Wind-induced instability of a suspension bridge: a tale of two frequencies

Kevin Daley[1], Vladimir Belykh[2], and Igor Belykh[1]

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SIAM Conference on Applications of Dynamical Systems, Snowbird, May 20, 2019

Outline

- Examples of wind-induced instabilities: The Tacoma Narrows Bridge, and the Volga Bridge

- Wind-induced vibrations of a bridge at a frequency **different** from the natural frequency of the bridge girder (the Tacoma bridge case).

- Parallels and differences between crowd and wind-induced synchrony.

- A synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements causes the onset of bridge oscillations and can explain the shift of the resonant frequency.

Tacoma Narrows (1940)

A transverse twisting mode emerged from **mild** winds (about 40 miles per hour)

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by hydraulic mass dampers

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Vortex Shedding and Instability

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Vortex-induced vibrations in a section model of Hardanger Bridge, Norway

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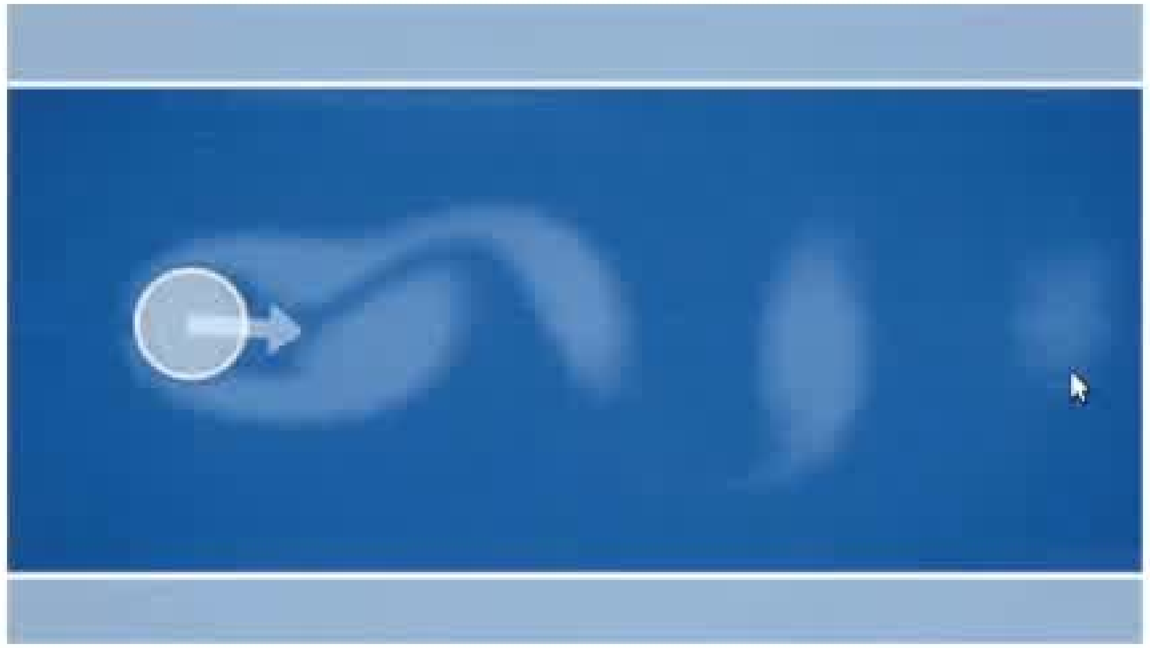

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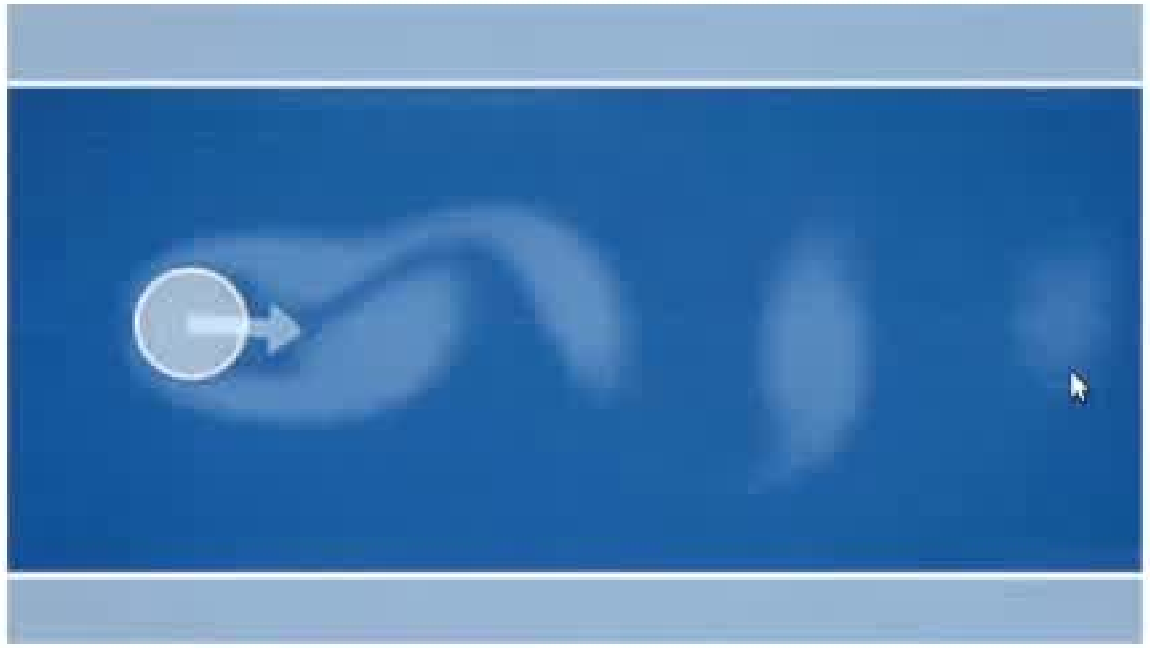

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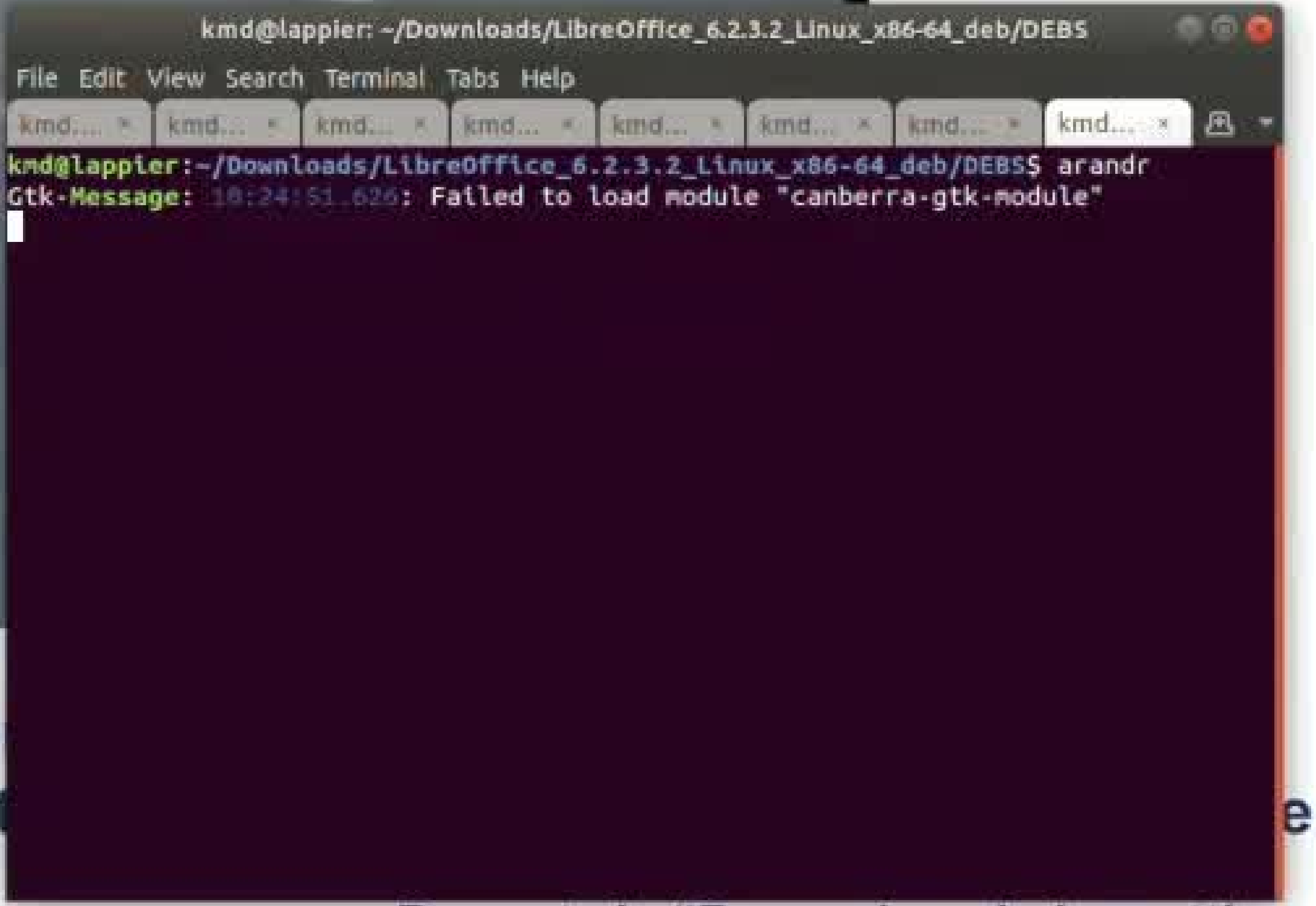

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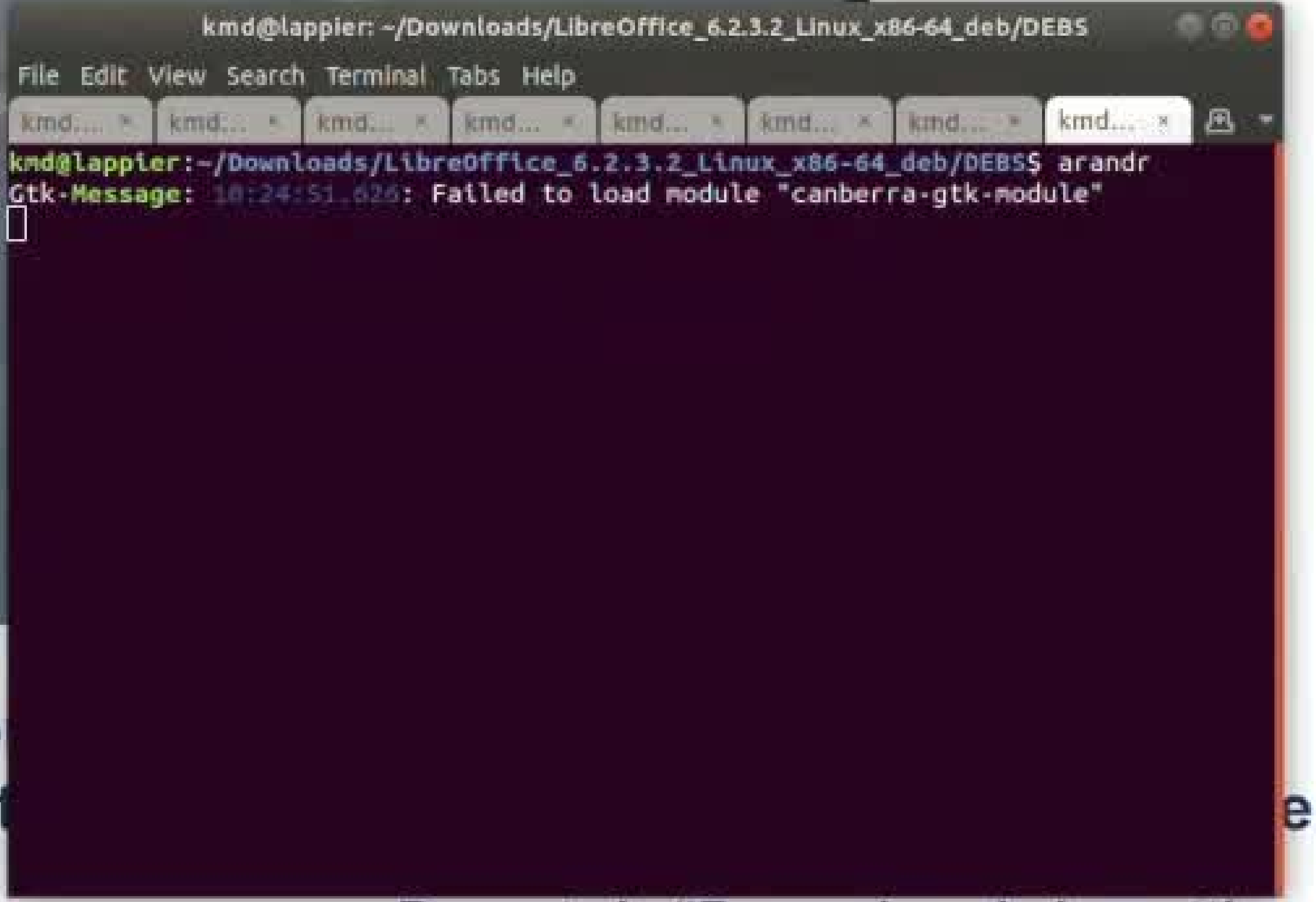

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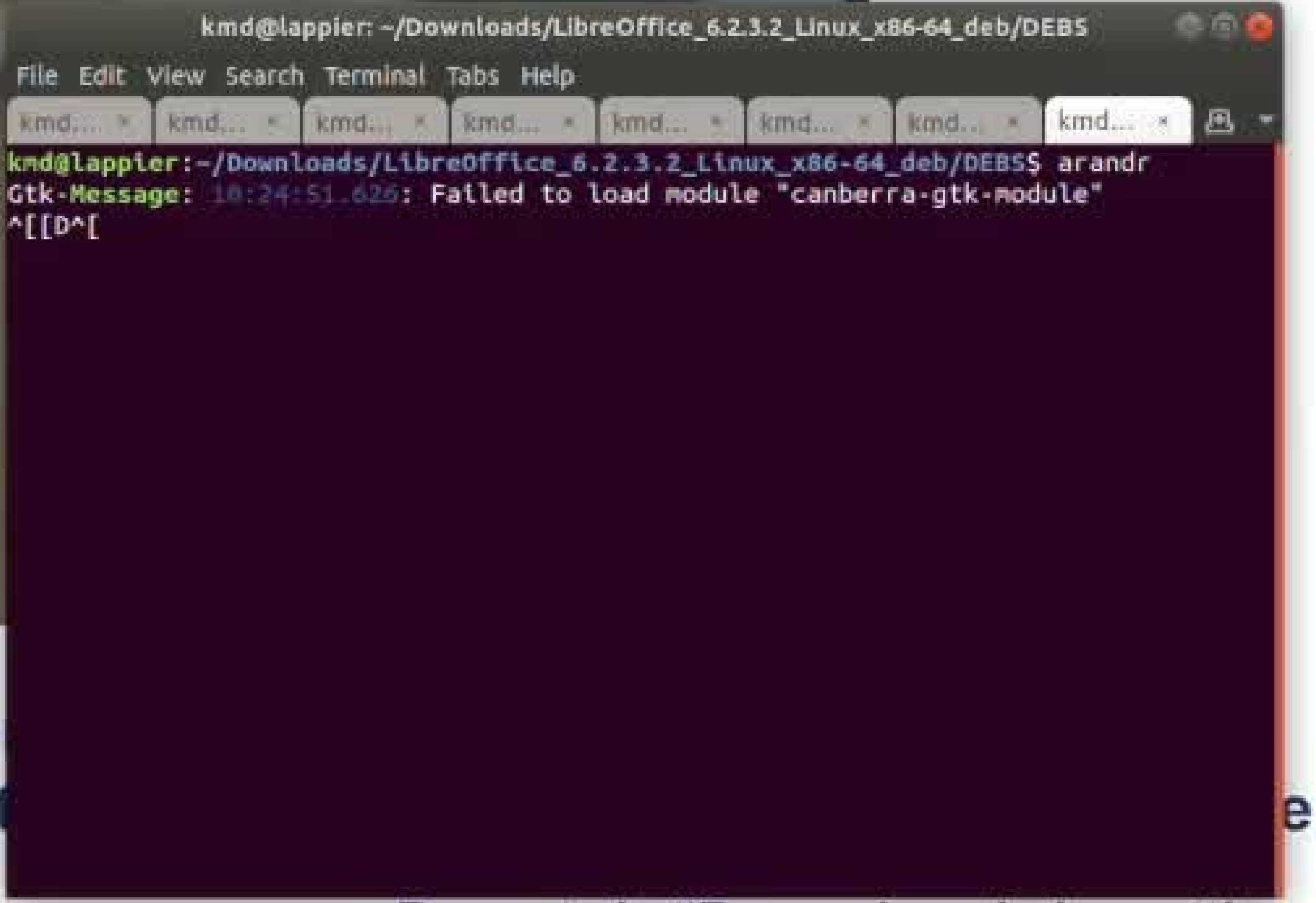

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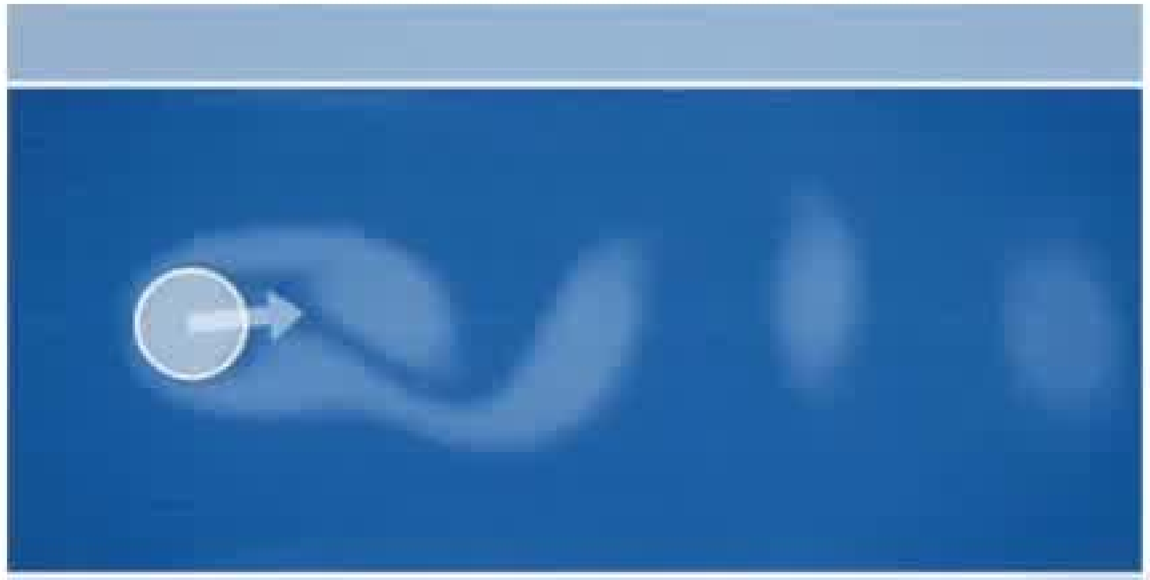

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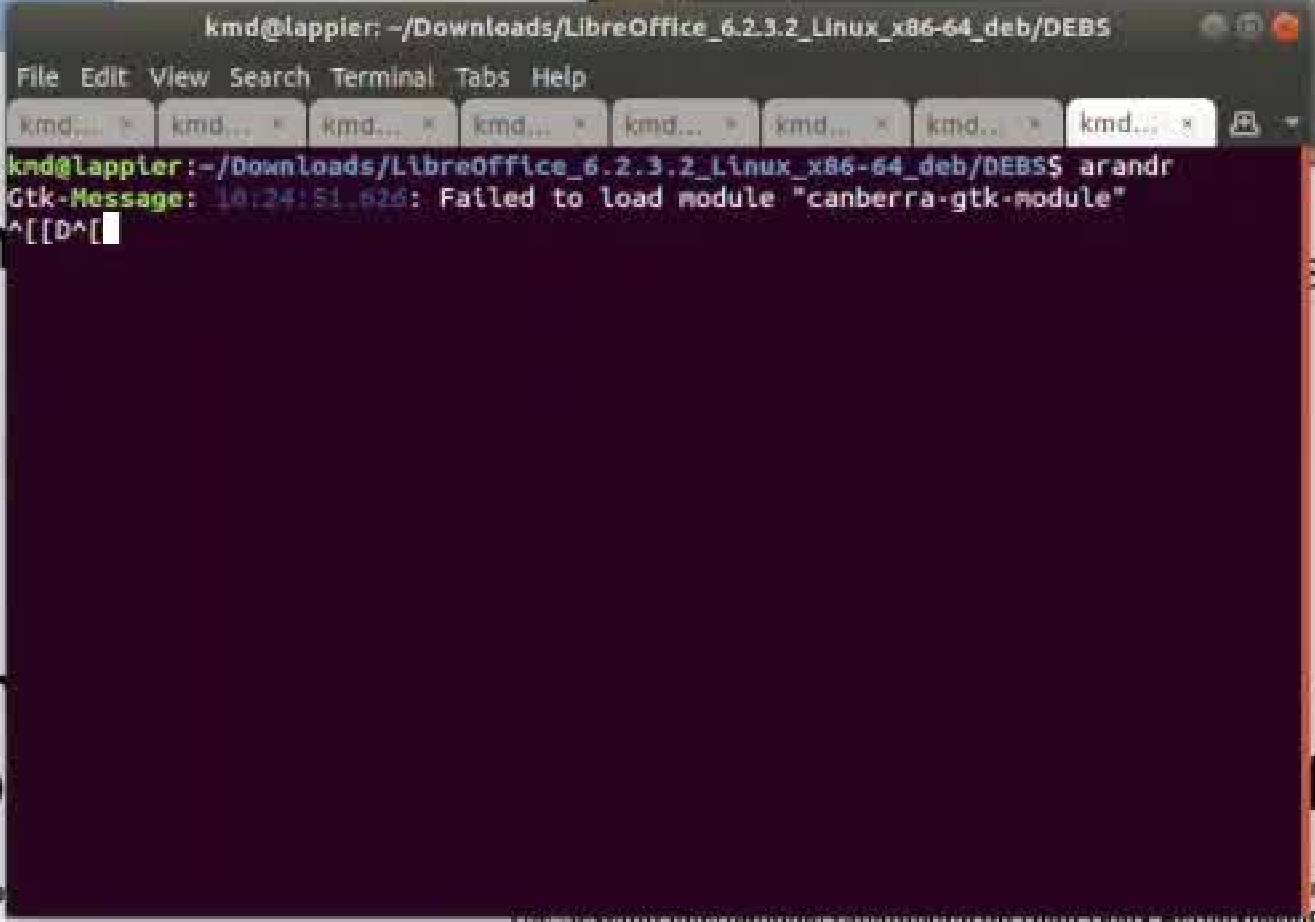

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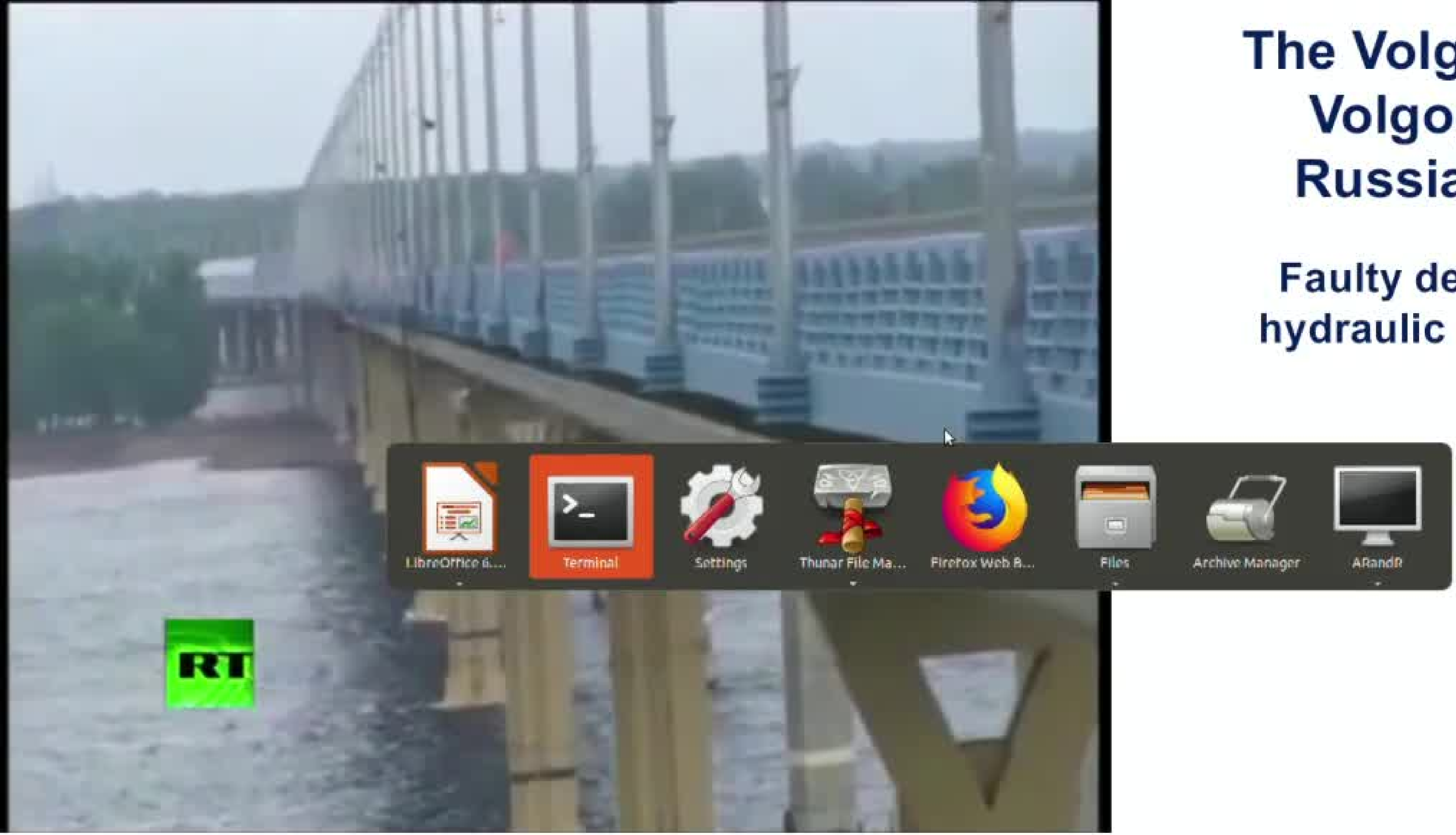

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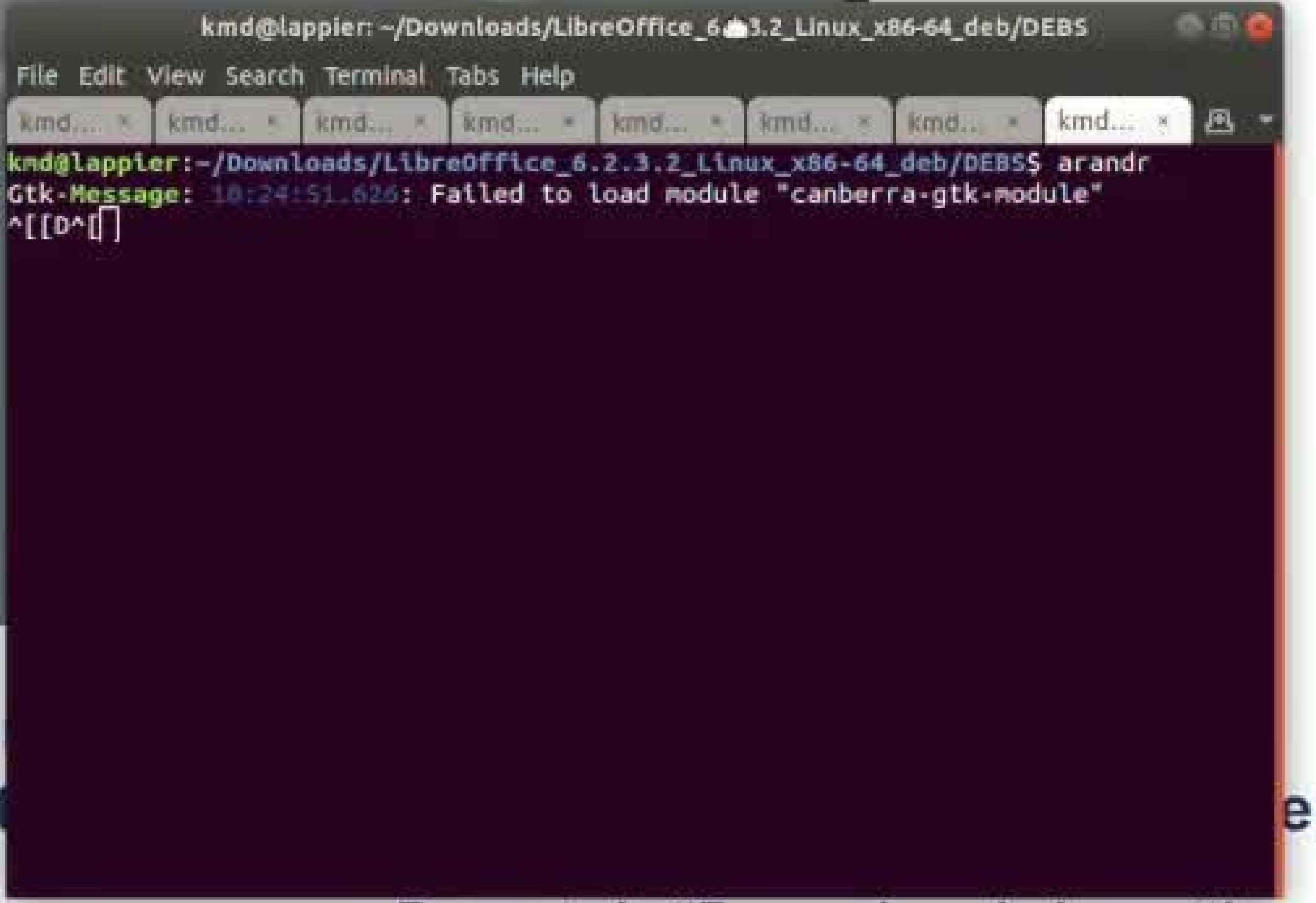

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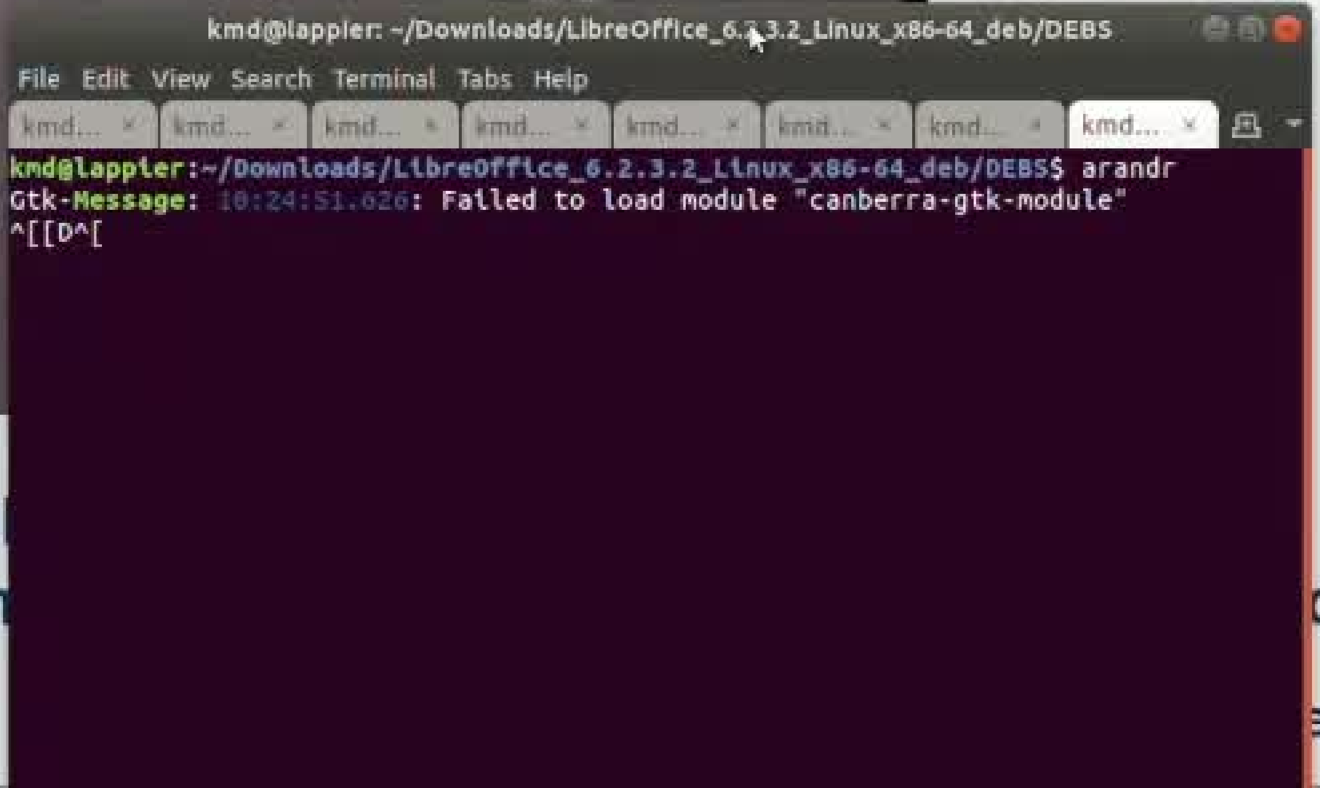

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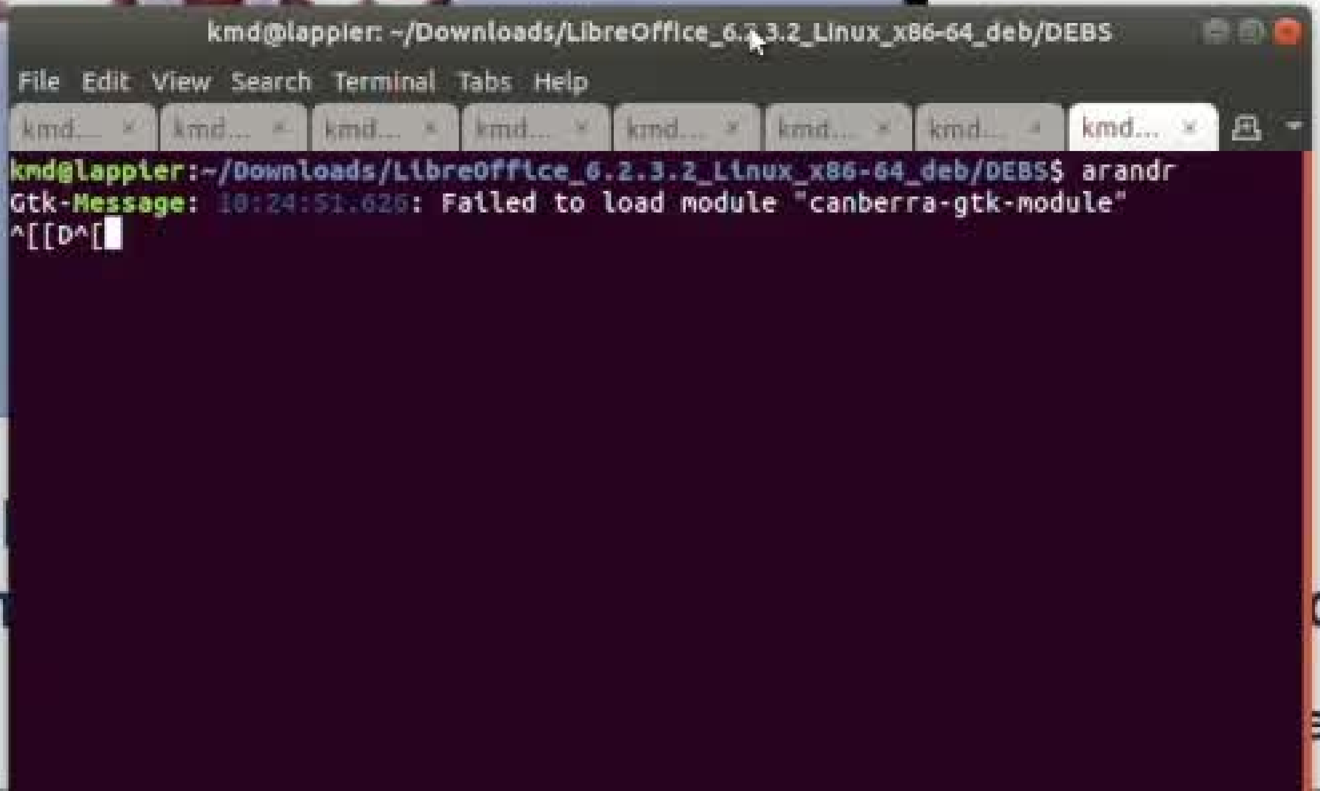

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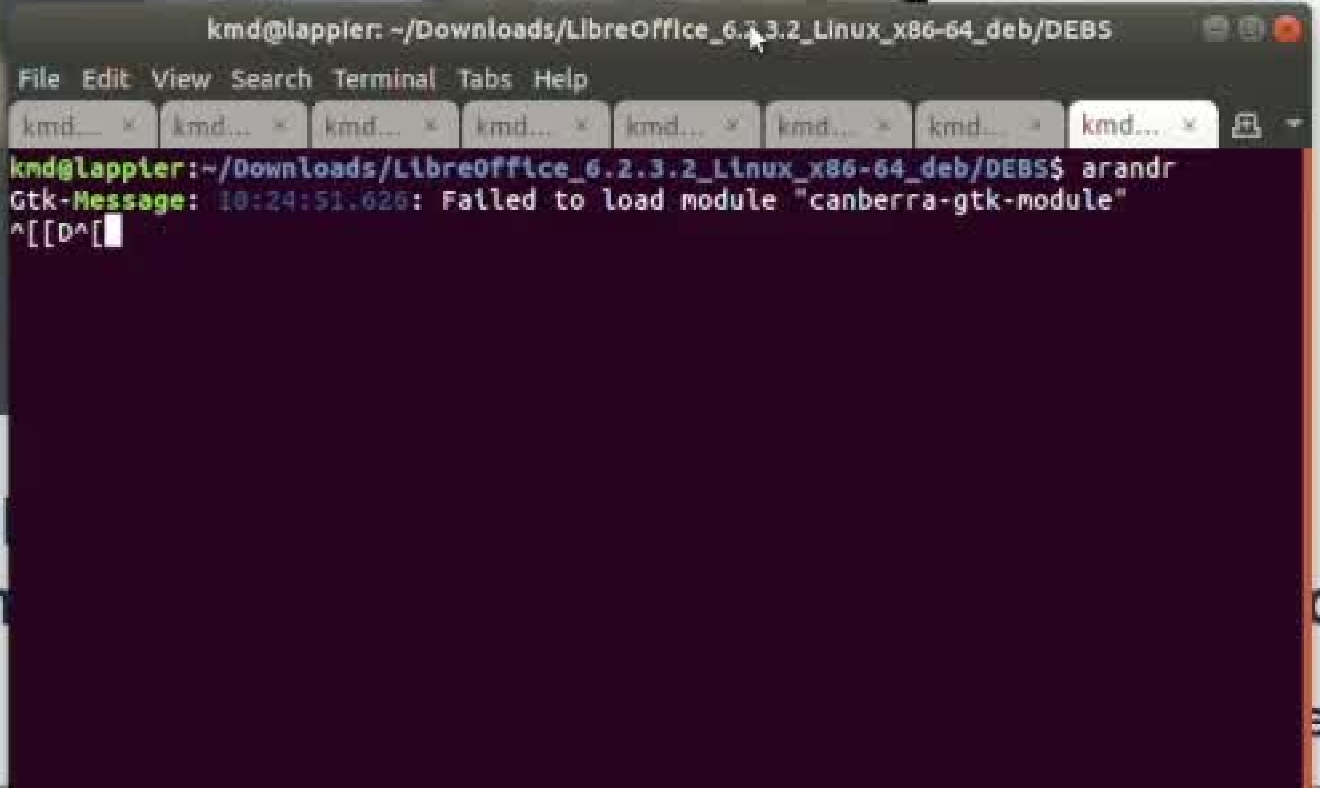

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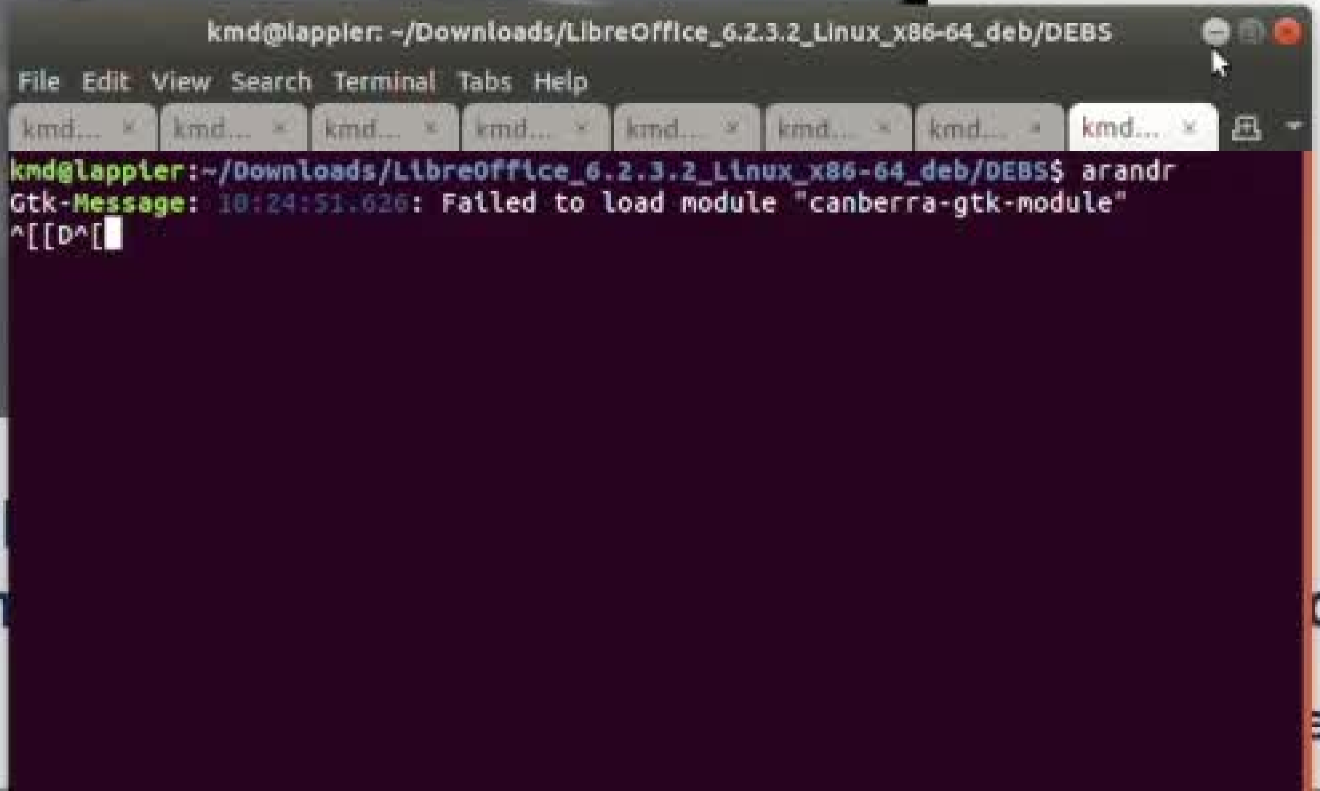

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Modeling lateral bridge vibrations

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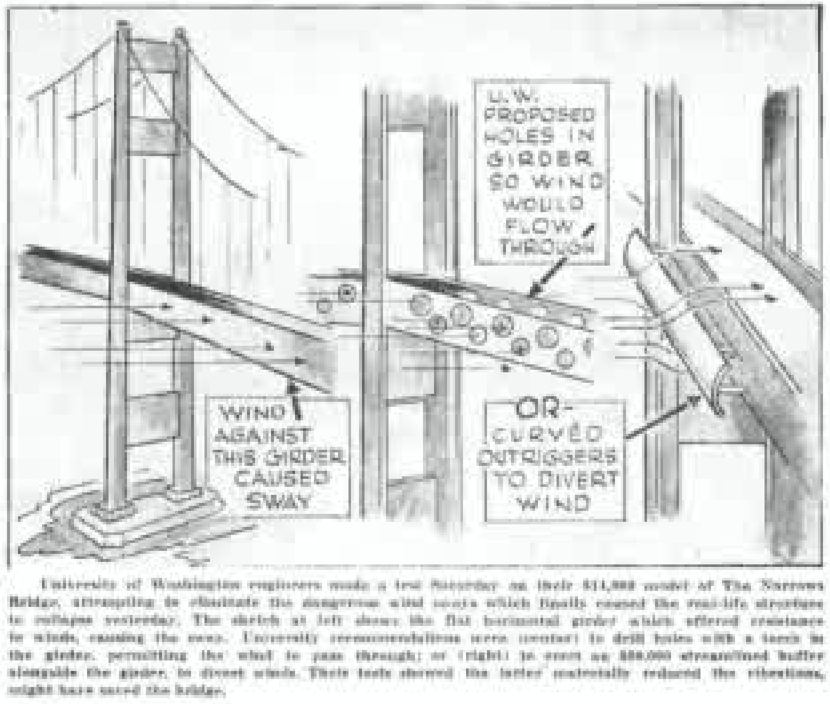

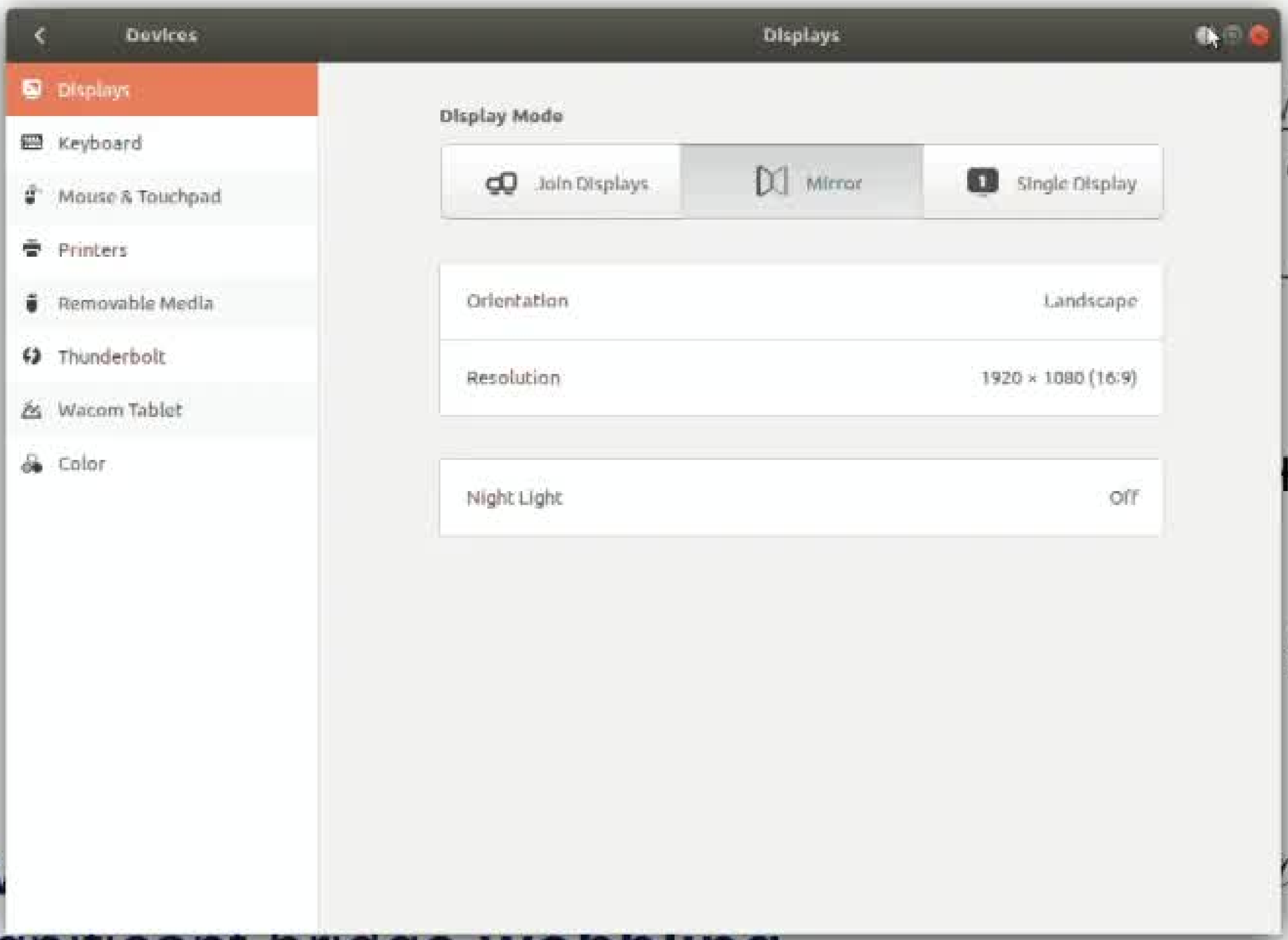

Display Mode: Mirror

Orientation Landscape

Resolution 1920 × 1080 (16:9)

Night Light Off

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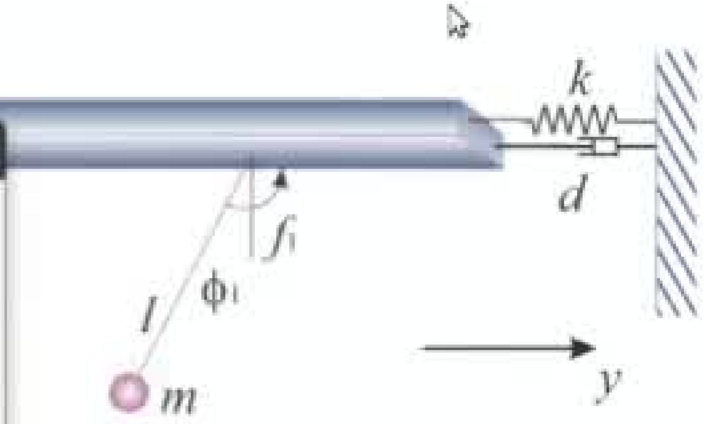

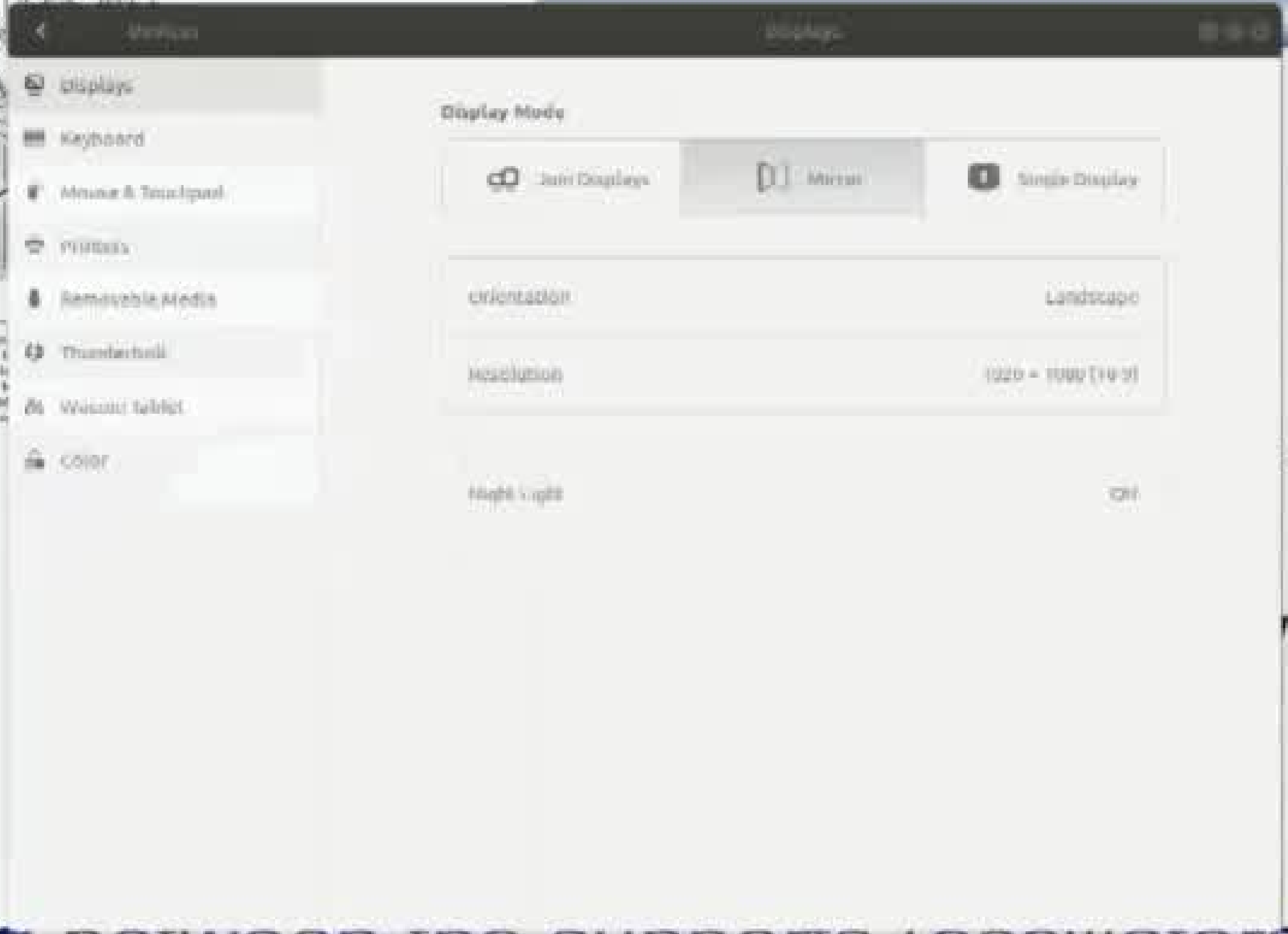

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Crowd vs. wind- load of a suspension bridge

Crowd loading (the London Millennium Bridge case): Phase-locking among pedestrians is **not** the cause of bridge wobbling[1], but rather a consequence. Pedestrians adjust their gaits to maintain balance and destabilize the bridge via the negative damping mechanism.

Wind loading (the Tacoma Bridge case): Our synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements can explain the shift of the resonant frequency.

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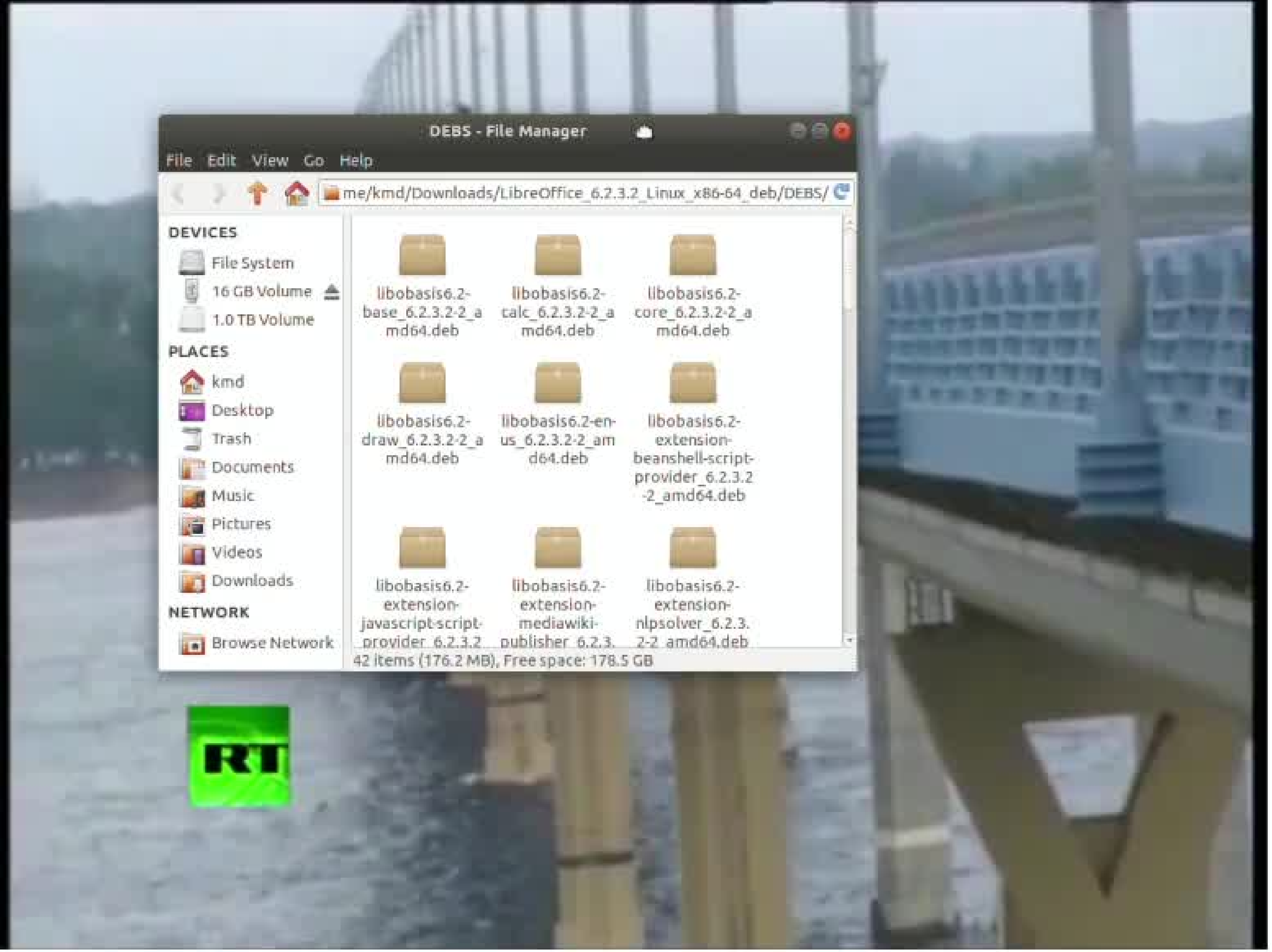

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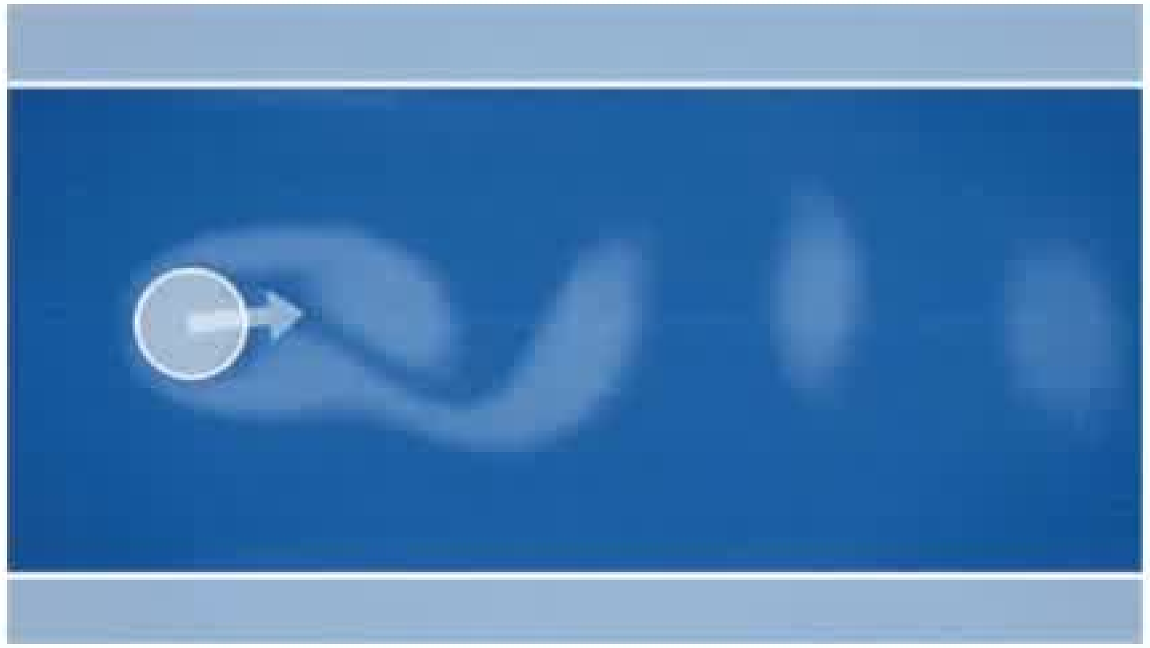

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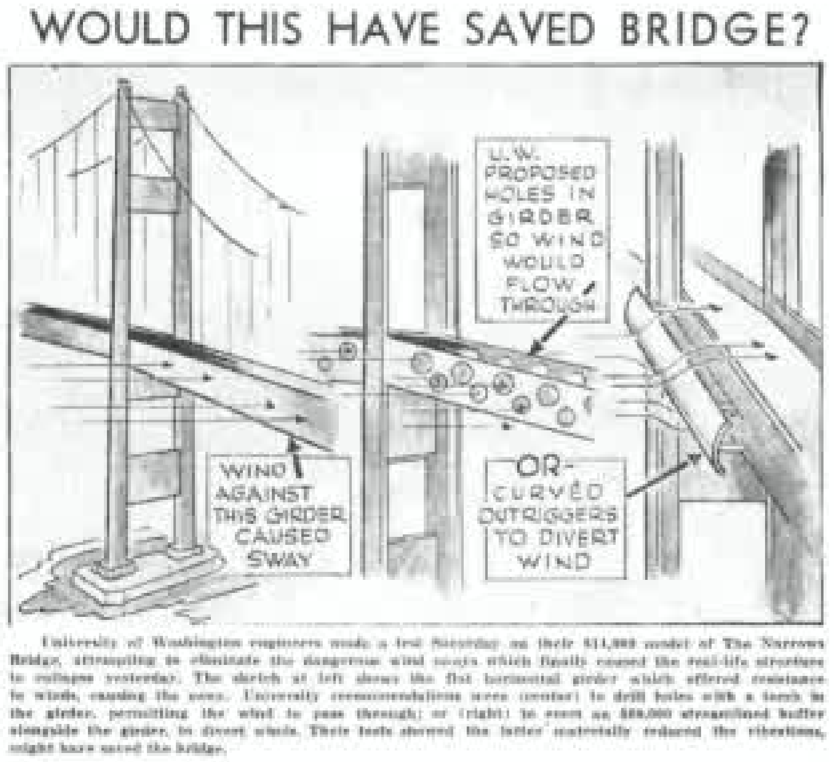

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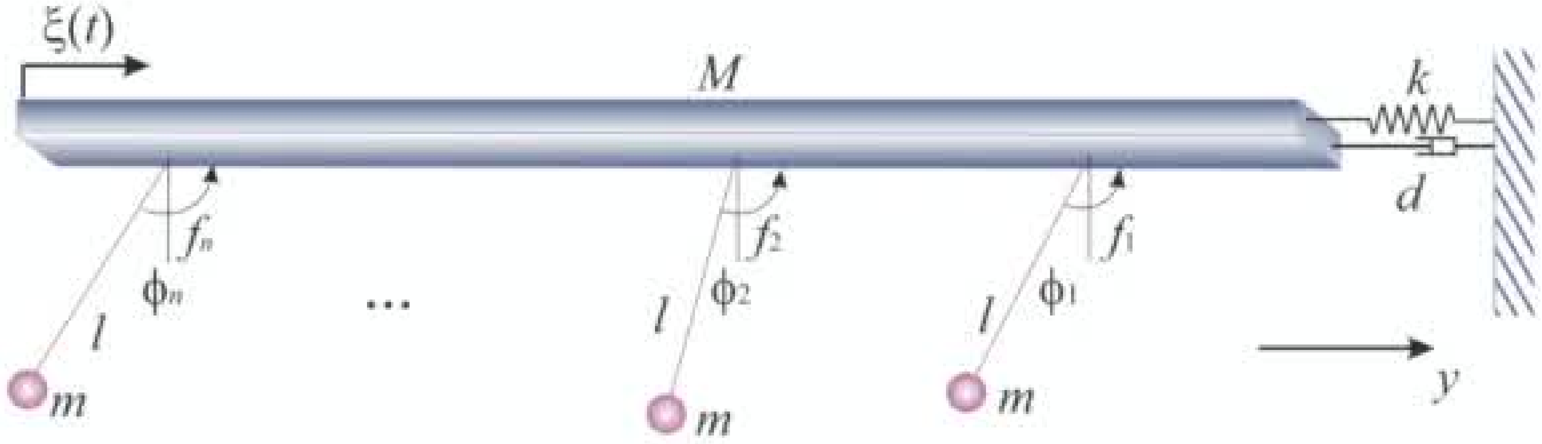

$$ml^2\ddot{\phi}_i + mgl\sin\phi_i + f(\phi_i,\ddot{\phi}_i) = -ml\ddot{y}\cos\phi_i + \xi(t),$$

$$(M + nm)\ddot{y} + d\dot{y} + ky = -ml\sum_{i=1}^{n}(\ddot{\phi}_i\cos\phi_i - \dot{\phi}_i\sin\phi_i) + \xi(t)$$

Phase-locking between the supports (oscillators): $|\varphi_i - \varphi_j| < \varphi_{syn} < \pi/2$
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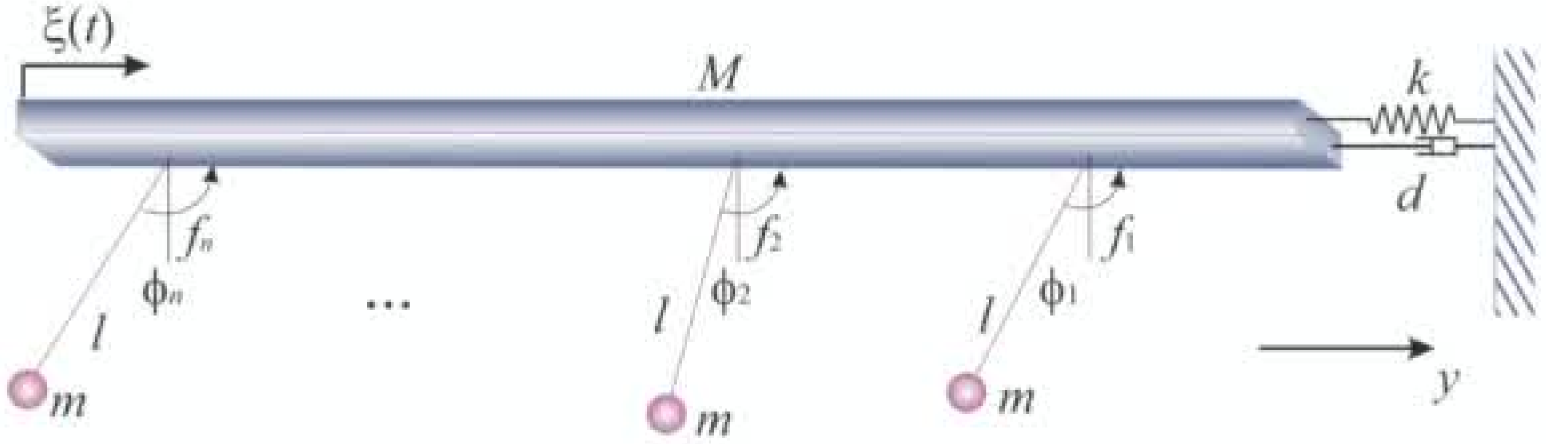

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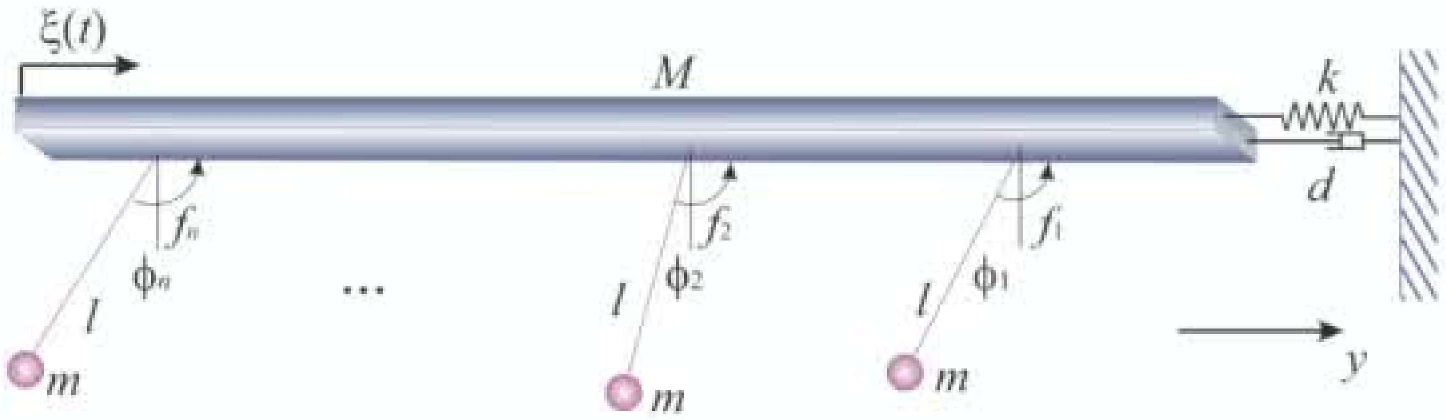

The angular displacement can be approximated by side-to-side motion:

$$x_i = l\phi_i$$

$$\ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = -\ddot{y} + \boxed{\gamma_1 \operatorname{sign}(\dot{x}),}$$

$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -r\sum_{i=1}^{n}\ddot{x}_i + \boxed{\gamma_2 \operatorname{sign}(\dot{y})}\ \boxed{+ A\sin(\beta t)}$$

force of wind gusts

force due to vortex shedding

The **signum terms** account for **von Kármán vortex shedding** behind the bridge, causing side-to-side vibrations.

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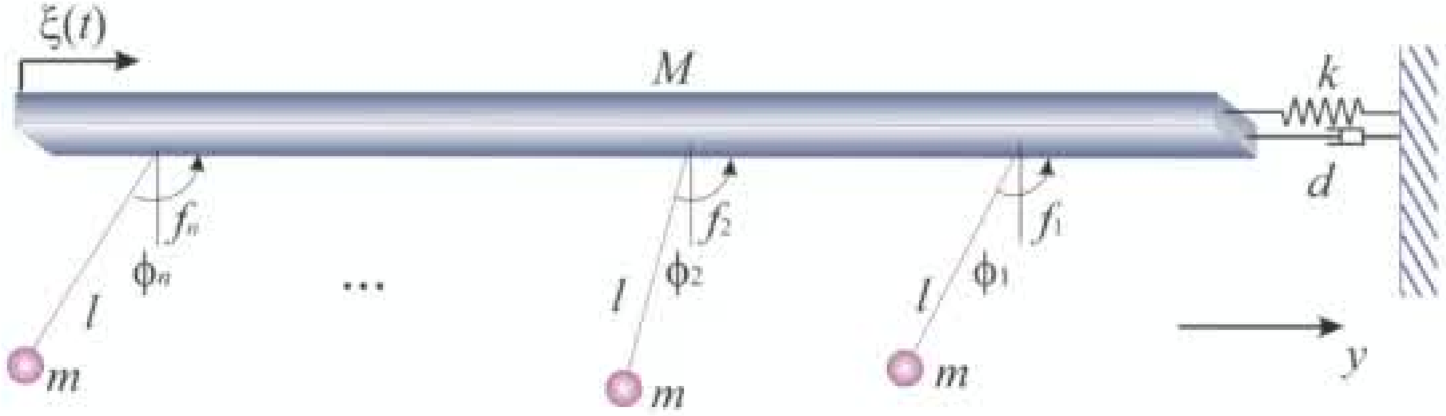

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No bridge movement *(y=0):*

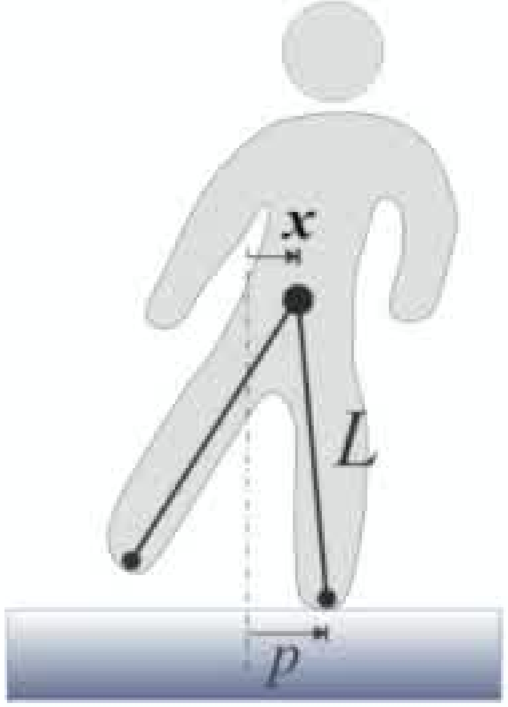
Supporting cables (or tall load bearing towers) are more flexible and can become oscillators prior to noticeable bridge wobbling.

Analogy with a walker on a bridge:

Piecewise-smooth system:

$$\ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = \gamma_1 \mathrm{sign}(\dot{x}),$$

$$\ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = -\gamma_1 \quad \Longleftrightarrow \quad \ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = \gamma_1$$



Close-form solutions for the "glued" limit cycle provide estimates of the oscillation frequency.

No bridge movement *(y=0):*

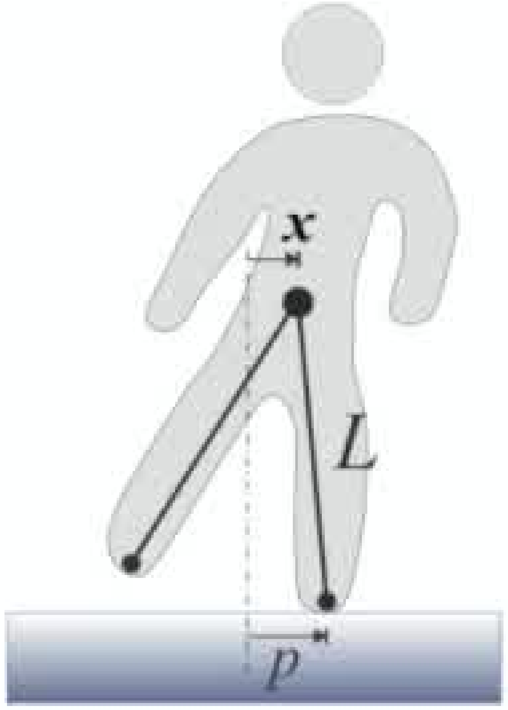
Supporting cables (or tall load bearing towers) are more flexible and can become oscillators prior to noticeable bridge wobbling.

Piecewise-smooth system:

$$\ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = \gamma_1 \mathrm{sign}(\dot{x}),$$

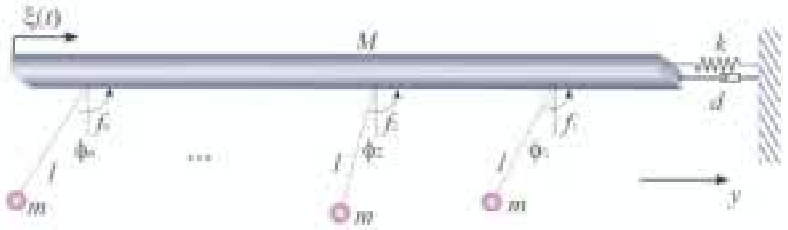
$$\ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = -\gamma_1 \quad \Longleftrightarrow \quad \ddot{x}_i + \lambda\dot{x}_i + \omega_i^2 x_i = \gamma_1$$

Analogy with a walker on a bridge:



Close-form solutions for the "glued" limit cycle provide estimates of the oscillation frequency.

Complete synchrony among the supports is governed by



$$\ddot{x} + \lambda\dot{x} + \omega^2 x = -\ddot{y} + \gamma_1 \operatorname{sign}(\dot{x}),$$

$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -rn\ddot{x} + \gamma_2 \operatorname{sign}(\dot{y}) + A\sin(t + \psi)$$

$$r = \frac{m}{M + nm}$$

The nonlinear system has **two characteristic frequencies** inherited from the linear system without the wind-induced terms:

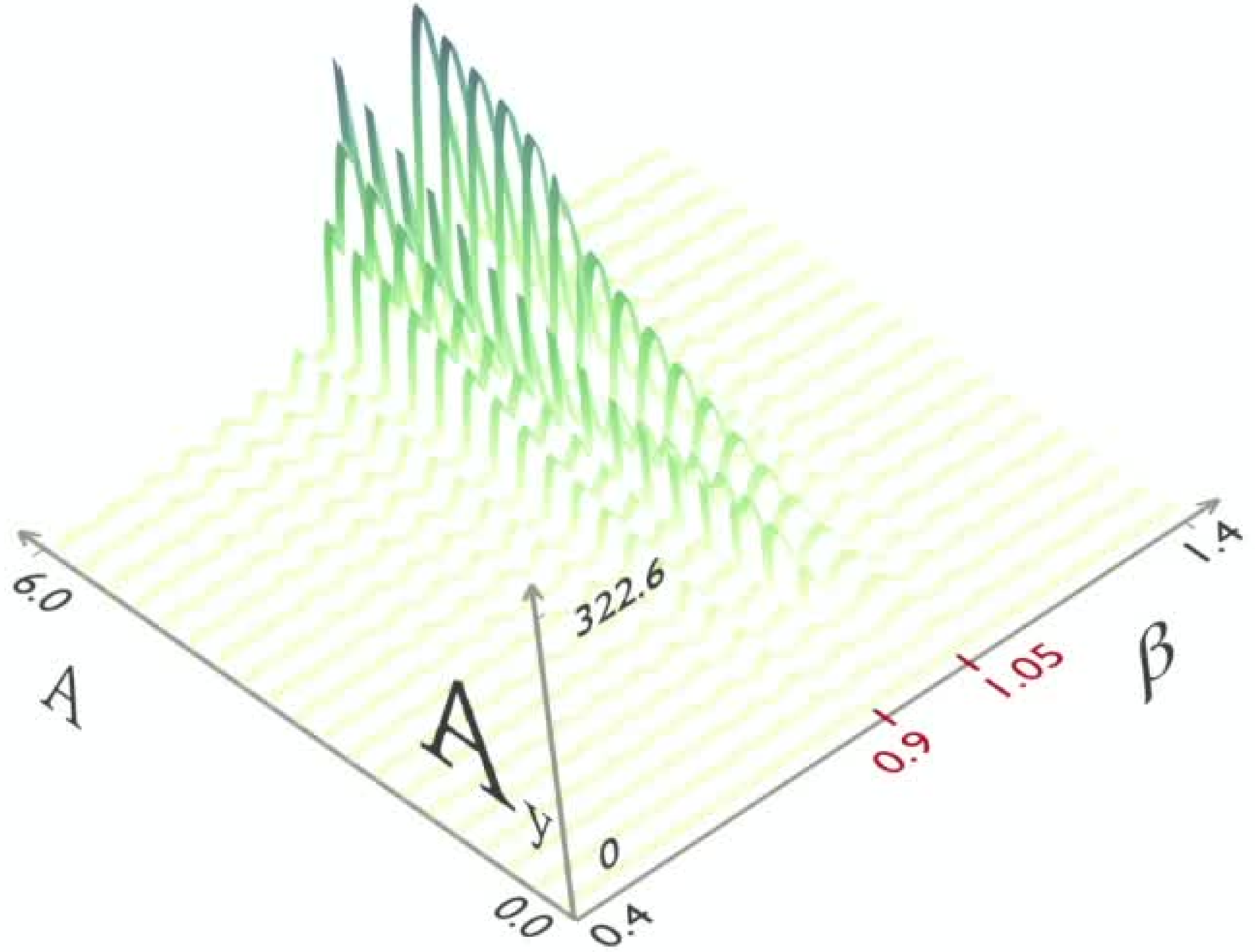
$$\ddot{x} + \lambda\dot{x} + \omega^2 x = -\ddot{y},$$

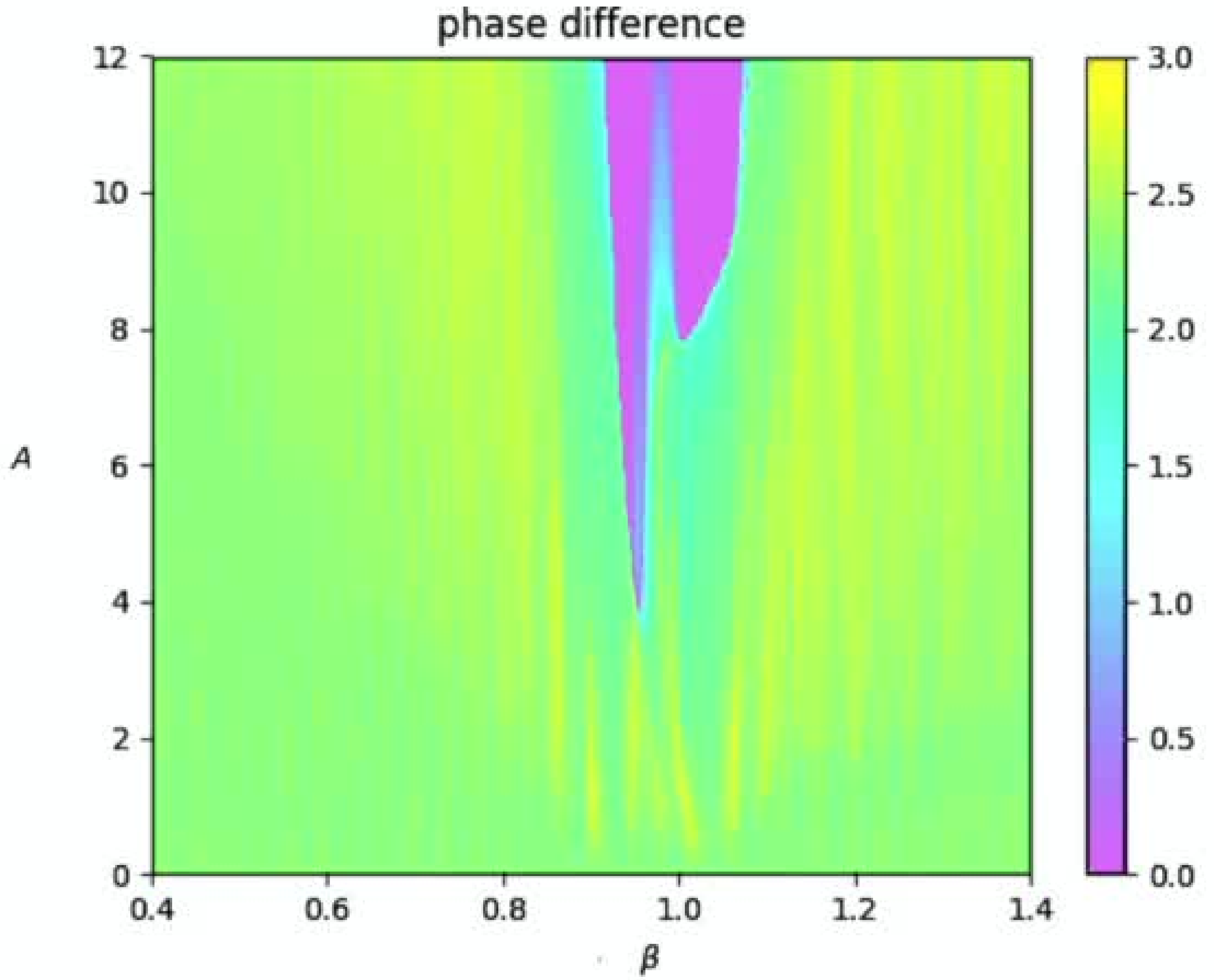
$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -rn\ddot{x}$$

Characteristic equation:

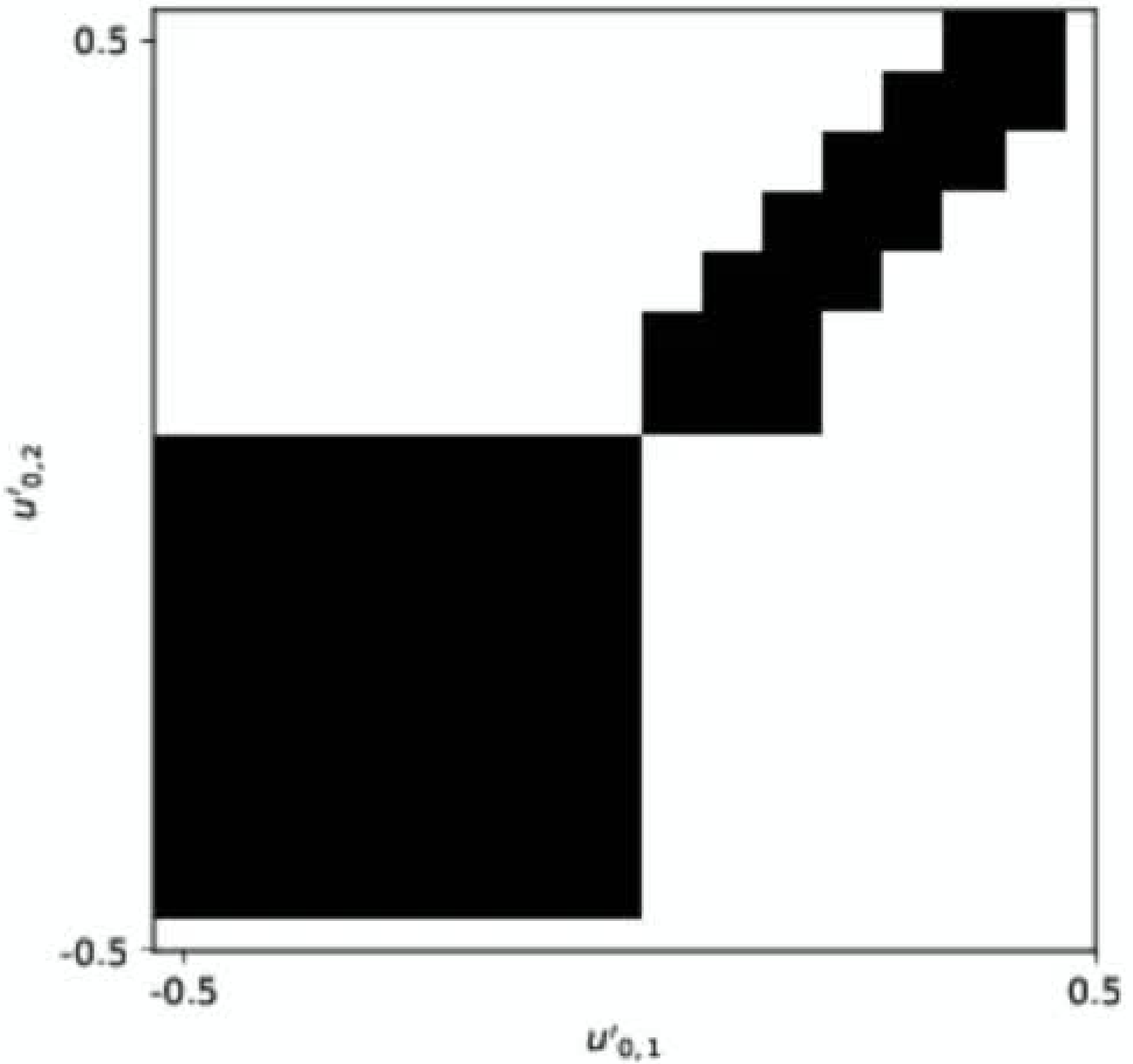
$$\mu^2 p^4 - \left(p^2 + \lambda p + \omega^2\right)\left(p^2 + hp + \Omega^2\right) = 0$$

Wobbling at **two** forcing frequencies



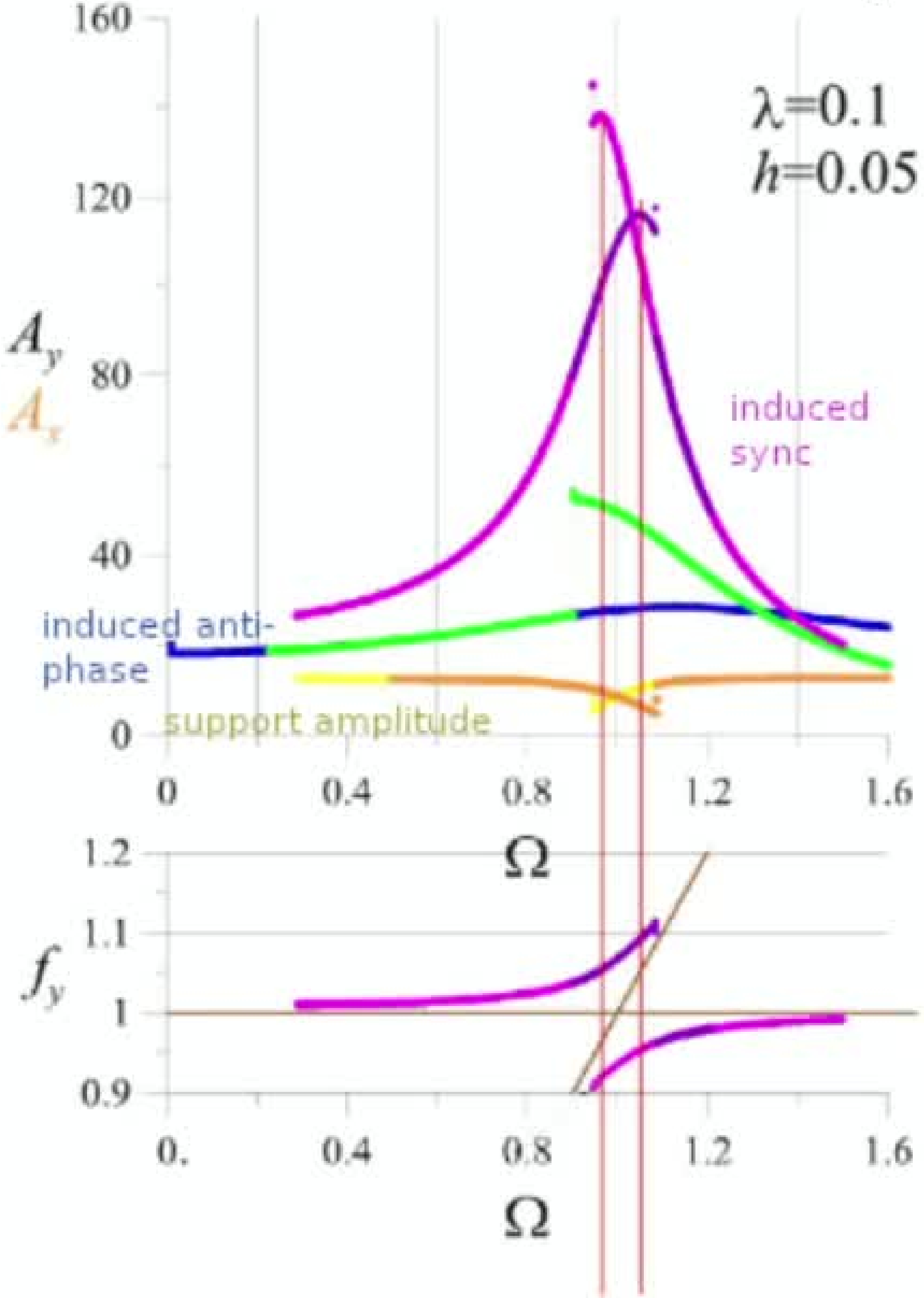


Wide **basin of attraction**



Even at low amplitudes!

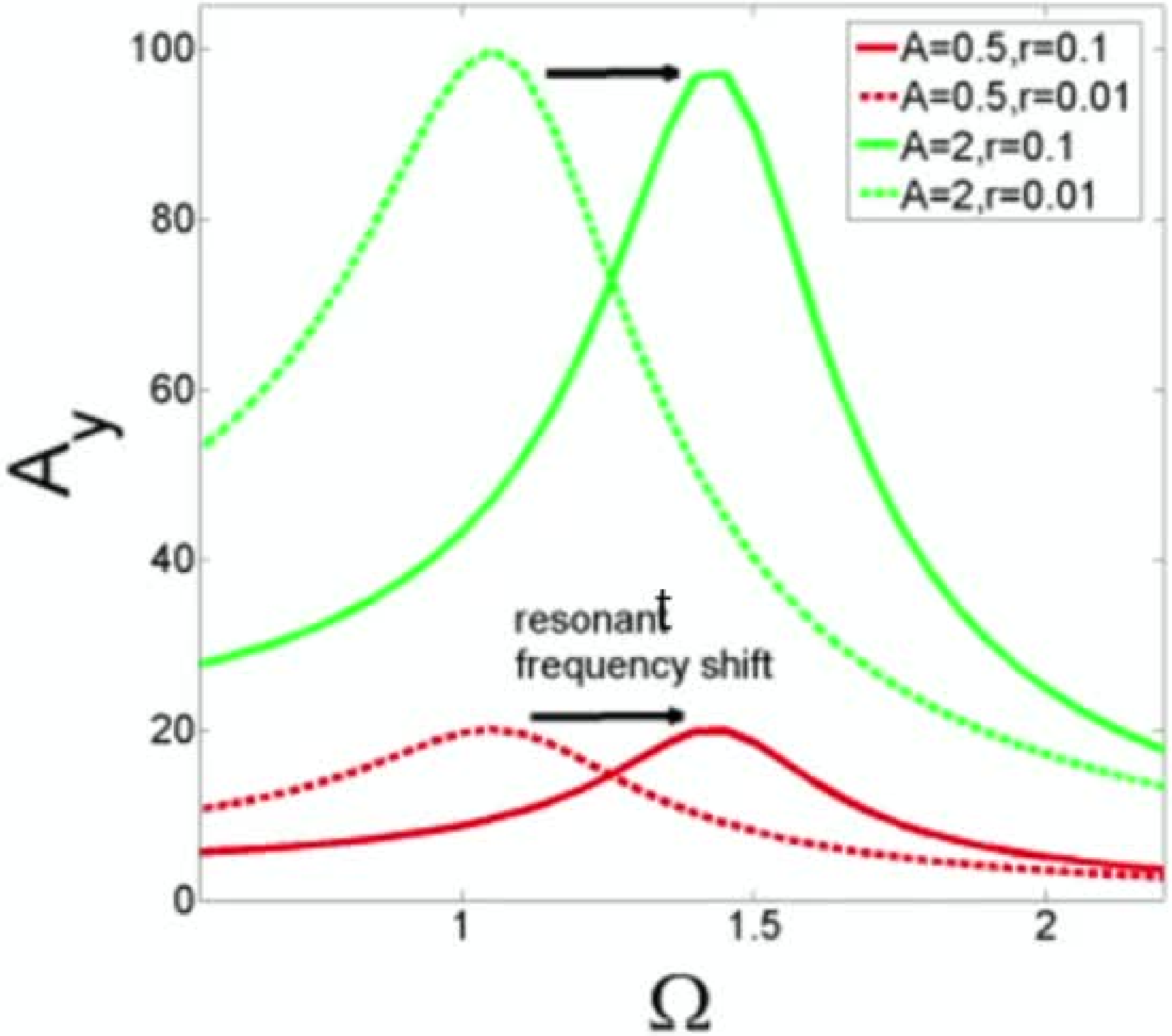
Co-existence of in-phase and out-of-phase states



Two emergent frequencies of bridge wobbling

$$\mu^2 p^4 - \left(p^2 + \lambda p + \omega^2\right)\left(p^2 + hp + \Omega^2\right) = 0$$

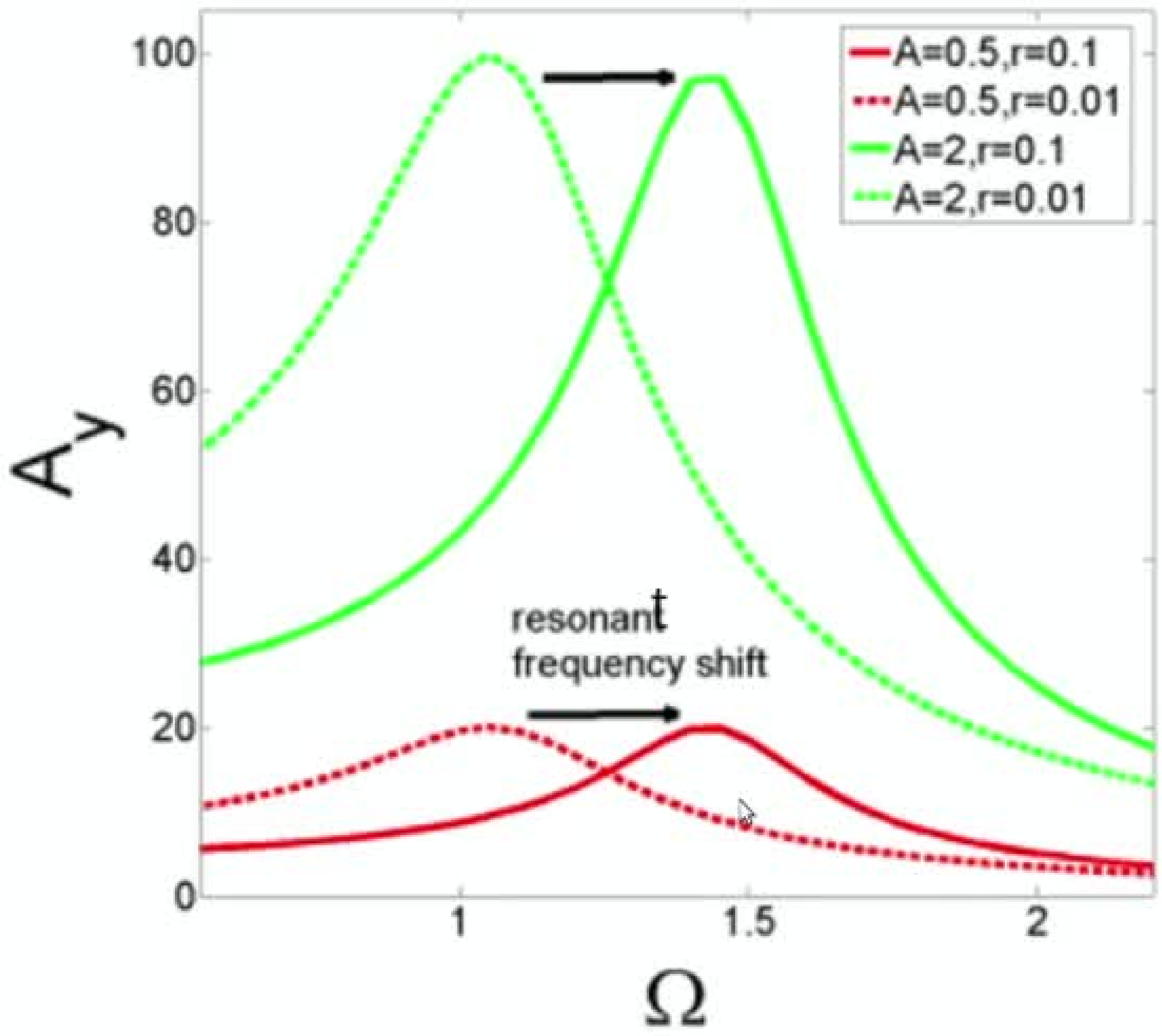
Resonant Frequency Shift



Resonance can be achieved at a frequency different from that of $A\sin(t+\psi)$

Varying γ_1 and γ_2 ($\gamma_1>\gamma_2$) helps shifting the resonant freqency even farther.

Resonant Frequency Shift



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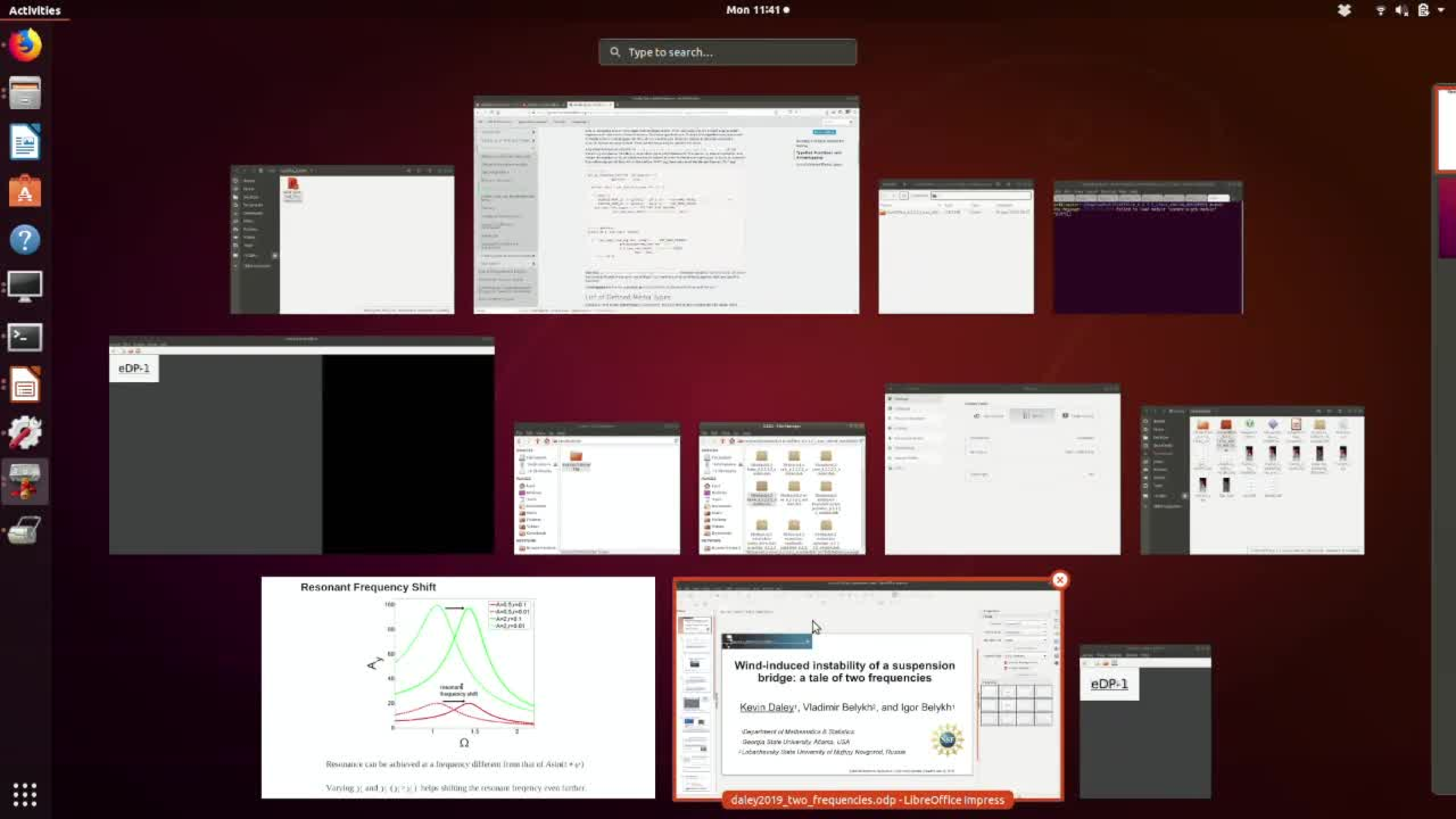


Resonant Frequency Shift

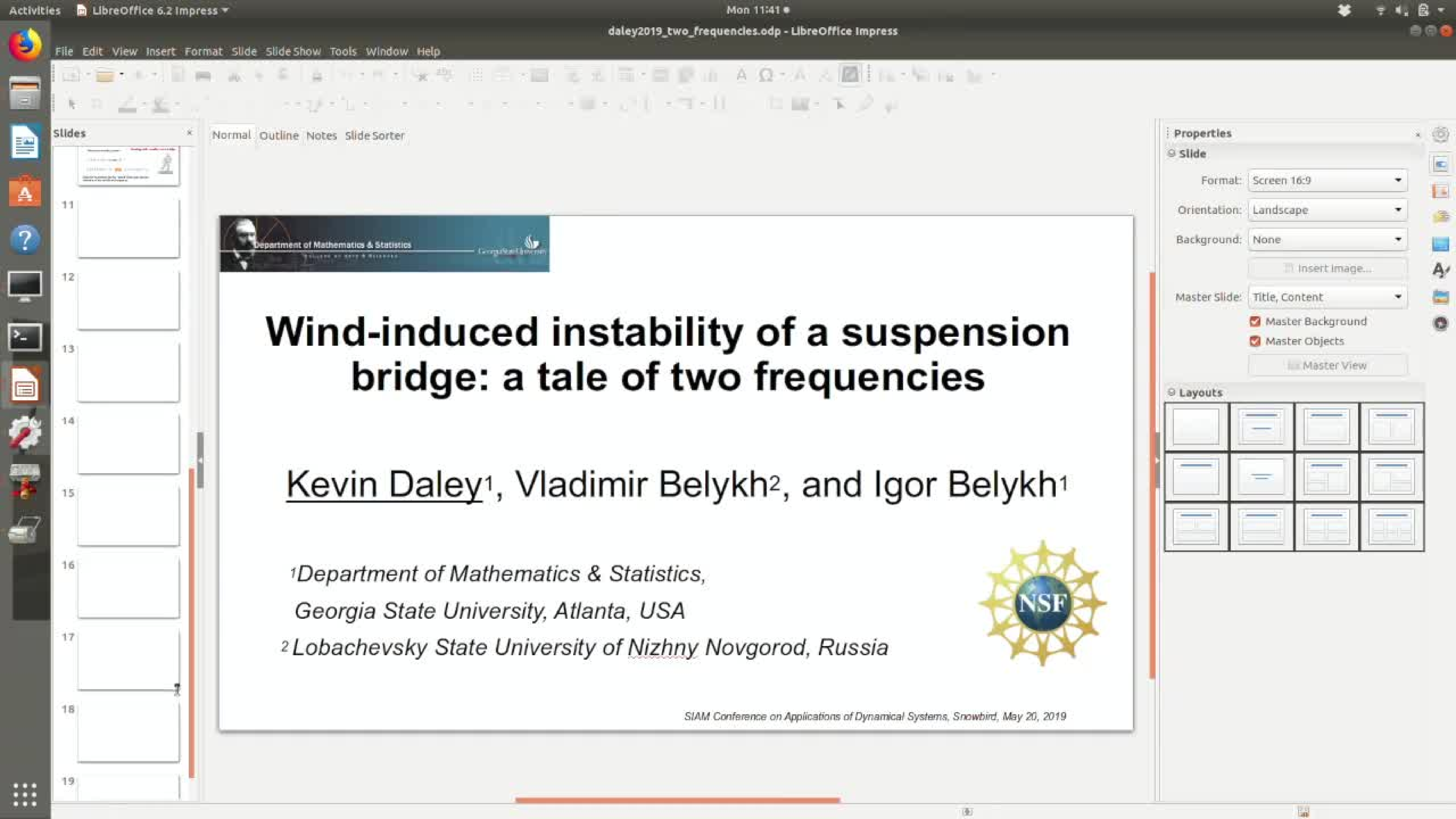
Resonance can be achieved at a frequency different from that of $A\sin(t+\psi)$

Varying γ_1 and γ_2 ($\gamma_1 \neq \gamma_2$) helps shifting the resonant frequency even further.

Wind-induced instability of a suspension bridge: a tale of two frequencies

Kevin Daley[1], Vladimir Belykh[2], and Igor Belykh[1]

[1]Department of Mathematics & Statistics
Georgia State University Atlanta, USA
[2]Lobachevsky State University of Nizhny Novgorod, Russia



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☑ Master Objects

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[1] *Department of Mathematics & Statistics,*
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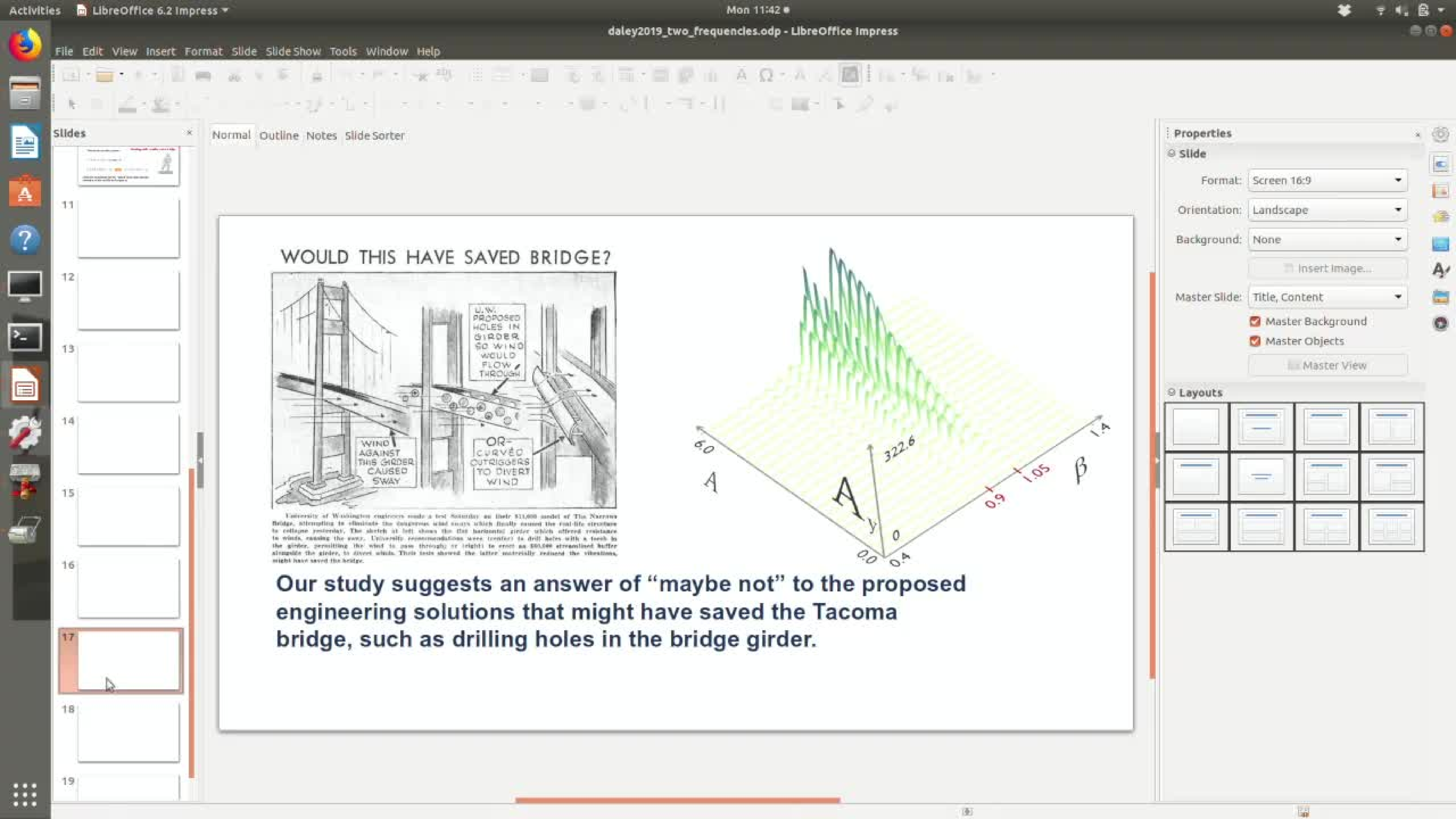
SIAM Conference on Applications of Dynamical Systems, Snowbird, May 20, 2019



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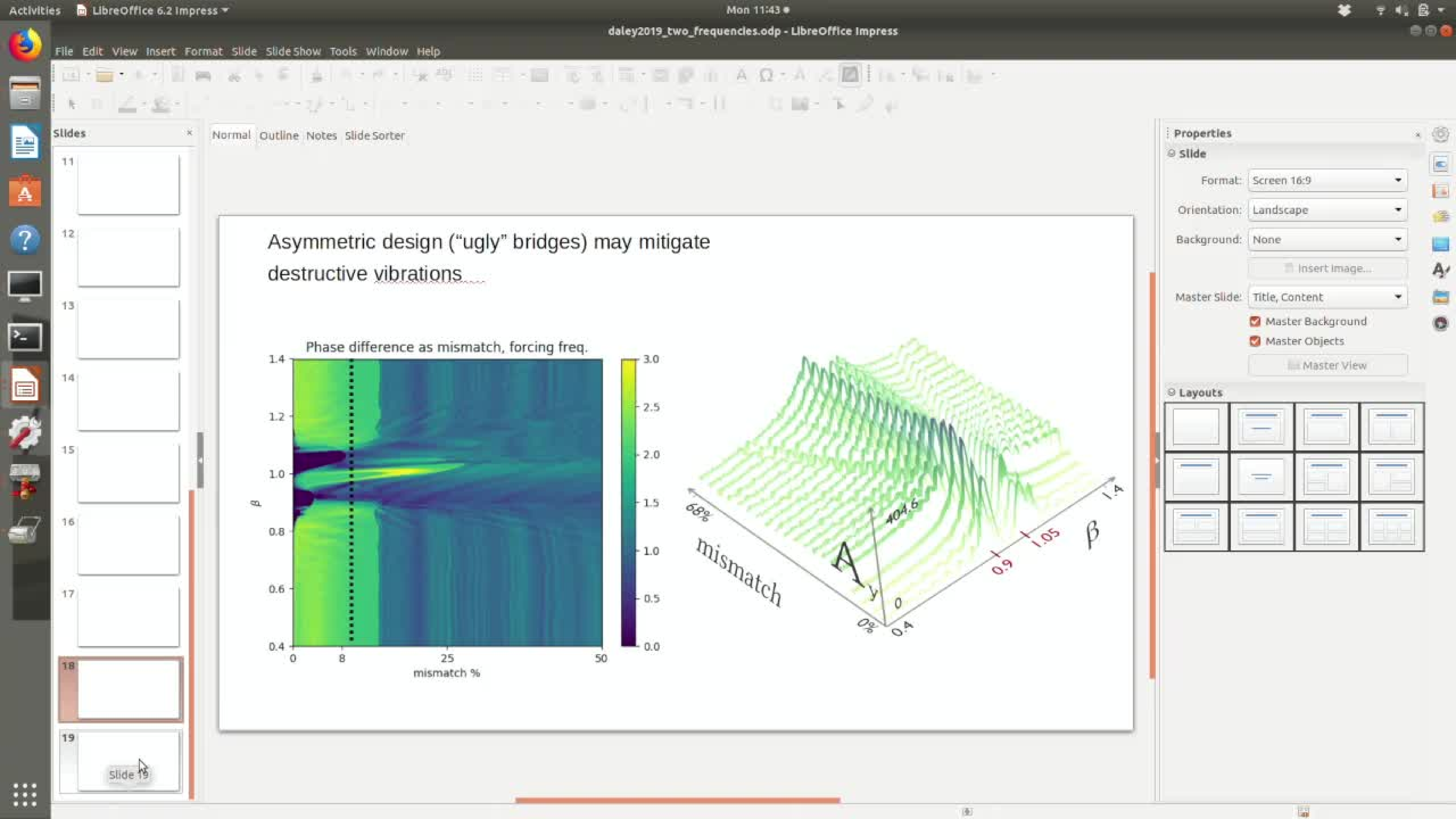
WOULD THIS HAVE SAVED BRIDGE?

Our study suggests an answer of “maybe not” to the proposed engineering solutions that might have saved the Tacoma bridge, such as drilling holes in the bridge girder.



Asymmetric design ("ugly" bridges) may mitigate destructive vibrations

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