Lawrence Livermore National Laboratory

Large scale simulations of earthquakes

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Anders Petersson

Center for Applied Scientific Computing

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Lawrence Livermore National Laboratory, P. O. Box 808, Livermore, CA 94551

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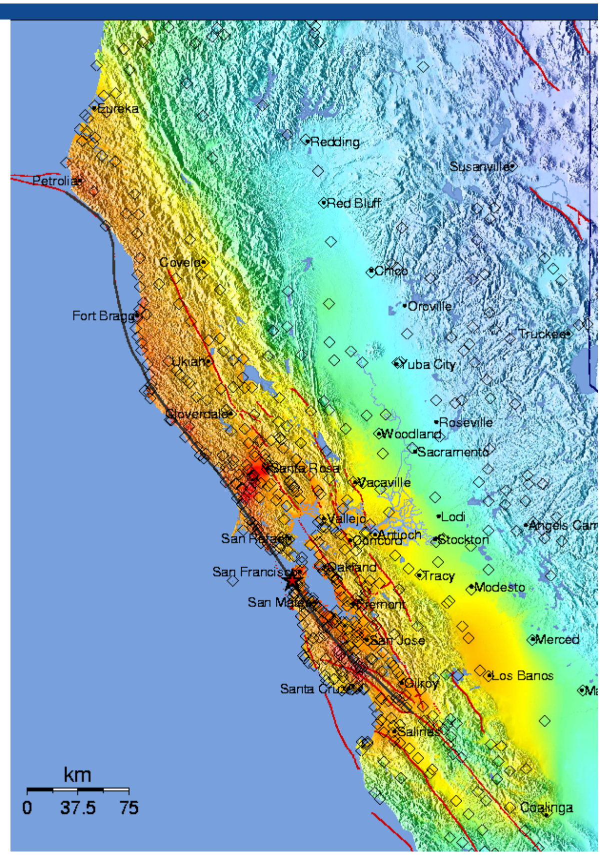
The great 1906 earthquake (M_W = 7.8 - 7.9) devastated San Francisco and caused damage throughout N. California

San Francisco city hall

Stockton Street from Union Square



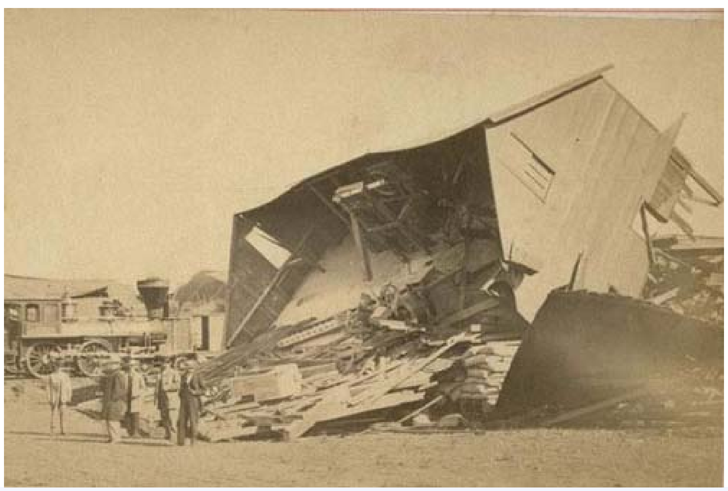
Shaking intensity on the Modified Mercalli scale (Boatwright, 2005)

Valencia Street Hotel before EQ…

… and after. Bottom 3 stories in a sink hole

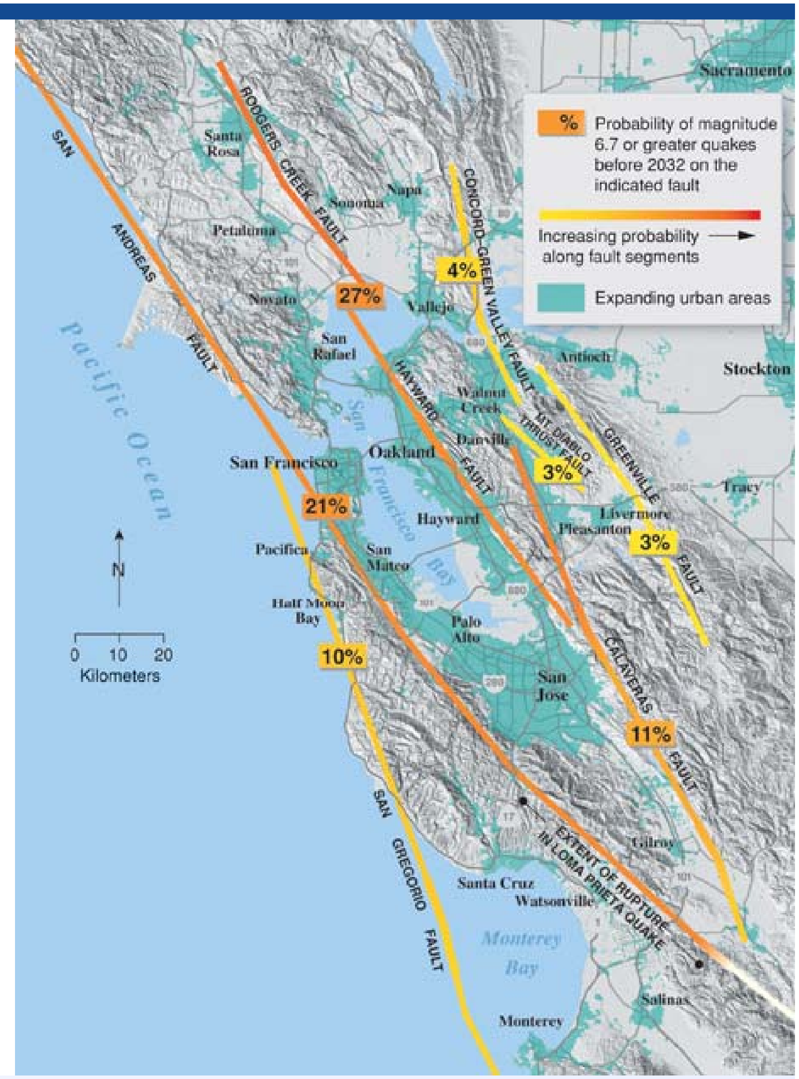
When will the next big earthquake hit?

- **USGS predicts a 62 % chance of a magnitude > 6.7 earthquake in the Bay Area during the next 25 years.**
- **The past 5 large earthquakes on the Hayward fault have occurred on average every 140 years.**
- **Last big one in October 1868 ($M_W \approx 6.8$).**
- **1868+140=2008**

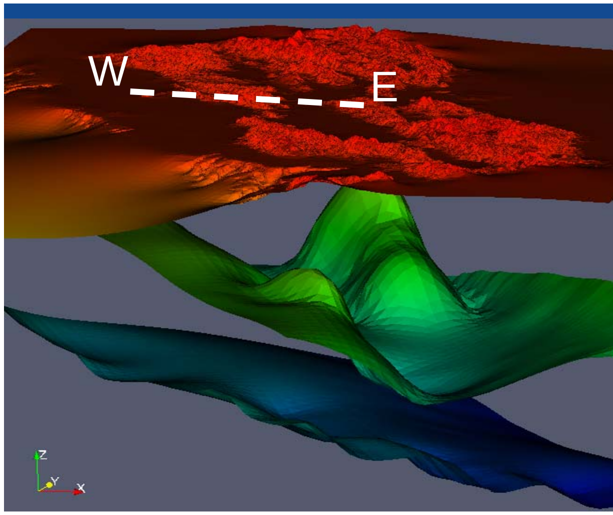


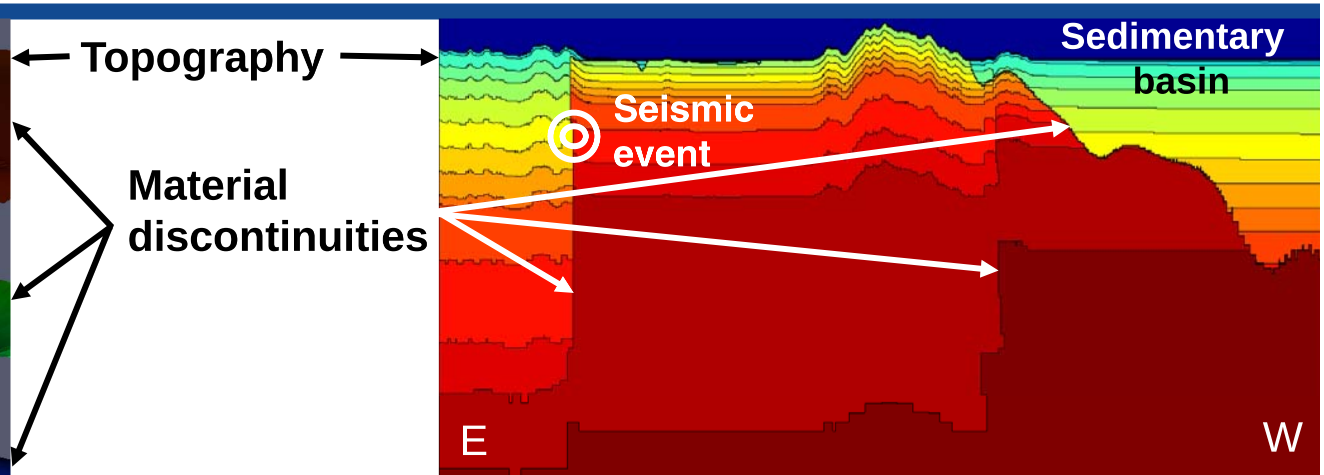
Flour mill in

Hayward, 1868



Seismic wave simulations are computationally challenging





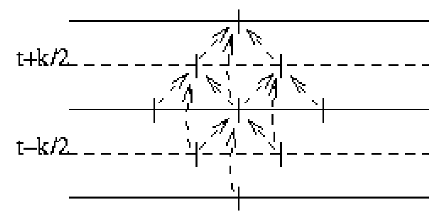
- **P & S-wave speed vary by factor 8**
- **Wave length ~ Wave speed**
- **Time step ~ h/V**
- **Smallest wave speed -> grid size**
- **Largest wave speed -> time step**
- **Min. res. 10 h = λ_{min}= Vs_{min}/freq**

Vs_{min}=500 m/s, Domain: 550 x 200 x 40 km, Single mesh WPP code, 1024 proc. cluster					
freq	h [m]	N_{GP}	δt [s]	T-steps	CPU [h]
0.2	250	256e6	2.2e-2	13,600	2
0.4	125	2e9	1.1e-2	27,300	26
0.5	100	4e9	8.7e-3	34,500	67
1.0	50	32e9	4.4e-3	68,200	1,061

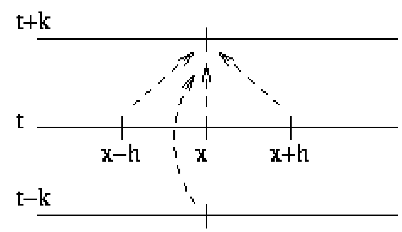
We solve the (an-)elastic wave equation in 2nd order displacement formulation

- **Staggered grid FD methods are popular for seismic wave propagation**
  - 1st order velocity-stress formulation
  - 9 dependent variables, 2 time levels = 18 per grid point
  - **Has not been (can't be?) generalized to non-planar topography**
- **Scalar wave eqn:**
  - Staggered grid discr. of 1st order system = node based discr. of 2nd order system
  - Solving the second order system "in disguise"
- **Second order displacement formulation**
  - All variables in the same location
  - 3 dependent variables, 3 time levels = 9 per grid point
  - Previous discretizations unstable with free surface bc
  - **New discretization stable for all V_P/V_S ratios**
  - **Generalizes to non-planar topography using curvilinear grids**

$$u_t = v_x$$
$$v_t = u_x$$



$$u_{tt} = u_{xx}$$



Summation by parts FD operators lead to a provably stable method for heterogeneous materials

- **Theory for 2nd order hyp. systems:**
  - Lemma: Solutions **v**(t) uniformly bounded in time iff eigenvalues of **A** are real and negative and eigenvectors form a complete set. Ok if **A=A***<0.

$$\mathbf{v}_{tt} = A\mathbf{v}, \quad \mathbf{v} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{pmatrix}$$

$$A\mathbf{e} = \zeta\mathbf{e}$$

- **New summation by parts scheme:**
  - Take cross terms one-sided on boundary and modify discretization of boundary condition
  - Stable with free surface bc and variable (discontinuous) wave speeds, any V_P/V_S ratio
  - 2nd order accurate for smooth materials
  - Fully discrete scheme stable if time-step satisfies Courant condition (non-trivial to estimate eigenvalues)

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} = \nabla\cdot\mathcal{T} + \mathbf{f},$$

$$\mathcal{T} = \mu\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right) + \lambda(\nabla\cdot\mathbf{u})I$$

$$\frac{V_P}{V_S} = \sqrt{2 + \frac{\lambda}{\mu}}$$

$$\max|\zeta|\delta_t^2 < 4$$

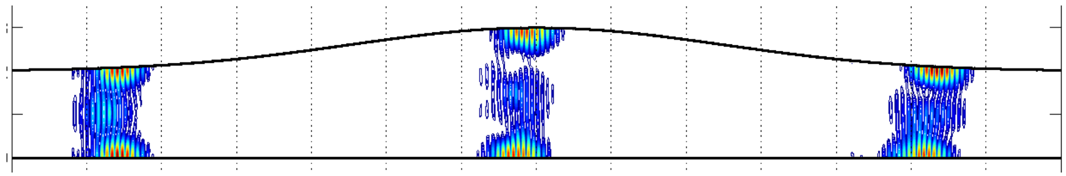
- **More info in publications:**

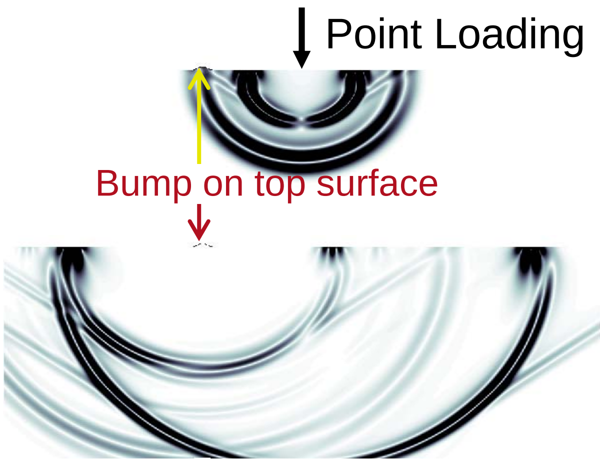
  - H.O. Kreiss, N.A Petersson and J. Ystrom, SIAM J. Numer. Anal., **40** (2002)
  - S. Nilsson, N.A Petersson, B. Sjogreen and H.O. Kreiss, SIAM J. Numer. Anal., **45** (2007)
  - D. Appelo and N.A Petersson, Comm. Comput. Phys. (2008) (to appear).

The summation by parts technique can be extended to non-planar free-surfaces using a curvilinear mesh

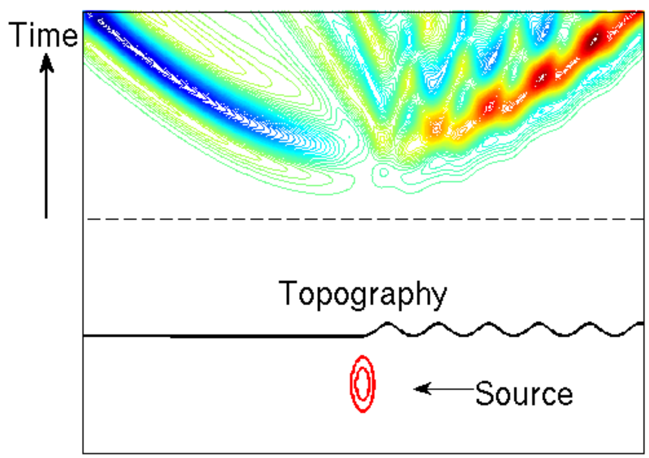
- **Topography becomes important as the wave length gets shorter (higher frequency)**

Curvature dependent wave speed



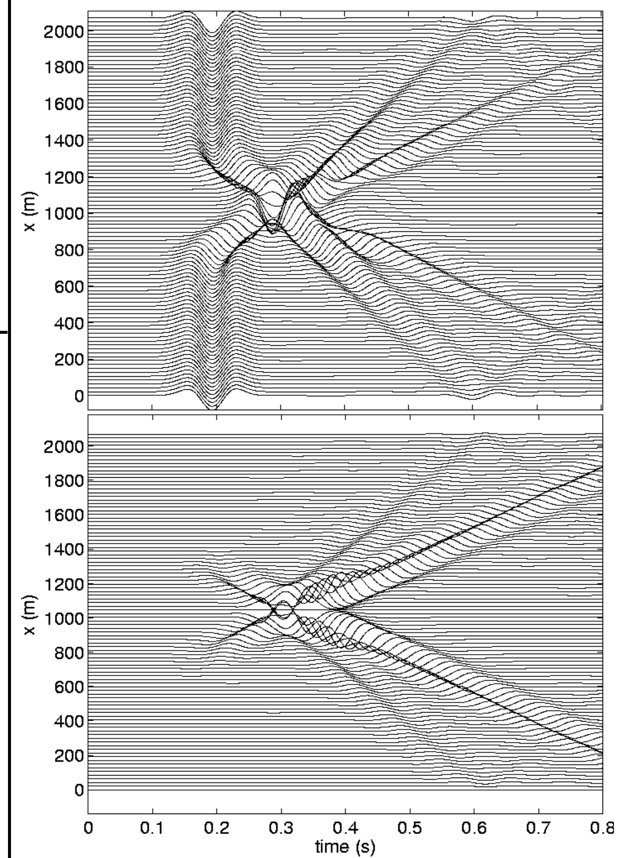


Back scattering



Complex interaction

Planar wave impinging on 3-D Gaussian hill

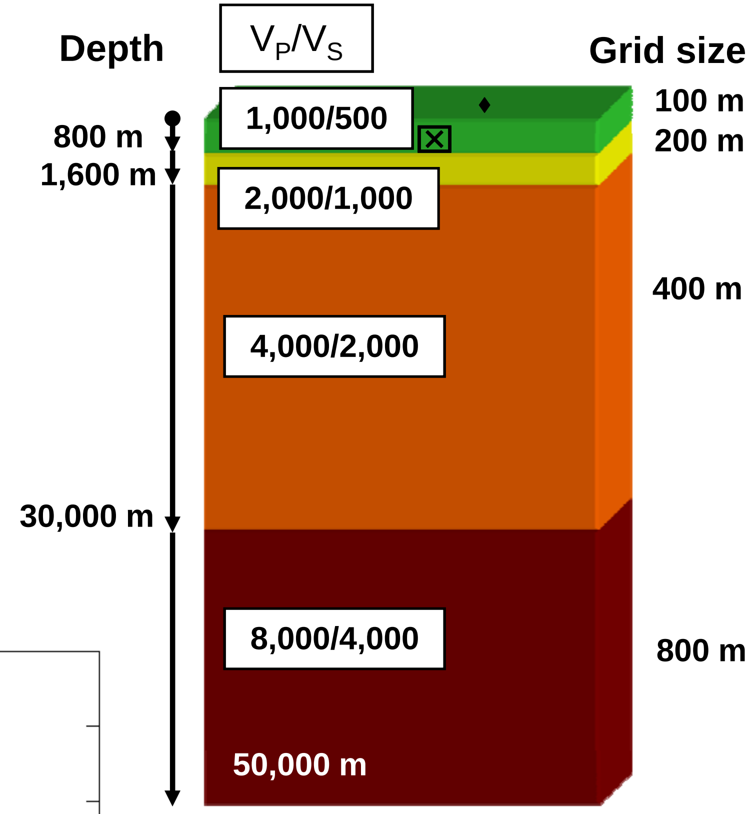


Mode to mode conversion

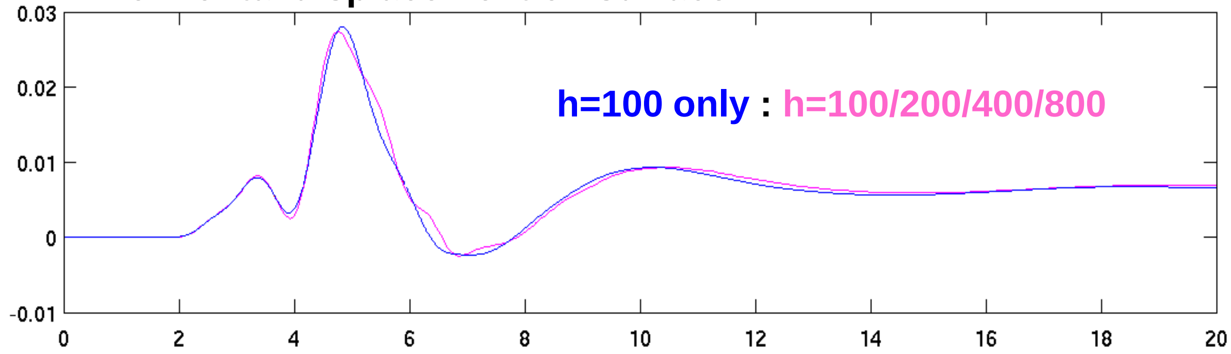
Local mesh refinement can speed up the calculations by over a factor 100

- **Ideal for materials with planar layers**
  - Grid size prop. to shear velocity
  - Time step $\delta t = \min(h/V)$
- **Peak performance:**

	Grid pts	t-steps	# Proc.	Total CPU [s]
1 mesh	47.8 e6	2449	64	106,000
Refined	1.8 e6	306	2	650
Ratio	27	8	32	162



Horizontal displacement on surface



Visco-Elastic modeling adds realistic attenuation

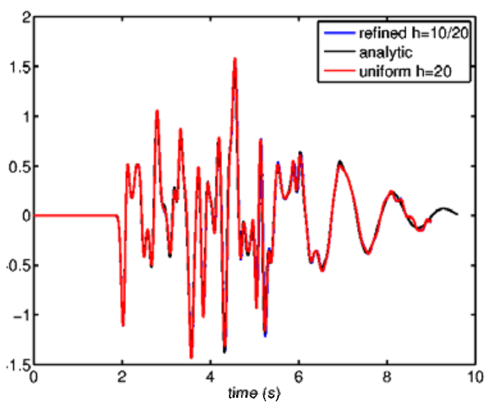
- **Constant quality factors Q_P and Q_S modeled by rational expression on Fourier side with "L" terms**
- **λ_q, μ_q and T_q through empirical relations with Q_P and Q_S**
- **From 48 to 24 memory variables $S^{(q)}$ by solving for div($S^{(q)}$)**
- **Comparison with analytic solution for a layered material (L=8):**

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \left(\mathcal{T} - \sum_{q=1}^{L} \mathcal{S}^{(q)} \right) + \mathbf{f},$$

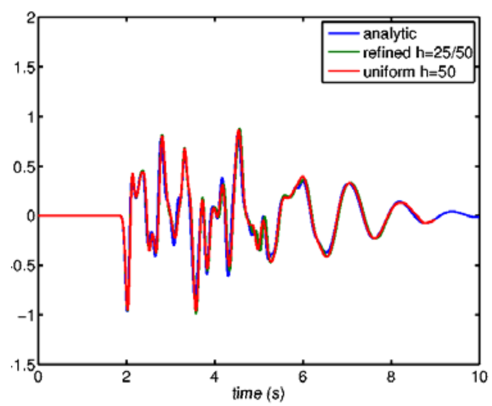
$$T_q \frac{d\mathcal{S}^{(q)}}{dt} = \mathcal{U}^{(q)} - \mathcal{S}^{(q)}, \quad q = 1, 2, \ldots, L,$$

$$\mathcal{U}^{(q)} = \lambda_q (\nabla \cdot \mathbf{u}) \mathbf{I} + \mu_q \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right)$$

$$\delta^{(q)} = \nabla \cdot \mathcal{S}^{(q)}$$



LOH.1:Elastic

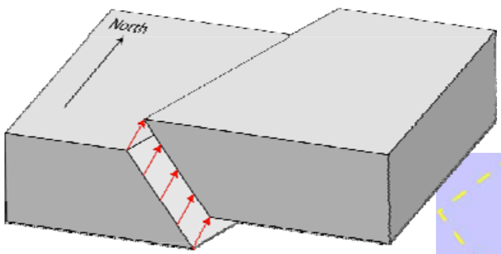


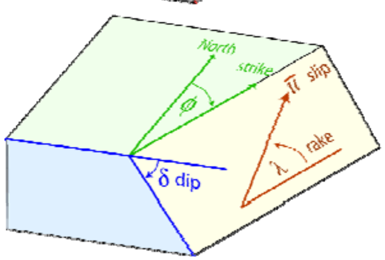
LOH.3:Visco-elastic

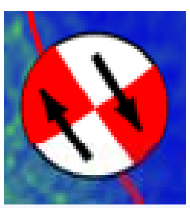
Recent moderate earthquakes can be used to validate the material model and the simulation code

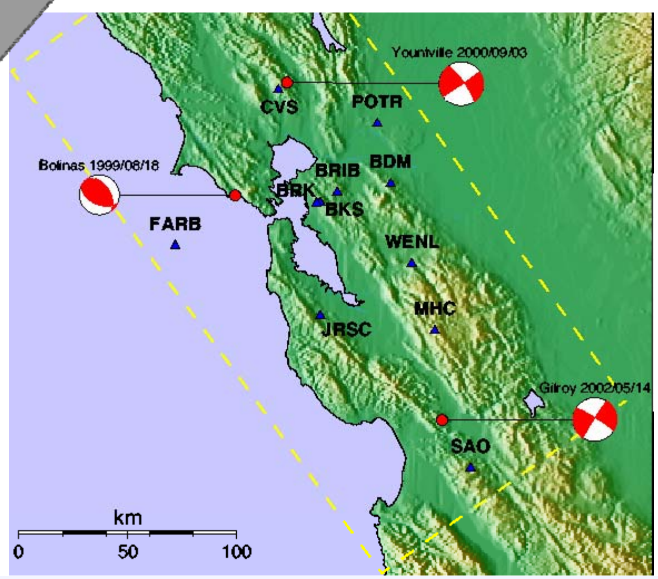
- **Smaller earthquakes have a simple focal mechanism**
- **Point moment tensor source**
- **Beach ball illustrates the strike, dip and rake angles**

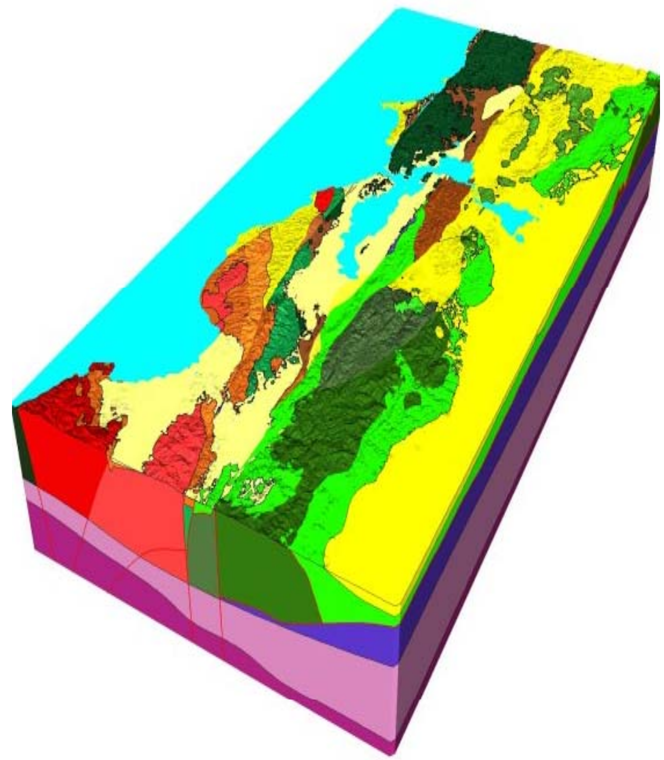
- **Material model (USGS)**
- **Compressional and shear wave speeds**
- **Density and attenuation (Q_P, Q_S)**



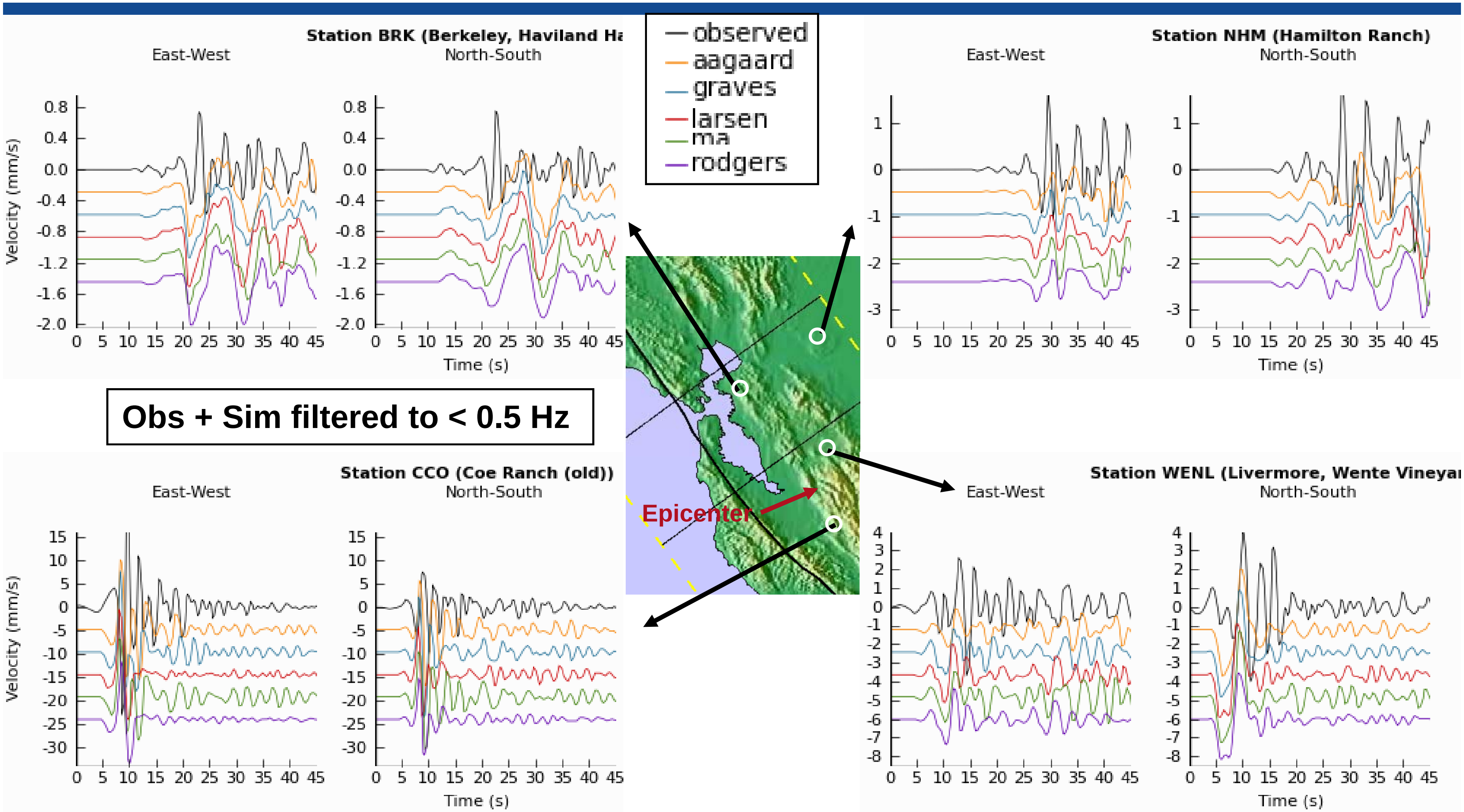




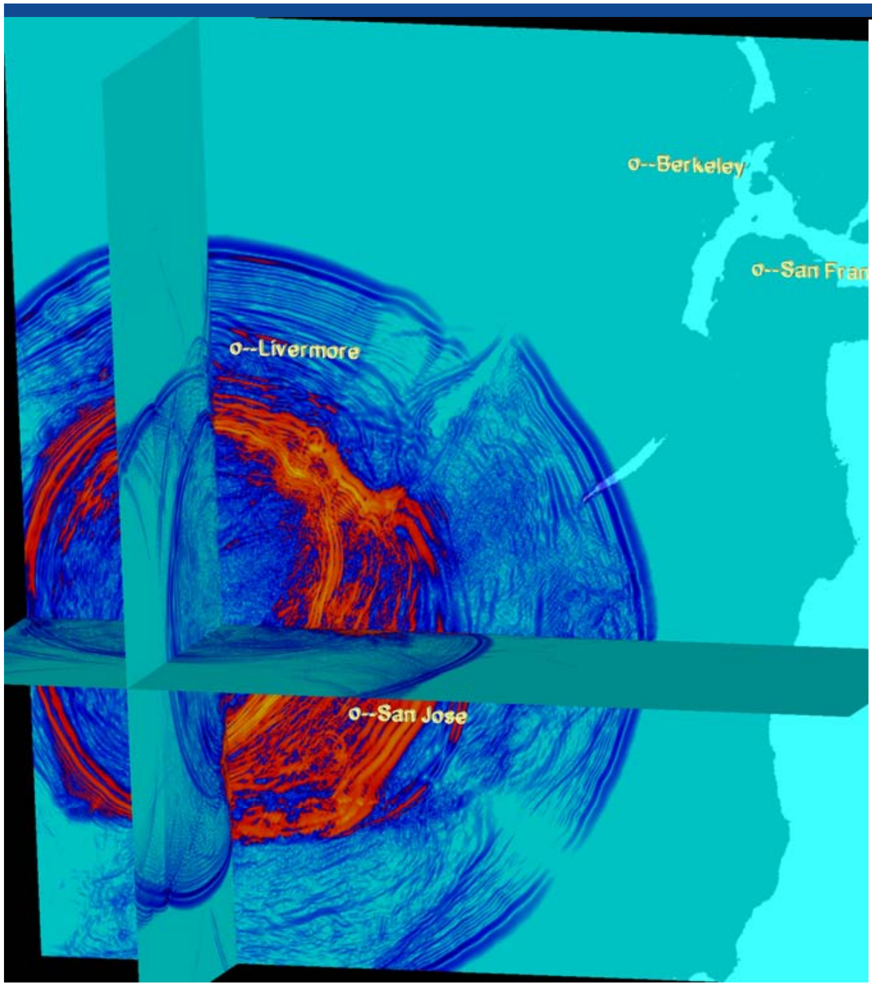




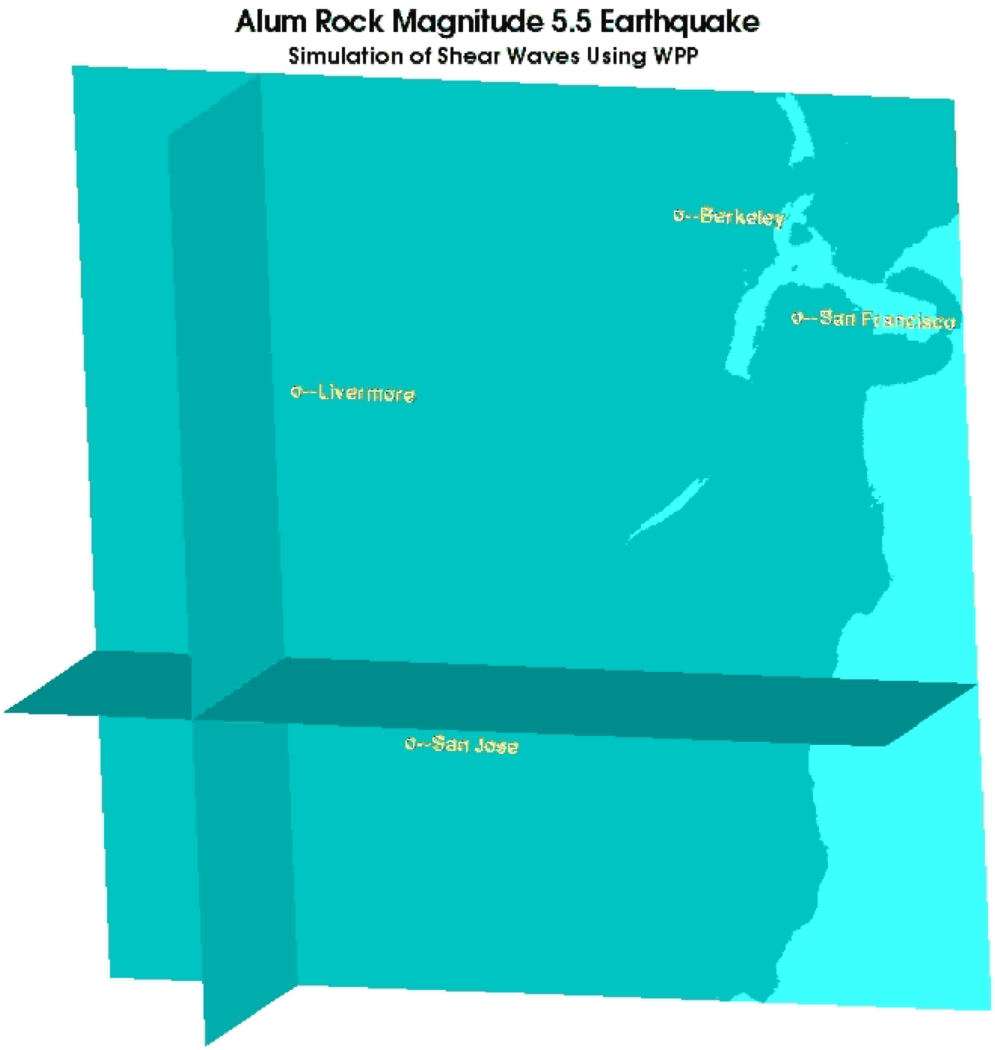
The October 2007, Alum Rock M_w = 5.4 earthquake was used in our validation effort



There is significant 3-D structure in the solution



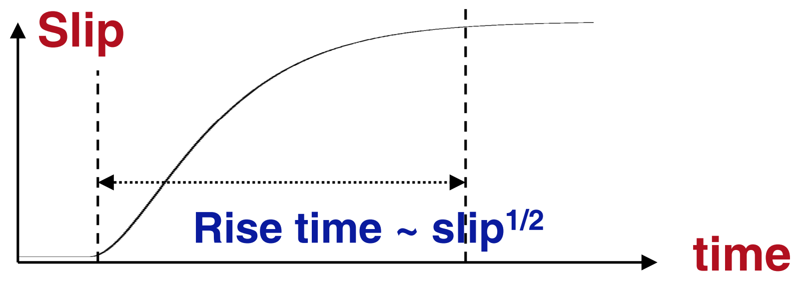
|Curl(velocity)|, bottom view

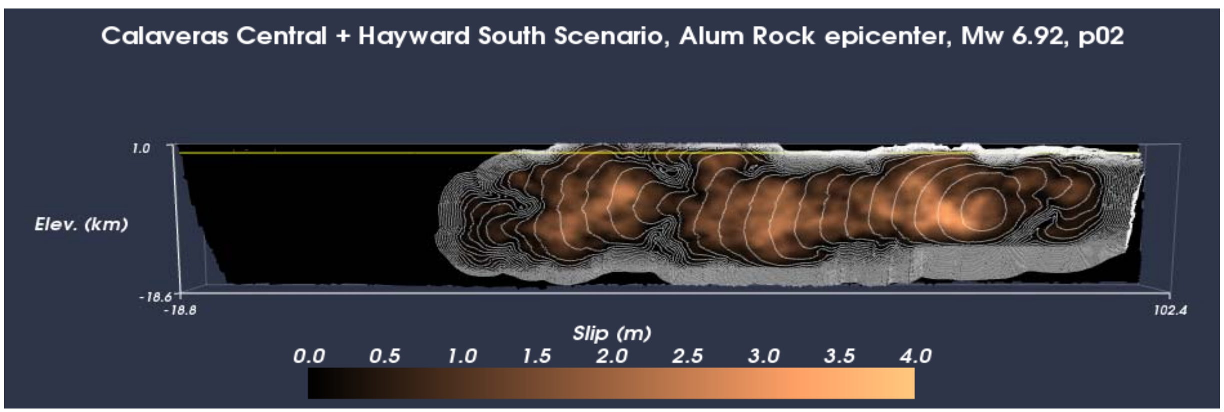


Larger earthquakes need a more sophisticated rupture model

Rupture model

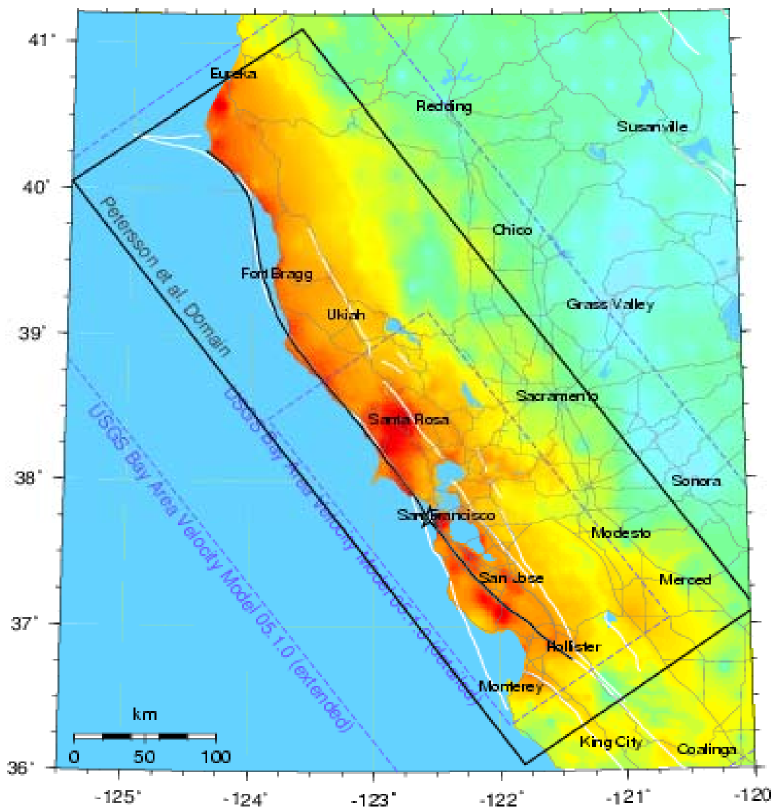
- **Distribution of slip along fault surface**
- **Rupture speed -> Slip initiation times**
- **Slip(time) function; Rise time**
- **Discretized by many point sources along fault surface**



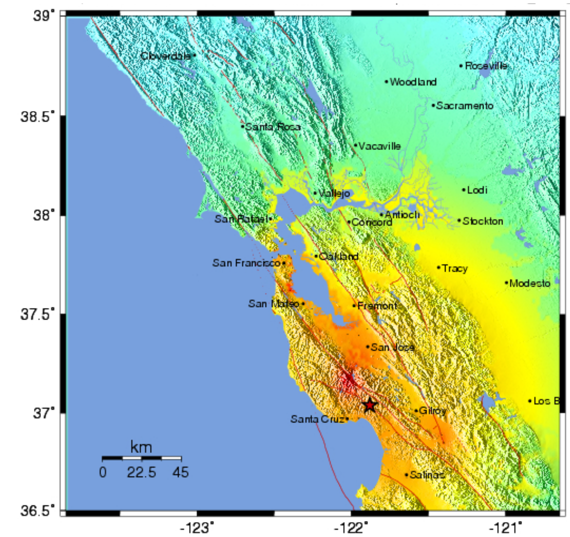


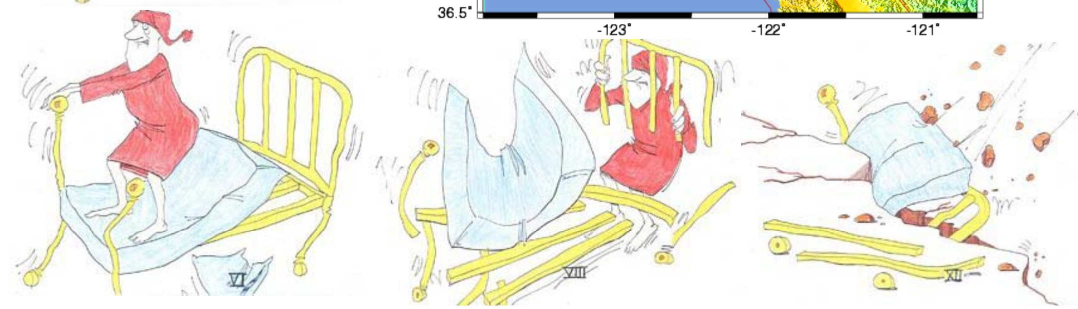
Historical data: The 1906 earthquake was much stronger than the 1989 Loma Prieta earthquake ($M_w = 6.9$)

1906 San Francisco



1989 Loma Prieta





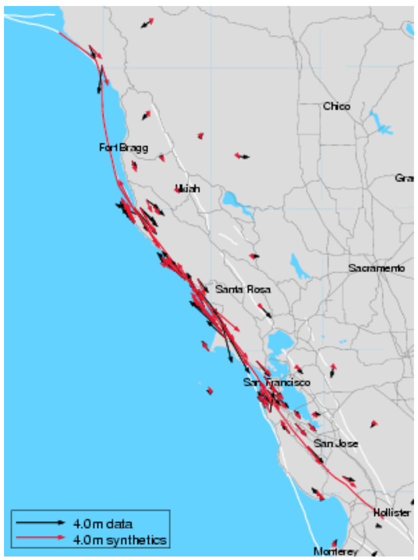
Can we simulate a repeat of the 1906 event?

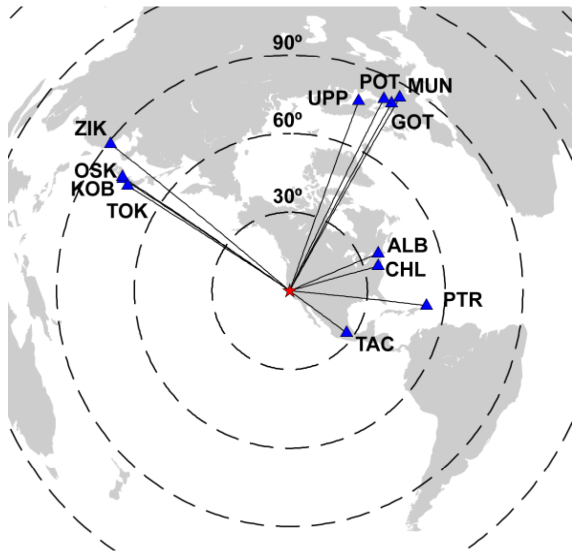
PERCEIVED SHAKING	Not felt	Weak	Light	Moderate	Strong	Very strong	Severe	Violent	Extreme
POTENTIAL DAMAGE	none	none	none	Very light	Light	Moderate	Moderate/Heavy	Heavy	Very Heavy
PEAK ACC.(%g)	<.17	.17-1.4	1.4-3.9	3.9-9.2	9.2-18	18-34	34-65	65-124	>124
PEAK VEL.(cm/s)	<0.1	0.1-1.1	1.1-3.4	3.4-8.1	8.1-16	16-31	31-60	60-116	>116
INSTRUMENTAL INTENSITY	I	II-III	IV	V	VI	VII	VIII	IX	X+

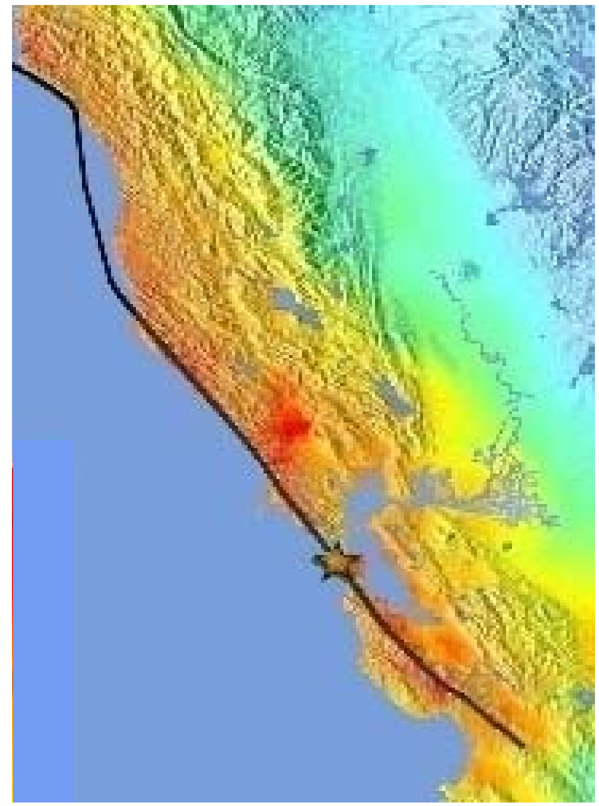
The rupture model for the 1906 event was reverse engineered from historical data

Lawson report (1908):

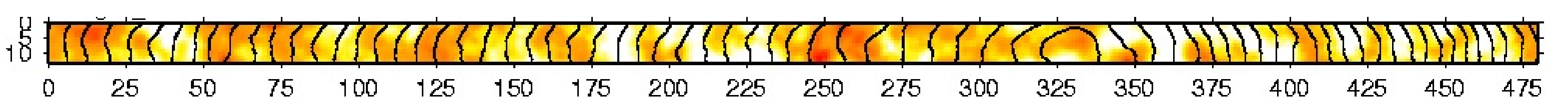
- **Geodetic Measurements (slip)**
- **Tele-seismic recordings (duration)**
- **Mercalli intensities (peak velocities)**





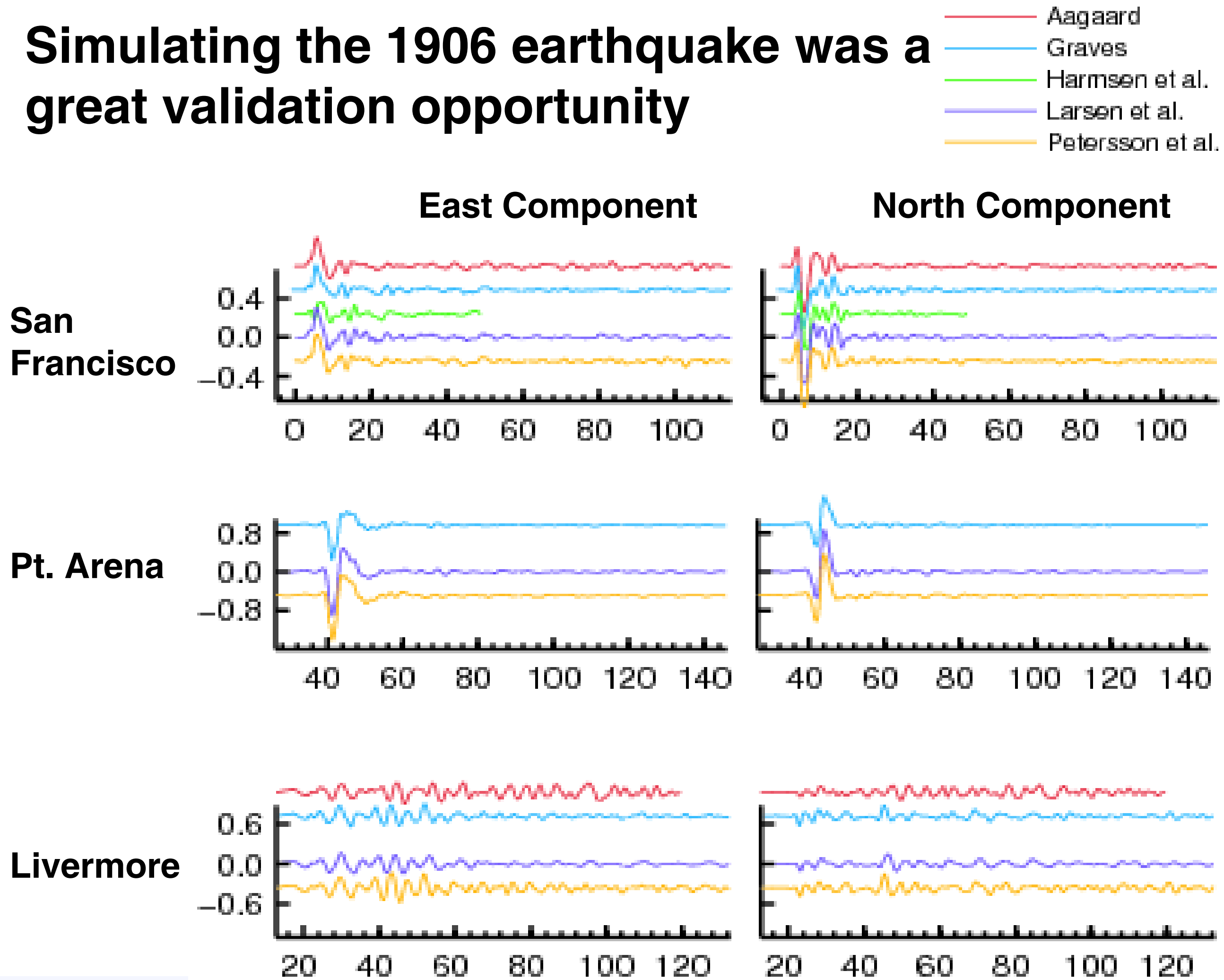


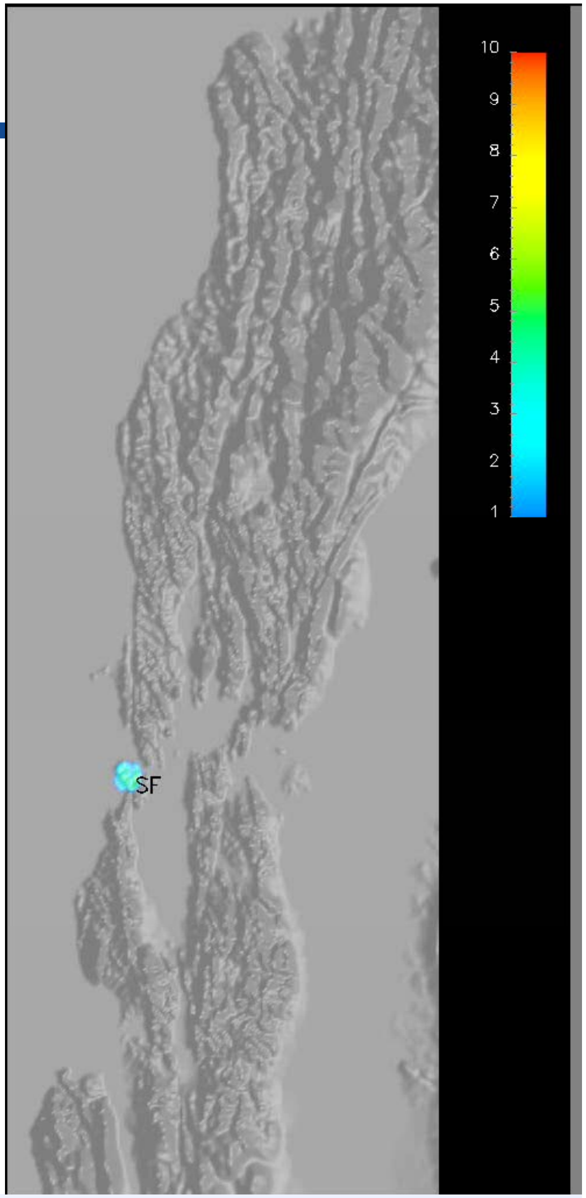
Song model 2c:



Our results compare well with historical data and other codes

Simulating the 1906 earthquake was a great validation opportunity





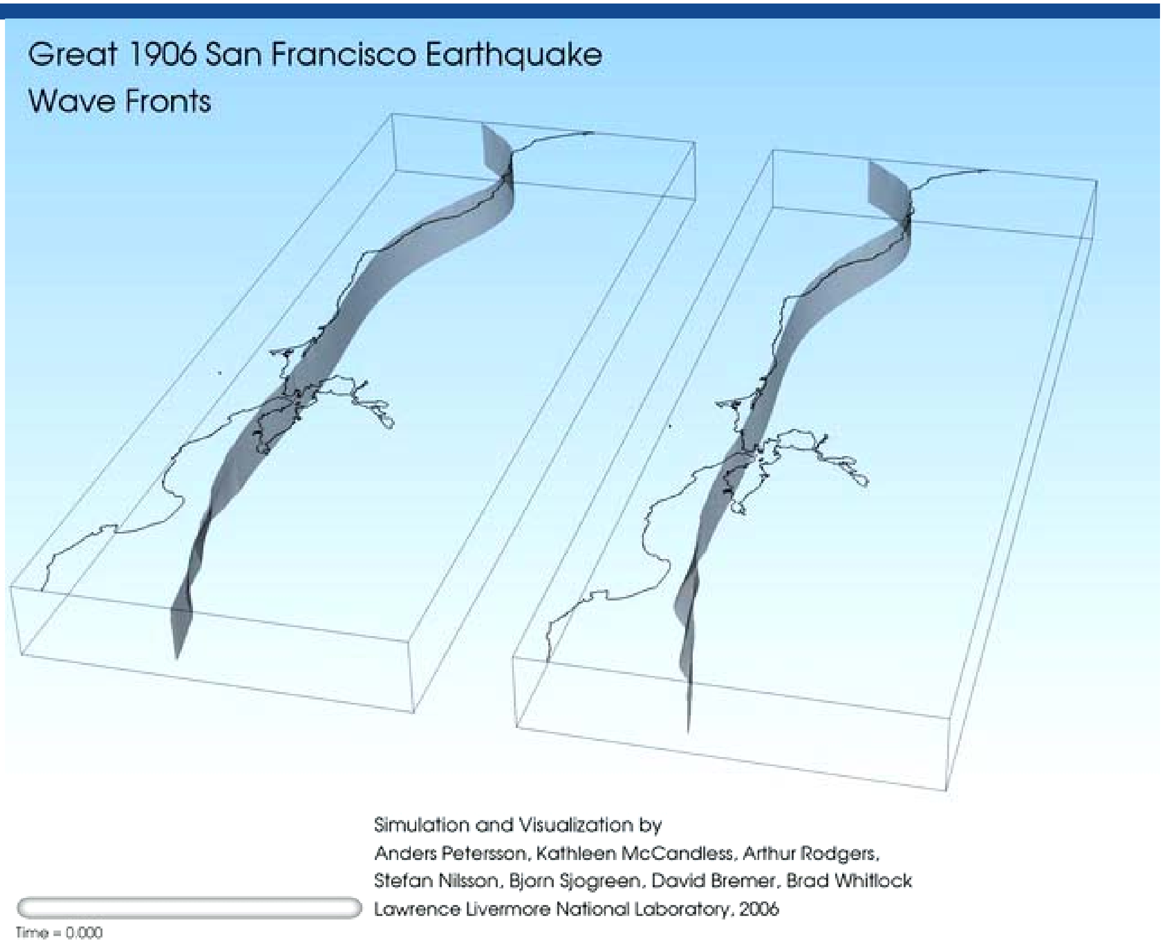
The volumetric data reveals interesting structure

Volumetric data compressed w/brick-of-wavelet technique

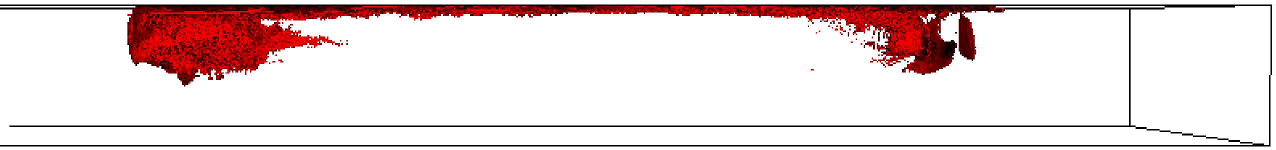
1.2 Tb of data rendered with VISIT on 64 proc.

Left: P-waves dilatation/compression

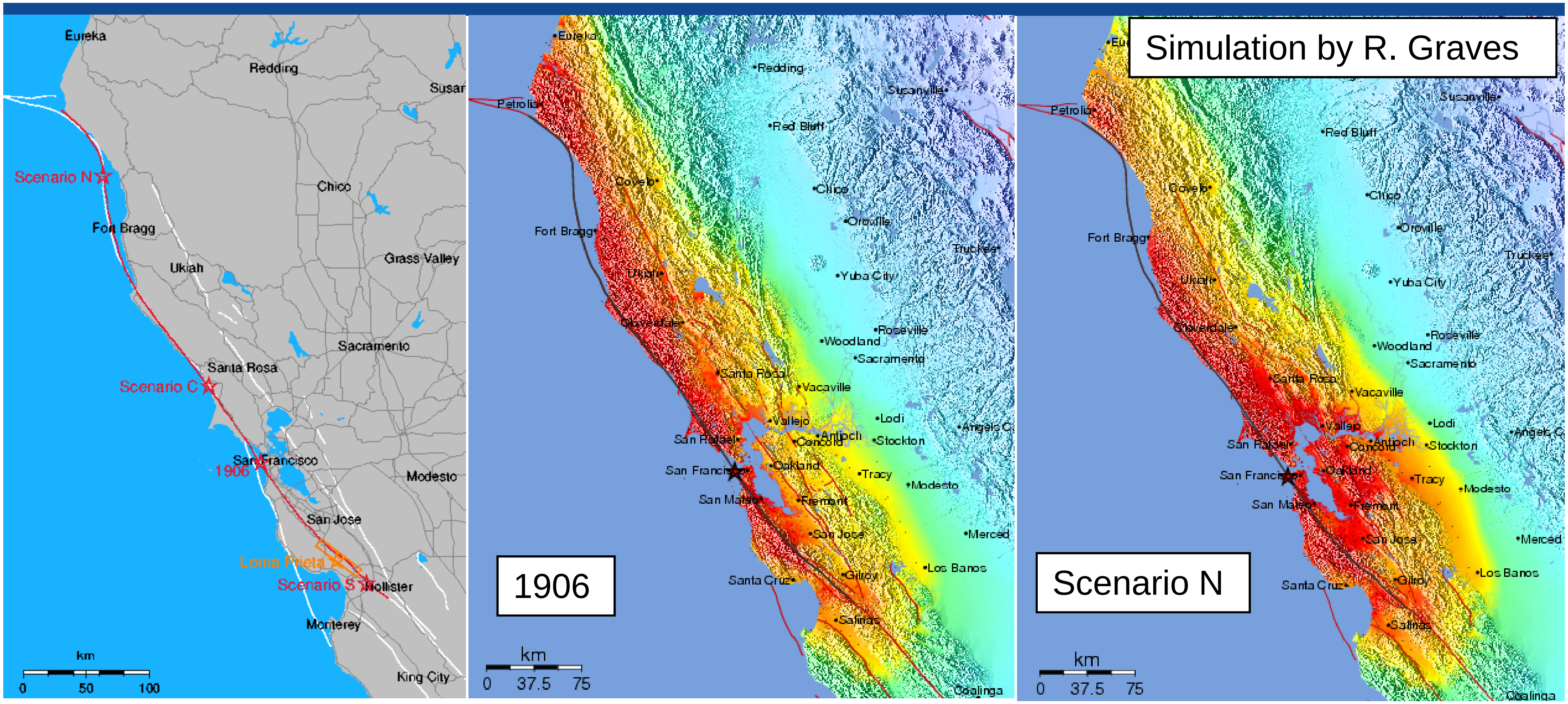
Right: S-waves |vorticity|



Side view: intense shaking at rupture front and surface



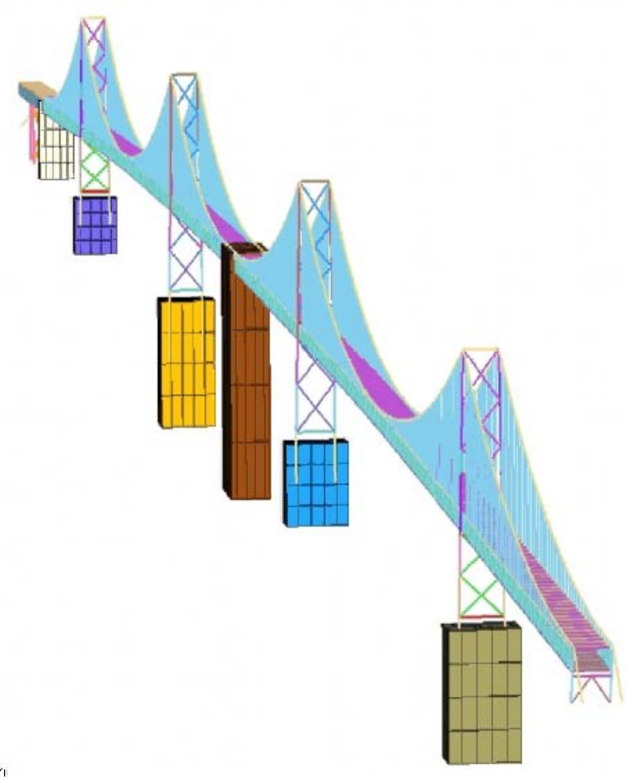
Other M_w=7.8 earthquakes on the San Andreas fault could cause more damage to the SF bay area



- **More info in publications:**
  - B. T. Aagaard, et. al, *Bull. Seism. Soc. Amer.* **98**, pp. 1012-1046 (2008)
  - A. Rodgers, N. A. Petersson, S. Nilsson, B. Sjogreen, K. McCandless, *Bull. Seism. Soc. Amer.* **98**, pp. 969-988 (2008)

Future work: couple earthquake ground motions to dynamic simulations of structures

- Higher frequencies important (1-2 Hz)
- Doubling the frequency:
  - Memory x8
  - CPU-time x16
- More accurate modeling
  - Topography
  - Attenuation / advanced soil models
- Mesh refinement will help
- Larger/faster clusters necessary
- Largest WPP simulation to date:
  - Grid: 9232 x 9232 x 309 = 26.3 Billion points, Time: 995 s = 41,350 steps
  - 32,768 proc on BG/L @ LLNL: 15 h, 32 min wall clock



Many people contributed to this work

Bjorn Sjogreen Artie Rodgers Kathleen McCandless

Anders Petersson Daniel Appelo Stefan Nilsson Heinz Kreiss

How much is my house going to shake?

- **Do your own wave simulations with WPP**
- **Distributed as open source software: https://computation.llnl.gov/casc/serpentine**