Ice Caps and Ice Belts:
the Effects of **Obliquity** on Ice-Albedo Feedback

Or, sometime there are still **new things to learn** from fully **analytical solutions** of simple models!

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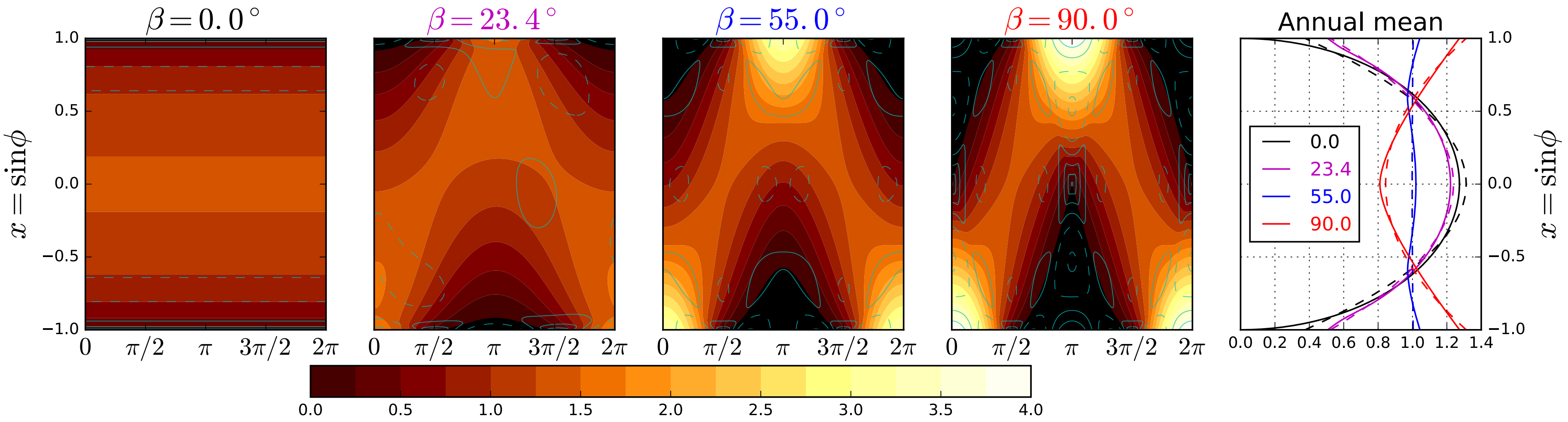
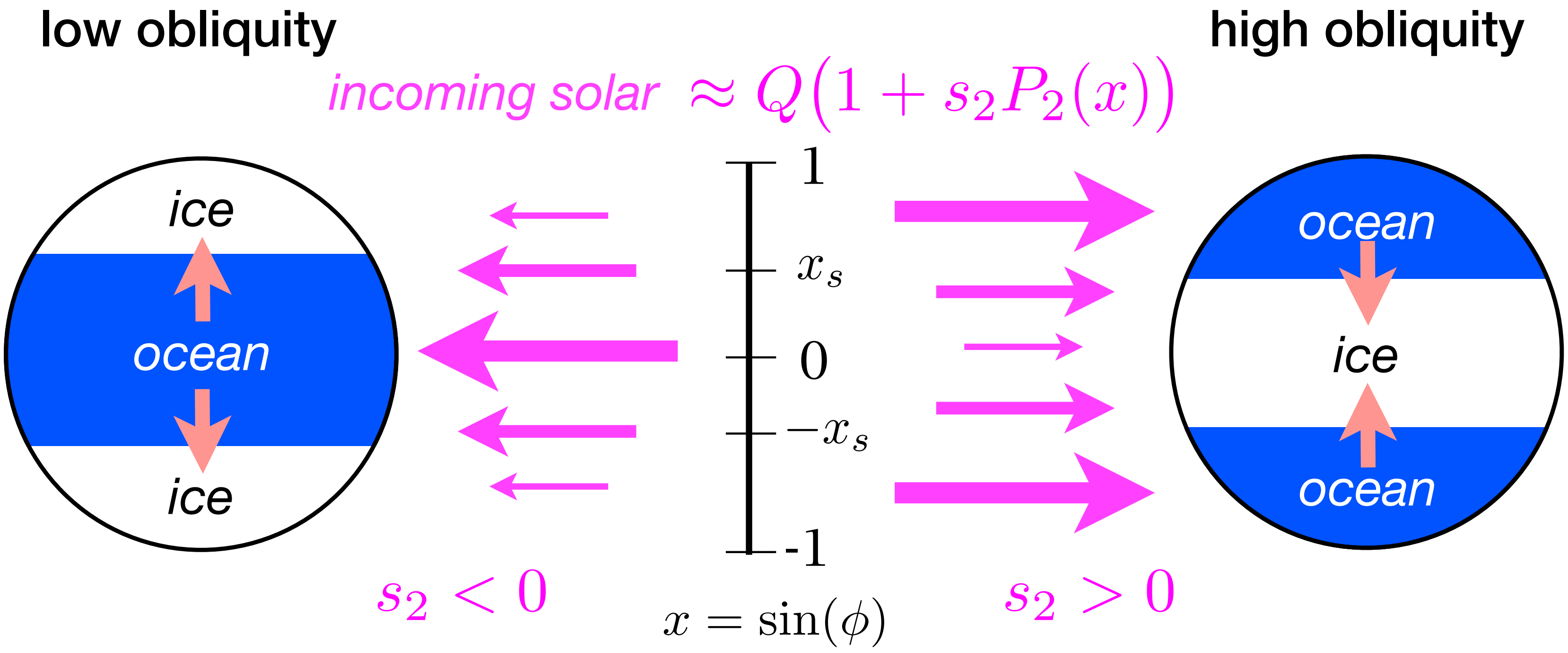
Effects of obliquity on insolation



Ice caps vs. Ice belts: the basic idea



The Energy Balance Model

Budyko (1969), Sellers (1969), North (1975)

$$C\frac{\partial T}{\partial t} = aQs(x,t) - [A + BT] + \frac{K}{R^2}\nabla^2 T$$

Seasonal heat storage

Absorbed solar radiation

Outgoing longwave radiation

Heat transport convergence

All the AOFD is here.

Key assumptions:

- Outgoing radiation parameterized as linear function of surface temperature
- Heat transport is diffusive – heat flows from warm to cold

Series expansion of insolation

$$S(x,t) = Qs(x,t) \qquad x \;=\; \sin\phi \;\; \text{an} \;\; \text{area-weighted} \;\; \text{latitude}$$

$$s(x,t) = \sum_{l=0,\;k=0} \left(a_{lk} \cos k\omega t + b_{lk} \sin k\omega t \right) P_l(x)$$

Truncated series for zero eccentricity (circular orbits)

$$s(x,t) = 1 + s_{11} \cos \omega t P_1(x) + \left(s_{20} + s_{22} \cos 2\omega t \right) P_2(x)$$

Coefficients are all simple functions of obliquity:

$$s_{20} = -\frac{5}{16} \left(2 - 3 \sin^2 \beta \right)$$

$$s_{11} = -2 \sin \beta$$

$$s_{22} = \frac{15}{16} \sin^2 \beta$$

The ice line albedo parameterization

$$a[T(x,t)] = a_{\lrcorner} = \begin{cases} a_0, & T(x,t) > T_0 \\ a_1, & T(x,t) < T_0 \end{cases}$$

The model becomes nonlinear (but still analytically tractable)

Consider the **deep-water** limit *(deep mixed layer and/or short solar year)* → use steady-state **annual mean** model

Non-dimensional form of the annual mean model

To identify minimal number of independent parameters, and explore broad departures from Earth-like conditions

$$\delta\nabla^2 T^* - T^* = -q\left[1 + s_{20}P_2(x)\right]\begin{cases} 1, & T^* > 1 \\ (1-\alpha), & T^* < 1 \end{cases}$$

$$T^*(x) = \frac{A + BT(x)}{A + BT_0}$$

non-dimensional temperature = 1 at the ice line

Four-dimensional parameter space:

$$s_{20} = -\frac{5}{16}\left(2 - 3\sin^2\beta\right)$$ *Insolation gradient (obliquity)*

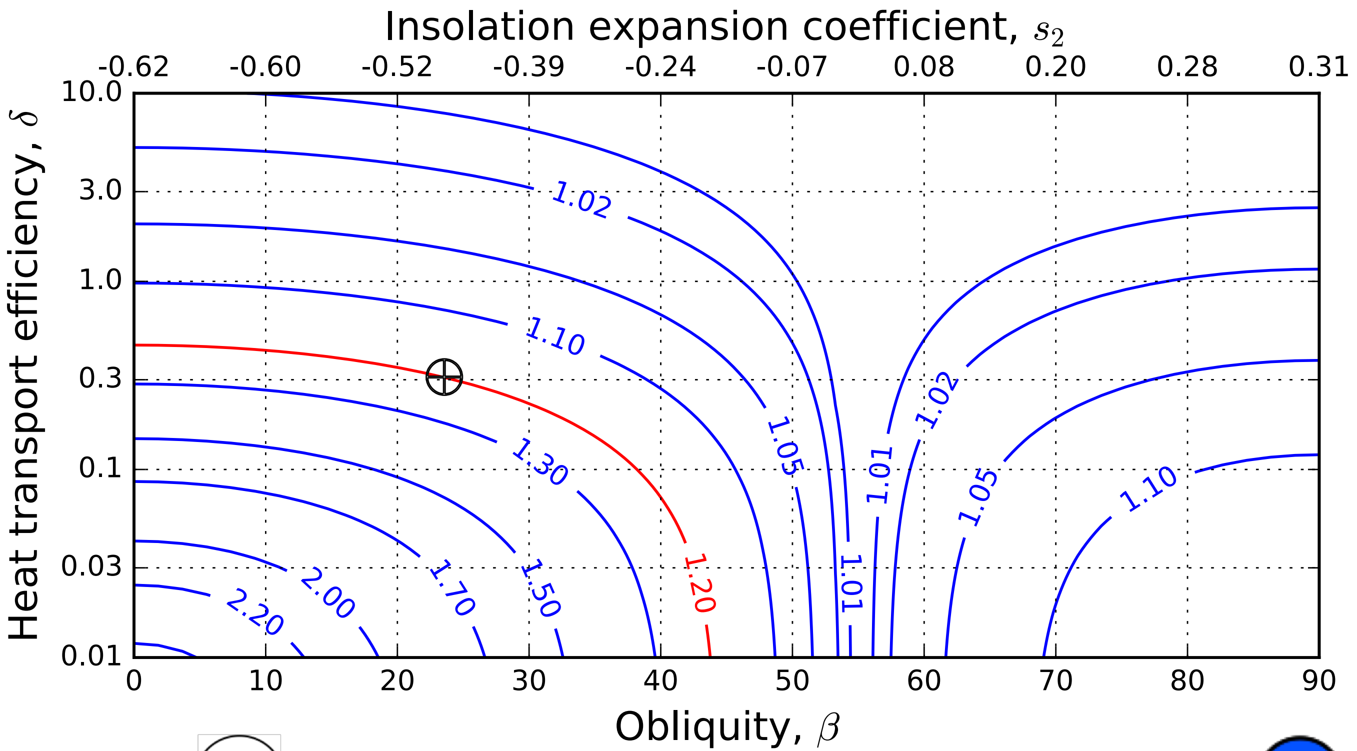
$$\delta = \frac{K}{R^2 B}$$ *efficiency of heat transport*

$$q = \frac{a_0 Q}{A + BT_0}$$ *radiative forcing*

$$\alpha = \frac{a_0 - a_1}{a_0}$$ *albedo contrast*

*We obtain a complete **analytical** solution,*
*extending **North (1975)** to the **high-obliquity** case and arbitrary parameters*

Minimum radiative forcing for an **ice-free planet**



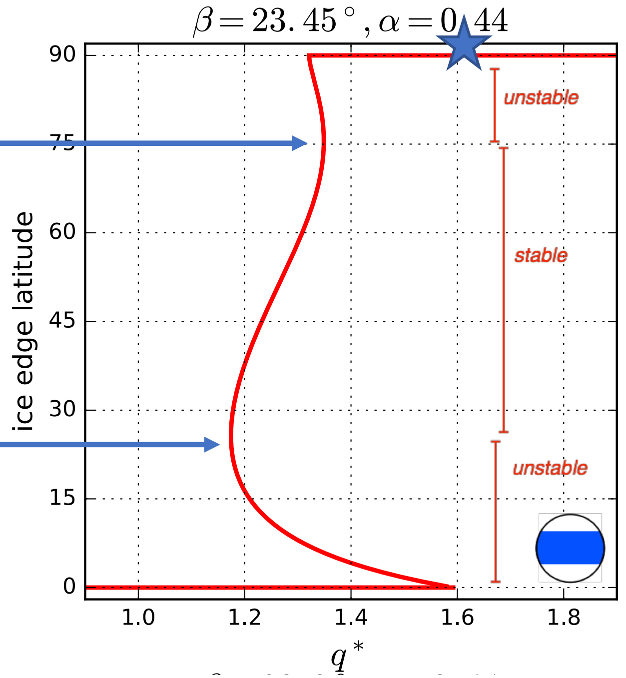
- Contours: minimum *q* to keep coldest regions above freezing
- All else equal, high-obliquity planets are ice-free at weaker insolation
- E.g. Earth at 90º obliquity is ice-free even with 10% reduction in insolation

Stability of ice caps and ice belts (1)

Graph of equilibrium ice edge position vs. radiative forcing (insolation) for one set of (quasi Earth-like) parameters (e.g. North 1975)

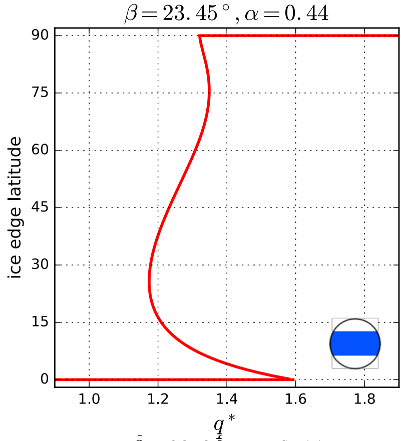
Small Ice Cap Instability (SICI)

Large Ice Cap Instability (LICI)

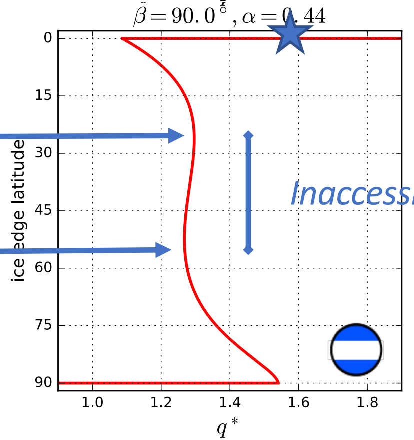


Hysteresis loop with gradual decrease and increase in global radiative forcing

Stability of ice caps and ice belts (2)



The solution for 90º obliquity



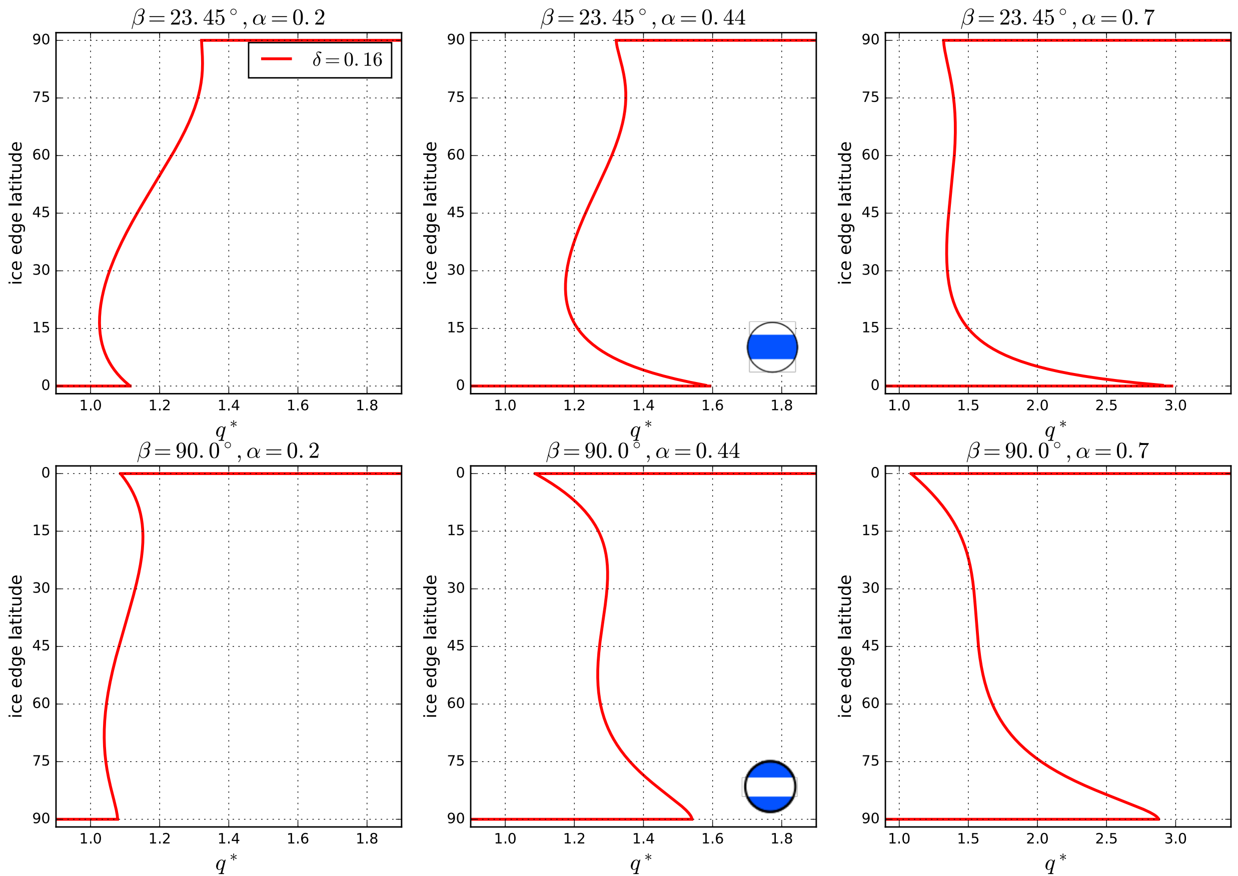
Small Ice Belt Instability (SIBI)

Large Ice Belt Instability (LIBI)

Inaccessible stable ice belt state

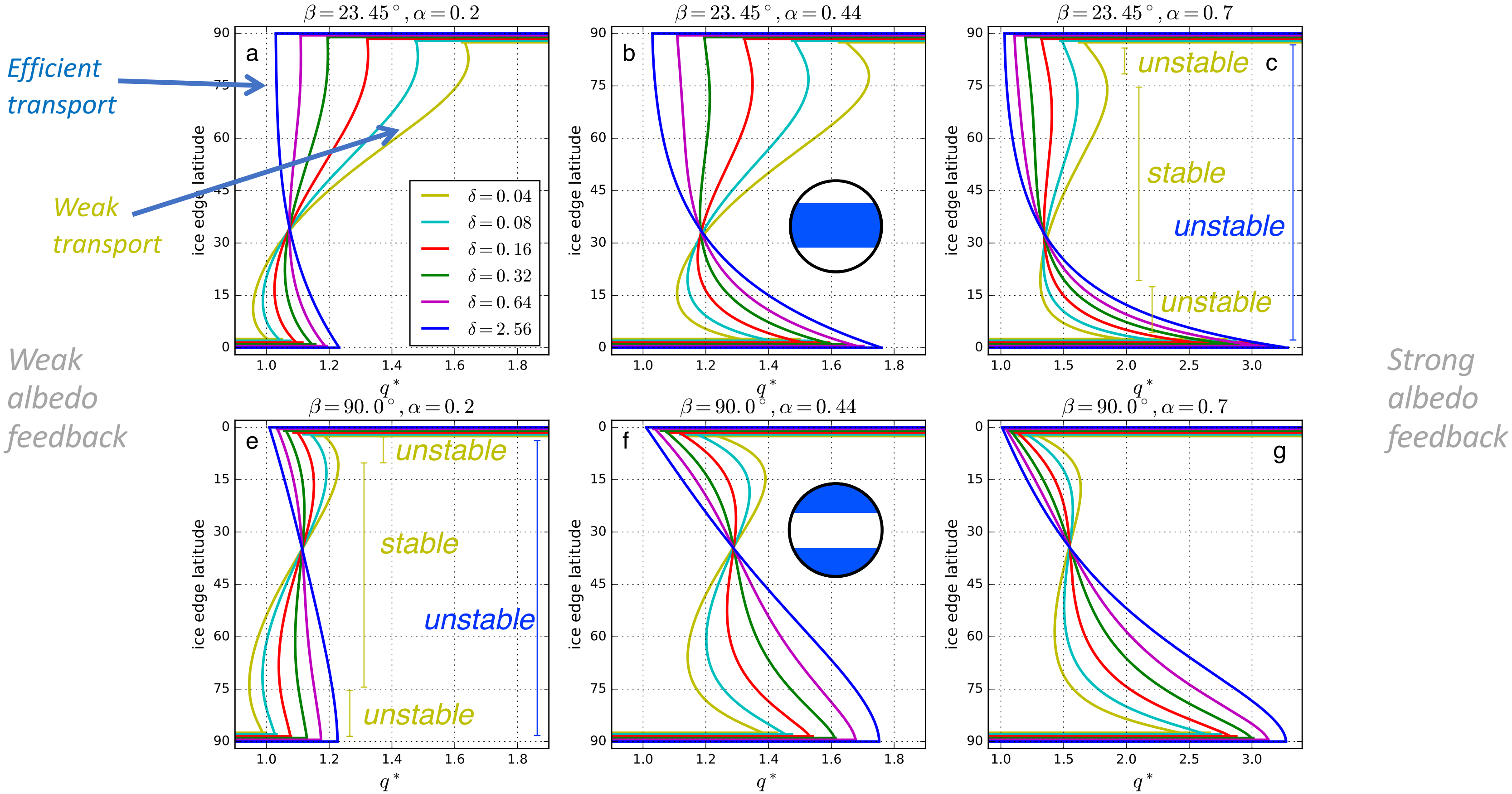
Stability of ice caps and ice belts (3)

Weak albedo feedback

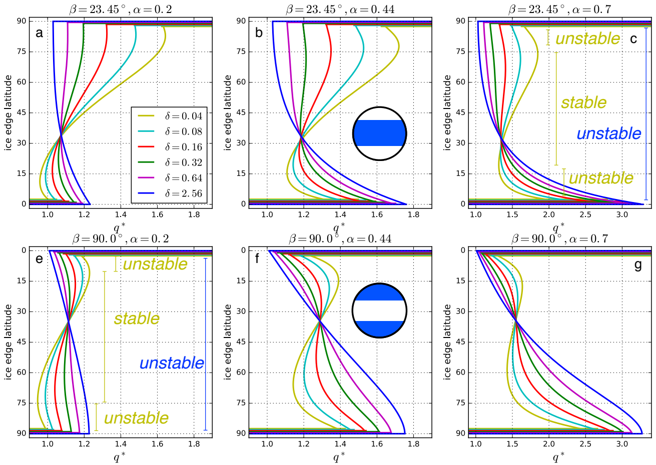


Strong albedo feedback

Stability of ice caps and ice belts (4)



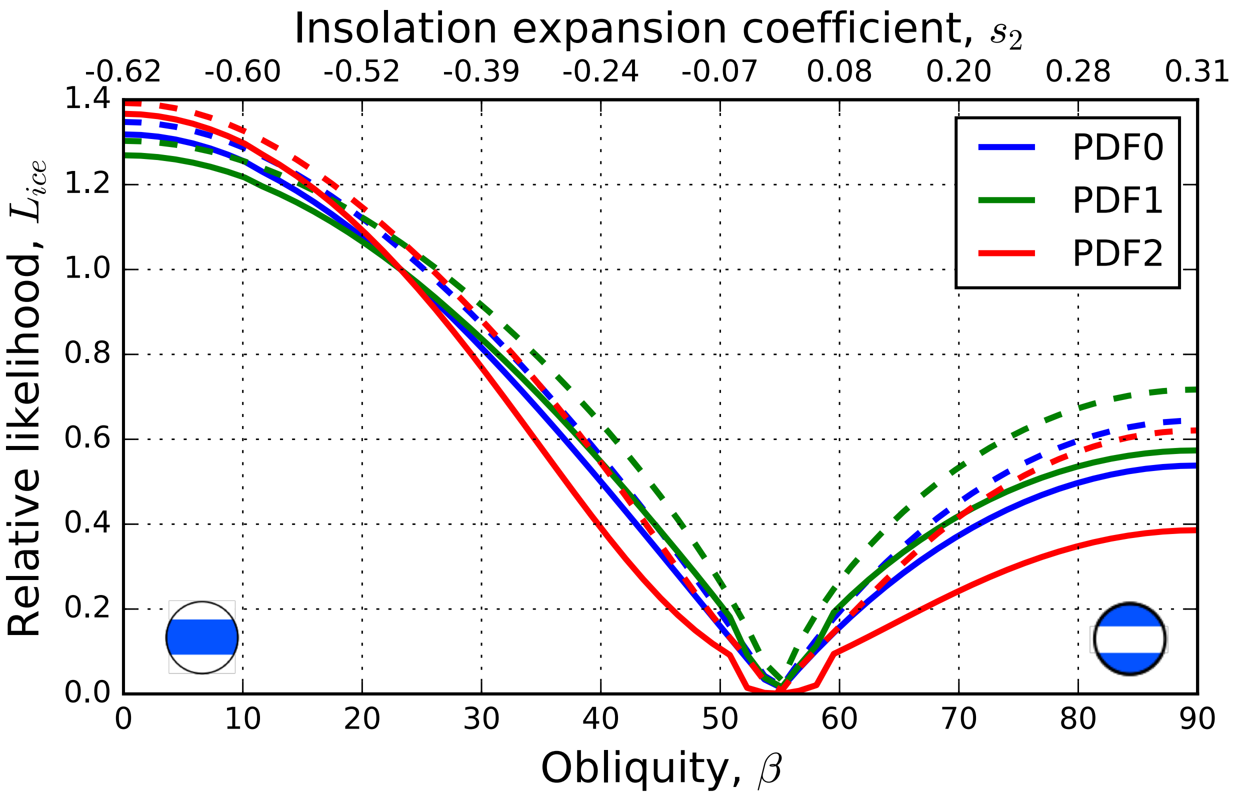
Stability of ice caps and ice belts (summary)



- Stable ice edges are far from universal in the parameter space
- Possible when neither **δ** (transport efficiency) or **α** (albedo contrast) is too large
- Conditions for **stable ice belt** are **more stringent** than for stable ice cap
- Stable ice belt states are frequently inaccessible through a hysteresis in radiative forcing
- In many cases, at high-obliquity the only viable solutions are **ice-free** and **Snowball** climate states

Implication: planets in **stable ice belt** states should be **harder to find** than stable ice caps!

Likelihood of finding stable ice edges (cap or belt) relative to Earth obliquity



- *Make plausible assumptions about PDFs of planetary parameters*
- *Compute probability of* ***stable*** *and* ***accessible*** *partial ice cover states*
- *55º obliquity → isothermal → zero probability*
- *BELTS always less probable than CAPS*
- *~4/5 of all* ***observable partial ice-covered planets*** *should be* ***CAPS****, not* ***BELTS****.*

Next step in the model hierarchy: the **seasonal cycle**

$$\gamma \frac{\partial T^*}{\partial \tau} - \delta \nabla^2 T^* + T^* = qs(x, \tau) \begin{cases} 1, & T^* > 1 \\ 1 - \alpha, & T^* < 1 \end{cases}$$

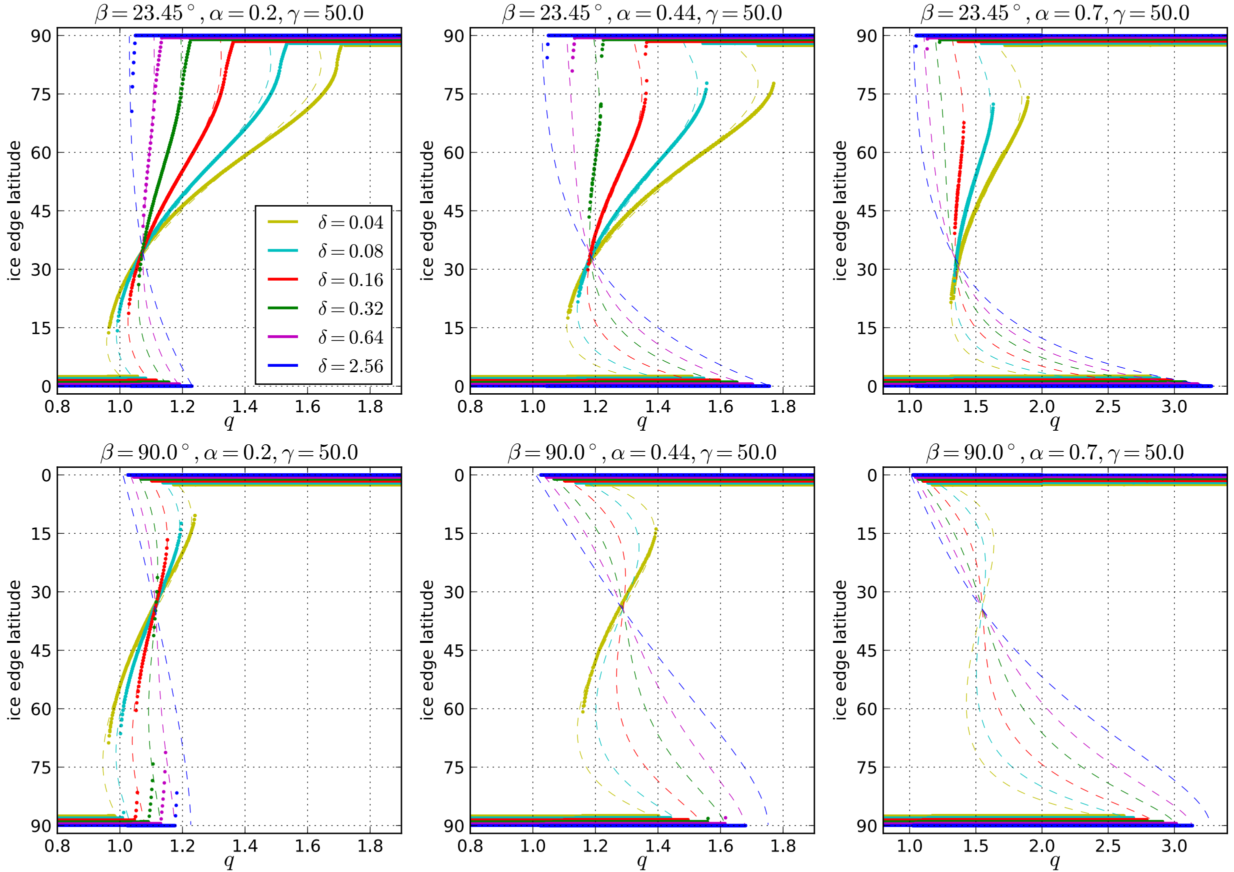
Non-dimensional seasonal model

We solve this numerically using the CLIMLAB software package

Stability of ice caps and ice belts: seasonal cycle

Deep water regime, gamma = 50

(mixed layer depth of 90 m for Earth parameters)



Seasonal EBM is solved numerically out to steady seasonal cycle.

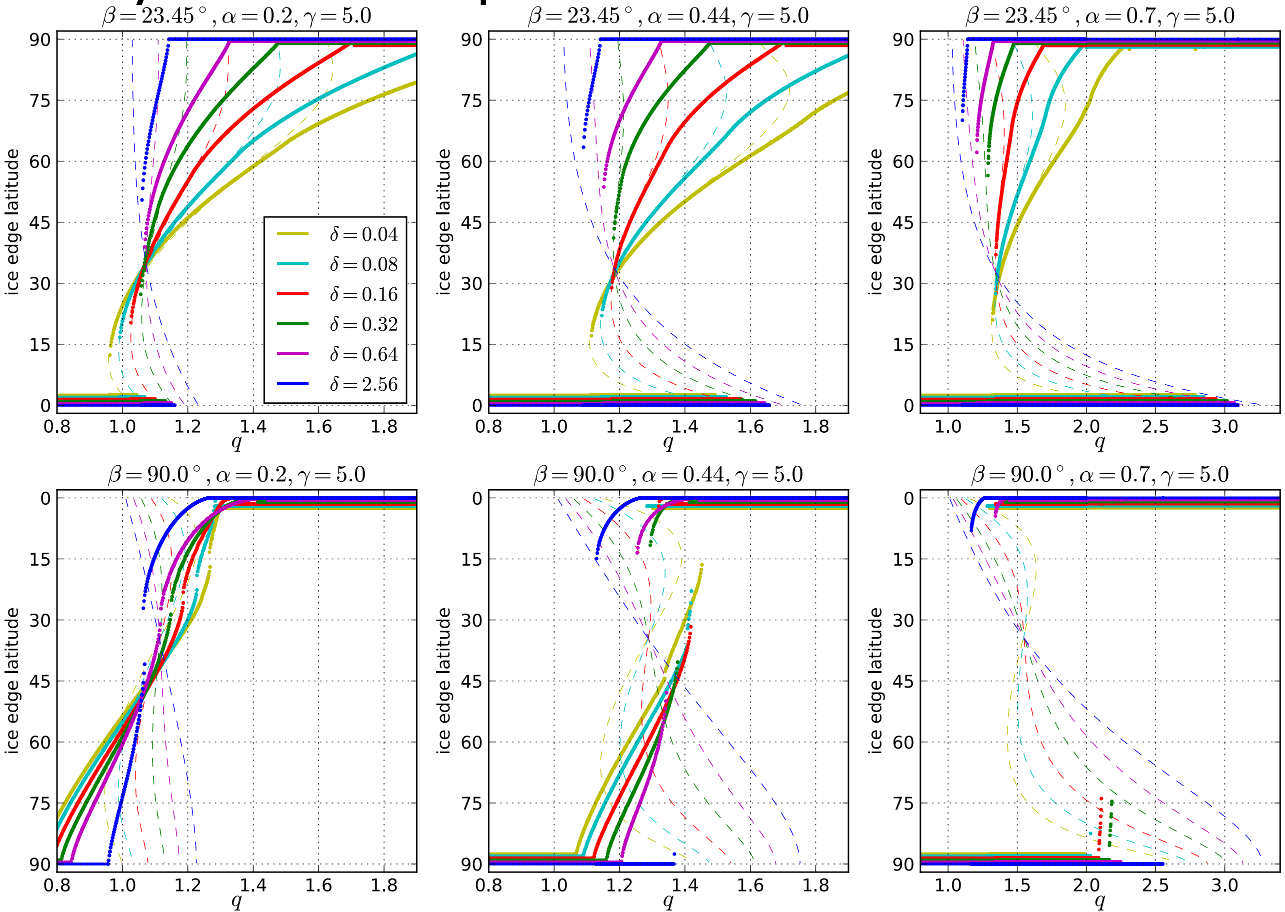
Annual mean stability diagrams generated with a large numerical parameter sweep of the seasonal model.

Results in the deep water regime are very consistent with the analytical annual model.

Stability of ice caps and ice belts: seasonal cycle

Intermediate depth regime, gamma = 5

(about 10 m of water)

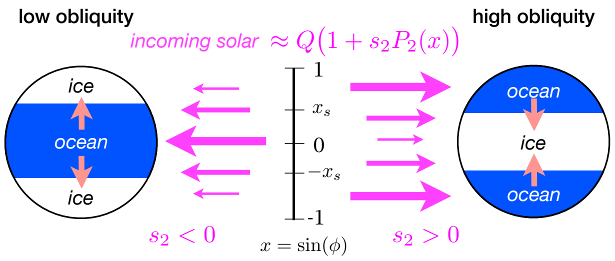


Results are completely different with a strong seasonal cycle!

Low obliquity: no SICI – gradual transition to ice-free

Very little agreement with the annual-mean model at high obliquity

Conclusion



- Four-parameter analytical EBM represents **spherical geometry, meridional heat transport**, and **ice-albedo feedback**, used to study stability of high-obliquity **ice belts** vs. low-obliquity **ice caps**.
- Three types of solution: **ice-free**, **Snowball**, and **partial ice cover** (cap or belt).
- **Multiple equilibria** exist over wide swaths of parameter space at both high and low obliquity.
- **Stable ice belts** are possible but exist over a smaller range of parameters than stable ice caps. Many *potentially stable* ice belt states are also **inaccessible** through any radiative hysteresis.
- Factors that favor **stable caps and belts** include:
  - Weak albedo contrast and weak heat transport efficiency
  - Large insolation gradients (i.e. obliquities not close to the critical value near 55º).
- The **Snowball catastrophe** is avoided in **two rather different ways**:
  1. Weak albedo feedback and inefficient heat transport *(stable cap or belt)*
  2. Efficient heat transport at high obliquity *(ice-free)*
- Results are robust to the seasonal cycle in the deep water limit
  - Role of seasonal ice line migrations in more strongly seasonal regimes needs more work!

Rose, Cronin and Bitz (2017), *The Astrophysical Journal* 846

Excluding inaccessible stable states

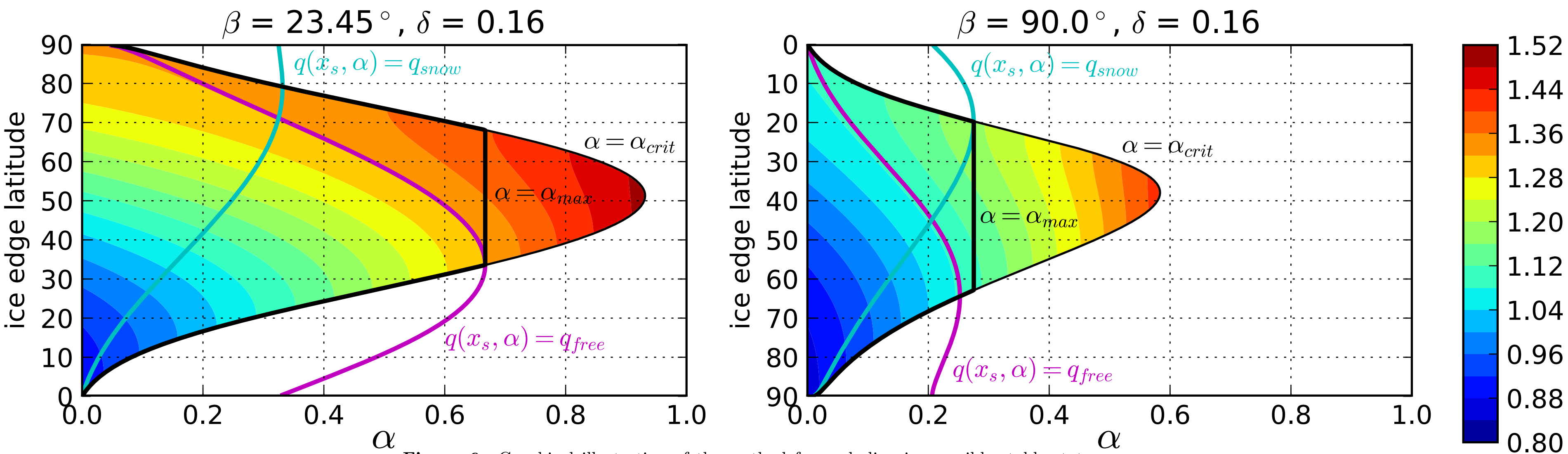


Figure 6. Graphical illustration of the method for excluding inaccessible stable states. In colors we contour $q(x_s, \alpha)$ from (25) for the stable region bounded by α_{crit} from (30). Magenta curve is the implicit solution of $q(x_s, \alpha) = q_{free}$ – the latitude to which the ice edge would jump in an unstable transition from ice-free conditions. Cyan contour is the implicit solution of $q(x_s, \alpha) = q_{snow}$ – the analogous ice edge latitude resulting from unstable transitions from the Snowball state. α_{warm} in (34) is the intersection of the magenta curve with α_{crit}. For $\alpha > \alpha_{warm}$, transitions from ice-free conditions would result directly in a Snowball. Similarly, α_{cold} is the intersection of the cyan curve with α_{crit}, giving the maximum α for which transitions from Snowball to stable ice edge are possible. The thick black contour illustrates α_{max} from (34). Inaccessible stable states lie between α_{max} and α_{crit}.

Planetary habitability and the Snowball transition

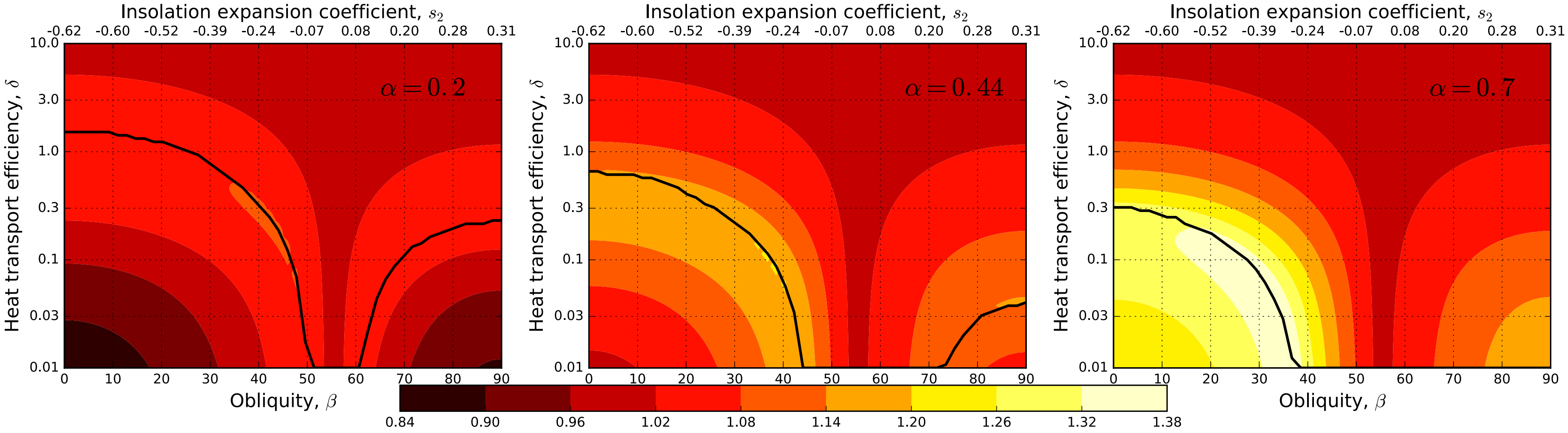
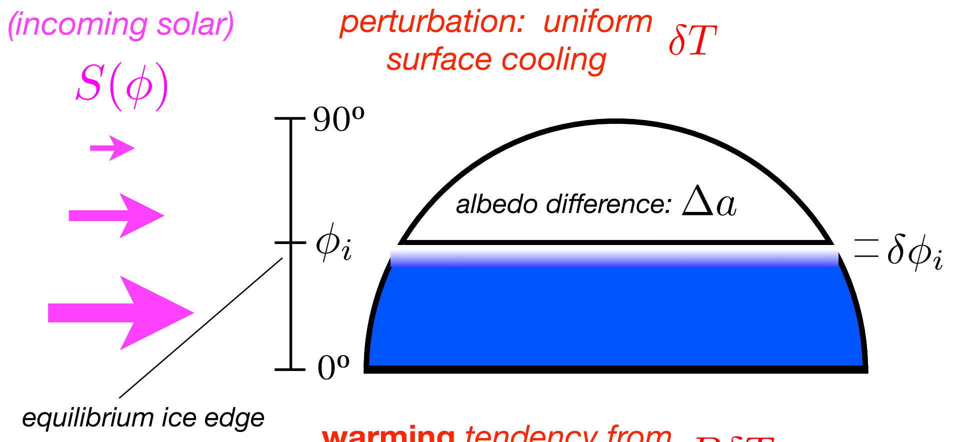


Figure 8. Contour plots of q_{hab}, the minimum q required for habitability (defined as the possibility of a non-Snowball climate). q_{hab}, defined by (36), is contoured for fixed albedo feedback parameter α as a function of obliquity and heat transport efficiency δ. Darker colors indicate smaller q_{hab}, i.e. a more habitable planet. The black contours indicate values of δ above which $q_{hab} = q_{free}$, i.e. the outer boundary of the habitable zone is an ice-free climate. For δ below this line, the outer boundary of the habitable zone is a partially ice-covered planet.

Geometrical basics of the Snowball Earth / runaway glaciation problem

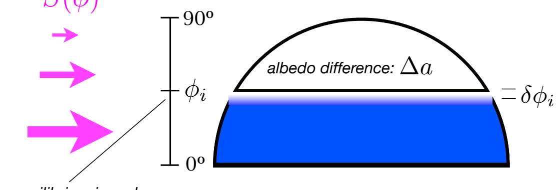


warming *tendency from anomalously low OLR:* $B\delta T$

cooling *tendency from increased albedo:* $\Delta a S(\phi_i) \cos\phi_i \delta\phi$

stable *if cooling < warming*

Large ice cap instability | A geometrical argument



warming *tendency from anomalously low OLR:* $B\delta T$

cooling *tendency from increased albedo:* $\Delta a S(\phi_i)\cos\phi_i \delta\phi$

stable *if cooling < warming*

for small perturbations:

$$\delta T = -\left.\frac{dT}{d\phi}\right|_{\phi_i} \delta\phi$$

ice edge is ***stable*** *if*

$$\frac{\Delta a S(\phi_i)\cos\phi_i}{B} < -\left.\frac{dT}{d\phi}\right|_{\phi_i}$$

Large at equator

Large in mid-lats, ~zero at equator

Ice edge must become unstable equatorward of some critical latitude

Large ice cap instability

A geometrical argument

History of the high obliquity / ice belt problem

- Williams (1975) put forward the high-obliquity hypothesis for early Earth, possible explanation for Neoproterozoic low-latitude glaciation
- Prompted a number of modeling studies (e.g., Hunt 1982; Oglesby & Ogg 1999; Chandler & Sohl 2000; Jenkins 2000, 2001, 2003; Donnadieu et al. 2002) – usually some form of atmospheric GCM, mixed-layer ocean, thermodynamic ice model
- More recently: high-obliquity exoplanets! (Williams & Kasting 1997; Williams & Pollard 2003; Spiegel et al. 2009; Abe et al. 2011; Armstrong et al. 2014; Ferreira et al. 2014; Wang et al. 2016)
- Many of these studies explicitly looked for ice belt states but did not find them!   WHY?

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Climate at high-obliquity

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A study with a fully coupled 3D atmosphere-ocean-sea ice GCM at 90º obliquity

Transition directly from ice-free to Snowball state

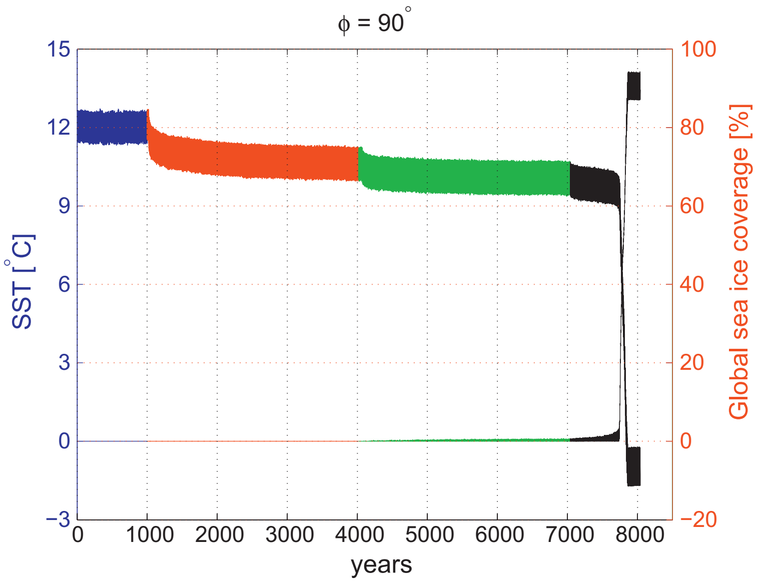

Fig. 12. SST (in °C, upper curve, left axis) and fraction of the globe covered with sea ice (in %, lower curve, right axis) in Aqua90 as the solar constant $S_o/4$ is decreased from 341.5 (blue) to 339.5 (red), 338.5 (green) and 338.0 (black) W m^{-2}.

In the spirit of model hierarchies…

- Let’s use a minimal climate/ albedo feedback model to compare low and high obliquity
- With a simple model, sample a wide range of different planetary characteristics