Dynamic Mode Decompositions and Koopman Analysis

Applications in Fluid Flow Analysis

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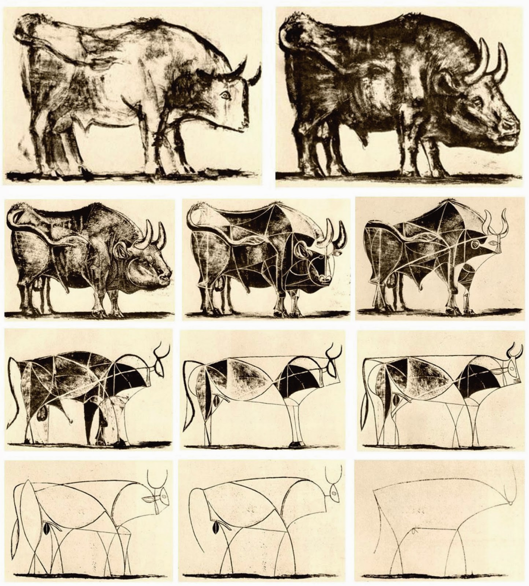
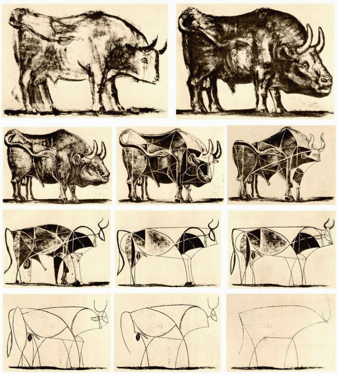
“A picture is a sum of destructions.” —Pablo Picasso, 1935



Bull (1945–1946)



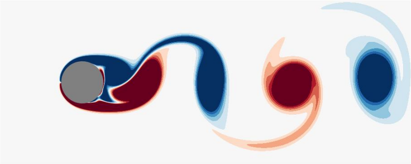
Bull (1945–1946)

Sparse Representations:
Descriptions based on a “minimal” set of “essential” features.

- Essential features can inform understanding.
- Significance of features to a description depends on context.
- Everything should be made as simple as possible,
  but not simpler.

Modes and Coherent Structures:

- Fluid flows have large (infinite) number of degrees of freedom, but most are “inactive”.
- Only a few interacting “active modes” dominate complex evolution of fluid flow.



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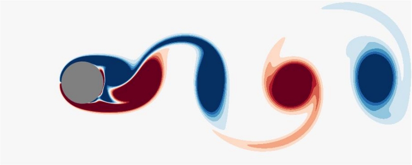


Extracting Sparse Representations from Data:

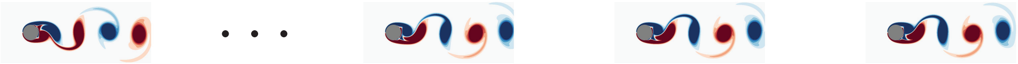
1. **Remove:** Identify the relevant variables, and ignore the rest.
2. **Consolidate:** Leverage dependencies to transform to lower-order representation.

“New dataset” should contain fewer variables, while preserving “interesting features” of original dataset.

Data-Informed Sparse Representations and POD/PCA

Consider a matrix of snapshot data $X = \begin{bmatrix} x_o & x_1 & x_2 & \dots & x_m \end{bmatrix} \in \mathbb{R}^{n \times m+1}$.

x_m $\quad\cdots\quad$ x_2 $\quad$ x_1 $\quad$ x_o



Use data covariance $C_X = \frac{1}{m} X X^{\mathrm{T}}$ to identify

- **relevancy:** large variances (i.e., diagonals in C_X) $\Rightarrow$ highly dynamic
- **redundancy:** large covariances (i.e., off-diagonals in C_X)$\Rightarrow$ highly redundant

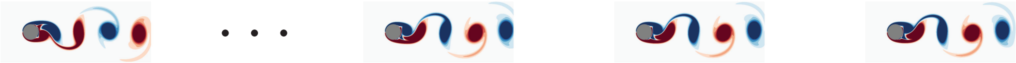
Diagonalizing C_X provides an ideal view of the data, since

- all redundancies will be removed, and
- directions with largest variance will be isolated and ordered.

A Matter of Perspective

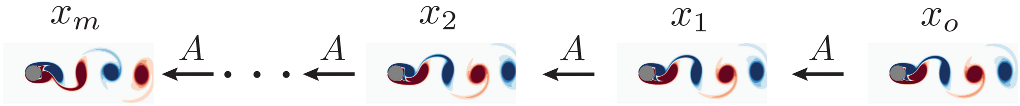
POD Perspective: A collection of snapshots.

x_m x_2 x_1 x_o



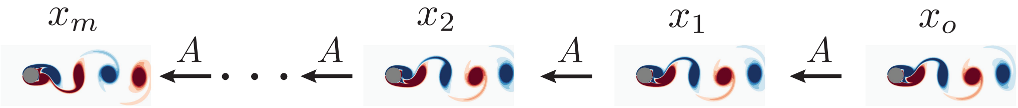
Taira et al., AIAA Journal, 2017.
Taira et al., arxiv:1903.05750.

DMD Perspective: A collection of snapshots related by a linear map (*dynamics*).



Rowley, Mezić, Bagheri, Schlatter, Henningson, J. Fluid Mechanics, 2009.

Schmid, J. Fluid Mechanics, 2010.



$$
\begin{aligned}
x_1 &= Ax_o \\
x_2 = Ax_1 &= A^2 x_o \\
x_3 = Ax_2 = A^2 x_1 &= A^3 x_o \\
&\vdots \\
x_k = Ax_{k-1} = A^2 x_{k-2} = \ldots &= A^k x_o
\end{aligned}
$$

Knowledge of A enables prediction.

$$x_k = A^k x_o$$

Prediction does not offer insight, so let's find an **interpretable** representation.

Express A in terms of its eigendecomposition

$$A = V \Lambda V^{-1}$$

where

- $V = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$ is a matrix of eigenvectors,
- $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$ is a diagonal matrix of eigenvalues.

Re-write the prediction in terms of the eigendecomposition of A,

$$\begin{aligned} x_k &= A^k x_o \\ &= (V \Lambda V^{-1})^k x_o \\ &= V \Lambda^k V^{-1} x_o \\ &= V \Lambda^k \alpha \end{aligned}$$

where $\alpha := V^{-1} x_o$.

Descriptive Modeling

Overall dynamics as the sum of contributions from simple *modal* dynamics

$$
\begin{aligned}
x_k &= V\Lambda^k\alpha \\
&= \sum_{j=1}^{n} v_j\lambda_j^k\alpha_j \\
&= \sum_{j=1}^{n} \underbrace{(\text{Mode } j)}_{\textbf{Space}}\,\underbrace{(\text{Eigenvalue } j)^k}_{\textbf{Time}}\;\underbrace{(\text{Amplitude } j)}_{\textbf{Relative Contribution}}
\end{aligned}
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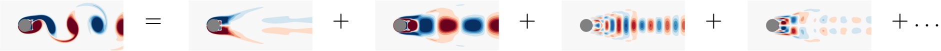
$$x_k = v_1\lambda_1^k\alpha_1 + v_2\lambda_2^k\alpha_2 + v_3\lambda_3^k\alpha_3 + v_4\lambda_4^k\alpha_4 + \dots$$



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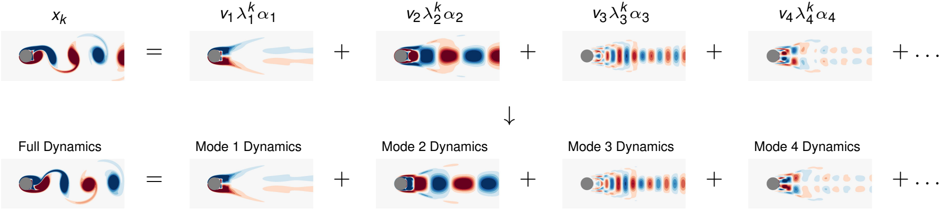

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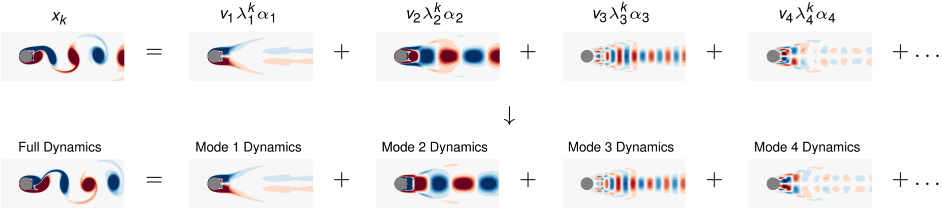

In practice, *A* unknown → Extract modes, eigenvalues, and amplitudes from snapshot data.

Dynamic Mode Decomposition (DMD)

Consider a discrete-time system

$$x \mapsto f(x) \in \mathbb{R}^n$$

with snapshot data matrices

$$\begin{aligned} X &:= \begin{bmatrix} x_1 & x_2 & \cdots & x_m \end{bmatrix} \\ Y &:= \begin{bmatrix} f(x_1) & f(x_2) & \cdots & f(x_m) \end{bmatrix} \end{aligned}$$

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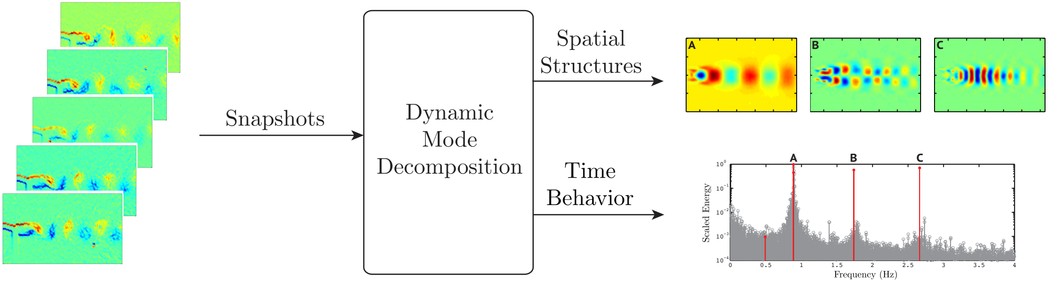
$$A := YX^+ \in \mathbb{R}^{n \times n}.$$

(Tu et al., 2014)

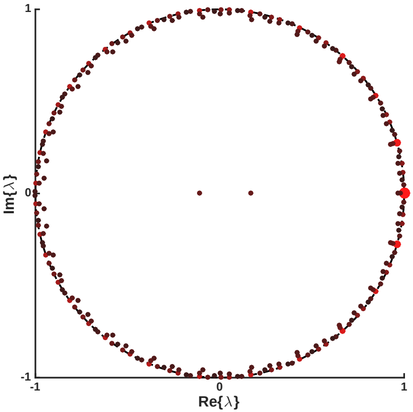
$$\begin{aligned} X &= U \Sigma V^\top \\ \tilde{A} &= U_r^\top Y V_r \Sigma_r^{-1} \in \mathbb{R}^{r \times r}, \\ A &= U_r \tilde{A} U_r^\top \in \mathbb{R}^{n \times n} \end{aligned}$$

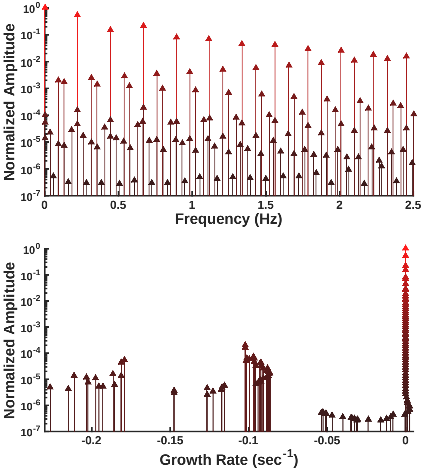


Example: Flow past a cylinder (Re=413)

DMD eigenvalues relate to **temporal characteristics** (i.e., simple modal dynamics).

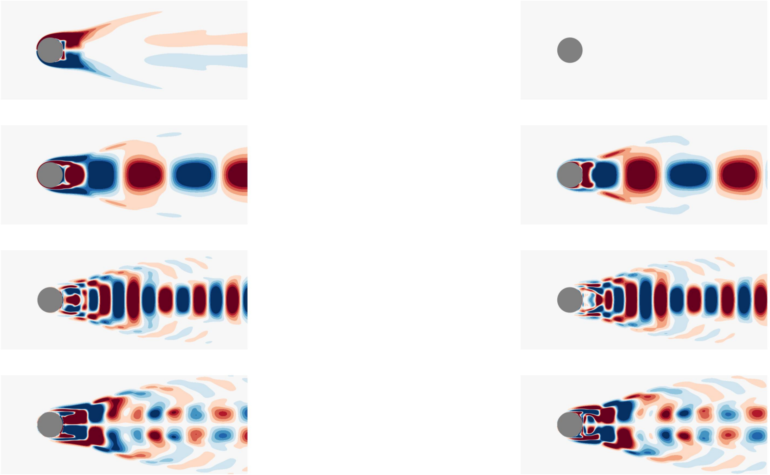
- Modal frequencies $\rightarrow$ phase angles
- Modal growth/decay rates $\rightarrow$ magnitudes





What **spatial structures** are involved in the modal dynamics?

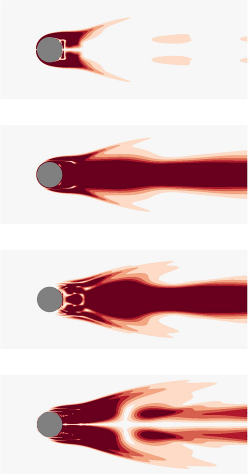
$Re\{v_j\}$ $Im\{v_j\}$



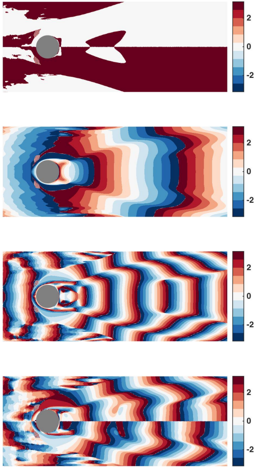
Example: Flow past a cylinder (Re=413)

What spatial regions are **active** in each mode?
What spatial regions **lead/lag** others in each mode?

Magnitude



Phase



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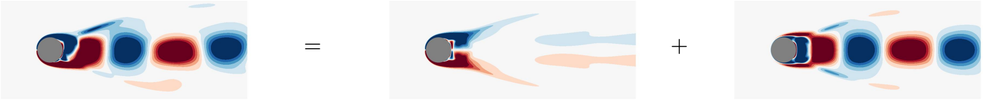
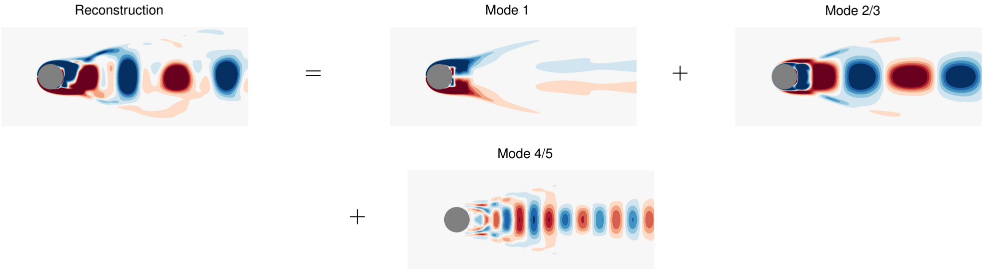
Reconstruction Mode 1 Mode 2/3



Reconstructing Fluid Dynamics



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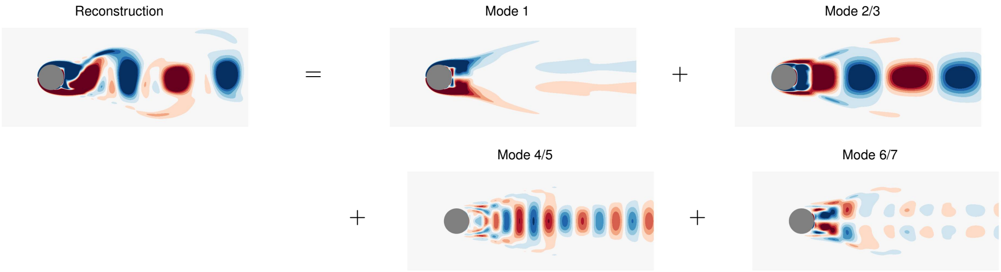
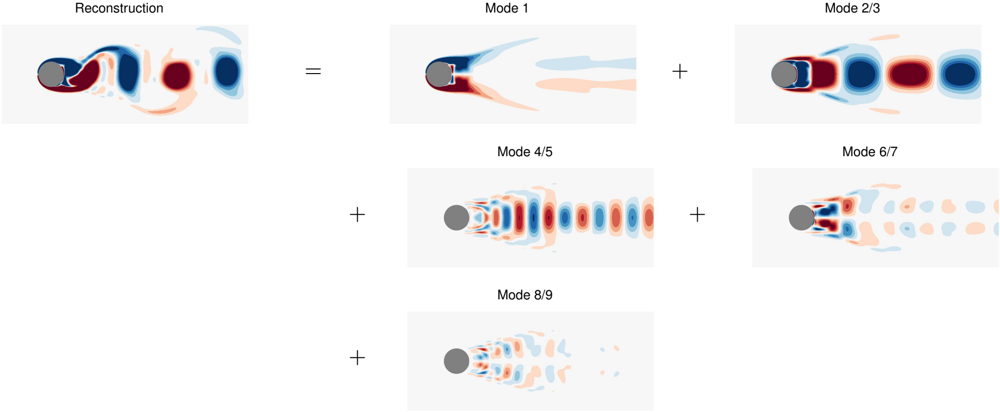
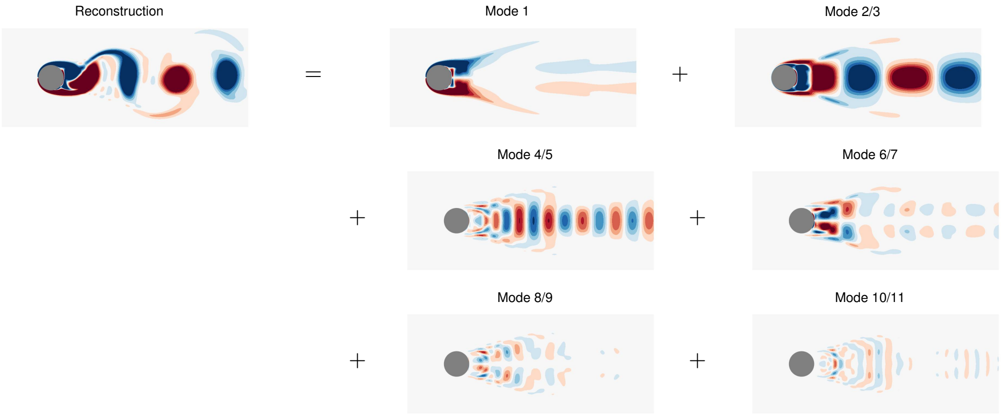



Reconstruction
=
Mode 1
+
Mode 2/3
+
Mode 4/5
+
Mode 6/7
+
Mode 8/9

Reconstructing Fluid Dynamics



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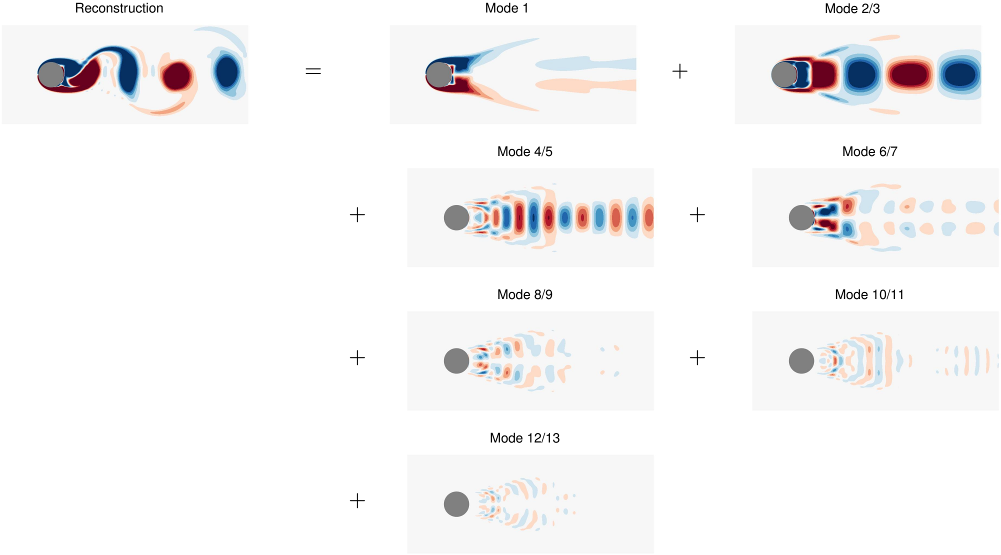

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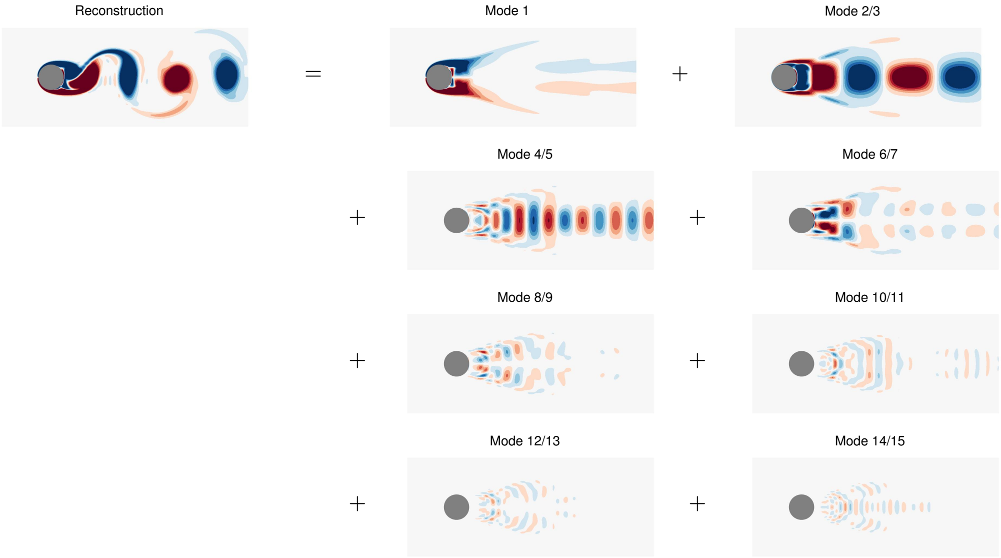

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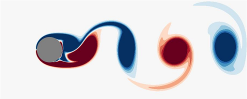
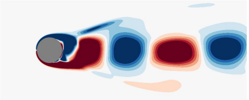
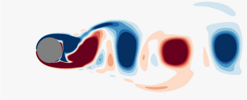


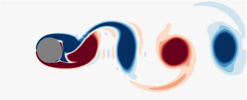
Original



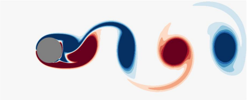
3 Modes



7 Modes



15 Modes



30 Modes

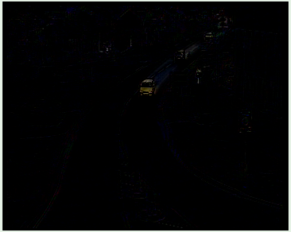
Deconstructing complex systems beyond fluid mechanics

- Video Processing
- Neuroscience
- Epidemiology
- Robotics
- Sustainable Buildings
- Power Systems
- . . .

Original Feed

Stationary

Moving



What are the limitations and weaknesses of DMD?

- Difficult in deciding which modes are important.
- Sensitivity to noisy data.
- Inability to model nonlinear dynamics.
- Inability to model the effects of actuation.
- Can be computationally expensive or intractable for large datasets.

Variants of Dynamic Mode Decomposition

Deciding which modes are important

- Selection of optimal subspace for projection
  - *Wynn et al., 2013*
  - *Chen at al., 2012*
- Selection of a sparse set of modes
  - *Jovanović et al. 2014*

Improving performance with noisy data

- Noise-robust and bias-free algorithms
  - *Dawson et al., 2016*
  - *Hemati et al., 2017*
  - *Askham & Kutz, 2017*
- Characterizing process noise effects
  - *Bagheri, 2014*

Modeling the effect of actuation

- DMD with control
  - *Proctor et al., 2016*

Dealing with big data

- *Streaming algorithms*
  - *Hemati et al., 2014*
- Parallelized algorithms
  - *Belson et al., 2013*
  - *Sayadi & Schmid, 2016*
  - *Anantharamu & Mahesh, 2019*
- Random projection methods
  - *Erichson & Donovan, 2016*
  - *Erichson et al., 2017*
- Random sampling methods
  - *Tu et al., 2014*
  - *Brunton et al., 2015*
  - *Erichson et al., 2016*

Modeling nonlinear dynamics

- Extended DMD
  - *Williams et al., 2015*

DMD can be customized to suit a variety of applications/datasets.

Software available at http://z.umn.edu/dmdtools

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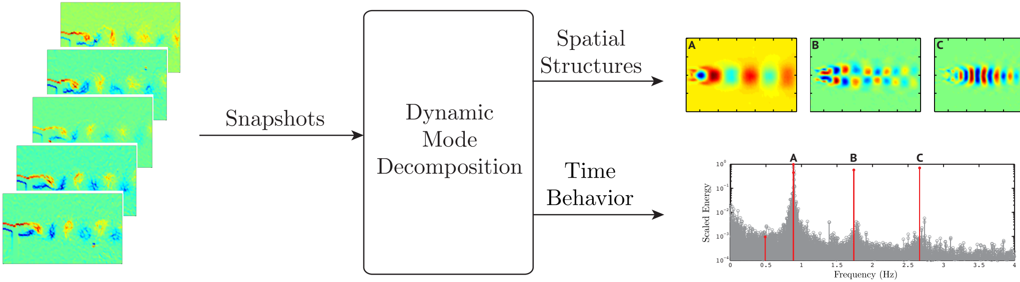

What is DMD really doing?

1. Compute an orthonormal basis for the image of X.
2. Construct a “small” proxy system to solve the eigenproblem.
3. Relate the eigenvectors and eigenvalues of the small problem to those of the full problem (i.e., $A = Q_X \tilde{A} Q_X^T$).

Standard DMD

$$\underbrace{\tilde{A}}_{r_X \times r_X} := \underbrace{Q_X^T}_{r_X \times n} \underbrace{Y}_{n \times m} \underbrace{X^+}_{m \times n} \underbrace{Q_X}_{n \times r_X},$$

Hemati, Williams, Rowley, Phys. Fluids, 2014.

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To design a streaming DMD method, assume:

1. Only one snapshot pair (x_i, y_i) can be stored at a given time (i.e., “single-pass”).
2. The data in X and Y are low-rank.

Hemati, Williams, Rowley, Phys. Fluids, 2014.

DMD for Large and Streaming Datasets

Standard DMD

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Re-write DMD as

$$\underbrace{\tilde{A}}_{r_X \times r_X} = \underbrace{Q_X^{\mathsf{T}}}_{r_X \times n} \underbrace{Q_Y}_{n \times r_Y} \underbrace{K}_{r_Y \times r_X} \underbrace{G_X^{+}}_{r_X \times r_X}.$$

- Q_X, Q_Y can be computed via a Gram-Schmidt procedure.
- $K := \tilde{Y}\tilde{X}^{\mathsf{T}}$, $G_X := \tilde{X}\tilde{X}^{\mathsf{T}}$, $\tilde{X} := Q_X^{\mathsf{T}} X$, $\tilde{Y} := Q_Y^{\mathsf{T}} Y$ can be dynamically updated.

Hemati, Williams, Rowley, Phys. Fluids, 2014.

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- (Optional) Maintain low-rank via POD compression.
  - Define $G_Y := \tilde{Y}\tilde{Y}^\top$ and make use of leading eigenvectors of G_X, G_Y.

Hemati, Williams, Rowley, Phys. Fluids, 2014.

DMD for Large and Streaming Datasets

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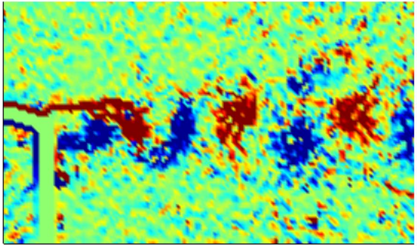
- $\mathcal{O}(nr^2)$ operations per iterate with mode computations.
- $\mathcal{O}(nr)$ operations per iterate without mode computations.
- $\mathcal{O}(nr)$ storage of matrix entries (single-pass method).

Hemati, Williams, Rowley, Phys. Fluids, 2014.

Example: PIV data for laminar flow past a cylinder (Re=413)

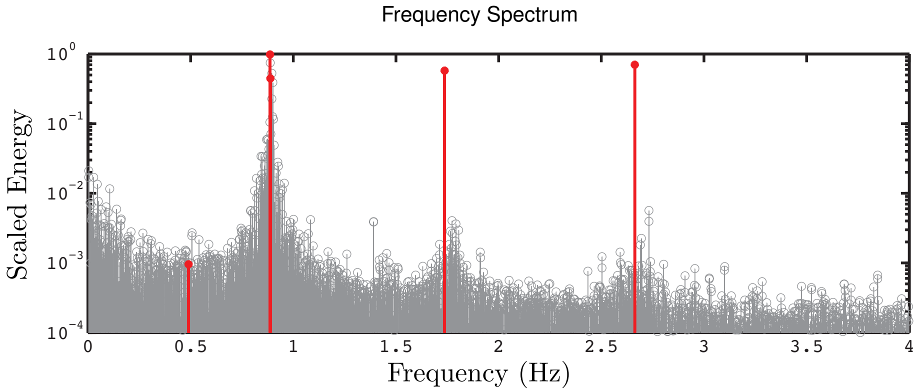


PIV data courtesy of Jessica Shang, U. Rochester.

Hemati, Williams, Rowley, Phys. Fluids, 2014.

Noise makes the data full-rank, regardless of the nature of the underlying dynamics.

$\rightarrow$ Apply POD Compression ($r = 25$)



Batch-Processed DMD: 3 Cores; Wall-clock $\sim \mathcal{O}$(hours)
Streaming DMD: My laptop; Wall-clock $\sim \mathcal{O}$(minutes)

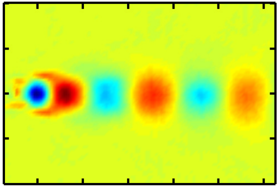
$n = 10800$, $m = 8000$

Hemati, Williams, Rowley, Phys. Fluids, 2014.

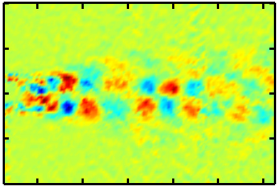
DMD for Large and Streaming Datasets

Batch-Processed DMD

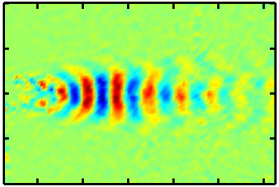
$f = 0.888$ Hz



$f = 1.744$ Hz

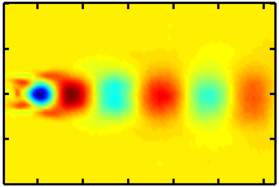


$f = 2.732$ Hz

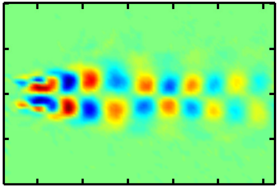


Streaming DMD

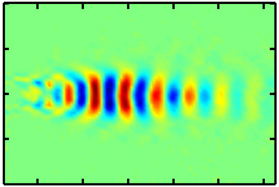
$f = 0.887$ Hz



$f = 1.737$ Hz



$f = 2.664$ Hz

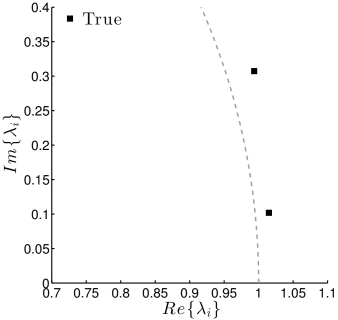


Hemati, Williams, Rowley, Phys. Fluids, 2014.

Prior to applying DMD to (noisy) experimental data, we *should* ask:

- How does measurement noise influence DMD analyses?
- Are such analyses representative of the “true” system dynamics?

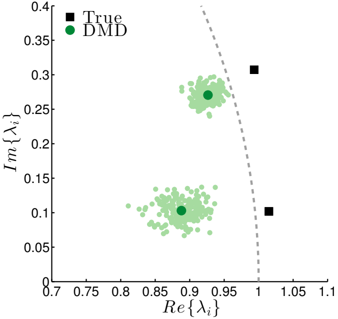
Example: A complex-valued linear system ($n = 250$, $r = 2$)



- Additive measurement noise $(\Delta X, \Delta Y) \sim \mathcal{CN}(0, 0.05)$.
- Computations repeated for 200 independent noise realizations.

DMD and Measurement Noise

Example: A complex-valued linear system ($n = 250$, $r = 2$)



$m = 100$

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Here, DMD identifies unstable eigenvalues as stable and decaying!

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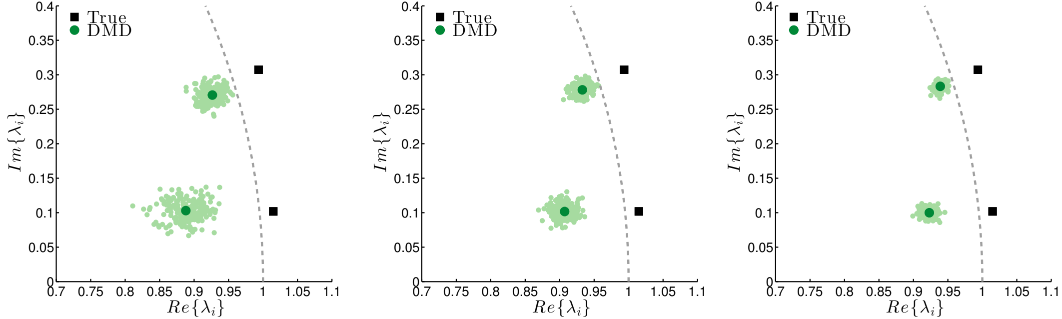

$m = 100$ $m = 200$ $m = 500$

- Additive measurement noise $(\Delta X, \Delta Y) \sim \mathcal{CN}(0, 0.05)$.
- Computations repeated for 200 independent noise realizations.

Here, DMD identifies unstable eigenvalues as stable and decaying!

Noise-Corrected DMD

Assume additive zero-mean i.i.d. noise with variance σ^2 on all snapshots (X, Y), and recall that $A = YX^+$ (or, $\tilde{A} = Q_X^\top YX^+ Q_X$).

For small noise, can correct for this error in DMD as

$$\tilde{A}_{\text{corrected}} = \tilde{A}\left(I - m\sigma^2\Sigma^{-2}\right)$$

$$\begin{aligned} m &:= \text{\# snapshots} \\ \Sigma &:= \text{matrix of non-zero singular values of } X \\ \sigma^2 &:= \text{measurement noise variance} \end{aligned}$$

Dawson, Hemati, Williams, Rowley, Exp. Fluids, 2016.

Why Does Measurement Noise Bias DMD?

Instead of relying on knowledge of the noise distribution, let's directly consider the *interpretation* of DMD as

$$A = YX^{+}.$$

Hemati, Rowley, Deem, Cattafesta, TCFD, 2017.

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In the over-constrained case ($i.e.$, $m > n$), this can be re-written as

$$\min_{A,\Delta Y} \|\Delta Y\|_F, \quad \text{subject to} \quad Y + \Delta Y = AX.$$

When snapshots are noisy, the residual ΔY can be interpreted as a "noise-correction."

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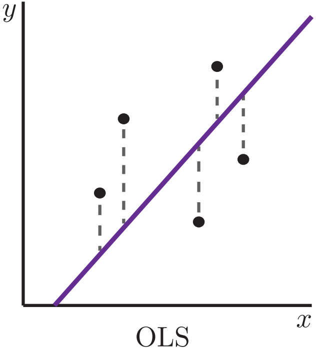
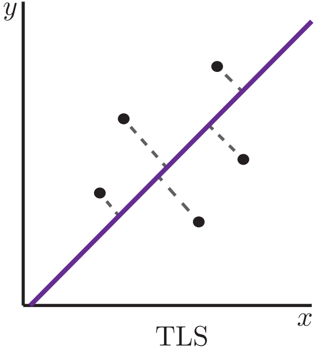
What about ΔX? $\longrightarrow$ Asymmetric treatment of noise!

Hemati, Rowley, Deem, Cattafesta, TCFD, 2017.

Instead, consider a problem of **total least-squares**:

$$\min_{A,\Delta X,\Delta Y} \left\| \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} \right\|_F, \quad \text{subject to} \quad Y + \Delta Y = A(X + \Delta X).$$





Hemati, Rowley, Deem, Cattafesta, TCFD, 2017.

A Noise-Robust "Total" DMD

A two-stage method for noise-robust "total" DMD (TDMD) analysis:

Stage 1: Subspace Projection

Define an augmented snapshot matrix $Z := \begin{bmatrix} X \\ Y \end{bmatrix}$,

then $\bar{Y} = Y\mathbb{P}_{Z_n^\top}$, $\bar{X} = X\mathbb{P}_{Z_n^\top}$,

where Z_n is the best rank-n approximation of Z.

*When the underlying dynamics are r-dimensional, replace n with r.
Results are "best" when $r \ll m$.

Stage 2: Operator Identification

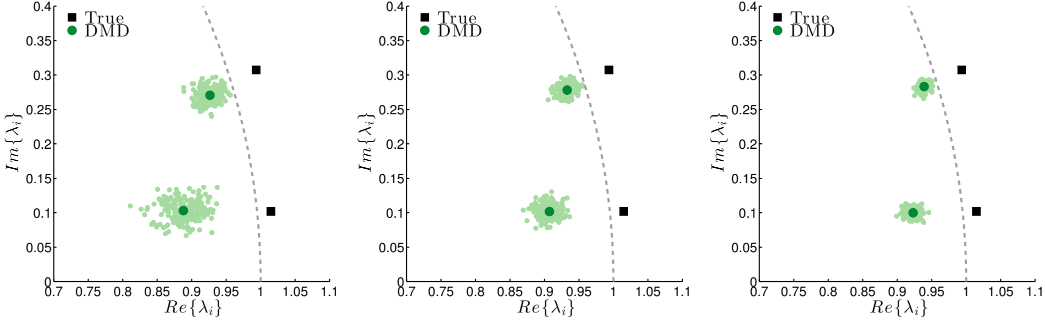
Perform DMD on the projected snapshots $\bar{X}$, $\bar{Y}$.

*Any variant of DMD can be used here (e.g., streaming DMD).

- the "de-biasing" occurs in the subspace projection stage.

Hemati, Rowley, Deem, Cattafesta, TCFD, 2017.

Example: A complex-valued linear system ($n = 250$, $r = 2$)



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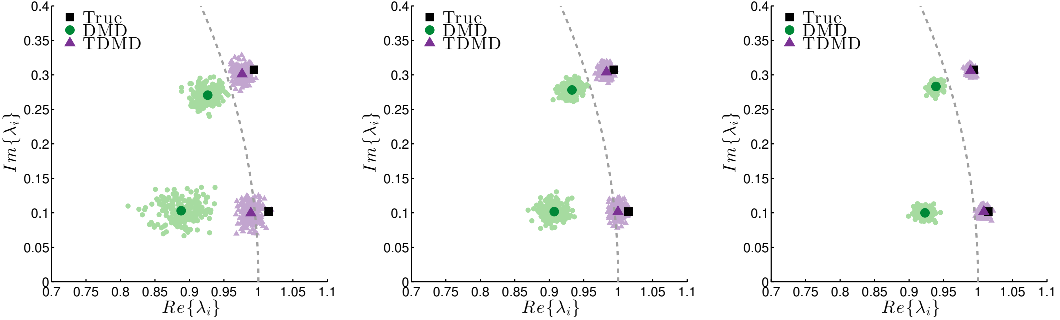

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Hemati, Rowley, Deem, Cattafesta, TCFD, 2017.


Image courtesy of www.dlr.de

Flow separation can degrade performance in many engineered systems:

- Decreased lift
- Increased drag
- Reduced efficiency
- Compromised control authority

Image courtesy of www.dlr.de

Flow separation can degrade performance in many engineered systems:

- Decreased lift
- Increased drag
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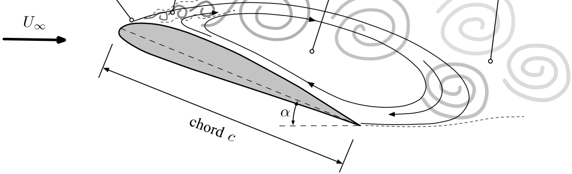
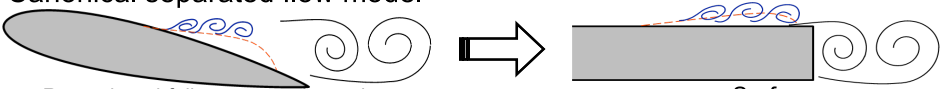
- Compromised control authority

Separated flow past an airfoil is characterized by frequencies associated with the

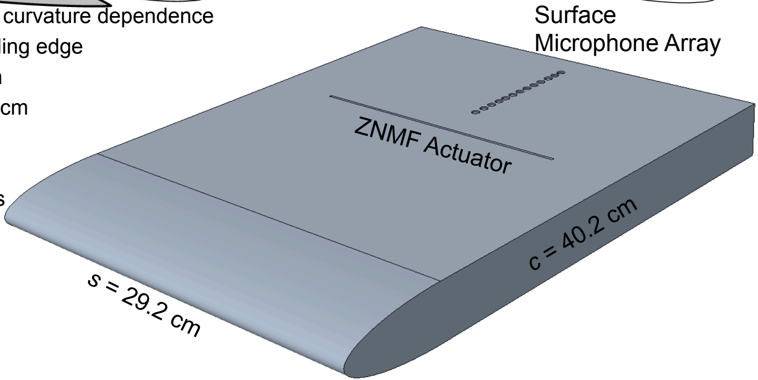
- wake
- shear layer (SL)
- separation bubble (SB)
- actuation (if applied)



Canonical separated flow model



- Removing airfoil curvature dependence
- 4:1 elliptical leading edge
- Chord = 40.2 cm
- Thickness = 3.8 cm
- Span = 29.2 cm
- $Re_c = 10^5$
- 13 WM-61A mics
- Spacing: 0.02c



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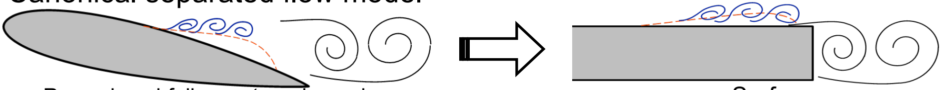

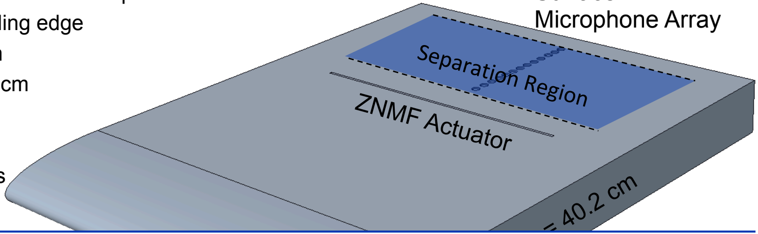

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Separation induced by blowing/suction on tunnel ceiling [1]

- Retains essential separation characteristics [2]
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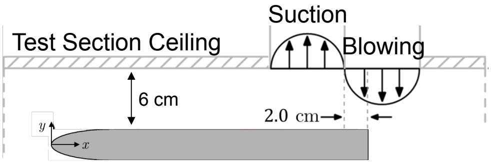
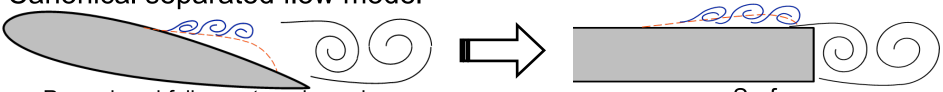
- Amenable to both simulations and experiments



[1] Na and Moin. "Direct numerical simulation of a separated turbulent boundary layer", J. Fluid Mech. 1998-370
[2] Mittal et al. "Numerical study of resonant interactions and flow control", AIAA 2005-1261

Canonical separated flow model



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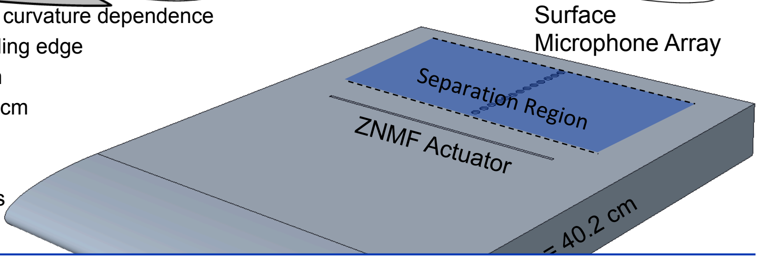

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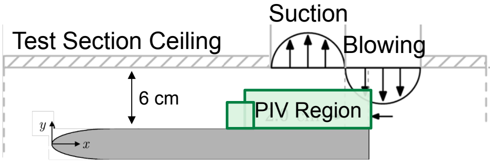

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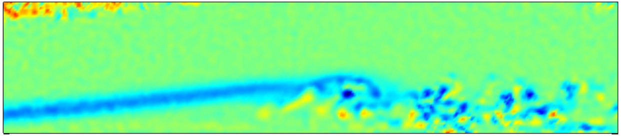
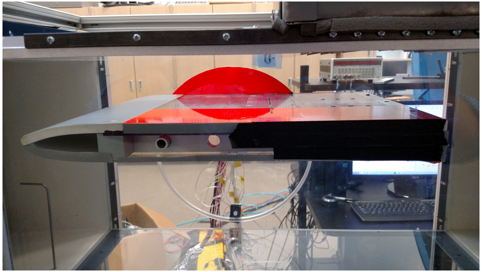
DMD analysis of TR-PIV data of canonically separated flow experiment ($Re = 10^5$)

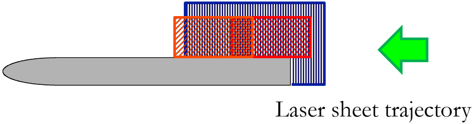


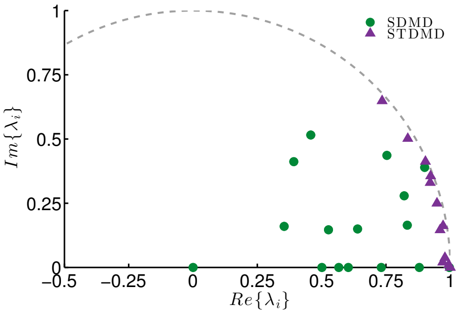
video slowed 40×

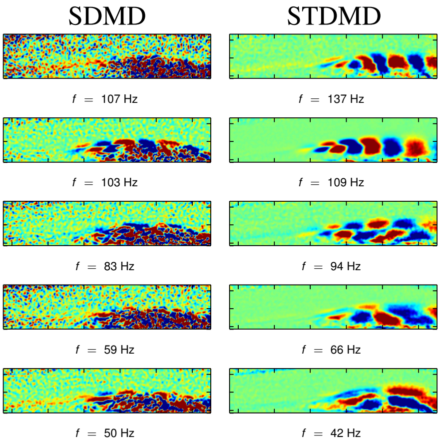
FSU Flow Control Wind Tunnel

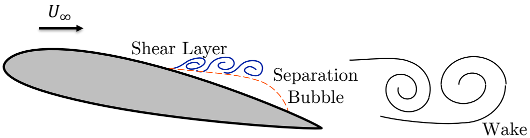


$n = 42\,976$, $m = 3000$
Experimental details in
Griffin et al. (AIAA Paper 2013-2968).



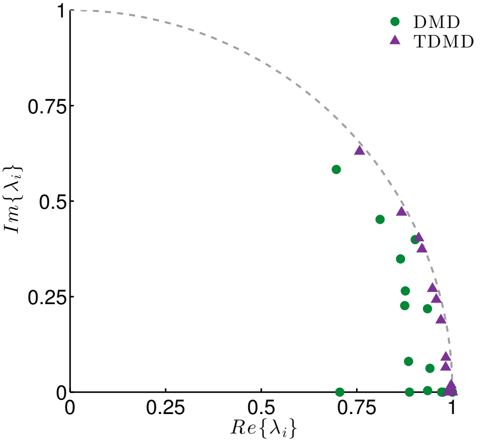


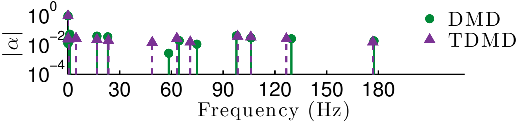


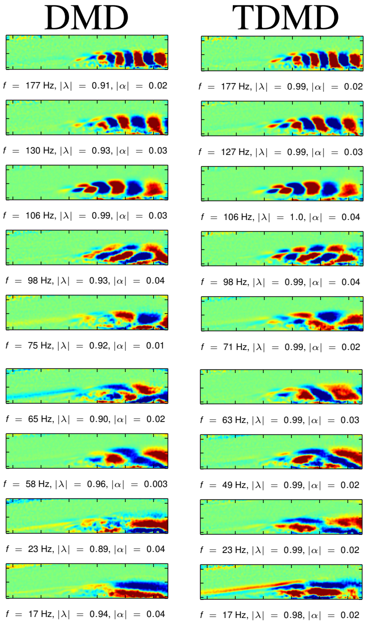


Hemati et al., AIAA Paper 2016-1103.

DMD Analysis of Canonically Separated Flow TR-PIV Data







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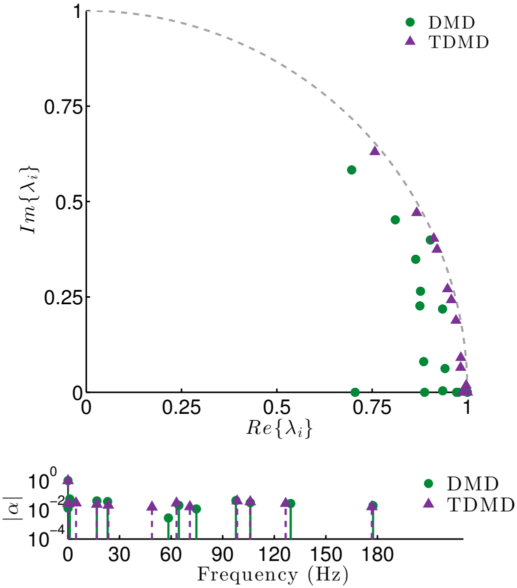
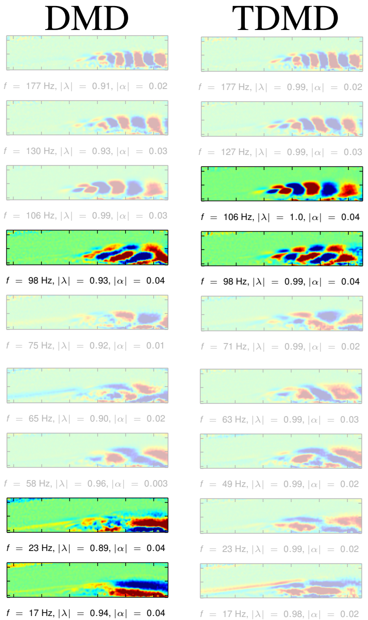
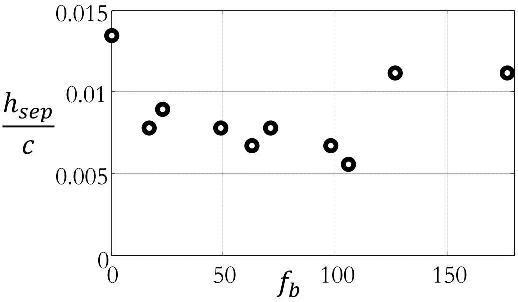


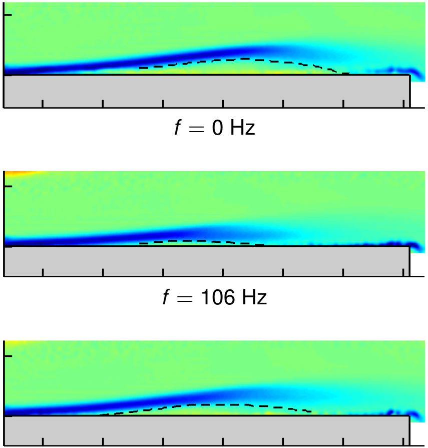


Targeted Open-Loop Actuation for Separation Control

Mean separation bubble height is **smallest** when ZNMF is forced at the **dominant DMD frequency** ($f_b = 106$ Hz).



Time-averaged vorticity



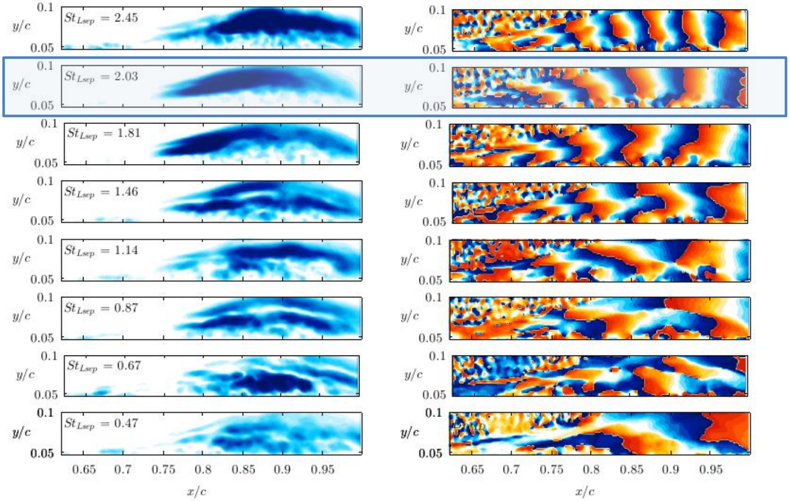
$f = 0$ Hz

$f = 106$ Hz

$f = 127$ Hz

TDMD of Baseline Separated Flow:

- Dimension of snapshot: n = 20,064, Number of snapshots: m = 10,000, Rank: r = 25



Deem et al., AIAA Paper 2018-1052.

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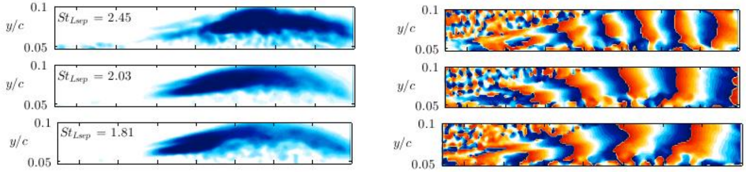
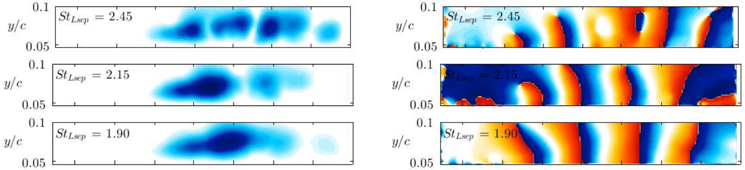


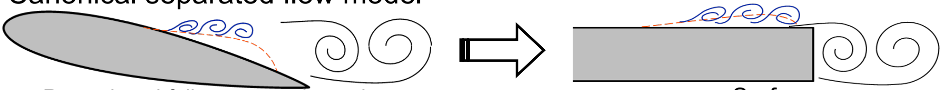
TDMD of Pressure Field:



Similar dynamical characteristics → Surface pressure based ROM

Deem et al., AIAA Paper 2018-1052.

Canonical separated flow model



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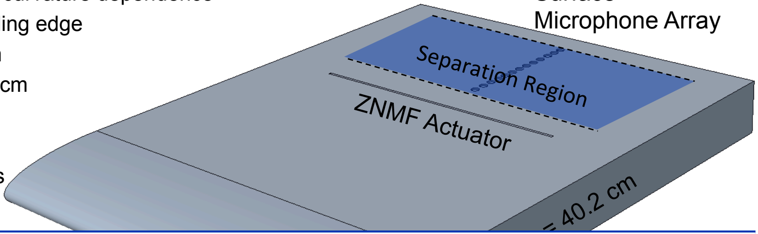

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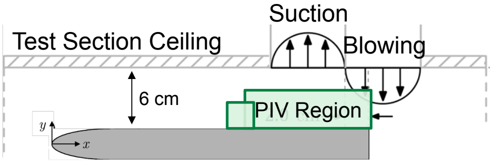

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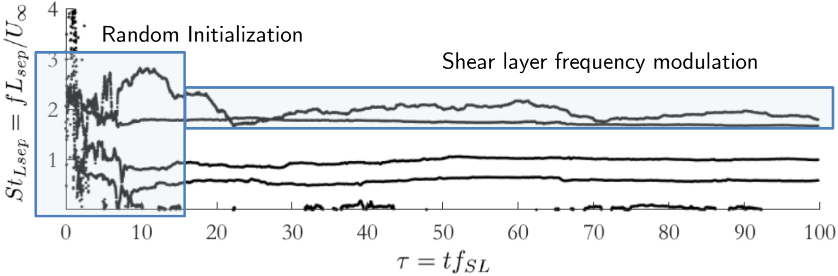
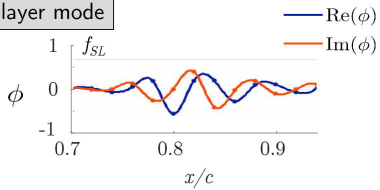
Deem et al., AIAA Paper 2018-1052.

Pressure measured by microphone array ($n = 13$)
Online DMD used to extract time-varying frequencies (Zhang et al., 2017)



Representative shear layer mode



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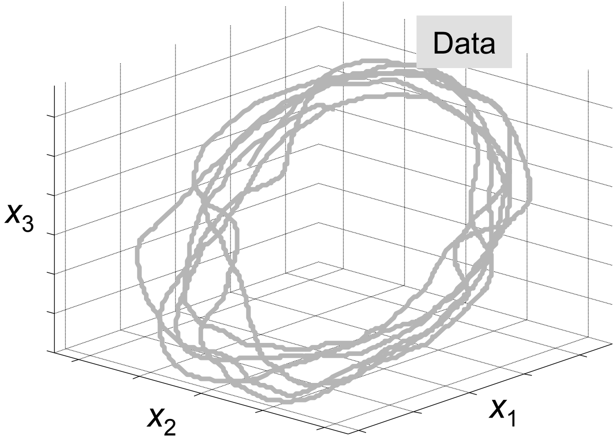
DMD for Adaptive/Model-Predictive Control



Deem et al., AIAA Paper 2018-1052.

Related work:

- DMD with Control (Proctor et al., 2016)
- Online DMD for TV Systems (Zhang et al., 2017)
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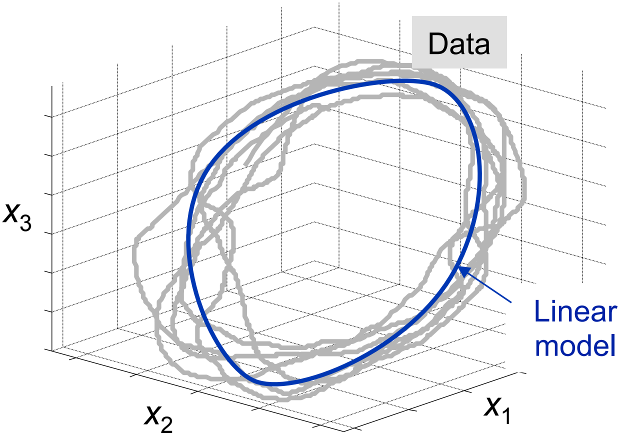

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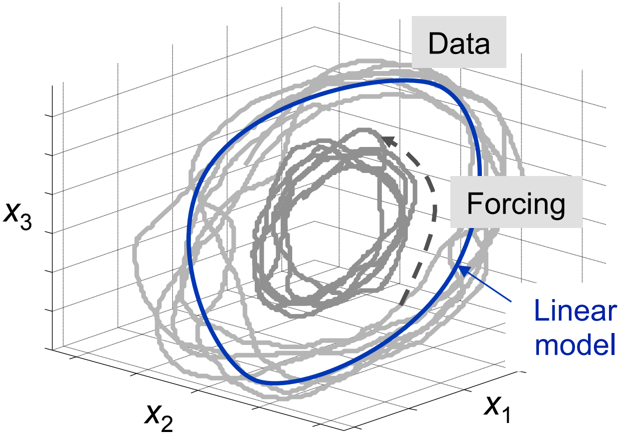

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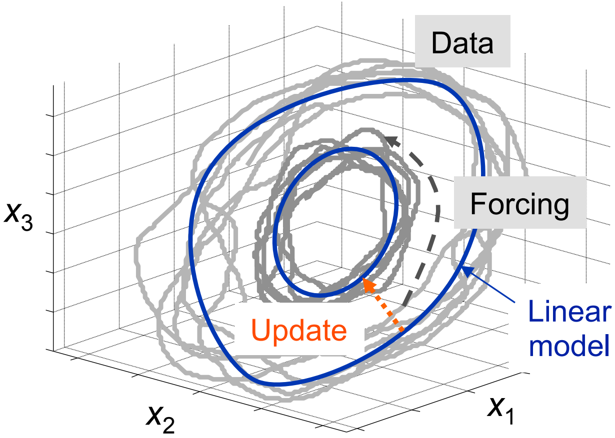

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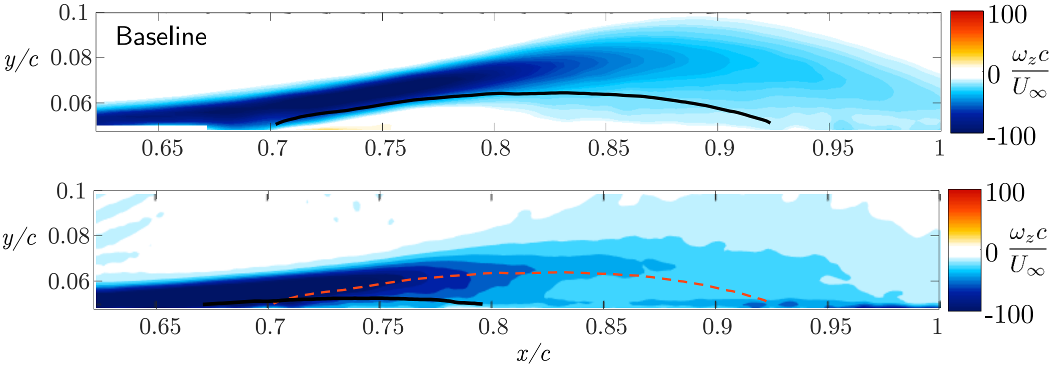
Adaptive/Model-Predictive Control of Separated Flow

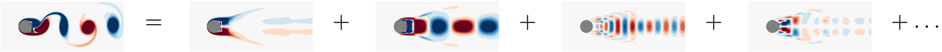
- Suppress surface pressure fluctuations
- Primary control loop rate: $125 f_{SL}$ (10 kHz)
- LQR gains updated at $4 f_{SL}$ (0.33 kHz)
- $Q_{lqr} = 20 \times I$, $R_{lqr} = 0.3$
- Online DMD weighting factor: $\kappa = 0.99995$
  $\rightarrow$ 50% snapshot attenuation after $\tau \approx 227$



Deem et al., AIAA Paper 2018-1052.

Conclusions

- DMD/Koopman offer powerful perspectives for analyzing fluid flows (and other systems).
- Volume, velocity, and veracity are practical challenges that must be considered in practice.



Software available at http://z.umn.edu/dmdtools

Acknowledgements:

- Lou Cattafesta (FSU)
- Scott Dawson (IIT)
- Eric Deem (Lockheed Martin)
- Clancy Rowley (Princeton)
- Matt Williams (Oceanit)
- AIAA DGs on Modal Analysis and Reduced-Complexity Modeling

Supported by:

- Air Force Office of Scientific Research

Q&A

- Theory
- Computations
- Applications

Software Resources:

- mathLab/PyDMD - GitHub
- dmdbook.com
- z.umn.edu/dmdtools