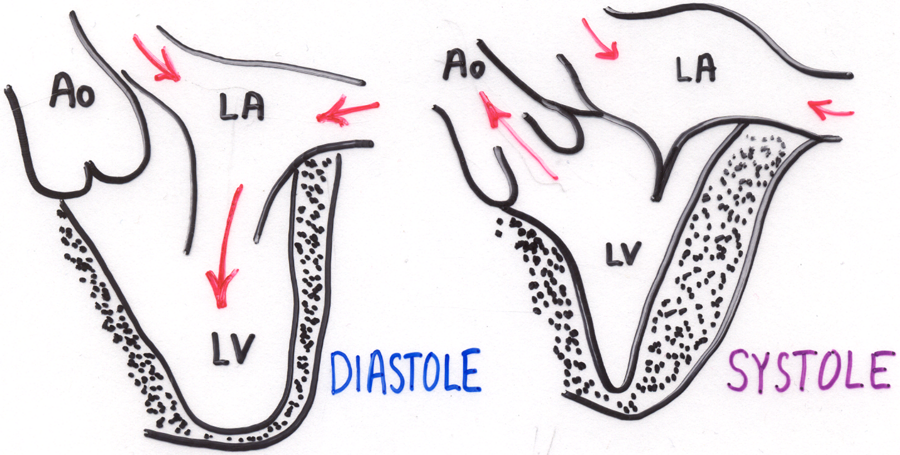
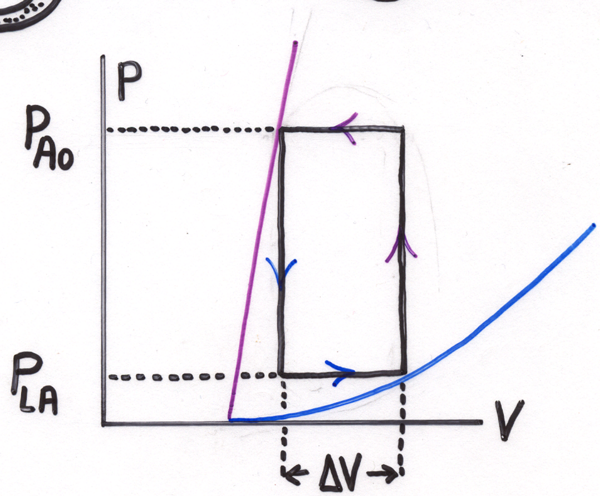
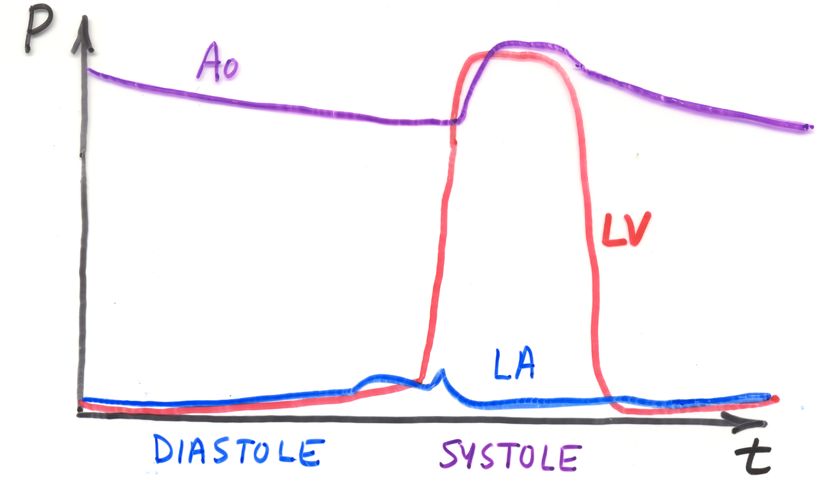
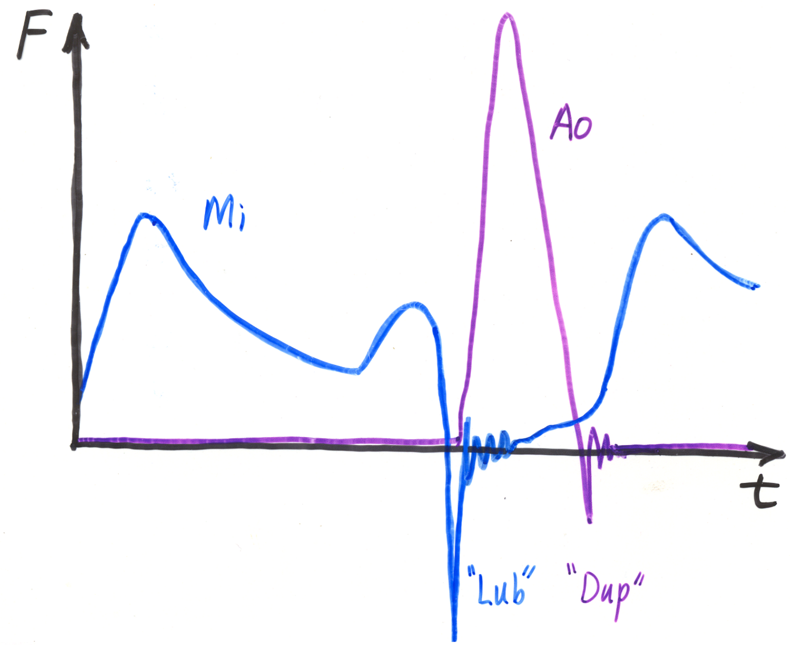

Ao
LA
LV
DIASTOLE
Ao
LA
LV
SYSTOLE


P
P_{Ao}
P_{LA}
V
ΔV


P
Ao
LV
LA
DIASTOLE
SYSTOLE
t


F
Ao
Mi
t
"Lub"
"Dup"

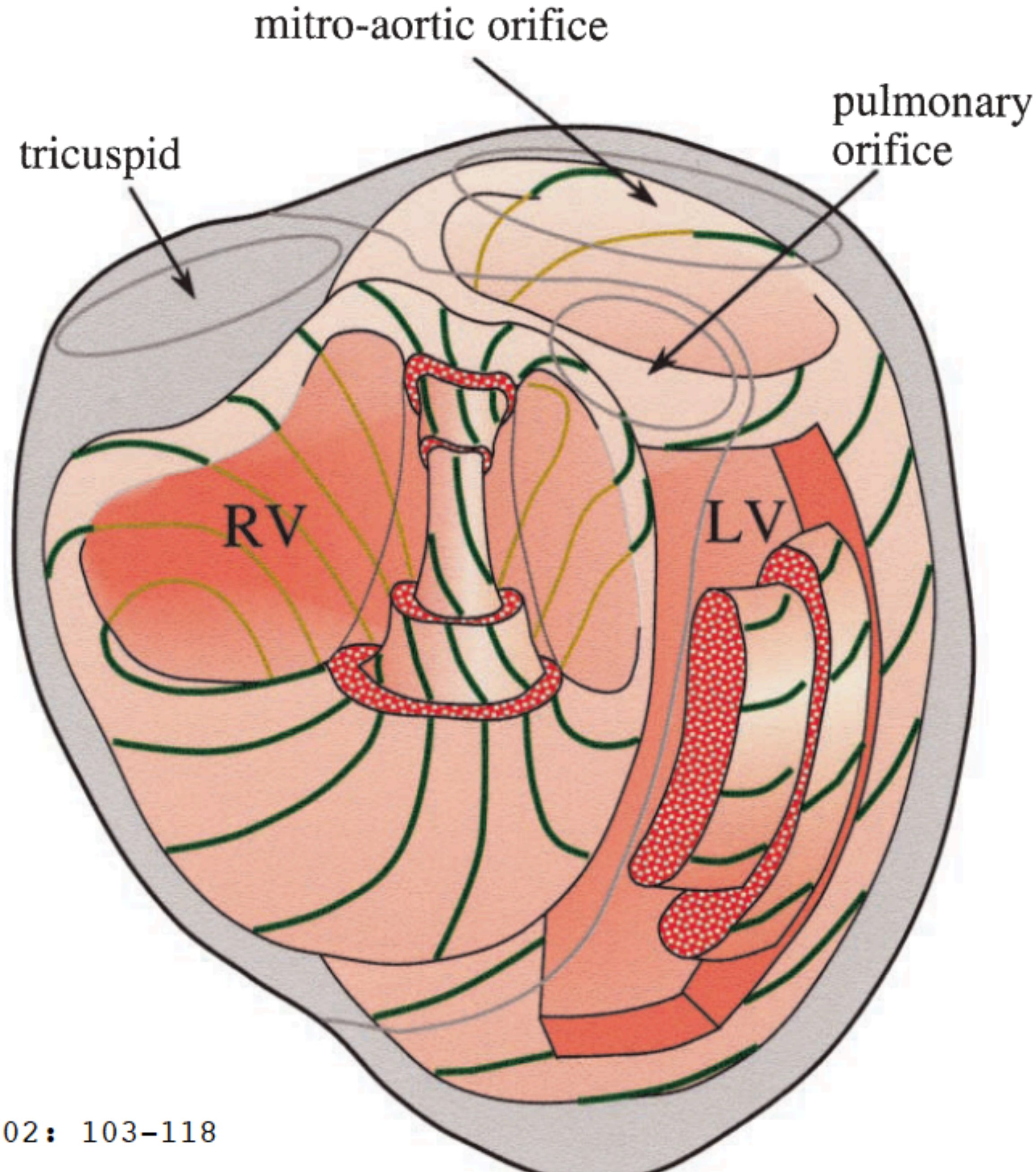
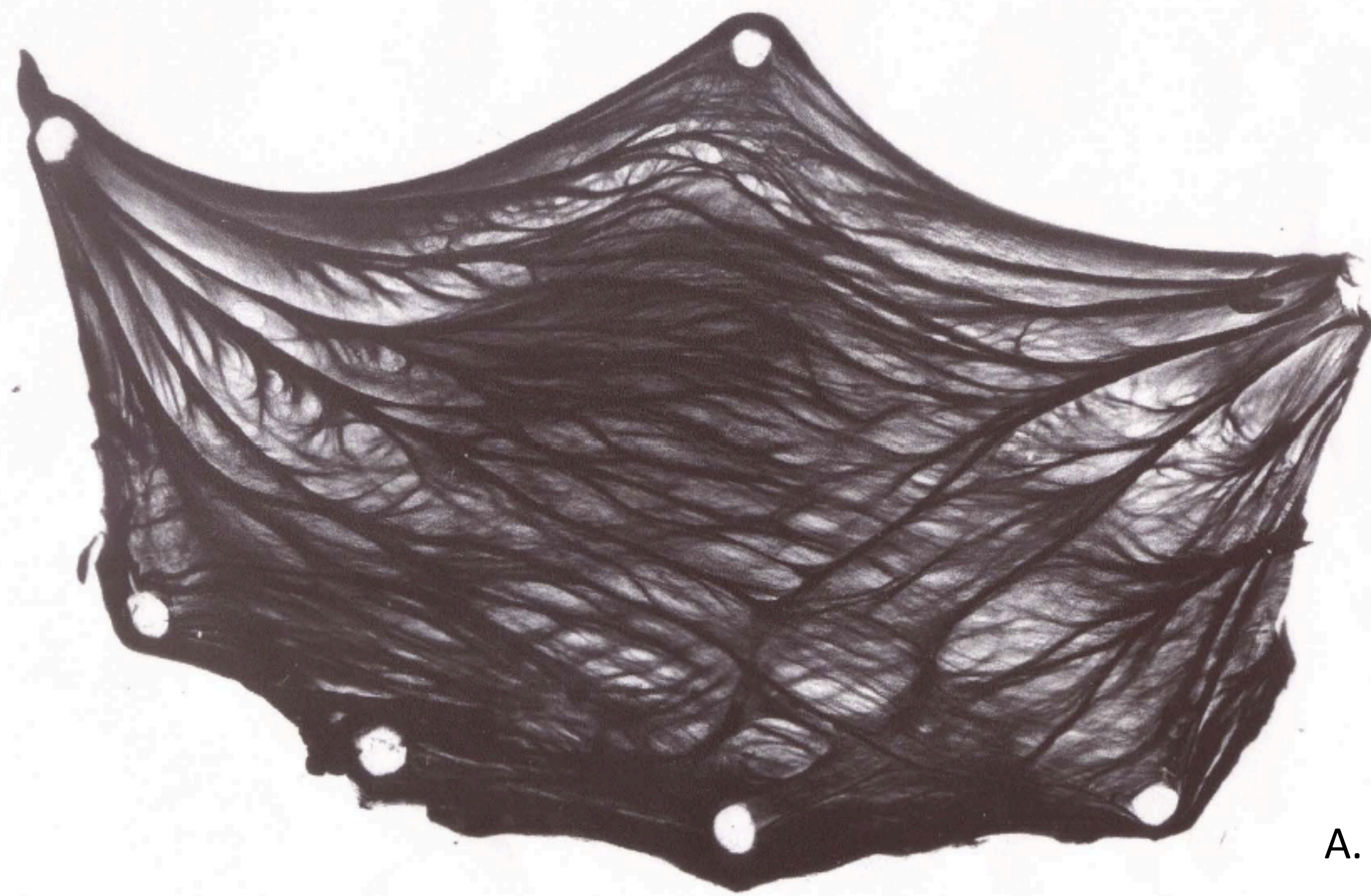


Fig. 9 Schematic description of the fibre architecture of the whole ventricular mass. The lines in green symbolize the geodesic trajectories of the fibres on the nested "pretzels". Wedges have been cut out of the right and left ventricular walls, in order to show the innermost hulls

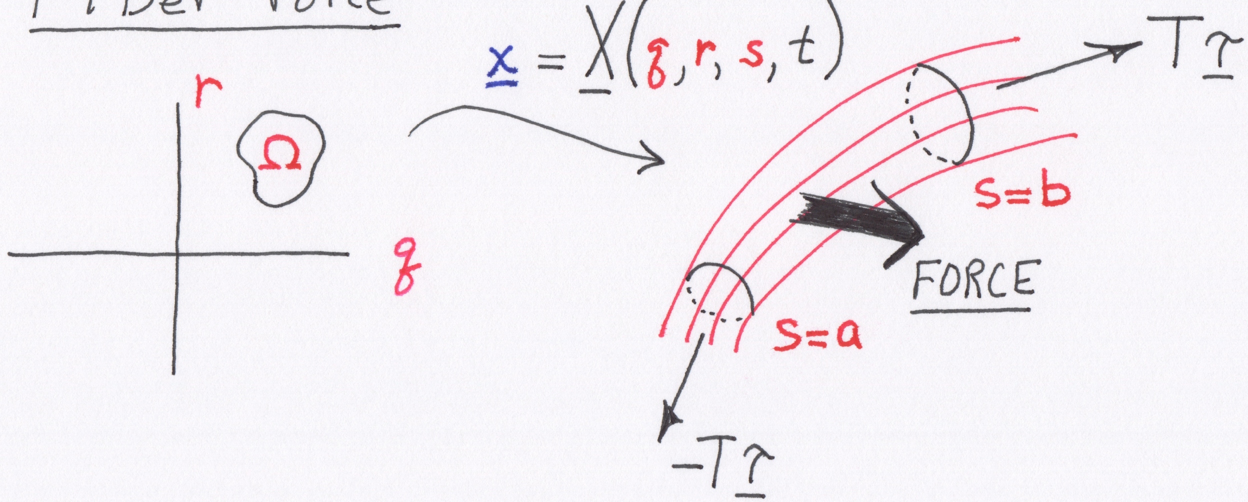
Jouk P-S, et al.
Anat.Embryol.(2000) 202: 103-118

Aortic Valve Leaflet Stained for Collagen



A. Sauren

Fiber Force



$$\underline{\tau}(q, r, s, t) = \frac{\partial \underline{X} / \partial s}{|\partial \underline{X} / \partial s|} \qquad T(q, r, s, t) = \sigma\left(\left|\frac{\partial \underline{X}}{\partial s}\right| - 1; q, r, s, t\right)$$

$$\underline{\text{FORCE}} = \int_{\Omega} T \underline{\tau} \, dq \, dr \Big|_a^b = \int_a^b \int_{\Omega} \frac{\partial}{\partial s}(T\underline{\tau}) \, dq \, dr \, ds$$

Equations of Motion

$$\rho\left(\frac{\partial \underline{u}}{\partial t} + \underline{u}\cdot\nabla\underline{u}\right) + \nabla p = \mu\Delta\underline{u} + \underline{f}(\underline{x},t)$$

$$\nabla\cdot\underline{u} = 0$$

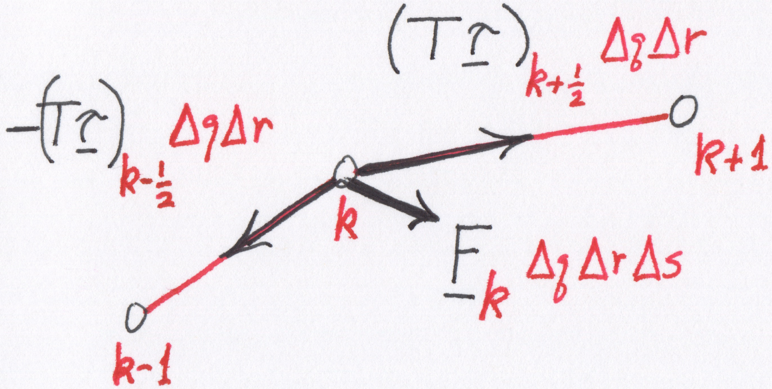
$$\underline{f}(\underline{x},t) = \int \frac{\partial}{\partial s}(T\underline{\tau})\,\delta\left(\underline{x} - \underline{X}(q,r,s,t)\right) dq\,dr\,ds$$

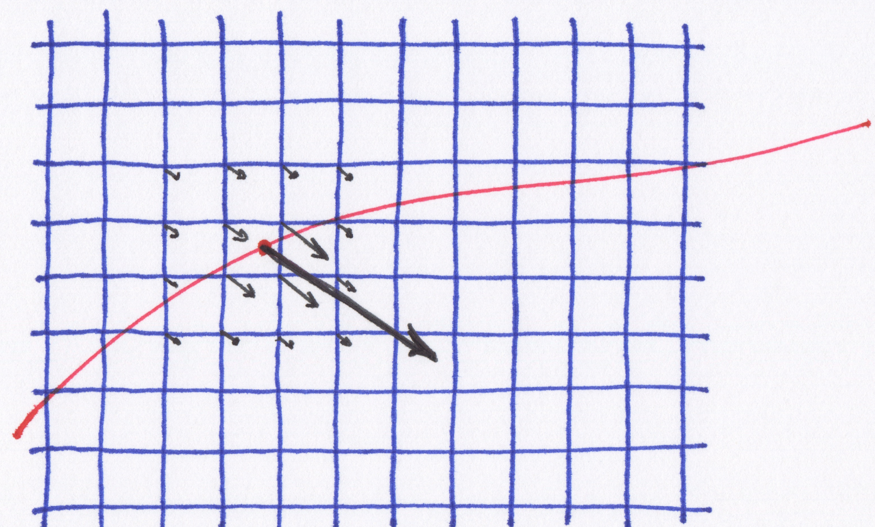
$$\frac{\partial \underline{X}}{\partial t}(q,r,s,t) = \underline{u}\left(\underline{X}(q,r,s,t),\, t\right)$$

$$= \int \underline{u}(\underline{x},t)\,\delta\left(\underline{x} - \underline{X}(q,r,s,t)\right) d\underline{x}$$

$$T = \sigma\left(\left|\frac{\partial \underline{X}}{\partial s}\right| - 1;\, q,r,s,t\right) \qquad \underline{\tau} = \frac{\partial \underline{X}/\partial s}{\left|\partial \underline{X}/\partial s\right|}$$

Spatial Discretization





Spatial Discretization : Fluid

$$\rho\left(\frac{\partial \underline{u}}{\partial t} + S_h(\underline{u})\,\underline{u}\right) + \nabla_h p = \mu\,\Delta_h u + \underline{f}$$

$$\nabla_h \cdot \underline{u} = 0$$

where

$$S_h(u)\,\phi = \frac{1}{2}\,\underline{u}\cdot\nabla_h\phi + \frac{1}{2}\,\nabla_h\cdot(\underline{u}\,\phi)$$

and ∇_h , Δ_h are central difference operators

(h = meshwidth)

Spatial Discretization: Fibers

$$\underline{\tau}_{k+\frac{1}{2}} = \frac{\underline{X}_{k+1} - \underline{X}_k}{|\underline{X}_{k+1} - \underline{X}_k|}$$

$$T_{k+\frac{1}{2}} = \sigma\left(\frac{|\underline{X}_{k+1} - \underline{X}_k|}{\Delta s} - 1 \,;\, s_{k+1/2}, t\right)$$

$$\underline{F}_k = \frac{(T\underline{\tau})_{k+1/2} - (T\underline{\tau})_{k-1/2}}{\Delta s}$$

(indices that label a fiber are not shown)

Spatial Discretization : Interaction

$$\underline{f}(\underline{x},t) = \sum_{q,r,s \in G_{\Delta q,\Delta r,\Delta s}} \underline{F}(q,r,s,t)\,\delta_h\big(\underline{x}-\underline{X}(q,r,s,t)\big)\,\Delta q\,\Delta r\,\Delta s$$

$$\frac{\partial \underline{X}}{\partial t}(q,r,s,t) = \sum_{\underline{x} \in g_h} \underline{u}(\underline{x},t)\,\delta_h\big(\underline{x}-\underline{X}(q,r,s,t)\big)\,h^3$$

Note that

$$\sum_{q,r,s \in G_{\Delta q,\Delta r,\Delta s}} \left(\underline{F}\cdot\frac{\partial \underline{X}}{\partial t}\right)(q,r,s,t)\,\Delta q\,\Delta r\,\Delta s = \sum_{\underline{x} \in g_h} (\underline{f}\cdot\underline{u})(\underline{x},t)\,h^3$$

Construction of δ_h

$$\delta_h(\underline{x}) = \frac{1}{h^3}\,\varphi\left(\frac{x_1}{h}\right)\varphi\left(\frac{x_2}{h}\right)\varphi\left(\frac{x_3}{h}\right)$$

where

$\varphi(r)$ is continuous

$\varphi(r) = 0$ for $|r| \geq 2$

$$\sum_{i \text{ even}} \varphi(r-i) = \sum_{i \text{ odd}} \varphi(r-i) = \frac{1}{2}, \quad \text{all } r$$

$$\sum_i \varphi(r-i)(r-i) = 0, \quad \text{all } r$$

$$\sum_i \varphi^2(r-i) = C, \quad \text{all } r$$

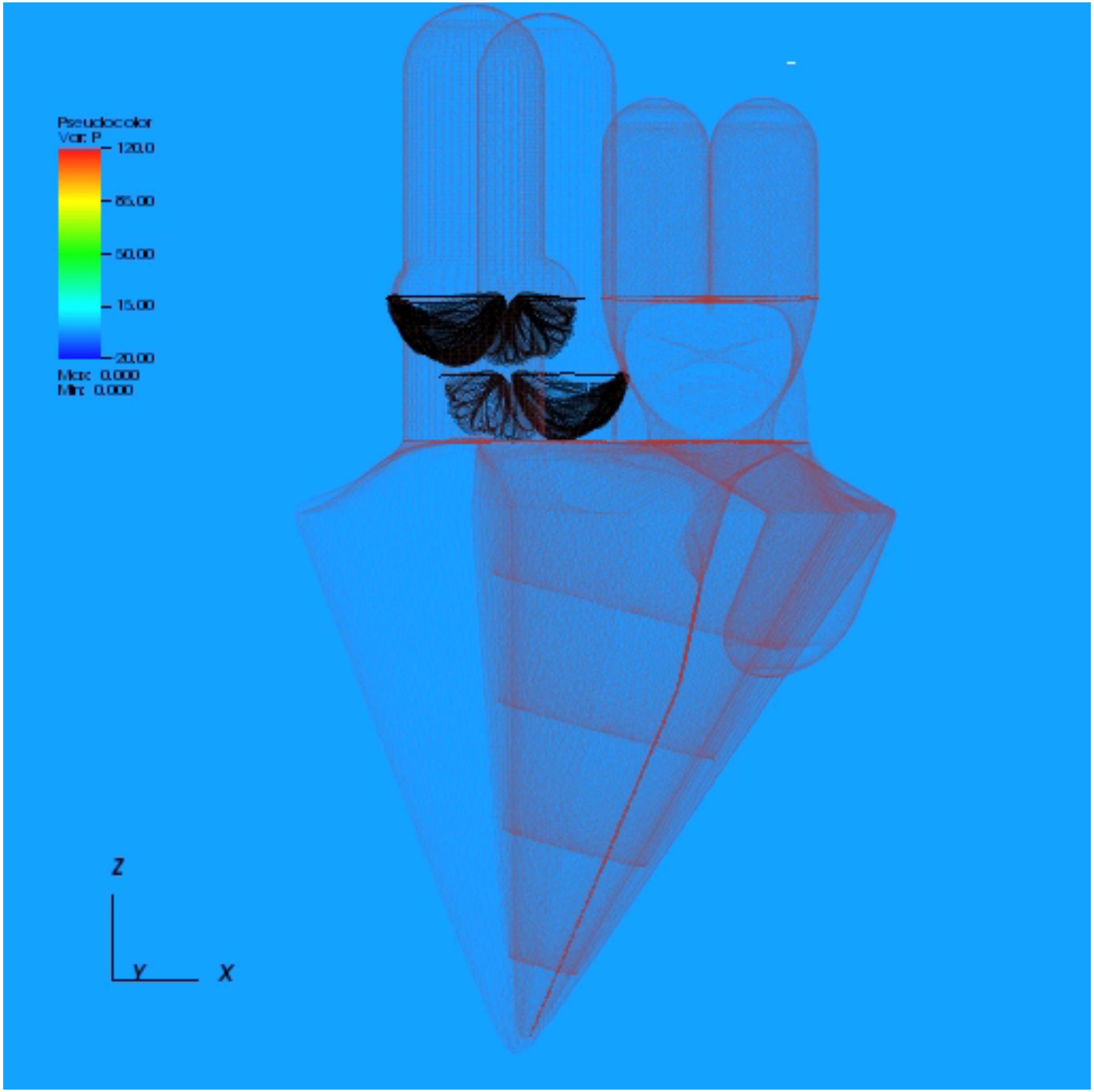
and it follows that

$$\sum_i \varphi(r_1 - i)\,\varphi(r_2 - i) \le \sum \varphi^2(r - i) = C = \frac{3}{8}$$

for all r_1, r_2, r

$$\varphi(r) = \begin{cases} \dfrac{3 - 2|r| + \sqrt{1 + 4|r| - 4r^2}}{8}, & |r| \le 1 \\[2ex] \dfrac{5 - 2|r| - \sqrt{-7 + 12|r| - 4r^2}}{8}, & 1 \le |r| \le 2 \\[2ex] 0, & 2 \le |r| \end{cases}$$

Note that $\varphi(r) = \varphi(-r)$, and $\varphi'(r)$ is continuous.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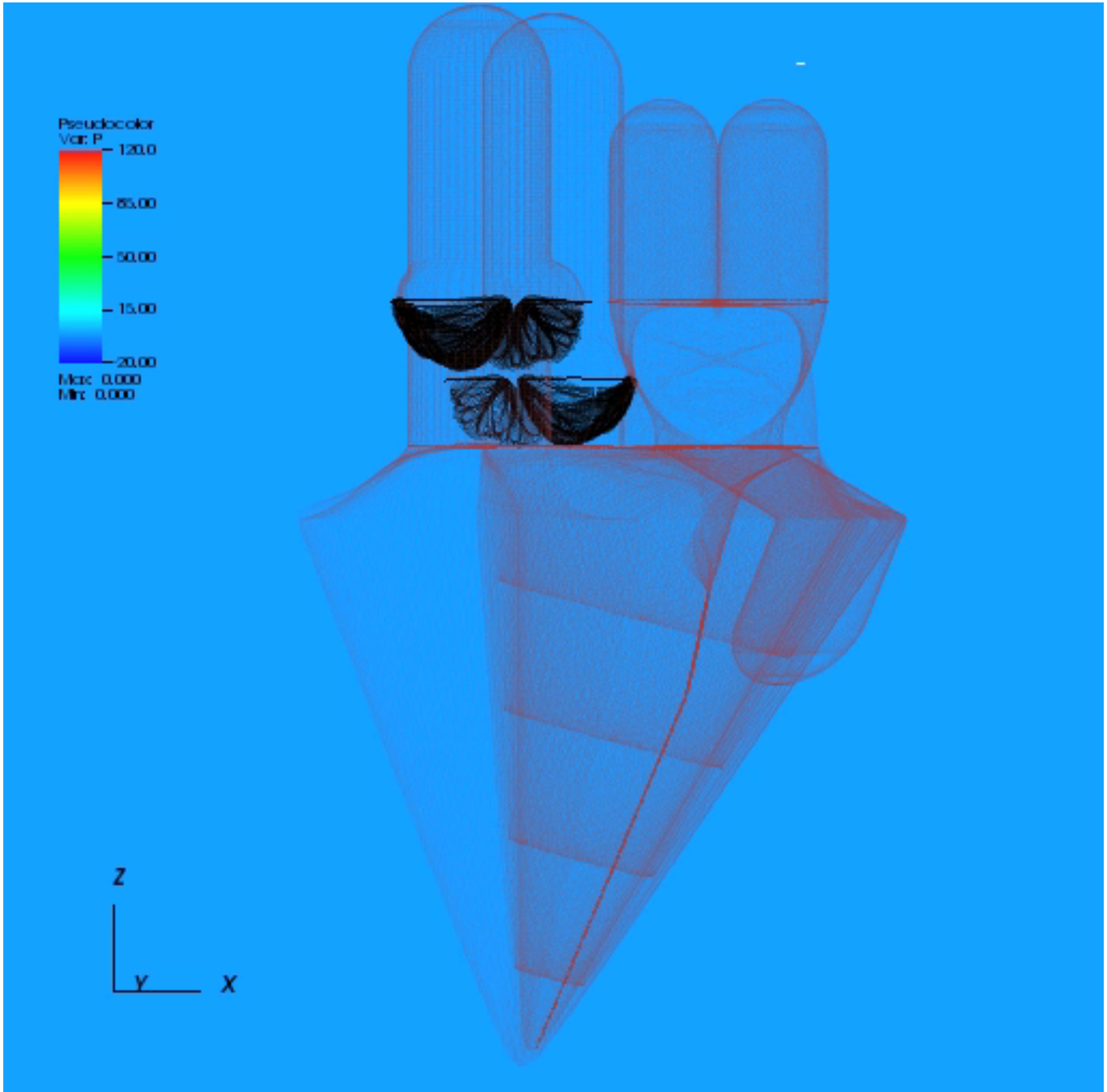
15.00
-20.00
Max: 0.000
Min: 0.000
Z
Y
X


Pseudocolor
Var: P
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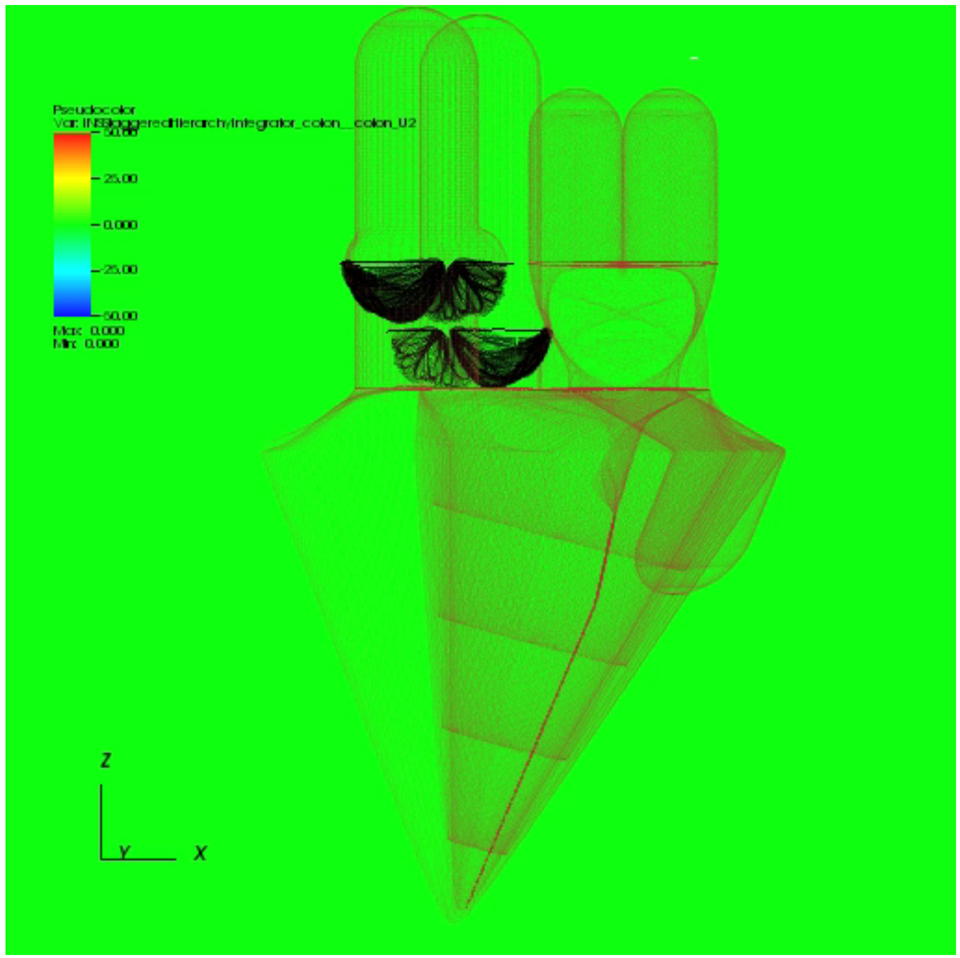
50.00
15.00
-20.00
Max: 0.000
Min: 0.000
Z
Y
X



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-50.00
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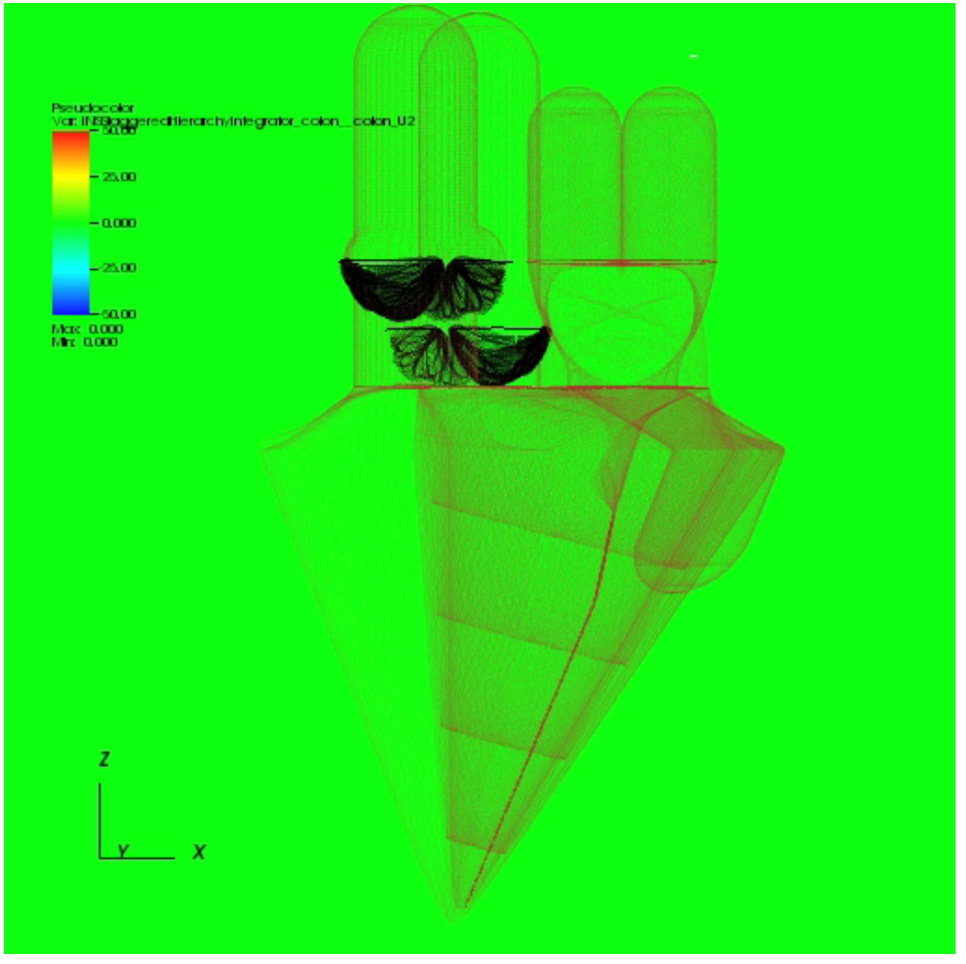
Min: 0.000
Z
Y
X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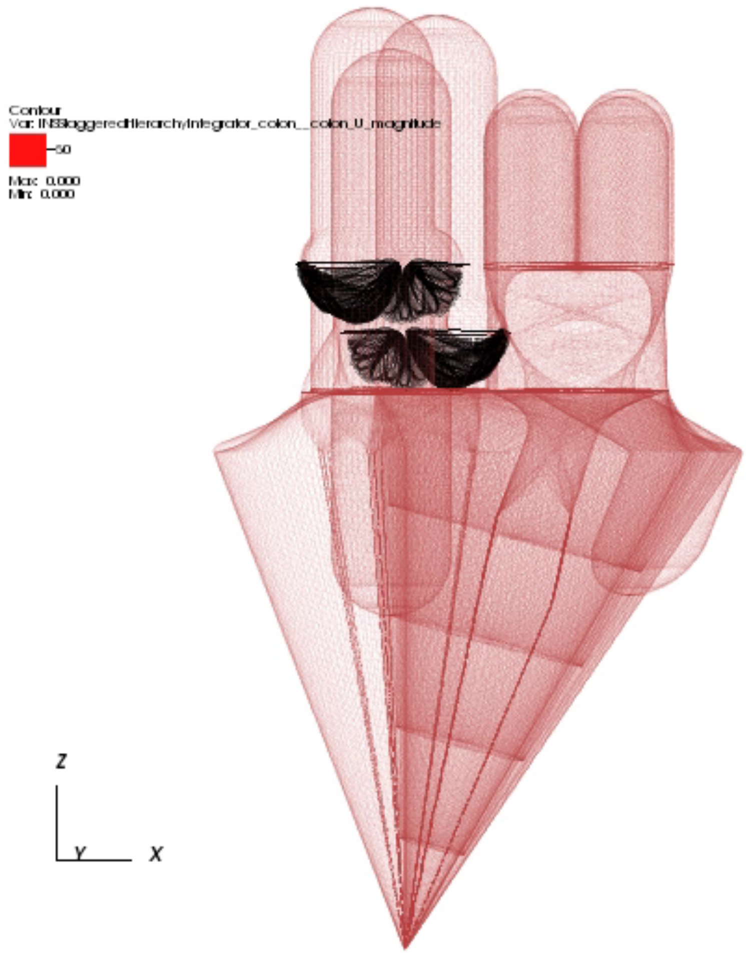
Z
Y
X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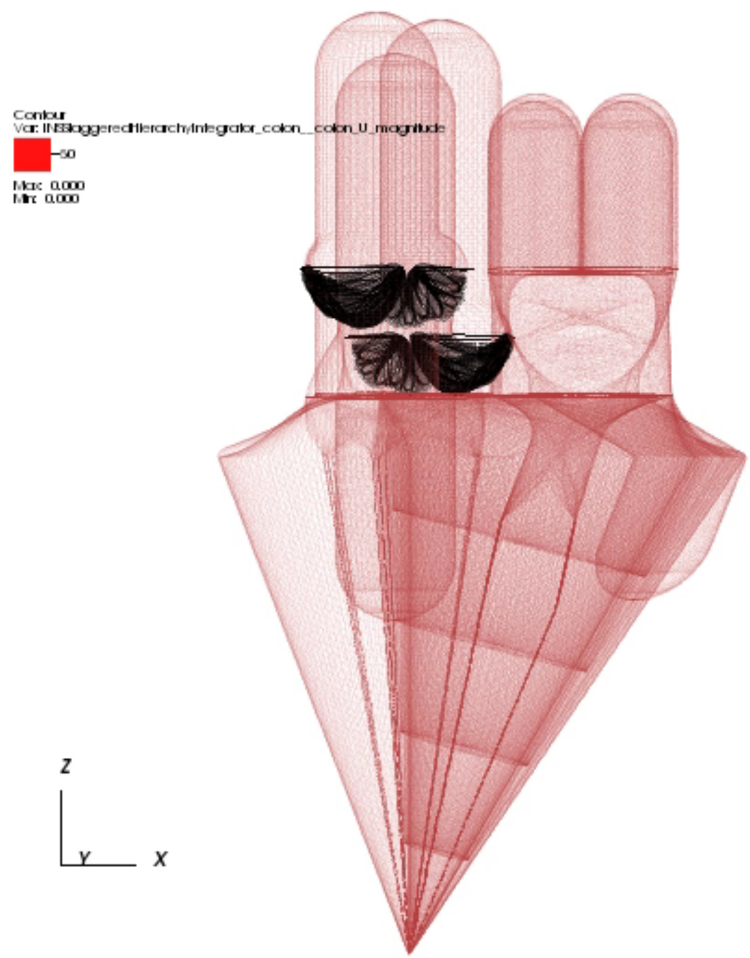
Max: 0.000
Min: 0.000
Z
Y
X



Contour
Var: INSStaggeredHierarchyIntegrator_colon__colon_U_magnitude
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Max: 0.000
Min: 0.000
Z
Y
X

The Bidomain Equations in Cartesian Coordinates

$$
\begin{aligned}
j_k^{\mathsf{i}} &= -\sigma_{kl}^{\mathsf{i}}(\mathbf{x},t)\frac{\partial v^{\mathsf{i}}}{\partial x_l} \\
j_k^{\mathsf{e}} &= -\sigma_{kl}^{\mathsf{e}}(\mathbf{x},t)\frac{\partial v^{\mathsf{e}}}{\partial x_l} \\
-\nabla\cdot\mathbf{j}^{\mathsf{i}} = \nabla\cdot\mathbf{j}^{\mathsf{e}} &= a(\mathbf{x},t)\left(C\frac{D(v^{\mathsf{i}}-v^{\mathsf{e}})}{Dt} + I_{\mathsf{ion}}(v^{\mathsf{i}}-v^{\mathsf{e}},w)\right) \\
\frac{Dw}{Dt} &= f_{\mathsf{w}}(v^{\mathsf{i}}-v^{\mathsf{e}},w,\lambda)
\end{aligned}
$$

The Bidomain Equations in Lagrangian/Eulerian Coordinates

$$J^{\mathsf{i}}_{\alpha}(q,t) = -S^{\mathsf{i}}_{\alpha\beta}(q,t)\frac{\partial V^{\mathsf{i}}}{\partial q_{\beta}}(q,t)$$

$$j^{\mathsf{e}}_{k}(\mathbf{x},t) = -\sigma^{\mathsf{e}}_{kl}(\mathbf{x},t)\frac{\partial v^{\mathsf{e}}}{\partial x_{l}}(\mathbf{x},t)$$

$$(\nabla\cdot\mathbf{j}^{\mathsf{e}})(\mathbf{X}(q,t),t)\det\left(\frac{\partial\mathbf{X}}{\partial q}\right) = -\frac{\partial J^{\mathsf{i}}_{\alpha}}{\partial q_{\alpha}}(q,t)$$

$$V^{\mathsf{e}}(q,t) = v^{\mathsf{e}}(\mathbf{X}(q,t),t)$$

$$A(q,t)\left(C\frac{\partial(V^{\mathsf{i}}-V^{\mathsf{e}})}{\partial t} + I_{\mathsf{ion}}(V^{\mathsf{i}}-V^{\mathsf{e}},W)\right) = -\frac{\partial J^{\mathsf{i}}_{\alpha}}{\partial q_{\alpha}}(q,t)$$

$$\frac{\partial W}{\partial t}(q,t) = f_{\mathsf{w}}(V^{\mathsf{i}}-V^{\mathsf{e}},W,\Lambda)$$

The Bidomain Equations in Immersed Boundary Form

$$-(\ell^{\mathsf{e}} v^{\mathsf{e}})(\mathbf{x},t) = \int (\mathcal{L}^{\mathsf{i}}(V+V^{\mathsf{e}}))(q,t)\delta\left(\mathbf{x}-\mathbf{X}(q,t)\right)dq$$

$$V^{\mathsf{e}}(q,t) = \int v^{\mathsf{e}}(\mathbf{x},t)\delta\left(\mathbf{x}-\mathbf{X}(q,t)\right)d\mathbf{x}$$

$$A\left(C\frac{\partial V}{\partial t} + I_{\mathsf{ion}}(V,W)\right) = \mathcal{L}^{\mathsf{i}}(V+V^{\mathsf{e}})$$

$$\frac{\partial W}{\partial t} = f_{\mathsf{w}}(V,W,\Lambda)$$

where

$$V = V^{\mathsf{i}} - V^{\mathsf{e}}$$

$$\begin{aligned}
\mathcal{L}^{\mathsf{i}}\Phi &= \frac{\partial}{\partial q_\alpha}\left(S^{\mathsf{i}}_{\alpha\beta}\frac{\partial \Phi}{\partial q_\beta}\right)\\
\ell^{\mathsf{e}}\phi &= \frac{\partial}{\partial x_k}\left(\sigma^{\mathsf{e}}_{kl}\frac{\partial \phi}{\partial x_l}\right)
\end{aligned}$$

Link-Based Discretization of Lagrangian Variables

$$\mathbf{F}_k(t)\left|\Delta^3 q\right|_k = \sum_{p=1}^{d(k)} T_{l_p(k)}(t)\left|\Delta^2 q\right|_{l_p(k)} \frac{\mathbf{X}_{k_p(k)}(t) - \mathbf{X}_k(t)}{\left|\mathbf{X}_{k_p(k)}(t) - \mathbf{X}_k(t)\right|}$$

$$\begin{aligned}
A_k\left|\Delta^3 q\right|_k \left(C\frac{dV_k}{dt} + I_{\mathsf{ion}}(V_k, W_k)\right) &= \sum_{p=1}^{d(k)} G_{l_p(k)}(t)\left(V^{\mathsf{i}}_{k_p(k)} - V^{\mathsf{i}}_k\right) \\
V^{\mathsf{i}} &= V + V^{\mathsf{e}} \\
\frac{dW_k}{dt} &= f_{\mathsf{w}}(V_k, W_k, \Lambda_k)
\end{aligned}$$

Deformation Dependence of Link Conductance

Fiber-oriented links (cytoplasmic resistance dominant):

$$G_l(t) = \sigma_{\text{cyt}} \frac{\left|\Delta^2 \mathbf{x}\right|_l (t)}{\left|\Delta \mathbf{x}\right|_l (t)}$$

$$\left|\Delta^2 \mathbf{x}\right|_l (t) \left|\Delta \mathbf{x}\right|_l (t) = \left|\Delta^2 \mathbf{x}\right|_l (0) \left|\Delta \mathbf{x}\right|_l (0)$$

$$G_l(t) = G_l(0) \left(\frac{\left|\Delta \mathbf{x}\right|_l (0)}{\left|\Delta \mathbf{x}\right|_l (t)} \right)^2$$

Cross-fiber links (gap-junctional resistance dominant):

$$G_l(t) = G_l(0)$$

Spatially Discretized Cardiac Mechanics

$$\rho\left(\frac{d\mathbf{u}}{dt} + s_h(\mathbf{u})\mathbf{u}\right) + \nabla_h p = \mu\Delta_h\mathbf{u} + \mathbf{f}$$

$$\nabla_h \cdot \mathbf{u} = 0$$

$$\mathbf{f}(\mathbf{x},t) = \sum_k \mathbf{F}_k(t)\delta_h\left(\mathbf{x} - \mathbf{X}_k(t)\right)\left|\Delta^3 q\right|_k$$

$$\frac{d\mathbf{X}_k}{dt}(t) = \sum_{\mathbf{x}\in g_{h,0}} \mathbf{u}(\mathbf{x},t)\delta_h\left(\mathbf{x} - \mathbf{X}_k(t)\right)h^3$$

$$\mathbf{F}_k(t)\left|\Delta^3 q\right|_k = \sum_{p=1}^{d(k)} T_{l_p(k)}(t)\left|\Delta^2 q\right|_{l_p(k)} \frac{\mathbf{X}_{k_p(k)}(t) - \mathbf{X}_k(t)}{\left|\mathbf{X}_{k_p(k)}(t) - \mathbf{X}_k(t)\right|}$$

Spatially Discretized Cardiac Electrophysiology

$$-(\sigma^{\mathsf{e}}\Delta_h v^{\mathsf{e}})(\mathbf{x},t) = \sum_k (L^{\mathsf{i}}(V+V^{\mathsf{e}}))_k(t)\delta_h\left(\mathbf{x}-\mathbf{X}_k(t)\right)\left|\Delta^3 q\right|_k$$

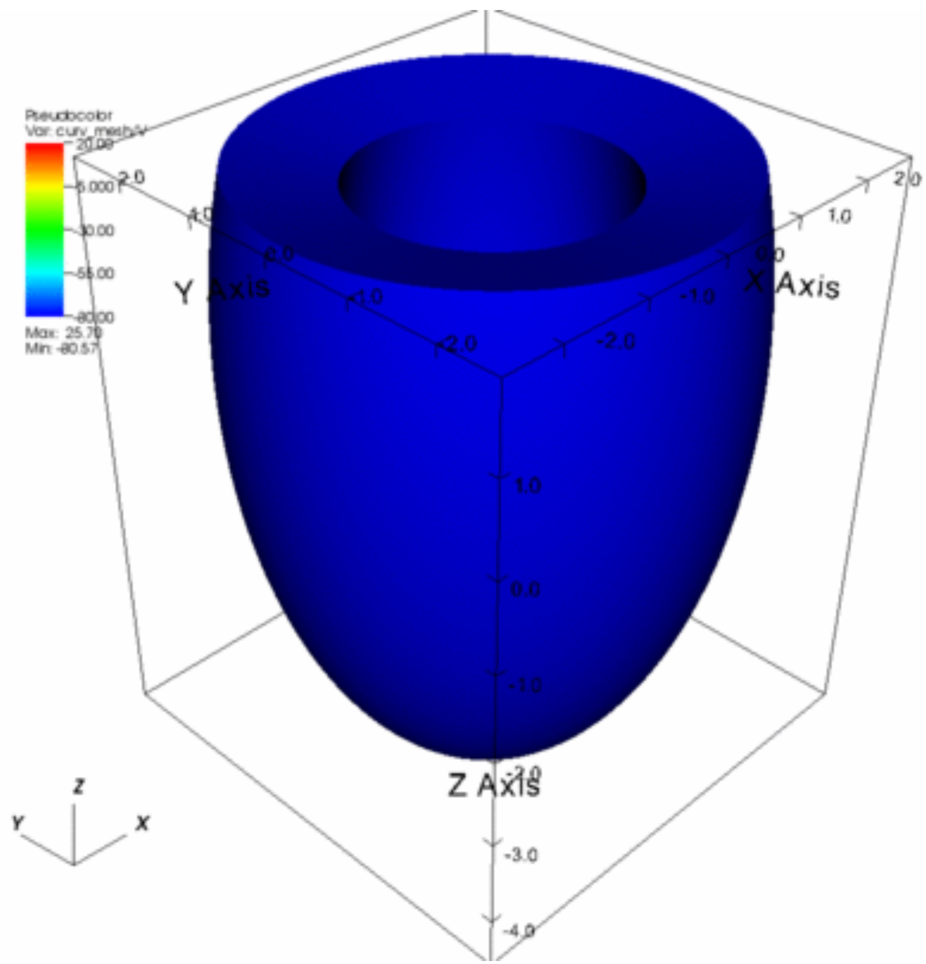
$$V_k^{\mathsf{e}}(t) = \sum_{\mathbf{x}\in g_{h,\mathbf{x}_{\mathsf{c}}}} v^{\mathsf{e}}(\mathbf{x},t)\delta_h\left(\mathbf{x}-\mathbf{X}_k(t)\right)h^3$$

$$A_k\left(C\frac{dV_k}{dt} + I_{\mathsf{ion}}(V_k,W_k)\right) = (L^{\mathsf{i}}(V+V^{\mathsf{e}}))_k$$

$$\frac{dW_k}{dt} = f_{\mathsf{w}}(V_k,W_k,\Lambda_k)$$

where the operator L^{i} is defined by

$$(L^{\mathsf{i}}\Phi)_k\left|\Delta^3 q\right|_k = \sum_{p=1}^{d(k)} G_{l_p(k)}\left(\Phi_{k_p(k)} - \Phi_k\right)$$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-55.00
-80.00
Max: 25.70
Min: -80.57
Y Axis
X Axis
Z Axis
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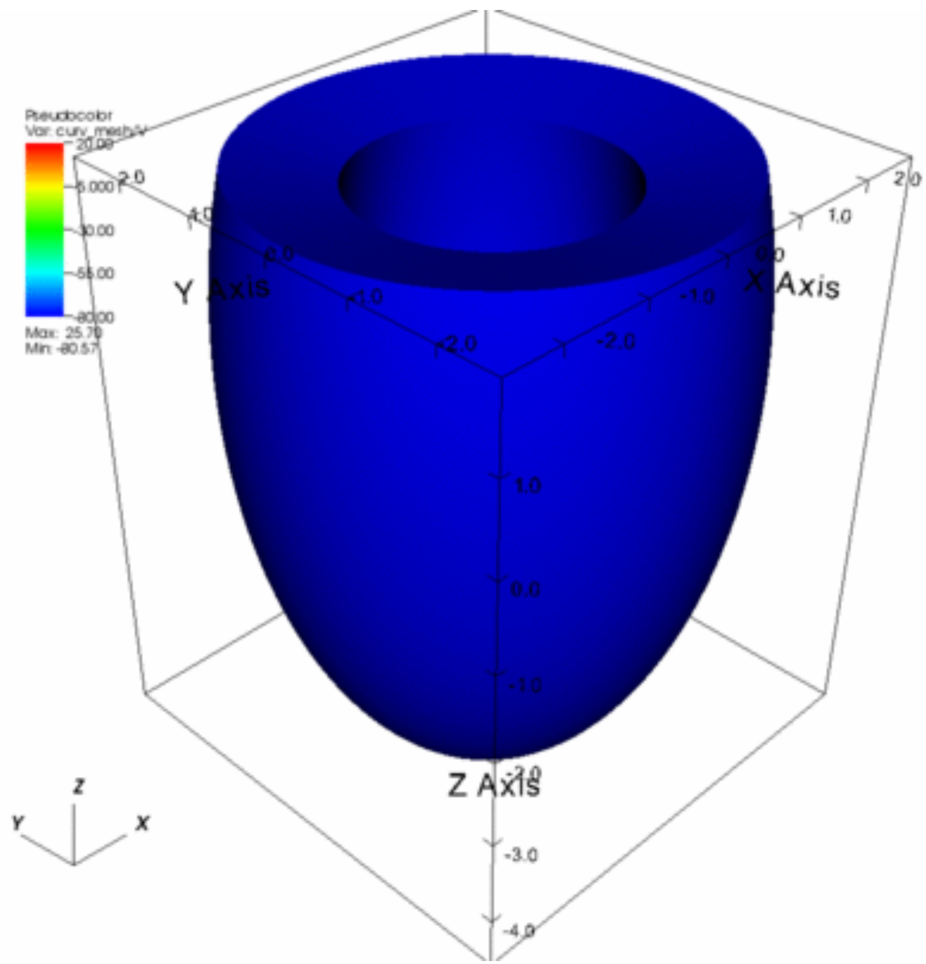
0.0
-1.0
-2.0
-3.0
-4.0
2.0
Z
Y
X



Pseudocolor
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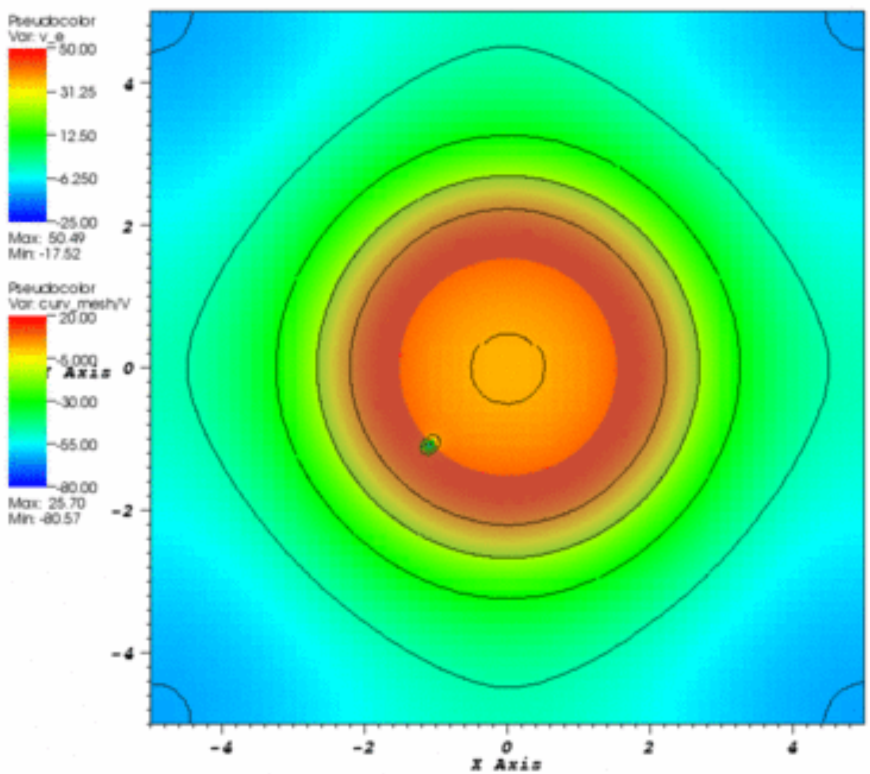
Min: -80.57
Y Axis
X Axis
Z Axis



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Max: 50.49
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Pseudocolor
Var: curv_mesh/V
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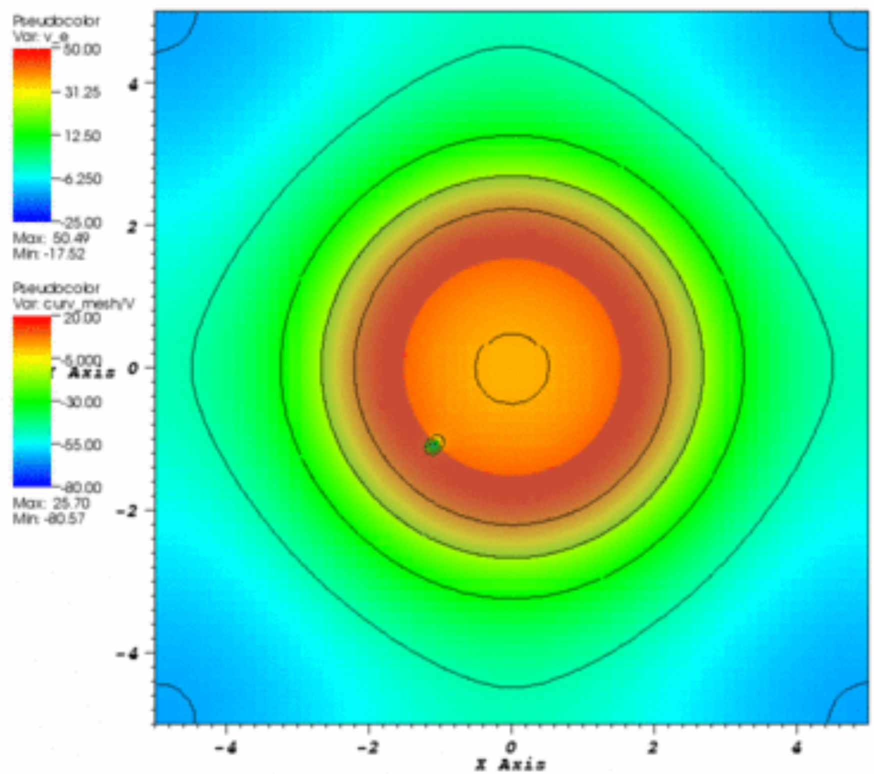
-5.000
-30.00
-55.00
-80.00
Max: 25.70
Min: -80.57
Y Axis
X Axis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Pseudocolor
Var: curv_mesh/V
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-5.000
-30.00
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-80.00
Max: 25.70
Min: -80.57
Y Axis
X Axis
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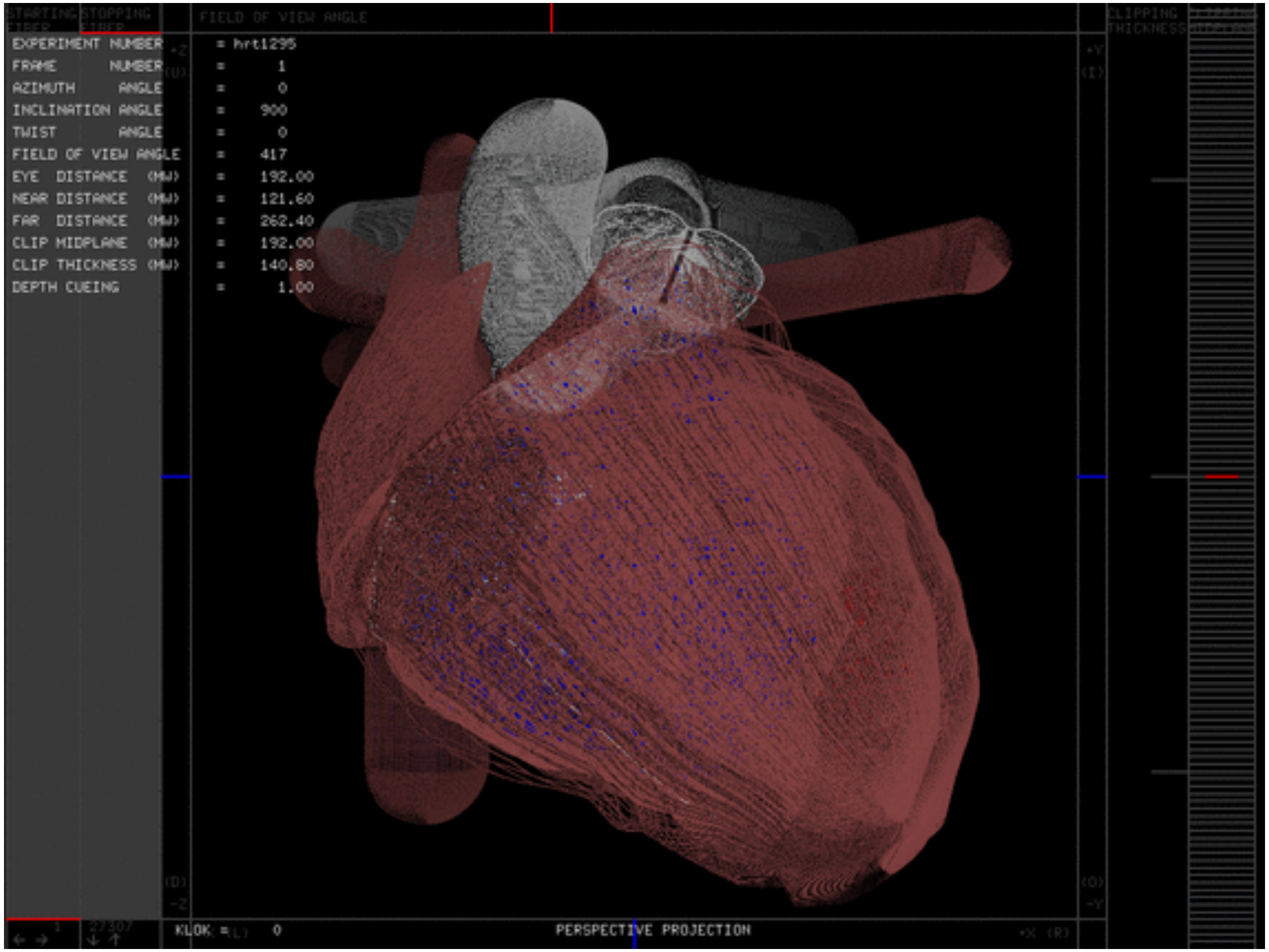
-2
0
2
4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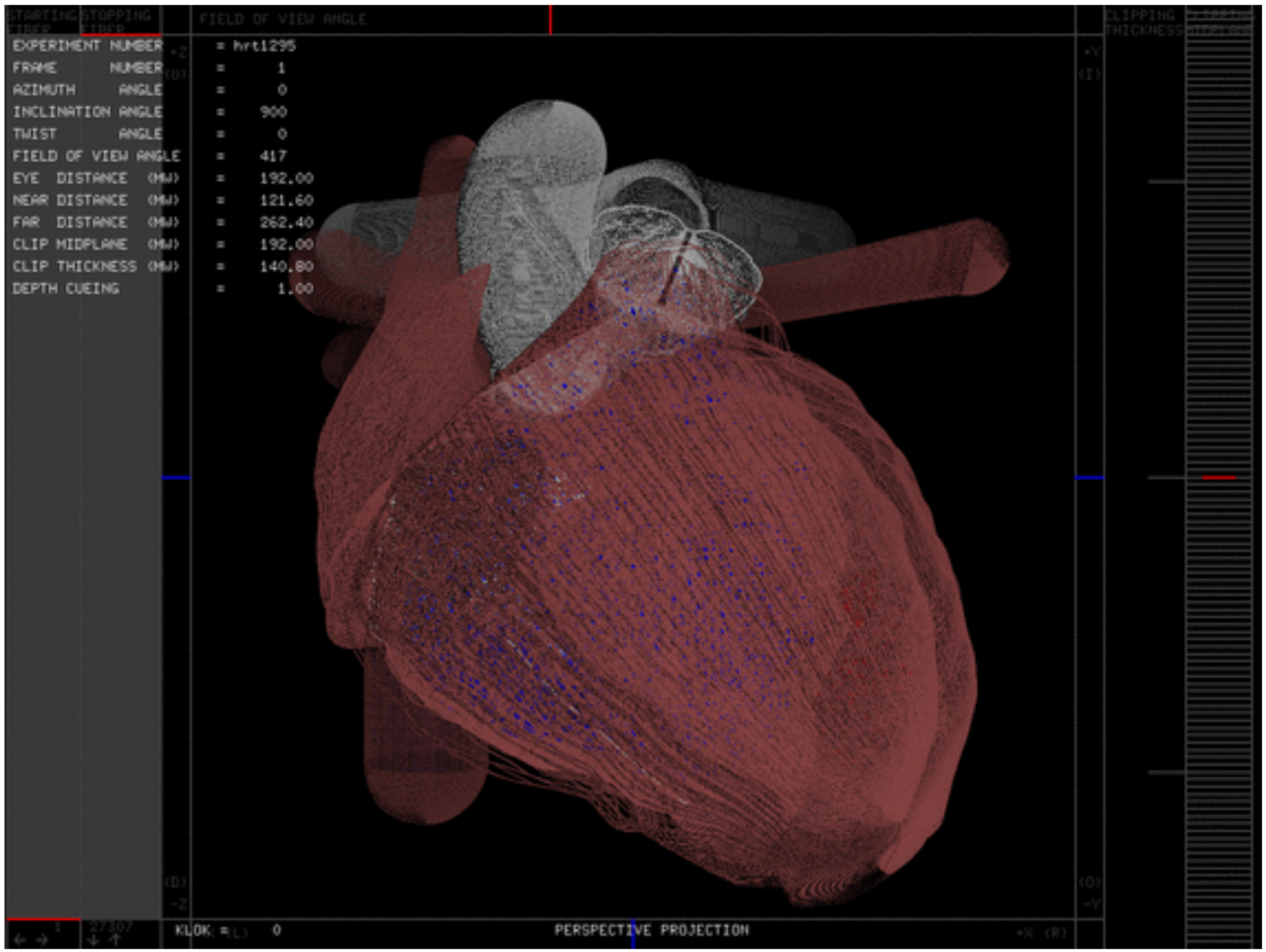
FAR DISTANCE (MW) = 262.40
CLIP MIDPLANE (MW) = 192.00
CLIP THICKNESS (MW) = 140.80
DEPTH CUEING = 1.00
KLOK = 0
PERSPECTIVE PROJECTION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NEAR DISTANCE (MW) = 121.60
FAR DISTANCE (MW) = 262.40
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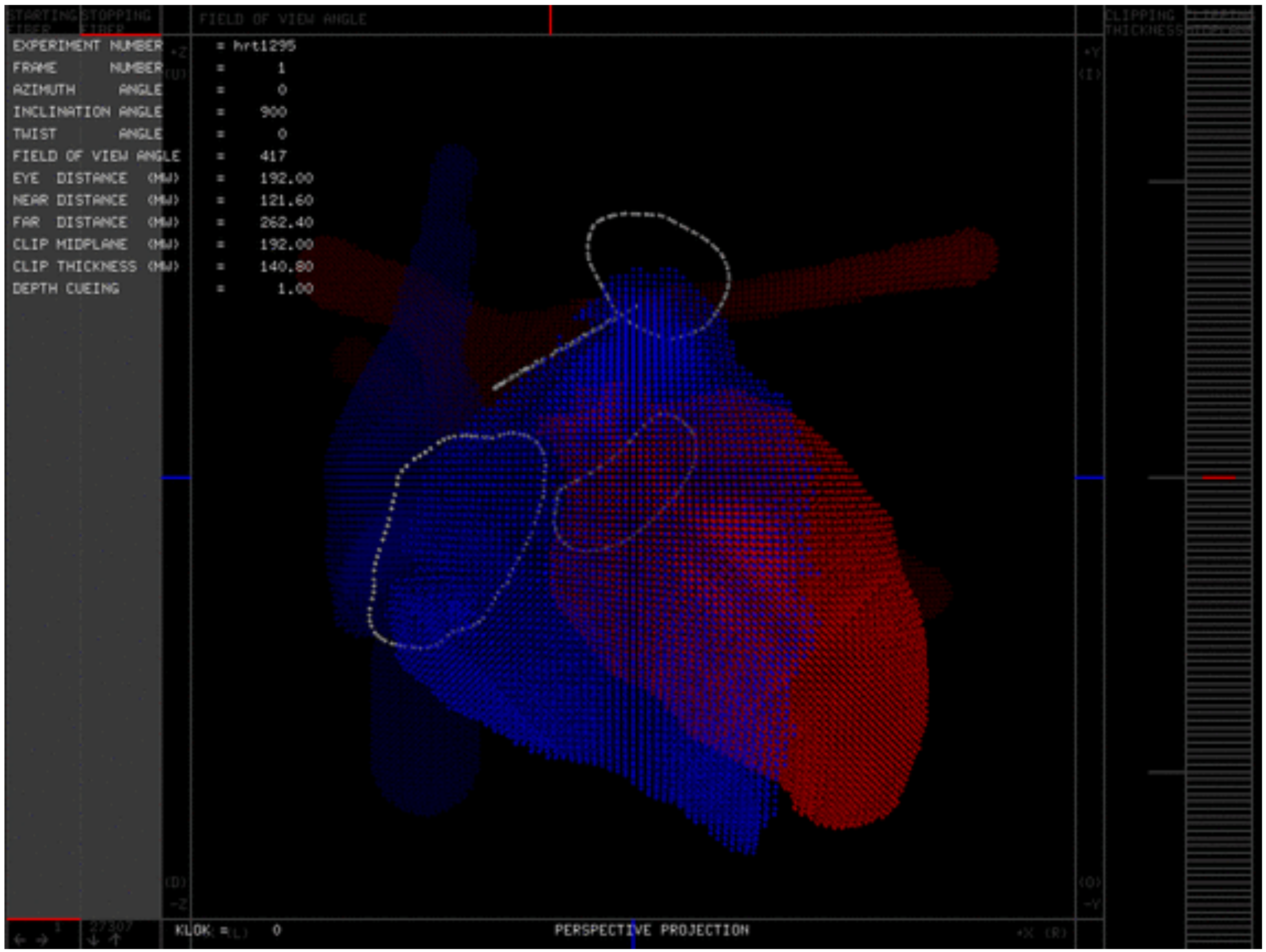
CLIP THICKNESS (MW) = 140.80
DEPTH CUEING = 1.00
KLOK = 0
PERSPECTIVE PROJECTION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STOPPING FIBER
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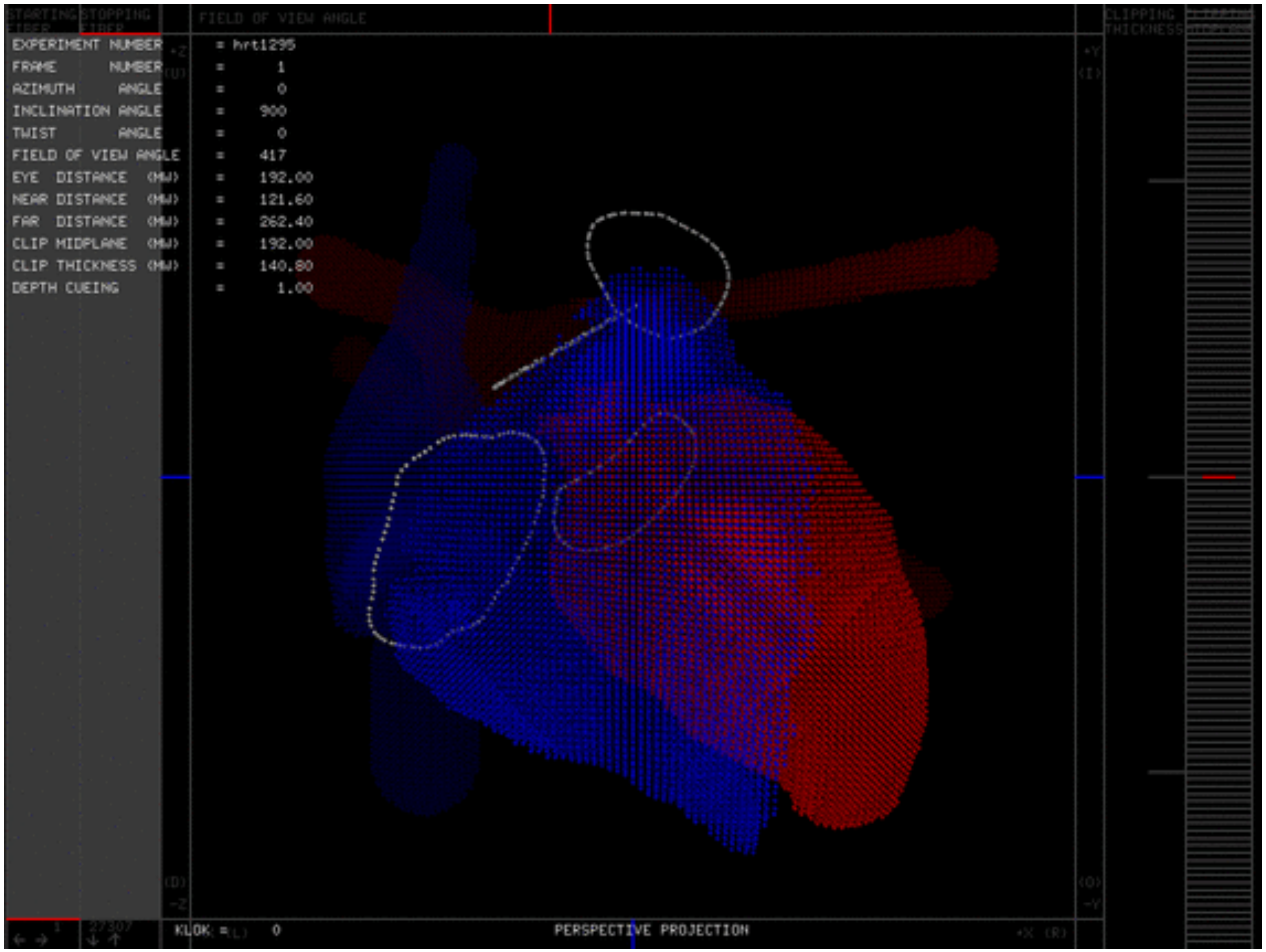
CLIP THICKNESS (MW) = 140.80
DEPTH CUEING = 1.00
KLOK = 0
PERSPECTIVE PROJECTION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FAR DISTANCE (MW) = 262.40
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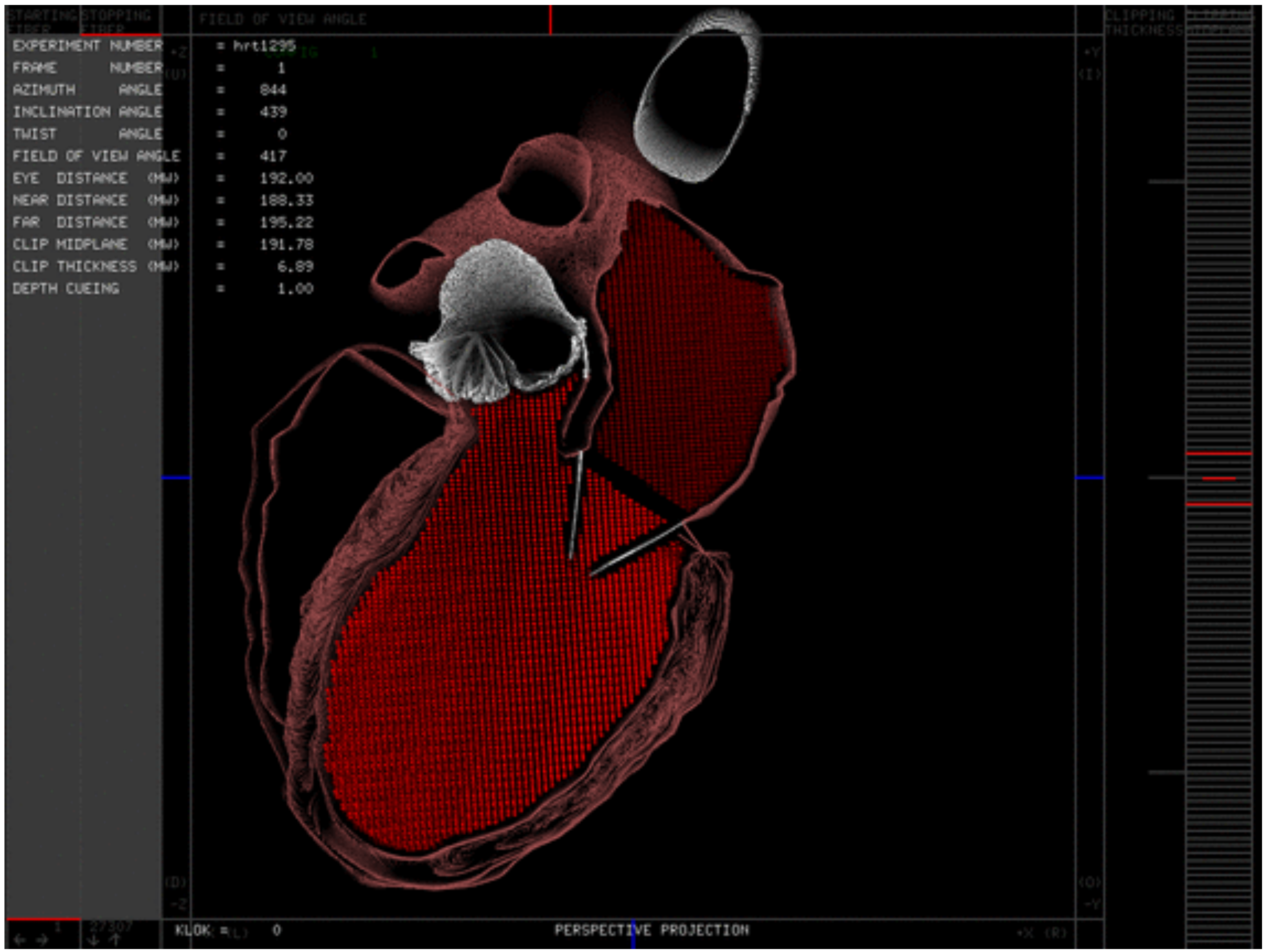
CLIP THICKNESS (MW) = 140.80
DEPTH CUEING = 1.00
KLOK = 0
PERSPECTIVE PROJECTION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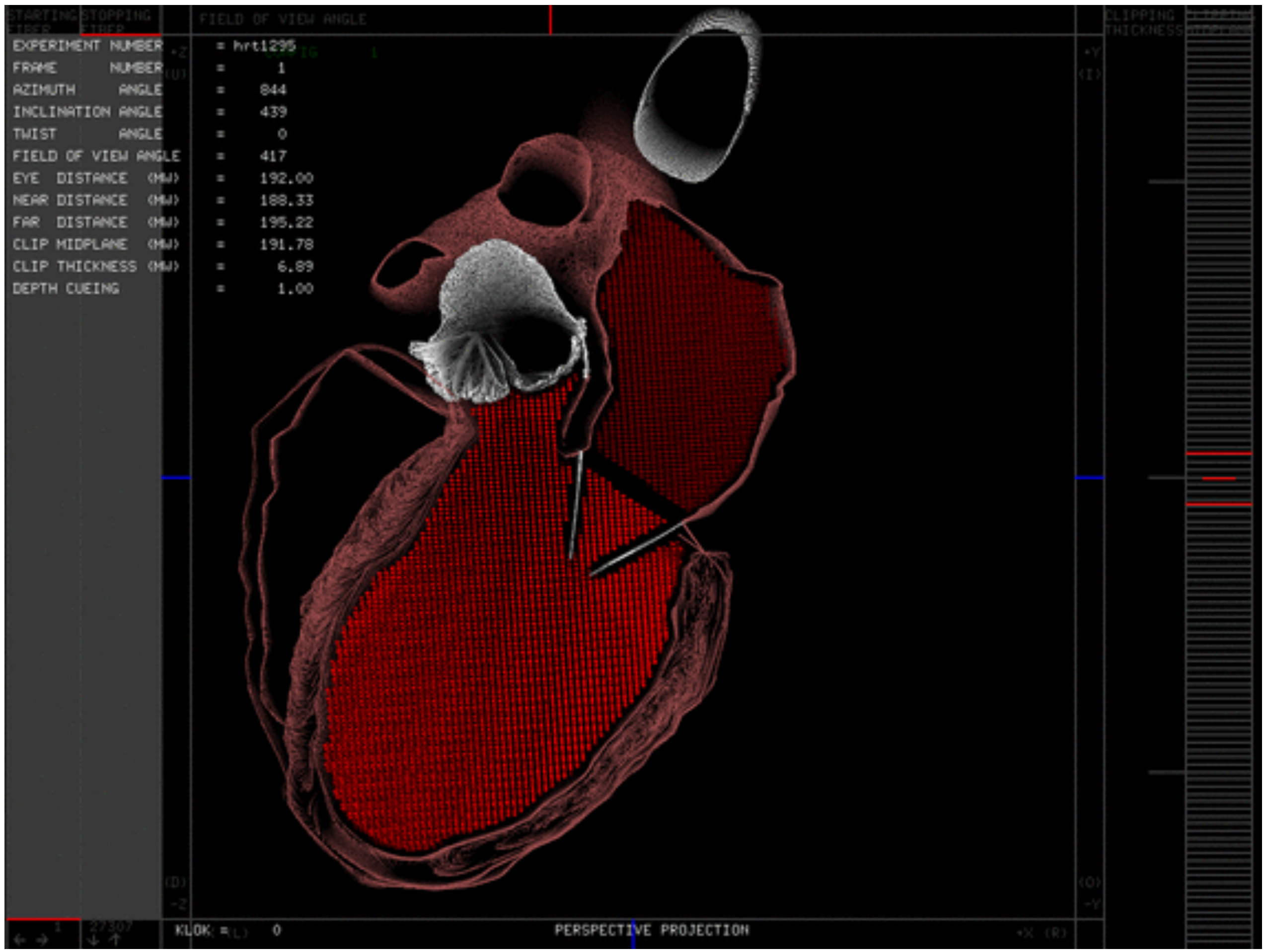
KLOK = 0
PERSPECTIVE PROJECTION



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STOPPING FIBER
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CLIPPING THICKNESS
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FRAME NUMBER = 1
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INCLINATION ANGLE = 439
TWIST ANGLE = 0
FIELD OF VIEW ANGLE = 417
EYE DISTANCE (MW) = 192.00
NEAR DISTANCE (MW) = 188.33
FAR DISTANCE (MW) = 195.22
CLIP MIDPLANE (MW) = 191.78
CLIP THICKNESS (MW) = 6.89
DEPTH CUEING = 1.00
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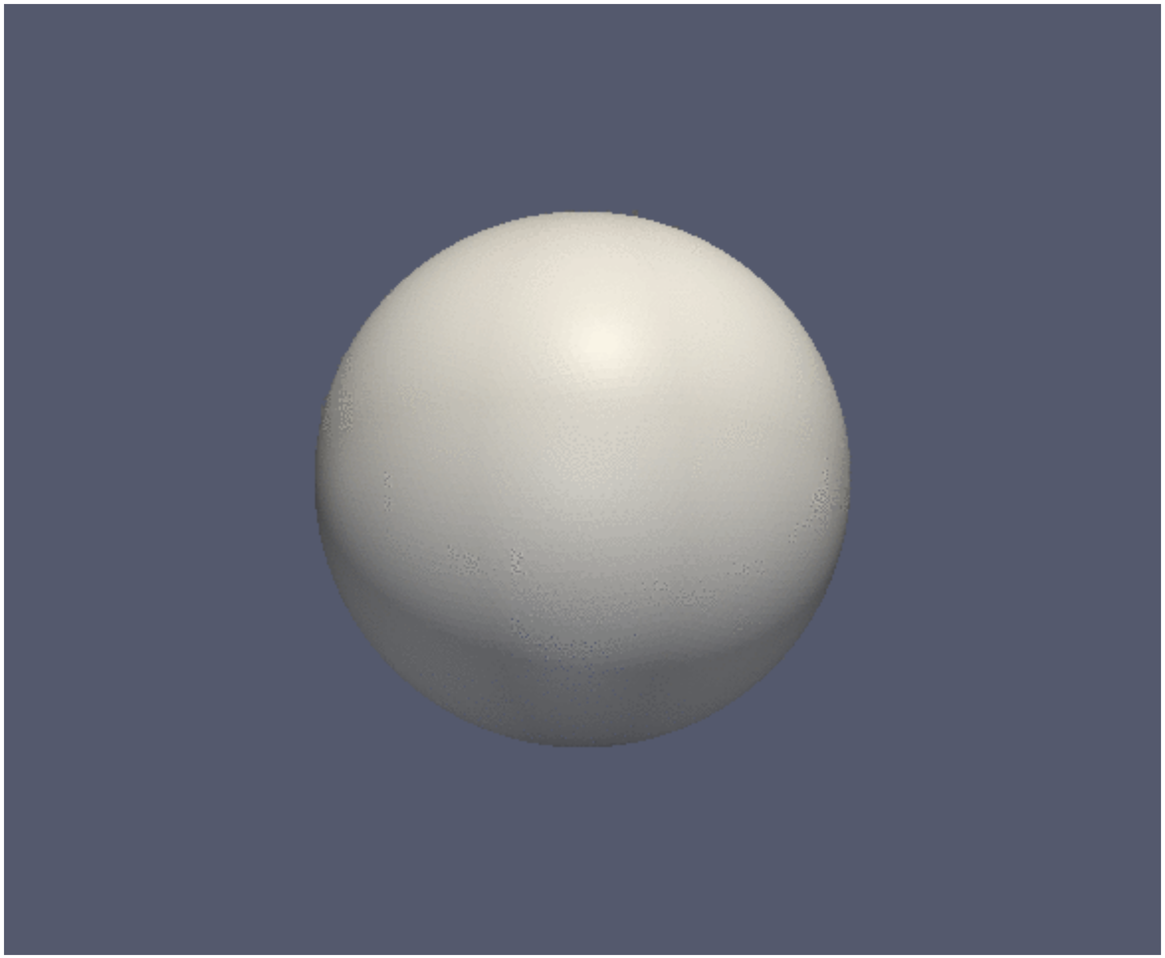
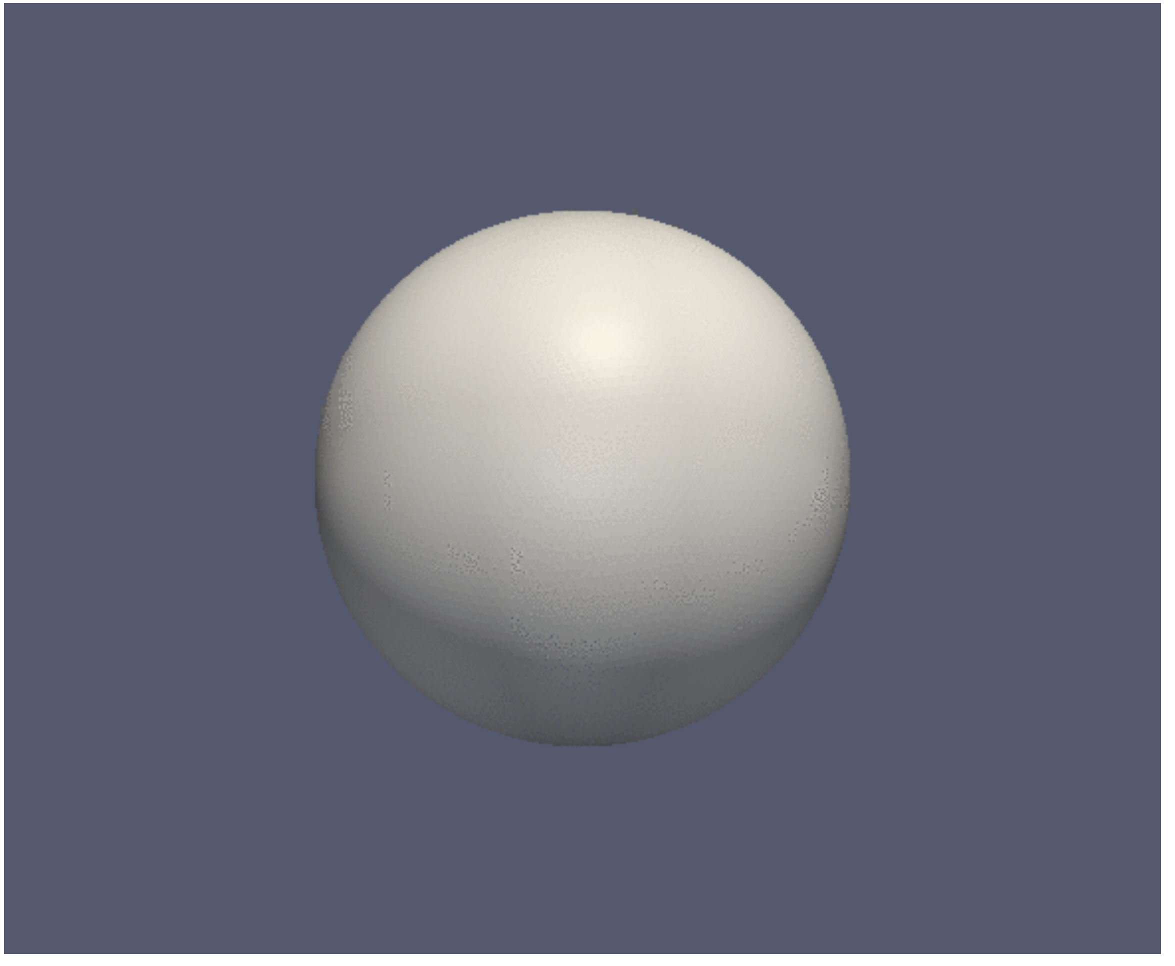
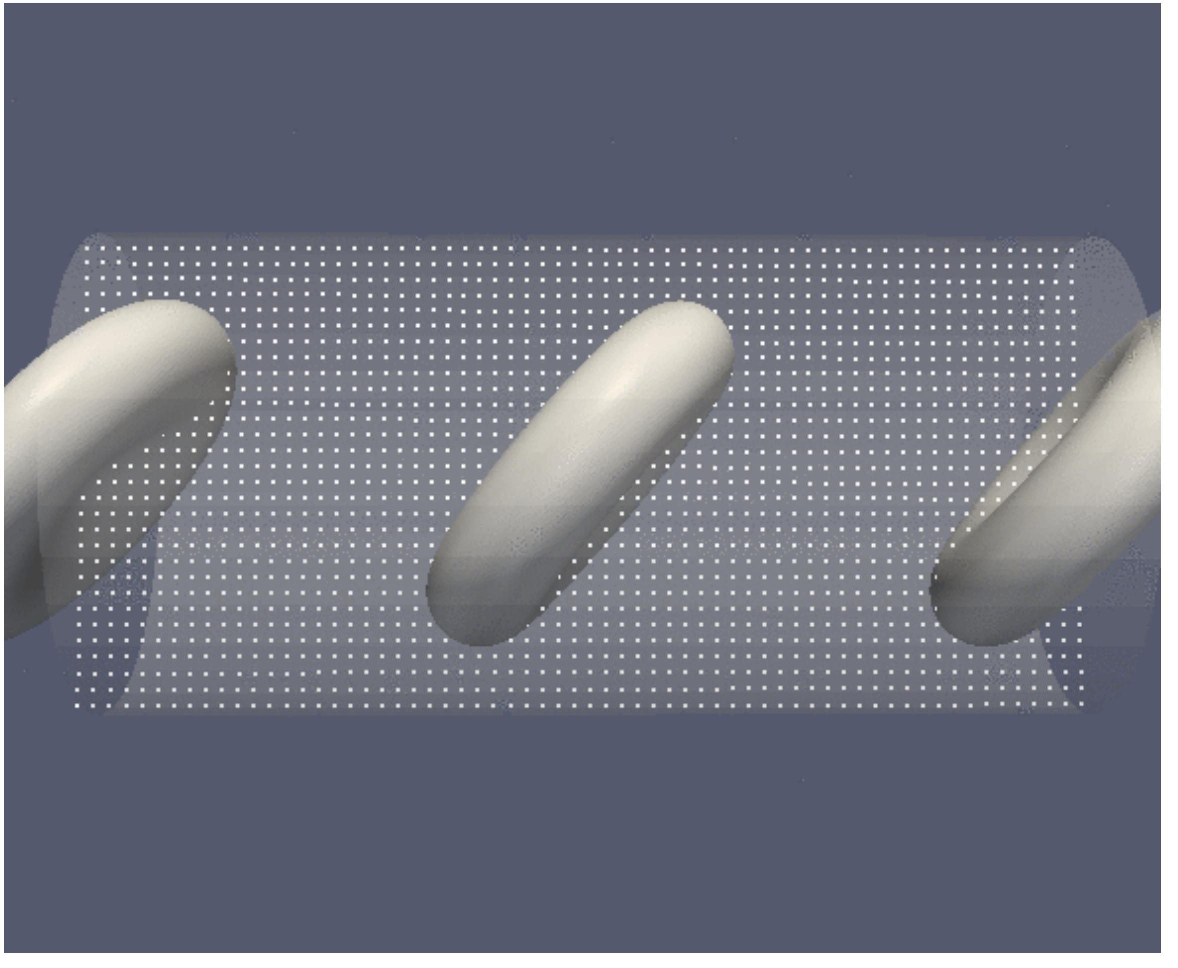
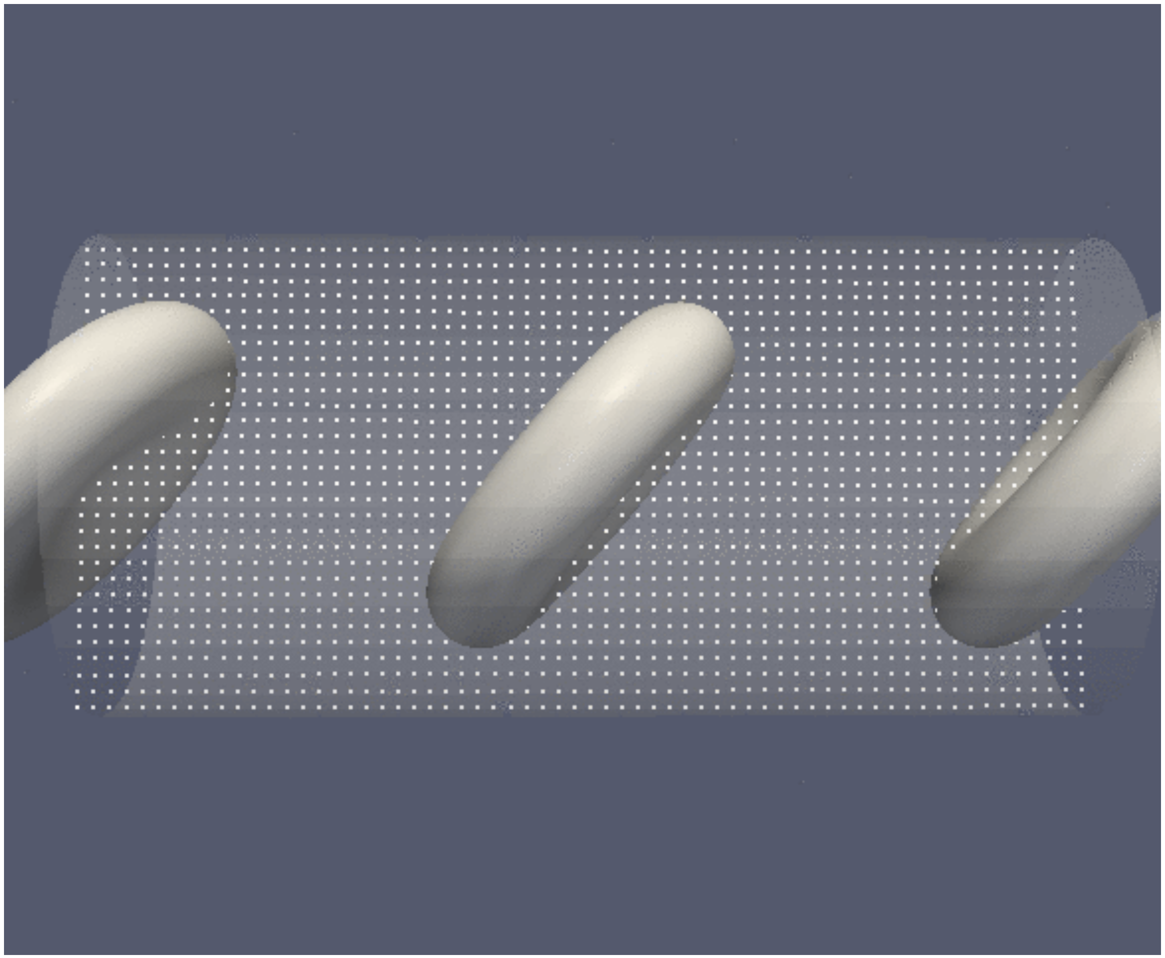
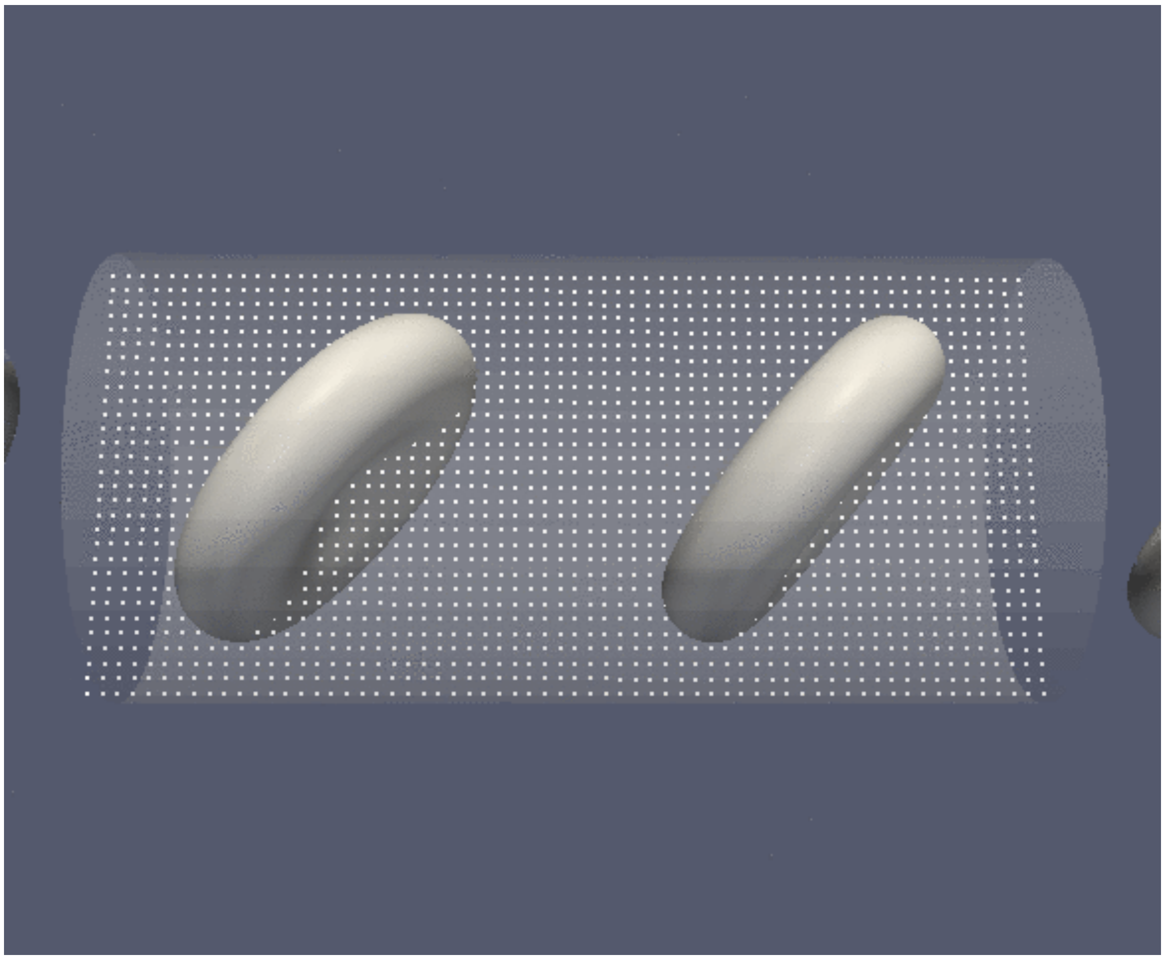
PERSPECTIVE PROJECTION

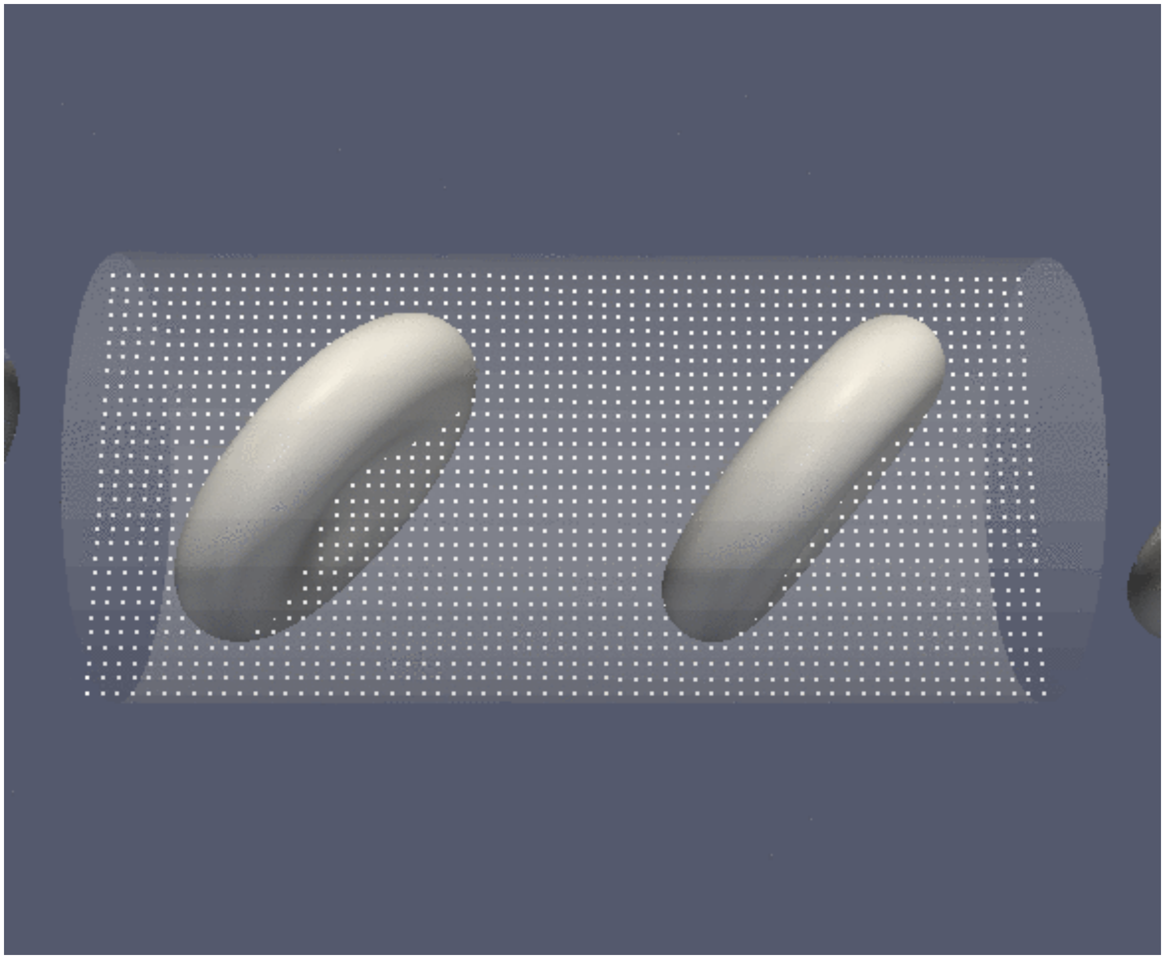


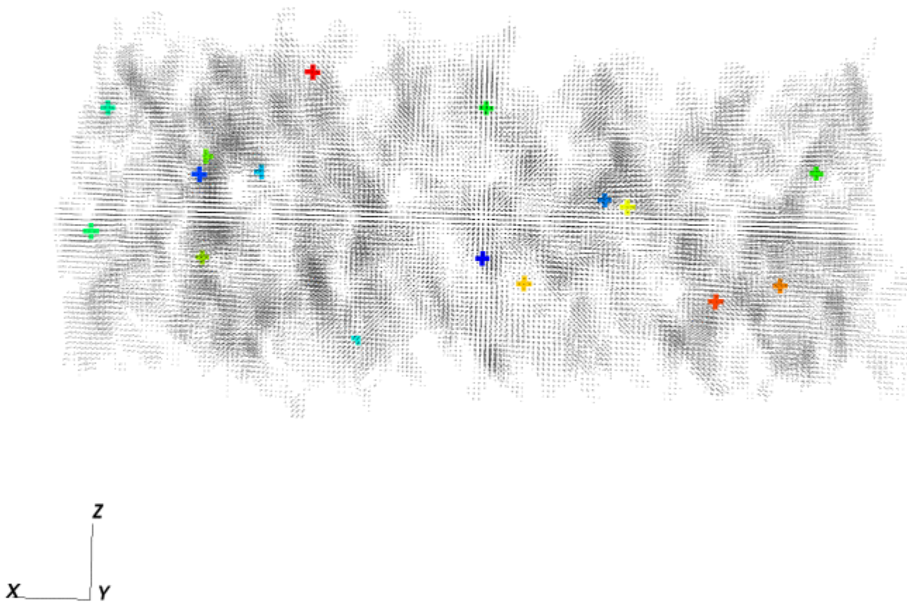








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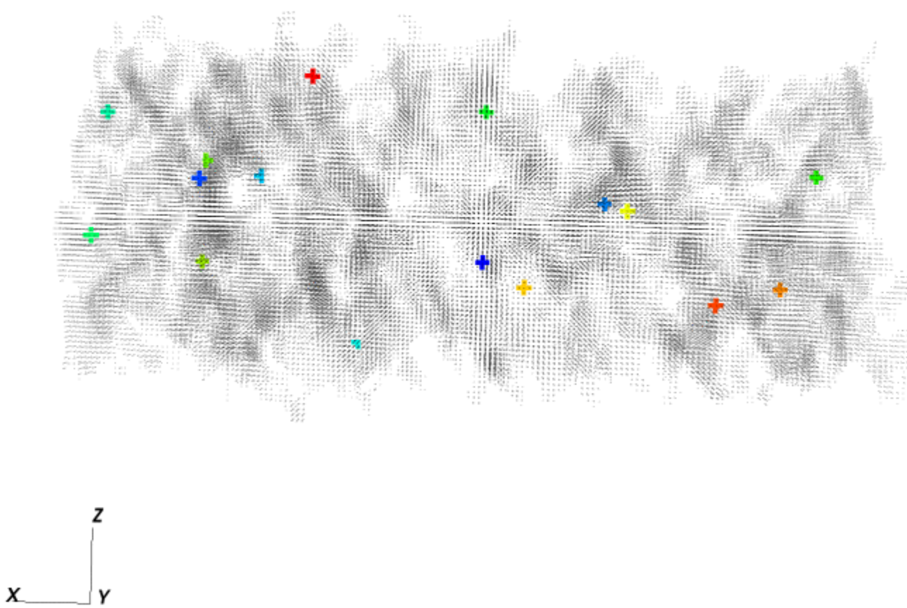
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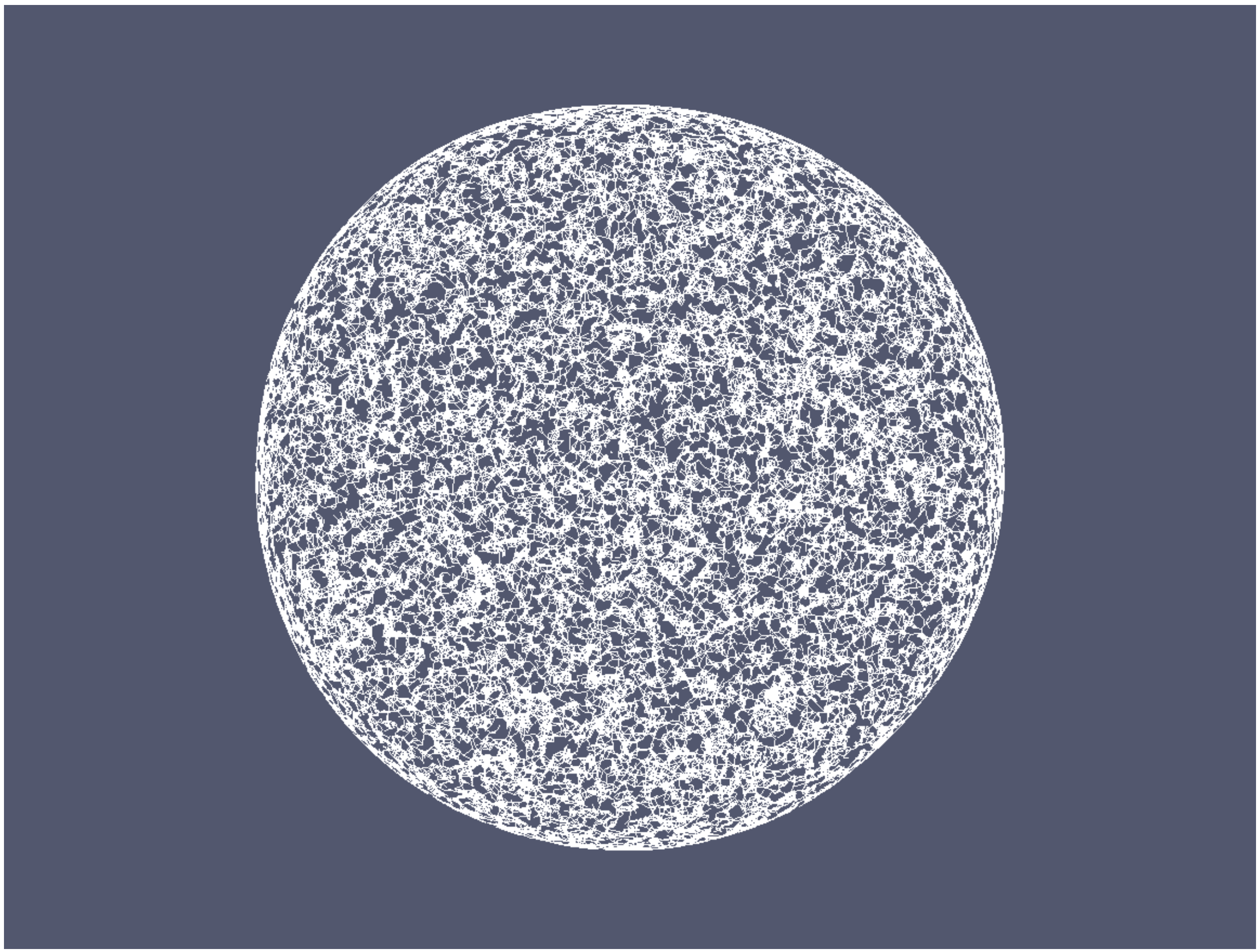

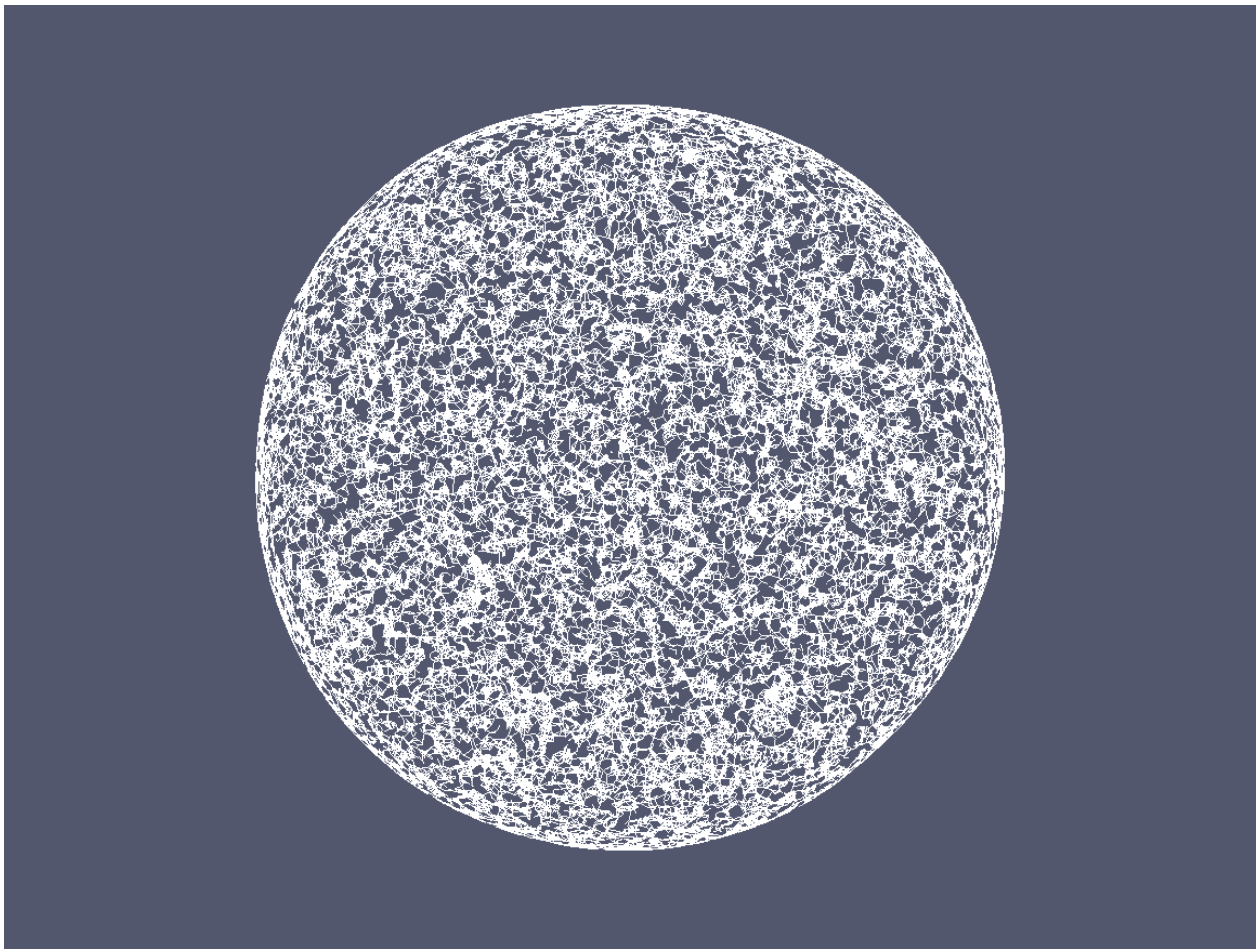

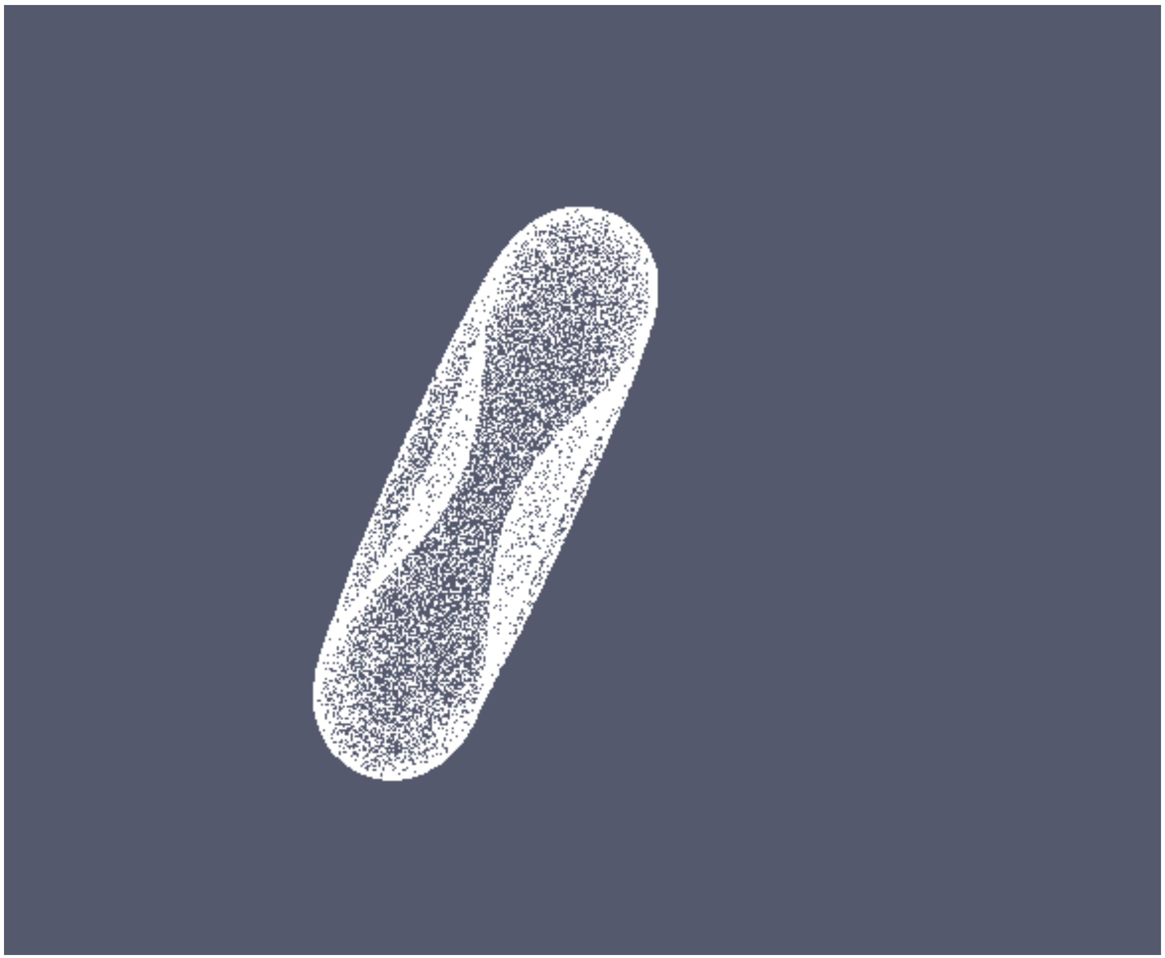

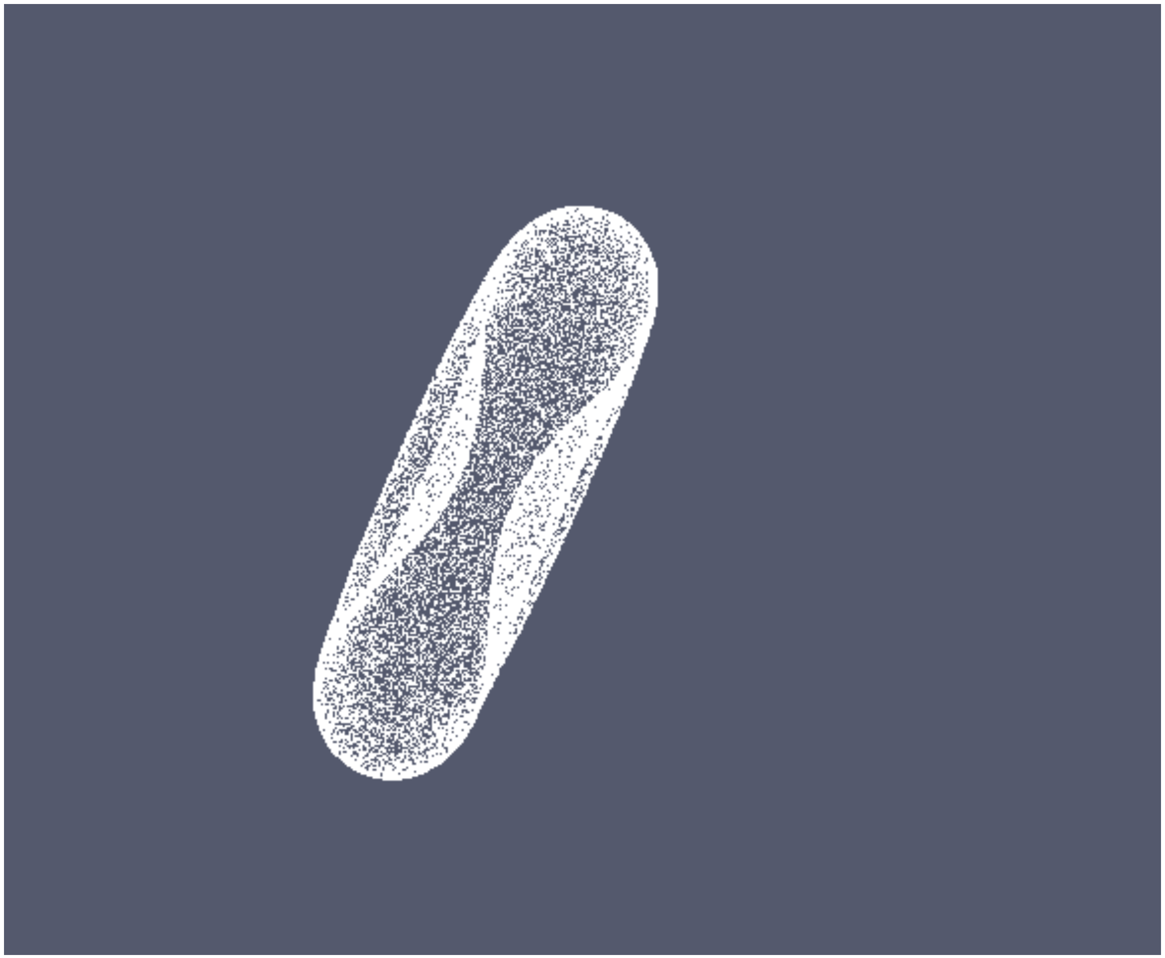

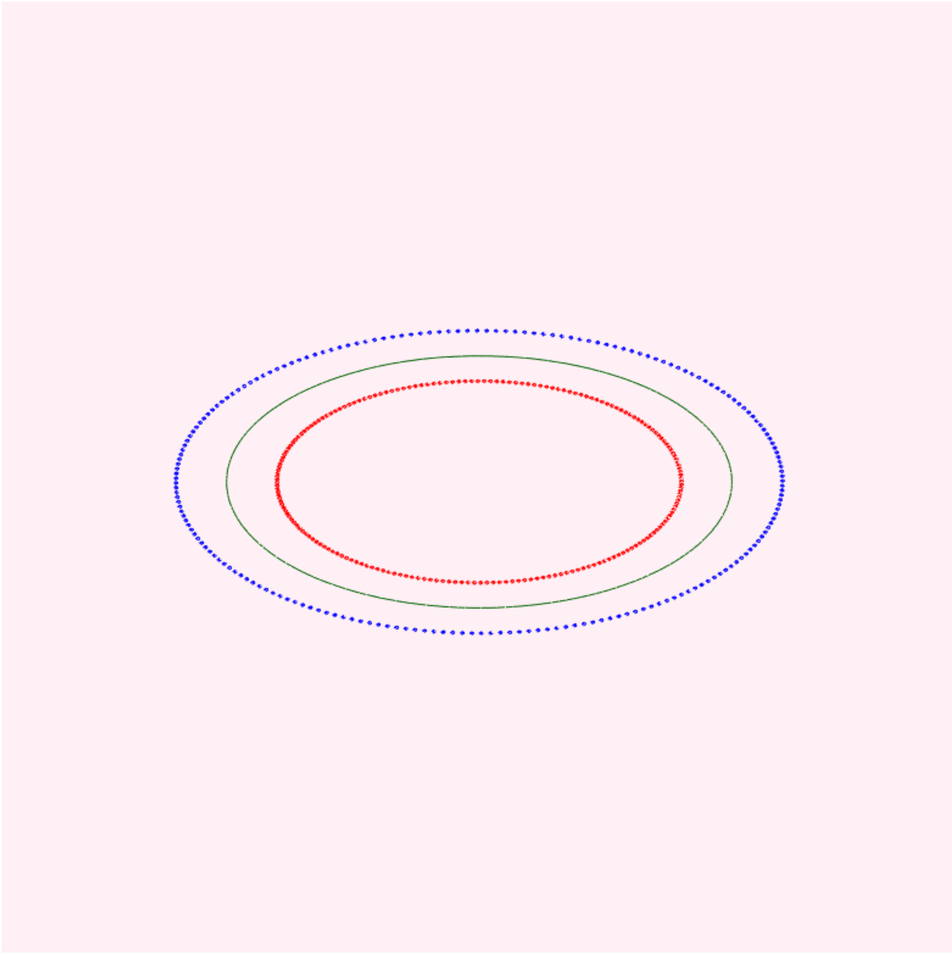

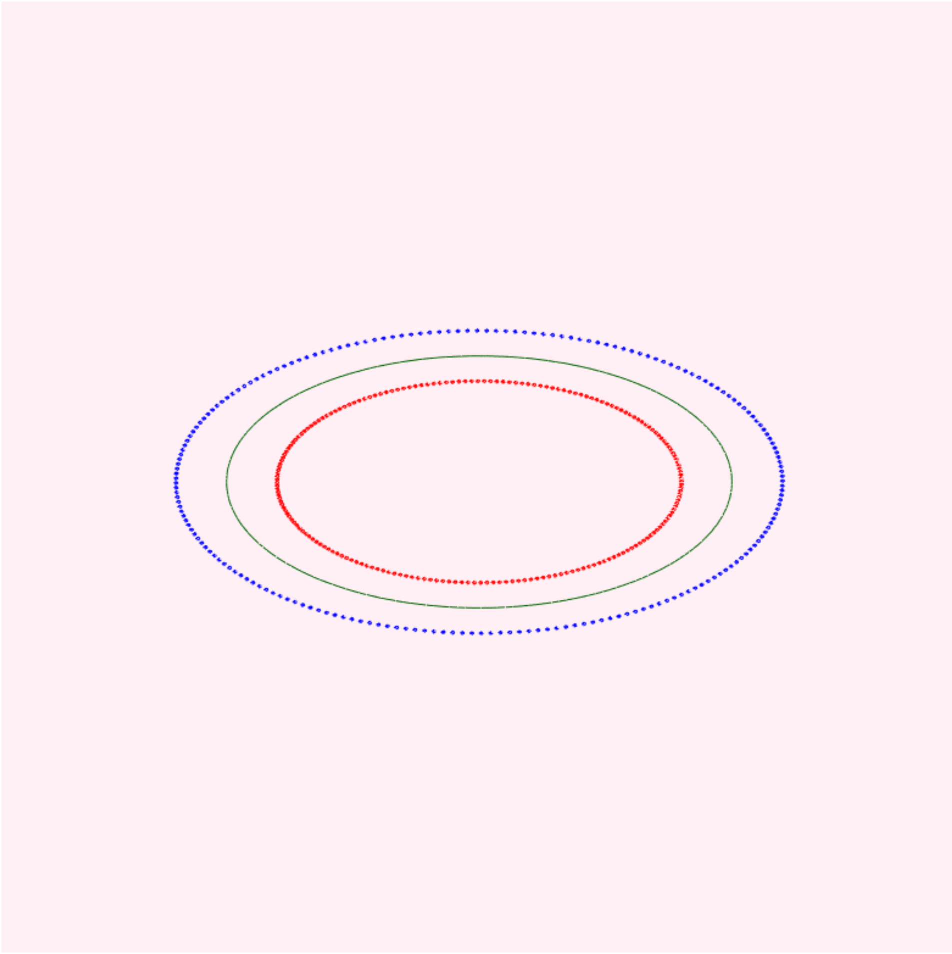

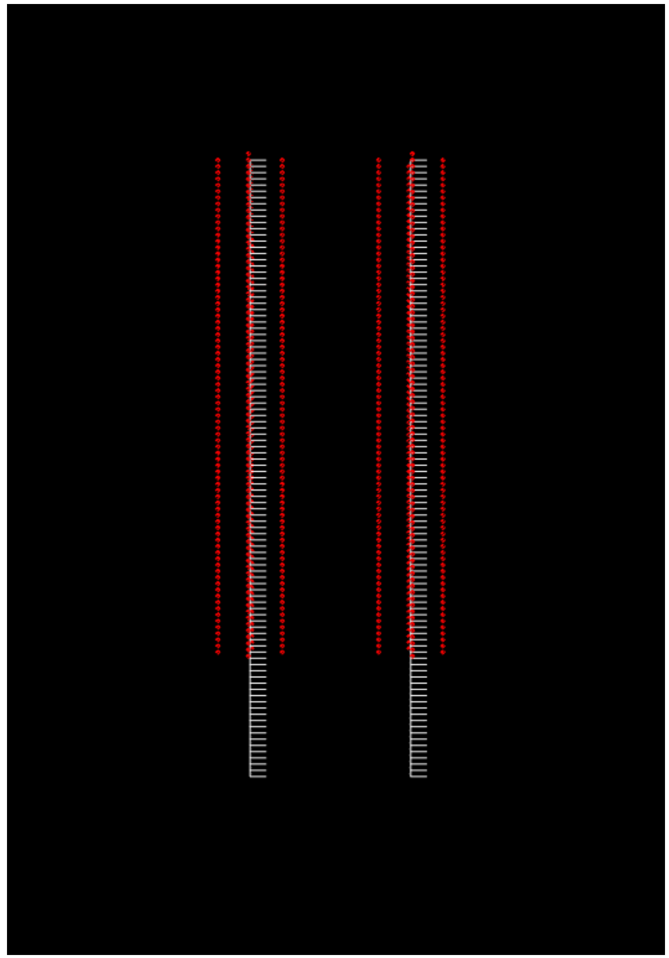

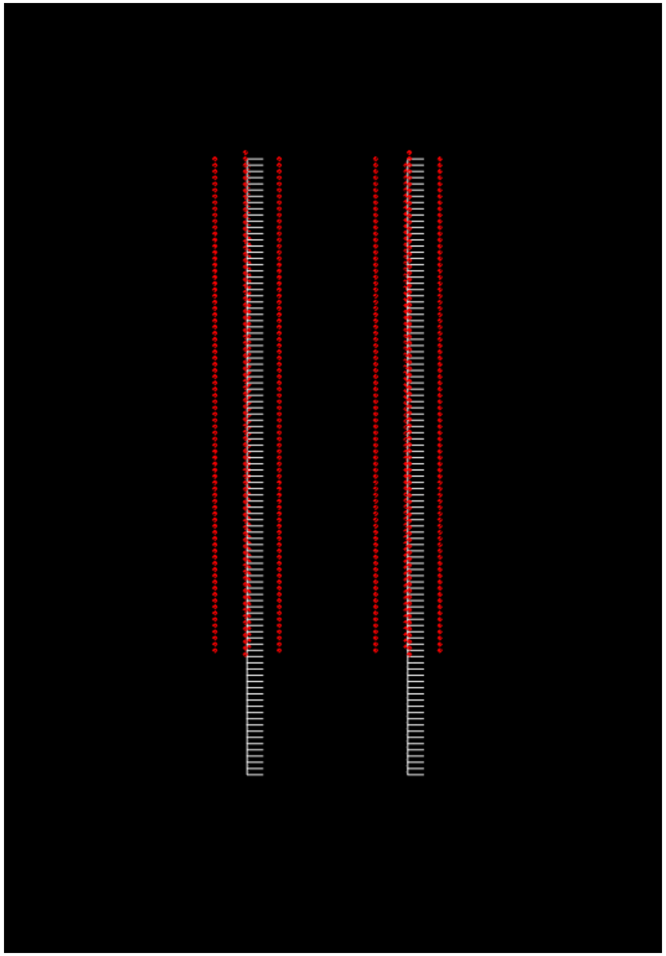

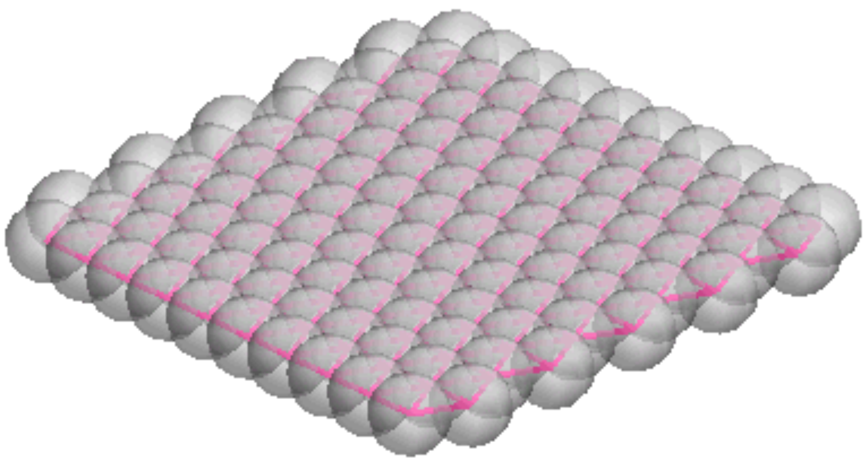

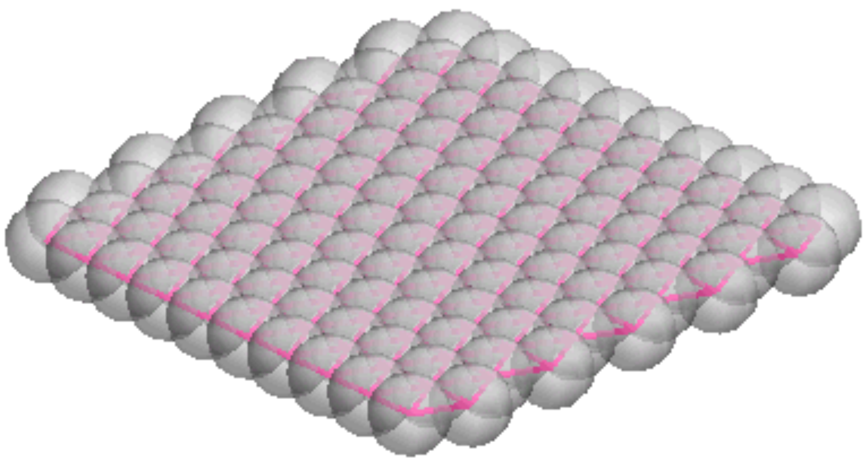

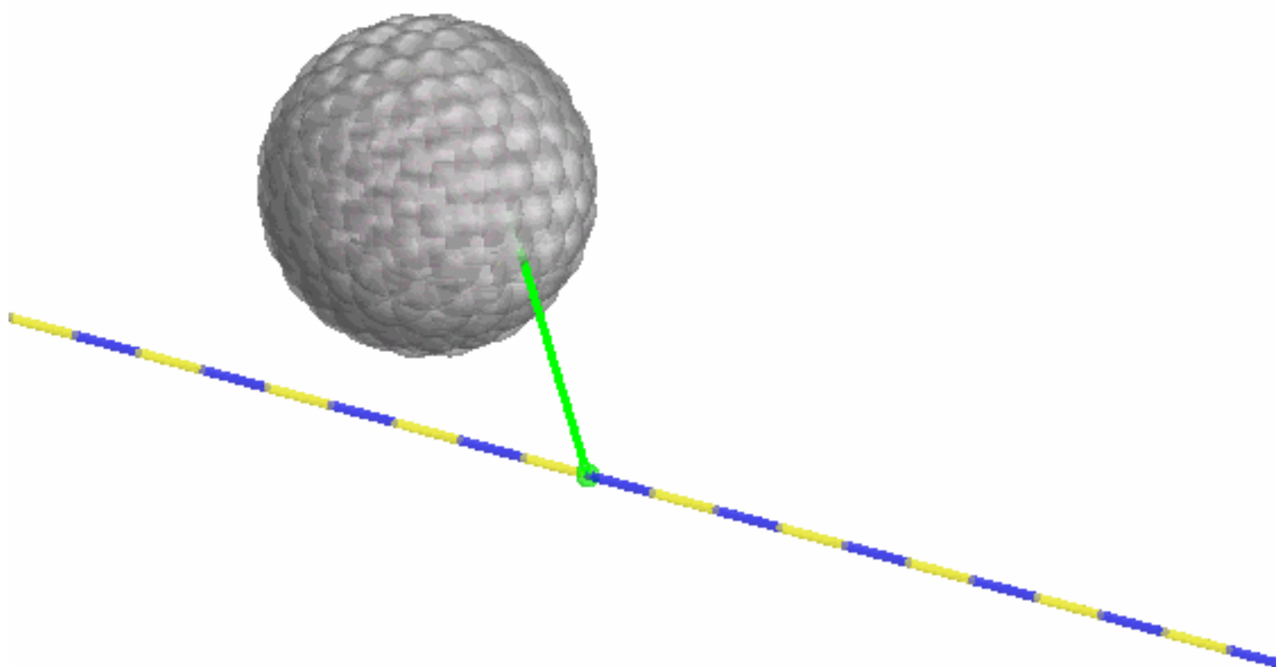

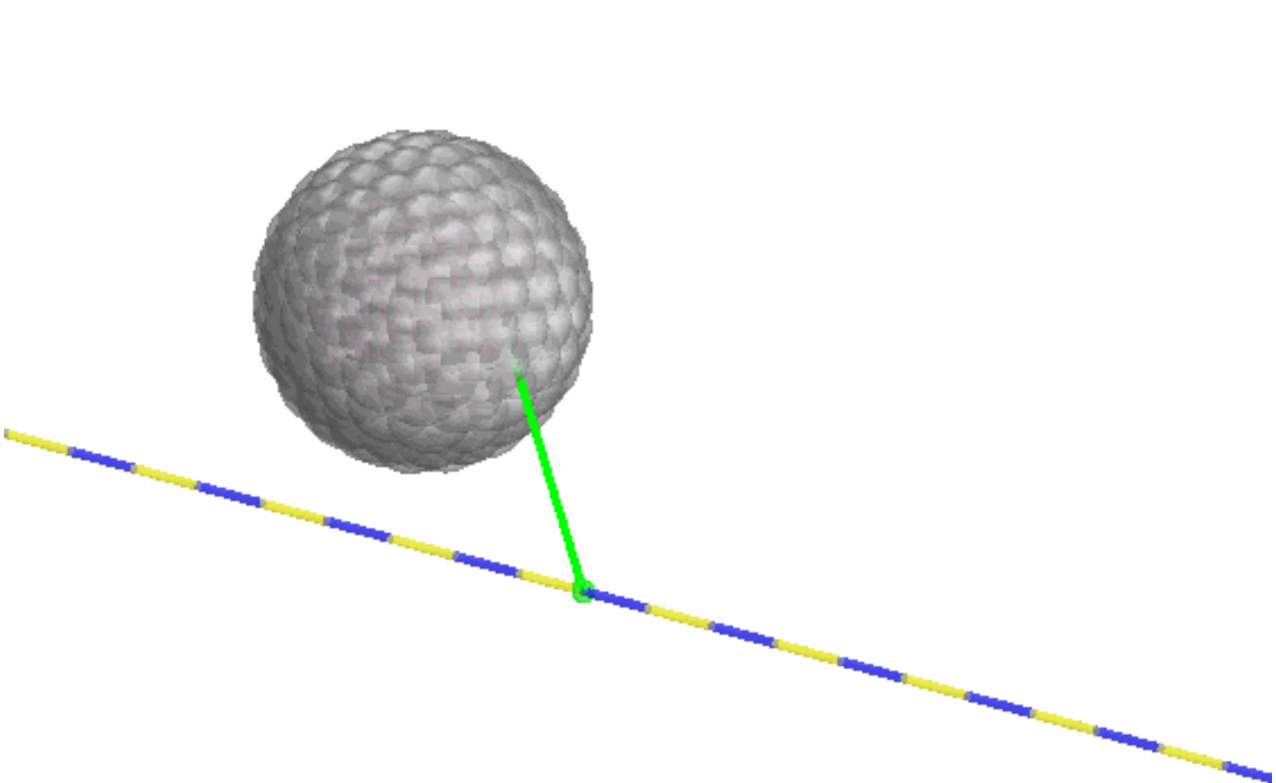

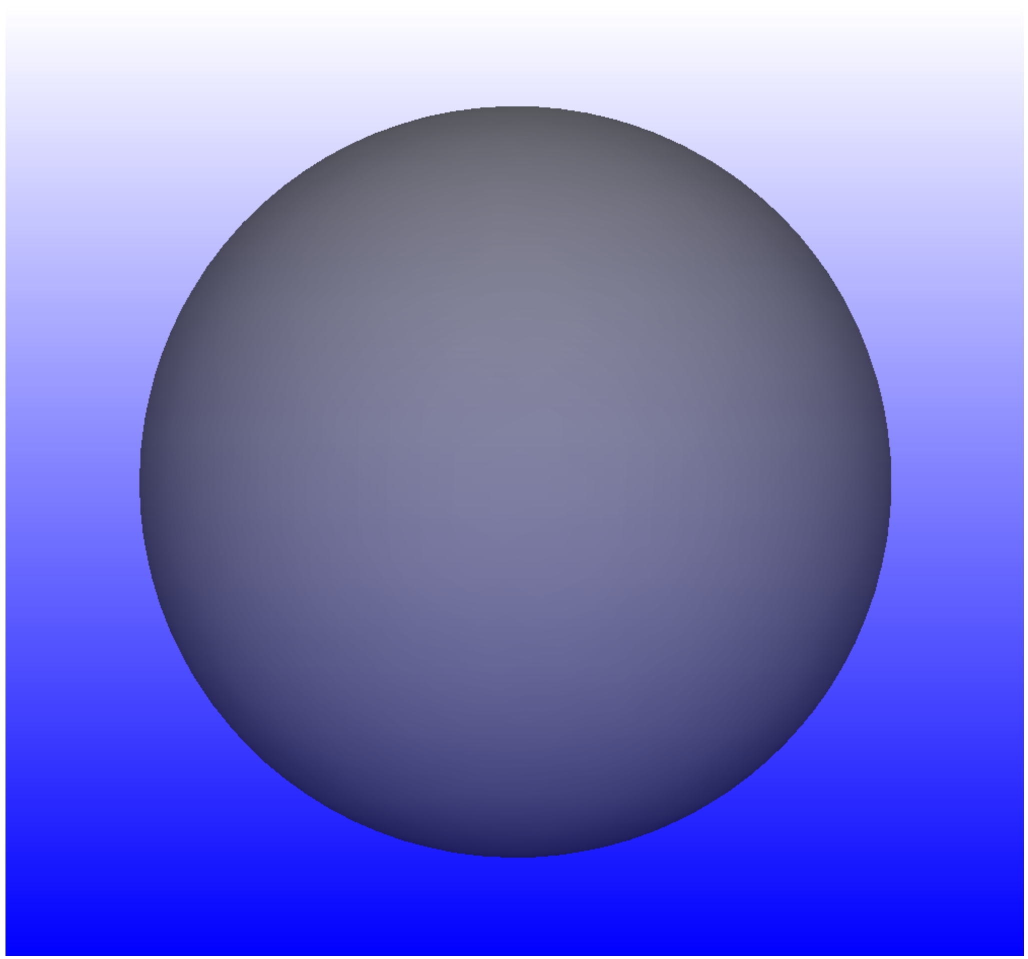

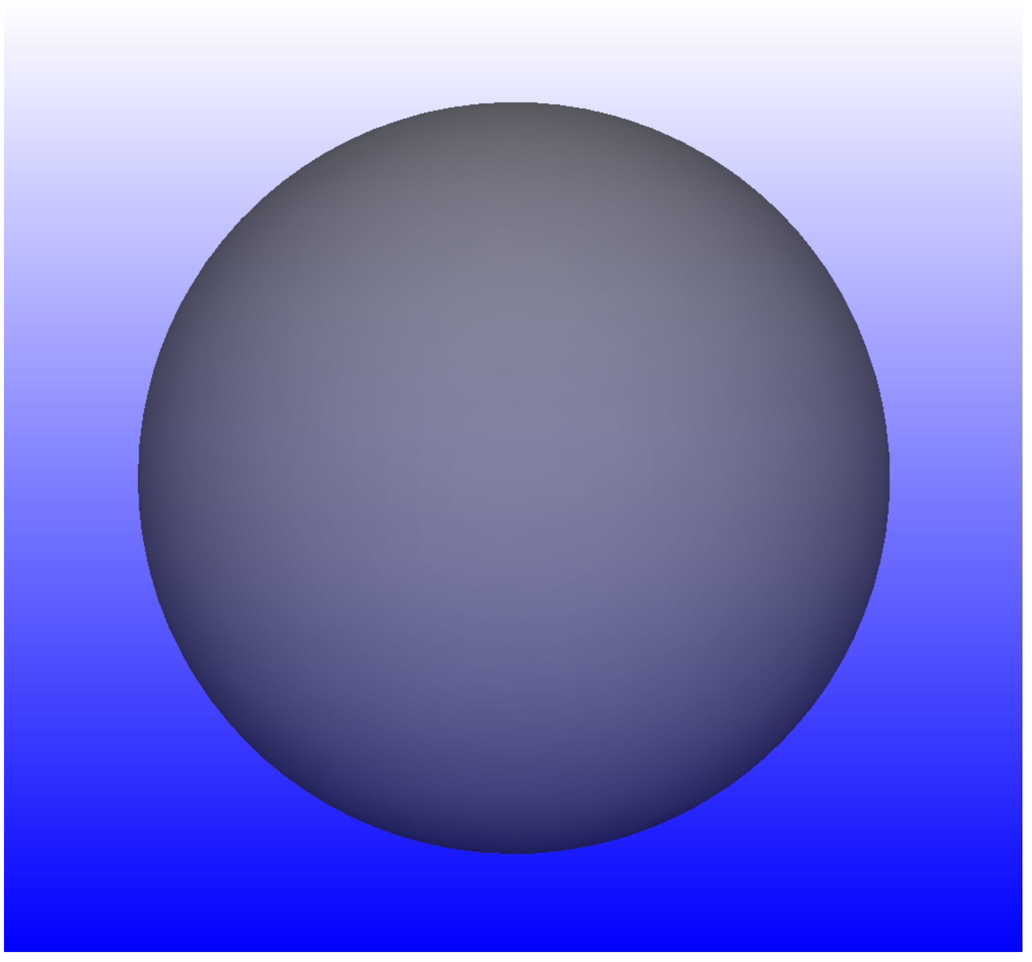

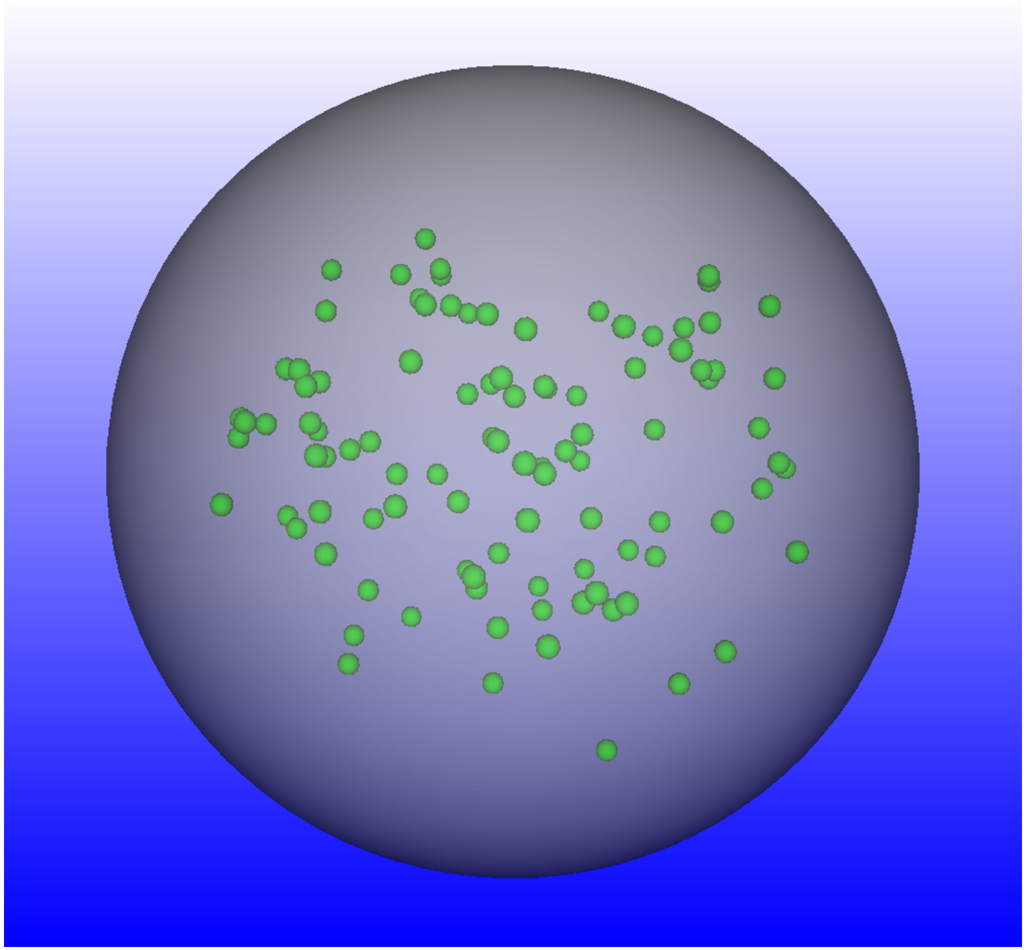

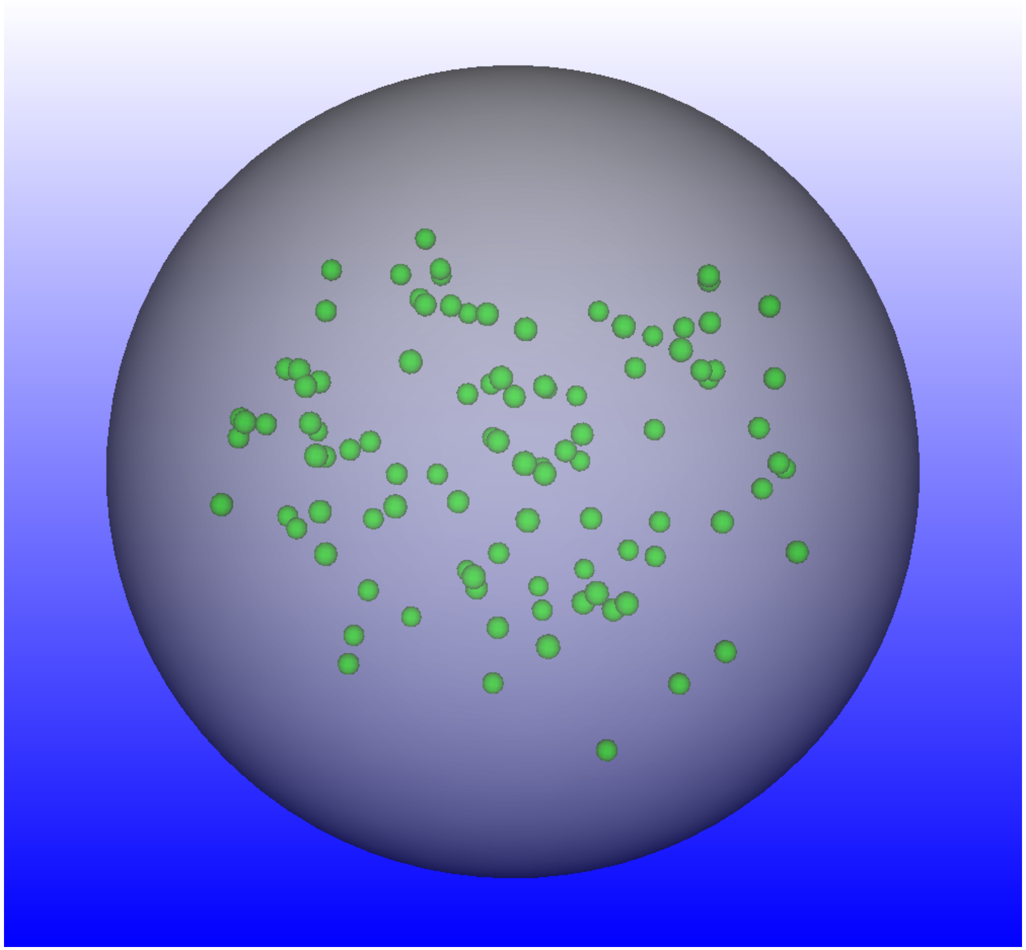

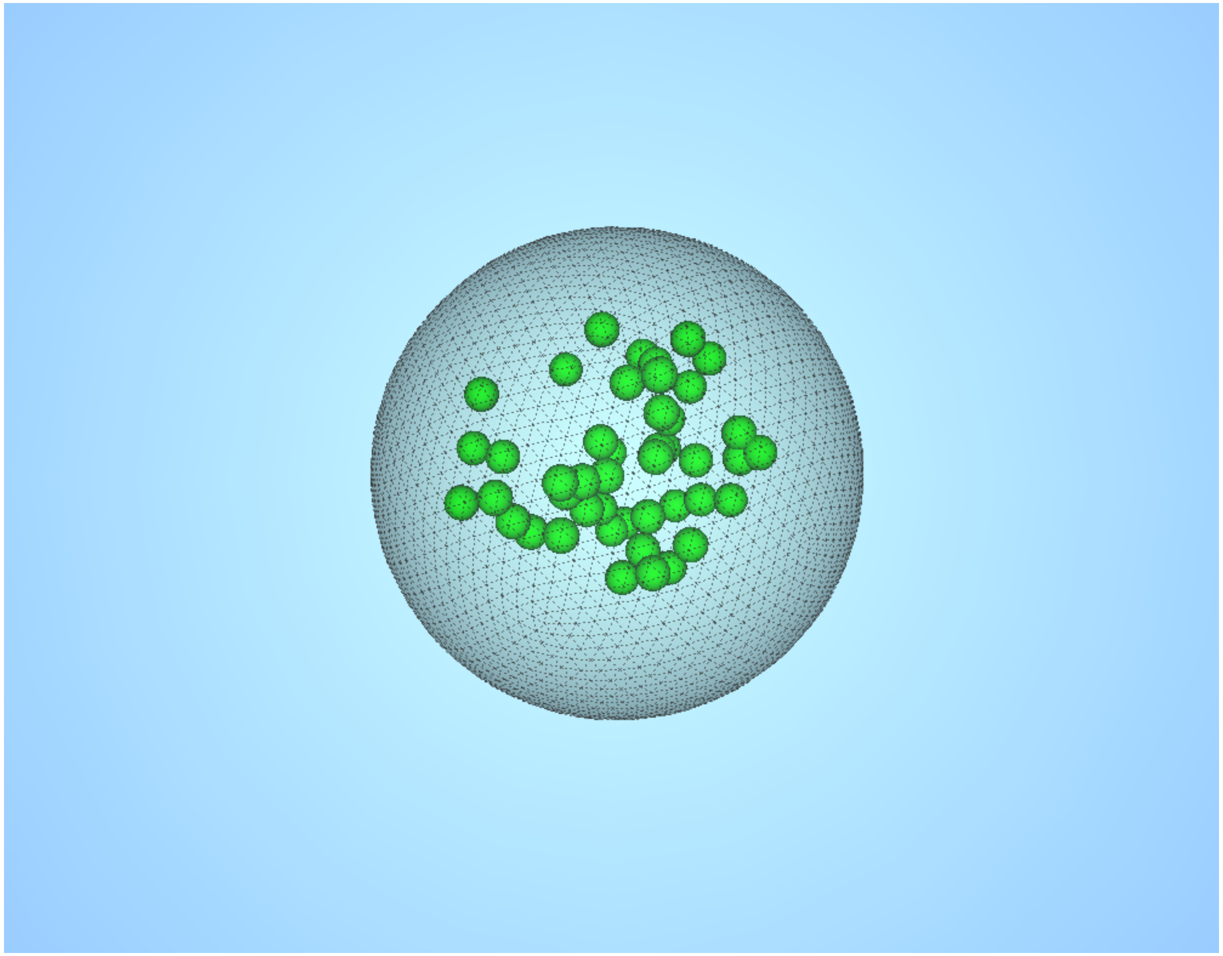

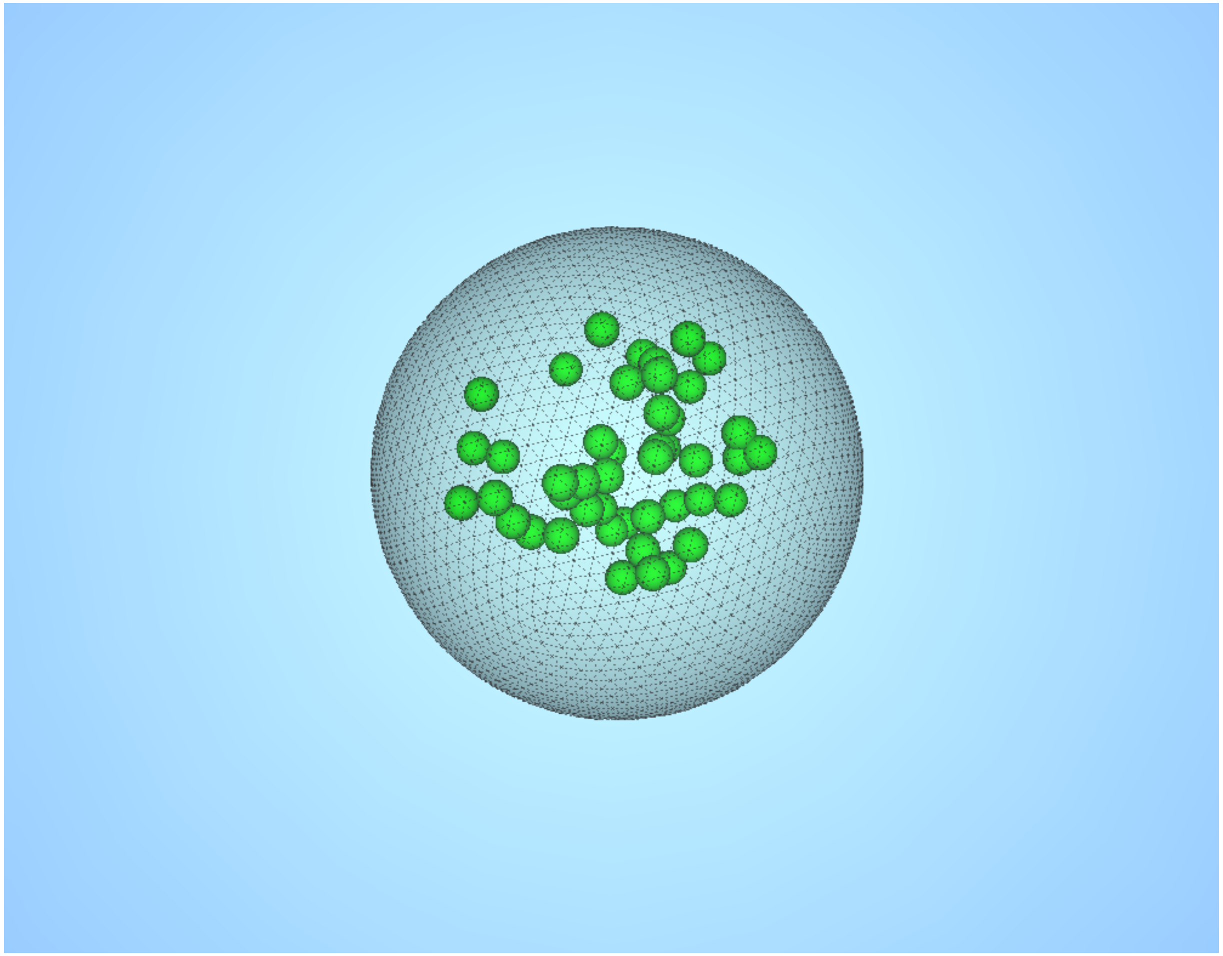

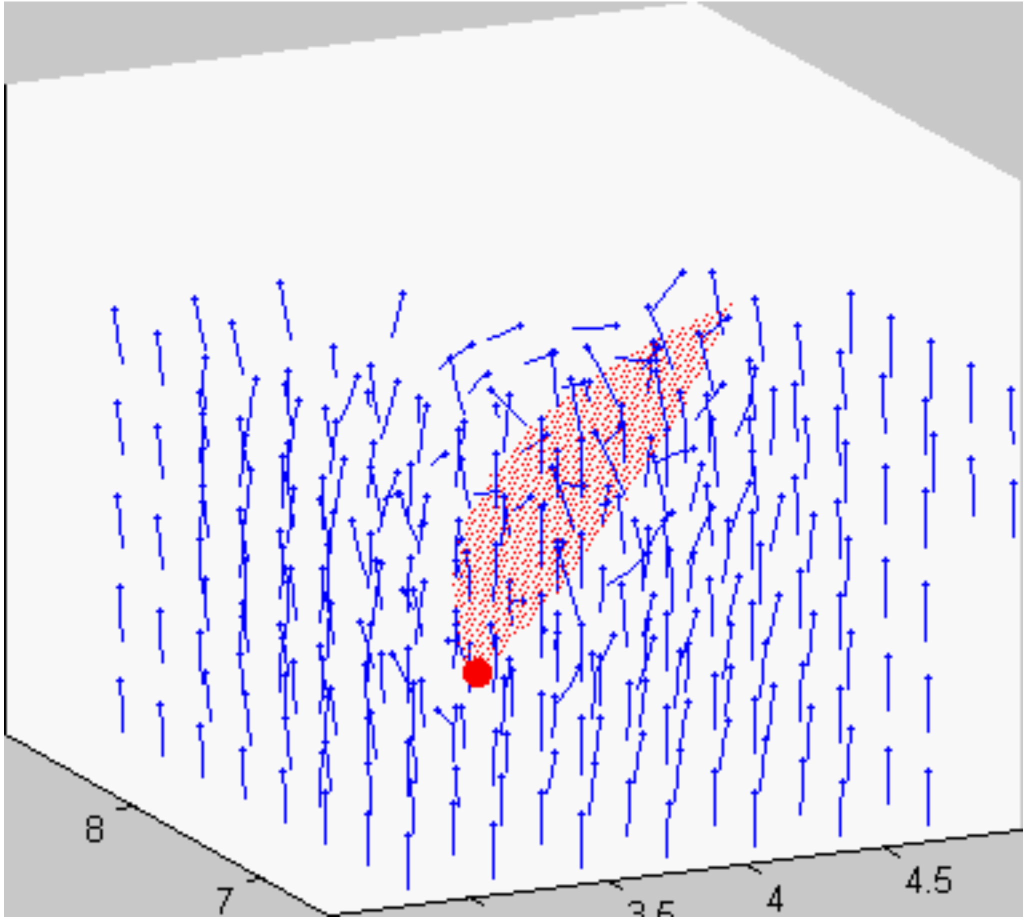

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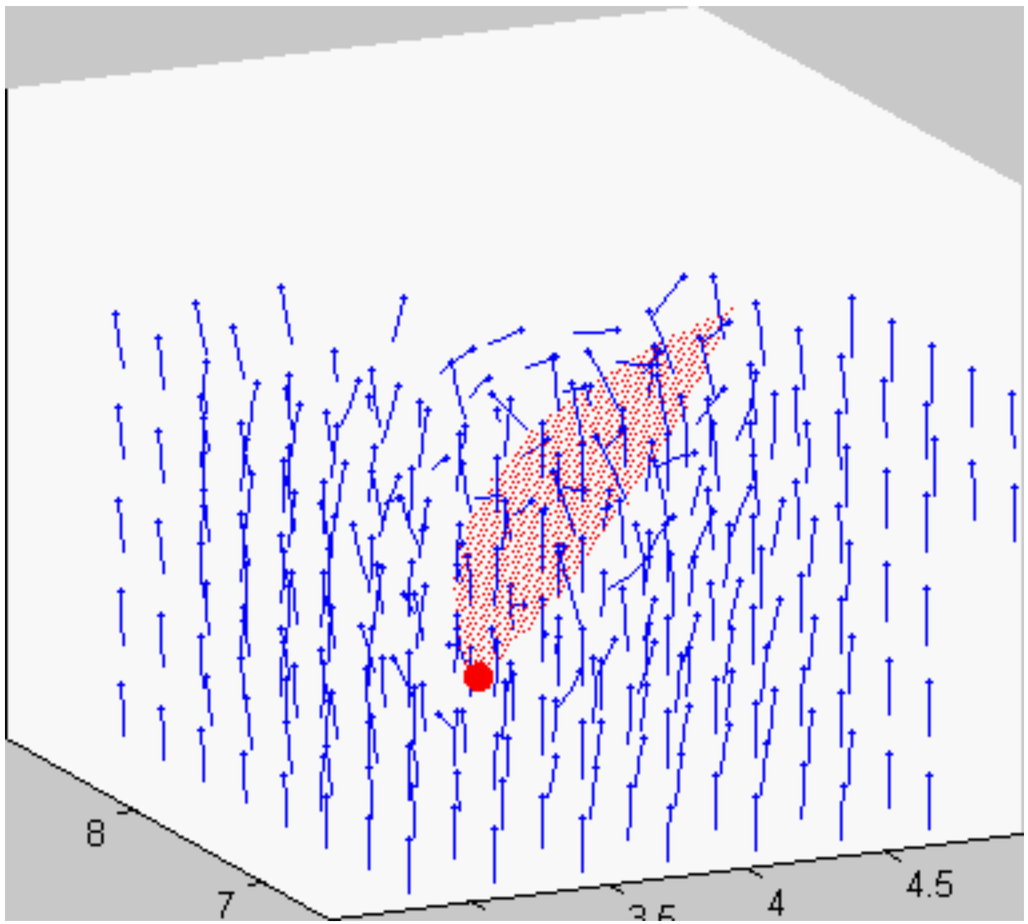
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7
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CREDITS

- HEART – Boyce Griffith & David McQueen
- RED BLOOD CELLS – Thomas Fai*
- TWISTED ROD, FLAGELLA – Sookkyung Lim
- THERMAL FLUCTUATIONS – Paul Atzberger
- OSMOTIC PRESSURE – Chen-Hung Wu
- MAPLE SEED – Yongsam Kim

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