Singular Values and Convex Programming for Power System Synchrophasor Data Management

Meng Wang

Assistant Professor
Dept. of ECSE
Rensselaer Polytechnic Institute

SIAM CSE 2015
March 18, 2015

Acknowledgement

- Mr. Pengzhi Gao (RPI)
- Prof. Joe Chow, Dr. Scott Ghiocel (RPI)
- Dr. Bruce Fardanesh, Dr. George Stefopoulos. (New York Power Authority)

nyserda
Energy. Innovation. Solutions.

CURENT

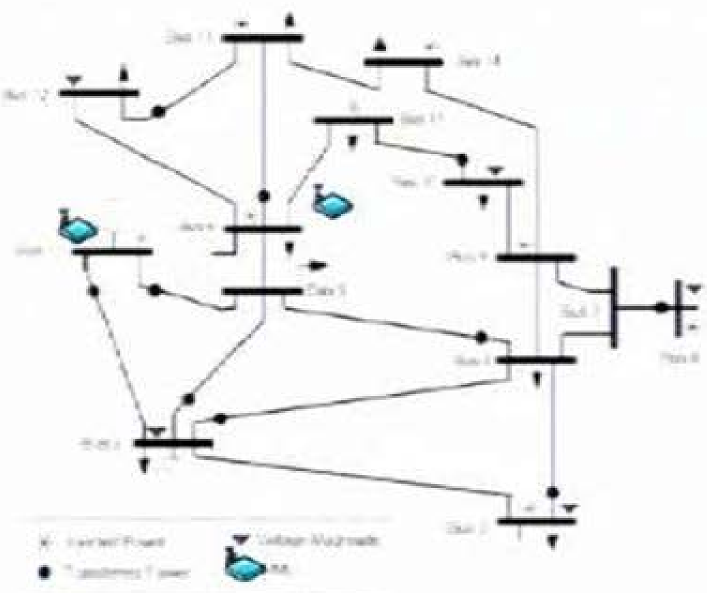
Outline

1. Background and Challenges of PMU Data Processing
2. A Low-rank Framework of PMU Data Processing
3. Missing Data Recovery
4. Detection of Cyber Data Attacks

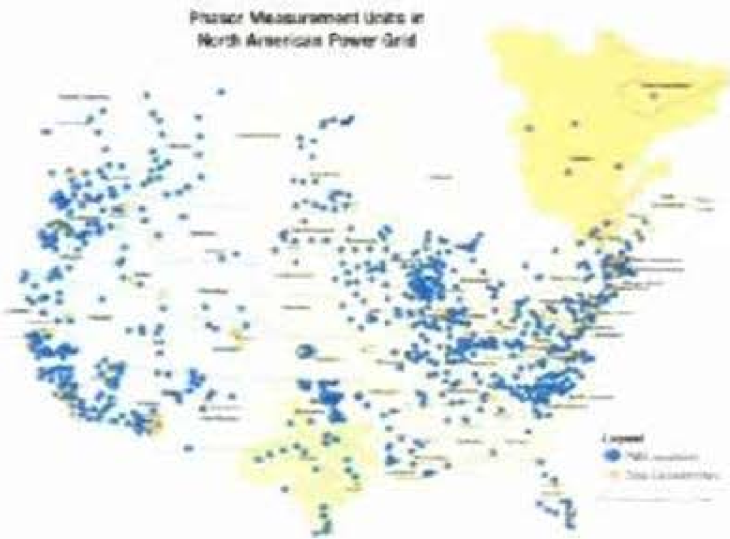
SIAM2015.pdf

Phasor Measurement Units

- PMUs (Phasor Measurement Units) provide synchronized phasor measurements at a sampling rate of 30 (or 60) samples per second.
- Multi-channel PMUs can measure bus voltage phasors, line current phasors, and frequency.



http://www.riteh.uniri.hr/zav_katd_sluz/zee/nzz/klub/images/IEEE14.JPG



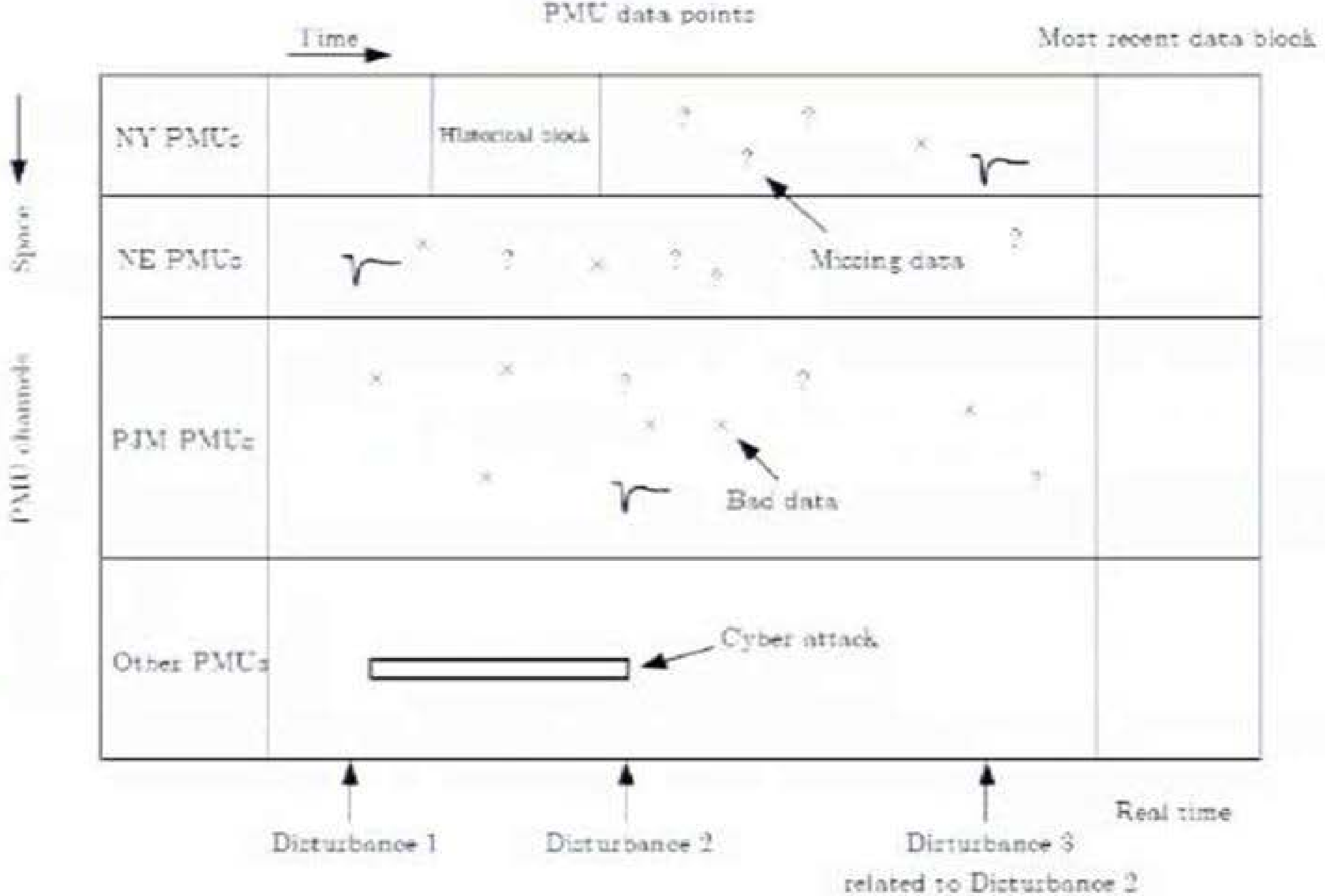
Installation (NASPI Oct. 2013)
1100+ PMUs, 150+ data concentrators

https://www.naspi.org/documents

Big Data in Power Systems

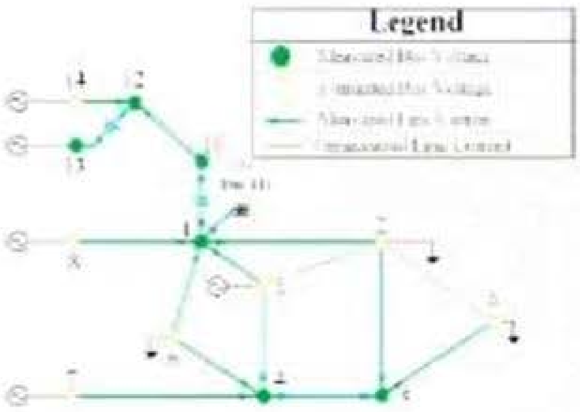
- PMU data is envisioned to provide the following capabilities:
  - Improved accuracy of power system state estimation
  - Disturbance location and recognition (what kind of disturbance, e.g., loss of generation, loss of line)
  - Assessing the severity of the disturbance and its impact on the power system
- PMU data is considered to be a source of Big Data in power systems.
  - Control regions such as New York and New England, will have about 40 PMUs each, with 6-12 data channels per PMU — averaging one PMU per 1,000 MW of generation.

Space-Time View of PMU Data

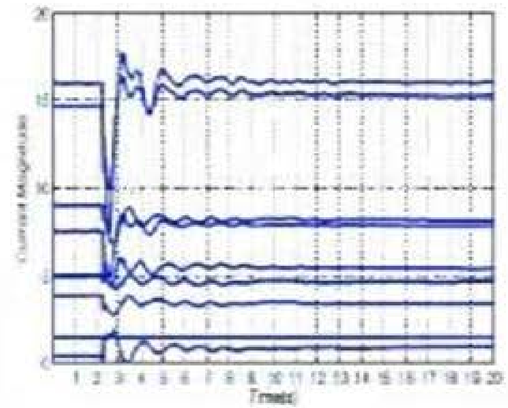


Low-rankness of PMU data blocks

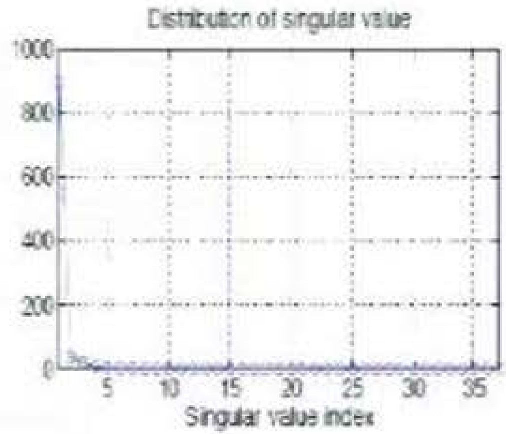
Low-rank Property of PMU data



PMUs in Central NY Power Systems



Current magnitudes of PMU data



Singular values of the PMU data matrix

- 6 PMUs measure 37 voltage/current phasors. 30 samples/second for 20 seconds.
- Singular values decay significantly. Mostly close to zero. Singular values can be approximated by a sparse vector.
- Low-dimensionality also used in Chen & Xie & Kumar 2013, Dahal & King & Madani 2012 for dimensionality reduction.

A Low-rank Approach of Data Processing

- Power system is an interconnected network–data measured at various buses will be driven by some underlying system condition.
- The system condition may change, but some consistent relationship between the PMU data from different nearby buses always exists.

A Low-rank Approach of Data Processing

- Power system is an interconnected network–data measured at various buses will be driven by some underlying system condition.
- The system condition may change, but some consistent relationship between the PMU data from different nearby buses always exists.
- Mathematical characterization: low-rankness of spatio-temporal PMU data blocks. No need of power system modeling.

Low-Rank Matrix Analysis for PMU Data

Process spatio-temporal blocks of PMU data for

- Missing PMU data recovery — low-rank matrix completion, missing data in correlated locations.
- Detection of cyber data attacks — matrix decomposition of a low-rank matrix and a transformed column-sparse matrix, convex-programming-based method.
- PMU data compression — inter-channel compression plus intra-channel compression.
- Disturbance detection — when dominating singular values change.

Wang, Chow, *et al.*, Hawaii International Conference on System Sciences 2015

Outline

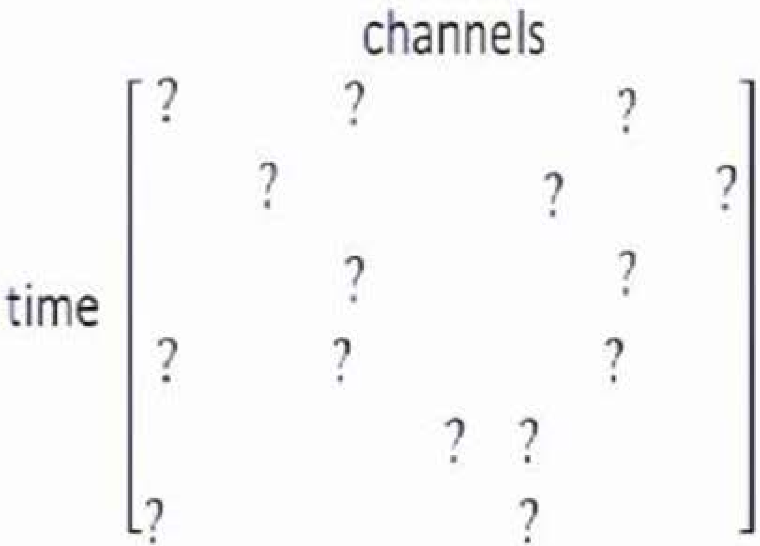
1. Background and Challenges of PMU Data Processing
2. A Low-rank Framework of PMU Data Processing
3. Missing Data Recovery
4. Detection of Cyber Data Attacks

Missing Data Recovery

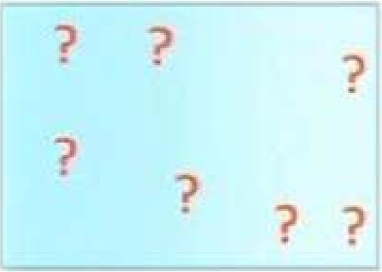
- Data losses happen due to PMU malfunction or communication congestion between PMUs and Phasor Data Concentrator (PDC).
- Existing missing data recovery: interpolation from measurements in the same channel.

Missing Data Recovery

- Data losses happen due to PMU malfunction or communication congestion between PMUs and Phasor Data Concentrator (PDC).
- Existing missing data recovery: interpolation from measurements in the same channel.
- Our approach: leverage low-rankness of PMU data blocks.

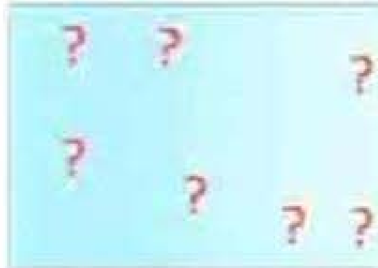


Low-rank Matrix Completion



Low-rank Matrix with Missing Entries

Low-rank Matrix Completion



Low-rank Matrix with Missing Entries

- Nuclear norm minimization (Fazel 2002), recover the missing data by solving a convex program.

$$\min_X \|X\|_* = \text{sum of singular values of } X$$

$$\text{s.t. } X \text{ is consistent with the observed entries.}$$

- Quite a few recovery algorithms exist, e.g., singular value thresholding (SVT) (Cai *et al.* 2010), information cascading matrix completion (ICMC) (Meka *et al.* 2009).
- Applications in collaborative filtering, computer vision, machine learning, remote sensing, and system identification.
- Applications in power systems: load forecasting (Mateos,Giannakis 2013), electricity market inference (Kekatos,Zhang,Giannakis 2014).

Low-rank Matrix Completion

Candés & Recht 09, Gross 11, Recht 11

All entries of a rank-r matrix $L \in \mathbb{C}^{n_1 \times n_2}$ can be corrected recovered, as long as $O(rn\log^2 n)(n = \max(n_1, n_2))$ randomly selected entries of L are observed.

- Significant saving in the number of observations when r is small.
- Existing analysis assumes that the locations of missing points are selected randomly.

Missing Data at Correlated Locations

The locations of missing PMU data are usually correlated.

- temporal correlation: loss of consecutive measurements in one PMU channel.
- spatial correlation: loss of measurements in multiple PMU channels simultaneously.

Missing Data at Correlated Locations

The locations of missing PMU data are usually correlated.

- temporal correlation: loss of consecutive measurements in one PMU channel.
- spatial correlation: loss of measurements in multiple PMU channels simultaneously.

Recovery guarantee of missing data at correlated locations?

Our Model of correlated missing points:

- temporal correlation: with prob. $1-p$, τ consecutive erasures staring from a fix data point in one channel.
- spatial correlation: with prob. $1-q$, all measurements in a d-channel PMU are lost at a fixed sampling instant.

Theoretical Results

Recovery of temporally correlated missing points

For every positive γ, there exists a positive constant $c(\gamma)$ such that if

$$p \geq \left(\frac{c(\gamma)\tau r \log n}{n}\right)^{\frac{1}{\tau(r+1)}} \qquad (1)$$

holds, then ICMC correctly recovers M with probability at least $1 - n^{-\gamma}$.

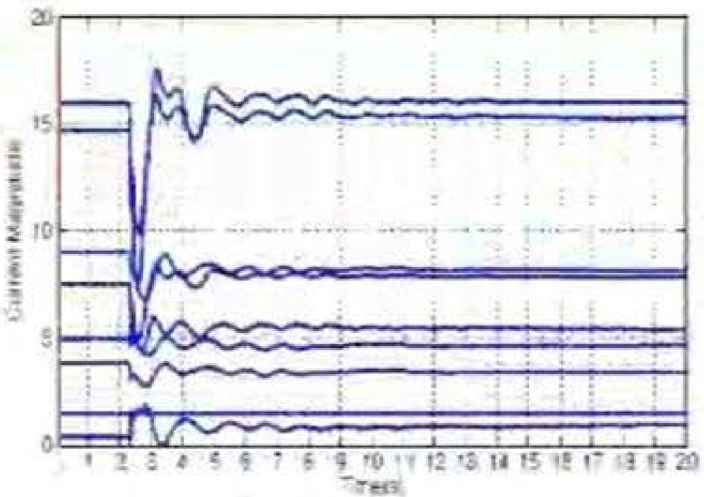
Correct recovery from partial measurements

Although the locations of the missing entries of a rank-r matrix are temporally or spatially correlated, all missing entries can be correctly recovered as long as $O(n^{2-\frac{1}{r+1}} r^{\frac{1}{r+1}} \log^{\frac{1}{r+1}} n)$ entries are observed.

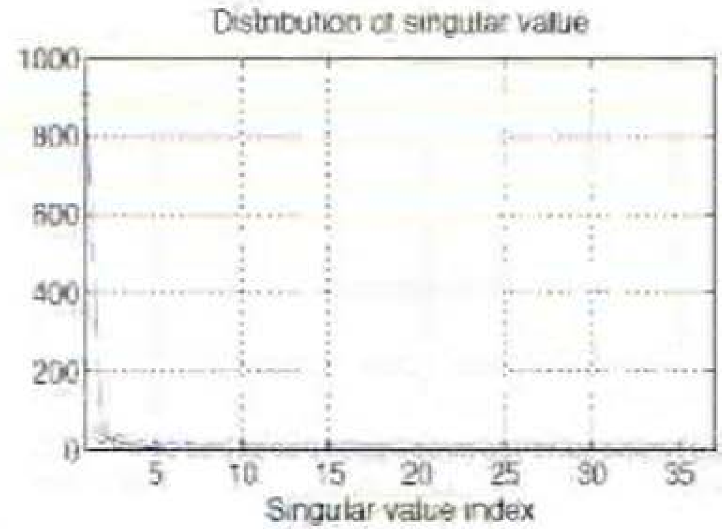
Gao, Wang, Ghiocel, Chow, IEEE Power & Energy Society General Meeting 2014, accepted to IEEE Trans. Power Systems 2015

Simulation

- 6 PMUs measuring 37 bus voltage phasors and line current phasors.
- 30 samples per second.



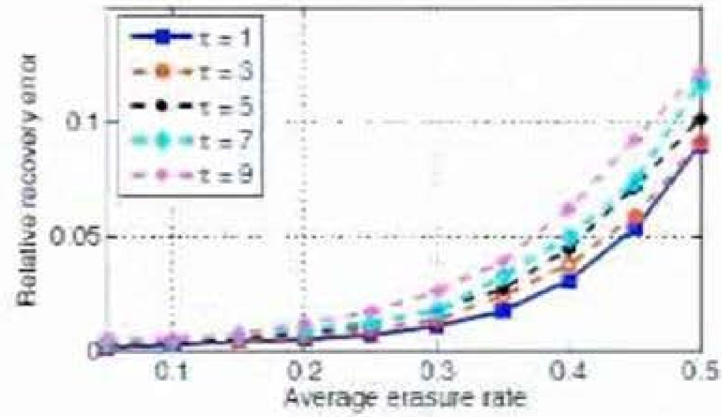
Current magnitudes of PMU data



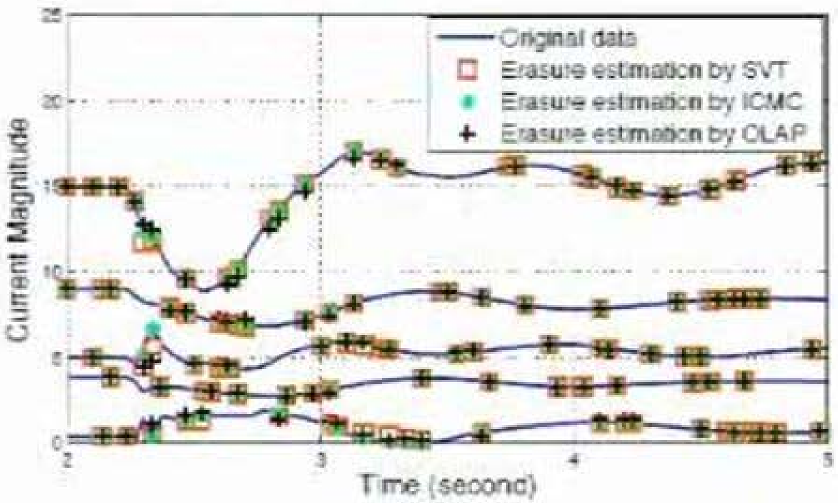
Singular values of the PMU data matrix

Recovery of Temporally Correlated Erasures

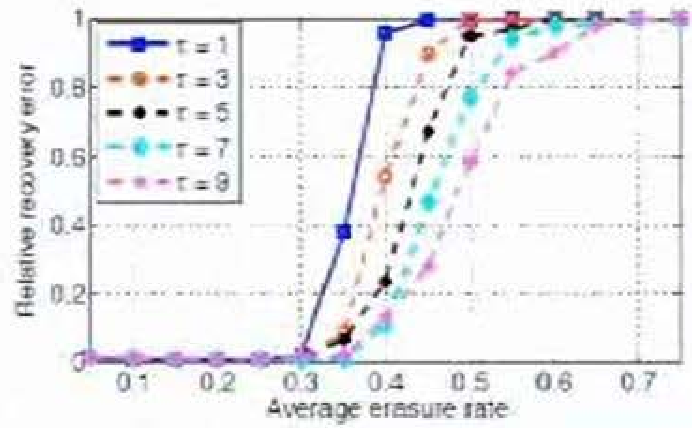
If a channel in a particular PMU is lost at a particular time, there is a probability that τ trailing data points will also be lost.



Relative recovery error of SVT



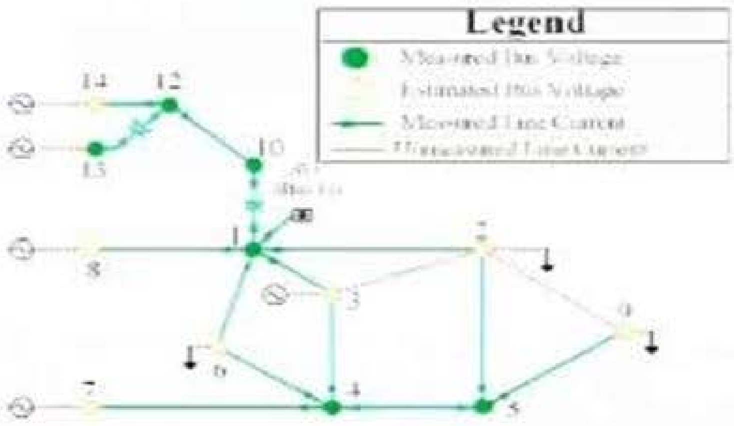
Missing data recovery by OLAP algorithm



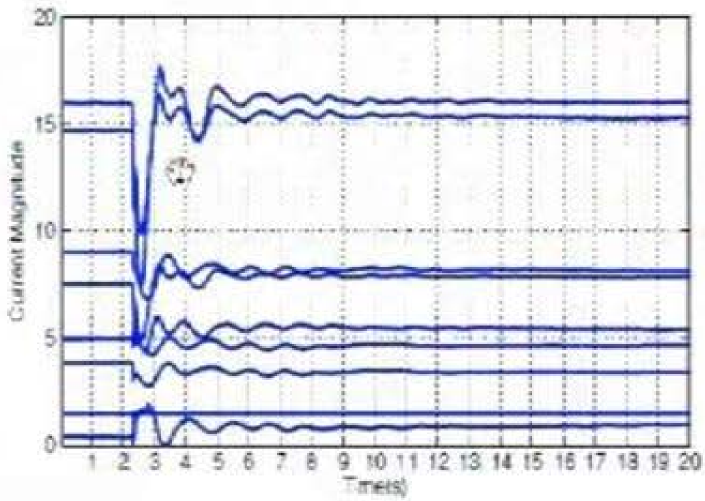
Relative recovery error of ICMC

Gao, Wang, Ghiocel, Chow, IEEE Power & Energy Society General Meeting 2014, submission to IEEE Trans. Power Systems 2014

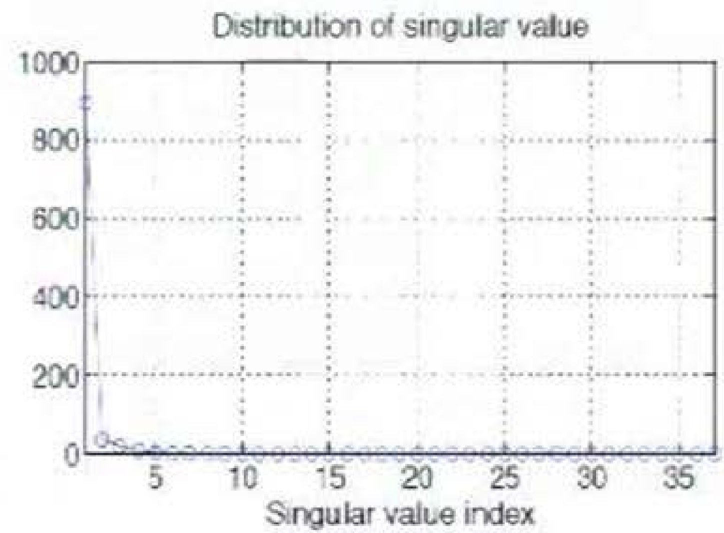
Simulation



- 6 PMUs measuring 37 bus voltage phasors and line current phasors.
- 30 samples per second.



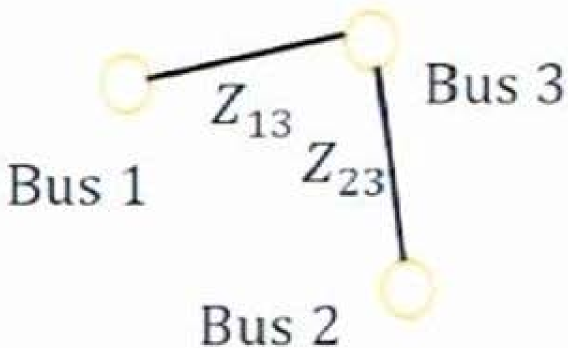
Current magnitudes of PMU data



Singular values of the PMU data matrix

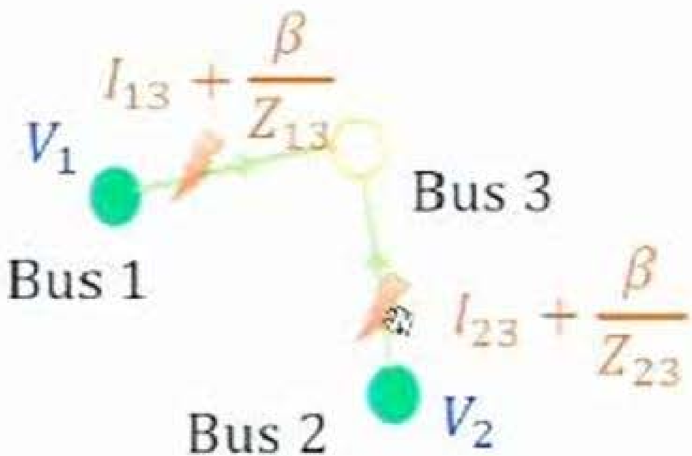
Cyber Data Attacks

The worst-case interacting bad data. (Liu & Ning & Reiter 11).



Cyber Data Attacks

The worst-case interacting bad data. (Liu & Ning & Reiter 11).



- Measurements V_1, V_2, I_{12}, and I_{13}. Estimate V_3.
- Redundancy in measurements can be used to detect bad data.
- Cyber data attack: manipulate I_{12} and I_{13} simultaneously.

Existing Approaches

Cyber attacks that are unobservable at one given sampling instant.

- Usually protect key measurement units to avoid these attacks. (Kosut & Jia & Thomas & Tong 10, Cui & Han & Kar & Kim & Poor & Tajer 12, Bobba *et al.* 10, Dán & Sandberg 10)

Existing Approaches

Cyber attacks that are unobservable at one given sampling instant.

- Usually protect key measurement units to avoid these attacks. (Kosut & Jia & Thomas & Tong 10, Cui & Han & Kar & Kim & Poor & Tajer 12, Bobba *et al.* 10, Dán & Sandberg 10)
- Detection of cyber data attacks in SCADA system. (Sedghi & Jonckheere 13, Liu & Esmalifalak & Ding & Emesih & Han 14)
  Assume the intruder attacks a different set of measurements at each time instant.

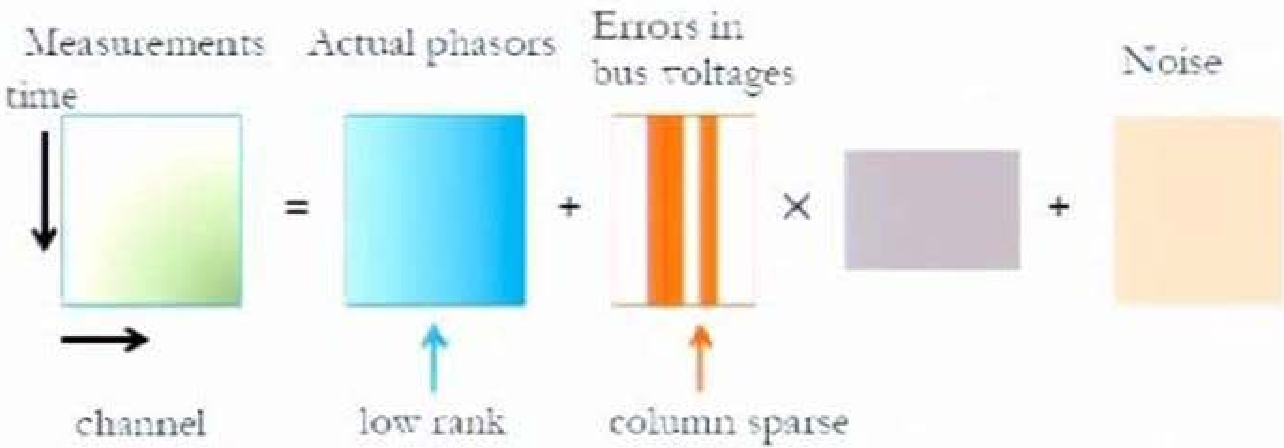
Our Contributions

- A new detection method of cyber data attacks when the intruder injects attacks to a fixed set of PMUs constantly.
- Bad Scenario: no correct information from affected PMUs.
- Intuition of detection: the injected dynamics is different from system dynamics.
- Method: the PMU channels under attack can be identified by solving a convex optimization problem.
- Advantage: the detection method can identify the attacks even when the system is under disturbance.

Wang, Gao, Ghiocel, Chow, Fardanesh, Stefopoulos, Razanousky, IEEE SmartGridComm 2014, submission to IEEE Trans. Signal Processing 2015

Problem Formulation

$$M = \bar{L} + \bar{C}W^T + N$$



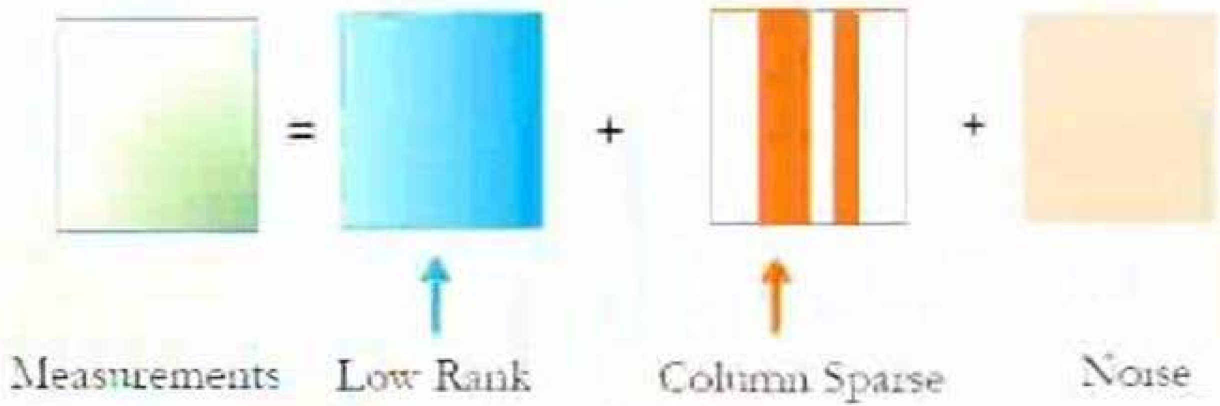
Measurements under attack

- $\bar{L}$: low-rank. From correlations in measurements.
- $\bar{C}$: column sparse. The intruder has limited access to the system.
- N: $\|N\|_F \leq \varepsilon$.

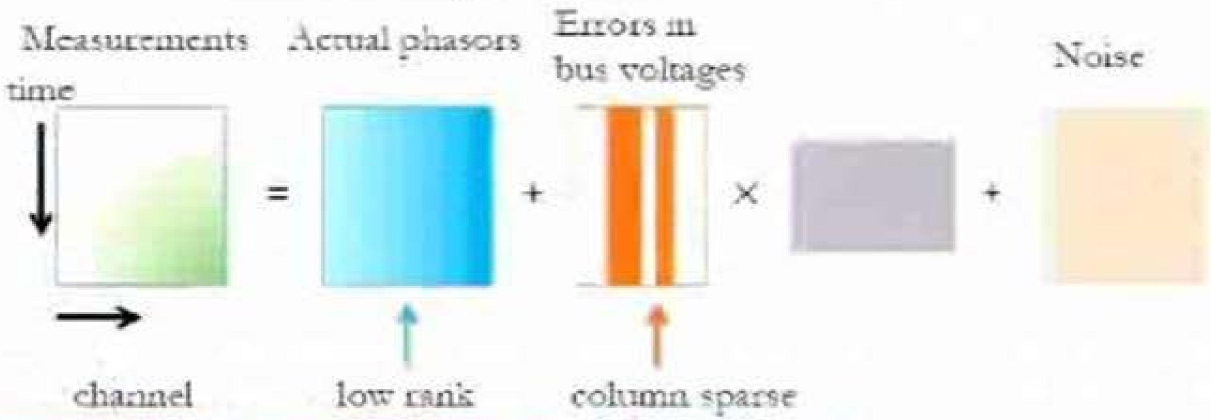
Given M and W, how could we identify $\bar{L}$ and $\bar{C}$?

Connection to Related Work

- Xu & Caramanis & Sanghavi 12: Decomposition of a low-rank matrix and a column-sparse matrix.

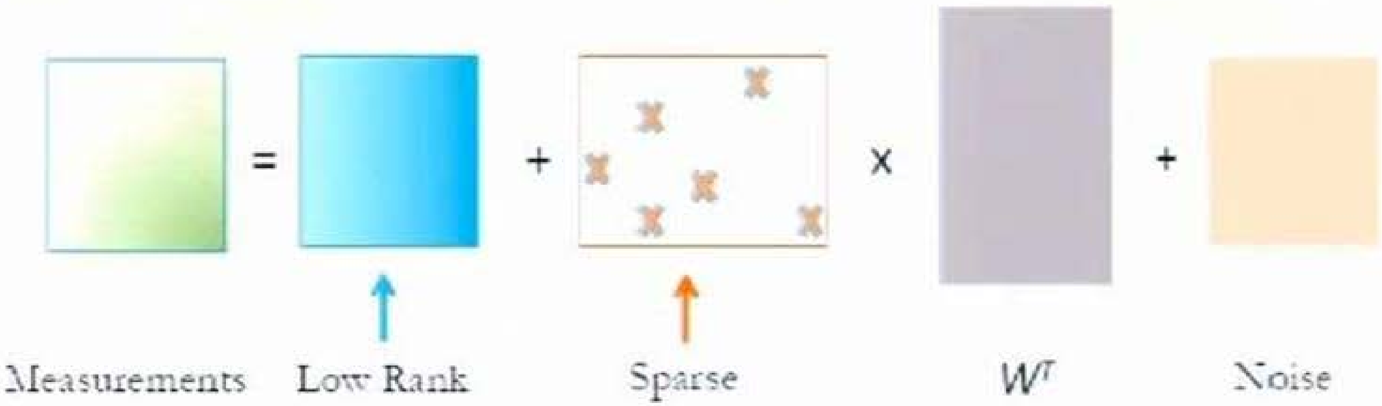


Our methods and proofs are built upon those in Xu & Caramanis & Sanghavi 12. Extension to general cases.

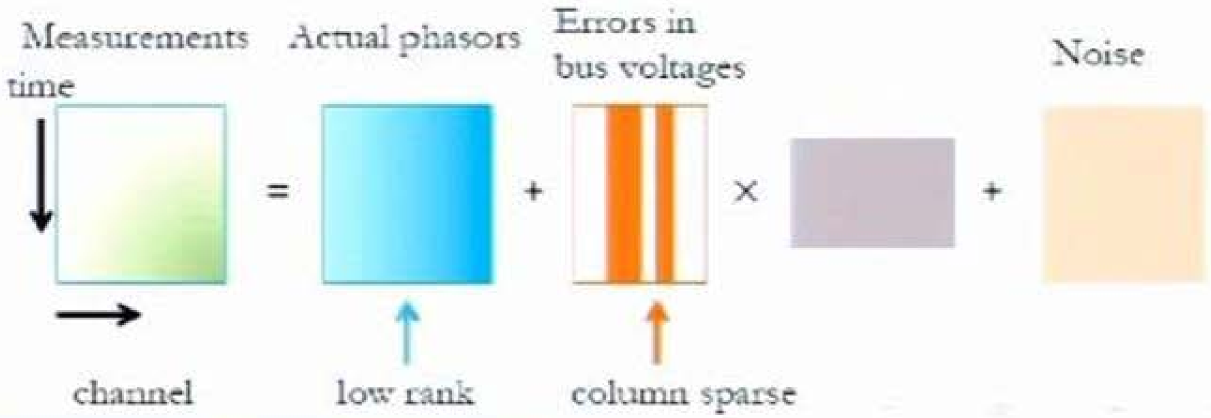


Connection to Related Work

- Mardani & Mateos & Giannakis 13: Decomposition of a low-rank matrix plus a compressed sparse matrix. Internet traffic anomaly detection.



Our focus: column-sparse matrices, W is arbitrary.



Our Approach

- Find (L^*, C^*), the optimum solution to the following optimization problem

$$\min_{L\in\mathbb{C}^{t\times p}, C\in\mathbb{C}^{t\times n}} \|L\|_* + \lambda\|C\|_{1,2} \quad \text{s.t.} \quad \|L + CW^T - M\|_F \leq \varepsilon \qquad (2)$$

  $\|L\|_*$: sum of singular values of L. $\|C\|_{1,2}$: sum of column norms of C.

- Compute the SVD of $L^* = U^*\Sigma^*V^{*\dagger}$.
- Find column support of $D^* = C^*W^T$, denoted by $\mathscr{I}^*$.
- Return $L^*_{\mathscr{I}^{*c}}$, U^* and $\mathscr{I}^*$.

(2) is convex and can be solved efficiently.

Theoretical Guarantee

Noiseless measurements, $N = \mathbf{0}$

If λ belongs to certain range, the solution returned by our method

1. identifies the PMU channels under attack.
2. identifies the measurements that are not attacked.
3. recovers the correct subspace spanned by actual phasors.

Wang, Gao, Ghiocel, Chow, Fardanesh, Stefopoulos, Razanousky, IEEE SmartGridComm 2014, submission to IEEE Trans. Signal Processing 2015

Theoretical Guarantee

Noiseless measurements, $N = 0$

If λ belongs to certain range, the solution returned by our method

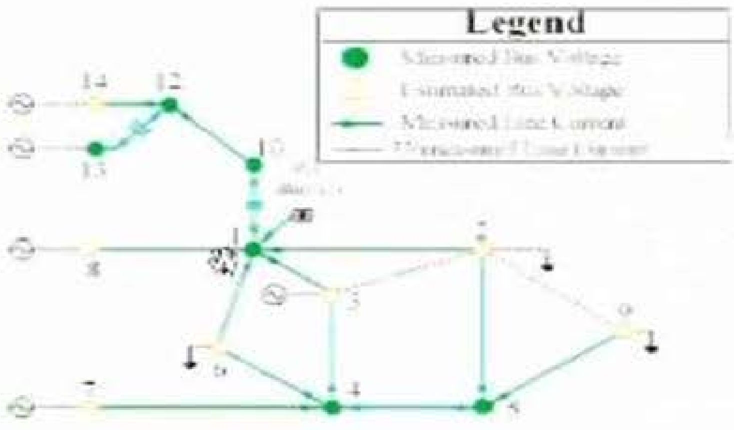
1. identifies the PMU channels under attack.
2. identifies the measurements that are not attacked.
3. recovers the correct subspace spanned by actual phasors.

Noisy measurements, $N \neq 0$

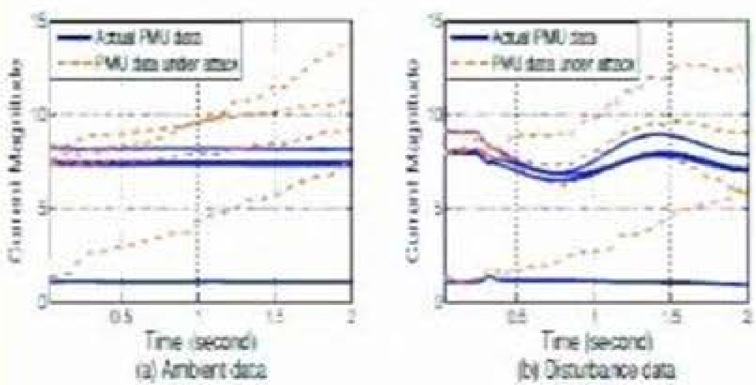
If λ belongs to certain range, the solution returned by our method is sufficiently close (with distance depending on the noise level) to a solution that meets 1-3.

Wang, Gao, Ghiocel, Chow, Fardanesh, Stefopoulos, Razanousky, IEEE SmartGridComm 2014, submission to IEEE Trans. Signal Processing 2015

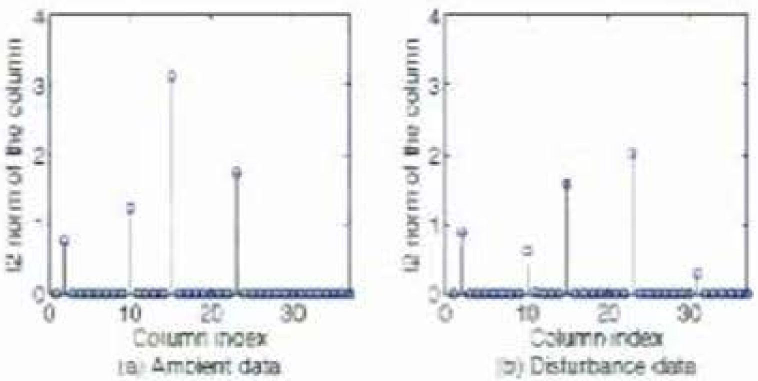
Numerical Results



- Simulate the case that the intruder alters the PMU channels that measure I^{12}, I^{52}, I^{13} and I^{43}.
- The voltage phasor estimates of Buses 2 and 3 are corrupted



Actual values and corrupted values



Column norms of the recovered error matrix

Conclusion

- A low-rank approach of PMU data analysis. Leverage the low-dimensional structures in various PMU data management tasks.
- Missing data recovery: theoretical guarantee of successful recovery when the locations of the missing points are correlated.
- Detection of cyber data attacks: theoretical guarantee of decomposition of a low-rank and a transformed column-sparse matrix using convex optimization.
- Ongoing work: disturbance classification and localization.