EFFICIENT ALGORITHMS FOR CONTINGENCY ANALYSIS OF POWER GRIDS

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CONTINGENCY ANALYSIS

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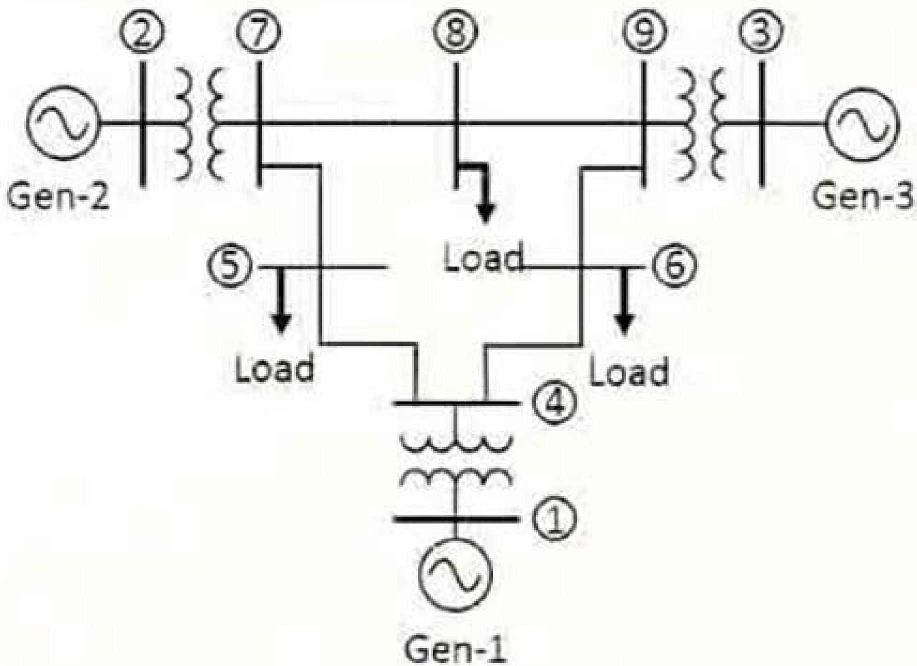
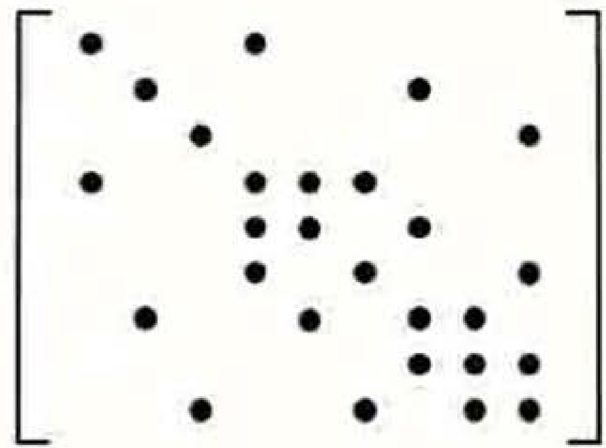
CONTINGENCY ANALYSIS

- $N - x$ contingency analysis on power flow evaluates the stability of a power system by simulating the failures of x transmission lines or generators. (N is the number of buses and x is the number of removed components)





CONTINGENCY ANALYSIS

- Currently each $N - x$ contingency analysis is performed independently, but since the number of cases increases exponentially with x, this is computationally impractical. (For example, if $N = 3120$ and $x = 5$, there are in total 2.46×10^{15} cases.)

- Heuristics are needed for choosing the cases to be analyzed. To be able to analyze more cases, we have to speed up the solution time for each case.

CONTINGENCY ANALYSIS

- We propose new algorithms for the problem by observing that only a small portion of the system is changed when a component is removed.

- We first analyze using the "DC" approximate equation:

$$B'\Delta\theta = P$$

where

B' is the imaginary part of the admittance matrix,

$\Delta\theta$ is the angular separation across a transmission circuit,

P is the real power flow.

AUGMENTED FORMULATION

AUGMENTED FORMULATION

Inspired by a surgery simulation project:

- Cutting a graphical mesh model of an organ to simulate a surgery using underlining physics-based system of equations.
- The matrix of the system changes while the mesh is being cut.
- Need fast update for interactive simulation.

Reference:

- Y.-H. Yeung, J. Crouch and A. Pothen, Interactively Cutting and Constraining Vertices in Meshes using Augmented Matrices. (Accepted by ACM Transaction on Graphics, under revision)

AUGMENTED FORMULATION

General ideas:

- If only few columns of the matrix change over time, preserve the original matrix and its factors while the changes occur
- Form equivalent augmented system of equations with appended rows and columns to compute the solutions of the changed system
- Use a hybrid (direct + implicit Krylov) solver or a direct solver for the augmented system of equations
- Exploit sparsity carefully in the implicit Krylov solver

AUGMENTED FORMULATION

Example

- K_G: original matrix; K_G': matrix after changes occurred
- 3rd column of $K_G \neq$ 3rd column of K_G'

$$\begin{bmatrix} \begin{bmatrix} & & & & \\ & & K_G & & \\ & & & & \end{bmatrix} & \begin{bmatrix} 3^{\text{rd}} \\ \text{col} \\ \text{of} \\ K_G' \end{bmatrix} \\ 0 \quad 0 \quad 1 \quad 0 \quad \cdots & 0 \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ z_3 \\ a_4 \\ \vdots \\ a_3 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ \vdots \\ 0 \end{bmatrix}$$

SOLUTION METHOD

The augmented system of equations can then be expressed as

$$\begin{bmatrix} K & J \\ H & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} f_G \\ 0 \end{bmatrix}$$

- α_1 is the solution vector of the unaffected buses.
- α_2 is the solution vector of the buses affected by the removal of components.

EXPLOITING SPARSITY IN VECTORS

EXPLOITING SPARSITY IN VECTORS

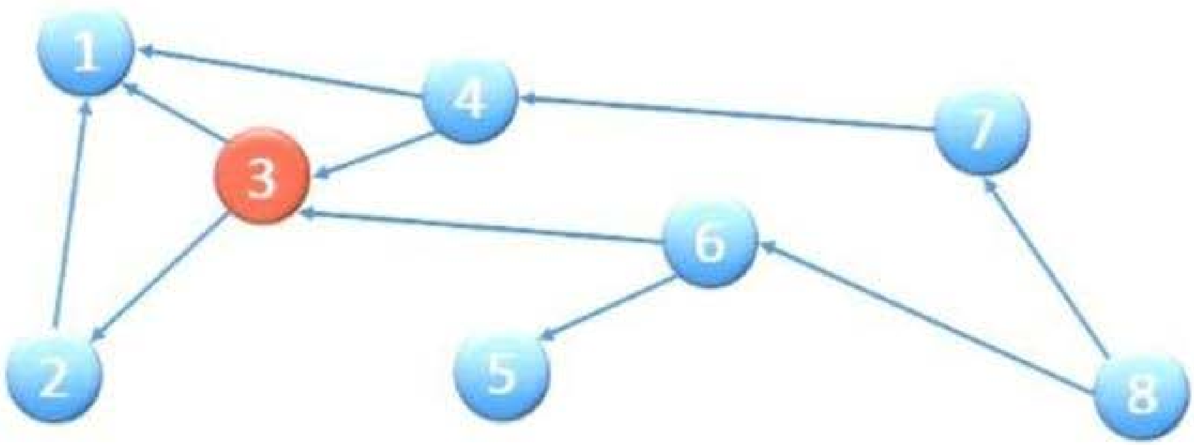
Consider the equation $z = Hy = H(LDL^T)^{-1}b$:

- Sparsity in b in solving with L (closure of nonzero indices of f in $G(L)$, will be defined in the next slide)

- H is a submatrix of an identity matrix, so $z = Hy$ selects few elements of y. Need to compute only components of y selected by H in solving with L^T (closure of selected components of y in $G(L)$)

EXPLOITING SPARSITY IN VECTORS

Closure in of a vector $\boldsymbol{v} = \boldsymbol{e}_3$ in a graph $G(L)$:

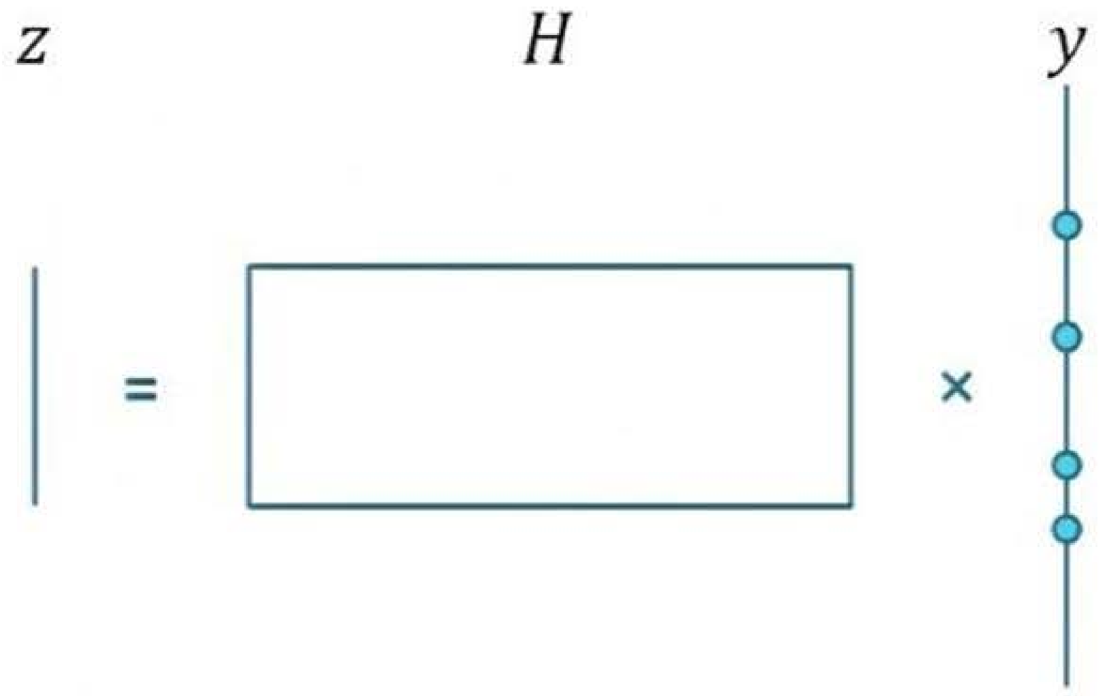


Reference:

- J. Gilbert, "Predicting structure in sparse matrix computations," SIAM Journal on Matrix Analysis and Applications, vol. 15, pp. 62–79, 1994.

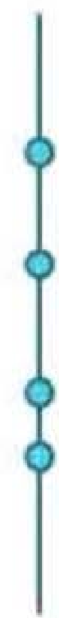
SPARSITY IN THE MATRIX H

Finding the needed entries of the vector y which are included in the vector z



SPARSITY: NEEDED SOLUTION COMPONENTS

Partial backward substitution by only computing columns of L^T induced by the needed entries of the solution vector



EXPLOITING SPARSITY

- For the GMRES step, copy submatrices of L and L^T for use in multiple iterations (for memory performance)
- For other steps, compute indices of nonzero components of the vectors to identify the rows and columns of L and L^T needed

USING DIRECT METHOD

When the augmented size is small, we could form the matrix $H(LDL^T)^{-1}J$ explicitly as follows:

Let $S = H(LDL^T)^{-1}J = HL^{-T}D^{-1}L^{-1}J$.

1. Solve $LZ = J$ for Z.
2. Solve $L^TY = H$ for Y.
3. Compute $S = Y^TD^{-1}Z$.

Sparsity can be exploited in solving Steps 1 & 2. All 3 steps can be implemented in parallel.

TIME COMPLEXITY

- Initialization step:
  1. Compute LDL^T factorization of K_G ($O(n^3)$ for general matrix or $O(n^{3/2})$ for planar graph)
- Update step:
  1. Update K_G. ($O(m)$)
  2. Compute matrices J and H. ($O(m)$)
  3. Solve $Ky = f_G$ and compute $z = Hy$. ($\sigma = O(|\mathrm{closure}(f_G)| + |\mathrm{closure}(\hat{y})|) \leq O(\mathrm{nnz}(L))$ where $\hat{y}$ is the components of y selected by H)

TIME COMPLEXITY

4. Solve $(HK^{-1}J)\alpha_2 = z$. ($O(\sigma \cdot n_{iter})$ for implicit GMRES where n_{iter} is the number of iterations; or $O(\sigma \cdot m + m^2 \cdot n + m^2)$ for serial implementation, $O(\sigma + n + m^2)$ for parallel implementation of direct method.
5. Solve $K\alpha_1 = f_G - J\alpha_2$. ($O(\mathrm{nnz}(L))$)

- Upper bound complexity is $O(nnz(L) \cdot n_{iter})$ for using GMRES implicitly.

RESULTS

TEST CASES

Problem size	nnz(B')	Condition number
9	24	8.99×10^1
30	107	9.62×10^2
118	463	4.85×10^3
300	1115	1.55×10^5
2736	9251	1.09×10^6
3120	10477	1.25×10^6

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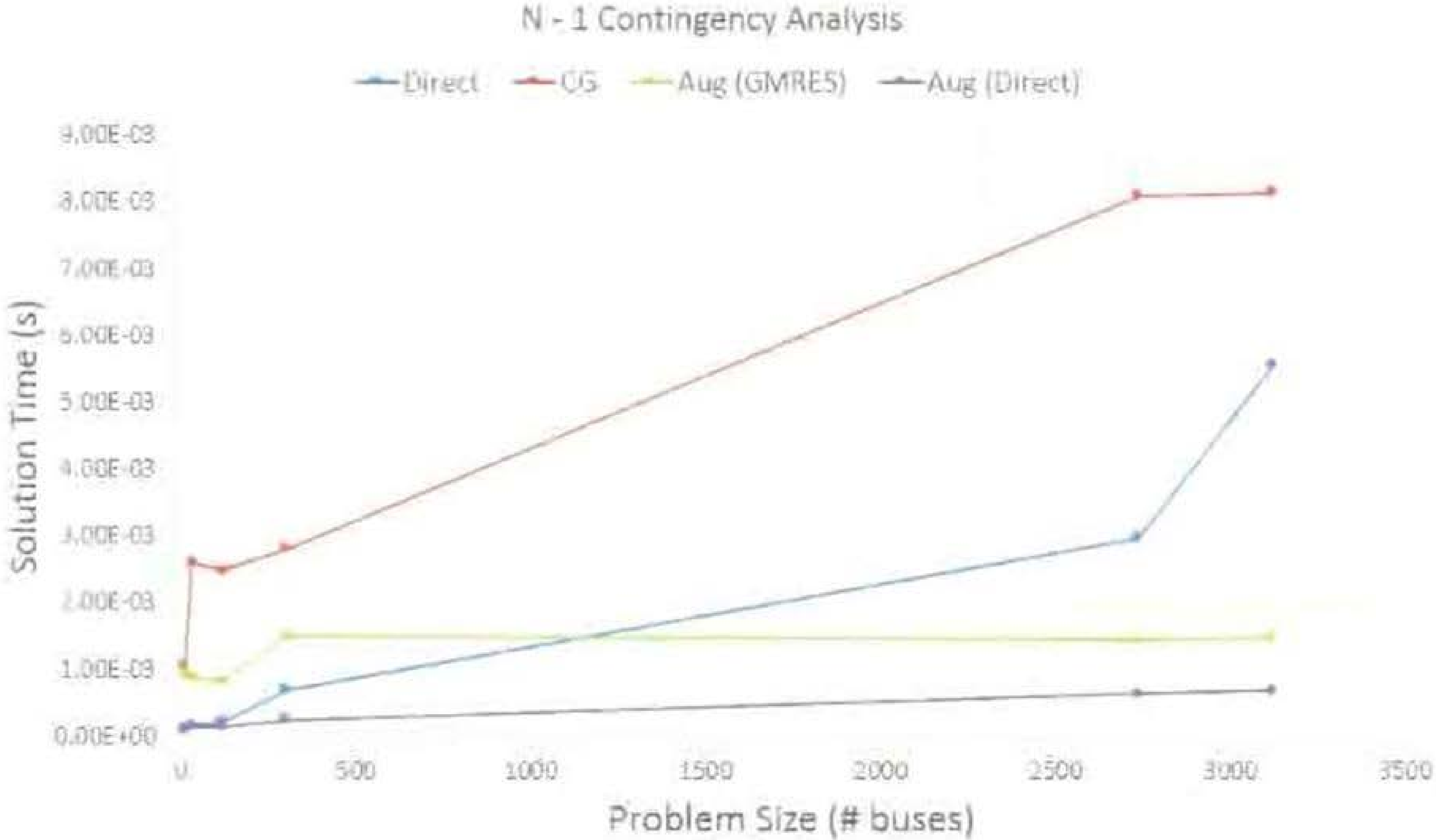
N – 1 CONTINGENCY ANALYSIS



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EXTENSION TO AC POWER FLOW

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EXTENSION TO AC POWER FLOW

Extension to AC Power Flow:

$$Y\boldsymbol{v} = \boldsymbol{b}^*(\boldsymbol{v})$$

$$\boldsymbol{b}^*(\boldsymbol{v})^T = \begin{bmatrix} \frac{s_1^*}{v_1^*} & \frac{s_2^*}{v_2^*} & \cdots & \frac{s_n^*}{v_n^*} \end{bmatrix}$$

Where

Y is the admittance matrix,

v_i is the voltage at each bus i,

s_i is the power at each bus i.

All values are complex.

EXTENSION TO AC POWER FLOW

Let $F(\boldsymbol{v}) = Y\boldsymbol{v} - \boldsymbol{b}^*(\boldsymbol{v}) = 0.$

Newton's method:

$$F_k' \Delta \boldsymbol{v} = -F(\boldsymbol{v}_k)$$

where

$$\Delta \boldsymbol{v} = \boldsymbol{v}_{k+1} - \boldsymbol{v}_k.$$

EXTENSION TO AC POWER FLOW

Equation becomes

$$[j(Y+S^*) \quad Y+S^*]\Delta\mathbf{y} = -F(\boldsymbol{v}_k)$$

where

$$S = \mathrm{diag}(\boldsymbol{s})\mathrm{diag}(\boldsymbol{v}_k)^{-2},$$
$$\Delta\boldsymbol{y} = \begin{bmatrix} \mathrm{diag}(\boldsymbol{v}_k) \\ \mathrm{diag}(\boldsymbol{v}_k)\mathrm{diag}(\boldsymbol{\xi}_k)^{-1} \end{bmatrix}\Delta\boldsymbol{x}.$$

EXTENSION TO AC POWER FLOW

Separating real and imaginary parts,

$$\underbrace{\begin{bmatrix} -\Im(Y) & \Re(Y) \\ \Re(Y) & \Im(Y) \end{bmatrix}}_{\text{Only in } Y\text{; constant across iterations}} \Delta \boldsymbol{y}_{k+1} = \underbrace{\begin{bmatrix} -\Im(S) & -\Re(S) \\ -\Re(S) & \Im(S) \end{bmatrix}}_{\text{changes every iteration; involves only matrix-vector multiplication}} \Delta \boldsymbol{y}_k - \begin{bmatrix} \Re(F(\boldsymbol{v}_k)) \\ \Im(F(\boldsymbol{v}_k)) \end{bmatrix}$$

Only in Y;
constant across iterations

changes every iteration;
involves only matrix-
vector multiplication

- We are currently working on the convergence problem.

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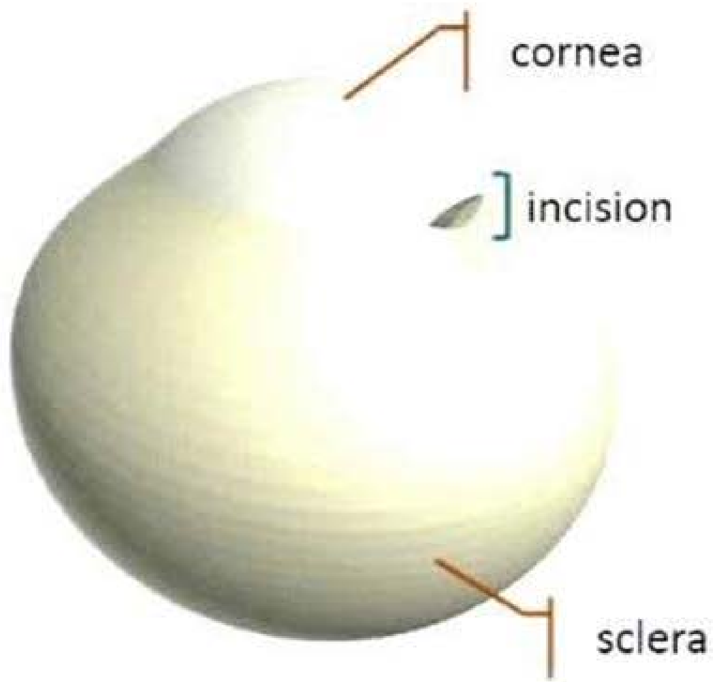
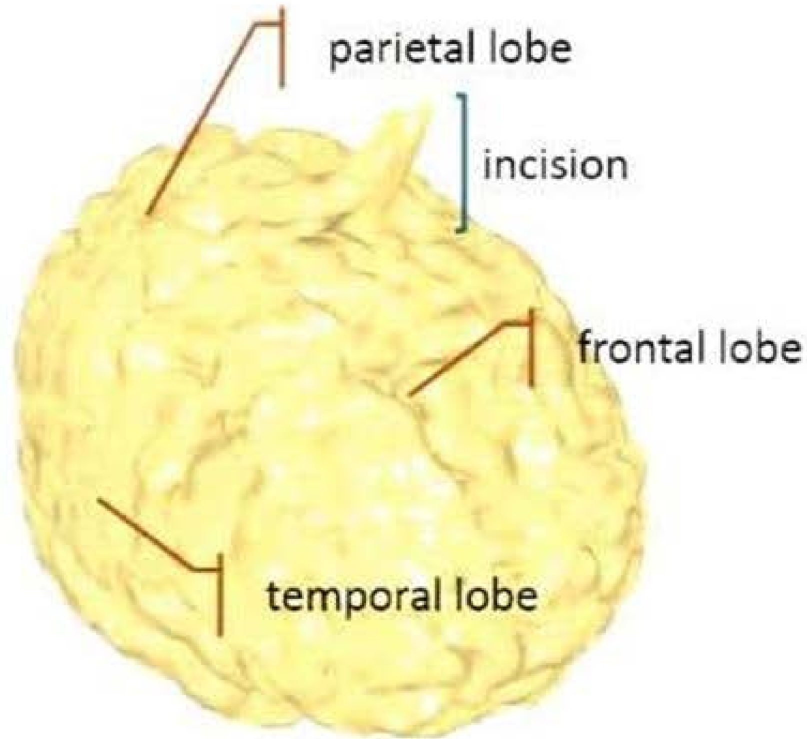
RESULTS OF SURGERY SIMULATION

SURGERY SIMULATION



Eye Model



Brain Model

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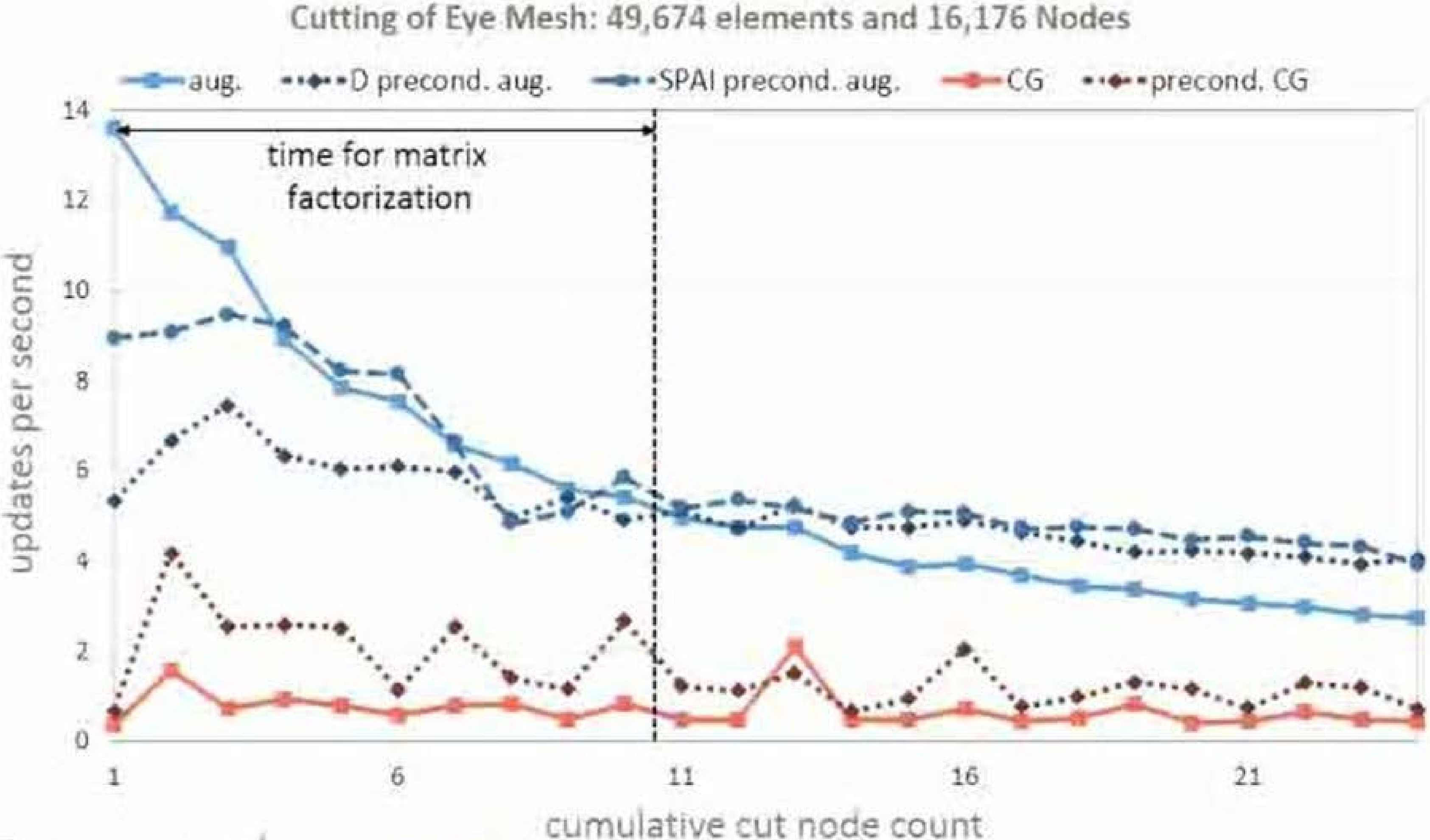
SURGERY SIMULATION

Condition number: 1.62×10^7



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SUMMARY

- We propose an augmented formulation for solving systems of equations undergoing small changes with time complexity linear in the number of nonzeros of the factors.
- Results from surgery simulation give promising speedups over conjugate gradient (CG) method.
- The admittance matrix across cases in contingency analysis of power flow system is quite similar to the stiffness matrix in finite element model.
- We showed that we can solve the problem about 8 times faster than direct and CG methods. Hence we can solve many more contingency analyses in a given time.

QUESTIONS & ANSWERS

THANK YOU!

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