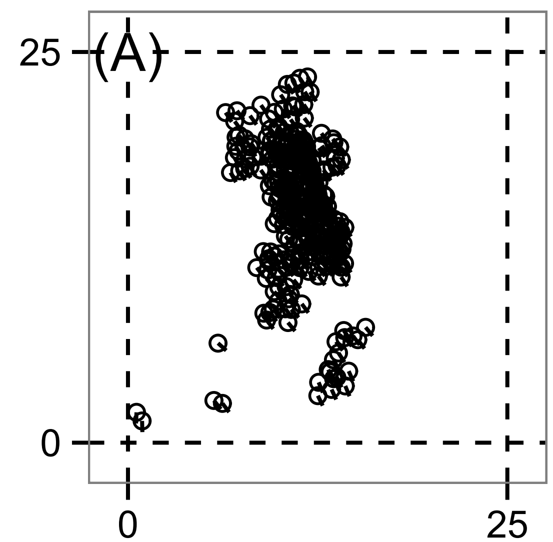
RESEARCH ARTICLE

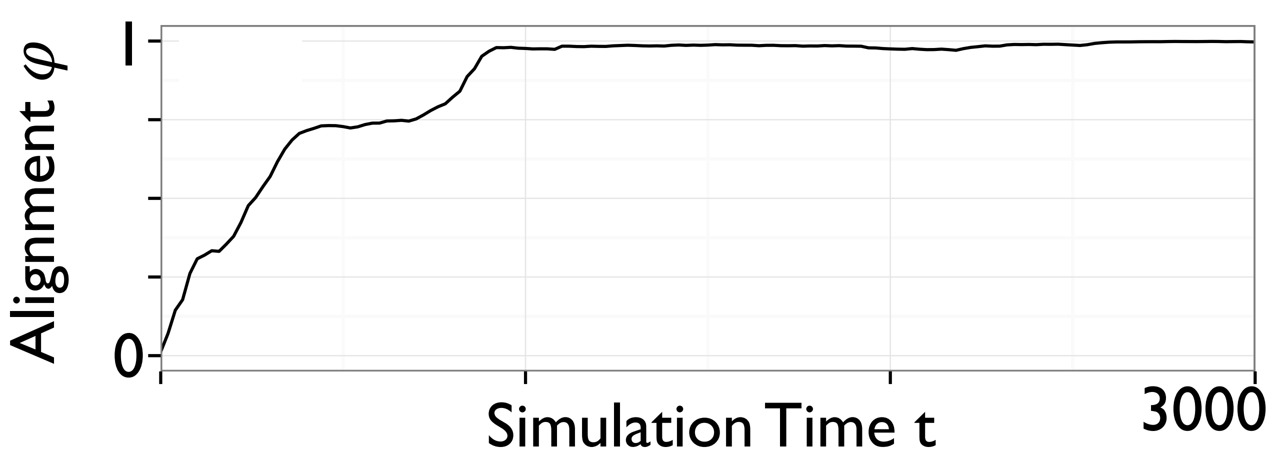
Topological Data Analysis of Biological Aggregation Models

Chad M. Topaz*, Lori Ziegelmeier, Tom Halverson

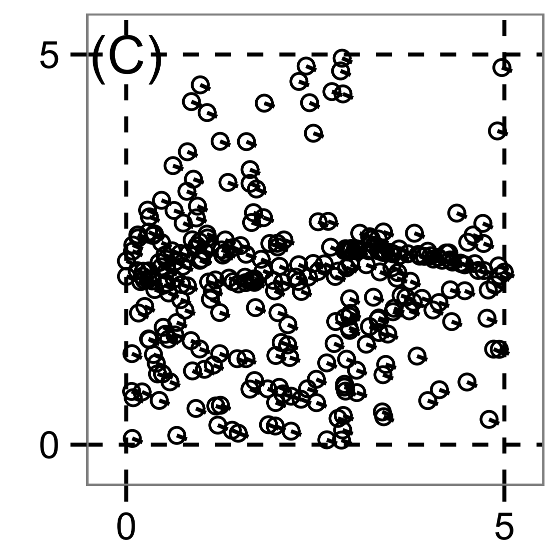
Order parameter time series that look similar…

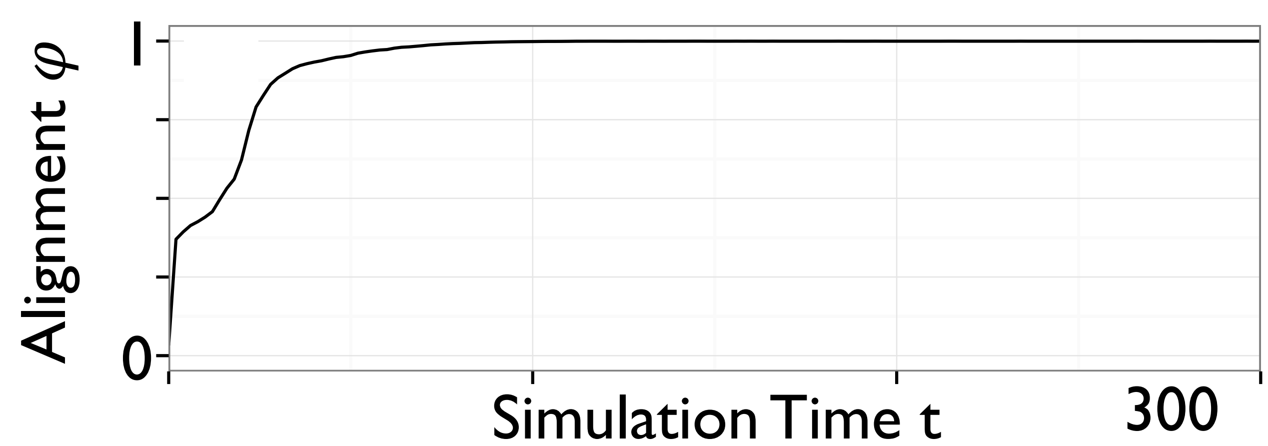
Parameter Set #1



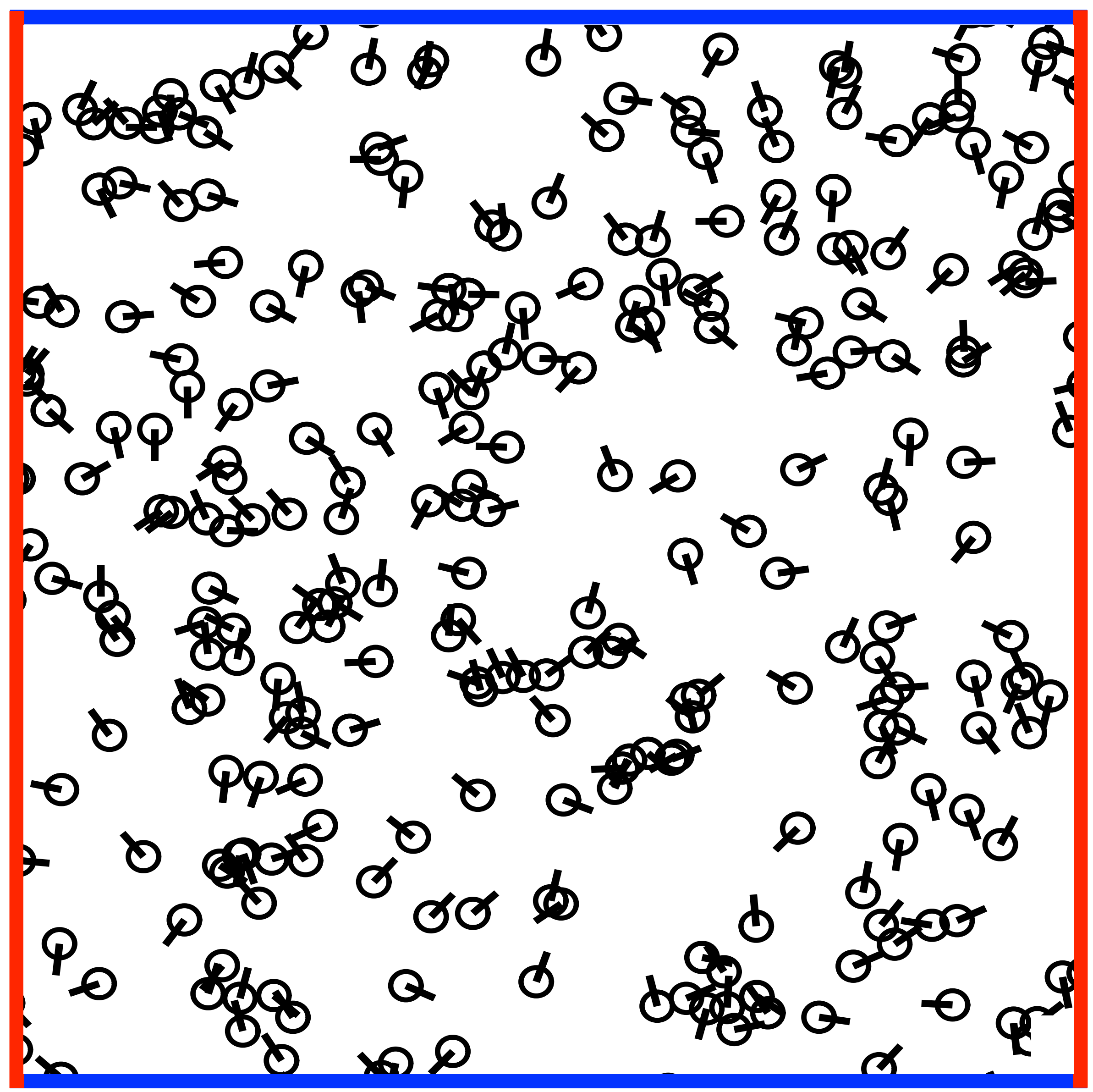


Parameter Set #2





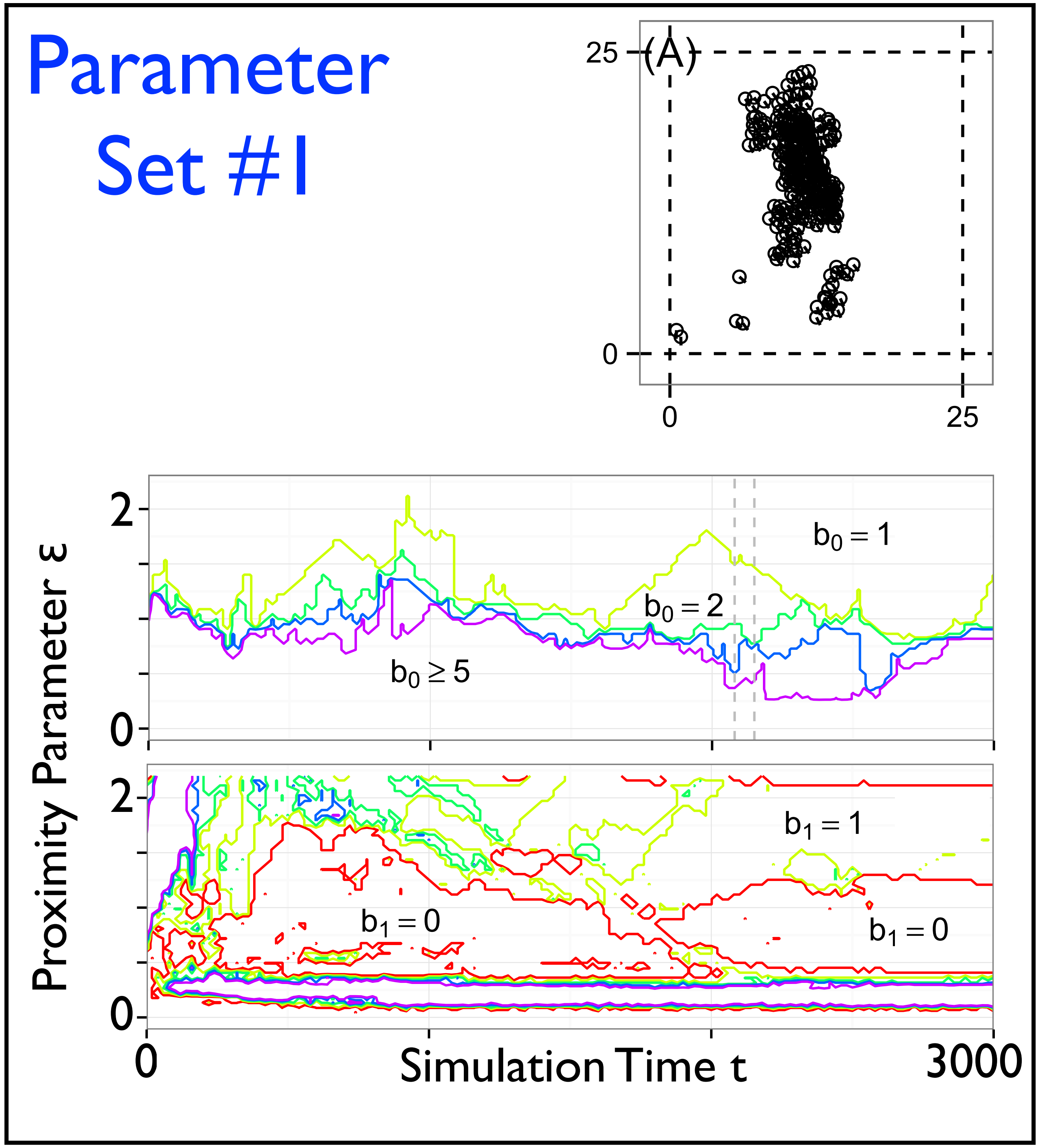
Random initial cond. for Vicsek model covers a three-torus.

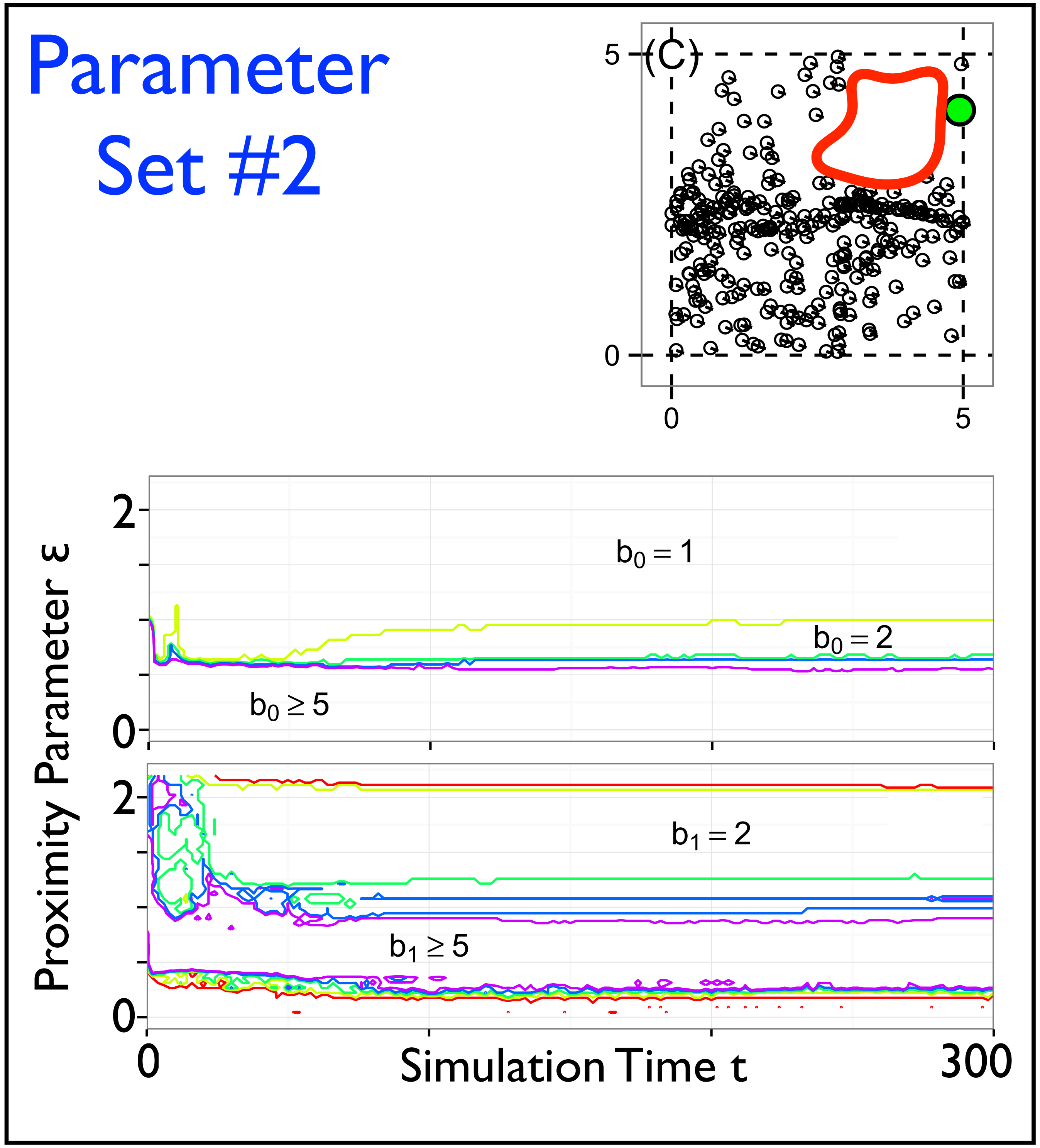


Three-torus T^3

$b = (1,3,3,1,0,\ldots)$

…can have drastically different topological signatures.





Main points of this tutorial-style talk:

- Topological data analysis (TDA) is a set of tools for computing and describing the shape of data ✓
- TDA of time series aids the classification of large data sets arising from collective motion ✓
- Topological time series of collective motion models have a coherent average


XII.P.
HECEST HORRENDA
CARIBDIS
F
D
TRODANES
DVVANES
LOFOT
VAST
NYGAVIK
DLA
HELGALA
TERRA NOBILIVM
HORV PISCI
CAPITIBVS
VTVNTVR
FISCA
STEK

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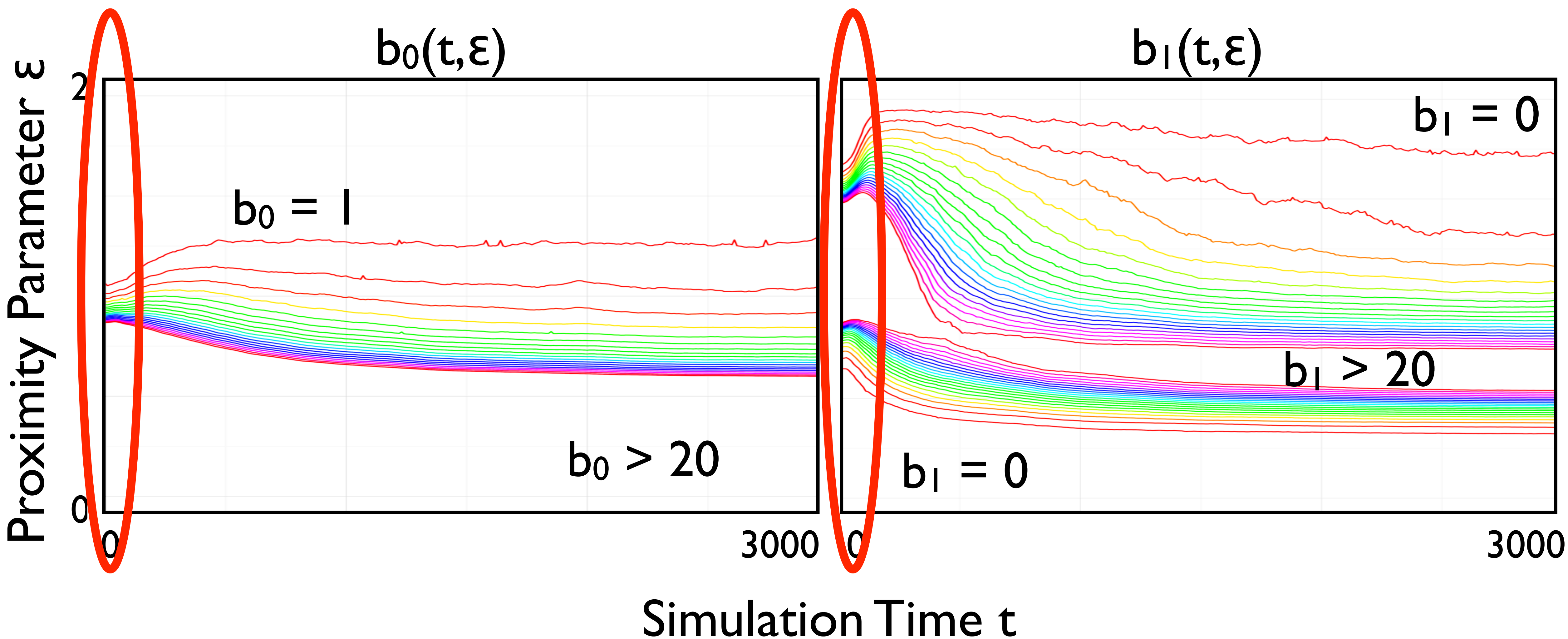
Do time series of random processes have average homology?

Vicsek model (naive) average over n = 1000 simulations



What is the homology of a random complex?

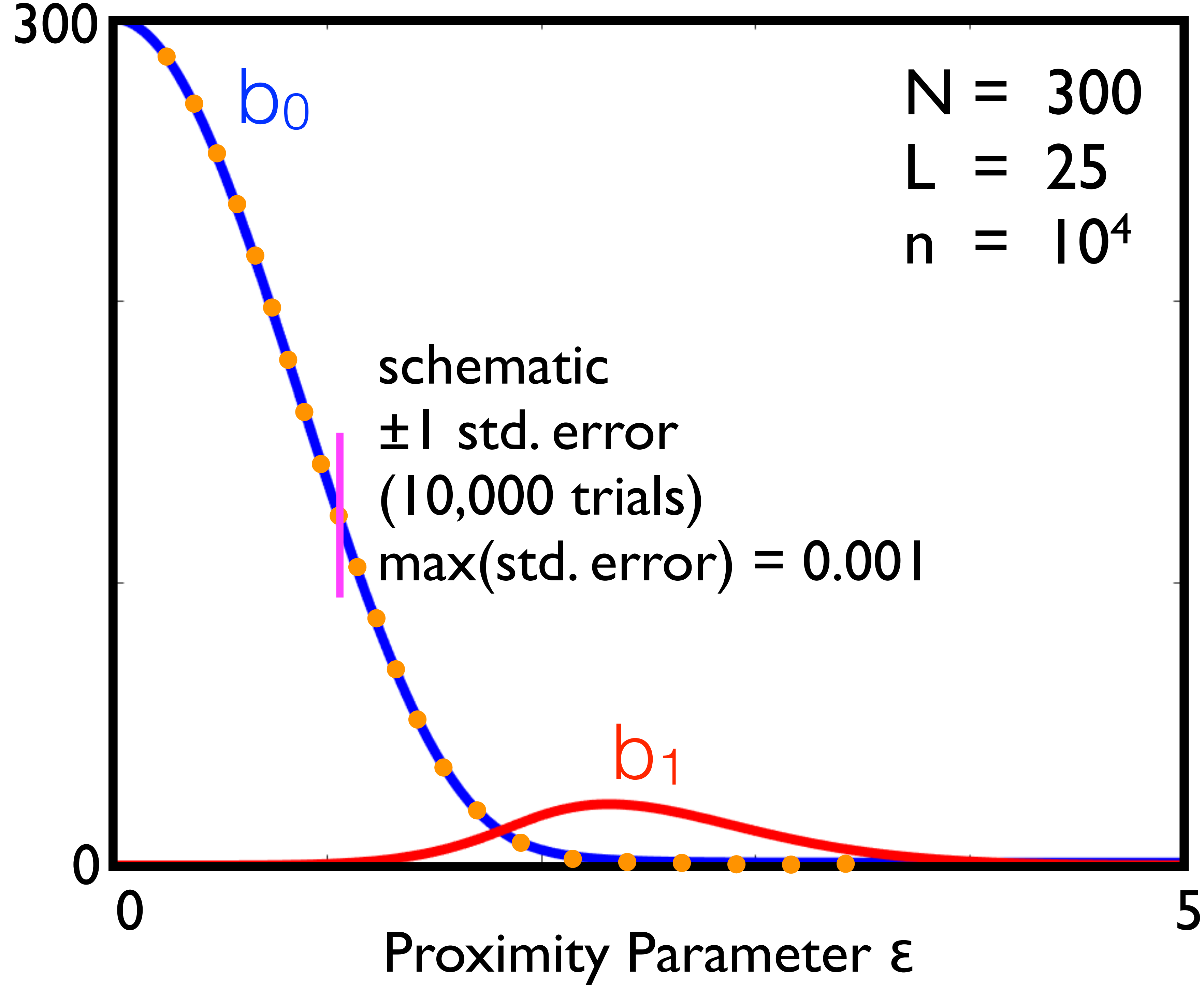
Topology of random simplicial complexes: A Survey (Matthew Kahle, preprint, 2014)

THEOREM 4.3. *Let $\alpha > 0$ be fixed, $p = n^{-\alpha}$, and $X \sim X(n,p)$. If $1/(k+1) < \alpha < 1/k$, then*

$$\frac{\mathbb{E}[\beta_k]}{\binom{n}{k+1} p^{\binom{k+1}{2}}} \to 1,$$

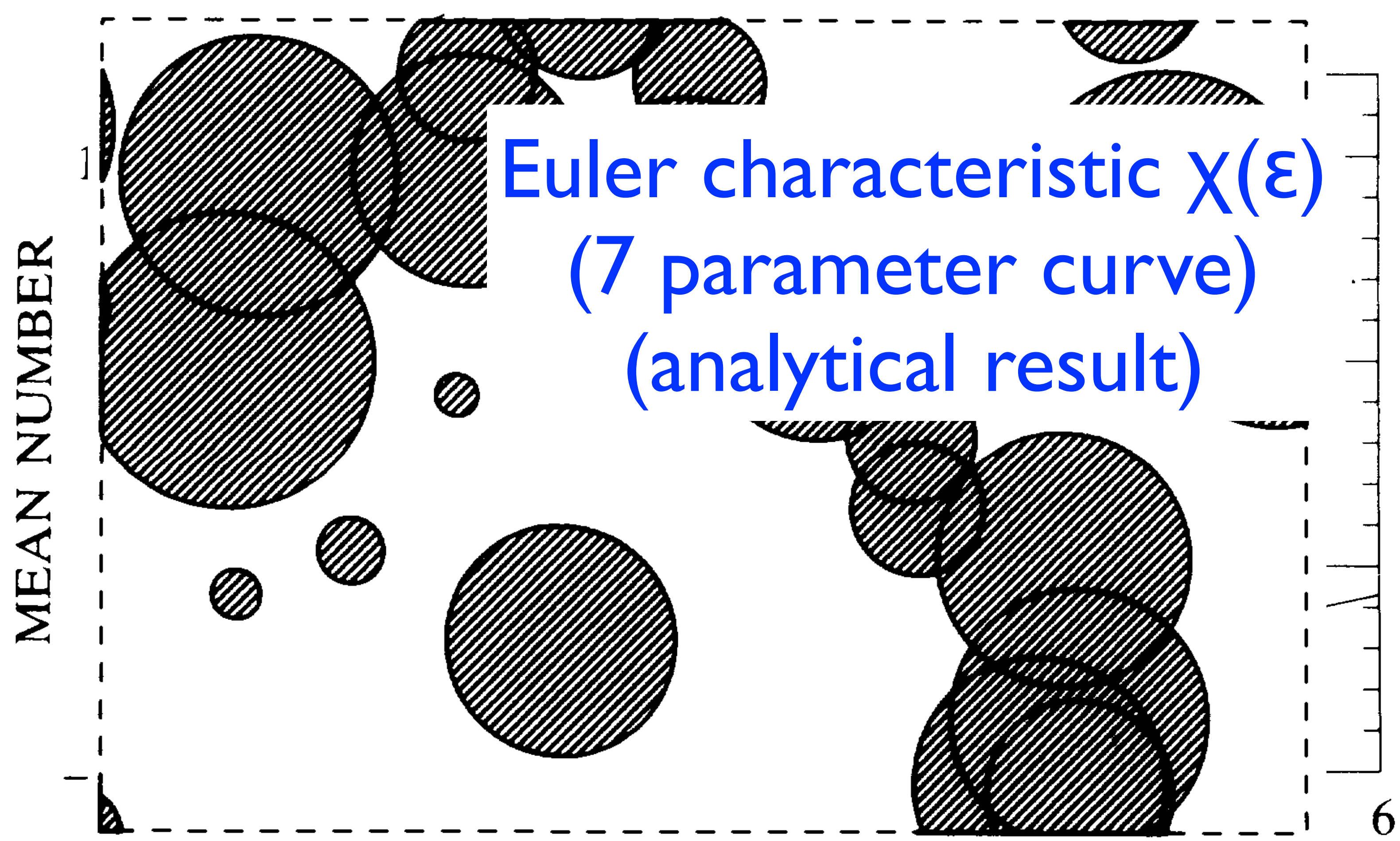
as $n \to \infty$.

What is the homology of a random complex with N finite?



Euler characteristic is known for poisson points in the plane.

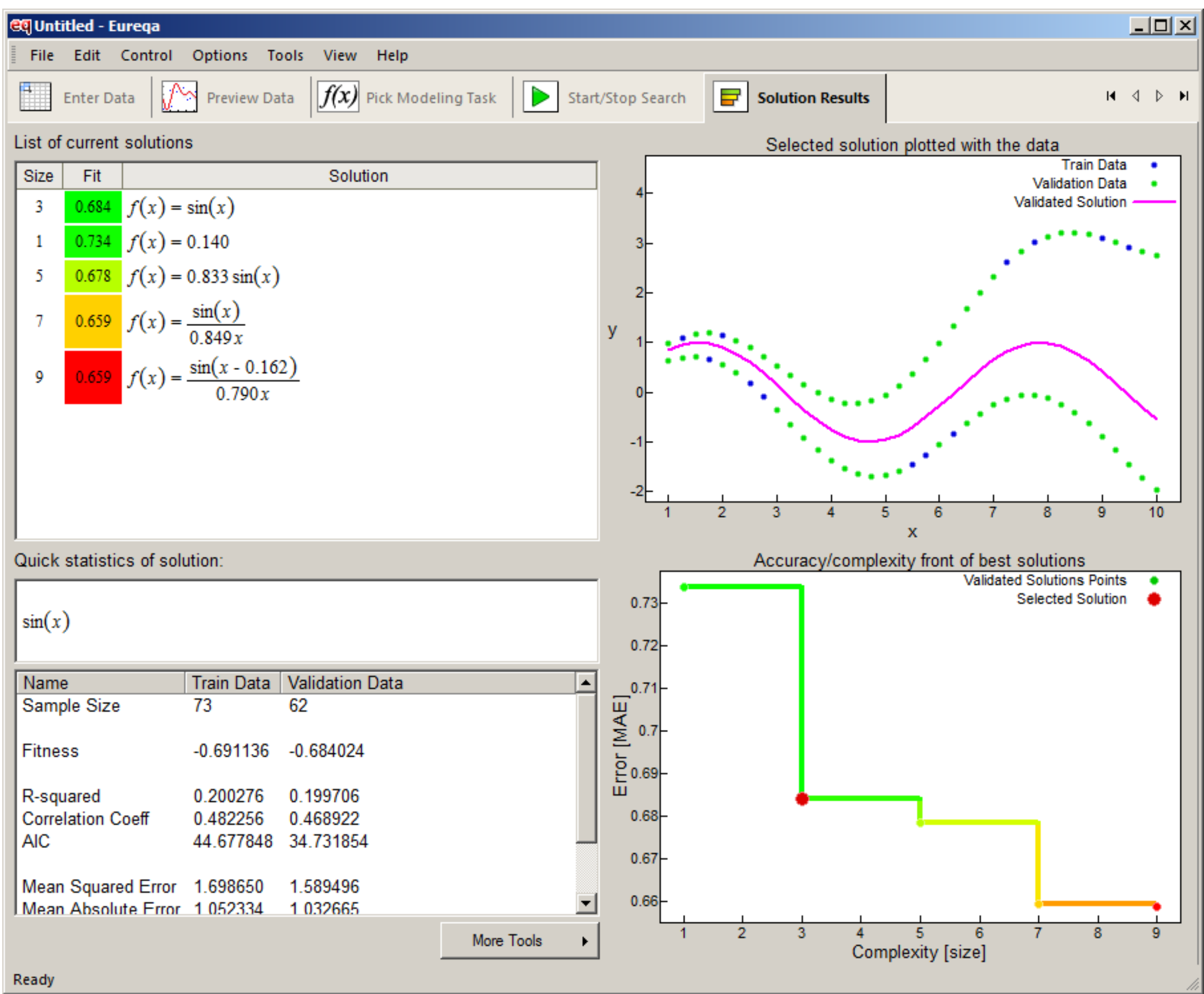
On the number of clumps resulting from
the overlap of randomly placed figures in a plane
(A.M. Kellerer, J. Appl. Prob., 1983)



Dimensional analysis + limiting behavior + machine learning?

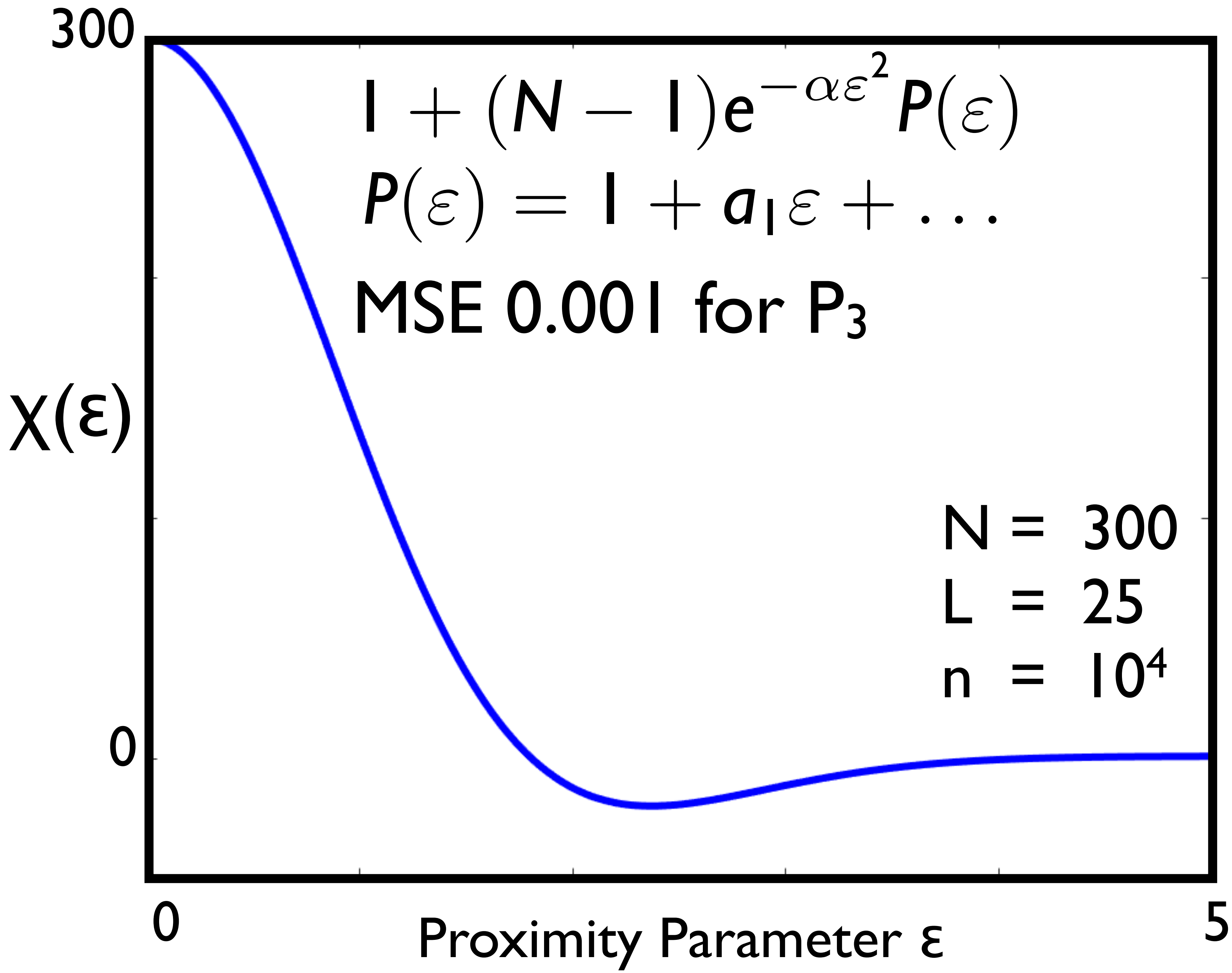
$$\chi(\varepsilon; N, L) = 1 + (N-1) f(\varepsilon/L, N)$$

$$f(0, N) = 1 \qquad \lim_{\varepsilon \to \infty} f(\varepsilon/L, N) = 0$$



Eureqa
see SIAM DS15 Plenary
“Automating Discovery”
by Hod Lipson

Computer-generated model provides a reasonable fit.



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- TDA of time series aids the classification of large data sets arising from collective motion ✓
- Topological time series of collective motion models have a coherent average… what is it? ✓

http://www.ams.org/profession/2018MRC-Agent

American Mathematical Society Mathematics Research Community

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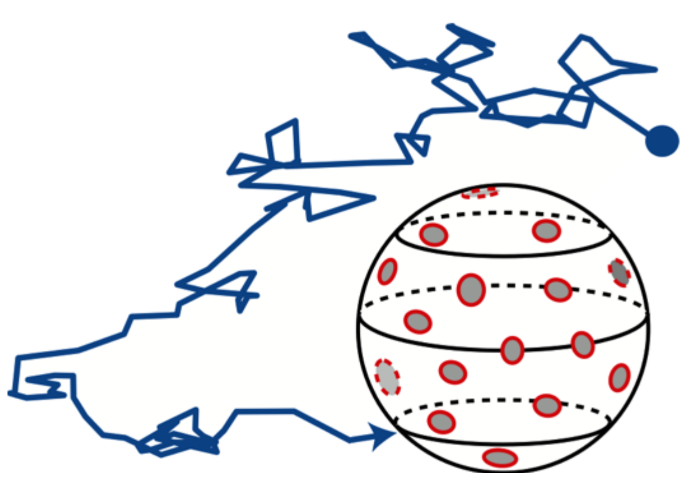
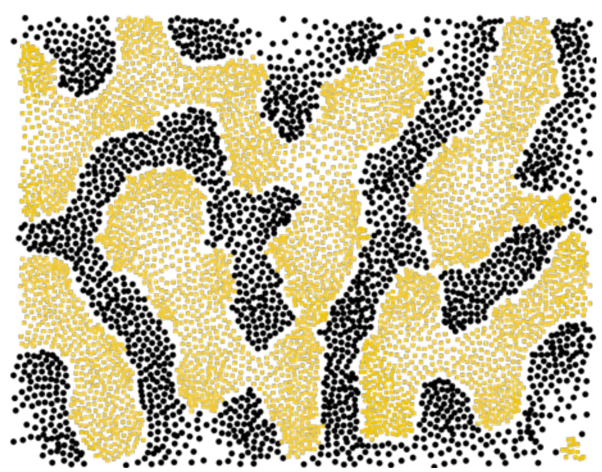
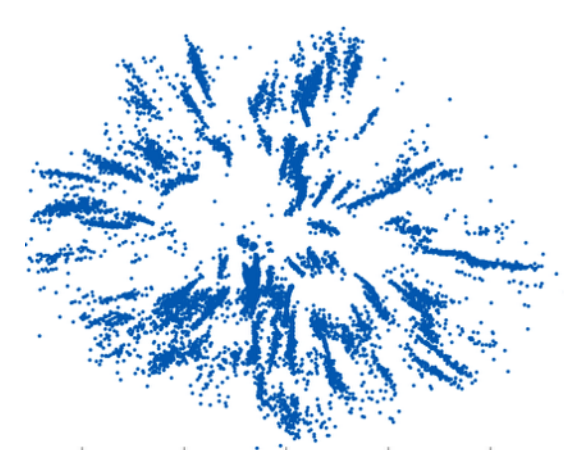


Diffusive signaling problems in chemoreception (Bernoff & Lindsay 2017)



Agent-based model of zebrafish stripes (Volkening & Sandstede 2015)



Agent-based model of locust hopper bands (Bernoff, Devore, Jones, Zhang & Topaz 2017)