

# SVD Approximations for Large Scale Imaging Problems

James Nagy

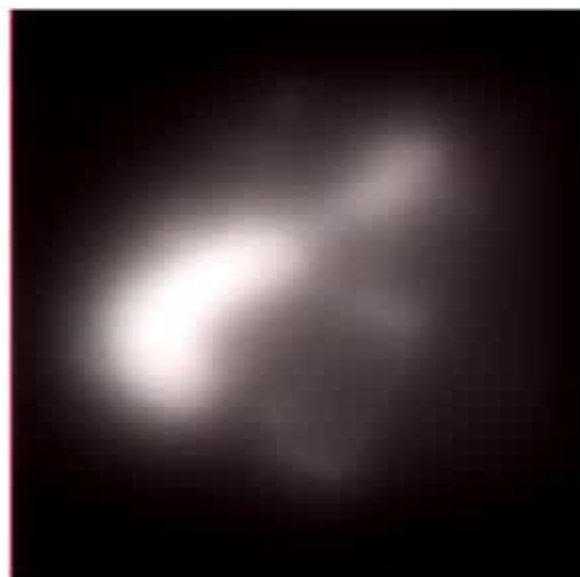
Mathematics and Computer Science  
Emory University  
Atlanta, GA USA

## Better SVD Approximations

- Compute decomposition:  $\mathbf{K} = \sum_{k=1}^r \mathbf{A}_k \otimes \mathbf{B}_k$
- Compute:  $[\mathbf{U}_a, \mathbf{\Sigma}_a, \mathbf{V}_a] = \text{svd}(\mathbf{A}_1)$ ,  $[\mathbf{U}_b, \mathbf{\Sigma}_b, \mathbf{V}_b] = \text{svd}(\mathbf{B}_1)$
- Let:  $\tilde{\mathbf{U}} = \mathbf{U}_a \otimes \mathbf{U}_b$ ,  $\tilde{\mathbf{V}} = \mathbf{V}_a \otimes \mathbf{V}_b$ ,  $\tilde{\mathbf{\Sigma}} = \mathbf{\Sigma}_a \otimes \mathbf{\Sigma}_b$
- Find a low rank approximation of  $\mathbf{K}$  using  $\tilde{\mathbf{U}}$  and  $\tilde{\mathbf{V}}$ :
 
$$\mathbf{K}_r = \tilde{\mathbf{U}}_r^T \mathbf{K} \tilde{\mathbf{V}}_r$$
- $\mathbf{K}_r$  is a “small”  $r \times r$  matrix, so compute its SVD:  $\mathbf{K}_r = \mathbf{U}_r \mathbf{\Sigma}_r \mathbf{V}_r^T$ .
- Then we can compute an SVD approximation of the original  $\mathbf{K}$  as:

$$\mathbf{K} \approx \tilde{\mathbf{U}}_r \mathbf{U}_r \mathbf{\Sigma}_r \mathbf{V}_r^T \tilde{\mathbf{V}}_r^T$$

## Image Deblurring Example



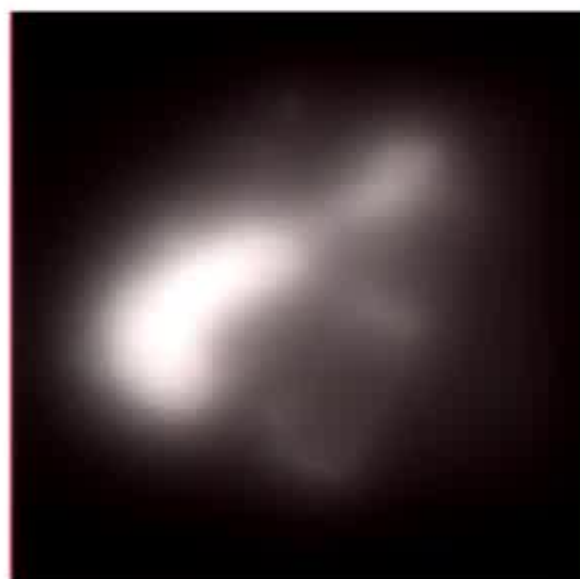
observed image

SVD approx  
1 Kron termSVD approx  
5 Kron terms

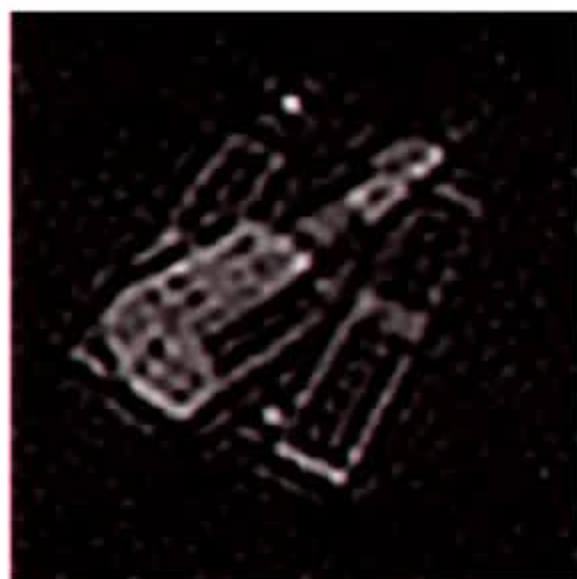
In this example:

- $\mathbf{K}$  is  $N \times N$ , with  $N = 65,536 \times 65,536$ .
- We used  $r = 2,601$ , which means that  $\mathbf{K}_r$  is an  $2,601 \times 2,601$  matrix.

## Image Deblurring Example



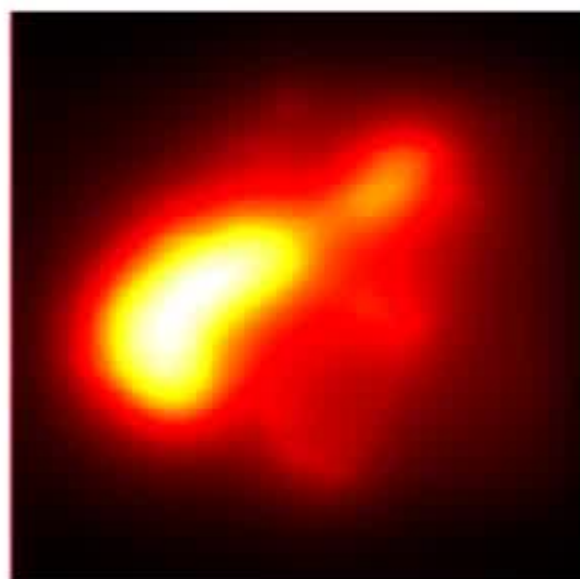
observed image

SVD approx  
1 Kron termSVD approx  
5 Kron terms

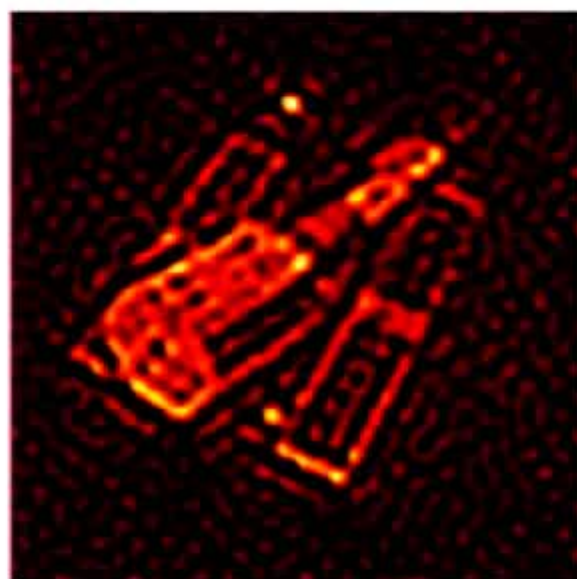
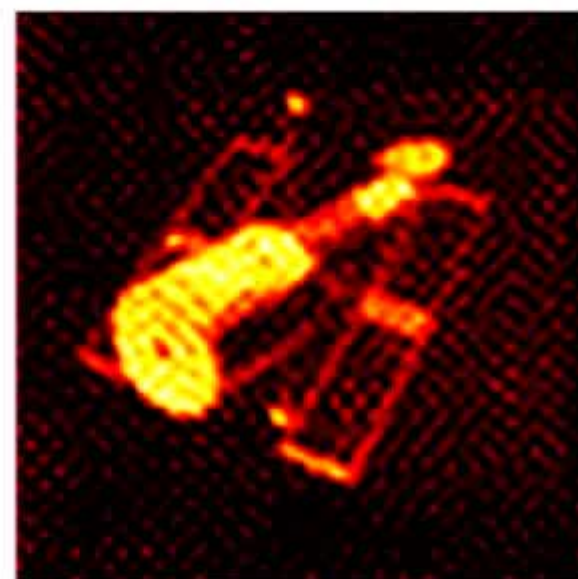
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## Image Deblurring Example



observed image

SVD approx  
1 Kron termSVD approx  
67 Kron terms

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