Distributed Algorithms for Wide-Area Monitoring of Power Systems

Theory, Experiments, and Open Problems

Aranya Chakrabortty
North Carolina State University

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Main trigger: 2003 Northeast Blackout

NYC before blackout

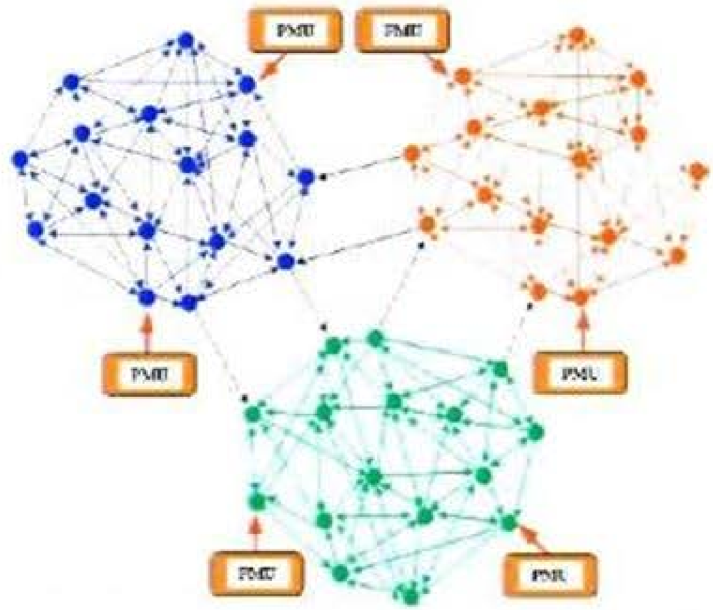
NYC after blackout

Hauer, Zhou & Trudnowsky, 2004
Kosterev & Martins, 2004

2 Main Lessons Learnt from the 2003 Blackout:

1. Need significantly higher resolution measurements

⇨ From traditional SCADA (System Control and Data Acquisition) to PMUs (Phasor Measurement Units)

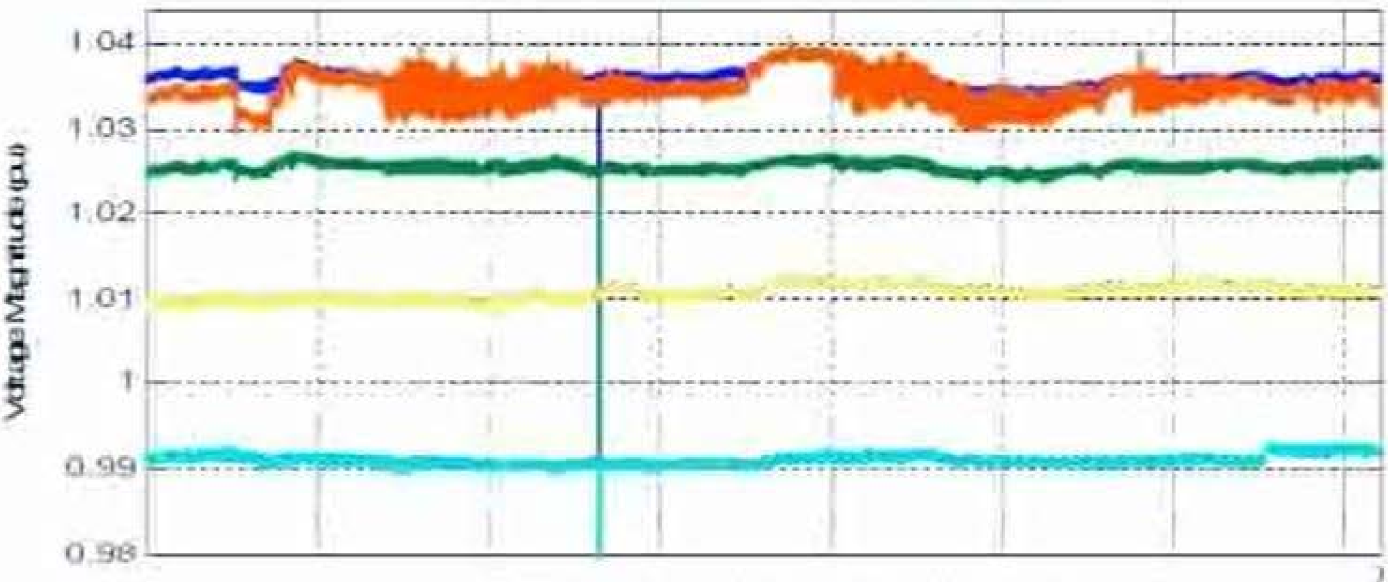


2. Local monitoring & control can lead to disastrous results

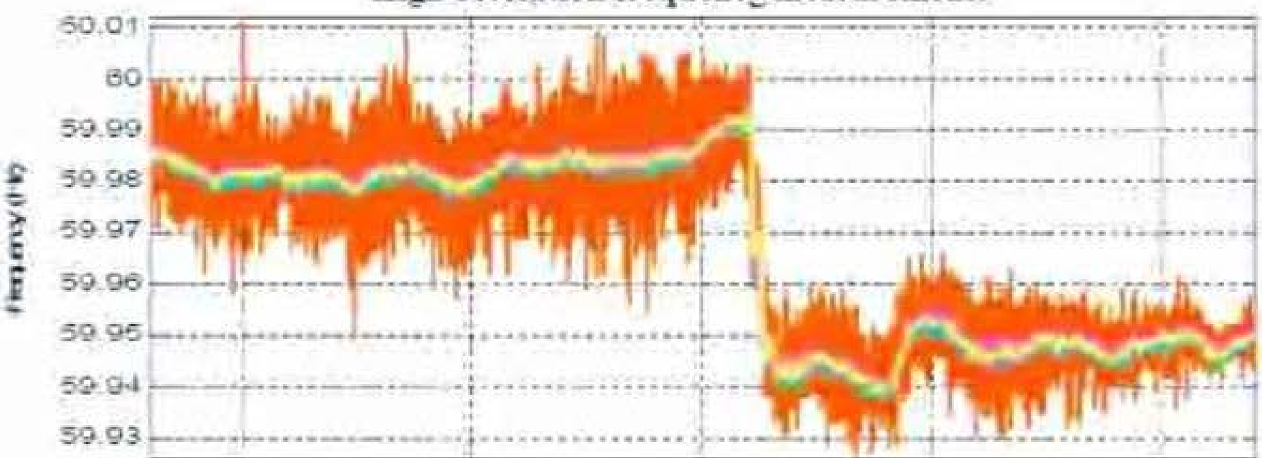
⇨ Coordinated control instead of selfish control

High-resolution PMU measurements from the US west coast grid

High-resolution voltage measurements

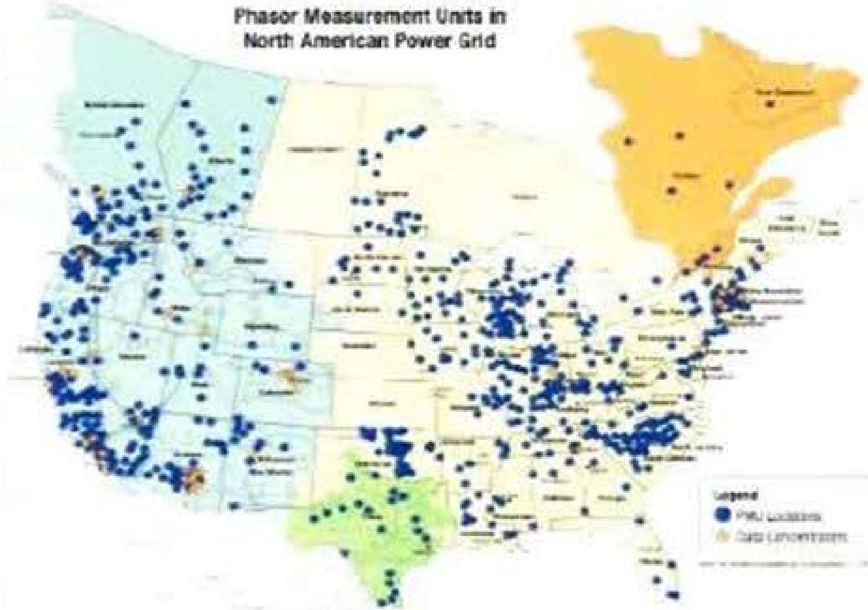


High-resolution frequency measurements



Increasing Volumes of PMU Data

2008: Only 40 PMUs in the entire east coast

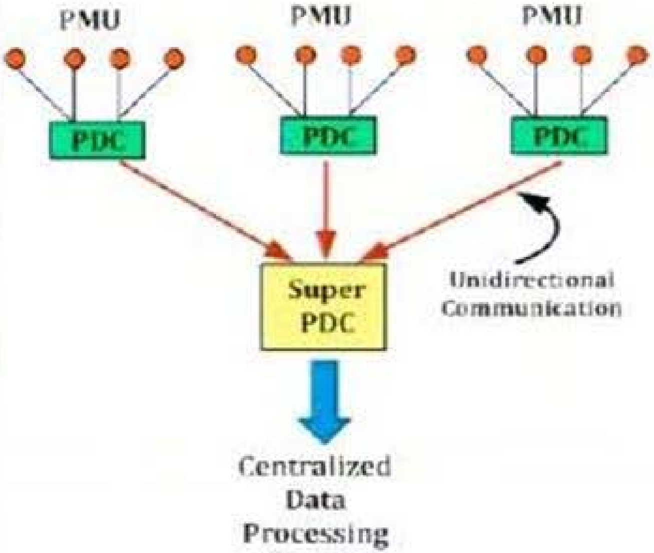


2015: More than 1000 PMUs across USA
(Nearly 52 PMUs only in North Carolina)

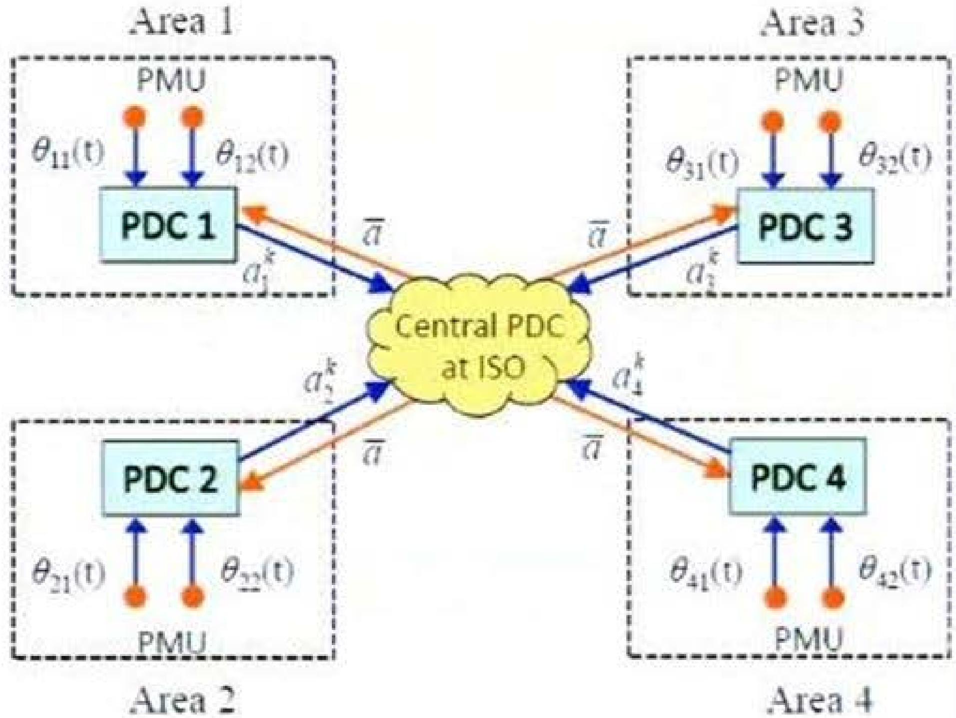
- Massive volumes of PMU data need to be transported from one part of the grid to another for monitoring and control
- Needs a highly reliable and resilient communication infrastructure
- Centralized processing will not be tenable
- Need combination of distributed monitoring spread over the entire system

Centralized vs Distributed Algorithms

Centralized RLS



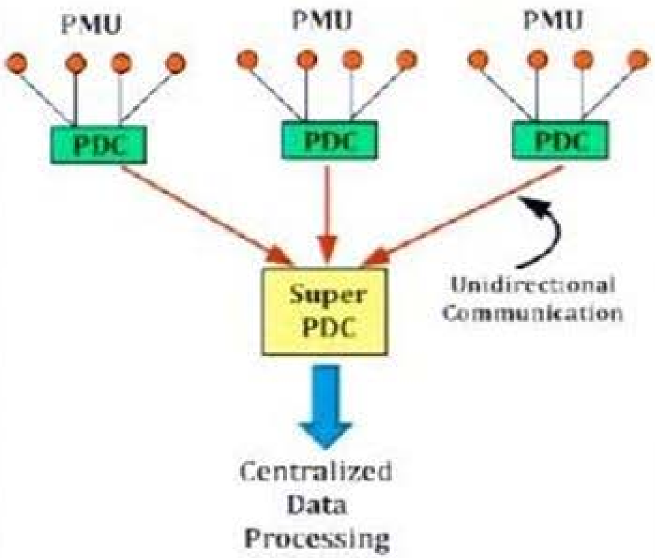
Semi-Distributed Prony



Control Room

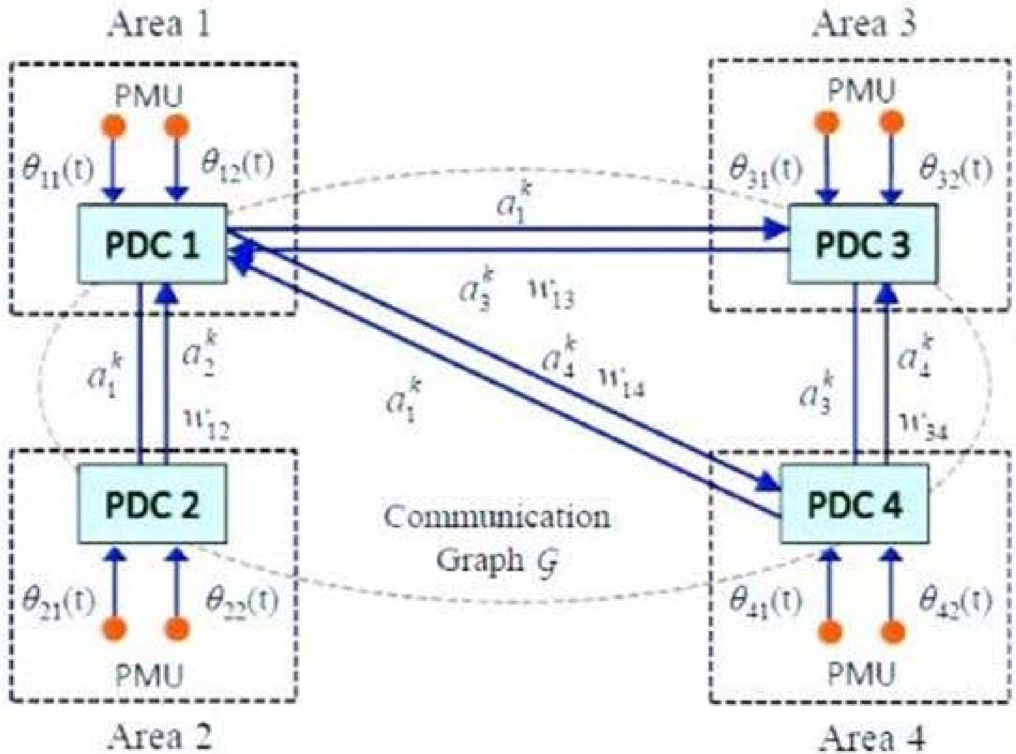
Centralized vs Distributed Algorithms

Centralized RLS



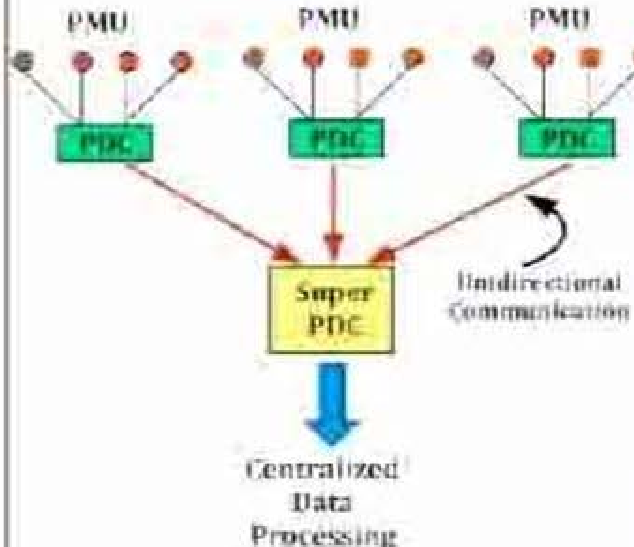
Control Room

Distributed Prony

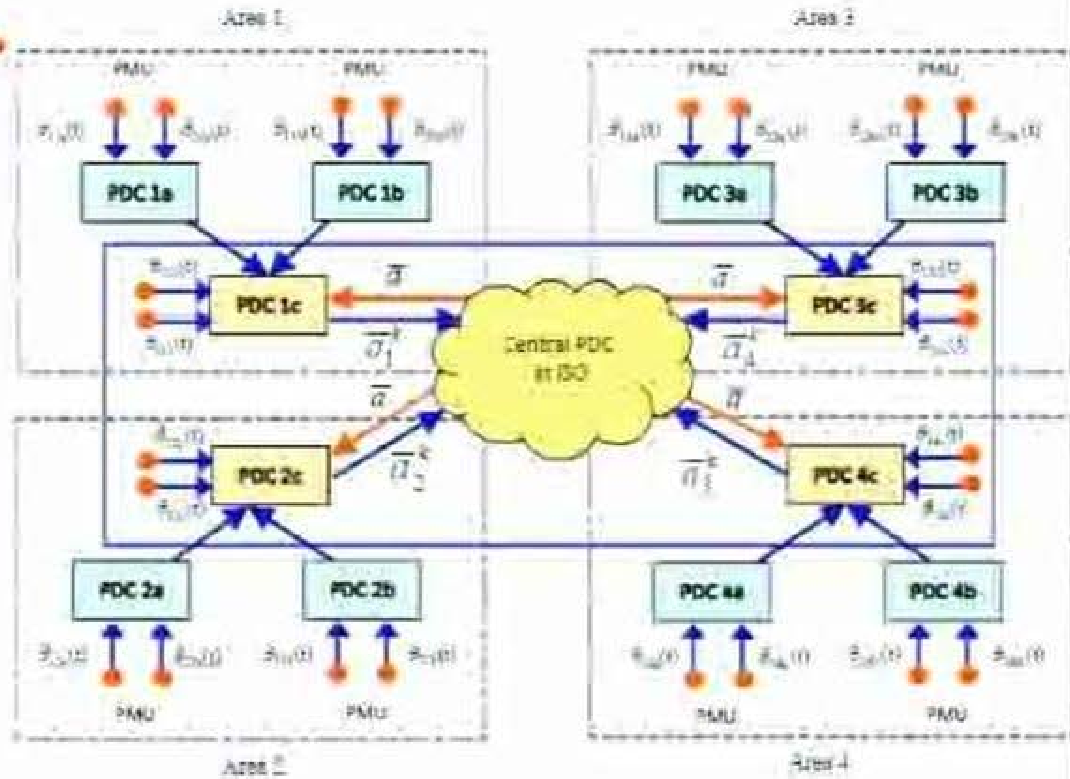


Centralized vs Distributed Algorithms

Centralized RLS



Heirarchically Distributed Prony



Control Room

Motivating the Wide-Area Oscillation Monitoring Problem:

• **Synchronous Generator Models**

$$\dot{\delta}_i = \omega_i - \omega_s$$

$$M_i\dot{\omega}_i = P_{mi} - D_i(\omega_i - \omega_s) - P_i^G$$

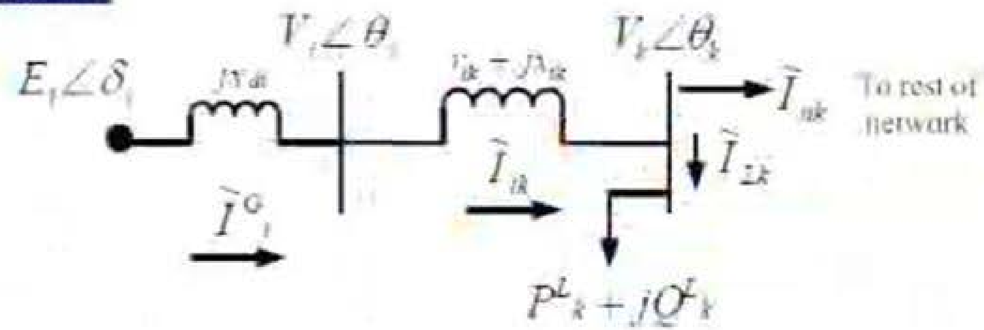
$$\tau_i \dot{E}_i = -\frac{x_{di}}{x'_{di}}E_i + \frac{x_{di} - x'_{di}}{x'_{di}}V_i \cos(\delta_i - \theta_i) + E_{Fi}$$

⇒ $E_{Fi} = \bar{E}_{Fi} + E_i$

Control input

Excitation voltage

• **Power Flow Equations**



$$P_i^G = \frac{E_iV_i}{x'_di}\sin(\delta_i - \theta_i) + \left(\frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}}\right)V_i^2\sin(2(\delta_i - \theta_i))$$

$$Q_i^G = \frac{E_iV_i}{x'_di}\cos(\delta_i - \theta_i) - \left(\frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}} - \frac{x'_{di} - x_{qi}}{2x_{qi}x'_{di}}\cos(2(\delta_i - \theta_i))\right)V_i^2,$$

⇒ Bus voltage and phase angle

Algebraic variables

Measured by PMU

Grid Dynamic Models

- **Load Models**

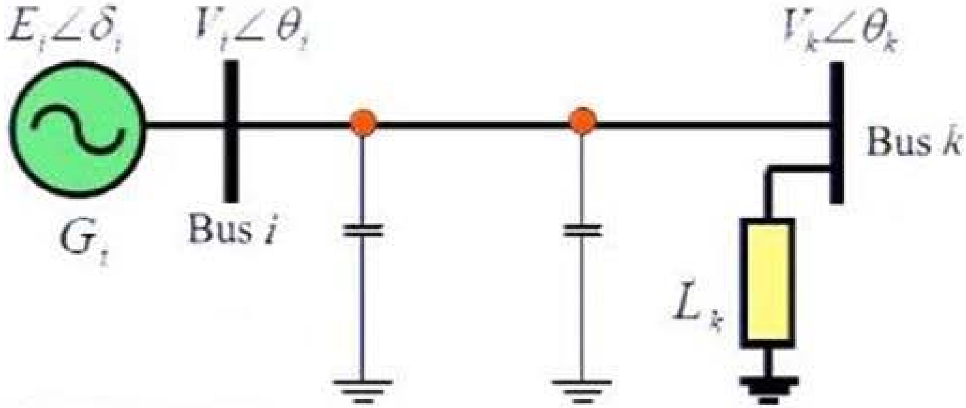
$$P_j^L = a_j V_j^2 + b_j V_j + c_j$$
$$Q_j^L = e_j V_j^2 + f_j V_j + g_j$$

a_j, e_j = constant impedance
b_j, f_j = constant current
c_j, g_j = constant power

- **Transmission Line Model**

$$P_{ij} = G_{ij} V_i^2 + B_{ij} V_i V_j \sin(\theta_i - \theta_j) - G_{ij} V_i V_j \cos(\theta_i - \theta_j)$$
$$Q_{ij} = (B_{ij} - B_{ij}^c) V_i^2 - B_{ij} V_i V_j \cos(\theta_i - \theta_j) - G_{ij} V_i V_j \sin(\theta_i - \theta_j).$$

Pi-model

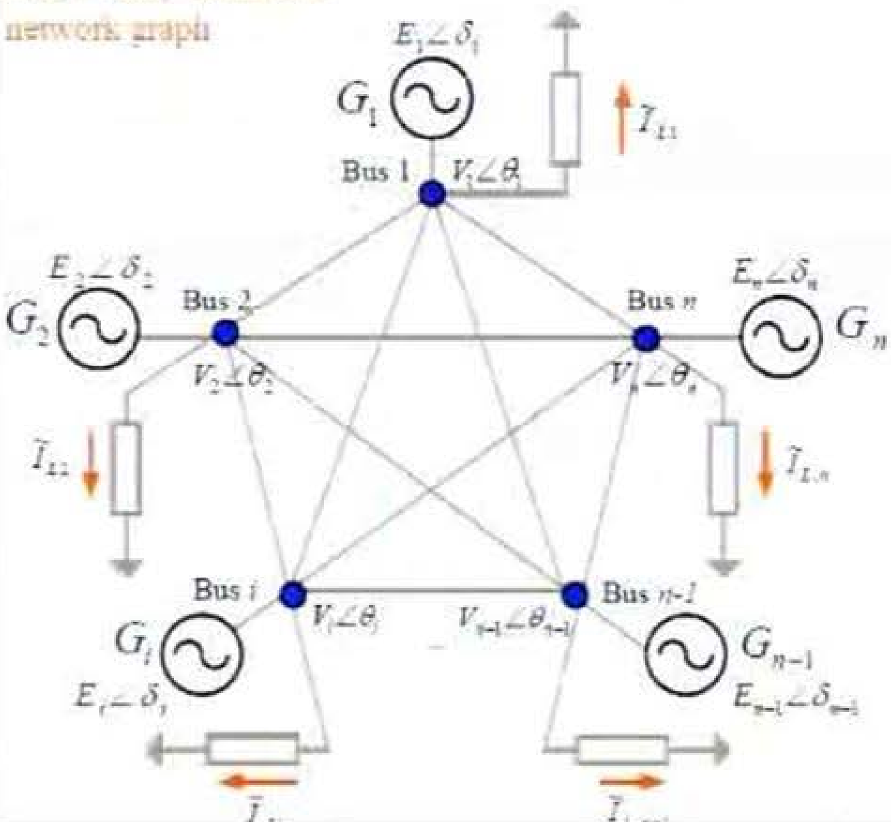


• Total Network Model

$$\begin{bmatrix} \Delta\dot{\delta} \\ M\Delta\dot{\omega} \\ \Delta\dot{E} \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ -L(G) & -D & -P \\ K & 0 & J \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \mathrm{col}_{i=1(1)n}(\gamma_i) \\ \mathrm{col}_{i=1(1)n}(\rho_i) \end{bmatrix}}_{\text{due to load}} + \begin{bmatrix} 0 & 0 \\ 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} \Delta P_m \\ \Delta E_F \end{bmatrix} \quad \ldots(1)$$

$L(G)$ = fully connected network graph

Controllable inputs



Output Equation

$$y = \mathrm{col}_{i\in S}(\Delta V_i,\ \Delta\theta_i). \quad \ldots(2)$$

Wide-Area Oscillation Estimation

PMU data → $y_j(t) = \Delta\theta_j(t) = \sum_{i=1}^{n} r_{j,i} e^{(-\sigma_i + j\Omega_i)t} + r_{j,i}^{*} e^{(-\sigma_i - j\Omega_i)t}$

$$\mathbf{y}_j(t) = \begin{bmatrix} \Delta\theta_1(t) \\ \vdots \\ \Delta\theta_p(t) \end{bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} r_{1,i} \\ \vdots \\ r_{p,i} \end{bmatrix} e^{(-\sigma_i + j\Omega_i)t} + \begin{bmatrix} r_{1,i}^{*} \\ \vdots \\ r_{p,i}^{*} \end{bmatrix} e^{(-\sigma_i - j\Omega_i)t}$$

- Our objective is to use PMU measurements $\boldsymbol{y}_j(t)$ to estimate σ_i, Ω_i, and $r_{i,j}$ for $i = 1, \ldots, n$.
- Least-Squares based Prony algorithm
- Let us consider the discrete-time transfer function from $d(t)$ to $\Delta\theta_i(t)$ assuming $d(t)$ to be an impulse

$$\Delta\theta_i(t) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{2n} z^{-2n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{2n} z^{-2n}}$$

Wide-Area Oscillation Estimation

Step 1. Find a_1 through a_{2n}

$$\underbrace{\begin{bmatrix} \Delta\theta_i(2n) \\ \Delta\theta_i(2n+1) \\ \vdots \\ \Delta\theta_i(2n+l) \end{bmatrix}}_{\mathbf{c}_i} = \underbrace{\begin{bmatrix} \Delta\theta_i(2n-1) & \cdots & \Delta\theta_i(0) \\ \Delta\theta_i(2n) & \cdots & \Delta\theta_i(1) \\ \vdots & & \vdots \\ \Delta\theta_i(2n+l-1) & \cdots & \Delta\theta_i(l) \end{bmatrix}}_{H_i} \underbrace{\begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_{2n} \end{bmatrix}}_{\mathbf{a}}$$

Finding the global **a** using all available measurements using simple linear LS:

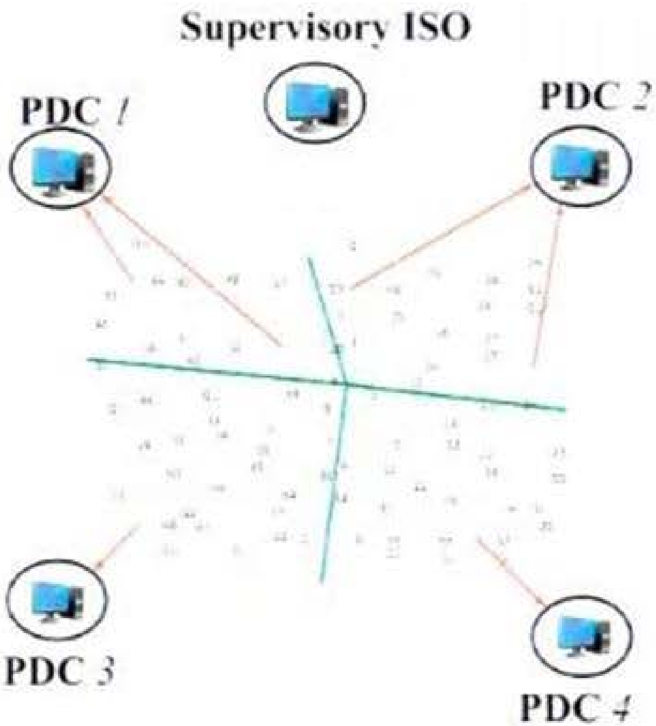
$$\theta_i \rightarrow (H_i, \mathbf{c}_i),\ i = 1, \ldots, p$$

$$\Rightarrow \begin{bmatrix} \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_p \end{bmatrix} = \begin{bmatrix} H_1 \\ \vdots \\ H_p \end{bmatrix} \mathbf{a}$$

$$\Rightarrow \mathbf{a} = \arg\min_{\mathbf{a}} \frac{1}{2} \left\| \begin{bmatrix} H_1 \\ \vdots \\ H_p \end{bmatrix} \mathbf{a} - \begin{bmatrix} \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_p \end{bmatrix} \right\|_2^2$$

→ Solve characteristic polynomial from $\boldsymbol{a}$

Distributing the Prony Algorithm via Consensus



Multiple Computational Areas

Area 1: $\hat{\theta}_1 = \{\theta_{30}, \theta_{66}\} \rightarrow (\hat{H}_1 = \begin{bmatrix} H_{30} \\ H_{66} \end{bmatrix}, \hat{\mathbf{c}}_1 = \begin{bmatrix} \mathbf{c}_{30} \\ \mathbf{c}_{66} \end{bmatrix})$

Area 2: $\hat{\theta}_2 = \{\theta_{16}, \theta_{53}\} \rightarrow (\hat{H}_1 = \begin{bmatrix} H_{16} \\ H_{53} \end{bmatrix}, \hat{\mathbf{c}}_1 = \begin{bmatrix} \mathbf{c}_{16} \\ \mathbf{c}_{53} \end{bmatrix})$

Area 3: $\hat{\theta}_3 = \{\theta_{68}\} \rightarrow (\hat{H}_3 = H_{68}, \hat{\mathbf{c}}_3 = \mathbf{c}_{68})$

Area 4: $\hat{\theta}_4 = \{\theta_{56}\} \rightarrow (\hat{H}_4 = H_{56}, \hat{\mathbf{c}}_4 = \mathbf{c}_{56})$

Global Consensus Problem:

$$\underset{\mathbf{a}_1, \ldots, \mathbf{a}_N, \mathbf{z}}{\text{minimize}} \sum_{i=1}^{N} \frac{1}{2} \left\| \hat{H}_i \mathbf{a}_i - \hat{\mathbf{c}}_i \right\|_2^2$$

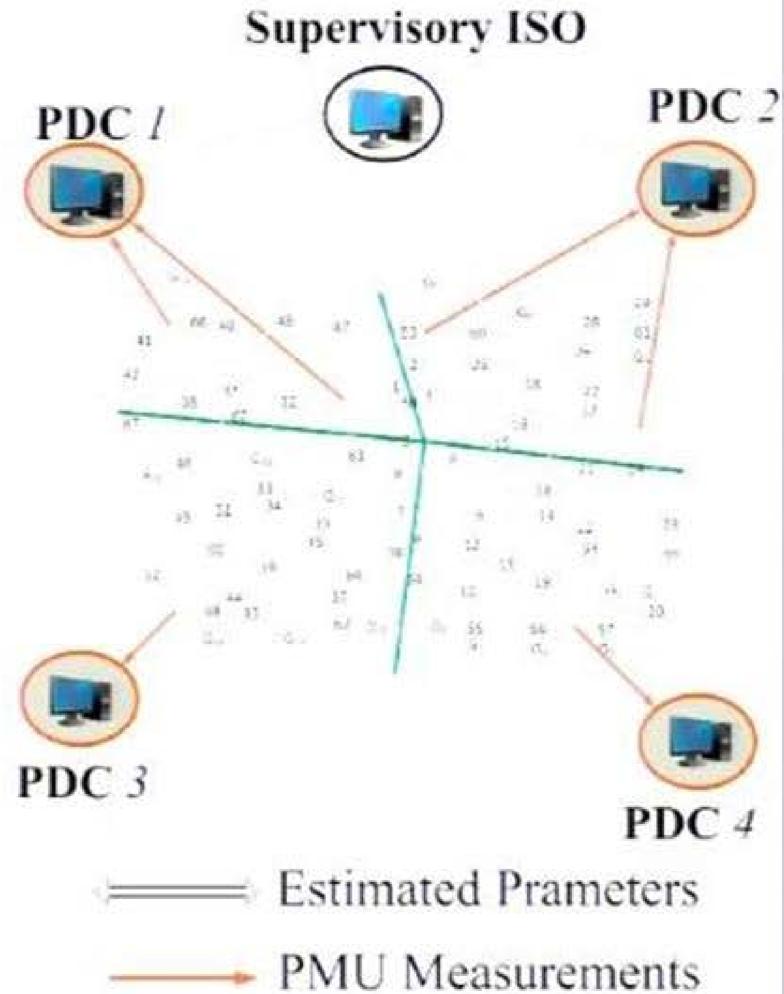
subject to $\mathbf{a}_i - \mathbf{z} = 0$, for $i = 1, \ldots, N$

Solve in a distributed way using:
Alternating Direction Method of Multipliers (ADMM)

Distributed Prony Using ADMM

Iteration 0

Initialize the primal variable $\mathbf{a}_i^{\mathbf{0}}$ and the dual variable $\mathbf{w}_i^{\mathbf{0}}$ at each local PDC i



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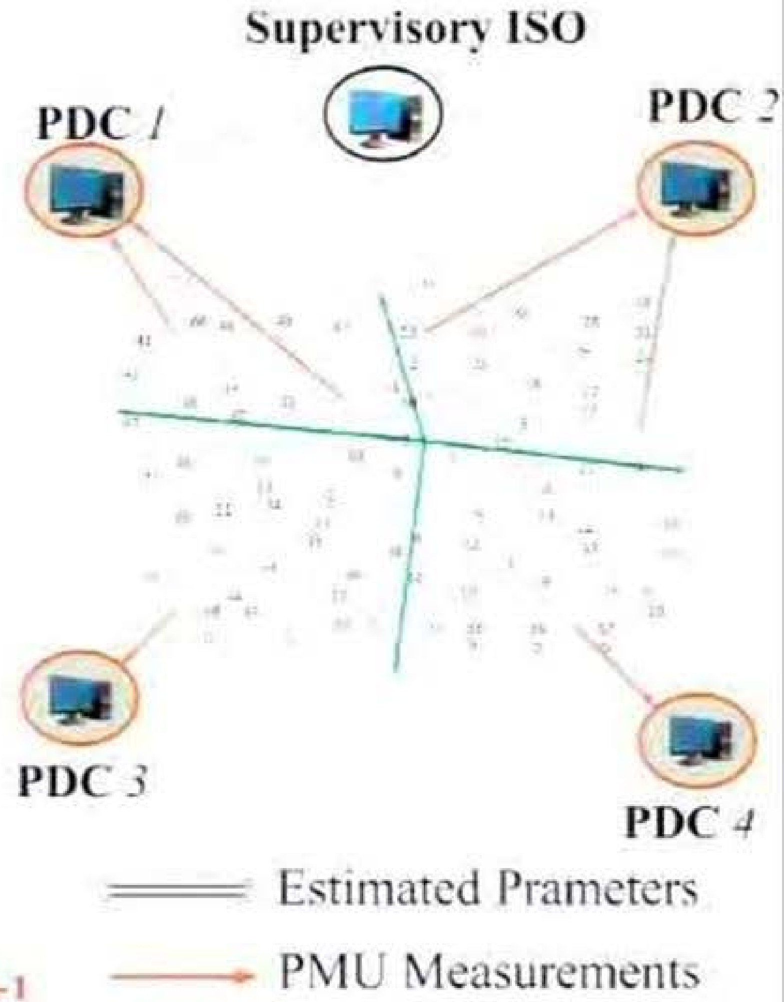
Distributed Prony Using ADMM

Iteration $k+1$

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC i

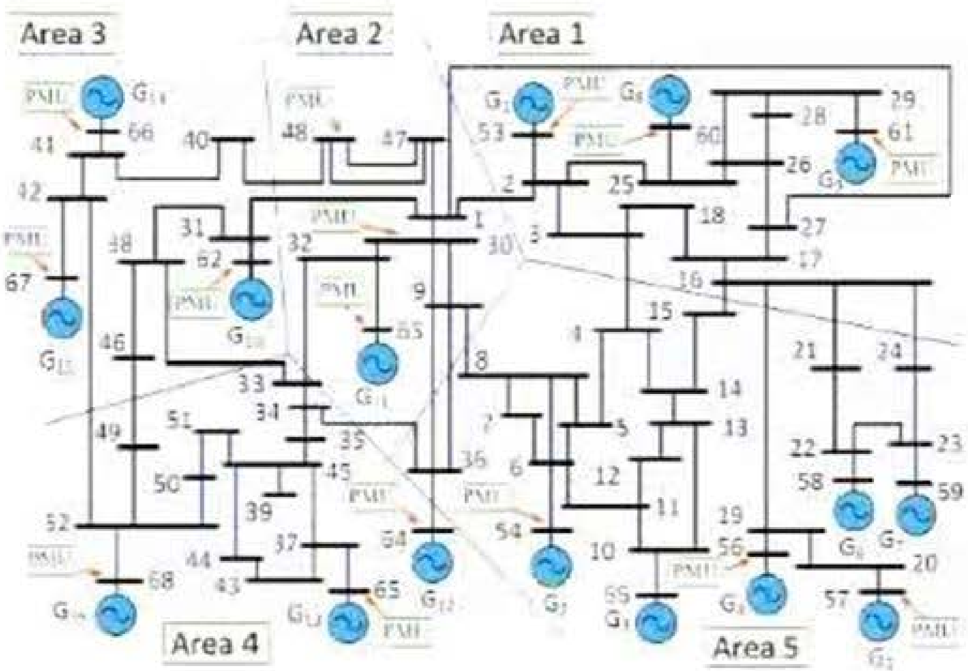
$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$
$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

- Step 2 Gather the values of $\mathbf{a}_i^{k+1}$ at the central PDC
- Step 3 Take the average of $\mathbf{a}_i^{k+1}$
- Step 4 Broadcast the average value ($\bar{\mathbf{a}}_i^{k+1}$) to local PDCs
- Step 5 Check the convergence
- Final Step Find the frequency Ω_i, and damping σ_i at each local PDC using $\bar{\mathbf{a}}_i^{k+1}$



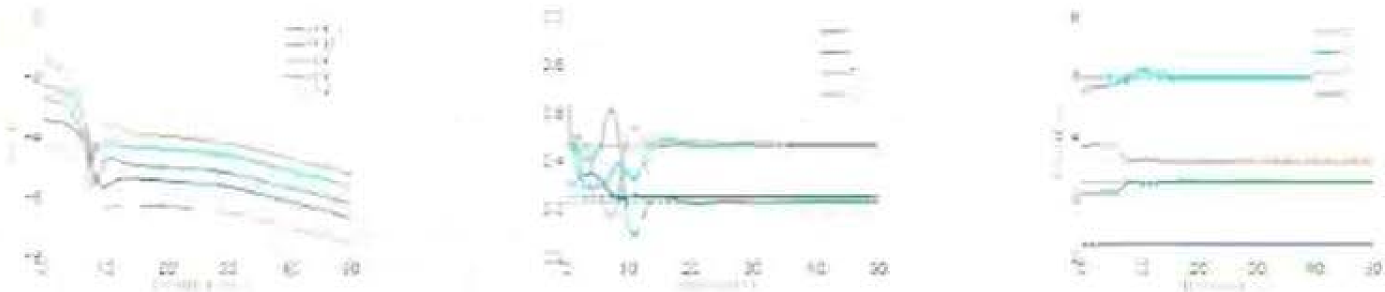
Simulation Results

IEEE-68 Bus Model (simplified model of the New-England power system)

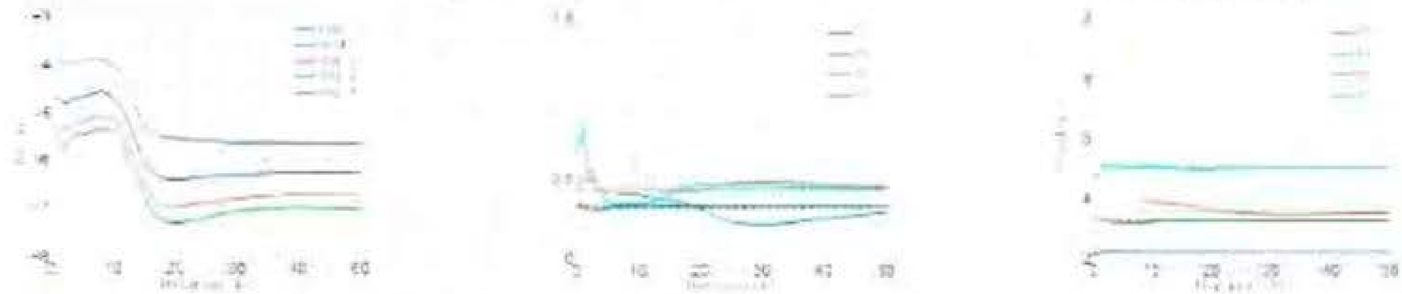


- 68 Bus, 16 Generators
- 5 Computational Areas
- Simulations are performed in Power System Toolbox (PST)
- A three-phase fault occurred at line connecting buses 1 and 2, started at t=0.1 (sec), cleared at bus 1 at t=0.15 (sec), and cleared at bus 2 at t=0.2 (sec).

Distributed Prony:



In Case of Communication Failure (1 healthy communication link in 10 iterations)



Actual value	Centralized Prony	Diistributed Prony	Distributed Prony with Comm Failure
-0.3256 □ j2.2262	-0.3250 □ j2.2230	-0.3247 □ j2.2230	-0.3243 □ j2.2225
-0.3143 □ j3.2505	-0.3146 □ j3.2531	-0.3153 □ j3.2525	-0.2808 □ j3.2560
-0.4312 □ j3.5809	-0.4318 □ j3.5849	-0.4328 □ j3.5855	-0.4443 □ j3.5106
-0.4301 □ j4.9836	-0.4308 □ j4.9865	-0.4294 □ j4.9798	-0.4361 □ j4.9853

Distributed Prony Using ADMM

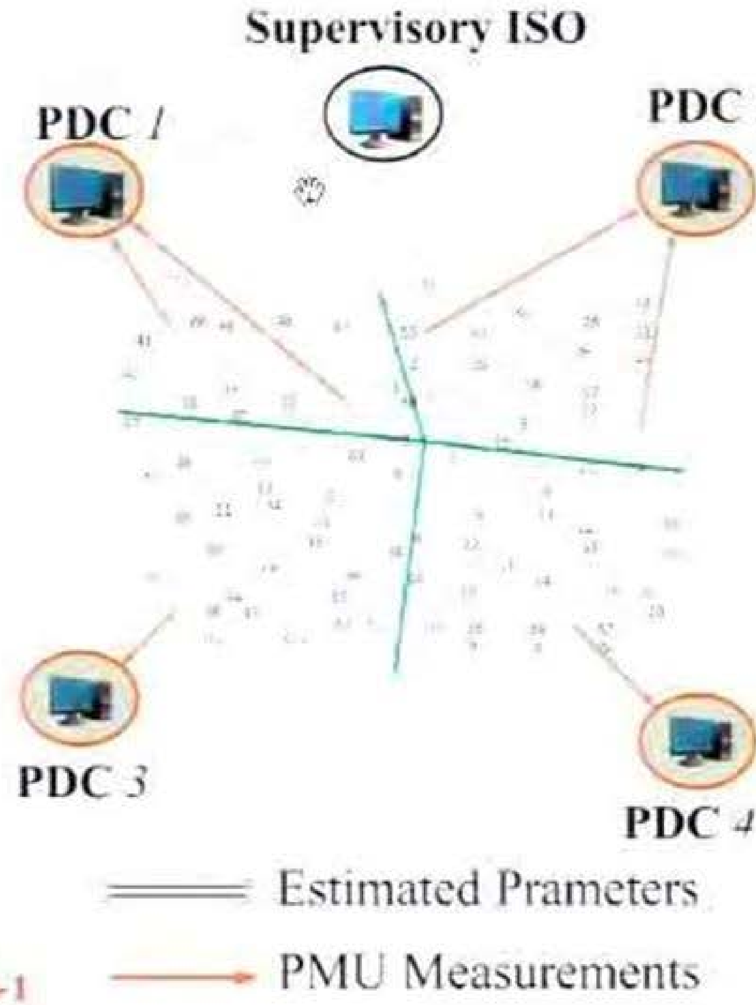
Iteration $k+1$

- Step 1 Update $\mathbf{a}_i$ and $\mathbf{w}_i$ locally at PDC i

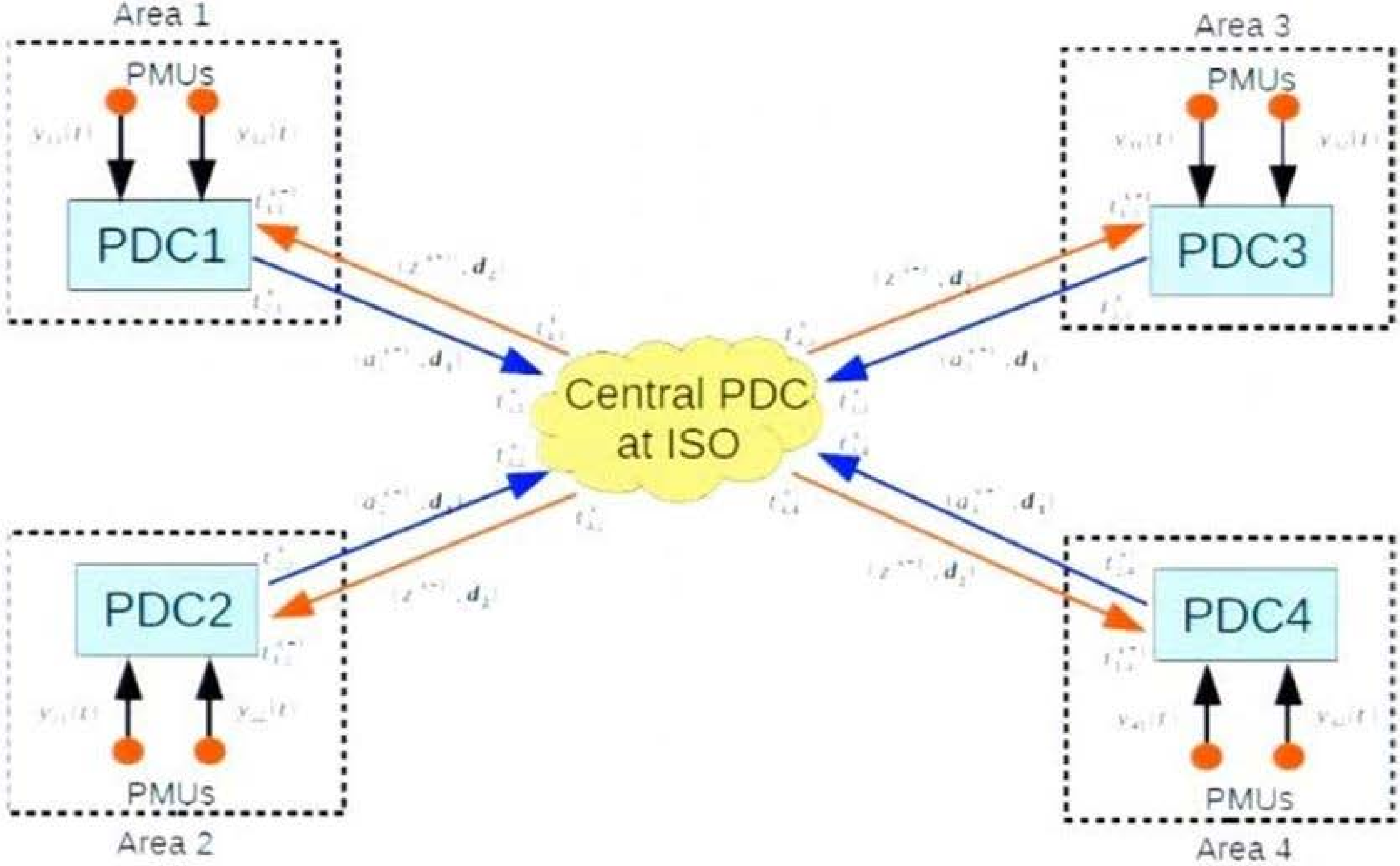
$$\mathbf{a}_i^{k+1} = ((H_i^k)^T H_i^k + \rho I)^{-1}((H_i^k)^T \mathbf{c}_i^k - \mathbf{w}_i^k + \rho \bar{\mathbf{a}}^k)$$

$$\mathbf{w}_i^{k+1} = \mathbf{w}_i^k + \rho(\mathbf{a}_i^{k+1} - \bar{\mathbf{a}}^{k+1})$$

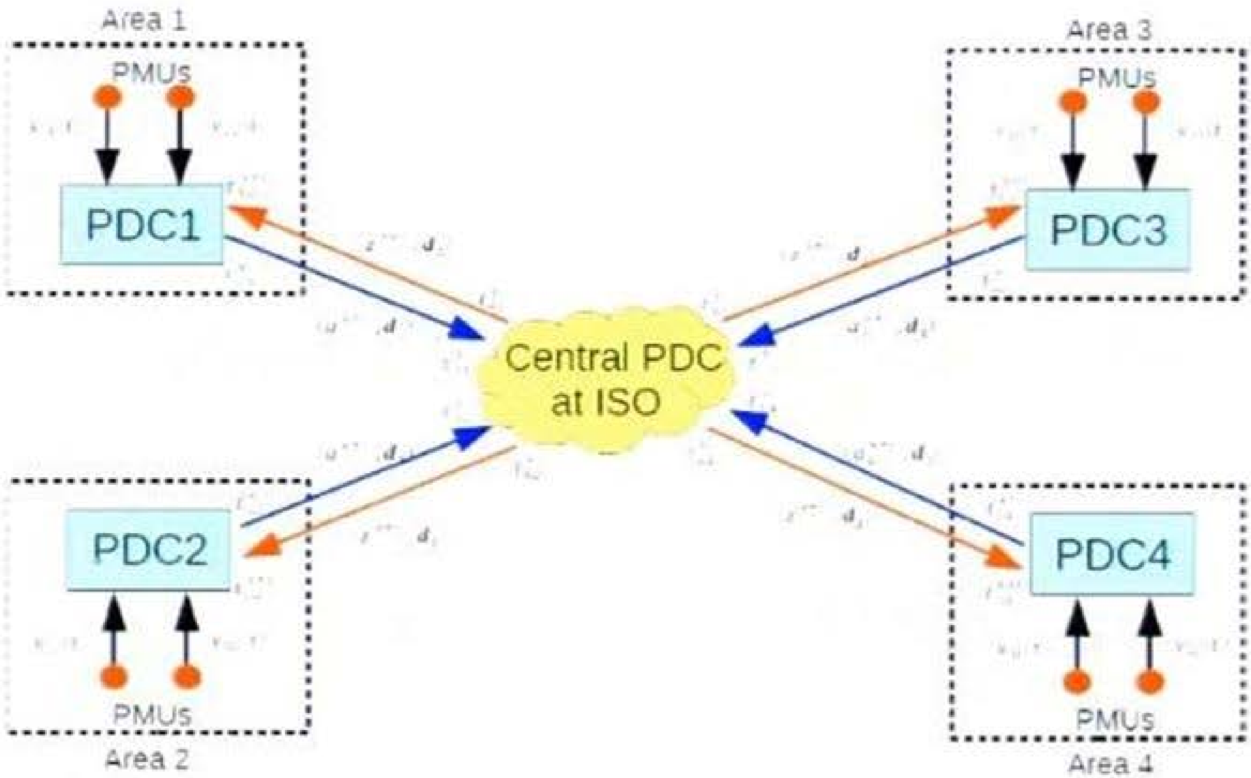
- Step 2 Gather the values of $\mathbf{a}_i{}^{k+1}$ at the central PDC
- Step 3 Take the average of $\mathbf{a}_i{}^{k+1}$
- Step 4 Broadcast the average value $(\bar{\mathbf{a}}_i{}^{k+1})$ to local PDCs
- Step 5 Check the convergence
- Final Step Find the frequency Ω_i, and damping σ_i at each local PDC using $\bar{\mathbf{a}}_i{}^{k+1}$



Incorporating Asynchronous Communication



Incorporating Asynchronous Communication



Traffic Models for Internet Delays:

$$P(t) = \frac{1}{2}[\mathrm{erf}(\frac{\mu}{\sqrt{2}\sigma}) + \mathrm{erf}(\frac{t-\mu}{\sqrt{2}\sigma})] + \frac{(1-p)}{N} e^{(\frac{1}{2}\lambda^2\sigma^2 + \mu\lambda)} [\mathrm{erf}(\frac{\lambda\sigma^2 + \mu}{\sqrt{2}\sigma}) + \mathrm{erf}(\frac{t - \lambda\sigma^2 - \mu}{\sqrt{2}\sigma})]$$

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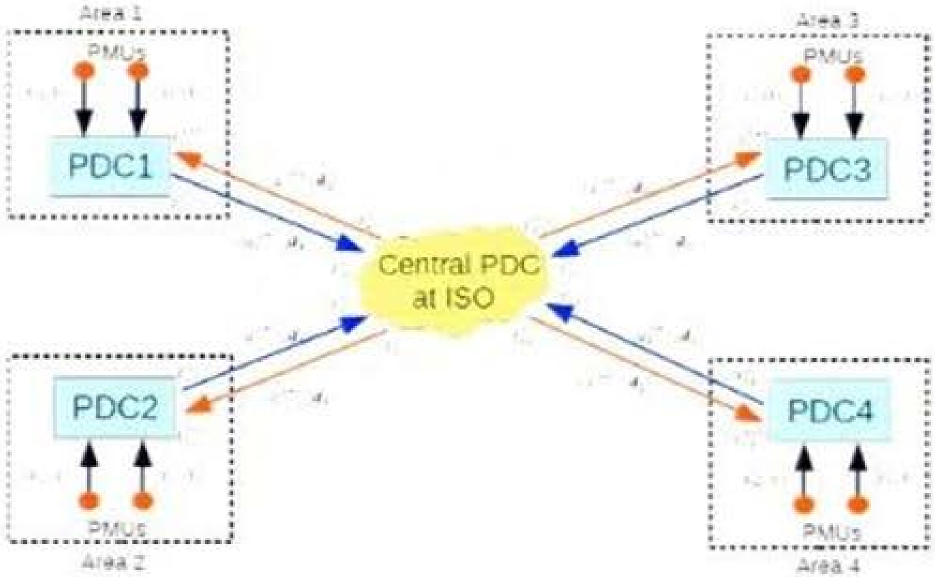
Incorporating Asynchronous Communication



IEEE PES General Meeting, 2015:

If message doesn't arrive at ISO by a delay threshold d_1*

- **Strategy 1:**

$$z^{(k+1)} = \frac{1}{|S_1^{(k)}|} \sum_{i \in S_1^{(k)}}^{N} (a_i^{(k+1)} + \frac{1}{\rho} w_i^{(k)})$$

→ Can easily lead to divergence

- **Strategy 2:**

$$z^{(k+1)} = \frac{1}{N} (\sum_{i \in S_1^{(k)}}^{N} (a_i^{(k+1)} + \frac{1}{\rho} w_i^{(k)}) + \sum_{i \in S_2^{(k)}}^{N} (a_i^{(k)} + \frac{1}{\rho} w_i^{(k-1)}))$$

Convergent, but slow

Substitute values from previous iteration

→ Modify dual update by ***a gradient term***: $w_i^{(k)} = w_i^{(k-1)} + \rho(a_i^{(k)} - (z^{(k-1)} + \gamma(z^{(k-1)} - z^{(k-2)})))$. $i \in S_2^{(k)}$

Testbed Integration

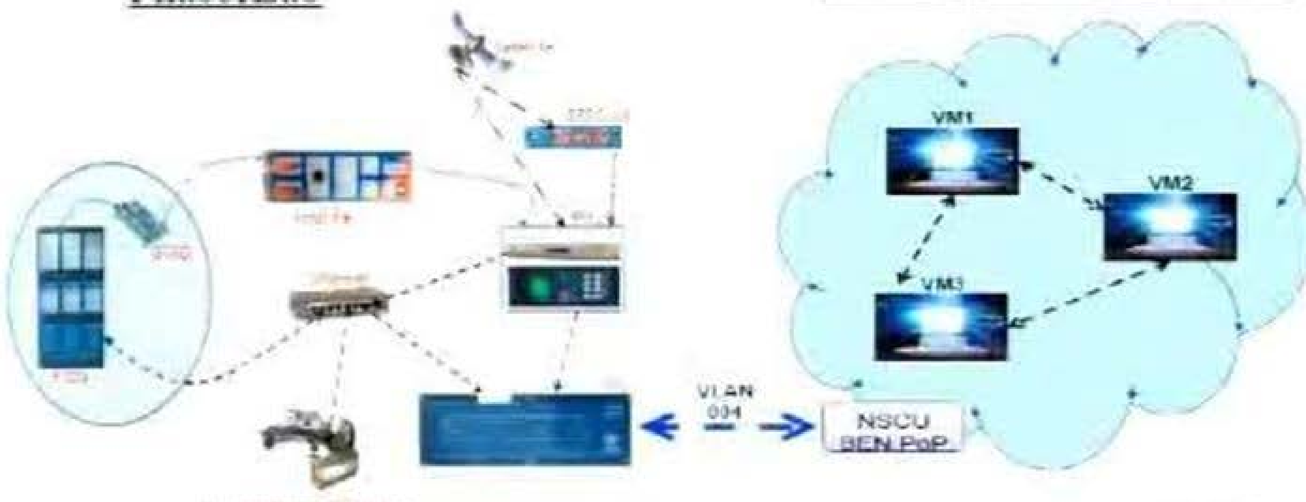
Smart America Challenge Summit June 2014

SMART AMERICA

Iowa State, NC State, USC, UNC
NI, Mitre, NREL, Scitor Corp.

PMU+ Real-Time Digital Simulators

PhasorLab | Exo-GENI + DETERLab



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Conclusions

1. WAMS is a tremendously promising technology for control researchers
2. Control + Communications + Computing (CPS) must merge
3. Plenty of new research problems – EE, Applied Math, Computer Science
4. Plenty of new distributed optimization and control problems
5. Both theory and testbed experiments must progress
6. Right time to think mathematically – Network theory is imperative electric grid
7. Needs participation of young researchers!
8. Promises to create jobs and provide impetus to power engineering