

NCSEA Structural Engineering Exam Review Course

Lateral Forces Review

Bridges – August 2017

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Topics

- Temperature and Shrinkage
- Centrifugal Forces
- Braking Forces
- Vehicle/Railway Collision Forces
- Stream Forces
- Ice Forces
- Wind
- Seismic

Temperature and Shrinkage

Problem 1: For the continuous flat slab three span superstructure shown, determine the horizontal unfactored load in each column due to temperature and shrinkage after 28 days and a 33.3°F temperature rise. Columns are very flexible compared to superstructure.

$$E_c = 3605 \text{ ksi}$$

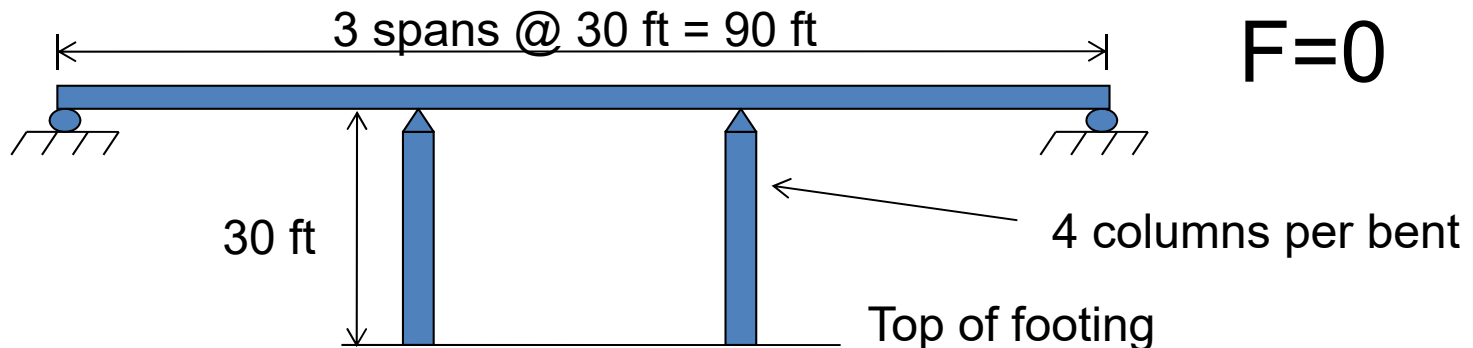
$$I_{column} = 520,000 \text{ in}^4$$

$$\alpha_{concrete} = 6 \times 10^{-6} / ^\circ F$$

$$\varepsilon_{sh,28 \text{ days}} = 0.0002 \text{ in / in}$$

$$\Delta_{bent} = \varepsilon_{sh,28 \text{ days}} (Middle \text{ Span}) / 2 + \alpha_{concrete} \Delta T (Middle \text{ Span}) / 2$$

$$\Delta_{bent} = 0.0002(30)(12) / 2 - 6 \times 10^{-6}(33.3)(30)(12) / 2 = 0 \text{ in.}$$



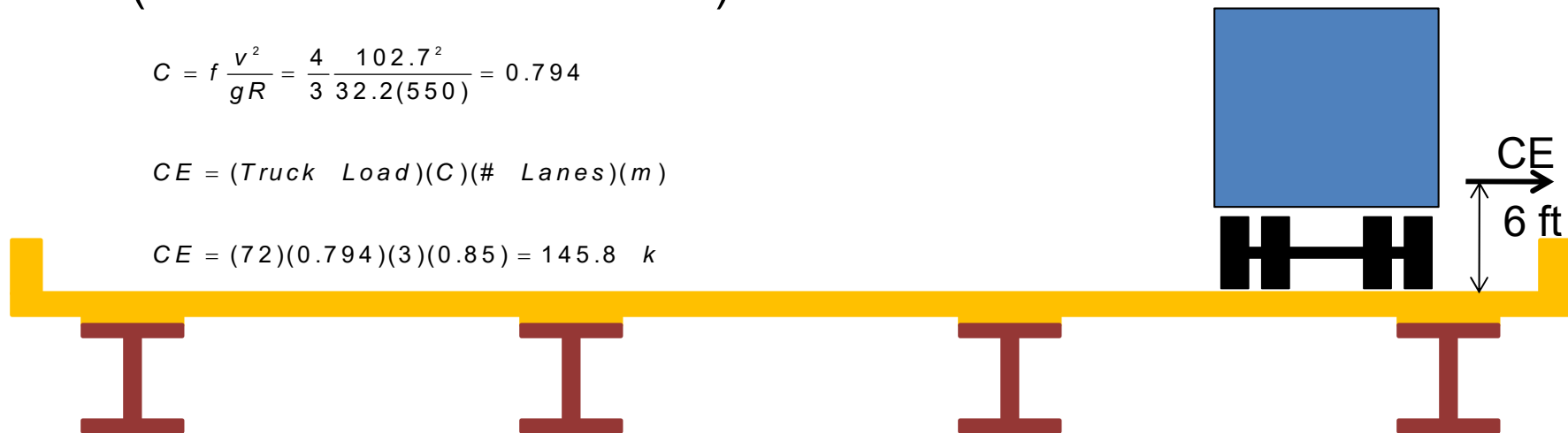
Centrifugal Forces

Problem 2: Determine the unfactored centrifugal force CE (AASHTO 3.6.3) for the structure shown. Assume N_L = three design lanes, R = 550 feet, highway design speed v = 70 mph = 102.7 ft/s, and fatigue is not being checked (i.e., $f=4/3$). m = 0.85 for three lanes loaded (see vertical review material).

$$C = f \frac{v^2}{gR} = \frac{4}{3} \frac{102.7^2}{32.2(550)} = 0.794$$

$$CE = (Truck \text{ Load})(C)(\# \text{ Lanes})(m)$$

$$CE = (72)(0.794)(3)(0.85) = 145.8 \text{ k}$$



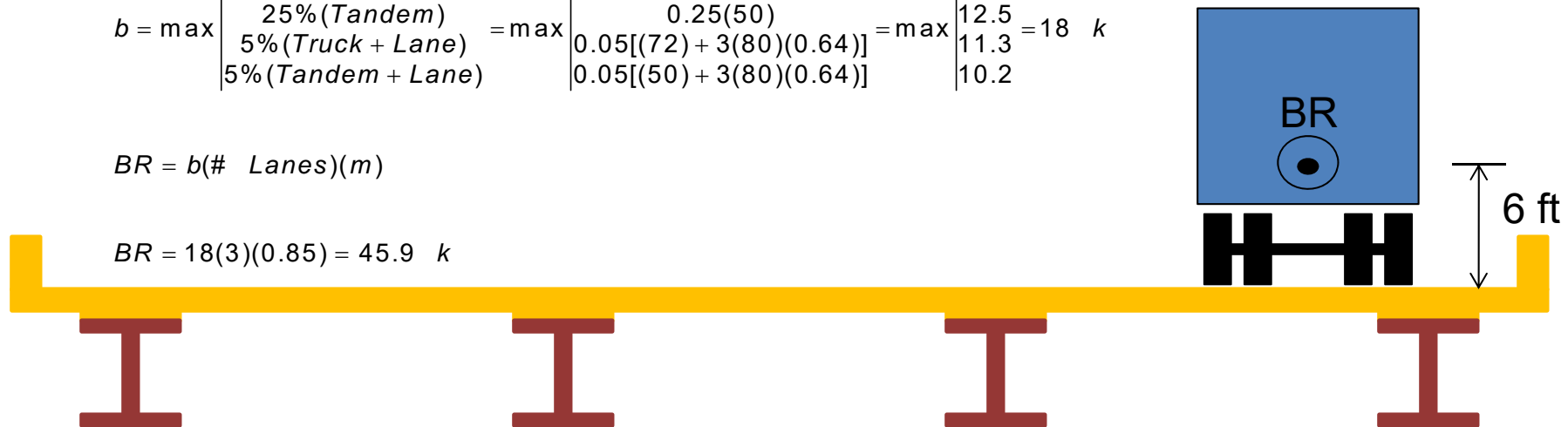
Braking Forces

Problem 3: For the one directional bridge shown, determine the unfactored braking force BR (AASHTO 3.6.4). Assume $N_L =$ three design lanes, bridge length = $3(80) = 240$ ft. $m = 0.85$ for three lanes loaded (see vertical review material).

$$b = \max \begin{vmatrix} 25\%(Truck) \\ 25\%(Tandem) \\ 5\%(Truck + Lane) \\ 5\%(Tandem + Lane) \end{vmatrix} = \max \begin{vmatrix} 0.25(72) \\ 0.25(50) \\ 0.05[(72) + 3(80)(0.64)] \\ 0.05[(50) + 3(80)(0.64)] \end{vmatrix} = \max \begin{vmatrix} 18 \\ 12.5 \\ 11.3 \\ 10.2 \end{vmatrix} = 18 \text{ } k$$

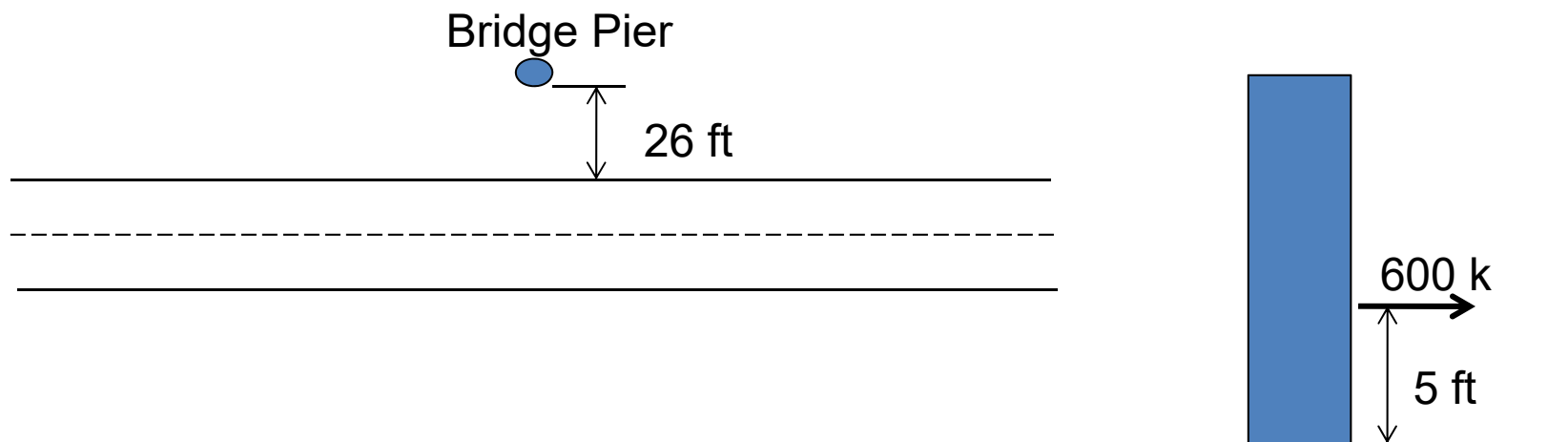
$$BR = b(\# \text{ Lanes})(m)$$

$$BR = 18(3)(0.85) = 45.9 \text{ } k$$



Vehicle/Railway Collision Forces

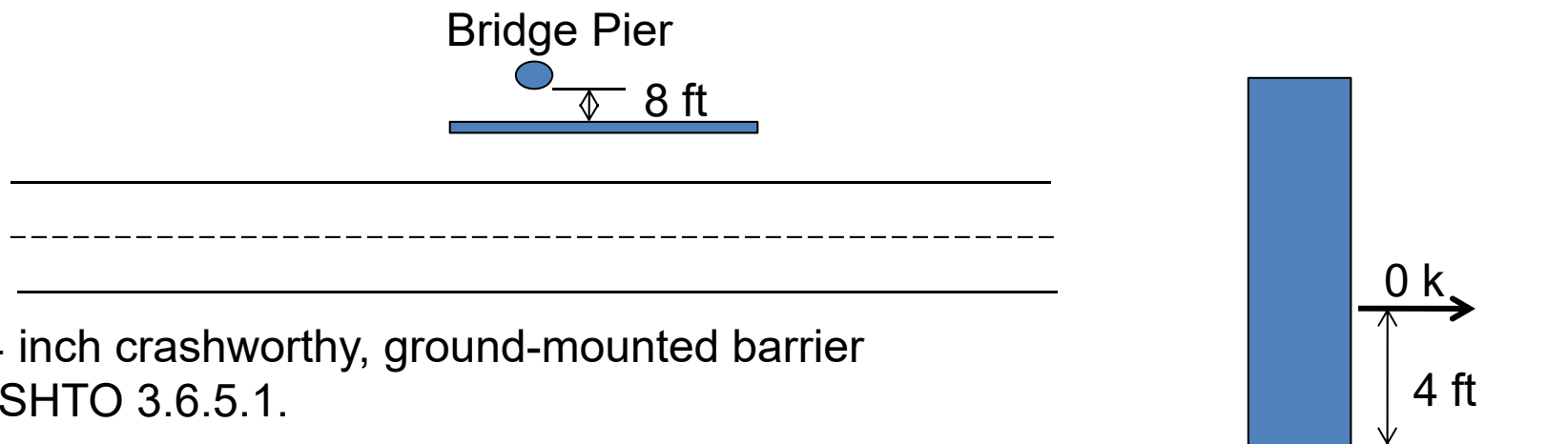
Problem 4: Determine the unfactored vehicular collision force CT (AASHTO 3.6.5.1) for an unprotected pier located 26 feet (i.e., within 30 feet) from the roadway edge.



Note that 600 k load is horizontal and application must consider all angles between 0 to 15 degrees with the edge of pavement.

Vehicle/Railway Collision Forces

Problem 5: Determine the required height of a barrier placed 8 feet (i.e., within 10 feet) from the bridge pier shown to avoid designing the pier for the 600 k load determined in the previous problem.



Use 54 inch crashworthy, ground-mounted barrier per AASHTO 3.6.5.1.

Note that a 42 inch high barrier is permitted when the separation distance exceeds 10 feet

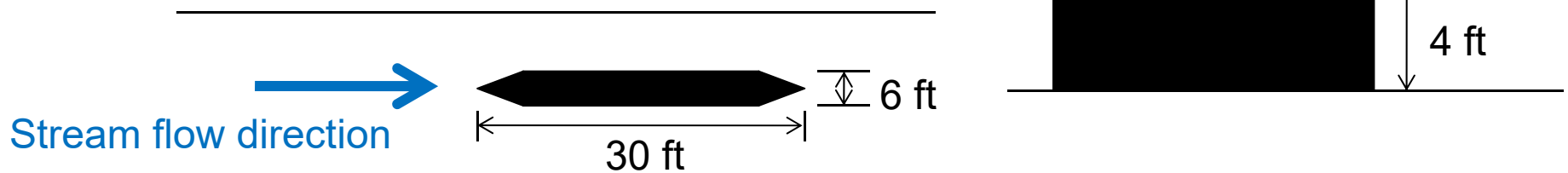
Stream Forces

Problem 6: Determine the unfactored stream pressure longitudinal force WA (AASHTO 3.7.3.1) acting on the bridge pier shown. Assume pier nose is an equilateral triangle ($\beta=60^\circ$), $V = 6$ ft/s, and design must account for debris lodged against the pier.

$$p = C_D \frac{V^2}{1000} = 1.4 \frac{6^2}{1000} = 0.0504 \text{ ksf}$$

$$WA = pA_{\text{projected, longitudinal}} = 0.0504(6)(6) = 1.81 \text{ k}$$

Water height = 6 ft



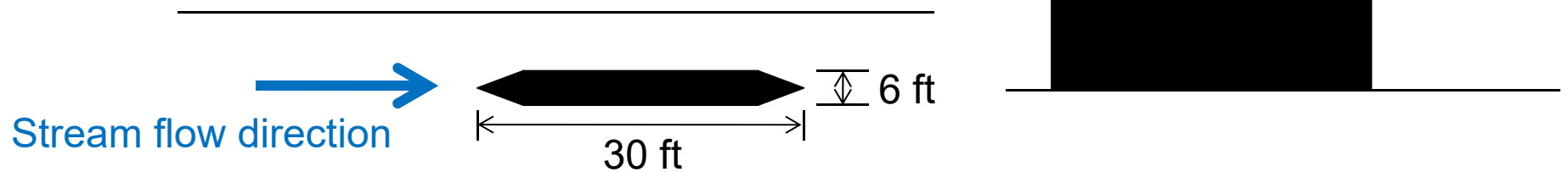
Stream Forces

Problem 7: Determine the unfactored stream pressure lateral force W_A (AASHTO 3.7.3.2) acting on the bridge pier shown. Assume pier nose is an equilateral triangle ($\beta=60^\circ$), $V=6$ ft/s, and design must account for debris lodged against the pier.

$$p = C_L \frac{V^2}{1000} = (0) \frac{6^2}{1000} = 0.0 \text{ ksf}$$

$$W_A = p A_{\text{projected, lateral}} = 0(30)(6) = 0 \text{ k}$$

Water height = 6 ft



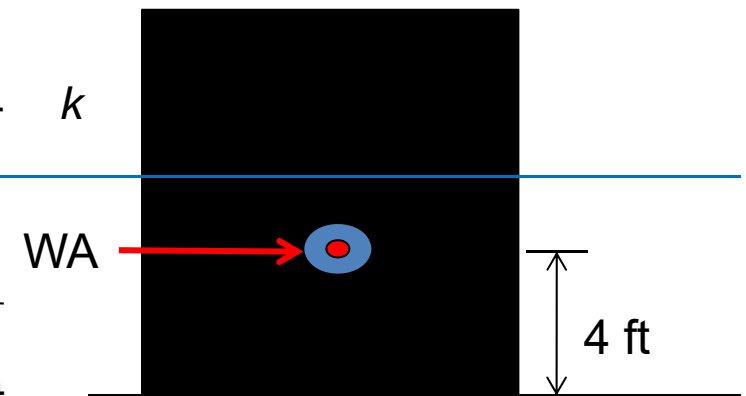
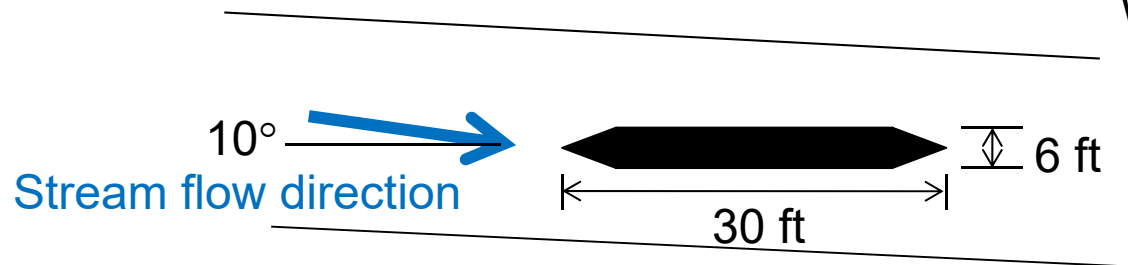
Stream Forces

Problem 8: Determine the unfactored stream pressure lateral force WA (AASHTO 3.7.3.2) acting on the bridge pier shown. Assume pier nose is an equilateral triangle ($\beta=60^\circ$), $V=6$ ft/s, and design must account for debris lodged against the pier.

$$p = C_L \frac{V^2}{1000} = (0.7) \frac{6^2}{1000} = 0.0252 \text{ ksf}$$

$$WA = p A_{\text{projected, lateral}} = 0.0252(30)(6) = 4.54 \text{ k}$$

Water height = 6 ft



Ice Forces

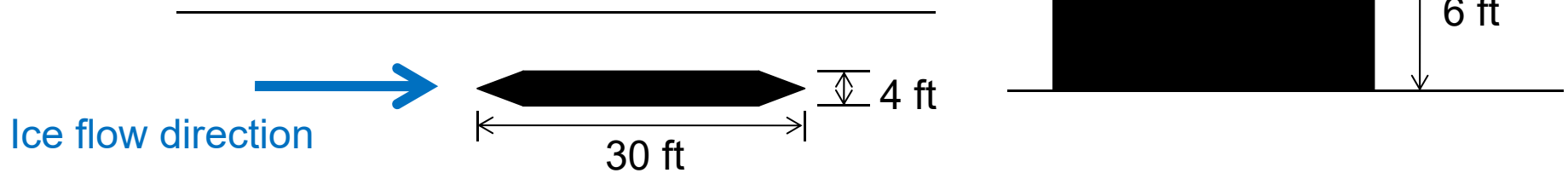
Problem 9: Determine the unfactored ice force IC (AASHTO 3.9.2.2) acting on the bridge pier shown. Assume pier nose is an equilateral triangle ($\beta=60^\circ$), $p = 24$ ksf, and $t = 0.8$ ft.

$$w / t = 4 / 0.8 = 5 < 6, \therefore,$$

$$C_a = (5t / w + 1)^{0.5} = [5(0.8) / 4 + 1]^{0.5} = 1.41$$

$$F = \min \begin{cases} F_c = C_a p t w = 1.41(24)(0.8)(4) = 108 \text{ k} \\ F_b = C_n p t^2 \quad (\text{NA}, \alpha = 0 < 15^\circ) \end{cases} = 108 \text{ k}$$

$$IC = 108 \text{ k}$$



Ice Forces

Problem 10: Determine the unfactored ice force IC (AASHTO 3.9.2.2) acting on the bridge pier shown. Assume pier nose is an equilateral triangle ($\beta=60^\circ$), $p = 24$ ksf, and $t = 0.8$ ft.

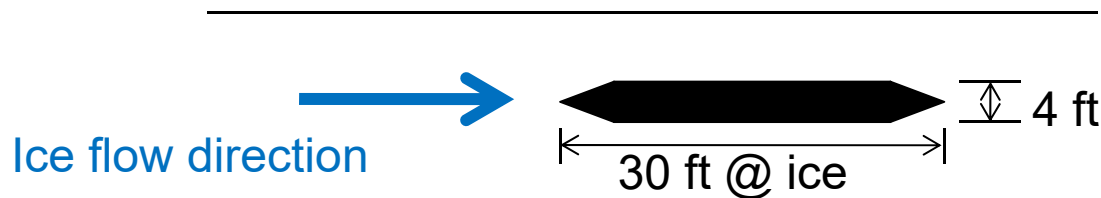
$$w / t = 4 / 0.8 = 5 < 6, \therefore,$$

$$C_a = (5t / w + 1)^{0.5} = [5(0.8) / 4 + 1]^{0.5} = 1.41$$

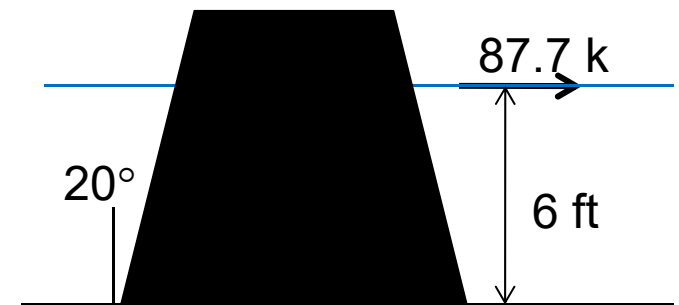
$$C_n = 0.5 / [\tan(\alpha - 15)] = 0.5 / [\tan(20 - 15)] = 5.71$$

$$F = \min \begin{cases} F_c = C_a p t w = 1.41(24)(0.8)(4) = 108 \text{ k} \\ F_b = C_n p t^2 = 5.71(24)(0.8)^2 = 87.7 \text{ k} \end{cases} = 87.7 \text{ k}$$

$$IC = 87.7 \text{ k}$$



Water height = 6 ft



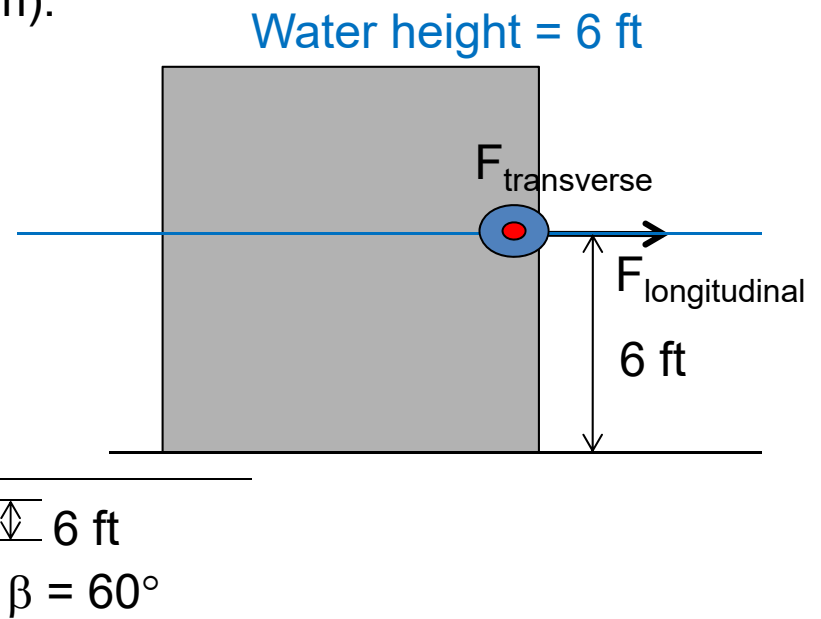
Ice Forces

Problem 11: Determine the combination of longitudinal and transverse unfactored ice forces IC (AASHTO 3.9.2.4) acting on the bridge pier shown. Assume pier nose is an equilateral triangle ($\beta=60^\circ$), $p = 24$ ksf, $t = 0.8$ ft, $\theta_f = 10^\circ$, and $F = 108$ k (see previous problem).

$$F_t = \frac{F}{2 \tan(\beta / 2 + \theta_f)} = \frac{108}{2 \tan(60 / 2 + 10)} = 64.4 \text{ k}$$

$$IC_{\text{case 1}} = \begin{cases} F_{\text{longitudinal}} = 108 \text{ k} \\ F_{\text{transverse}} = 0.15F = 0.15(108) = 16.2 \text{ k} \end{cases}$$

$$IC_{\text{case 2}} = \begin{cases} F_{\text{longitudinal}} = 0.5F = 0.5(108) = 54 \text{ k} \\ F_{\text{transverse}} = 64.4 \text{ k} \end{cases}$$



Ice Forces

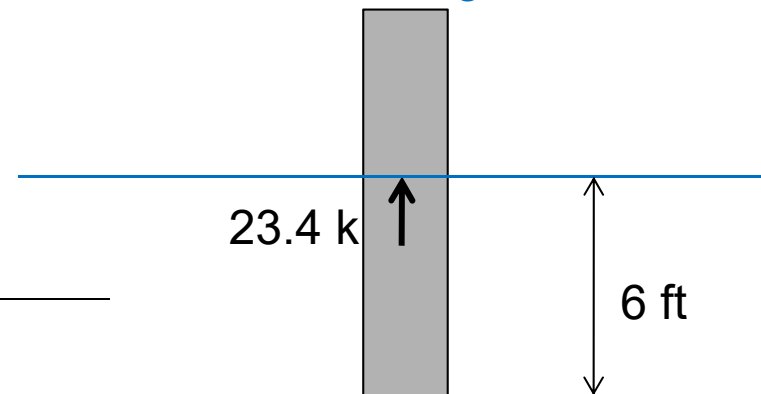
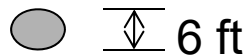
Problem 12: Determine the unfactored vertical ice force IC (AASHTO 3.9.5) acting on a 6-foot diameter pier. Assume $t = 0.8$ feet.

$$F_v = 80t^2 \left(0.35 + \frac{0.03R}{t^{0.75}} \right) = 80(0.8)^2 \left(0.35 + \frac{0.03(3)}{(0.8)^{0.75}} \right) = 23.4 \text{ k}$$

Water height = 6 ft

$$IC = 23.4 \text{ k}$$

Ice flow direction

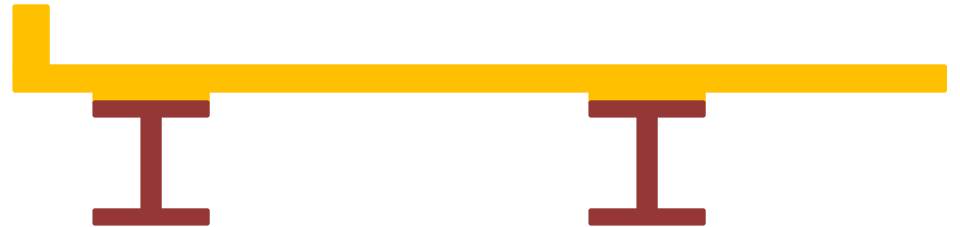


Wind

Problem 13: Determine the design wind velocity V_{DZ} for a slab on steel beam bridge for $Z = 25$ ft.

$$V_{DZ} = 100 \text{ mph}$$

Note that unless $Z > 30$ ft, the design wind velocity is the base wind velocity of 100 mph per AASHTO 3.8.1.1.



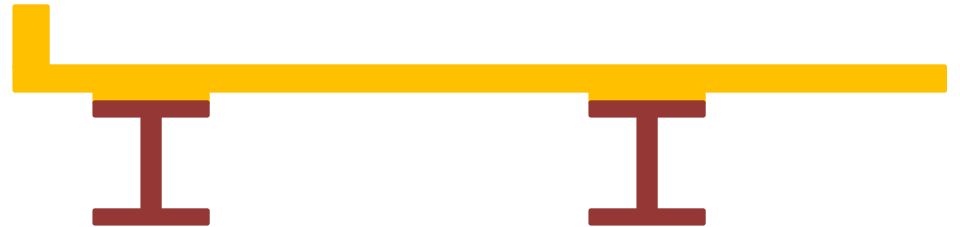
Wind

Problem 14: Determine the design wind velocity V_{DZ} for a slab on steel beam bridge for $Z = 45$ ft. Assume $V_{30} = V_B = 100$ mph and the upstream surface condition is “Suburban.”

$$V_0 = 10.9 \text{ mph (AASHTO Table 3.8.1.1-1)}$$

$$Z_0 = 3.28 \text{ ft (AASHTO Table 3.8.1.1-1)}$$

$$V_{DZ} = 2.5V_0 \left(\frac{V_{30}}{V_B} \right) \ln \left(\frac{Z}{Z_0} \right) = 2.5(10.9) \left(\frac{100}{100} \right) \ln \left(\frac{45}{3.28} \right) = 71.36 \text{ mph}$$



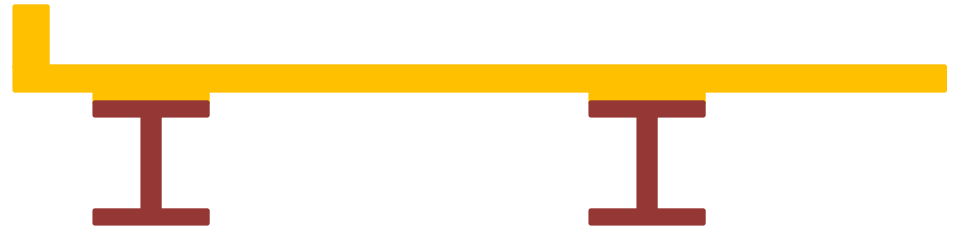
Wind

Problem 15: Determine the design wind velocity V_{DZ} for a slab on steel beam bridge for $Z = 45$ ft. Assume $V_{30} = V_B = 100$ mph and the upstream surface condition is “Open Country.”

$$V_0 = 8.2 \text{ mph (AASHTO Table 3.8.1.1-1)}$$

$$Z_0 = 0.23 \text{ ft (AASHTO Table 3.8.1.1-1)}$$

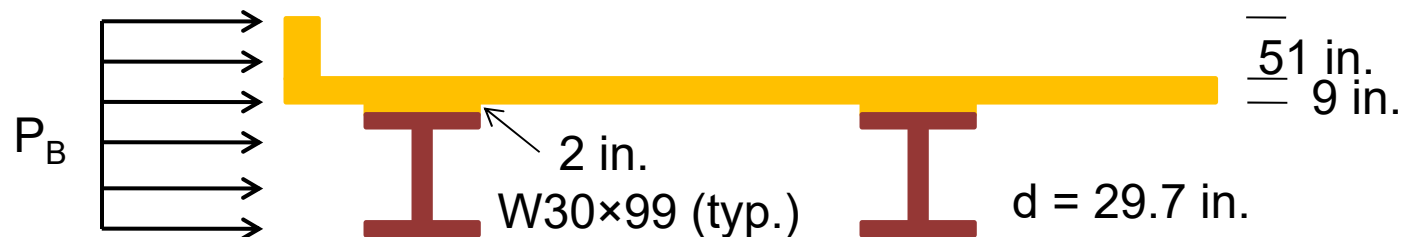
$$V_{DZ} = 2.5 V_0 \left(\frac{V_{30}}{V_B} \right) \ln \left(\frac{Z}{Z_0} \right) = 2.5(8.2) \left(\frac{100}{100} \right) \ln \left(\frac{45}{0.23} \right) = 108.2 \text{ mph}$$



Wind

Problem 16: Determine the base wind pressure P_B for a slab on steel beam bridge for $Z = 25$ ft.

$$P_B = 0.050 \text{ ksf (AASHTO Table 3.8.1.2.1-1)}$$



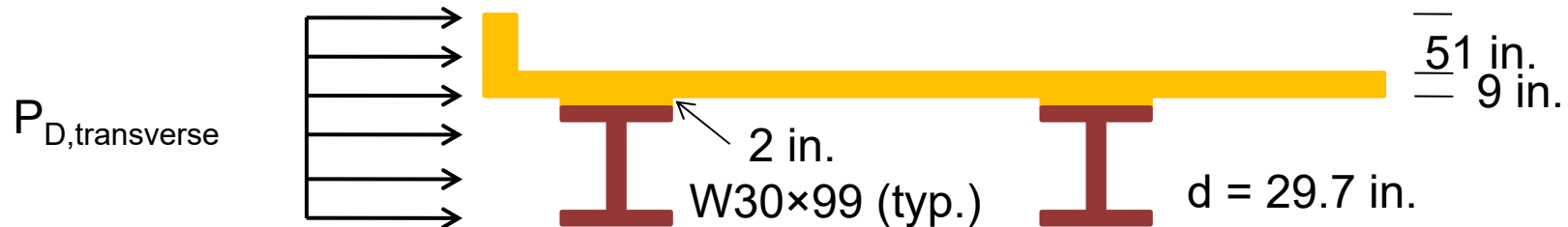
Wind

Problem 17: Determine the design wind pressures P_D for a slab on steel beam bridge for $Z = 25$ ft.

$$P_{D,\text{transverse}} = 0.050 \text{ ksf (AASHTO 3.8.1.2.2)}$$

$$P_{D,\text{longitudinal}} = 0.012 \text{ ksf (AASHTO 3.8.1.2.2)}$$

Note that AASHTO 3.8.1.2.1 provides minimum total design wind loading for girder spans as 0.30 k/ft. $0.050(51 + 9 + 2 + 29.7) / 12 = 0.382$ (ok).



Wind

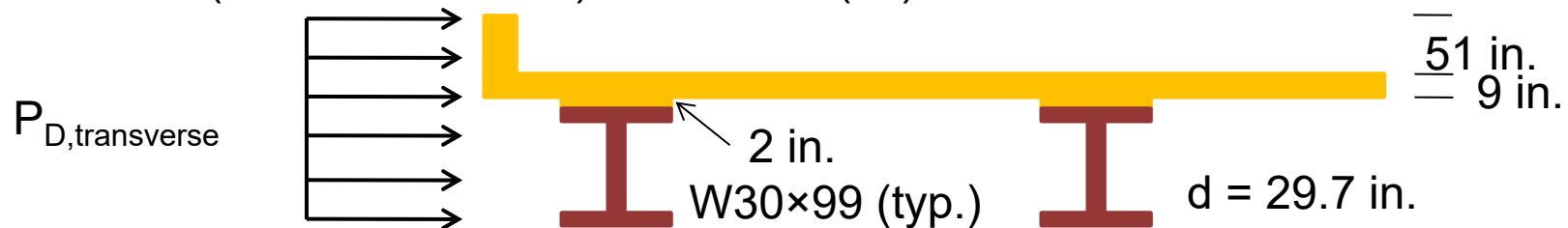
Problem 18: Determine the design wind pressures P_D for a slab on steel beam bridge for $Z = 45$ ft. Assume $V_{30} = V_B = 100$ mph and the upstream service condition is "Open Country." $V_{DZ} = 108$ mph. Only consider skew angle = 0° case.

$$P_{D,transverse} = P_B \frac{V_{DZ}^2}{10,000} = 0.050 \frac{108^2}{10,000} = 0.0583 \text{ ksf}$$

$$P_{D,longitudinal} = P_B \frac{V_{DZ}^2}{10,000} = 0 \frac{108^2}{10,000} = 0 \text{ ksf}$$

Note that AASHTO 3.8.1.2.1 provides minimum total design wind loading for girder spans of 0.30 k/ft (0.35 k/ft with increase for elevation).

$0.0583(51 + 9 + 2 + 29.7)/12 = 0.446$ (ok).

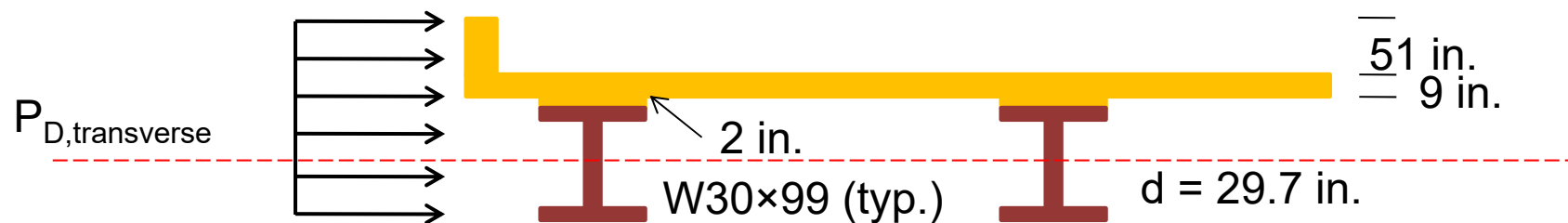


Wind

Problem 19: Determine the unfactored transverse design wind pressure distribution to the concrete diaphragm and the beam bottom flange (AASHTO 4.6.2.7) for a slab on steel beam bridge for $Z = 25$ ft. $P_{D,transverse} = 0.050$ ksf.

$$W_{D,diaphragm} = 0.050(51 + 9 + 2 + 29.7 / 2) / 12 = 0.321 \text{ k / ft}$$

$$W_{D,bottom \text{ flange}} = 0.050(29.7 / 2) / 12 = 0.0619 \text{ k / ft}$$

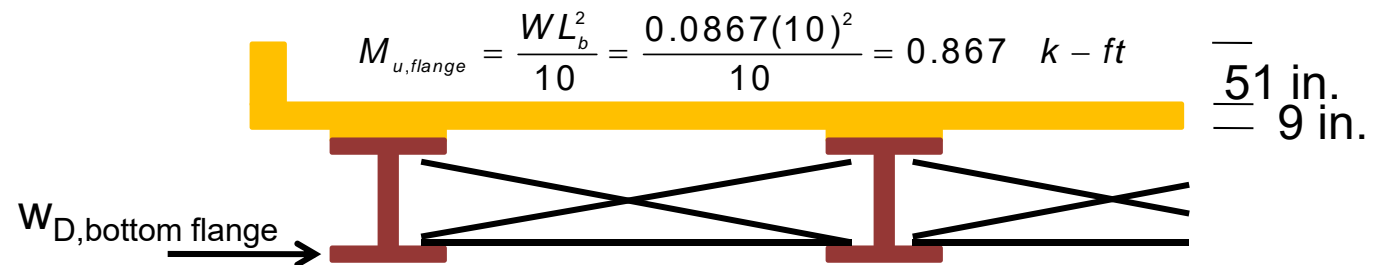


Wind

Problem 20: Determine the factored transverse design wind force for the beam bottom flange and the maximum moment in the beam flange (AASHTO 4.6.2.7) for a slab on steel beam bridge for $Z = 25$ ft. $P_{D,transverse} = 0.050$ ksf and cross frames are spaced at 10 feet o.c. Strength III limit state applies and $\eta_i = 1.0$.

$$w_{D,bottom\ flange} = 0.0619\ k / ft\ (previous)$$

$$W = \eta_i \gamma w_D = 1(1.4)(0.0619) = 0.0867\ k / ft$$

$$M_{u,flange} = \frac{WL_b^2}{10} = \frac{0.0867(10)^2}{10} = 0.867\ k - ft$$


51 in.
9 in.

Wind

Problem 21: Determine the total design wind forces WS for a slab on steel beam bridge for $Z = 25$ ft. The bridge is simply supported and spans 40 feet.

$$F_{transverse} = 0.050(40)(51 + 9 + 2 + 29.7) / 12 = 15.3 \text{ k}$$

$$F_{longitudinal} = 0.012(40)(51 + 9 + 2 + 29.7) / 12 = 3.67 \text{ k}$$

Note that these forces are applied simultaneously.

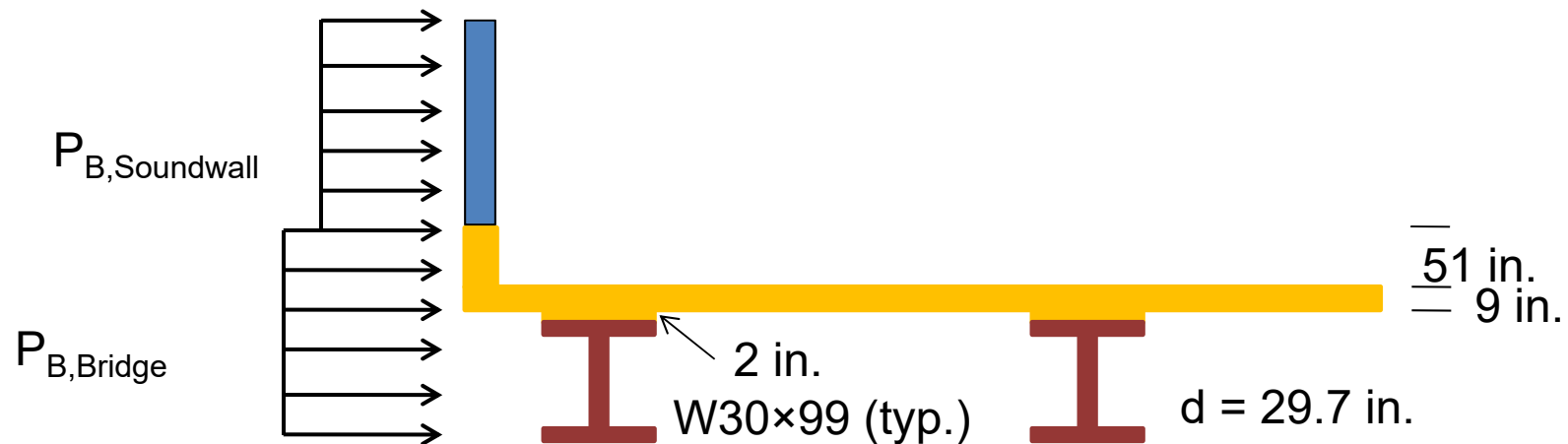


Wind

Problem 22: Determine the base wind pressure P_B for a slab on steel beam bridge and the attached soundwall for $Z < 30$ ft.

$$P_{B, \text{Soundwall}} = 0.040 \text{ ksf (AASHTO Table 3.8.1.2.1-1)}$$

$$P_{B, \text{Bridge}} = 0.050 \text{ ksf (AASHTO Table 3.8.1.2.1-1)}$$



Wind

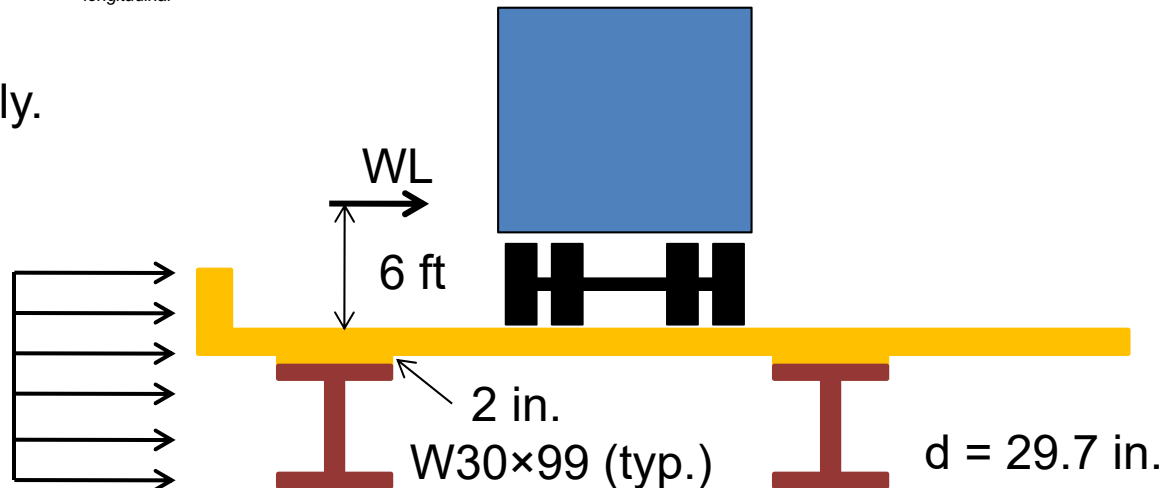
Problem 23: Determine the design wind forces WL for a slab on steel beam bridge for $Z = 25$ ft. The bridge is simply supported and spans 40 feet.

$$WL_{transverse} = 0.10 \text{ k / ft (AASHTO 3.8.1.3)}$$

$$WL_{longitudinal} = 0.04 \text{ k / ft (AASHTO 3.8.1.3)}$$

Note that these forces are applied simultaneously.
Load factor of 0.4 in Strength V reduces wind on structure.

$P_{D,transverse}$

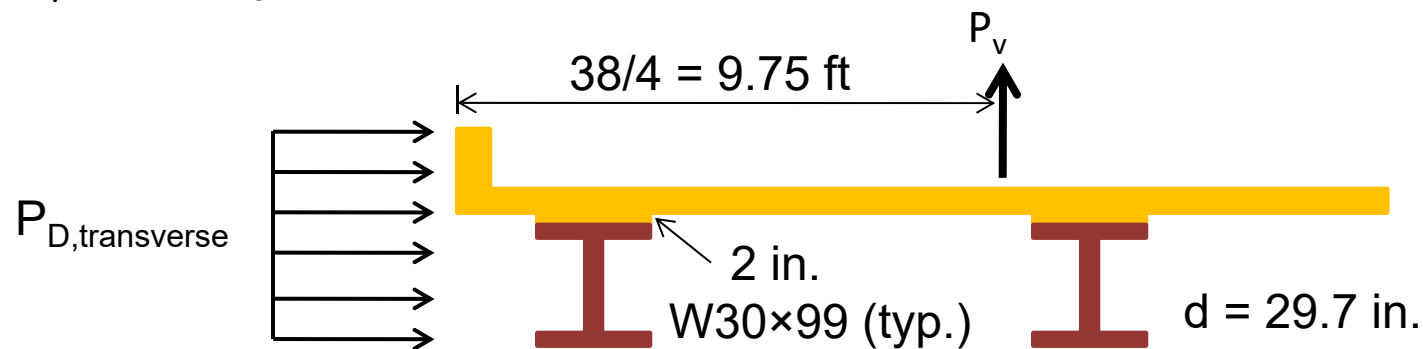


Wind

Problem 24: Determine the design vertical wind force for a slab on steel beam bridge for $Z = 25$ ft. The bridge is simply supported and spans 40 feet. The gross width of the bridge is 38 feet.

$$P_v = 0.020(38) = 0.76 \text{ k / ft (AASHTO 3.8.2)}$$

Note that P_v is only used in Strength III and Service IV limit states.
 P_v is not adjusted for elevation.



Wind

Problem 25: Determine the base wind pressure for calculating the design wind force on a bridge substructure element.

$$P_{substructure} = 0.040 \text{ ksf (AASHTO 3.8.1.2.3)}$$

Seismic

Problem 26: Discuss when a site specific procedure is required for determining the seismic hazard at a bridge site.

- Site within 6 miles of an active fault
- Site Class F
- Long duration earthquakes expected
- Longer return period justified by bridge importance

AASHTO 3.10.2

Seismic

Problem 27: Determine the 1000 year return period peak values for PGA, S_s , and S_1 in the proximity of Charleston, SC.

$$PGA = 0.60$$

$$S_s = 1.19$$

$$S_1 = 0.27$$

AASHTO Figure 3.10.2.1-13 and 3.10.2.1-14

Seismic

Problem 28: Determine the Site Class for a site classified as “highly organic clays” for a soil thickness of 15 feet (i.e., H greater than 10 feet).

Site Class F (AASHTO Table 3.10.3.1-1)

Seismic

Problem 29: Determine the Site Class for a site where the shear wave velocity is measured as 700 feet/second in the upper 100 feet.

Site Class D (AASHTO Table 3.10.3.1-1)

Seismic

Problem 30: Discuss what information may be used to determine Site Class.

- Shear wave velocity (Method A)
- Standard Penetration Test (blow counts, Method B)
- Undrained shear strengths (Method C)

Seismic

Problem 31: Determine S_{DS} , S_{D1} , A_S (AASHTO 3.10.4.1), and other response spectrum values for a Site Class D site with the following spectral accelerations: $PGA = 0.60$, $S_S = 1.19$, and $S_1 = 0.27$.

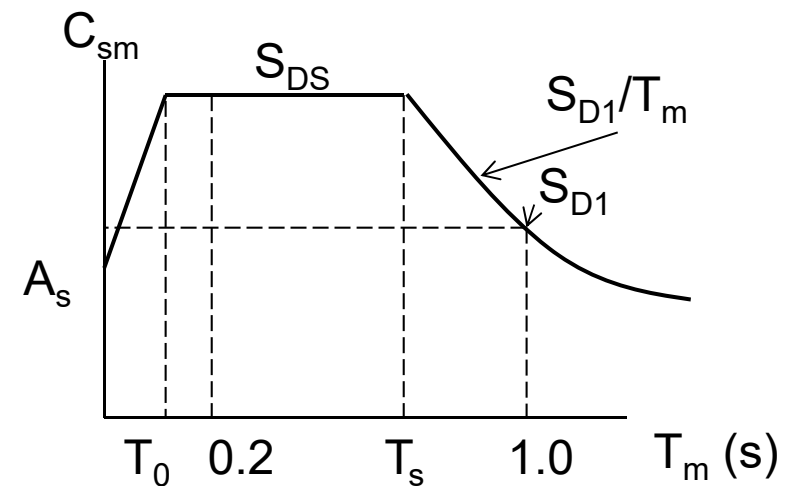
$$S_{DS} = F_a S_s = (1.02)(1.19) = 1.21$$

$$S_{D1} = F_v S_1 = (1.86)(0.27) = 0.502$$

$$A_s = F_{pga} PGA = (1.0)(0.6) = 0.6$$

$$T_s = S_{D1} / S_{DS} = 0.502 / 1.21 = 0.415 \text{ s}$$

$$T_0 = 0.2T_s = 0.2(0.415) = 0.083 \text{ s}$$



Seismic

Problem 32: Define the three seismic importance categories per AASHTO 3.10.5.

- Critical bridges
- Essential bridges
- Other bridges

The bridge importance category is based on social/survival and security/defense requirements.

Critical bridges should remain open to all traffic after 1000 year earthquake and for emergencies only after a 2500 year earthquake.

Essential bridges should be usable (emergencies only) after a 1000 year earthquake.

Seismic

Problem 33: Determine the Seismic Performance Zone for a bridge if $S_{D1} = 0.502$.

SPZ 4 (AASHTO Table 3.10.6-1, $S_{D1} > 0.5$)

Seismic

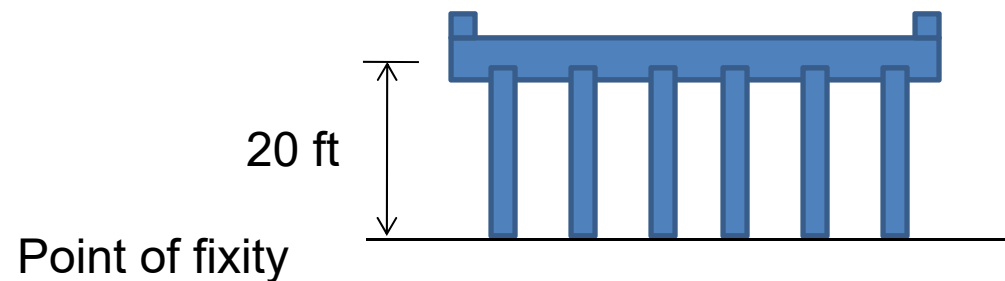
Problem 34: Discuss the impact of SPZ on bridge design.

- Permitted methods of analysis
- Minimum support lengths
- Column design details
- Foundation and abutment design procedures

Seismic

Problem 35: Determine the response modification factor for pile design for an essential bridge with a reinforced concrete vertical pile bent substructure.

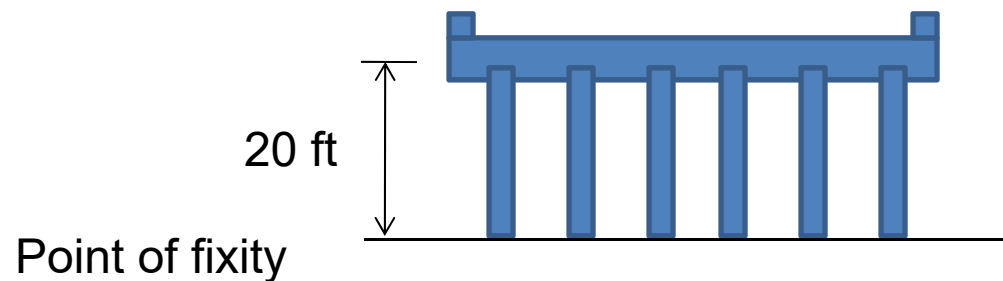
$$R = 2.0 \text{ (AASHTO Table 3.10.7.1-1)}$$



Seismic

Problem 36: Determine the response modification factor for pile bent to superstructure connection design for an essential bridge with a vertical pile bent substructure.

$$R = 1.0 \text{ (AASHTO Table 3.10.7.1-2)}$$

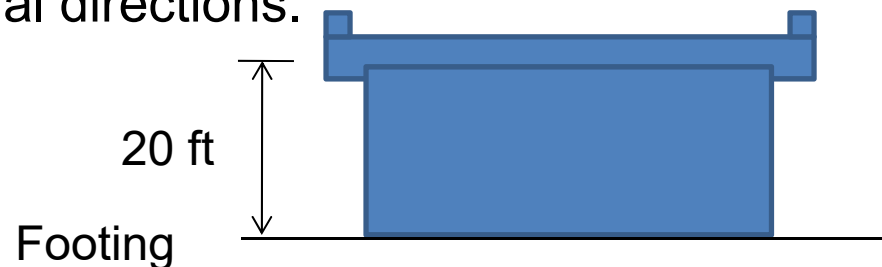


Seismic

Problem 37: Determine the response modification factor for weak direction wall-type pier design for an essential bridge.

$$R = 2.0 \text{ (AASHTO 3.10.7.2 and Table 3.10.7.1-1)}$$

Note that $R = 1.5$ for the strong direction. For most structures, R is the same in both orthogonal directions.



Seismic

Problem 38: Define combination of seismic force effects per AASHTO 3.10.8.

1. $| 100\% \text{ force effects } X | + | 30\% \text{ force effects } Y |$
2. $| 100\% \text{ force effects } Y | + | 30\% \text{ force effects } X |$

Seismic

Problem 39: For a single span bridge (AASHTO 3.10.9.1) in any Seismic Zone, determine the minimum unfactored seismic connection force effect at the abutment.

The minimum seismic force at the abutment is A_s times the tributary permanent load.

Also note that minimum support lengths are also required for single span bridges per AASHTO 4.7.4.4.

Seismic

Problem 40: For a multispan bridge in Seismic Zone 1 (AASHTO 3.10.9.2) with $A_s = 0.04$, determine the minimum unfactored seismic connection force effect to be considered in the restrained direction at the superstructure to substructure connection.

The minimum seismic force at the abutment when $A_s < 0.05$ is equal to 0.15 times the tributary permanent load (as vertical reaction) and the tributary live load (usually 0) assumed to be present during the earthquake.

The minimum seismic force at the abutment when $A_s \geq 0.05$ is equal to 0.25 times the tributary permanent load (as vertical reaction) and the tributary live load (usually 0) assumed to be present during the earthquake.

Seismic

Problem 41: For a multispan bridge in Seismic Zone 2 (AASHTO 3.10.9.3), discuss the determination of seismic forces for all components except for foundations (i.e., piles under columns and footings).

The appropriate seismic force is determined using UL, SM, MM, or TH (AASHTO 4.7.4.1 and 4.7.4.3) and dividing by R.

Note that the appropriate seismic force for foundations is obtained by dividing R by $2 > 1.0$ to conservatively account for overstrength in a simplified manner.

Seismic

Problem 42: For a multispan bridge in Seismic Zone 3 or 4 (AASHTO 3.10.9.4), discuss the determination of seismic forces for all components except for foundations (i.e., piles under columns and footings).

Design for the lesser of the following two cases:

- CASE 1: The appropriate seismic force is determined using UL, SM, MM, or TH (AASHTO 4.7.4.1 and 4.7.4.3) and dividing by R .
Note that the appropriate seismic force for foundations is obtained by dividing by $R = 1.0$ to account for maximum demand as an elastic response.
- CASE 2: The appropriate seismic force is determined using inelastic hinging (AASHTO 3.10.9.4.3) at the top and/or bottom of the columns with overstrength.

Seismic

Problem 43: Determine if a three span bridge with adjacent span lengths of 50 feet, 150 feet, and 50 feet is regular or irregular (AASHTO Table 4.7.4.3.1-2).

Irregular—for three spans, the maximum span length ratio for adjacent spans is 2 [$150/50 = 3 > 2$ (ng)]

Note that curved bridges and adjacent bent/pier stiffness ratios (excluding the abutment stiffness) may also result in irregular classifications.

Seismic

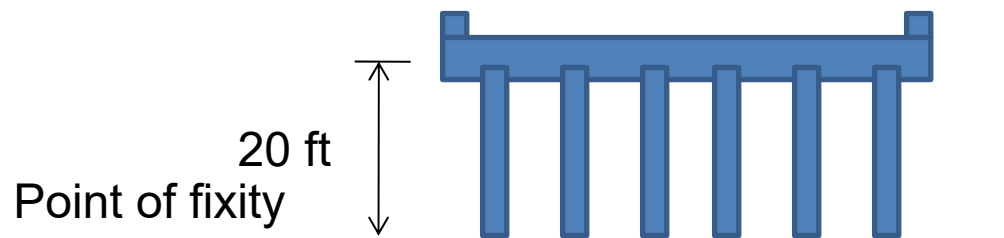
Problem 44: Determine the minimum permitted analysis method for seismic effects for an irregular multispan essential bridge in Seismic Zone 3 (AASHTO Table 4.7.4.3.1-1).

MM—The multimode elastic method is the minimum permitted method; however, the time history method (TH) is also permitted.

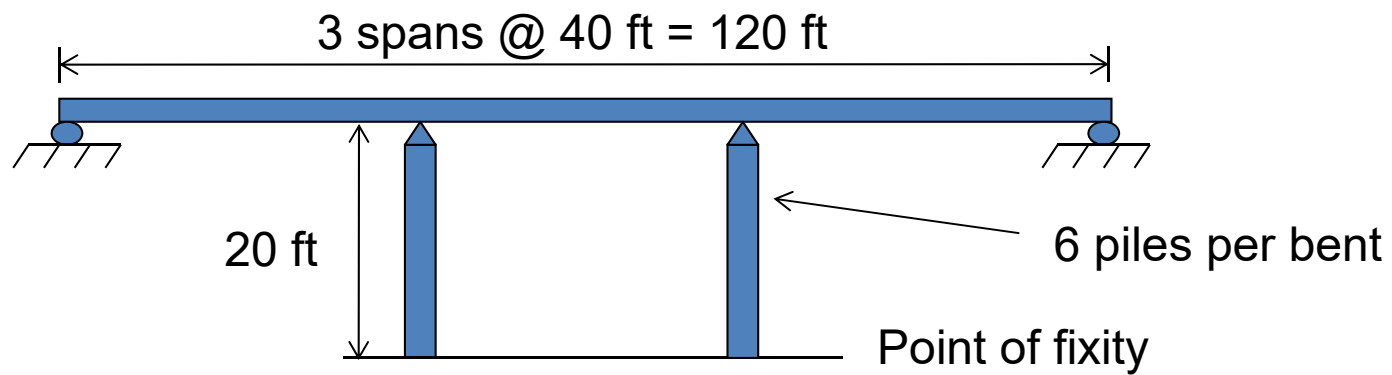
Other methods allowed for other bridges include the uniform load elastic method (UL) and the single-mode elastic method (SM).

Seismic

Introduction for subsequent problems: The subject bridge is a three span bridge with a total length of $3(40 \text{ ft}) = 120 \text{ ft}$. The bridge is supported by two interior six-column bents and two abutments. The columns (20 inch precast prestressed square piles) are continuous from the deck elevation to the point of fixity 20 feet below the deck elevation (i.e., the piles can be considered 20 feet long, fixed at the bottom, free at the top in longitudinal direction, and fixed at the top in the transverse direction). Assume the bridge deck is free to move in the longitudinal direction at the abutments and pinned at the abutments in the transverse direction. Assume $E = 4,000 \text{ ksi} = 576,000 \text{ ksf}$ and $I = 0.5I_g$ for the piles (AASHTO 4.7.1.3) and $I = 6,480 \text{ ft}^4$ for the superstructure (20 inch slab).



Seismic



Seismic

Problem 45: Determine the bridge's stiffness in response to longitudinal and transverse forces.

$$I_g = 20(20)^3 / 12 = 13,300 \text{ in}^4 = 0.643 \text{ ft}^4$$

$$K_{fixed} = \frac{12EI}{h^3}$$

$$I = (0.5)0.643 = 0.322 \text{ ft}^4$$

$$K_{fixed} = \frac{12(576,000)(0.322)}{(20)^3} = 278 \text{ k / ft (per pile)}$$

$$K_{free} = \frac{3EI}{h^3}$$

$$K_e = 6(278) = 1,668 \text{ k / ft (per bent)}$$

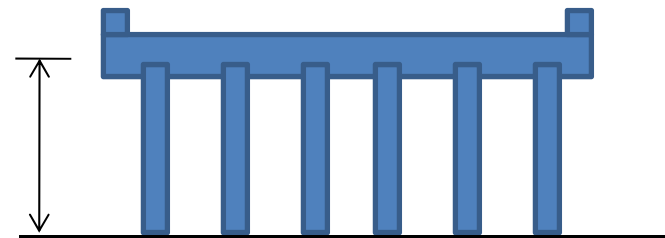
$$K_{free} = \frac{3(576,000)(0.322)}{(20)^3} = 69.5 \text{ k / ft (per pile)}$$

Note abutments assumed infinitely stiff in the transverse direction.

$$K_e = 12(69.5) = 834 \text{ k / ft}$$

Point of fixity

20 ft



Seismic

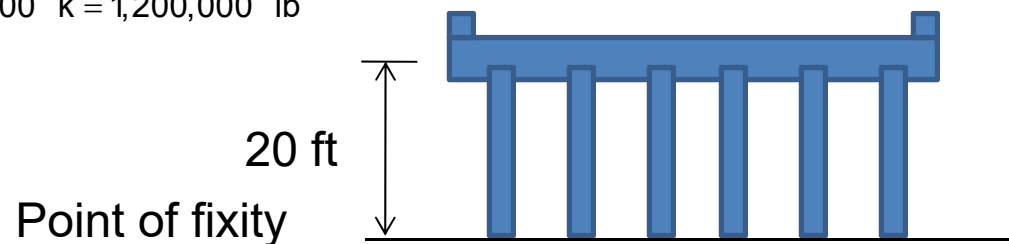
Problem 46: Using the uniform load method (AASHTO 4.7.4.3.2c) for the longitudinal response, determine the static displacement $v_s(x)$ for the uniform load p_0 , the bridge lateral stiffness K for the maximum displacement, and the total seismic weight of the bridge W . Assume $w(x) = 10$ k/ft.

$$K_e = 834 \text{ k / ft}$$

$$v_s(x) = 120 / 834 = 0.144 \text{ ft (assumes a unit load of } p_0 = 1 \text{ k / ft at deck)}$$

$$K = \frac{p_0 L}{v_{s,\max}} = \frac{(1)(120)}{0.144} = 834 \text{ k / ft} = 834,000 \text{ lb / ft}$$

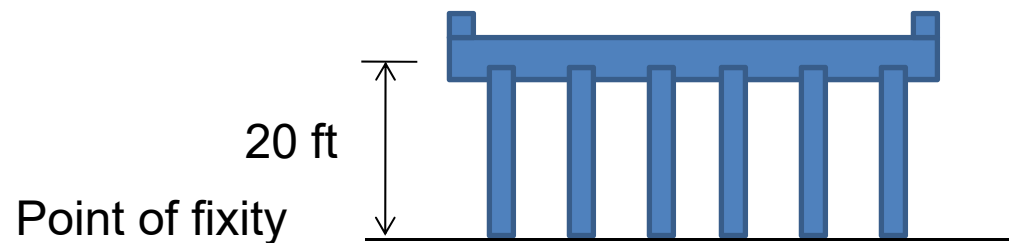
$$W = \int w(x) dx = w(x)L = 10(120) = 1,200 \text{ k} = 1,200,000 \text{ lb}$$



Seismic

Problem 47: Using the uniform load method for the longitudinal response, determine the natural period of the bridge.

$$T_m = 2\pi \sqrt{\frac{W}{gK}} = 2\pi \sqrt{\frac{1,200,000}{32.2(834,000)}} = 1.33 \text{ s}$$



Seismic

Problem 48: Using the uniform load method for the longitudinal response, determine C_{sm} for the bridge if $T_m = 1.33$ s.

$$S_{DS} = F_a S_s = (1.02)(1.19) = 1.21$$

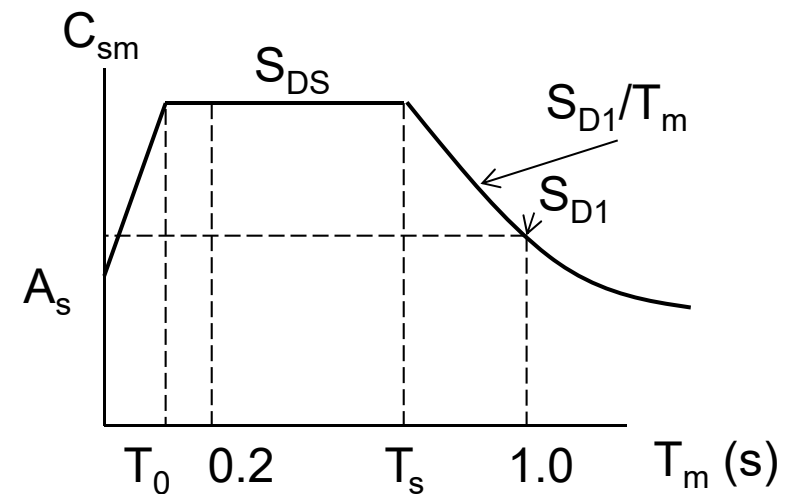
$$S_{D1} = F_v S_1 = (1.86)(0.27) = 0.502$$

$$A_s = F_{pga} PGA = (1.0)(0.6) = 0.6$$

$$T_s = S_{D1} / S_{DS} = 0.502 / 1.21 = 0.415 \text{ s}$$

$$T_0 = 0.2T_s = 0.2(0.415) = 0.083 \text{ s}$$

$$C_{sm} = S_{D1} / T_m = 0.502 / 1.33 = 0.377$$

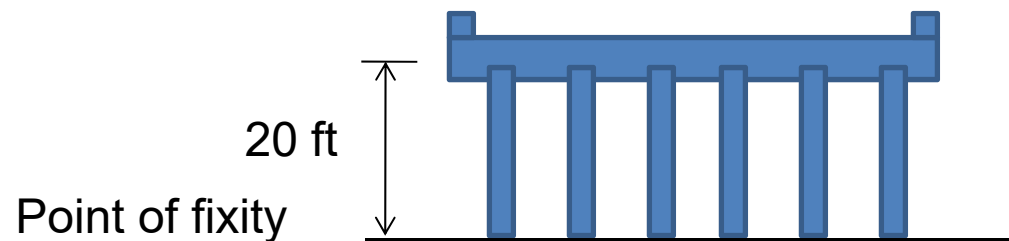


Seismic

Problem 49: Using the uniform load method for the longitudinal response, determine the equivalent seismic load.

$C_{sm} = 0.377$, $W = 1,200$ k, and $L = 120$ ft.

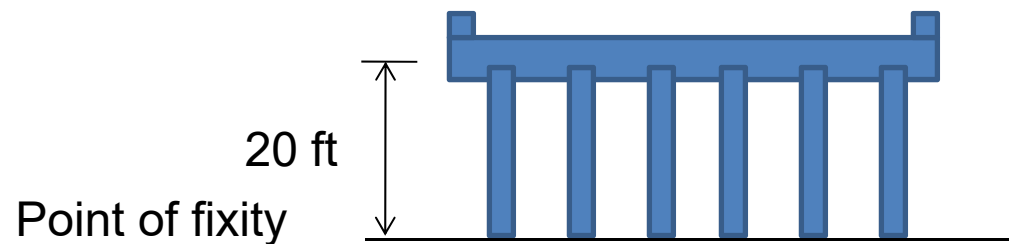
$$p_e = \frac{C_{sm} W}{L} = \frac{0.377(1,200)}{120} = 3.77 \text{ k / ft}$$



Seismic

Problem 50: Using the uniform load method for the longitudinal response, determine the longitudinal displacement due to the equivalent seismic load. $v_s(x) = 0.144$ ft and $p_e = 3.77$ k/ft.

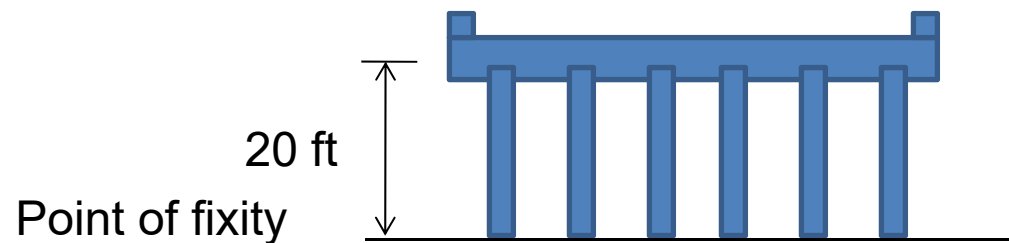
$$v_e(x) = v_s(x)p_e = 0.144(3.77) = 0.543 \text{ ft}$$



Seismic

Problem 51: Using the uniform load method for the longitudinal response, determine the elastic shear in a column due to the equivalent seismic load. $v_e(x) = 0.543$ ft and $K_e = 69.5$ k/ft (per pile).

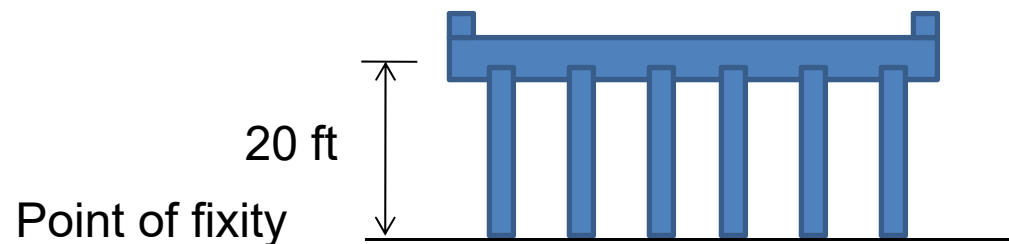
$$V = 0.543(69.5) = 37.7 \text{ k}$$



Seismic

Problem 52: Using the uniform load method for the longitudinal response, determine the elastic moment at the base of the column due to the equivalent seismic load. $V = 37.7$ k at top of one pile.

$$M = 37.7(20) = 754 \text{ k} - \text{ft}$$



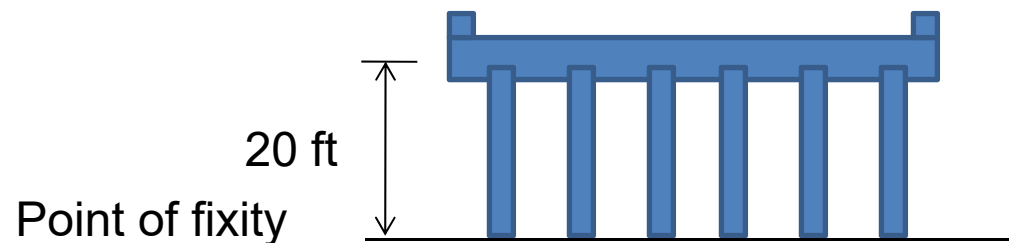
Seismic

Problem 53: Determine the seismic design moment at the base of the column due to the equivalent seismic load. Assume an importance category of “essential.” (AASHTO Table 3.10.7.1-1)

$$R = 2.0$$

$$M = 754 \text{ k-ft (previous slide)}$$

$$M_{R, \text{longitudinal}} = 754 / 2 = 377 \text{ k-ft}$$



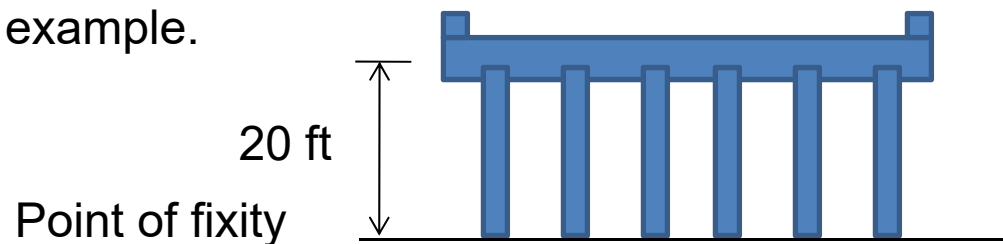
Seismic

Problem 54: Determine the seismic design moment at the base of the column due to the equivalent seismic load. Assume that subsequent transverse analysis results in $M_{R,transverse} = 1,500$ k-ft. (AASHTO 3.10.8)

$$M_{R,longitudinal} = 377 \text{ k-ft (previous slide)}$$

$$M_R = \sqrt{[377(0.3)]^2 + 1,500^2} = 1,504 \text{ k-ft}$$

The vector sum is more applicable to circular sections, and biaxial bending with $0.3(377) = 113$ k-ft combined with 1,500 k-ft may be more appropriate for this example.



Seismic

Problem 55: Using the single-mode spectral method (AASHTO 4.7.4.3.2b) for the longitudinal response, determine the static displacement $v_s(x)$ for the uniform load p_0 and other relevant analysis variables. Assume $w(x) = 10$ k/ft.

$$K_e = 834 \text{ k / ft}$$

$$v_s(x) = 120 / 834 = 0.144 \text{ ft (assumes a unit load of } p_0 = 1 \text{ k / ft at deck)}$$

$$\alpha = \int_0^L v_s(x) dx = 0.144(120) = 17.28 \text{ ft}^2$$

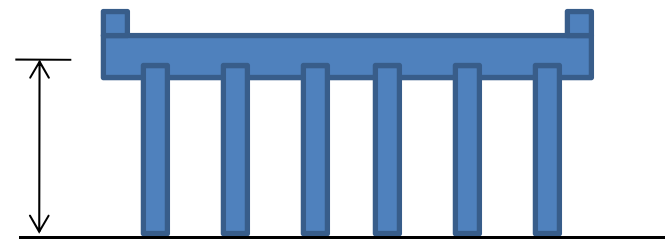
$$T_m = 2\pi \sqrt{\frac{\gamma}{p_0 g \alpha}} = 2\pi \sqrt{\frac{24.9}{(1)(32.2)17.28}} = 1.33 \text{ s}$$

$$\beta = \int_0^L w(x) v_s(x) dx = (10)(0.144)(120) = 172.8 \text{ k - ft}$$

$$\gamma = \int_0^L w(x) v_s^2(x) dx = 10(0.144)^2(120) = 24.9 \text{ k - ft}^2$$

20 ft

Point of fixity



Seismic

Problem 56: Using the single-mode spectral method for the longitudinal response, determine the equivalent seismic load.

$$\beta = 172.8 \text{ k-ft (previous slide)}$$

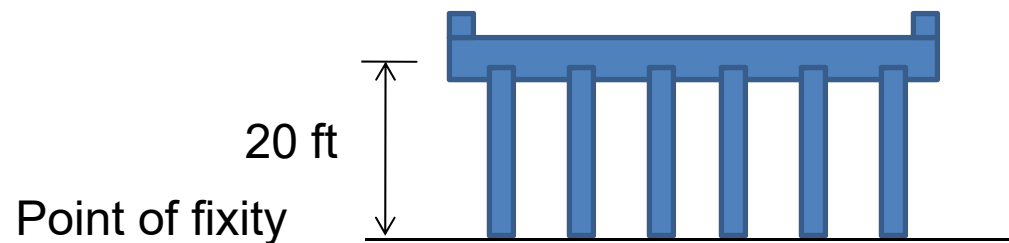
$$w(x) = 10 \text{ k/ft (previous slide)}$$

$$v_s(x) = 0.144 \text{ ft (previous slide)}$$

$$C_{sm} = 0.377 \text{ (previous slide)}$$

$$\gamma = 24.9 \text{ k-ft}^2 \text{ (previous slide)}$$

$$p_e(x) = \frac{\beta C_{sm} w(x) v_s(x)}{\gamma} = \frac{(172.8)(0.377)(10)(0.144)}{24.9} = 3.77 \text{ k/ft}$$



Seismic

Problem 57: Using the uniform load method (AASHTO 4.7.4.3.2c) for the transverse response, determine the maximum static displacement $v_{s,max}$ for the uniform load p_0 , and the bridge lateral stiffness K for the maximum displacement. Assume $E = 576,000$ ksf, $K_e = 1,668$ k/ft (per bent), and $I = 6,480$ ft⁴ for the superstructure.

$$\Delta_{deck,max} = \frac{5p_0L^4}{384E_{sup\ erstructure}I_{sup\ erstructure}} = \frac{5(1)(120)^4}{384(576,000)(6,480)} = 0.000723 \text{ ft}$$

$$\Delta_{piles,max} = \frac{23V_{bent}L^3}{648E_{sup\ erstructure}I_{sup\ erstructure}} = -\frac{23V_{bent}(120)^3}{648(576,000)(6,480)} = -0.0000164V_{bent} \text{ ft}$$

$$\Delta_{deck,bent} = \frac{22p_0L^4}{1,944E_{sup\ erstructure}I_{sup\ erstructure}} = \frac{22(1)(120)^4}{1,944(576,000)(6,480)} = 0.000629 \text{ ft}$$

$$\Delta_{piles,bent} = \frac{20V_{bent}L^3}{648E_{sup\ erstructure}I_{sup\ erstructure}} = -\frac{20V_{bent}(120)^3}{648(576,000)(6,480)} = -0.0000143V_{bent} \text{ ft}$$

Seismic

Problem 58:

$$\Delta_{\text{deck,max}} = 0.000723 \text{ ft}$$

$$\Delta_{\text{bent,max}} = -0.0000164 V_{\text{bent}} \text{ ft}$$

$$\Delta_{\text{deck,bent}} = 0.000629 \text{ ft}$$

$$\Delta_{\text{bent,bent}} = -0.0000143 V_{\text{bent}} \text{ ft}$$

$$V_{\text{bent}} = K_c v_{\text{s,bent}} = 1,668 v_{\text{s,bent}}$$

$$v_{\text{s,max}} = \Delta_{\text{deck,max}} + \Delta_{\text{bent,max}} = 0.000723 - 0.0000164 V_{\text{bent}}$$

$$v_{\text{s,bent}} = \Delta_{\text{deck,bent}} + \Delta_{\text{bent,bent}} = 0.000629 - 0.0000143 V_{\text{bent}} = 0.000629 - 0.0000143 (1,668) v_{\text{s,bent}}$$

$$v_{\text{s,bent}} = 0.000614 \text{ ft}$$

$$V_{\text{bent}} = 1.024 \text{ k}$$

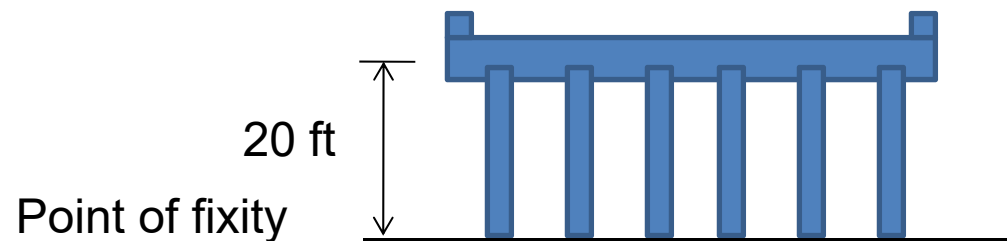
$$v_{\text{s,max}} = 0.000706 \text{ ft}$$

$$K = \frac{p_0 L}{v_{\text{s,max}}} = \frac{(1)(120)}{0.000706} = 169,970 \text{ k / ft} = 169,970,000 \text{ lb / ft}$$

Seismic

Problem 59: Using the uniform load method for the transverse response, determine the natural period of the bridge.

$$T_m = 2\pi \sqrt{\frac{W}{gK}} = 2\pi \sqrt{\frac{1,200,000}{32.2(169,970,000)}} = 0.093 \text{ s}$$



Seismic

Problem 60: Using the uniform load method for the transverse response, determine C_{sm} for the bridge if $T_m = 0.093$ s.

$$S_{DS} = F_a S_s = (1.02)(1.19) = 1.21$$

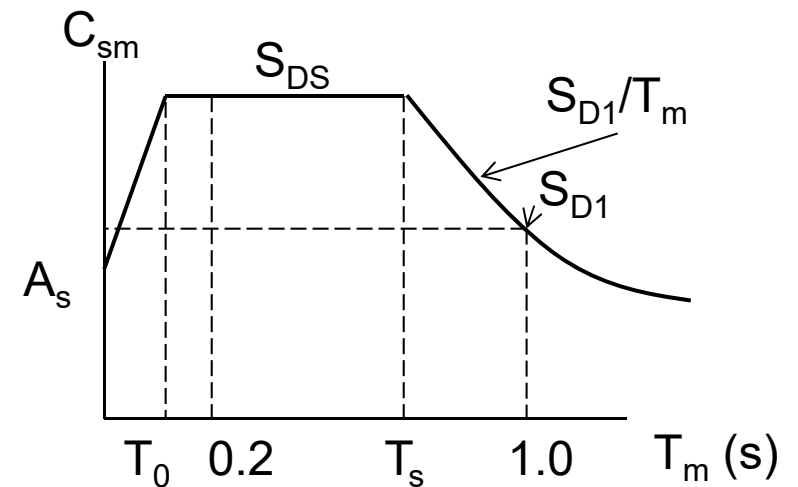
$$S_{D1} = F_v S_1 = (1.86)(0.27) = 0.502$$

$$A_s = F_{pga} PGA = (1.0)(0.6) = 0.6$$

$$T_s = S_{D1} / S_{DS} = 0.502 / 1.21 = 0.415 \text{ s}$$

$$T_0 = 0.2 T_s = 0.2(0.415) = 0.083 \text{ s}$$

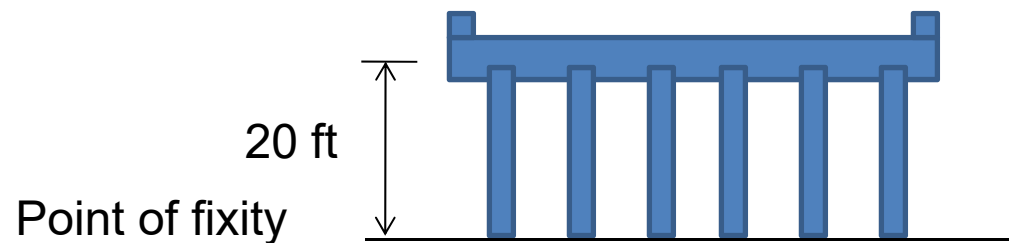
$$C_{sm} = S_{DS} = 1.21$$



Seismic

Problem 61: Using the uniform load method for the transverse response, determine the equivalent seismic load. $C_{sm} = 1.21$, $W = 1,200$ k, and $L = 120$ ft.

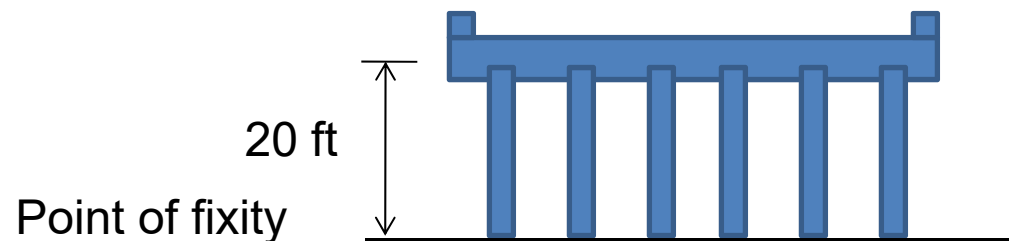
$$p_e = \frac{C_{sm} W}{L} = \frac{1.21(1,200)}{120} = 12.1 \text{ k / ft}$$



Seismic

Problem 62: Using the uniform load method for the transverse response, determine the transverse displacement of the bents due to the equivalent seismic load. $v_{s,bent} = 0.000614$ ft and $p_e = 12.1$ k/ft.

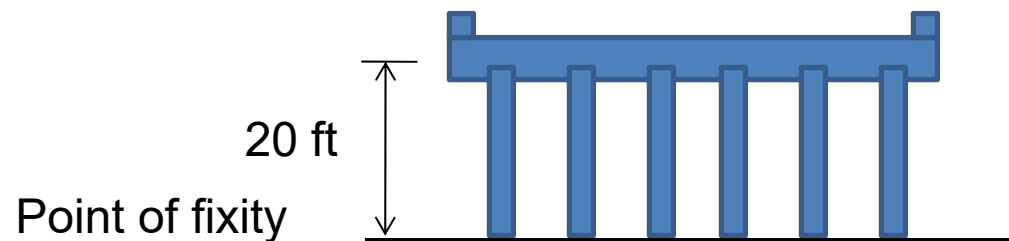
$$V_{e,bent} = v_{s,bent} p_e = 0.000614(12.1) = 0.00743 \text{ ft}$$



Seismic

Problem 63: Using the uniform load method for the transverse response, determine the elastic shear in a column due to the equivalent seismic load. $v_{e,bent} = 0.00743$ ft and $K_e = 278$ k/ft (per pile).

$$V = 0.00743(278) = 2.065 \text{ k}$$



Seismic

Problem 64: Using the single-mode spectral method (AASHTO 4.7.4.3.2b) for the transverse response, determine the static displacement $v_s(x)$ for the uniform load p_0 and other relevant analysis variables. Assume $w(x) = 10$ k/ft. Assume pile bent stiffness is negligible based on results of uniform load method.

$$v_s(x) = \frac{p_0 x}{24EI} (L^3 - 2Lx^2 + x^3) = \frac{x}{24(576,000)(6,480)} (120^3 - 2(120)x^2 + x^3)$$

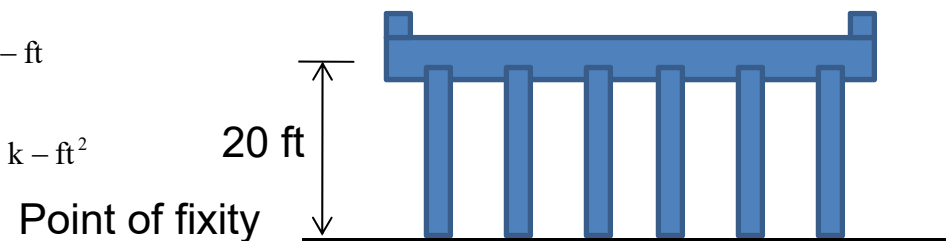
$$v_s(x) = 1.929(10)^{-5} x - 2.679(10)^{-9} x^3 + 1.116(10)^{-11} x^4$$

$$\alpha = \int_0^{120} v_s(x) dx = 0.0555 \text{ ft}^2$$

$$\beta = \int_0^{120} w(x) v_s(x) dx = (10) v_s(x) dx = 0.555 \text{ k-ft}$$

$$\gamma = \int_0^{120} w(x) v_s^2(x) dx = 10 v_s^2(x) dx = 0.000316 \text{ k-ft}^2$$

$$T_m = 2\pi \sqrt{\frac{\gamma}{p_0 g \alpha}} = 2\pi \sqrt{\frac{0.000316}{(1)(32.2)0.0555}} = 0.0836 \text{ s}$$



Seismic

Problem 65: Using the single-mode spectral method for the transverse response, determine the equivalent seismic load.

$$v_s(x) = 1.929(10)^{-5} x - 2.679(10)^{-9} x^3 + 1.116(10)^{-11} x^4$$

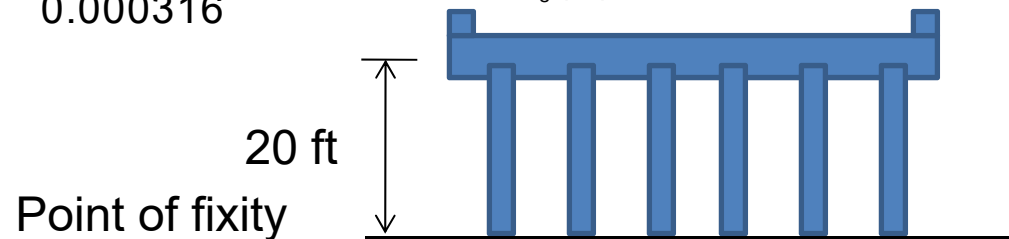
$$\beta = 0.555 \quad k - ft \quad (\text{previous slide})$$

$$w(x) = 10 \quad k / ft \quad (\text{previous slide})$$

$$C_{sm} = 1.21 \quad (\text{previous slide})$$

$$\gamma = 0.000316 \quad k - ft^2 \quad (\text{previous slide})$$

$$p_e(x) = \frac{\beta C_{sm} w(x) v_s(x)}{\gamma} = \frac{(0.555)(1.21)(10) v_s(x)}{0.000316} = 21,251 v_s(x) \quad k / ft$$

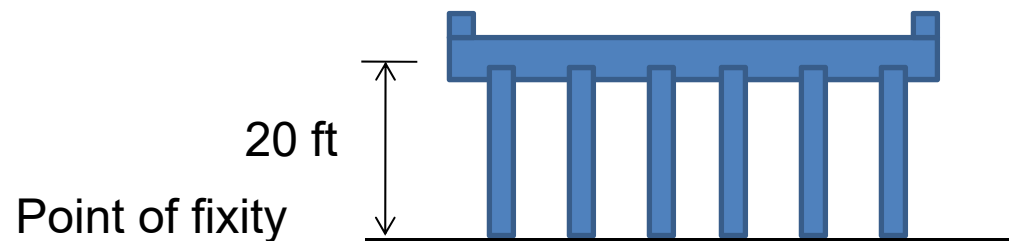


Seismic

Problem 66: Use simple structural dynamics to determine the natural period of the bridge in the transverse direction. Assume pile bent stiffness is negligible based on results of uniform load method.

$$\omega = \frac{\pi^2}{L^2} \sqrt{\frac{EI}{m}} = \frac{\pi^2}{(120)^2} \sqrt{\frac{576,000,000(6,480)}{10,000 / 32.2}} = 75.14 \text{ rad / s}$$

$$T_m = \frac{2\pi}{\omega} = \frac{2\pi}{75.14} = 0.0836 \text{ s}$$

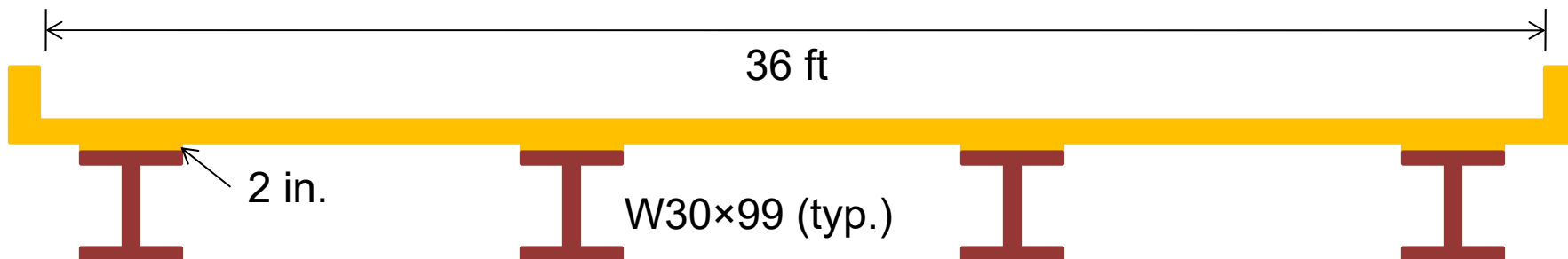


Seismic

Problem 67: Determine the required seat-width for the stringers shown. Assume that there is no skew (i.e., $S=0$). $SPZ = 4$. Expansion joints are placed only at the abutments. (AASHTO 4.7.4.4). $L = 120$ feet and $H = 20$ feet.

$$N = (12 + 0.03L + 0.12H)(1 + 0.000125S^2)$$

$$N = [12 + 0.03(120) + 0.12(20)][1 + 0.000125(0)^2] = 18 \text{ in.}$$



Structural Design Standards Relevant for Lateral Forces

- In order of precedence
 - AASHTO LRFD Bridge Design Specifications (7th Edition, 2014)

Recommended References and Additional Study Materials

- Structural: Sample Questions + Solutions (NCEES, 2014)
- Structural Engineering Reference Manual (Williams, PPI, 2014)
- Design of Highway Bridges: An LRFD Approach (Barker and Puckett, Wiley, 2013)