

SIAM Conference on Computational Science and Engineering

**MS27:**

*Featured Minisymposium:*

Fast Multipole Methods Maturing at 30 Years

**Prof. Lorena A. Barba**

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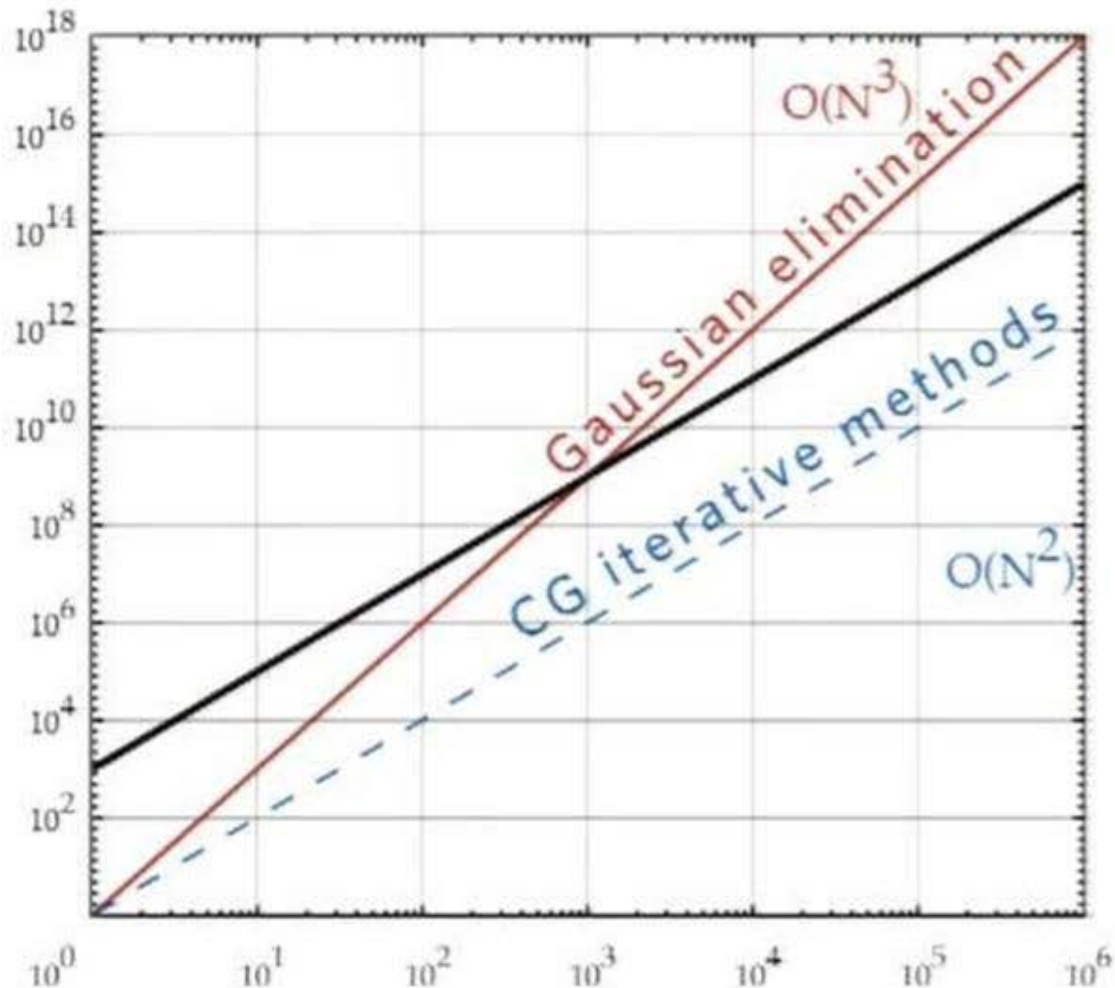




**“Complexity trumps hardware.”**

# The story of the conjugate gradient method

- ▶ CG iterations bring the  $O(N^3)$  cost to  $O(N^2)$
- ▶ 1950s —  $N$  too small for CG to be competitive
- ▶ 1970s — becomes popular





"a fast algorithm for particle simulations"

Scholar

About 2,140 results (0.11 sec)

### **A fast algorithm for particle simulations**

L Greengard, V Rokhlin - *Journal of computational physics*, 1987 - Elsevier

Abstract An algorithm is presented for the rapid evaluation of the potential and force fields in systems involving large numbers of particles whose interactions are Coulombic or gravitational in nature. For a system of  $N$  particles, an amount of work of the order  $O(N^2)$  ...

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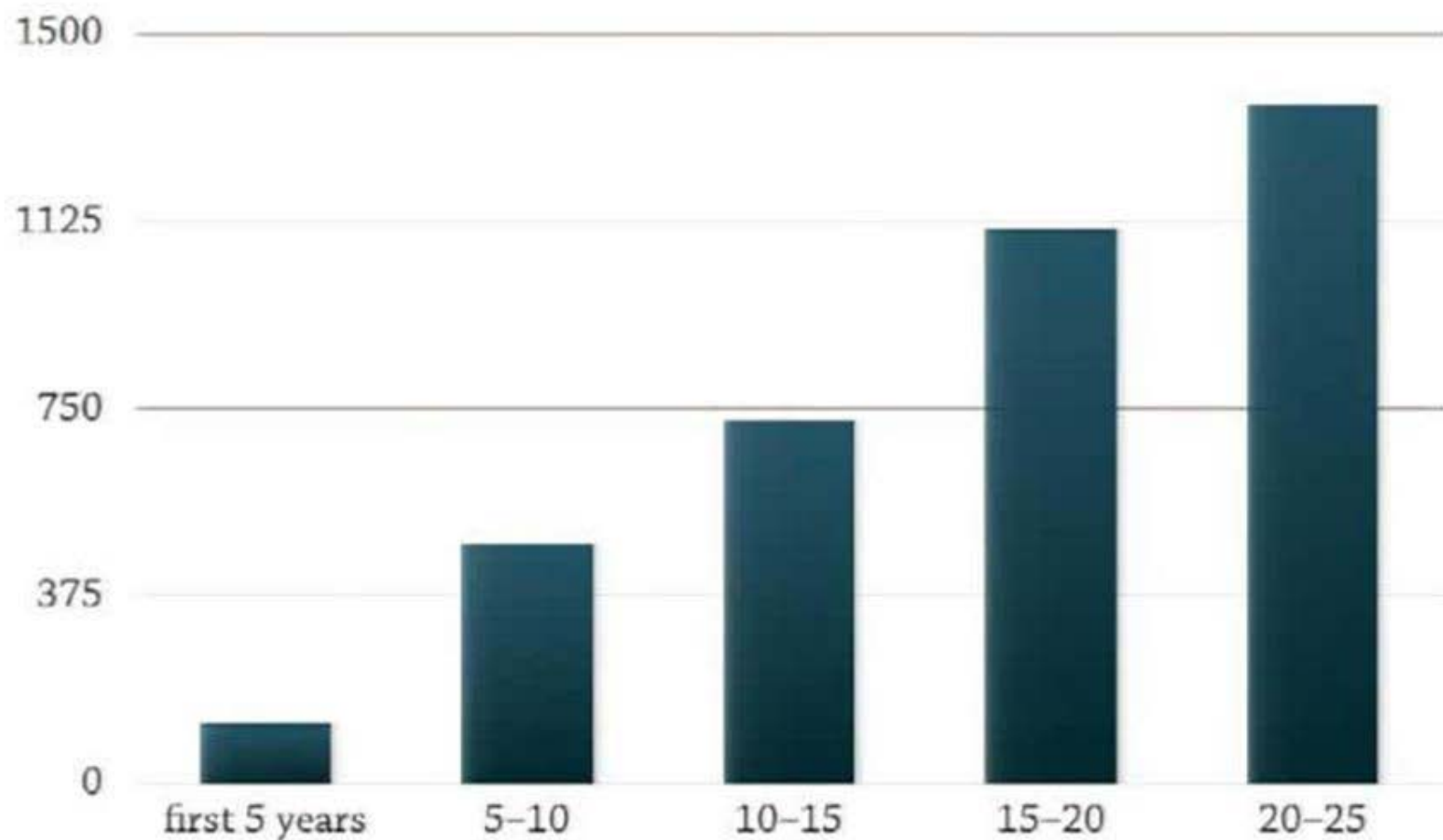
### **A fast algorithm for particle simulations**

L Greengard, V Rokhlin - *Journal of Computational Physics*, 1997 - Elsevier

An algorithm is presented for the rapid evaluation of the potential and force fields in systems involving large numbers of particles whose interactions are Coulombic or gravitational in nature. For a system of  $N$  particles, an amount of work of the order  $O(N^2)$  has traditionally ...

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# Number of citations in the years after the publication of Greengard & Rocklin, 1987



**SIAM Conference on  
Computational Science  
and Engineering**



Figure courtesy Thomas A. Brunner and Tzanio V. Kolev,  
*SISC*, Vol.33, 5-6

**February 25-March 1, 2013  
The Westin Boston Waterfront  
Boston, Massachusetts, USA**

# SIAM CSE 13

- ▶ **MS11**—Fast Algorithms for Integral Equations Methods and Their Applications  
- *Organizer:* Prabir Daripa
- ▶ **MS31**—Fast Algorithms in Potential Theory (2 Parts)  
- *Organizers:* Bryan D. Quaife, George Biros
- ▶ **MS72**—Integral Equation Methods in Complex Geometry (3 Parts)  
- *Organizers:* Leslie Greengard, Andreas Kloeckner
- ▶ **MS132**—Applications and New Developments in Fast-multipole and Tree-based Methods (2 Parts)  
- *Organizer:* Lorena A. Barba, Rio Yokota, Cris R. Cecka



A photograph of a large server room. In the background, there are long rows of black server racks. A few people are visible in the distance, walking through the aisle. The floor is light-colored with a grid pattern. The ceiling has recessed lighting.

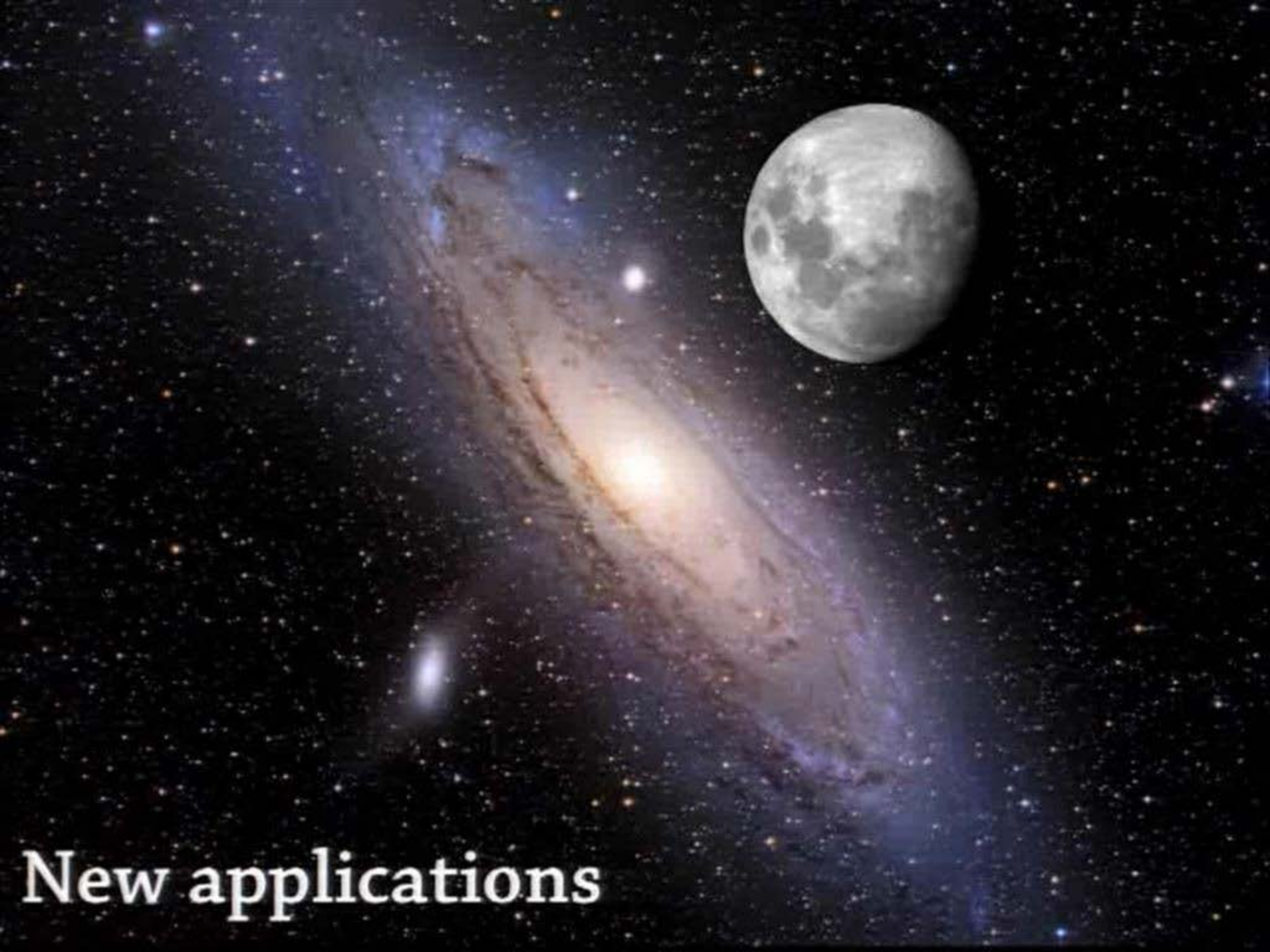
Lloyd Trefethen, 1998:

**Predictions for Scientific Computing  
50 Years From Now**

#7

“Multipole methods and their descendants will be ubiquitous.”

—L. N. Trefethen, 1998



**New applications**

# A biomolecular electrostatics solver using Python, GPUs and boundary elements that can handle solvent-filled cavities and Stern layers

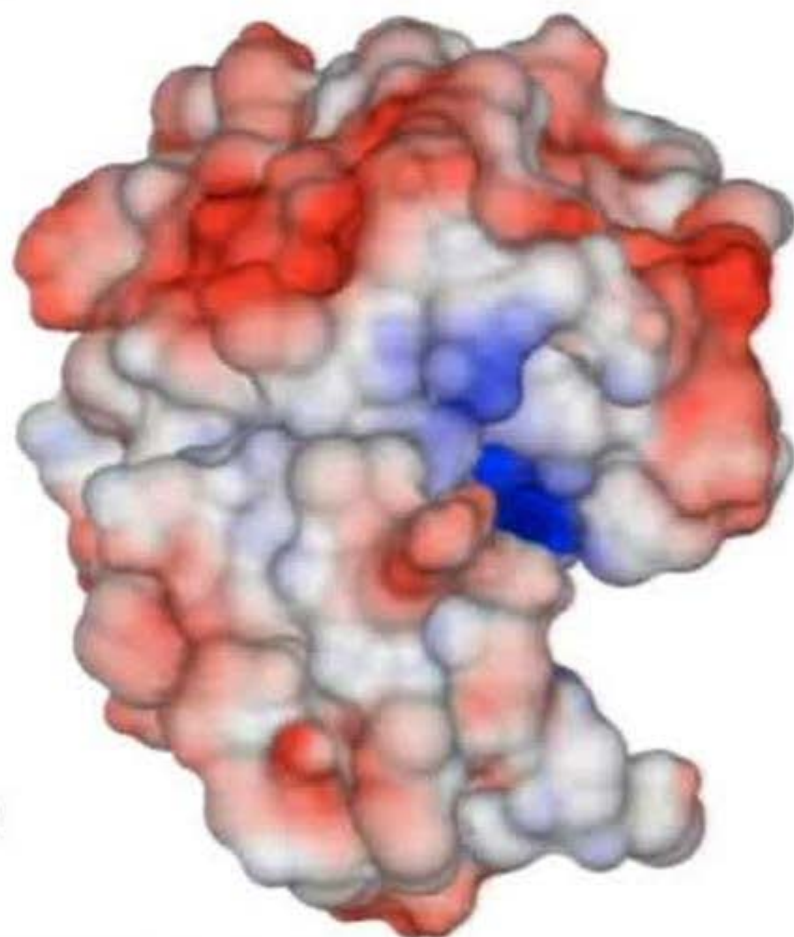
Christopher D. Cooper<sup>a</sup>, Jaydeep P. Bardhan<sup>b</sup>, L.A. Barba<sup>a,\*</sup>

<sup>a</sup> Mechanical Engineering, Boston University, Boston, MA 02215, USA

<sup>b</sup> Electrical and Computer Engineering, Northeastern University, Boston, MA 02115, USA



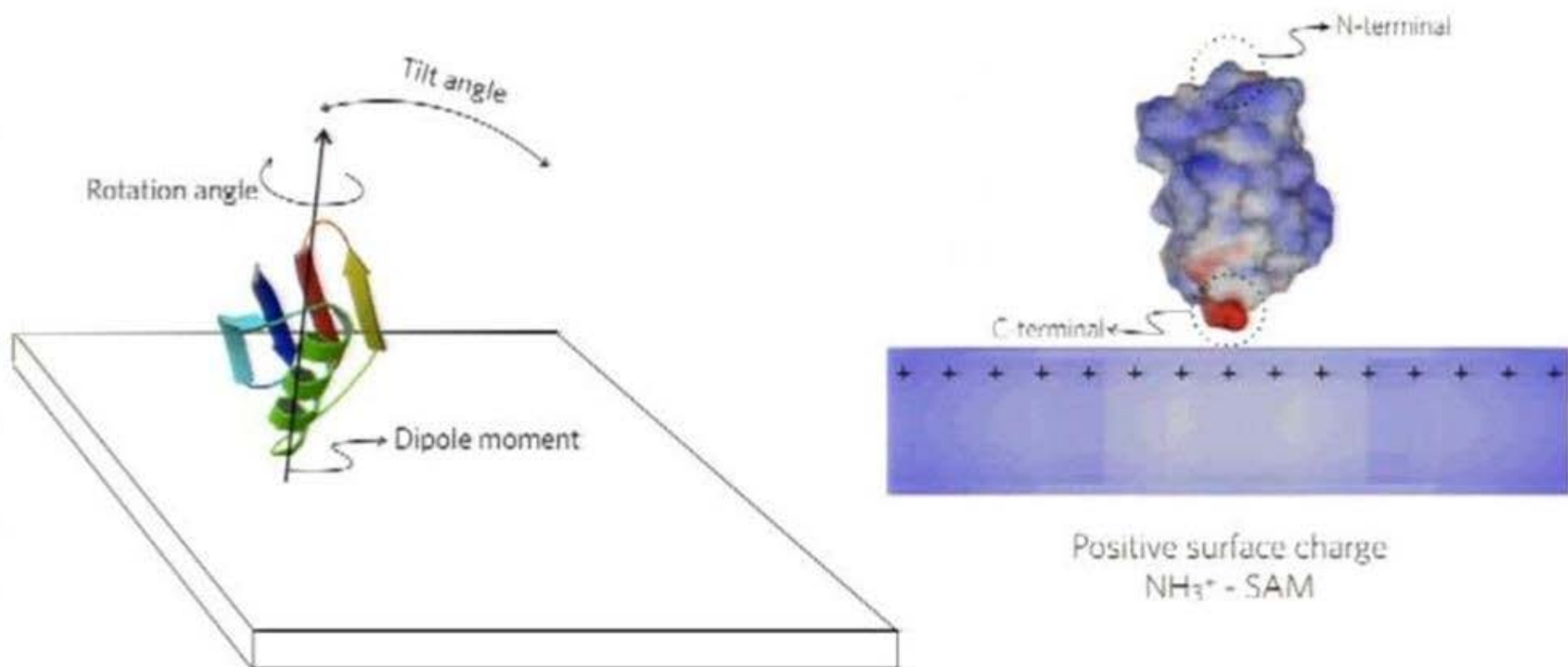
<https://github.com/barbagroup/pygbe>



Preprint:

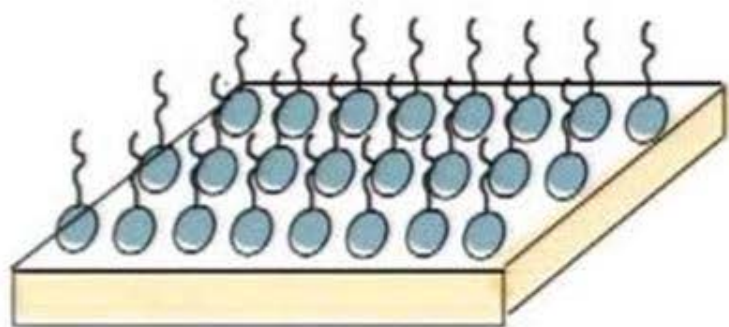
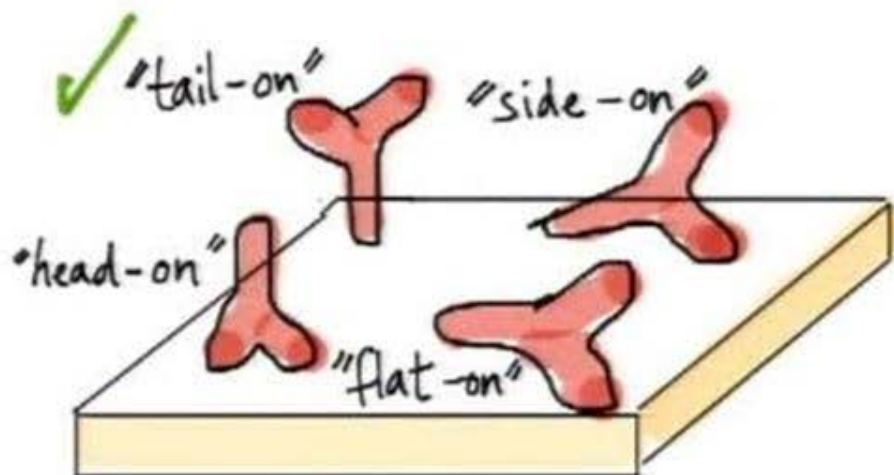
## Probing protein orientation near charged surfaces with an implicit-solvent model and the PyGBe code

Christopher D. Cooper<sup>1, a)</sup> and Lorena A. Barba<sup>2, b)</sup>

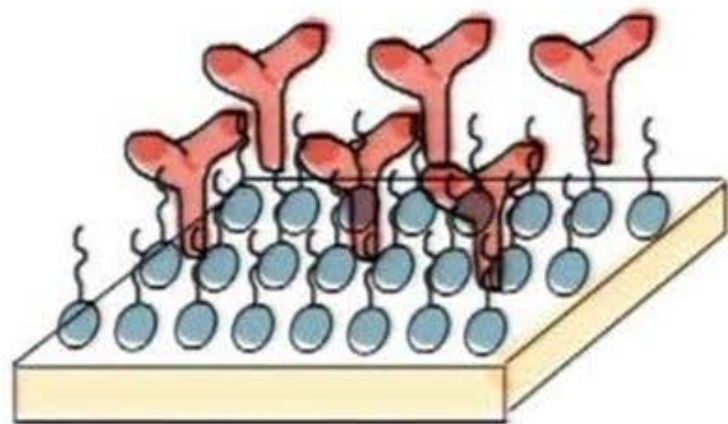





# Antibody-based sensors



Self-assembled Monolayer  
on a gold electrode.

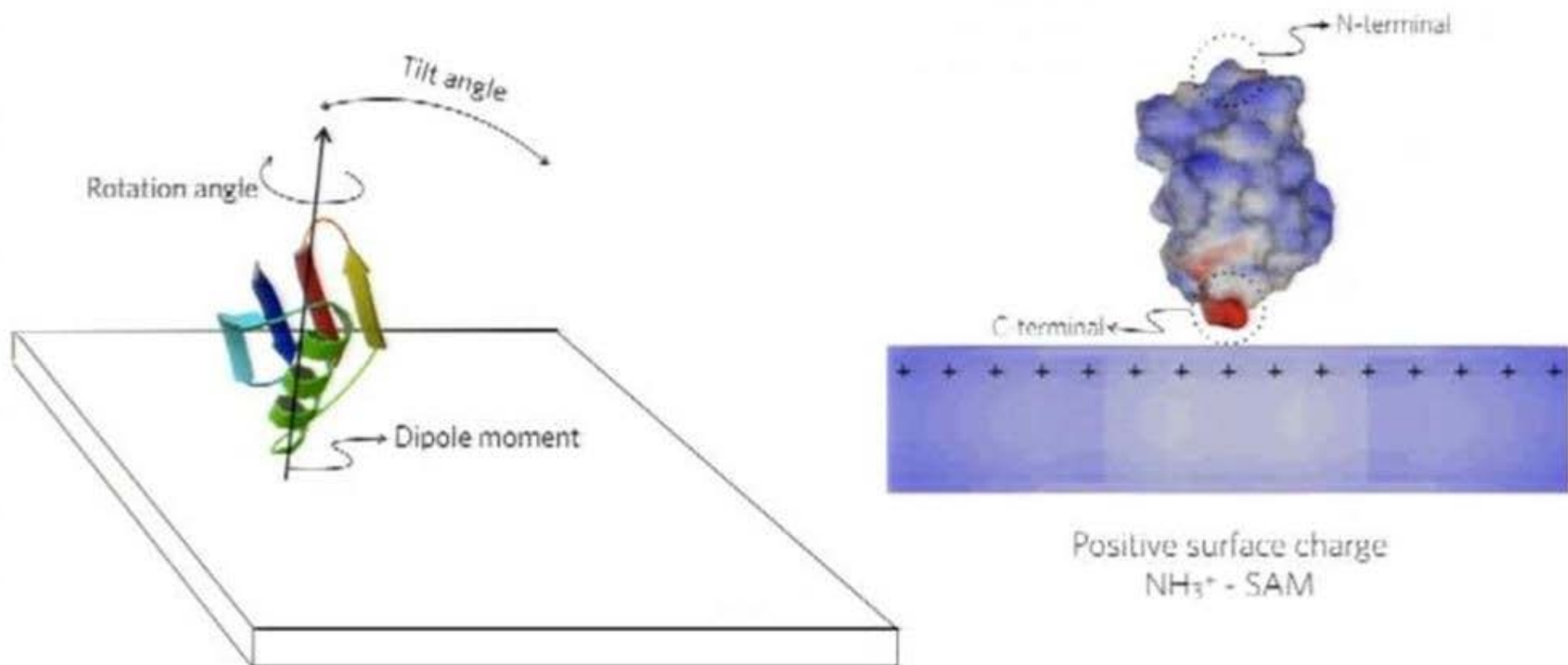


 Antibody  
(or antibody-binding protein)

Preprint:

## Probing protein orientation near charged surfaces with an implicit-solvent model and the PyGBe code

Christopher D. Cooper<sup>1, a)</sup> and Lorena A. Barba<sup>2, b)</sup>





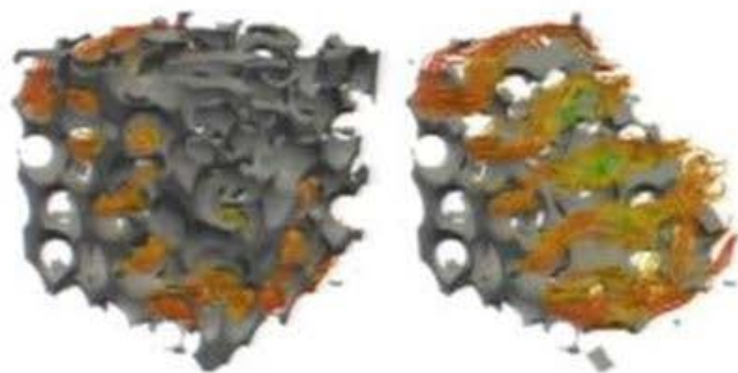
SC14: International Conference for High Performance Computing, Networking, Storage and Analysis

## A volume integral equation Stokes solver for problems with variable coefficients

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**FAR-FIELD COMPRESSION FOR FAST KERNEL SUMMATION METHODS IN  
HIGH DIMENSIONS**

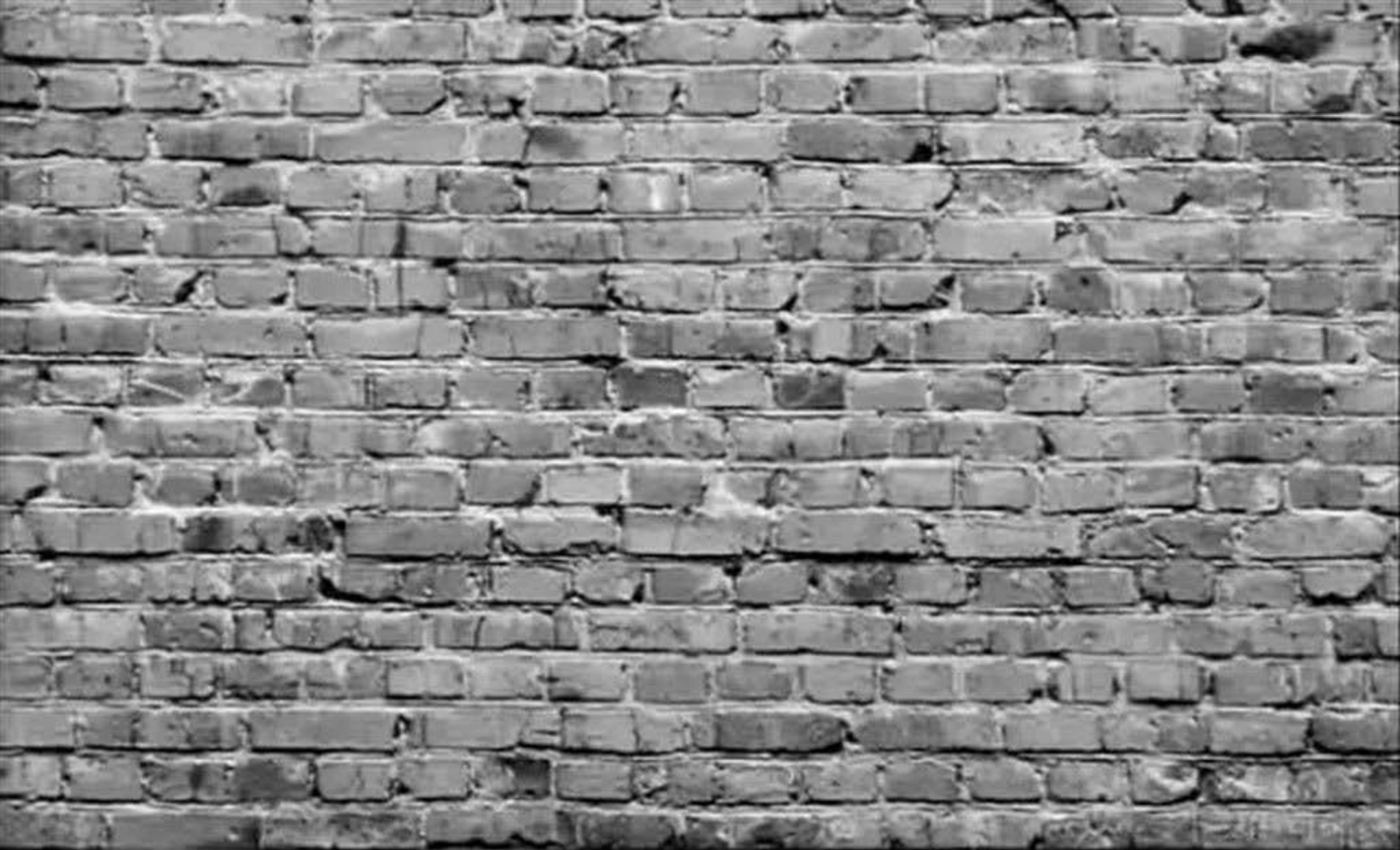
WILLIAM B. MARCH\* AND GEORGE BIROS

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**ASKIT: APPROXIMATE SKELETONIZATION  
KERNEL-INDEPENDENT TREECODE IN HIGH DIMENSIONS**

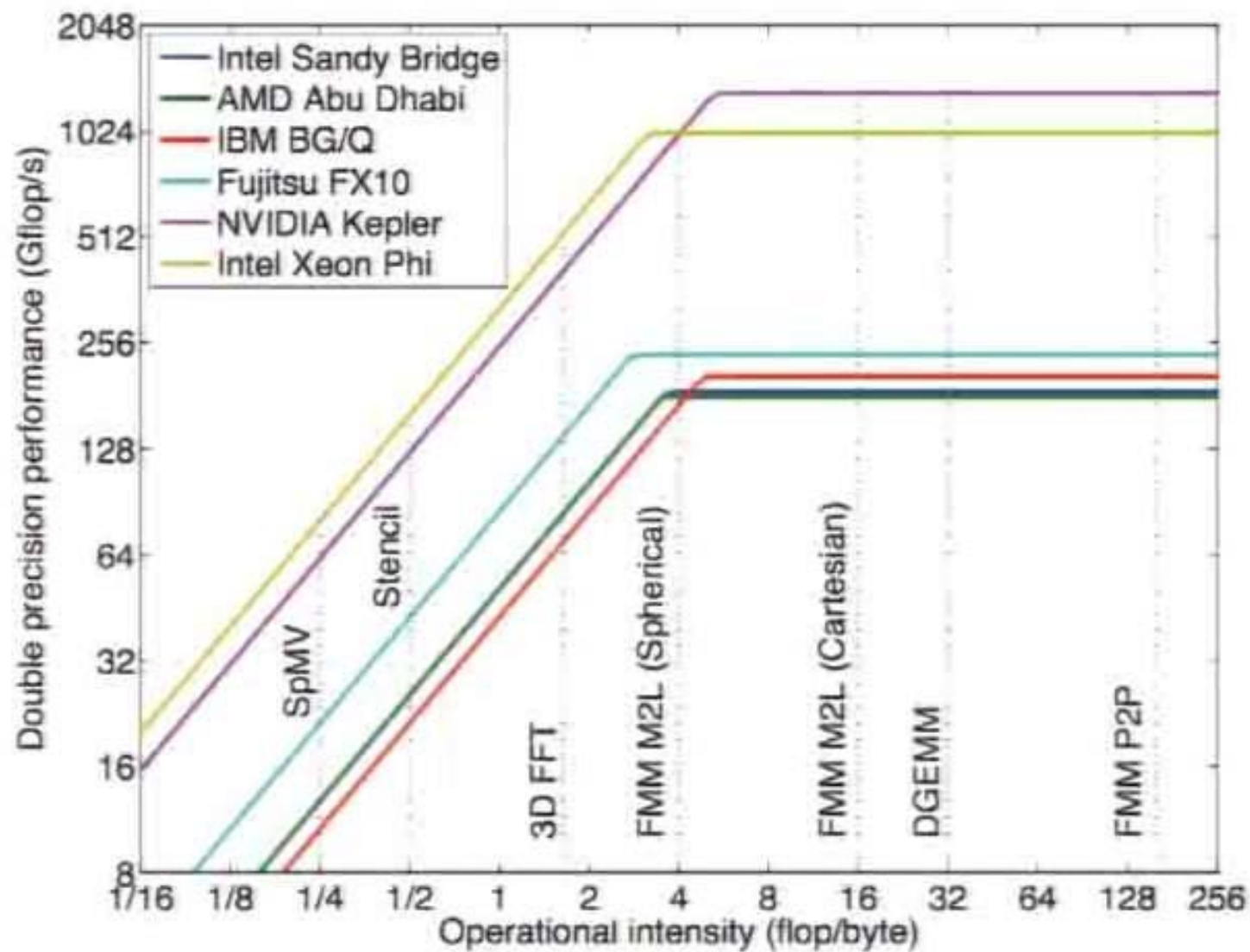
WILLIAM B. MARCH\*, BO XIAO\*, AND GEORGE BIROS\*

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**Computer science & HPC aspects**

“Intensity trumps sparsity.”



# Parallel FMM

- ▶ 1990 G&G — shared memory
- ▶ “A parallel adaptive fast multipole method”, J. P. Singh, C. Holt, J. L. Hennessy, A. Gupta, Supercomputing '93 Proceedings of the 1993 ACM/IEEE Conference
- ▶ “Load Balancing and Data Locality in Adaptive Hierarchical N-Body Methods ...”, J.P. Singh, C. Holt, T. Totsuka, A. Gupta, J. Hennessy, *Journal of Parallel and Distributed Computing*, 27(2):118-141 (1995)
- ▶ “A new parallel kernel-independent fast multipole method”, L. Ying, G. Biros, D. Zorin, H. Langston, Supercomputing, 2003 ACM/IEEE Conference (*Best Student Paper Award*)

## ... jump to 2014

- ▶ Hybrid architectures & computational models

### **A CPU–GPU Hybrid Implementation and Model-Driven Scheduling of the Fast Multipole Method**

Jee Choi<sup>1</sup>, Aparna Chandramowliswaran<sup>3</sup>, Kamesh Madduri<sup>4</sup>, Richard Vuduc<sup>2</sup>

### **A Performance Model for the Communication in Fast Multipole Methods on HPC Platforms**

Huda Ibeid, Rio Yokota, and David Keyes

## Communication Complexity of the Fast Multipole Method and its Algebraic Variants

*Rio Yokota<sup>1</sup>, George Turkiyyah<sup>1</sup>, David Keyes<sup>1</sup>*

Reference	Processes		Data per Process		Communication complexity
Teng [32]	$\mathcal{O}(P)$		$\mathcal{O}((N/P)^{2/3}(\log N + \mu)^{1/3})$		$\mathcal{O}(P(N/P)^{2/3}(\log N + \mu)^{1/3})$
Lashuk <i>et al.</i> [27]	$\mathcal{O}(\sqrt{P})$		$\mathcal{O}((N/P)^{2/3})$		$\mathcal{O}(\sqrt{P}(N/P)^{2/3})$
Ibeid <i>et al.</i> [21]	Global	Local	Global	Local	Global + Local
	$\mathcal{O}(\log P)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}((N/P)^{2/3})$	$\mathcal{O}(\log P + (N/P)^{2/3})$



From *SIAM News*, Volume 46, Number 6, July/August 2013

*CSE 2013*

## How Will the Fast Multipole Method Fare in the Exascale Era?

By *Lorena A. Barba and Rio Yokota*

Another cause for optimism is the new spirit of collaboration emerging in the field. Leading up to the SIAM CSE conference, several groups were exchanging ideas about building a set of standard benchmark tests for FMM codes and, in due course, developing a community software library. A handful of open-source codes are already available, but adoption of multipole algorithms would thrive if we had a BLAS-like col-

# Open-source codes

► 2004, updated 2006

## KIFMM3d Download and Installation

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The Kernel-Independent Fast Multipole (FMM) 3D code (kifmm3d) requires the installation of several libraries. Before downloading the kifmm3d code, make sure to install the necessary libraries listed on the [links page](#).

### Download

The most current version of the code and documentation can be downloaded here

- Code: [kifmm3d.tar.gz](#)
- HTML Documentation: [kifmm3d\\_doc.tar.gz](#)(Not yet active)

# Open-source codes

► 2010

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING

*Int. J. Numer. Meth. Engng* 2011; **85**:403–428

Published online 1 September 2010 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/nme.2972

## PetFMM—A dynamically load-balancing parallel fast multipole library

Felipe A. Cruz<sup>1</sup>, Matthew G. Knepley<sup>2</sup> and L. A. Barba<sup>3,\*,†</sup>

Atlassian

 Bitbucket

Features Pricing



petfmm

Member since February 2010

# Open-source codes

► 2012: ExaFMM.org

Boston University Mechanical Engineering  
ExaFMM

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# Open-source codes

► 2013

## A CPU-GPU Hybrid Implementation and Model-Driven Scheduling of the Fast Multipole Method

Jee Choi<sup>1</sup>, Aparna Chandramowliswaran<sup>3</sup>, Kamesh Madduri<sup>4</sup>, Richard Vuduc<sup>2</sup>



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jeewhanchoi / kifmm--hybrid--double-only

CPU-GPU hybrid implementation of KIFMM in double-precision only

# Open-source codes

► 2014: PvFMM.org

## A volume integral equation Stokes solver for problems with variable coefficients

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George Biros  
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Austin, TX 78712



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Can we coalesce an open community?