Designing noisy networks

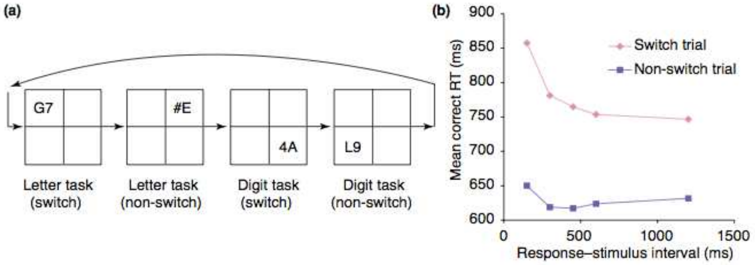
Claire Postlethwaite

with: Peter Ashwin (University of Exeter)

Department of Mathematics
University of Auckland

Task switching

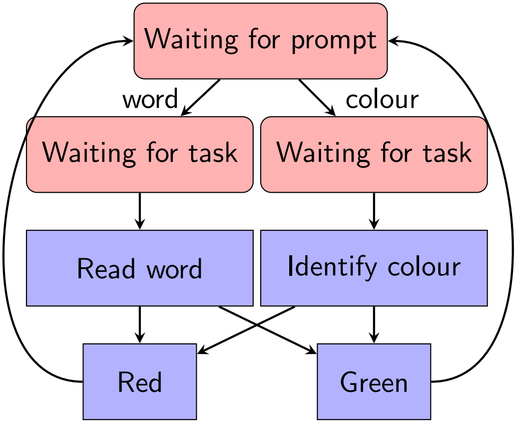
Switch cost



Can we model this in a useful way?

Image from: Rogers, R.D. and Monsell, S. (1995) The costs of a predictable switch between simple cognitive tasks. *J. Exp. Psychol. Gen.* 124, 207–231

Task switching



Outline

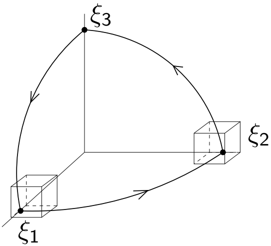
Heteroclinic cycles

Consider the ODE:

$$\dot{x} = f(x), \qquad x \in \mathbb{R}^n. \tag{1}$$

- An equilibrium ξ of (1) satisfies $f(\xi) = 0$.
- A solution ϕ_j of (1) is a heteroclinic connection from ξ_j to ξ_{j+1}, if it is backward asymptotic to ξ_j and forward asymptotic to ξ_{j+1}.

- A heteroclinic cycle is a set of equilibria $\{\xi_1, \ldots, \xi_m\}$ and orbits $\{\phi_1, \ldots, \phi_m\}$, where ϕ_j is a heteroclinic connection between ξ_j and ξ_{j+1}, and $\xi_1 \equiv \xi_{m+1}$.



Noisy heteroclinic cycles

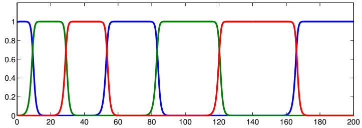
Stone and Holmes, 1990

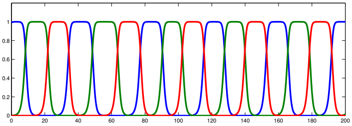
- Consider additive noise to a heteroclinic cycle, i.e. the SDE

$$dx_t = f(x_t) + \eta dW_t, \; x_t \in \mathbb{R}^n$$

  - W_t is n-dimensional Brownian motion
  - η is noise amplitude.
- Mean passage time past an equilibrium

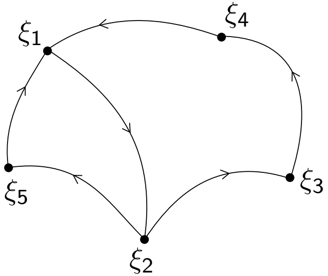
$$T \sim \frac{\log \eta}{\lambda_u}$$





Heteroclinic networks

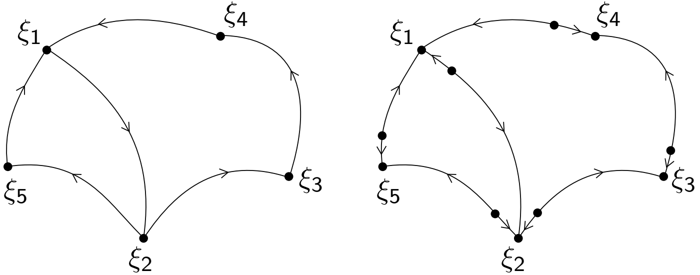
- A heteroclinic network is a connected union of heteroclinic cycles.



- Stability conditions of the network as a whole may be quite complicated.
- Nearby trajectories may switch between different sub-cycles of the network.
- Noise can have unexpected effects.

Excitable networks

- Consider as a pitchfork bifurcation from a heteroclinic network.
- Original equilibria are now all stable.
- Small amplitude perturbations can push trajectories between equilibria.



Outline

Designing networks

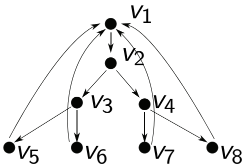
Consider using heteroclinic networks to model neural processes. Design a graph structure describing the process, each node corresponds to an equilibrium.

Some questions:

- For a given graph, can we construct an ODE which contains that graph embedded as a heteroclinic or excitable network?
- Can we control the residence times near the equilibria?
- Can we control the switching probabilities between nodes?

Network construction

Ashwin and P, 2014, 2015

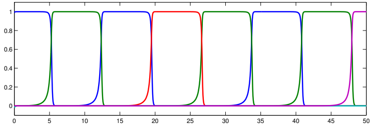


- Consider a graph with n_v vertices and n_e edges.
- Several construction methods:
  - Simplex network: in $\mathbb{R}^{n_v}$, each vertex is an equilibrium on a coordinate axes, connections live in two-dimensional coordinate planes.
  - Cylinder network: in $\mathbb{R}^{n_e+1}$, vertices lie in a line, each connecting orbit lives in a two-dimensional plane.
  - Excitation-inhibition network: in $\mathbb{R}^{n_e+n_v}$, reminiscent of neuronal systems.
  - Others...

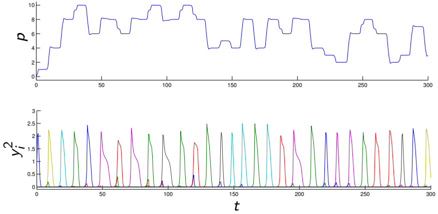
Network construction: Time series

Ashwin and P, 2014, 2015

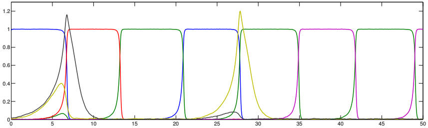
- Simplex method:



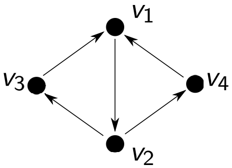
- Cylinder method:



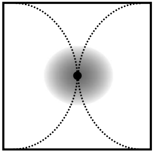
- Excitation-inhibition:

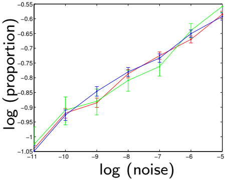


Armbruster, Stone and Kirk, 2003



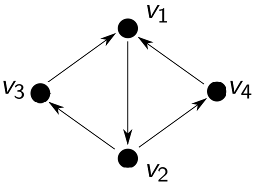
- How does noise affect the dynamics near the network?
    - Residence times
    - Switching rates
- Consider a probability density function of trajectories, assume it is centered at the origin.
- Proportion of times each cycle visited proportional to shaded area.



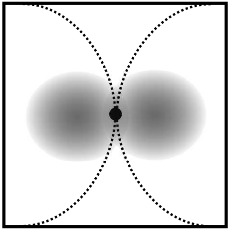


Network statistics: Noisy Kirk and Silber network

Armbruster, Stone and Kirk, 2003



- For certain parameter sets, noise ellipse can move into basin of attraction of one cycle or the other.
- This is termed *lift-off*.
- Lift-off can *reduce* switching, and cause *memory*.



Escape rates

- To compute mean residence times near equilibria, we compute *escape rates* from a potential well.
- Consider

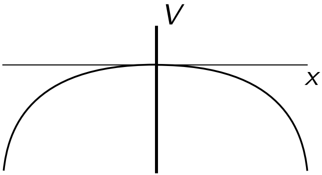
$$dx = -V'(x)dt + \eta dW_t$$

with potential

$$V(x) = \frac{\nu x^2 - x^4}{2}$$

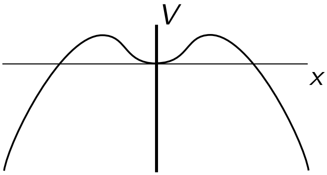
$\nu < 0$

Heteroclinic connection



$\nu > 0$

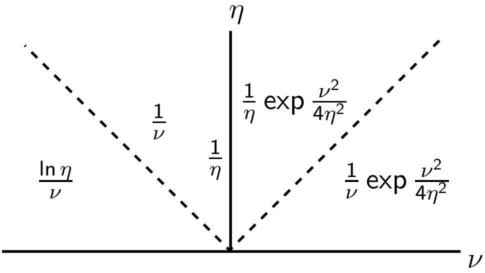
Excitable connection



Mean residence times

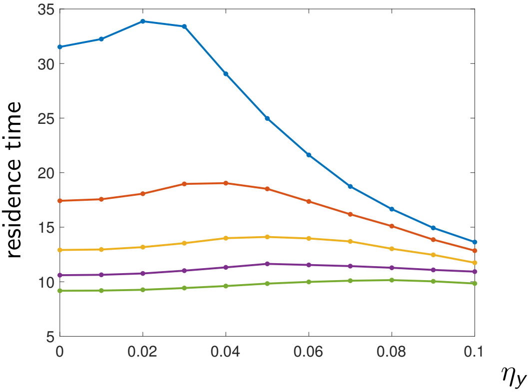
- Compute the length of time spent near $x = 0$ when η and ν are small.

$$\overline{T}(\nu, \eta) = \frac{2}{\eta^2} \int_{z=0}^{a} \int_{y=0}^{z} \exp \frac{\nu(z^2 - y^2) + (y^4 - z^4)}{\eta^2} \, dy \, dz.$$



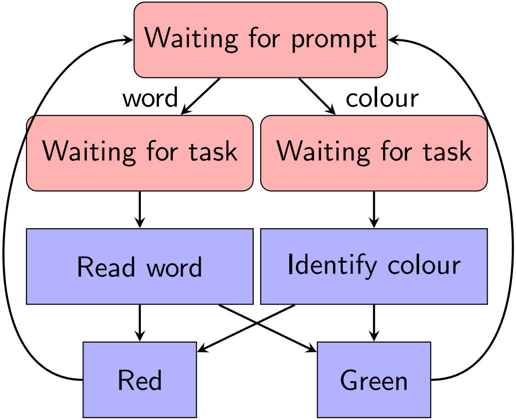
Mean residence times with forcing

- Consider a similar system, now non-autonomous, and ‘forcing’ can push the trajectories in a desired direction around the network.



Outline

Task switching



Outline

Summary

- We can design heteroclinic and excitable networks to embed any specified graph in phase space.
- We can adjust noise parameters to fit mean residence times.
- Switching probabilities, residence time distributions, anisotropic noise....

Acknowledgements

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