Advances in Reduced Order Methods for Computational Fluid Dynamics and Fluid-Structure Interaction

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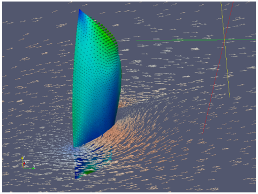
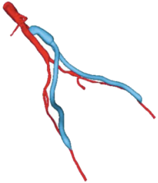
Leading Motivation: Computational Sciences challenges

- **Simulation-based sciences** is a quickly emerging field for mathematics and computational modelling.
- Present and future efforts: towards **multiphysics** problems, as well as systems characterized by **multiple spatial and temporal scales**.
- Growing demand of
  - **efficient computational tools** for
  - **many query** and **real time** computations,
  - **parametrized formulations**,
  - simulations of increasingly **complex systems** with uncertain scenarios,

  by **industrial and clinical** research partners.
- The need of a computational collaboration rather than a competition between **High Performance Computing** (HPC) and **Reduced Order Methods** (ROM), as well as Full/High Order and Reduced Order Methods.





Overview: our current efforts, aims and perspectives

A team developing **Advanced Reduced Order Modelling** techniques with special focus on **Computational Fluid Dynamics**

Overview: our current efforts, aims and perspectives

A team developing **Advanced Reduced Order Modelling** techniques with special focus on **Computational Fluid Dynamics**:

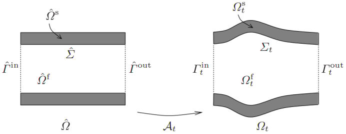
- to face and overcome **several limitations** of the **state of the art** for ROM in CFD;
- to improve capabilities of reduced order methodologies for **more demanding applications** in **industrial, medical and applied sciences settings**;
- to carry out important methodological developments in **Numerical Analysis**, with special emphasis on mathematical modelling and a more extensive exploitation of **Computational Science and Engineering**;
- focus on **Computational Fluid Dynamics** as a central topic to enhance broader applications in **multiphysics** and **coupled settings**, as well as more realistic models (e.g. aeronautical, mechanical, naval, off-shore, wind, sport, biomedical engineering and also cardiovascular surgery planning).



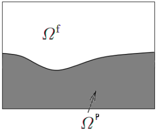
Current efforts

- Efficient management of (physical and numerical) interfaces and subdomains in a ROM setting:
  - physical interfaces:

FSI

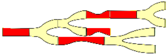


Stokes-Darcy [Martini, Haasdonk, R., 2015]



  - non-physical (computational) interfaces:

networks



domain decomposition

(including same physics with different mathematical models, e.g. viscous-potential coupling).

- Aim:
  - accurate coupling of physics,
  - keep low number of parameters,
  - dealing with moving boundaries, interfaces and domains.

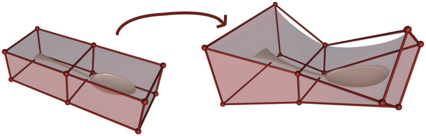
Shape parametrization for ROM

- need to combine solutions defined on different domains because (i) domain is moving and also (ii) initial configuration is parametrized,
- definition of a map

$$\Omega_o(\boldsymbol{\mu}) = \boldsymbol{T}(\Omega; \boldsymbol{\mu})$$

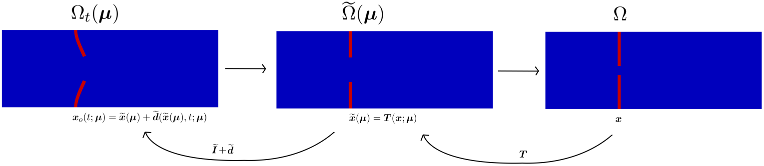
Shape parameterization for ROM

- Free-Form Deformations (FFD) [Lassila, Rozza, CMAME, 2010], [Salmoiraghi *et al.*, AMSES, 2016].
- Radial Basis Functions (RBF) [Manzoni *et al.*, IJNMBE, 2011].
- Transfinite Mapping (TM) [Løvgren, Maday, Rønquist, 2006], [Iapichino *et al.*, CMAME, 2012].
- Vascular shape parametrization [Ballarin *et al.*, JCP, 2016].
- Reduced inverse Distance Weighting [ongoing 2016].



#FSI

Monolithic ROMs for FSI problems
Joint work with Francesco Ballarin.



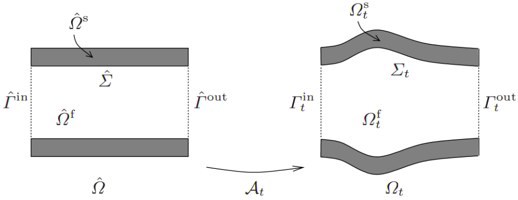
Formulation of FSI problems

- Fluid variables: $(\boldsymbol{u}_f, p, \boldsymbol{d}_f)$,
- Structure variables: $(\boldsymbol{u}_s, \boldsymbol{d}_s)$,
- Fluid-structure interaction problem three-fields formulation:

$$\begin{cases} F(\boldsymbol{u}_f, p, \boldsymbol{d}_f; \boldsymbol{d}_s) = 0, & \text{Fluid} \\ S(\boldsymbol{u}_s, \boldsymbol{d}_s) = 0, & \text{Structure} \\ I(\boldsymbol{d}_f, \boldsymbol{d}_s) = 0, & \text{Interface} \end{cases}$$

subject to interface (coupling) conditions

$$\begin{cases} \boldsymbol{d}_s - \boldsymbol{d}_f = 0 & \text{on } \Sigma, & \text{geometric continuity} \\ \boldsymbol{u}_s - \boldsymbol{u}_f = 0 & \text{on } \Sigma, & \text{velocity continuity} \\ \sigma_f \cdot n_f + \sigma_s \cdot n_s = 0 & \text{on } \Sigma, & \text{balance of normal forces.} \end{cases}$$



Previous reduced order approaches to FSI problems

- [Lassila *et al.*, 2012] and [Lassila *et al.*, 2013] (1D structural model, parametric interface coupling to RB fluid problem Stokes/Navier-Stokes, respectively, axialsymmetry),
- [Colciago, Ph.D. thesis, 2014] (fixed domain and thin-walled structure, RB for fluid problem with generalized Robin boundary conditions),
- [Bertagna, Veneziani, 2014] (1D structural model, POD–Galerkin),
- [Forti, Rozza, 2014] (efficient geometrical parametrization of interfaces, modal greedy),
- [Lieu *et al.*, 2006], ..., [Amsallem *et al.*, 2013], [Amsallem *et al.*, 2015] (aeroelasticity),

Our approach:

- no simplifications for structural model,
- POD–Galerkin method for **global** variables $\boldsymbol{u}, p, \boldsymbol{d}$ (monolithic approach), time dependent,
- capability to parametrize the initial configuration (geometry).

Reduced order monolithic formulation of FSI problems

Truth Finite Element discretization (P2-P1 Taylor-Hood)

For $\boldsymbol{\mu} \in \mathcal{D}$, solve — large $\mathcal{N}$

$$F^{\mathcal{N}}(\boldsymbol{u}_f^{\mathcal{N}}(\mu), p^{\mathcal{N}}(\mu), \boldsymbol{d}_f^{\mathcal{N}}(\mu); \boldsymbol{d}_s^{\mathcal{N}}(\mu); \mu) = 0 \quad \text{Fluid}$$

$$S^{\mathcal{N}}(\boldsymbol{u}_s^{\mathcal{N}}(\mu), \boldsymbol{d}_s^{\mathcal{N}}(\mu); \mu) = 0 \quad \text{Structure}$$

$$I^{\mathcal{N}}(\boldsymbol{d}_f^{\mathcal{N}}(\mu), \boldsymbol{d}_s^{\mathcal{N}}(\mu); \mu) = 0 \quad \text{Interface}$$

coupling conditions

OFFLINE – Space construction and matrices assembling

- Space construction by **Proper Orthogonal Decomposition** for **global** variables.
- Additional computations related to inf-sup stabilization procedure by means of supremizer enrichment → accurate pressure recovery for balance of normal forces. [Ballarin et al., 2015], [Rozza *et al.*, 2012], [Rozza, Veroy, 2007].

ONLINE – Galerkin projection over the enriched space

For $\boldsymbol{\mu} \in \mathcal{D}$, solve — $\mathbf{N} \ll \mathcal{N}$

$$F^{\mathbf{N}}(\boldsymbol{u}_f^{\mathbf{N}}(\mu), p^{\mathbf{N}}(\mu), \boldsymbol{d}_f^{\mathbf{N}}(\mu); \boldsymbol{d}_s^{\mathbf{N}}(\mu); \mu) = 0 \quad \textbf{Reduced}\text{ fluid}$$

$$S^{\mathbf{N}}(\boldsymbol{u}_s^{\mathbf{N}}(\mu), \boldsymbol{d}_s^{\mathbf{N}}(\mu); \mu) = 0 \quad \textbf{Reduced}\text{ structure}$$

$$I^{\mathbf{N}}(\boldsymbol{d}_f^{\mathbf{N}}(\mu), \boldsymbol{d}_s^{\mathbf{N}}(\mu); \mu) = 0 \quad \textbf{Reduced}\text{ interface}$$

coupling conditions

POD-Galerkin ROM for parametrized (Navier)-Stokes problems

Truth Finite Element discretization

For $\boldsymbol{\mu} \in \mathcal{D}$, solve

$$\begin{cases} a(\boldsymbol{u}_f^{\mathcal{N}}(\boldsymbol{\mu}), \boldsymbol{w}^{\mathcal{N}}; \boldsymbol{\mu}) + \\ \qquad c(\boldsymbol{u}_f^{\mathcal{N}}(\boldsymbol{\mu}), \boldsymbol{u}_f^{\mathcal{N}}(\boldsymbol{\mu}; \boldsymbol{\mu}), \boldsymbol{w}^{\mathcal{N}}) + b(p^{\mathcal{N}}(\boldsymbol{\mu}), \boldsymbol{w}^{\mathcal{N}}; \boldsymbol{\mu}) = F(\boldsymbol{w}^{\mathcal{N}}; \boldsymbol{\mu}) \qquad \forall \boldsymbol{w}^{\mathcal{N}} \in V_{\mathcal{N}} \\ b(\tau^{\mathcal{N}}, \boldsymbol{u}_f^{\mathcal{N}}(\boldsymbol{\mu}); \boldsymbol{\mu}) = 0 \qquad \forall \tau^{\mathcal{N}} \in Q_{\mathcal{N}} \end{cases}$$

OFFLINE – Space construction and matrices assembling

- Space construction by **Proper Orthogonal Decomposition**.
- Parameter independent matrices assembling under the assumption of affine parameter dependence (recovered, in general, through EIM).
- Additional computations to setup stabilization procedures.

ONLINE – Galerkin projection

For $\boldsymbol{\mu} \in \mathcal{D}$, solve $\qquad \mathbf{N} \ll \dim(V_{\mathcal{N}}) + \dim(Q_{\mathcal{N}})$

$$\begin{cases} a(\boldsymbol{u}_f^{\mathbf{N}}(\boldsymbol{\mu}), \boldsymbol{w}^{\mathbf{N}}; \boldsymbol{\mu}) + \\ \qquad c(\boldsymbol{u}_f^{\mathbf{N}}(\boldsymbol{\mu}), \boldsymbol{u}_f^{\mathbf{N}}(\boldsymbol{\mu}), \boldsymbol{w}^{\mathbf{N}}; \boldsymbol{\mu}) + b(p^{\mathbf{N}}(\boldsymbol{\mu}), \boldsymbol{w}^{\mathbf{N}}; \boldsymbol{\mu}) = F(\boldsymbol{w}^{\mathbf{N}}; \boldsymbol{\mu}) \qquad \forall \boldsymbol{w}^{\mathbf{N}} \in V_{\mathbf{N}} \\ b(\tau^{\mathbf{N}}, \boldsymbol{u}_f^{\mathbf{N}}(\boldsymbol{\mu}); \boldsymbol{\mu}) = 0, \qquad \forall \tau^{\mathbf{N}} \in Q_{\mathbf{N}} \end{cases}$$

Reduced order monolithic formulation of FSI problems

OFFLINE – Space construction

- Reduced pressure space:
$$Q_N = POD(\{p^{\mathcal{N}}(\boldsymbol{\mu}^i)\}_{i=1}^{N_{train}}; N),$$
- Reduced velocity space:
$$V_N = POD(\{\boldsymbol{u}_f^{\mathcal{N}}(\boldsymbol{\mu}^i)\}_{i=1}^{N_{train}}; N) \quad \oplus \quad POD(\{T^{\boldsymbol{\mu}^i} p^{\mathcal{N}}(\boldsymbol{\mu}^i)\}_{i=1}^{N_{train}}; N),$$

where $T^{\boldsymbol{\mu}} : Q_{\mathcal{N}} \to V_{\mathcal{N}}$ is the **supremizer operator** given by

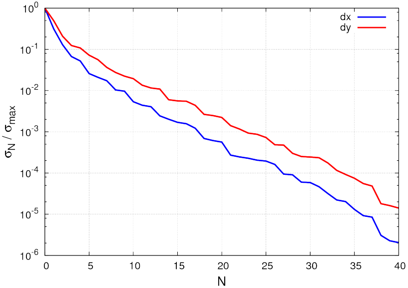
$$(T^{\boldsymbol{\mu}} q, \boldsymbol{w})_X = b(q, \boldsymbol{w}; \boldsymbol{\mu}), \quad \forall \boldsymbol{w} \in V_{\mathcal{N}}.$$

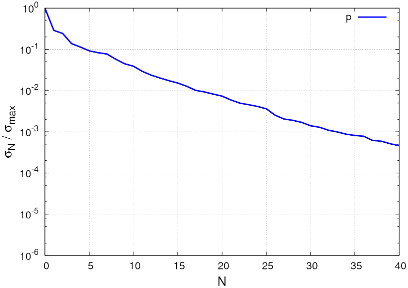
Online *inf-sup* condition

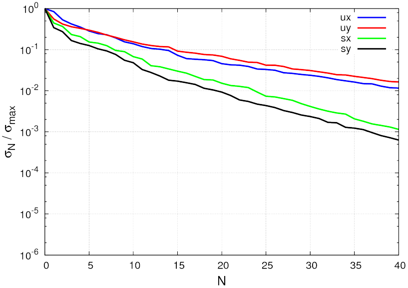
$$\inf_{q_N \in Q_N} \sup_{\boldsymbol{v}_N \in V_N} \frac{b(q_N, \boldsymbol{v}_N; \boldsymbol{\mu})}{\|\boldsymbol{v}_N\|_V \|q_N\|_Q} =: \beta_N(\boldsymbol{\mu}) \geq \beta_{\mathcal{N}}(\boldsymbol{\mu}) > 0$$

[Ballarin et al., 2015], [Rozza *et al.*, 2012], [Rozza, Veroy, 2007].

Reduced order monolithic formulation of FSI problems: results







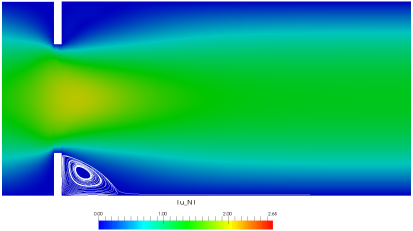
POD singular values for (global) displacement, pressure, velocity and supremizers.

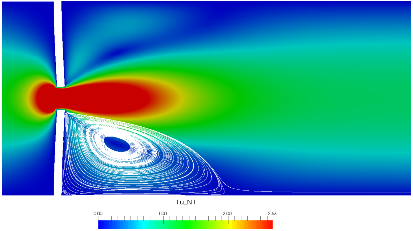
Fastest decay: displacement (top left).

Slower decay for velocity and supremizers modes (bottom left).

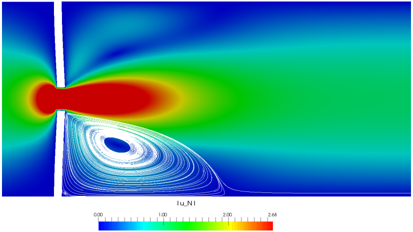
Ongoing applications to cardiovascular modelling (Coanda, MVR)

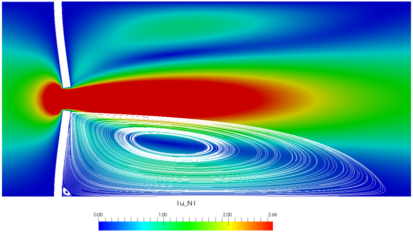
Increase leaflet length: *(same inlet velocity)*





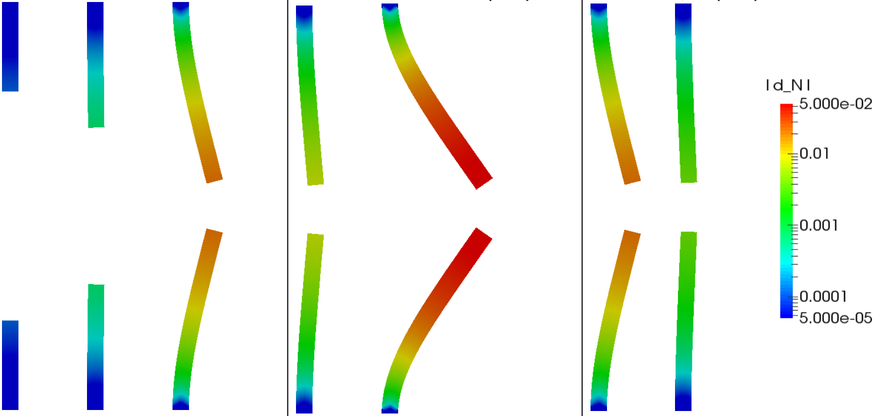
Increase inlet velocity: *(same leaflet length)*





Ongoing applications to cardiovascular modelling (Coanda, MVR)

Increase leaflet length: | *Increase inlet vel. (5×):* | *Increase μ_s (8×):*

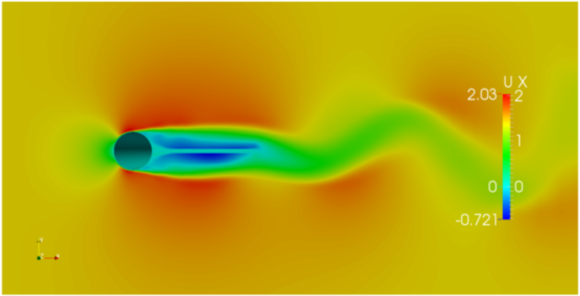


(same inlet velocity, same material properties) | *(same leaflet length, same material properties)* | *(same leaflet length, same inlet velocity)*

[Ballarin, Rozza. IJNMF, 2016]; [Pitton, Quaini, Rozza, sub., 2016]; in progress.

#FSI 2

Partitioned ROMs for FSI problems
Joint work with Francesco Ballarin and Yvon Maday

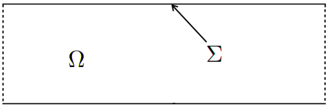


FSI problem with semi-implicit scheme

- **monolithic scheme** requires an **extensive preprocessing** of parametrized tensors, but is capable of handling **geometrical parametrization**, both as variation of the initial configuration and as interface coupling;
- goal: investigate reduced-order **semi-implicit operator-splitting** schemes;
- simplifying assumptions: **Stokes** equations, on a **fixed** fluid domain, under **thin wall** assumption and generalized string model for the **vertical** motion of the structure.

find fluid velocity $\boldsymbol{u}(t) : \Omega \to \mathbb{R}^2$, fluid pressure $p(t) : \Omega \to \mathbb{R}$ and structure (**vertical**) displacement $\eta(t) : \Sigma \to \mathbb{R}$ such that

$$\begin{cases} \rho_f \partial_t \boldsymbol{u} - \mathrm{div}(\boldsymbol{\sigma}_f(\boldsymbol{u}, p)) = \boldsymbol{0} & \text{in } \Omega \times (0, T], \\ \mathrm{div}\, \boldsymbol{u} = 0 & \text{in } \Omega \times (0, T], \\ \boldsymbol{u} = \partial_t \eta\, \boldsymbol{n} & \text{on } \Sigma \times (0, T], \\ \rho_s h_s \partial_{tt} \eta - c_1 \partial_{xx} \eta + c_0 \eta = -\boldsymbol{\sigma}(\boldsymbol{u}, p)\boldsymbol{n} \cdot \boldsymbol{n} & \text{on } \Sigma \times (0, T]. \end{cases}$$



High-fidelity semi-implicit operator-splitting scheme

The high-fidelity discretization is based on the **Chorin-Temam** projection scheme:

1. Explicit step (fluid viscous part): find $\boldsymbol{u}^{k+1} : \Omega \to \mathbb{R}^2$ such that:

$$\begin{cases} \rho_f \frac{\boldsymbol{u}^{k+1} - \boldsymbol{u}^k}{\Delta t} - 2\mu_f \,\mathrm{div}\, \boldsymbol{\varepsilon}(\boldsymbol{u}^{k+1}) = -\nabla p^k & \text{in } \Omega, \\ \boldsymbol{u}^{k+1} = D_t \eta^k \,\boldsymbol{n} & \text{on } \Sigma. \end{cases}$$

2. Implicit step:
   2.1. Fluid projection substep: find $p^{k+1} : \Omega \to \mathbb{R}$ such that:

$$\begin{cases} -\,\mathrm{div}(\nabla p^{k+1}) = -\frac{\rho_f}{\Delta t} \,\mathrm{div}\, \boldsymbol{u}^{k+1} & \text{in } \Omega, \\ \frac{\partial}{\partial \boldsymbol{n}} p^{k+1} = -\rho_f D_{tt} \eta^{k+1} & \text{on } \Sigma. \end{cases}$$

   2.2. Structure substep: find $\eta^{k+1} : \Sigma \to \mathbb{R}$ such that:

$$\rho_s h_s D_{tt} \eta^{k+1} - c_1 \partial_{xx} \eta^{k+1} + c_0 \eta^{k+1} = -\boldsymbol{\sigma}(\boldsymbol{u}^{k+1}, p^{k+1}) \boldsymbol{n} \cdot \boldsymbol{n} \quad \text{on } \Sigma.$$

[Fernández, Gerbeau, Grandmont, IJNME, 2007]
[Astorino, Chouly, Fernandez, SISC, 2010]

High-fidelity semi-implicit operator-splitting scheme (II)

We further discretize in space by the **finite element method**:

1_h. Explicit step (fluid viscous part): find $\boldsymbol{u}_h^{k+1} \in V_h$ such that:

$$\int_\Omega \frac{\rho_f}{\Delta t} \boldsymbol{u}_h^{k+1} \cdot \boldsymbol{v}_h \, d\boldsymbol{x} + \int_\Omega 2\mu_f \boldsymbol{\varepsilon}(\boldsymbol{u}_h^{k+1}) : \nabla \boldsymbol{v}_h \, d\boldsymbol{x} = \int_\Omega \frac{\rho_f}{\Delta t} \boldsymbol{u}_h^{k} \cdot \boldsymbol{v}_h \, d\boldsymbol{x} - \int_\Omega \nabla p_h^k \cdot \boldsymbol{v}_h \, d\boldsymbol{x}$$

for all $\boldsymbol{v}_h \in V_h$, subject to the coupling condition

$$\boldsymbol{u}_h^{k+1} = D_t \eta_h^k \, \boldsymbol{n} \quad \text{on } \Sigma \times [0, T],$$

2_h. Implicit step: for any $j = 0, \ldots,$ until convergence:

2.1_h. Fluid projection substep: find $p_h^{k+1} \in Q_h$ such that:

$$\int_\Omega \nabla p_h^{k+1} \cdot \nabla q_h \, d\boldsymbol{x} = - \int_\Omega \frac{\rho_f}{\Delta t} \operatorname{div} \boldsymbol{u}_h^{k+1} \, q_h \, d\boldsymbol{x} - \int_\Sigma \rho_f D_{tt} \eta_h^{k+1,j} \, q_h \, ds$$

for all $q_h \in Q_h$.

2.2_h. Structure substep: find $\eta_h^{k+1} \in E_h$ such that:

$$\int_\Sigma \frac{\rho_s h_s}{\Delta t^2} \, \eta_h^{k+1} \, \zeta_h \, ds + \int_\Sigma c_1 \partial_x \eta_h^{k+1} \, \partial_x \zeta_h \, ds + \int_\Sigma c_0 \eta_h^{k+1} \, \zeta_h \, ds =$$

$$\int_\Sigma \frac{\rho_s h_s}{\Delta t^2} \eta_h^k \, \zeta_h \, ds + \int_\Sigma \frac{\rho_s h_s}{\Delta t} D_t \eta_h^k \, \zeta_h \, ds - \int_\Sigma \boldsymbol{\sigma}(\boldsymbol{u}^{k+1}, p^{k+1}) \boldsymbol{n} \cdot \zeta_h \boldsymbol{n} \, ds$$

for all $\zeta_h \in E_h$.

An operator-splitting ROM scheme: advantages and questions

- as for the high-fidelity method, online systems (for explicit and implicit steps) have smaller dimensions than a monolithic approach;

- thanks to the Chorin-Temam projection scheme, no supremizer enrichment (see *[Ballarin et al., IJNME, 2015]*, *[Rozza, Veroy, CMAME, 2007]*) is required, resulting in a smaller online dimension for the explicit step;

- in order to enhance the convergence of the implicit step, a Robin-Neumann scheme must be adopted (*[Astorino, Chouly, Fernandez, SISC, 2010]*). However, since it requires on the evaluation of a mass matrix on the interface, it is straightforward to adapt it to a reduced order setting;

- how will the number of iterations of the implicit-step at the reduced-order level compare to the one at the high-fidelity? Will it increase?

- for matching meshes the interface condition

$$\boldsymbol{u}^{k+1} = D_t \eta^k \, \boldsymbol{n} \quad \text{on } \Sigma.$$

is easy to impose at the high-fidelity level. How to efficiently impose it at the reduced-order level since $\boldsymbol{u}$ and η belong to different reduced spaces?

ROM FSI-1: velocity continuity by Lagrange multipliers

Offline stage:

- collect snapshots of the high-fidelity approximation of the FSI problem, and build reduced basis spaces $V_N^{(1)}, Q_N^{(1)}, E_N^{(1)}$ carrying out a Proper Orthogonal Decomposition for each unknown.
- moreover, also collect snapshots of the residual of the fluid viscous part (explicit step) for test functions that do *not* vanish on the interface, denoted by λ_k. Carry out a Proper Orthogonal Decomposition, that will serve as reduced basis space $L_N^{(1)}$ for Lagrange multipliers to enforce velocity continuity.

Online stage:

$1_N^{(1)}$. Explicit step (fluid viscous part): find $(\boldsymbol{u}_N^{k+1}, \lambda_N^{k+1}) \in V_N^{(1)} \times L_N^{(1)}$ such that:

$$\begin{cases} \int_\Omega \frac{\rho_f}{\Delta t} \boldsymbol{u}_N^{k+1} \cdot \boldsymbol{v}_N \, d\boldsymbol{x} & + \int_\Omega 2\mu_f \varepsilon(\boldsymbol{u}_N^{k+1}) : \nabla \boldsymbol{v}_N \, d\boldsymbol{x} \\ & + \int_\Sigma \lambda_N^{k+1} \boldsymbol{n} \cdot \boldsymbol{v}_N \, ds = \int_\Omega \frac{\rho_f}{\Delta t} \boldsymbol{u}_N^{k} \cdot \boldsymbol{v}_N \, d\boldsymbol{x} \\ & - \int_\Omega \nabla p_N^k \cdot \boldsymbol{v}_N \, d\boldsymbol{x}, \\ \int_\Sigma \boldsymbol{u}_N^{k+1} \cdot \Upsilon_N \boldsymbol{n} \, ds & = \int_\Sigma D_t \eta_h^k \, \Upsilon_N \, ds, \end{cases}$$

for all $(\boldsymbol{v}_N, \Upsilon_N) \in V_N^{(1)} \times L_N^{(1)}$.

$2_N^{(1)}$. Implicit step: standard Galerkin projection on $Q_N^{(1)} \times E_N^{(1)}$.

ROM FSI-1: velocity continuity by Lagrange multipliers

☺ offline-online decomposition is straightforward, thanks to the simplifying assumptions of this model problem. In a more general setting, one can resort to EIM, as done in *[Ballarin, Rozza, IJNMF, 2016]* for monolithic FSI problems.

☹ even though it is not necessary to enrich the velocity space by (LBB) supremizers, online we still end up solving a saddle point problem due to the imposition of the interface condition by Lagrange multipliers:

- the advantage (in terms of online system dimension) of using a reduced Chorin-Temam approach is squandered;
- we may still need to add supremizers for the velocity-Lagrange multipliers formulation! (although not needed in practice).

ROM FSI-2: velocity continuity by change of variable for fluid velocity

The idea: online, aim at approximating the following *auxiliary* fluid velocity

$$\boldsymbol{z}^{k+1} = \boldsymbol{u}^{k+1} - D_t \widehat{\eta}^k \, \boldsymbol{n}, \qquad \Rightarrow \qquad \boldsymbol{z}^{k+1} = \boldsymbol{0} \quad \text{on } \Sigma.$$

Here $\widehat{\eta}^k$ is an **harmonic extension** of the displacement η^k.

Offline stage:

- load all velocity and displacement snapshots to compute the auxiliary fluid velocity $\boldsymbol{z}^{k+1}$, compute a POD and store the first modes in the (auxiliary) velocity space $V_N^{(2)}$.
- reduced pressure and displacement spaces are unchanged, $Q_N^{(2)} := Q_N^{(1)}$ and $E_N^{(2)} := E_N^{(1)}$.

Online stage:

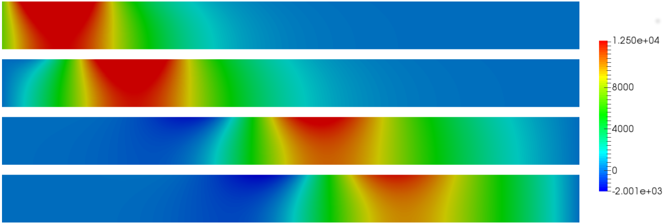
formally rewrite the weak formulation problem in terms of the unknowns $(\boldsymbol{z}, p, \eta)$ and carry out a standard Galerkin projection over the reduced space $V_N^{(2)} \times Q_N^{(2)} \times E_N^{(2)}$.

ROM FSI-2: velocity continuity by change of variable for fluid velocity

- ☺ offline-online decomposition is still straightforward.
- ☺ there is no need to enlarge the system size for the reduced explicit step, neither for supremizer enrichment nor for Lagrange multipliers
- ☺ the interface velocity continuity condition is imposed strongly also at the reduced-order level.
- ☺ the harmonic extension does not require any additional online problem. Indeed, each displacement basis function can be harmonically extended once and for all during the offline stage, and then linearly combined once the solution of the structure problem has been computed without any Galerkin projection for the extension problem.

Numerical test case

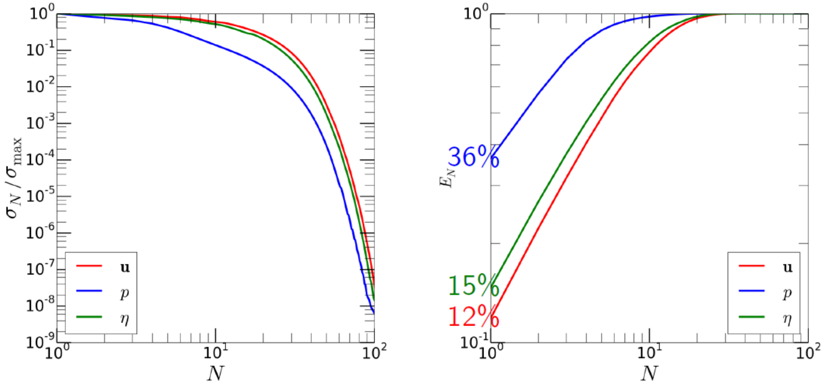
We compare the accuracy and efficiency of the proposed ROMs in a test case characterized by the propagation of a pressure wave inside the fluid domain.



RBniCS

[Formaggia, Gerbeau, Nobile, Quarteroni, CMAME, 2001]

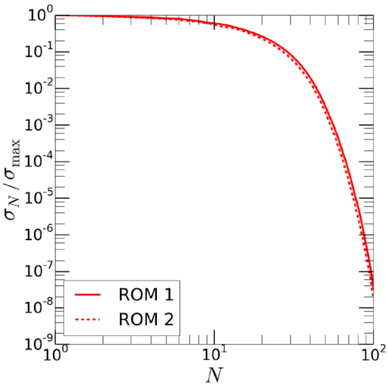
POD singular values for ROM FSI-1



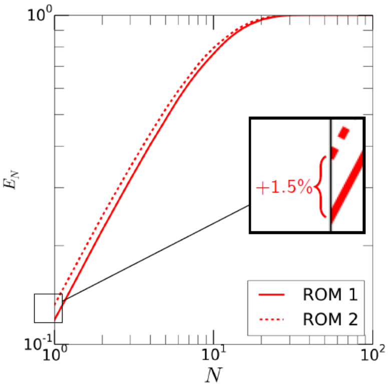
(a) POD singular values.

(b) POD retained energy.

POD singular values for ROM FSI-2 vs ROM FSI-1 (velocity only)



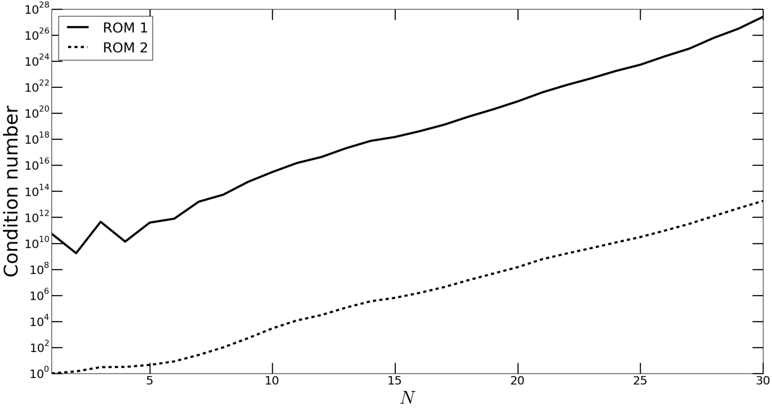
(a) POD singular values.



(b) POD retained energy.

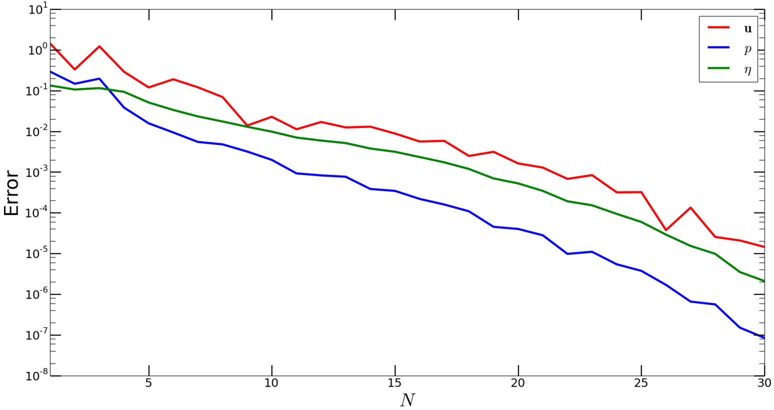
☺ the first (auxiliary) velocity mode of ROM FSI-2 retains more energy than the corresponding mode of ROM FSI-1.

Condition number of the reduced explicit step, ROM FSI-2 vs ROM FSI-1

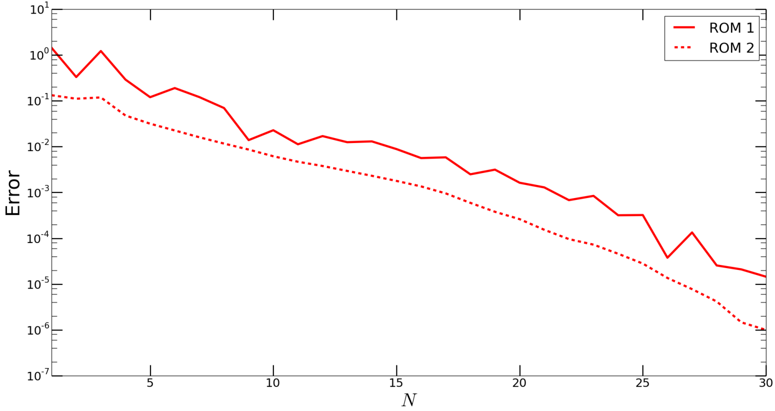


☺ ROM FSI-2 is characterized by a condition number of at least 10 orders of magnitude lower than ROM FSI-1.

Error analysis of ROM FSI-1

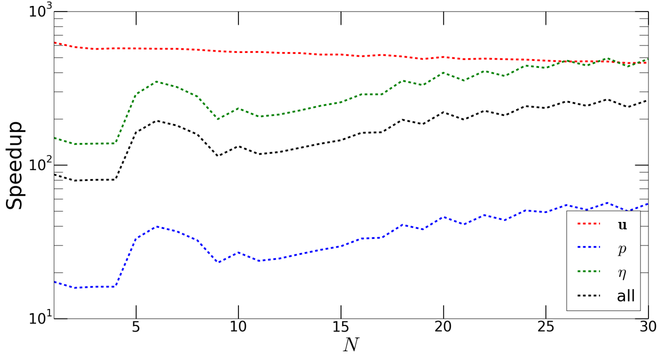


Error analysis of ROM FSI-2 vs ROM FSI-1 (velocity only)



☺ thanks to the lower condition number, strong imposition of interface conditions and higher retained energy, ROM FSI-2 is more accurate (1 order of magnitude) than ROM FSI-1.

Speedup analysis, ROM FSI-2



☺ The overall ROM/HF speedup is of two orders of magnitude. Furthermore, the speedup increases with N because a lower number of iterations is required in the implicit step.

- F. Ballarin, G. Rozza, *POD–Galerkin monolithic reduced order models for parametrized fluid-structure interaction problems*. International Journal for Numerical Methods in Fluids, in press, 2016.
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Thanks for your attention! New post-doc open position very soon!