

Wind-induced instability of a suspension bridge: a tale of two frequencies

Kevin Daley¹, Vladimir Belykh², and Igor Belykh¹

¹*Department of Mathematics & Statistics,
Georgia State University, Atlanta, USA*

²*Lobachevsky State University of Nizhny Novgorod, Russia*



Outline

- Examples of wind-induced instabilities: The Tacoma Narrows Bridge, and the Volga Bridge
- Wind-induced vibrations of a bridge at a frequency **different** from the natural frequency of the bridge girder (the Tacoma bridge case).
- Parallels and differences between crowd and wind-induced synchrony.
- A synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements causes the onset of bridge oscillations and can explain the shift of the resonant frequency.

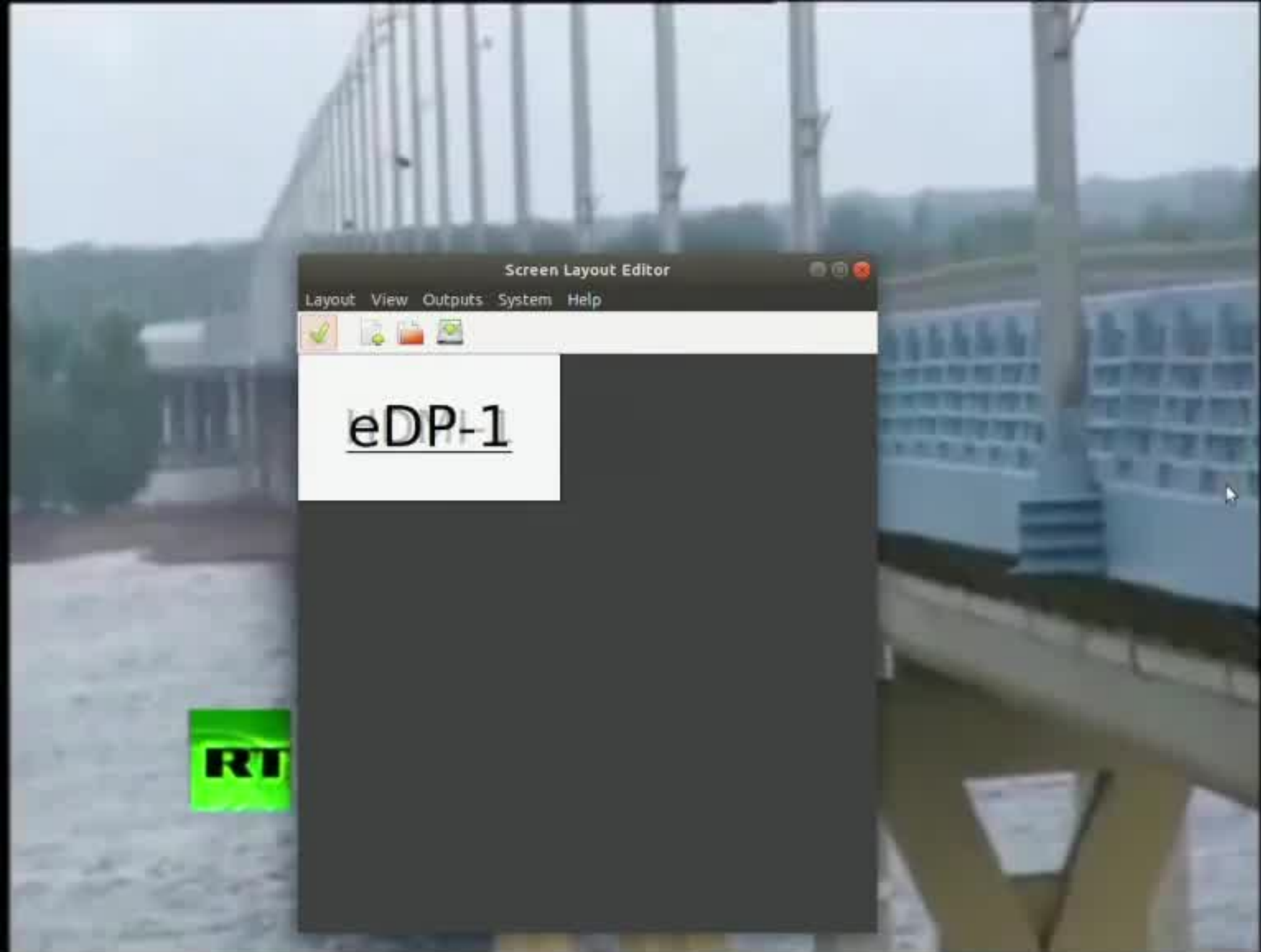
Tacoma Narrows (1940)



A transverse twisting mode emerged from **mild** winds (about 40 miles per hour)

The Volga Bridge: Volgograd, Russia, 2011

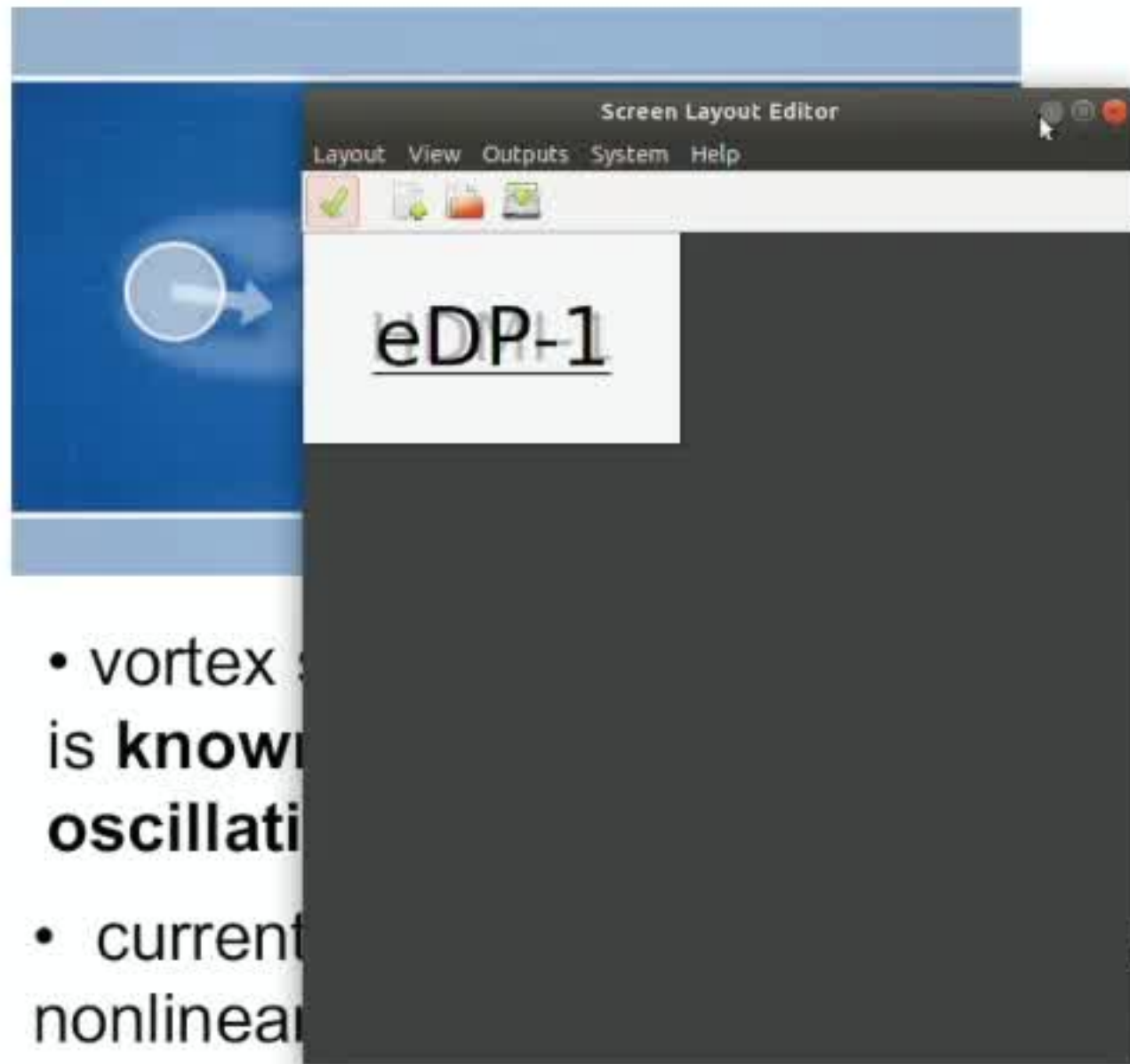
Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

Vortex Shedding and Instability



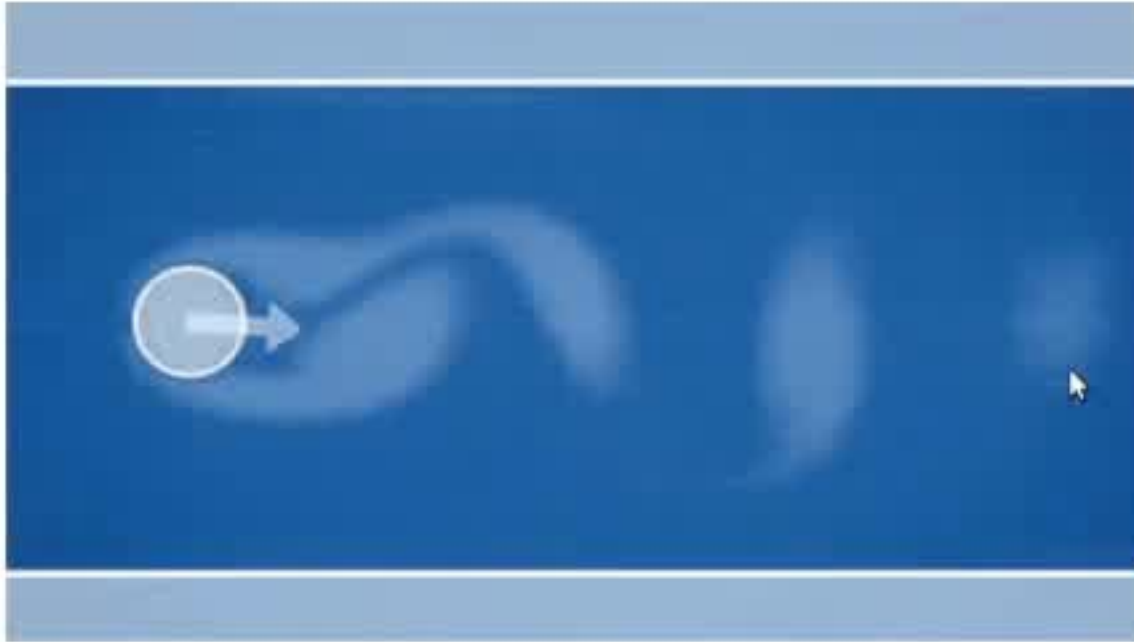
Vortex-induced vibrations in a section model of Hardanger Bridge, Norway

- vortex shedding is **known to cause large oscillations**
- current design is **specific** due to poorly-understood nonlinearities
- requires extensive **wind-tunnel testing** and often **only a section of the deck** is tested (full models **only for long-span bridges** and usually **highly simplified miniatures**)².

¹ J.S.Owen, A.M.Vann, J.P.Davies and A.Blakeborough, "The prototype testing of Kessock Bridge: response to vortex shedding." *Journal of Wind Engineering and Industrial Aerodynamics* Volume 60, April 1996, Pages 91-108

² Giorgio Diana and Giuseppe Fiammenghi. "Wind tunnel tests and numerical approach for long span bridges: the Messina bridge" *The Seventh International Colloquium on Bluff Body Aerodynamics and its Applications (BBAA7)* Shanghai, China; September 2-6, 2012

Vortex Shedding and Instability



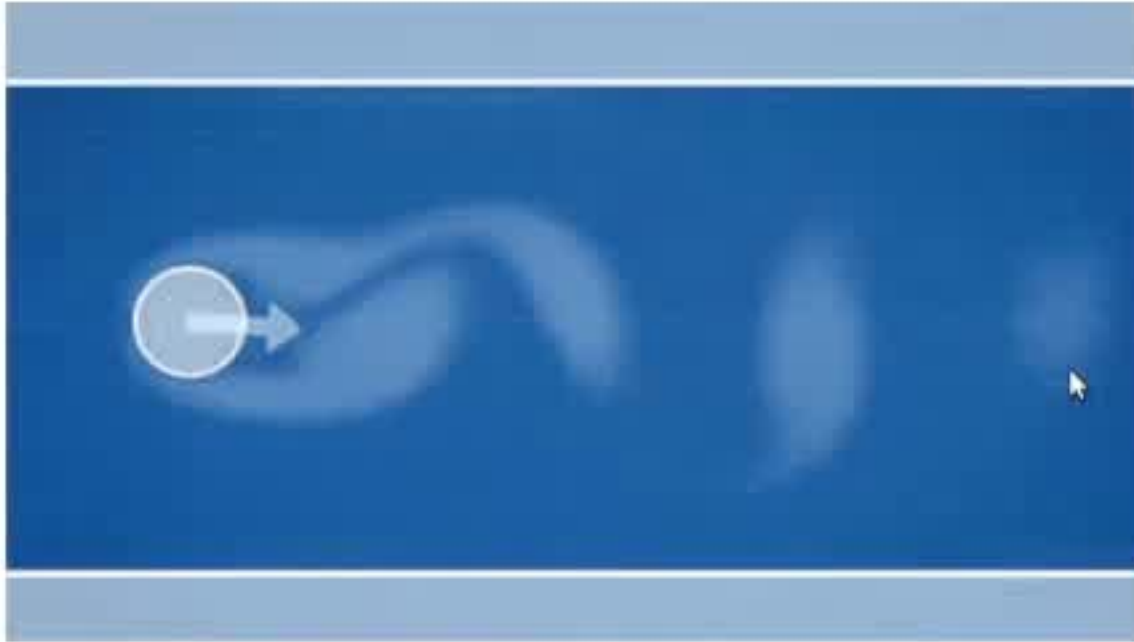
Vortex-induced vibrations in a section model of Hardanger Bridge, Norway

- vortex shedding is **known to cause damaging oscillations**¹
- current models are **structure-specific** due to poorly-understood nonlinearity.
- requires extensive **wind-tunnel testing** and often **only a section of the deck** is tested (full models **only for long-span bridges** and usually **highly simplified miniatures**)².

¹ J.S.Owen, A.M.Vann, J.P.Davies and A.Blakeborough, "The prototype testing of Kessock Bridge: response to vortex shedding." *Journal of Wind Engineering and Industrial Aerodynamics* Volume 60, April 1996, Pages 91-108

² Giorgio Diana and Giuseppe Fiammenghi. "Wind tunnel tests and numerical approach for long span bridges: the Messina bridge" *The Seventh International Colloquium on Bluff Body Aerodynamics and its Applications (BBAA7)* Shanghai, China; September 2-6, 2012

Vortex Shedding and Instability



Vortex-induced vibrations in a section model of Hardanger Bridge, Norway

- vortex shedding is **known to cause damaging oscillations**¹
- current models are **structure-specific** due to poorly-understood nonlinearity.
- requires extensive **wind-tunnel testing** and often **only a section of the deck** is tested (full models **only for long-span bridges** and usually **highly simplified miniatures**)².

¹ J.S.Owen, A.M.Vann, J.P.Davies and A.Blakeborough, "The prototype testing of Kessock Bridge: response to vortex shedding." *Journal of Wind Engineering and Industrial Aerodynamics* Volume 60, April 1996, Pages 91-108

² Giorgio Diana and Giuseppe Fiammenghi. "Wind tunnel tests and numerical approach for long span bridges: the Messina bridge" *The Seventh International Colloquium on Bluff Body Aerodynamics and its Applications (BBAA7)* Shanghai, China; September 2-6, 2012

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@lappier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd:... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x
kmd@lappier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 18:24:51.626: Failed to load module "canberra-gtk-module"
```

- Long-wavelength resonance vib
- Bridge vibrations initiated from t

er)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@lappier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd:... x kmd:... x kmd:... x kmd:... x kmd:... x kmd:... x kmd:... x kmd:... x
kmd@lappier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 10:24:51.626: Failed to load module "canberra-gtk-module"
```

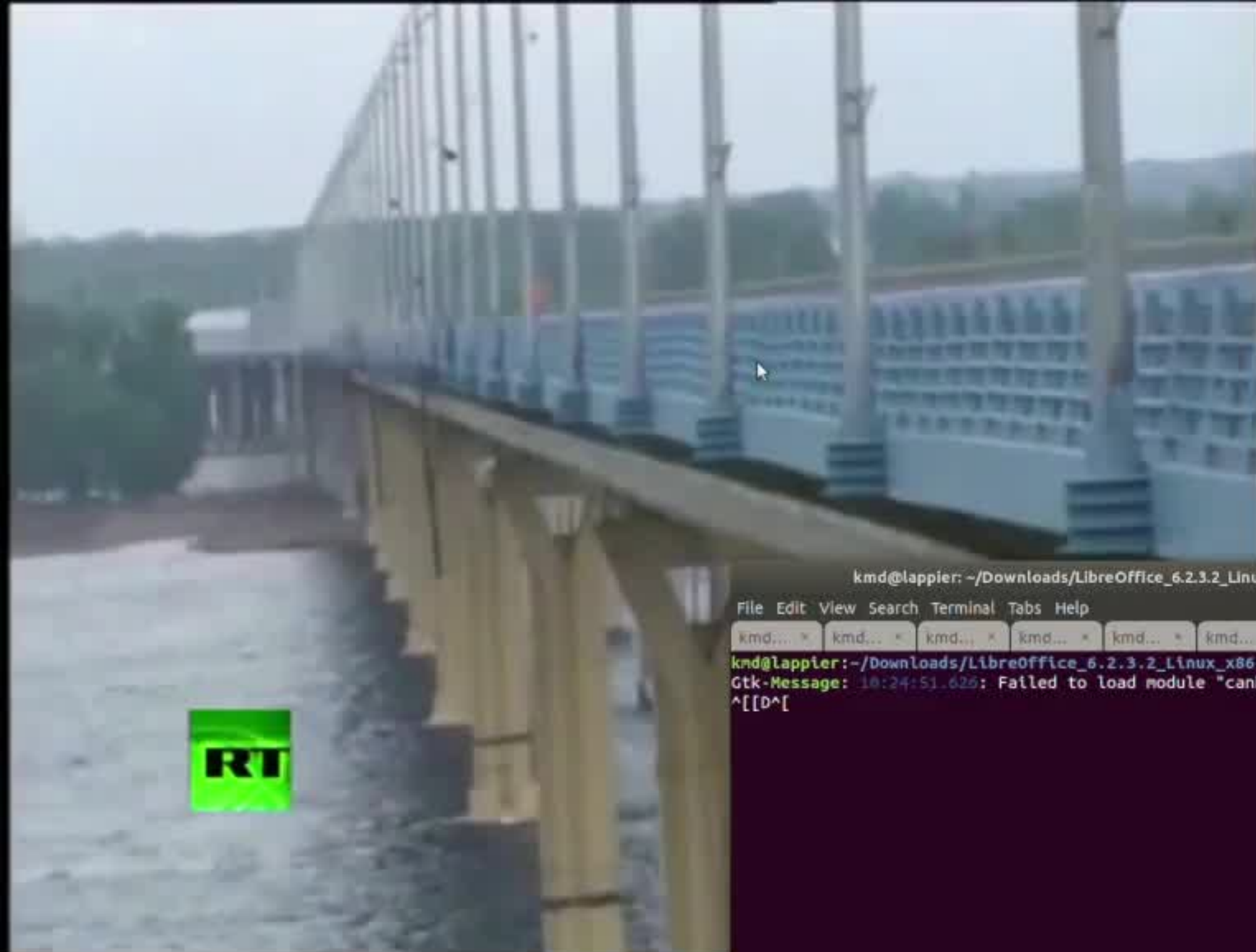
- Long-wavelength resonance vibrations
- Bridge vibrations initiated from traffic

(er)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@lappier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd@lappier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 10:24:51.626: Failed to load module "canberra-gtk-module"
^[[D^]
```

- Long-wavelength resonance vibrations
- Bridge vibrations initiated from traffic

(er)!

Brun et al., "Bypassing shake, rattle, and roll," *Physics World*, 2013.

Vortex Shedding and Instability



- vortex shedding is known to cause damaging oscillations¹
- current models are structurally nonlinear.
- requires extensive wind-tunnel testing (full models only for long-span)

section model of Hardanger

of the deck is tested using scaled (scaled miniatures)².

```
kmd@lappier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd@lappier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 10:24:51.626: Failed to load module "canberra-gtk-module"
^[[D^[[
```

¹ J.S.Owen, A.M.Vann, J.P.Davies and A.Blakeborough, "The prototype testing of the Messina Bridge: response to vortex shedding." Journal of Wind Engineering and Industrial Aerodynamics Volume 60, April 1996, Pages 91-108

The Seventh International Colloquium on Bluff Body Aerodynamics and its Applications (BBAA7) Shanghai, China; September 2-6, 2012

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@lappier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x
kmd@lappier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 18:24:51.626: Failed to load module "canberra-gtk-module"
^[[D^[]
```

- Long-wavelength resonance vib
- Bridge vibrations initiated from t

er)!

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibrations
- Bridge vibrations initiated from

der)!

e, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@laplier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x
kmd@laplier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 10:24:51.626: Failed to load module "canberra-gtk-module"
^[[D^[[
```

- Long-wavelength resonance vibrations
- Bridge vibrations initiated from

der)!
e, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@laplier: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x
kmd@laplier:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 10:24:51.626: Failed to load module "canberra-gtk-module"
^[[D^[[
```

- Long-wavelength resonance vibrations
- Bridge vibrations initiated from

der)!

e, and roll,” Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



```
kmd@lappler: ~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS
File Edit View Search Terminal Tabs Help
kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x kmd... x
kmd@lappler:~/Downloads/LibreOffice_6.2.3.2_Linux_x86-64_deb/DEBS$ arandr
Gtk-Message: 10:24:51.626: Failed to load module "canberra-gtk-module"
^[[D^[[
```

- Long-wavelength resonance vibrations
- Bridge vibrations initiated from

der)!

e, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.



The Volga Bridge: Volgograd, Russia, 2011

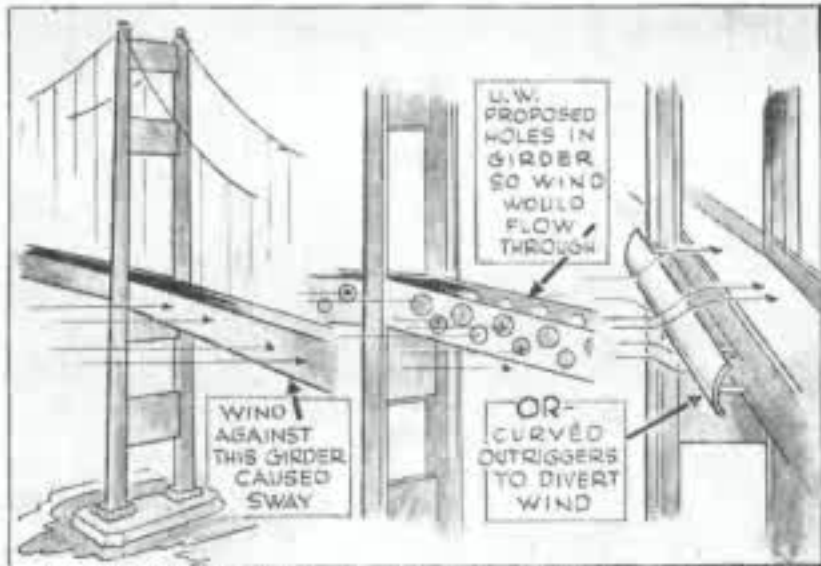
Faulty design fixed by
hydraulic mass dampers

- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

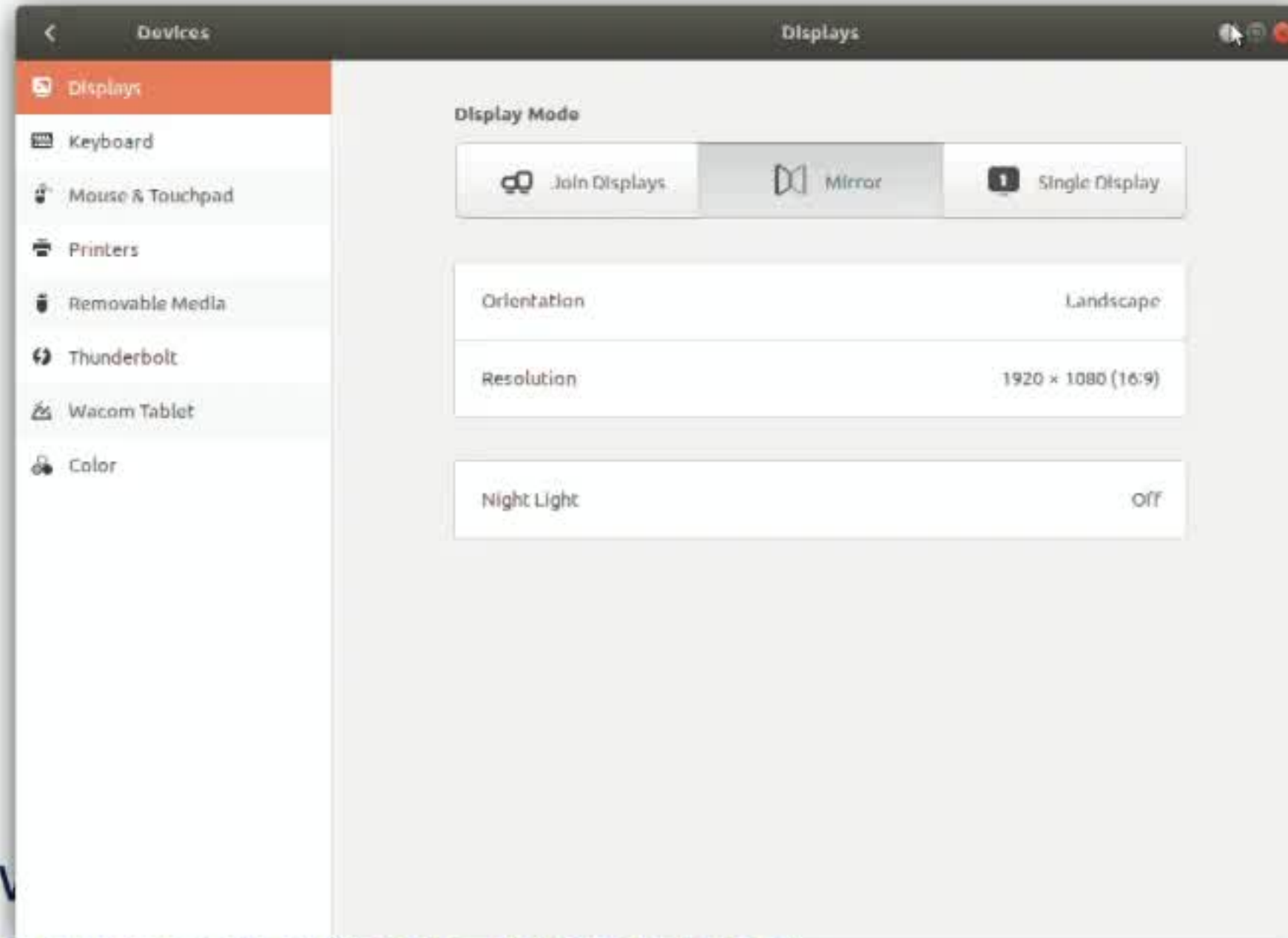
Brun et al., "Bypassing shake, rattle, and roll," *Physics World*, 2013.

Modeling lateral bridge vibrations

WOULD THIS HAVE SAVED BRIDGE?



Engineers of Washington engineers made a test model on their \$1,000 model at The Narrows Bridge, attempting to eliminate the dangerous wind waves which finally caused the real-life structure to collapse yesterday. The sketch at left shows the flat horizontal girder which offered resistance to wind, causing the sway. Unusually recommendations were (center) to drill holes with a inch in the girder, permitting the wind to pass through; or (right) to erect an \$80,000 streamlined buffer alongside the girder, to divert winds. Tests showed the latter materially reduced the vibrations, might have saved the bridge.



$$+ \xi(t),$$

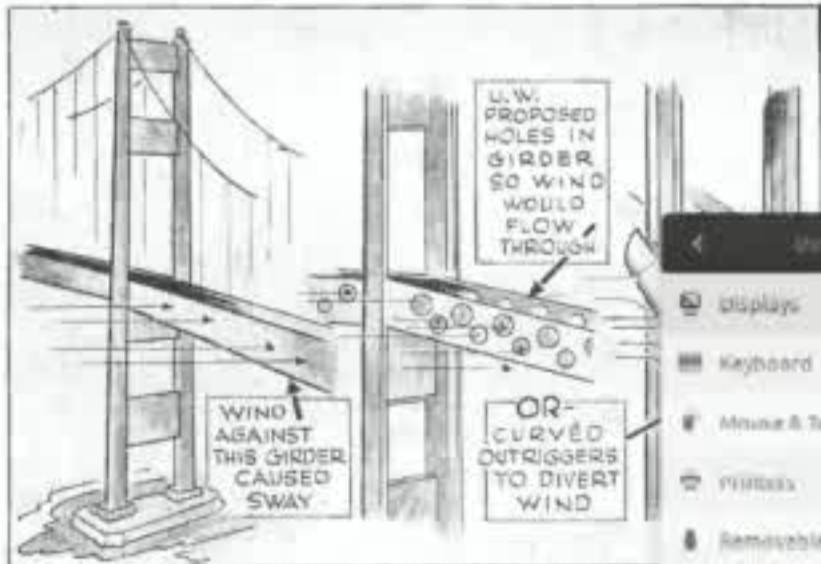
$$\dot{\phi}_i \sin \phi_i) + \xi(t)$$

$$\phi_{syn} < \pi/2$$

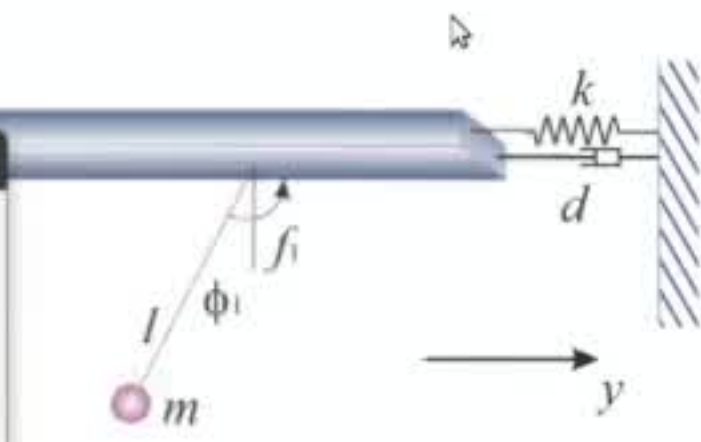
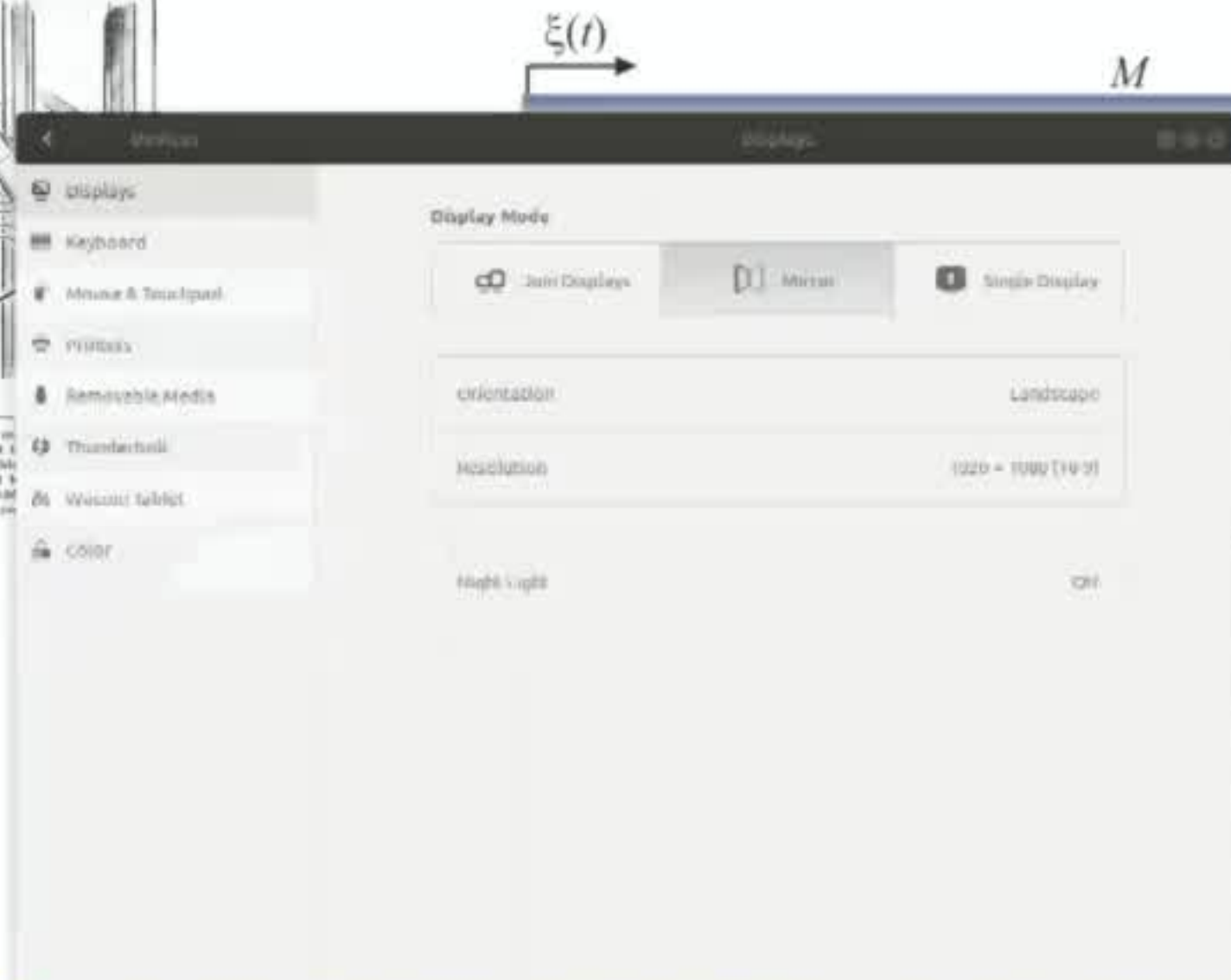
Phase-locking between
can induce significant bridge **wobbling**

Modeling lateral bridge vibrations

WOULD THIS HAVE SAVED BRIDGE?



University of Washington engineers made a test Monday on their \$11,000 model bridge, attempting to eliminate the dangerous wind swirls which finally caused it to collapse yesterday. The sketch at left shows the flat horizontal girder which caused the swirl. Engineers' recommendations were (a) to drill holes in the girder, permitting the wind to pass through; or (right) to erect an outrigger alongside the girder, to divert winds. These tests showed the latter' practically as might have saved the bridge.



$$= -m\ddot{y} \cos \phi_i + \xi(t),$$

$$\sum_{i=1}^n (\ddot{\phi}_i \cos \phi_i - \dot{\phi}_i \sin \phi_i) + \xi(t)$$

Phase-locking between the supports (oscillators): $|\varphi_i - \varphi_j| < \varphi_{syn} < \pi/2$
 can induce significant bridge **wobbling**

Crowd vs. wind- load of a suspension bridge

Crowd loading (the London Millennium Bridge case): Phase-locking among pedestrians is **not** the cause of bridge wobbling¹, but rather a consequence. Pedestrians adjust their gaits to maintain balance and destabilize the bridge via the negative damping mechanism.

Wind loading (the Tacoma Bridge case): Our synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements can explain the shift of the resonant frequency.

1. "On the Millennium Bridge Synchronization Myth."
(first talk in this minisymposium)



Vortex Shedding and Instability

4



Vortex-induced vibrations in a section model of Hardanger Bridge, Norway

- vortex is known to cause oscillations

- current models are **structure-specific** due to poorly-understood nonlinearity.

- requires extensive **wind-tunnel testing** and often **only a section of the deck** is tested (full models **only for long-span bridges** and usually **highly simplified miniatures**)².

¹ J.S.Owen, A.M.Vann, J.P.Davies and A.Blakeborough, "The prototype testing of Kessock Bridge: response to vortex shedding." *Journal of Wind Engineering and Industrial Aerodynamics* Volume 60, April 1996, Pages 91-108

² Giorgio Diana and Giuseppe Fiammenghi. "Wind tunnel tests and numerical approach for long span bridges: the Messina bridge" *The Seventh International Colloquium on Bluff Body Aerodynamics and its Applications (BBAA7)* Shanghai, China; September 2-6, 2012

Crowd vs. wind- load of a suspension bridge

Crowd loading (the London Millennium Bridge case): Phase-locking among pedestrians is **not** the cause of bridge wobbling¹, but rather a consequence. Pedestrians adjust their gaits to maintain balance and destabilize the bridge via the negative damping mechanism.

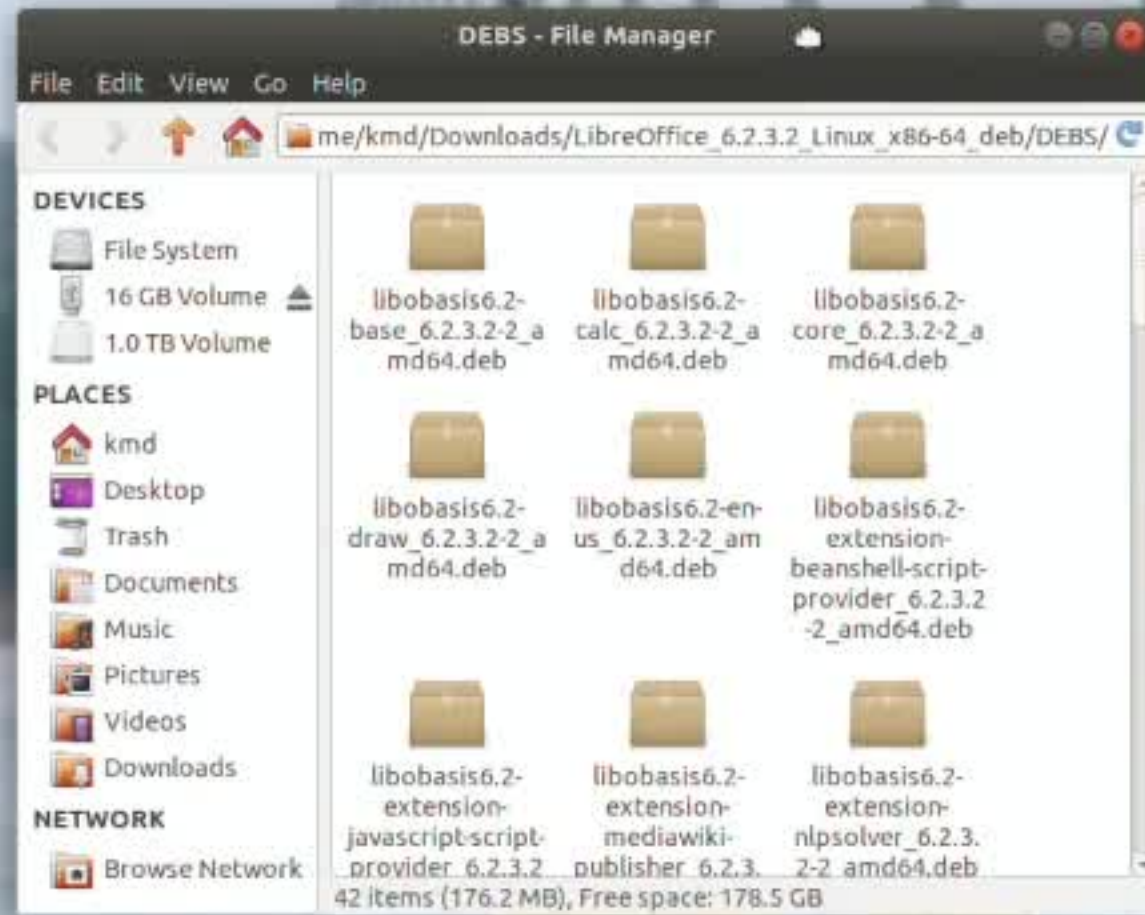
Wind loading (the Tacoma Bridge case): Our synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements can explain the shift of the resonant frequency.

¹ “On the Millenium Bridge Synchronization Myth.”
(first talk in this minisymposium)



The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.



The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers

- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," *Physics World*, 2013.

The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers



- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," Physics World, 2013.



The Volga Bridge: Volgograd, Russia, 2011

Faulty design fixed by
hydraulic mass dampers

- Long-wavelength resonance vibration due to mild winds
- Bridge vibrations initiated from the vibration of the supports (not the girder)!

Brun et al., "Bypassing shake, rattle, and roll," *Physics World*, 2013.

Crowd vs. wind- load of a suspension bridge

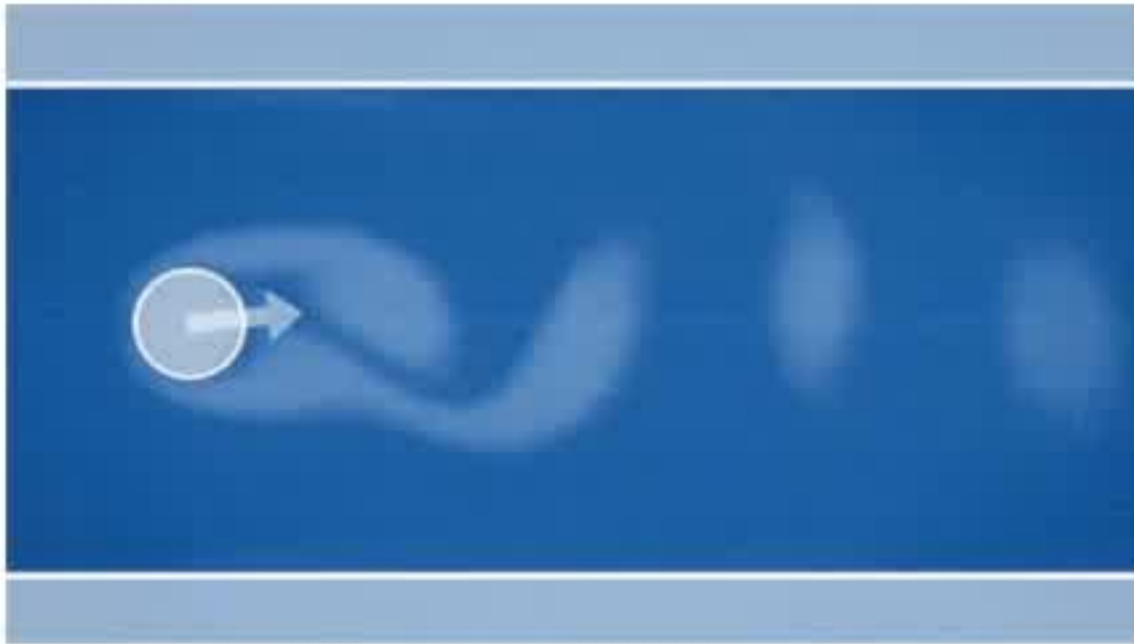
Crowd loading (the London Millennium Bridge case): Phase-locking among pedestrians is **not** the cause of bridge wobbling¹, but rather a consequence. Pedestrians adjust their gaits to maintain balance and destabilize the bridge via the negative damping mechanism.

Wind loading (the Tacoma Bridge case): Our synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements can explain the shift of the resonant frequency.

1. "On the Millenium Bridge Synchronization Myth."
(first talk in this minisymposium)



Vortex Shedding and Instability



Vortex-induced vibrations in a section model of Hardanger Bridge, Norway

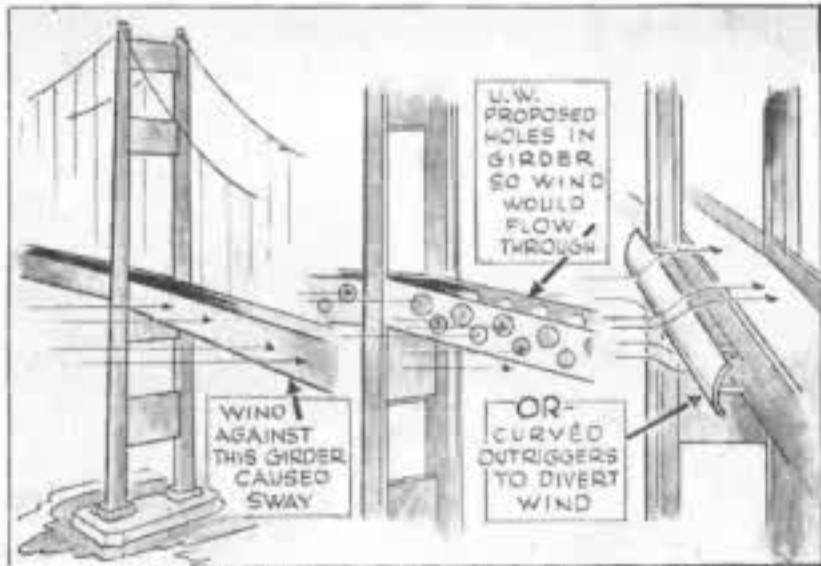
- vortex shedding is **known to cause damaging oscillations**¹
- current models are **structure-specific** due to poorly-understood nonlinearity.
- requires extensive **wind-tunnel testing** and often **only a section of the deck** is tested (full models **only for long-span bridges** and usually **highly simplified miniatures**)².

¹ J.S.Owen, A.M.Vann, J.P.Davies and A.Blakeborough, "The prototype testing of Kessock Bridge: response to vortex shedding." *Journal of Wind Engineering and Industrial Aerodynamics* Volume 60, April 1996, Pages 91-108

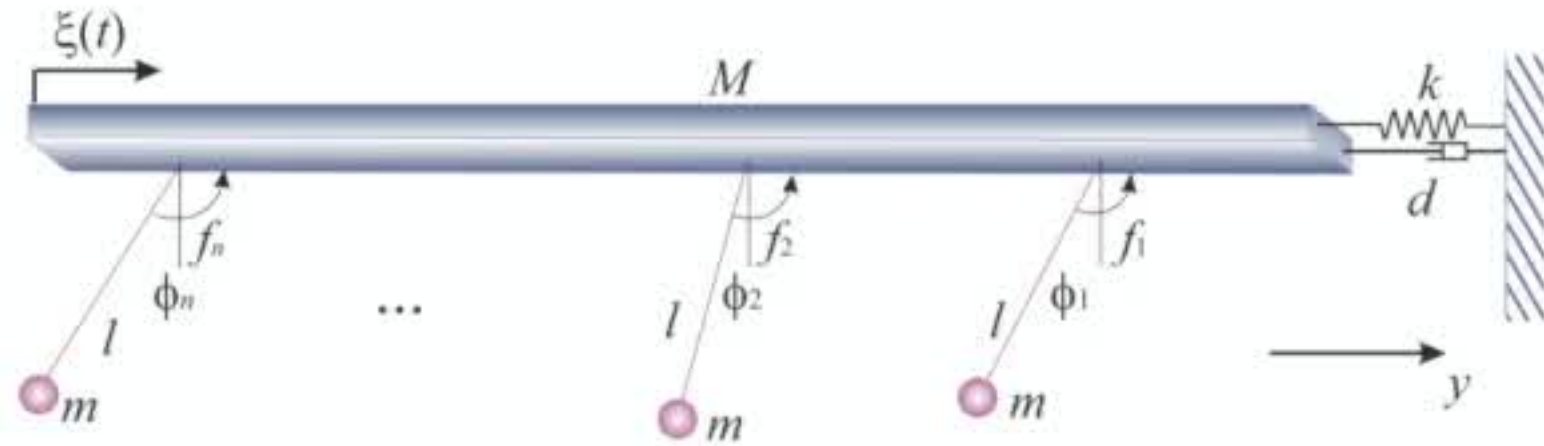
² Giorgio Diana and Giuseppe Fiammenghi. "Wind tunnel tests and numerical approach for long span bridges: the Messina bridge" *The Seventh International Colloquium on Bluff Body Aerodynamics and its Applications (BBAA7)* Shanghai, China; September 2-6, 2012

Modeling lateral bridge vibrations

WOULD THIS HAVE SAVED BRIDGE?



University of Washington engineers made a test facility on their \$1,000 model of The Narrows Bridge, attempting to eliminate the dangerous wind sways which finally caused the real-life structure to collapse yesterday. The sketch at left shows the flat horizontal girder which offered resistance to wind, causing the sway. University recommendations were (center) to drill holes with a larch in the girder, permitting the wind to pass through; or (right) to erect an \$89,000 streamlined buffer alongside the girder, to divert winds. Their tests showed the latter materially reduced the vibrations, might have saved the bridge.



$$ml^2 \ddot{\phi}_i + mgl \sin \phi_i + f(\phi_i, \dot{\phi}_i) = -ml\dot{y} \cos \phi_i + \xi(t),$$

$$(M + nm)\ddot{y} + d\dot{y} + ky = -ml \sum_{i=1}^n (\ddot{\phi}_i \cos \phi_i - \dot{\phi}_i \sin \phi_i) + \xi(t)$$

Phase-locking between the supports (oscillators): $|\varphi_i - \varphi_j| < \varphi_{syn} < \pi/2$
 can induce significant bridge **wobbling**

Crowd vs. wind- load of a suspension bridge

Crowd loading (the London Millennium Bridge case): Phase-locking among pedestrians is **not** the cause of bridge wobbling¹, but rather a consequence. Pedestrians adjust their gaits to maintain balance and destabilize the bridge via the negative damping mechanism.

Wind loading (the Tacoma Bridge case): Our synchronization hypothesis: wind-induced synchronization of suspension/load bearing elements can explain the shift of the resonant frequency.

1. "On the Millenium Bridge Synchronization Myth."
(first talk in this minisymposium)

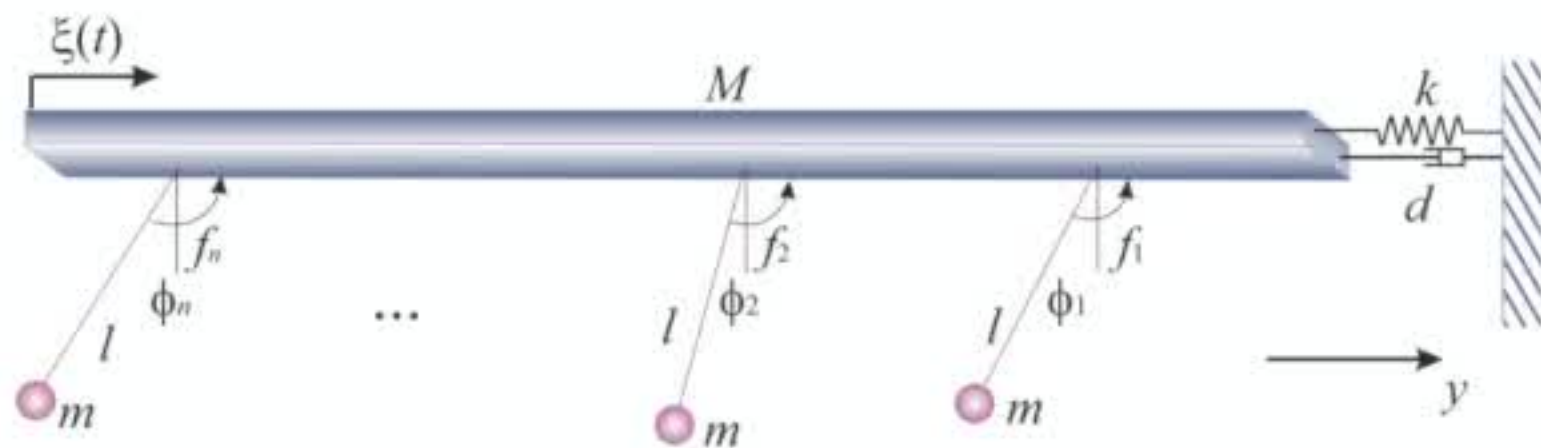


Modeling lateral bridge vibrations

WOULD THIS HAVE SAVED BRIDGE?



University of Washington engineers made a test recording on their \$1,000 model at The Narrows Bridge, attempting to eliminate the dangerous wind waves which finally caused the real-life structure to collapse yesterday. The sketch at left shows the flat horizontal girder which offered resistance to wind, causing the sway. University reconstructions were (center) to drill holes with a larch in the girder, permitting the wind to pass through; or (right) to erect an \$80,000 streamlined buffer alongside the girder, to divert winds. Their tests showed the latter materially reduced the vibrations, might have saved the bridge.

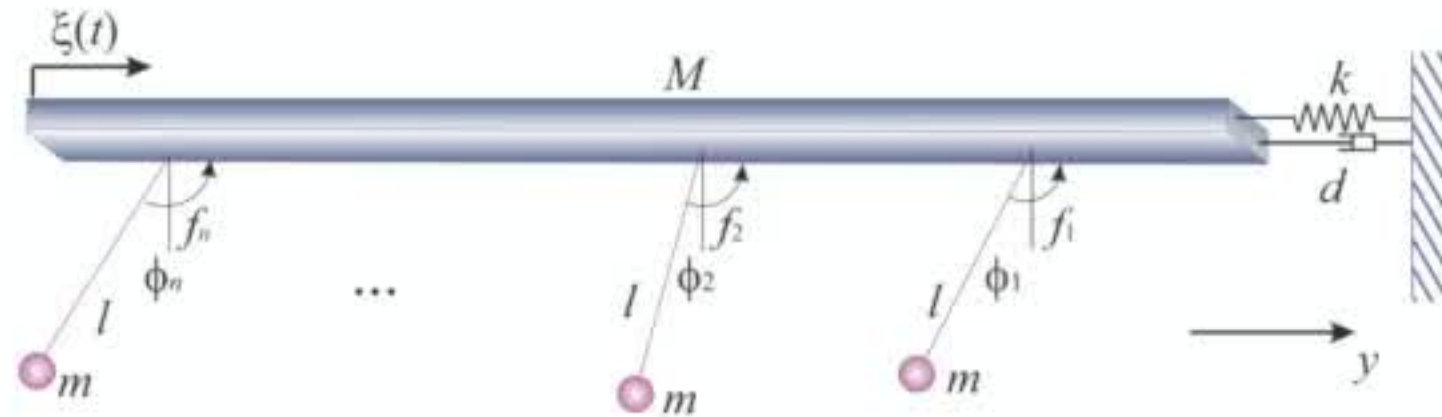


$$ml^2 \ddot{\phi}_i + mgl \sin \phi_i + f(\phi_i, \dot{\phi}_i) = -ml\dot{y} \cos \phi_i + \xi(t),$$

$$(M + nm)\ddot{y} + d\dot{y} + ky = -ml \sum_{i=1}^n (\ddot{\phi}_i \cos \phi_i - \dot{\phi}_i \sin \phi_i) + \xi(t)$$

Phase-locking between the supports (oscillators): $|\varphi_i - \varphi_j| < \varphi_{syn} < \pi/2$
 can induce significant bridge **wobbling**

Adding vortex shedding



The angular displacement can be approximated by side-to-side motion:

$$x_i = l\phi_i$$

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = -\ddot{y} + \gamma_1 \text{sign}(\dot{x}),$$

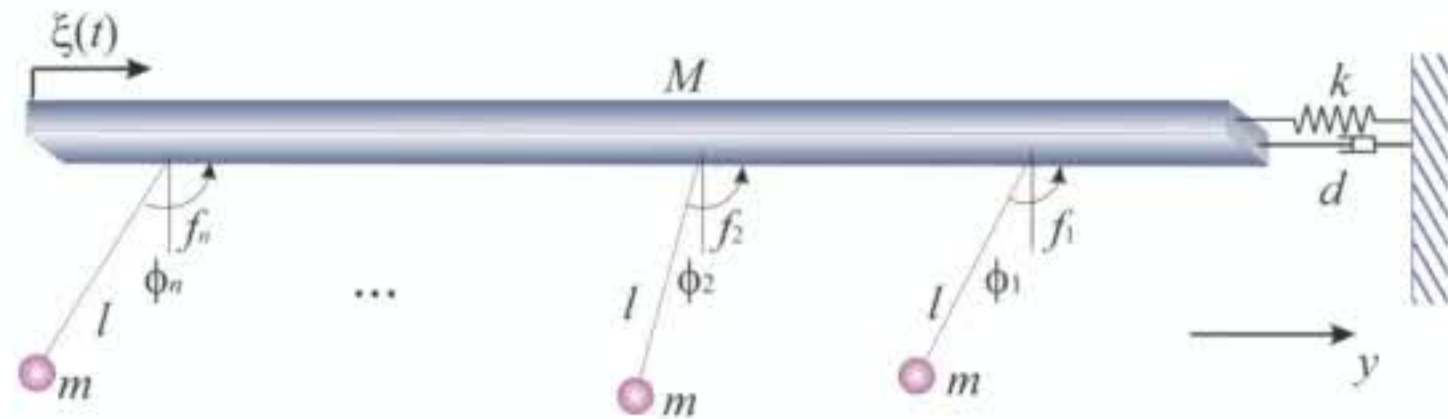
$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -r \sum_{i=1}^n \ddot{x}_i + \gamma_2 \text{sign}(\dot{y}) + A \sin(\beta t)$$

force due to vortex shedding

force of wind gusts

The **signum terms** account for **von Kármán vortex shedding** behind the bridge, causing side-to-side vibrations.

Adding vortex shedding



The angular displacement can be approximated by side-to-side motion:

$$x_i = l\phi_i$$

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = -\ddot{y} + \gamma_1 \text{sign}(\dot{x}),$$

$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -r \sum_{i=1}^n \ddot{x}_i + \gamma_2 \text{sign}(\dot{y}) + A \sin(\beta t)$$

force of wind gusts

force due to vortex shedding

The **signum terms** account for **von Kármán vortex shedding** behind the bridge, causing side-to-side vibrations.

No bridge movement ($y=0$):

Supporting cables (or tall load bearing towers) are more flexible and can become oscillators prior to noticeable bridge wobbling.

Piecewise-smooth system:

Analogy with a walker on a bridge:

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = \gamma_1 \text{sign}(\dot{x}),$$

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = -\gamma_1 \iff \ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = \gamma_1$$



Close-form solutions for the “glued” limit cycle provide estimates of the oscillation frequency.

No bridge movement ($y=0$):

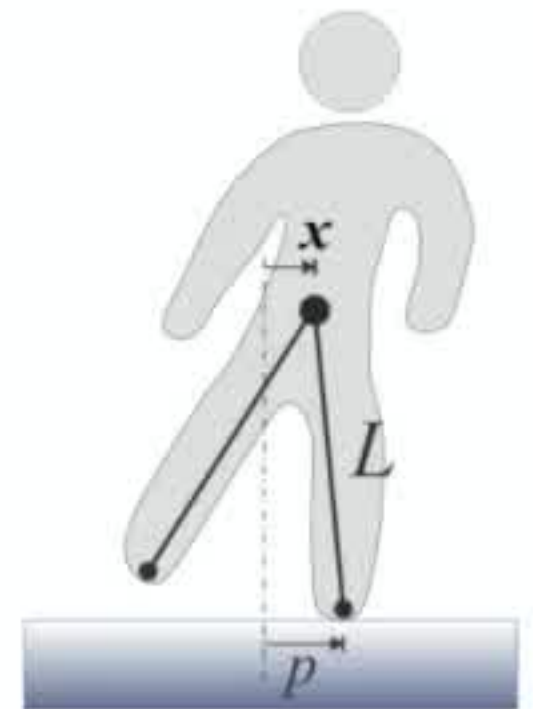
Supporting cables (or tall load bearing towers) are more flexible and can become oscillators prior to noticeable bridge wobbling.

Piecewise-smooth system:

Analogy with a walker on a bridge:

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = \gamma_1 \text{sign}(\dot{x}),$$

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = -\gamma_1 \iff \ddot{x}_i + \lambda \dot{x}_i + \omega_i^2 x_i = \gamma_1$$

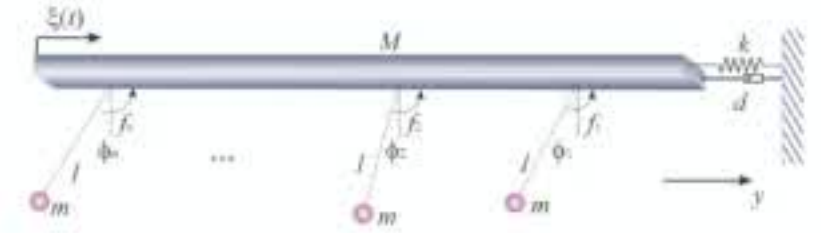


Close-form solutions for the “glued” limit cycle provide estimates of the oscillation frequency.

Complete synchrony among the supports is governed by

$$\ddot{x} + \lambda \dot{x} + \omega^2 x = -\ddot{y} + \gamma_1 \text{sign}(\dot{x}),$$

$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -rn\ddot{x} + \gamma_2 \text{sign}(\dot{y}) + A \sin(t + \psi)$$



$$r = \frac{m}{M + nm}$$

The nonlinear system has **two characteristic frequencies** inherited from the linear system without the wind-induced terms:

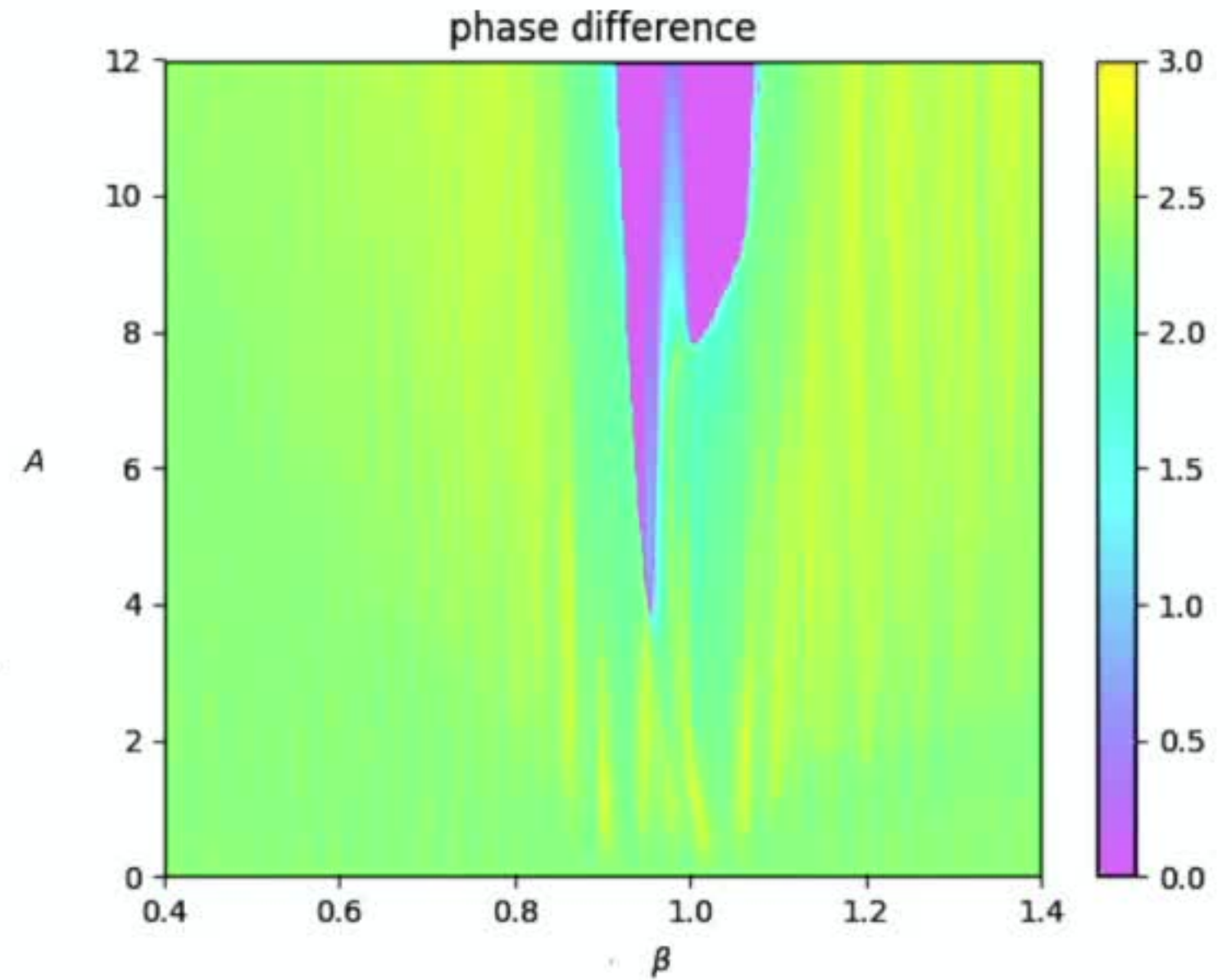
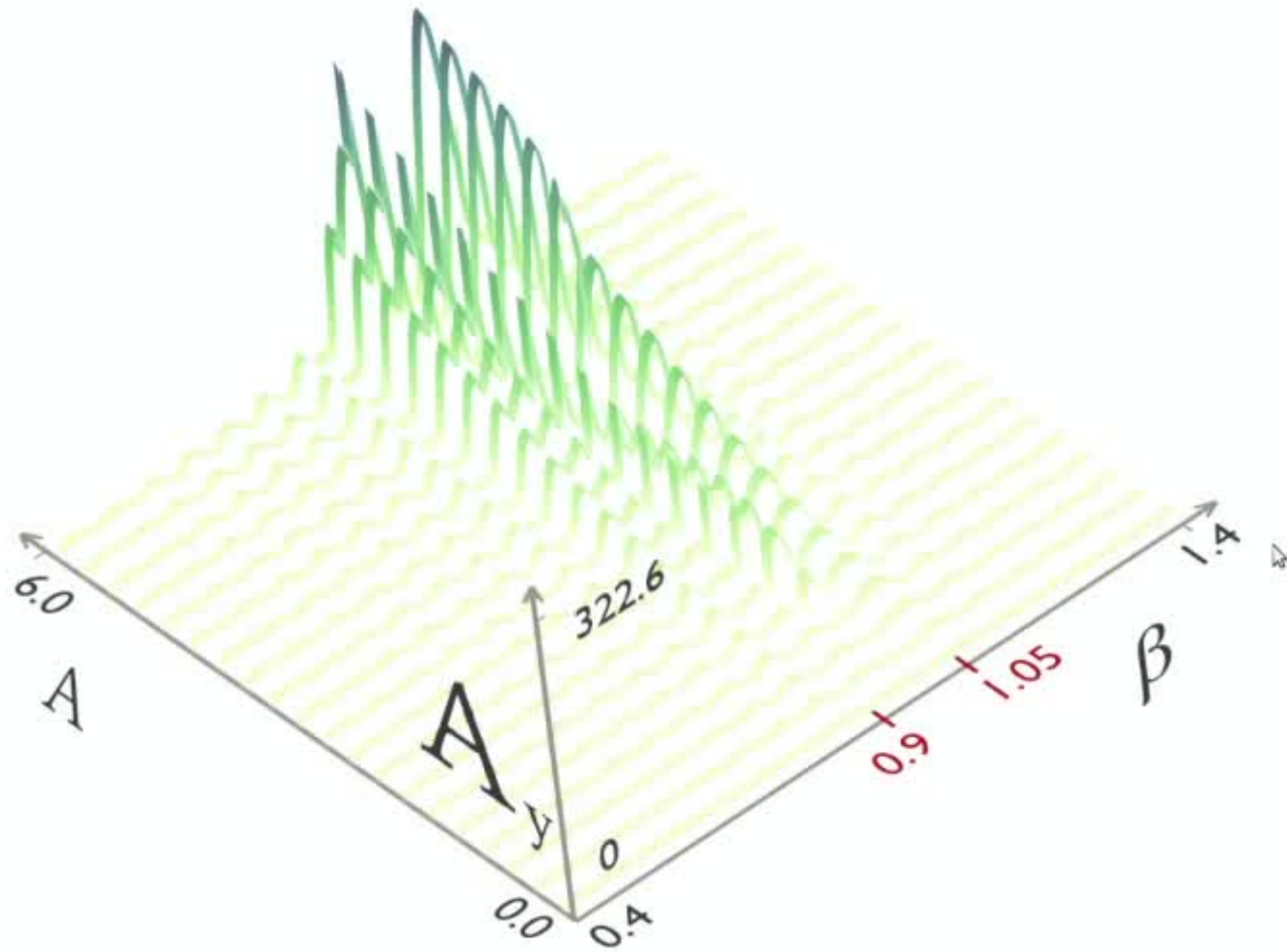
$$\ddot{x} + \lambda \dot{x} + \omega^2 x = -\ddot{y},$$

$$\ddot{y} + 2h\dot{y} + \Omega^2 y = -rn\ddot{x}$$

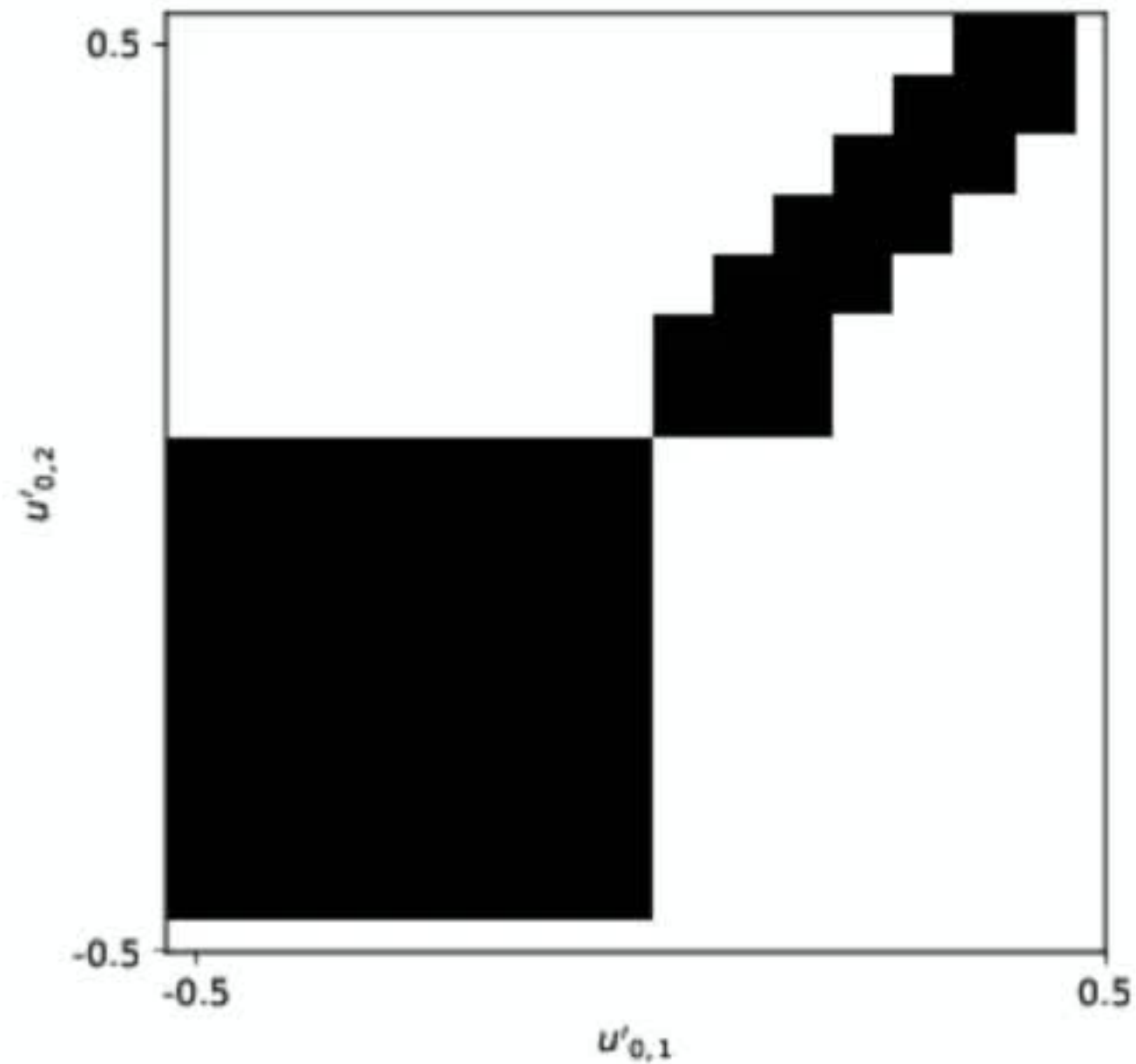
Characteristic equation:

$$\mu^2 p^4 - (p^2 + \lambda p + \omega^2) (p^2 + hp + \Omega^2) = 0$$

Wobbling at **two** forcing frequencies

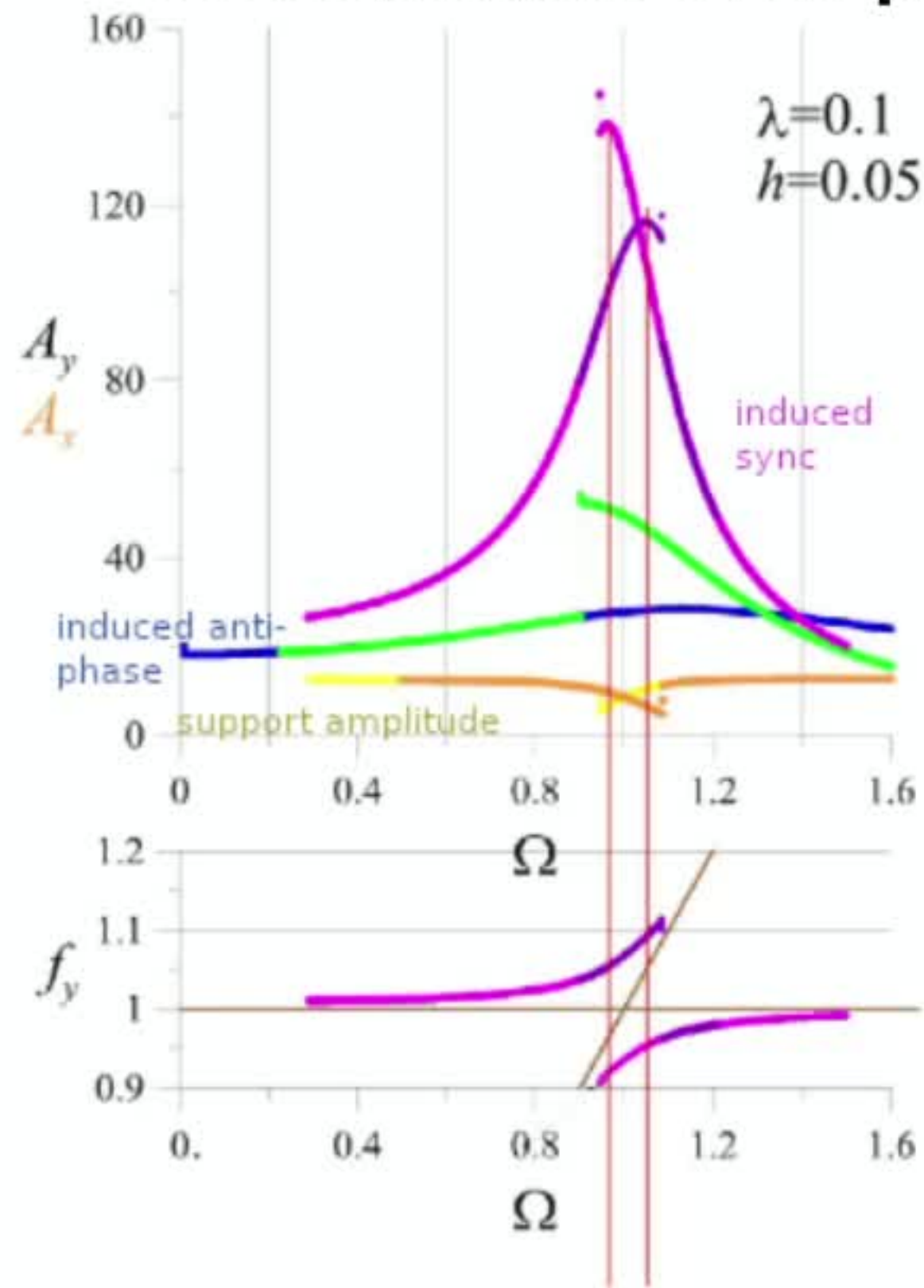


Wide basin of attraction



Even at low amplitudes!

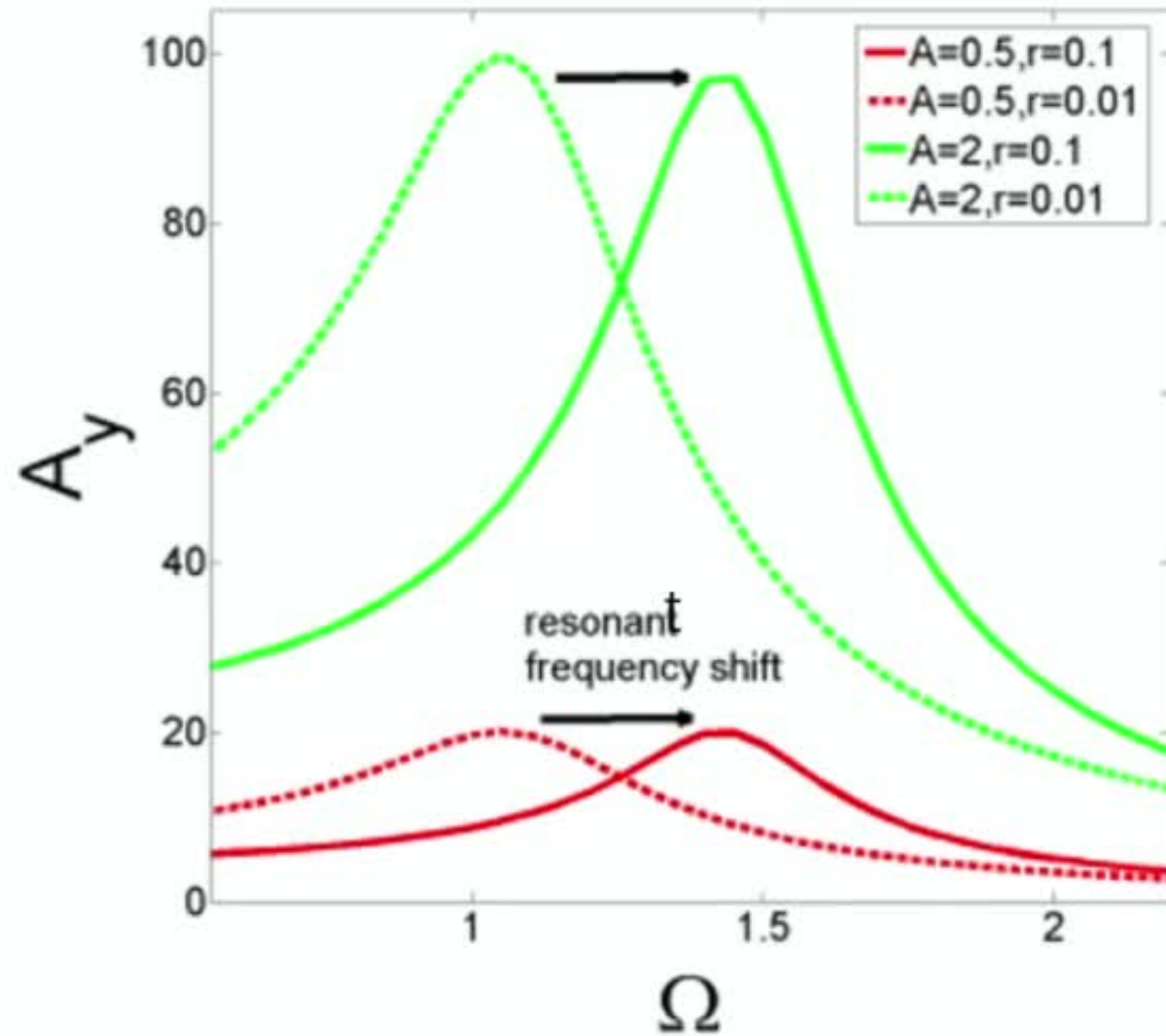
Co-existence of in-phase and out-of-phase states



Two emergent frequencies of bridge wobbling

$$\mu^2 p^4 - (p^2 + \lambda p + \omega^2) (p^2 + hp + \Omega^2) = 0$$

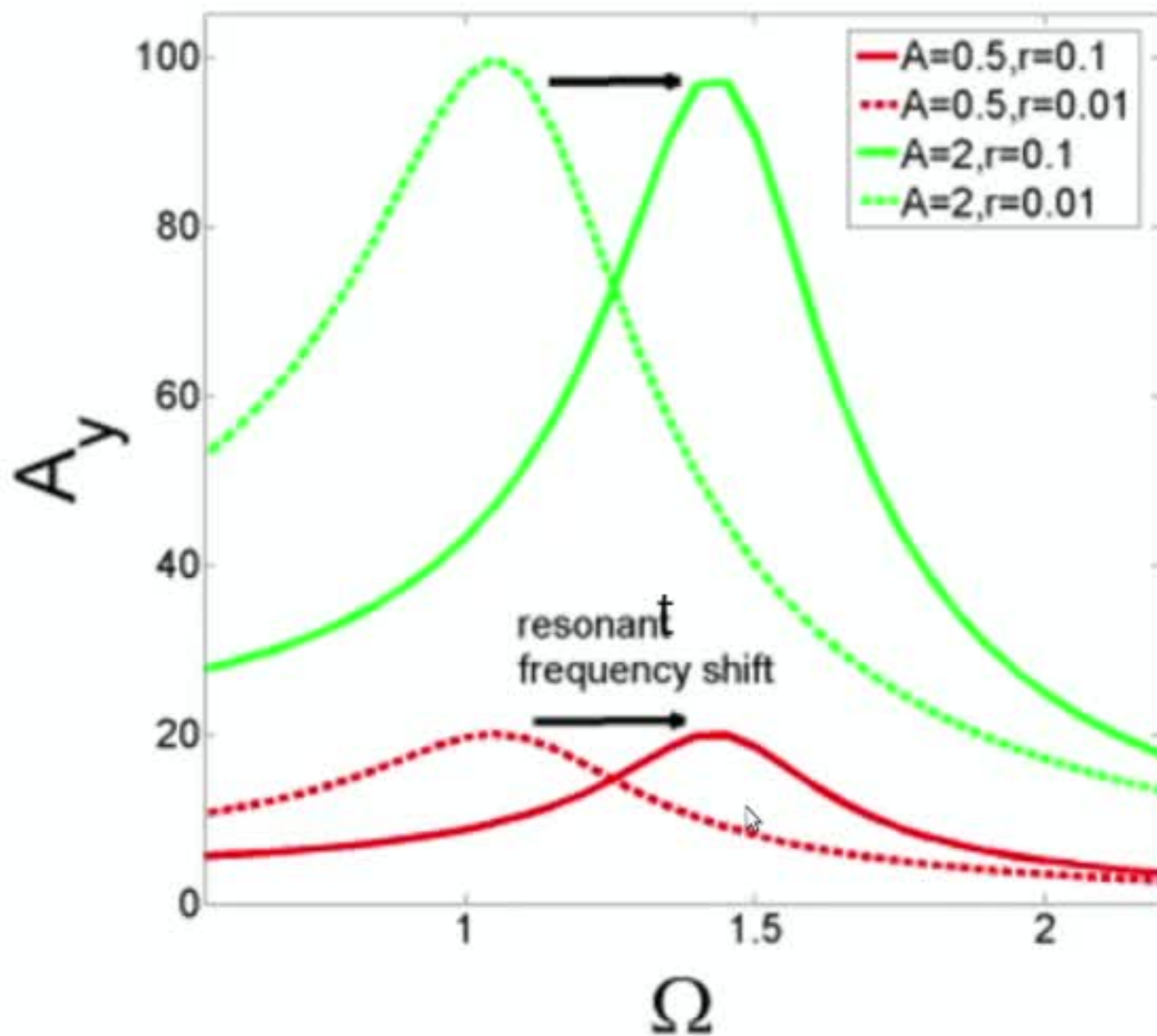
Resonant Frequency Shift



Resonance can be achieved at a frequency different from that of $A\sin(t + \psi)$

Varying γ_1 and γ_2 ($\gamma_1 > \gamma_2$) helps shifting the resonant frequency even farther.

Resonant Frequency Shift



Resonance can be achieved at a frequency different from that of $A\sin(t + \psi)$

Varying γ_1 and γ_2 ($\gamma_1 > \gamma_2$) helps shifting the resonant frequency even farther.

Type to search...

Terminal window showing code:

```

#!/usr/bin/perl

my $file = "test.txt";
my $content = "Hello World!";

open(FILE, ">$file");
print FILE $content;
close(FILE);

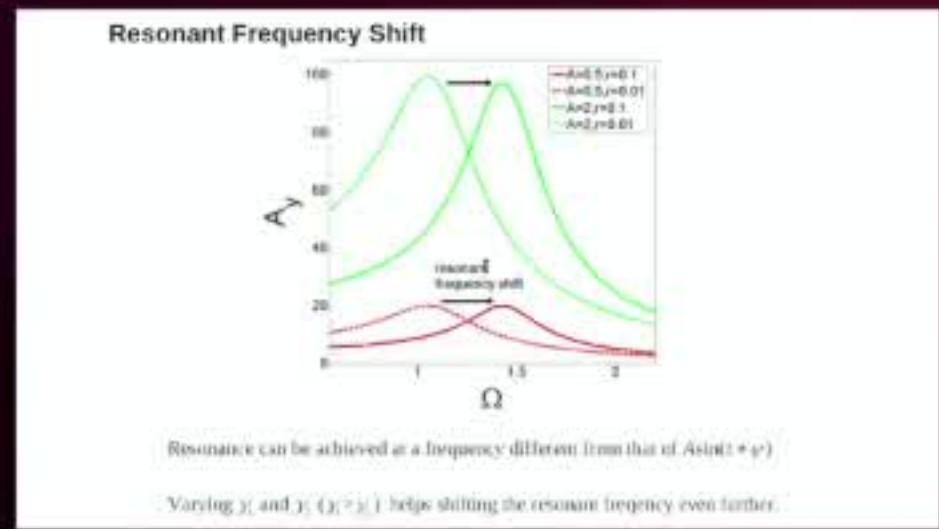
```

File manager window showing a folder named 'test' containing 'test.txt'.

Terminal window with title 'eDP-1' and a dark background.

File manager windows showing folder contents:

- Folder 'test' containing 'test.txt'.
- Folder 'test2' containing 'test2.txt'.
- Folder 'test3' containing 'test3.txt'.
- Folder 'test4' containing 'test4.txt'.
- Folder 'test5' containing 'test5.txt'.



Wind-induced instability of a suspension bridge: a tale of two frequencies

Kevin Daley¹, Vladimir Belykh², and Igor Belykh¹

¹Department of Mathematics & Statistics
Georgia State University, Atlanta, USA

²Lobachevsky State University of Nizhny Novgorod, Russia

daley2019_two_frequencies.odp - LibreOffice Impress

Terminal window with title 'eDP-1' and a dark background.



Slides

Normal Outline Notes Slide Sorter



Wind-induced instability of a suspension bridge: a tale of two frequencies

Kevin Daley¹, Vladimir Belykh², and Igor Belykh¹

¹*Department of Mathematics & Statistics,
Georgia State University, Atlanta, USA*

²*Lobachevsky State University of Nizhny Novgorod, Russia*



SIAM Conference on Applications of Dynamical Systems, Snowbird, May 20, 2019

Properties

Slide

Format: Screen 16:9

Orientation: Landscape

Background: None

Insert Image...

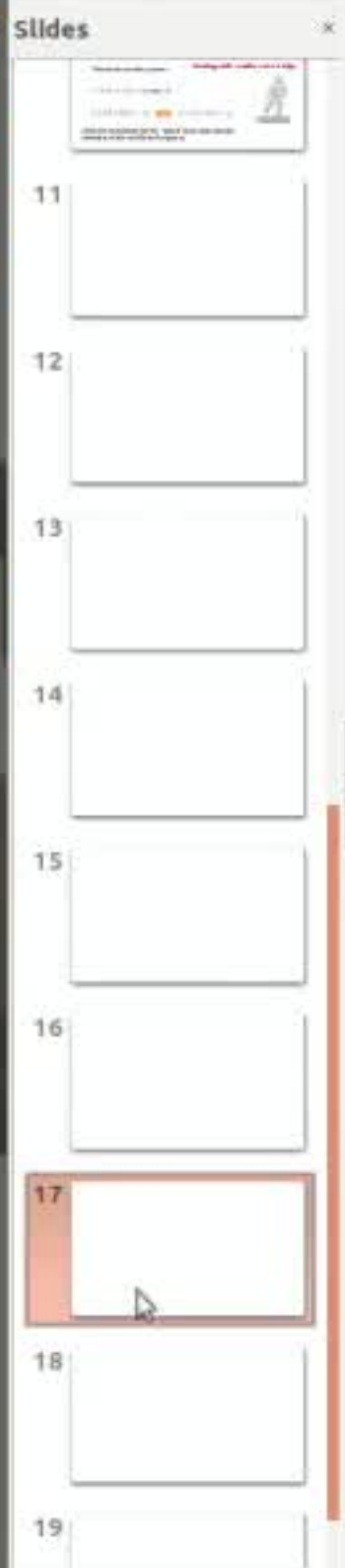
Master Slide: Title, Content

 Master Background Master Objects

Master View

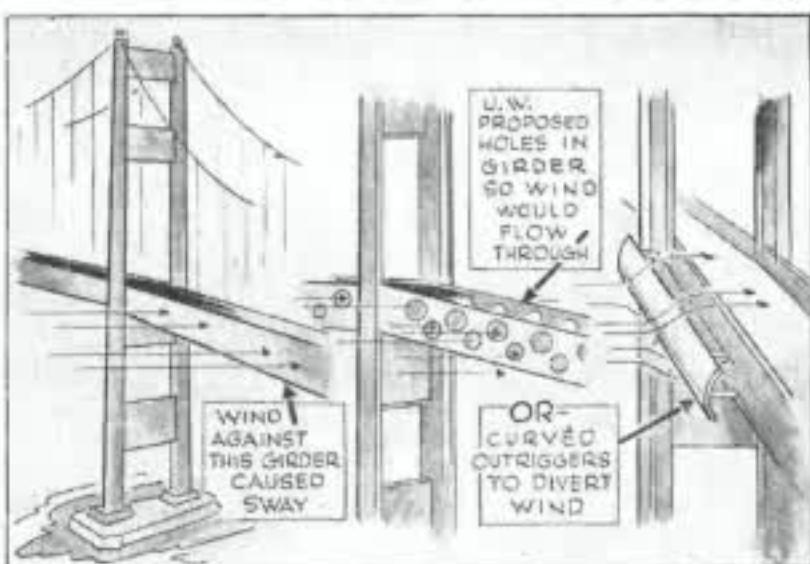
Layouts



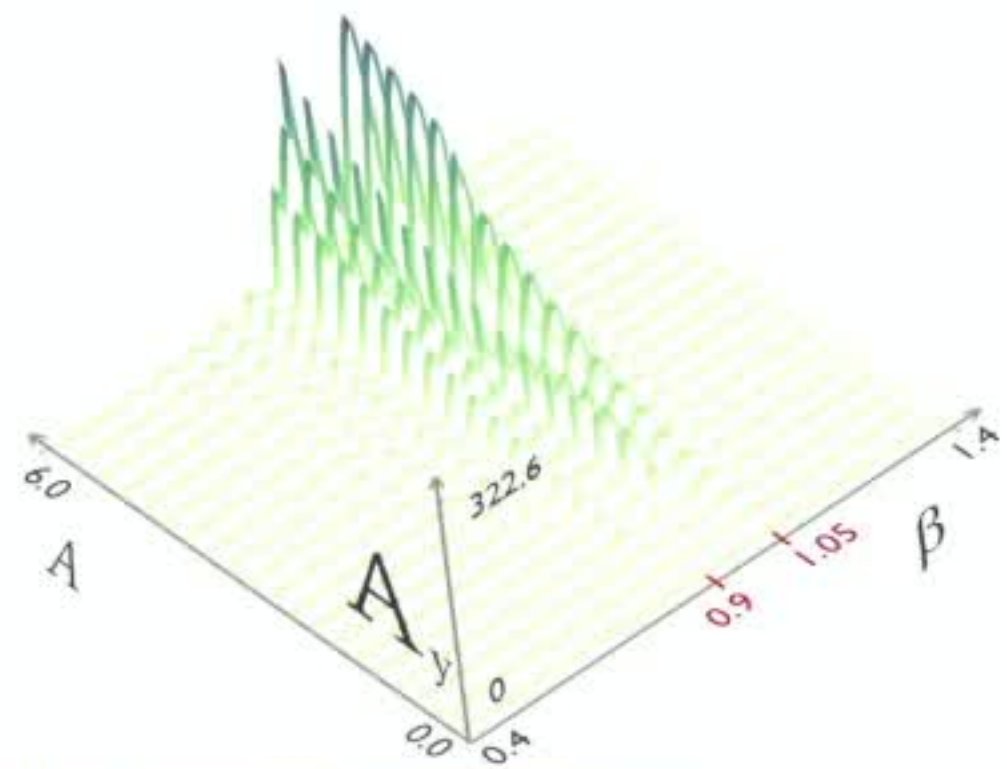


Normal Outline Notes Slide Sorter

WOULD THIS HAVE SAVED BRIDGE?



University of Washington engineers made a test Saturday on their \$11,000 model of the Tacoma Narrows Bridge, attempting to eliminate the dangerous wind sways which finally caused the real-life structure to collapse yesterday. The sketch at left shows the flat horizontal girder which offered resistance to winds, causing the sway. University recommendations were (center) to drill holes with a tooth in the girder, permitting the wind to pass through; or (right) to erect an \$80,500 streamlined buffer alongside the girder, to divert winds. These tests showed the latter materially reduced the vibrations, might have saved the bridge.



Our study suggests an answer of "maybe not" to the proposed engineering solutions that might have saved the Tacoma bridge, such as drilling holes in the bridge girder.

Properties

Slide

Format: Screen 16:9

Orientation: Landscape

Background: None

Insert Image...

Master Slide: Title, Content

Master Background

Master Objects

Master View

Layouts

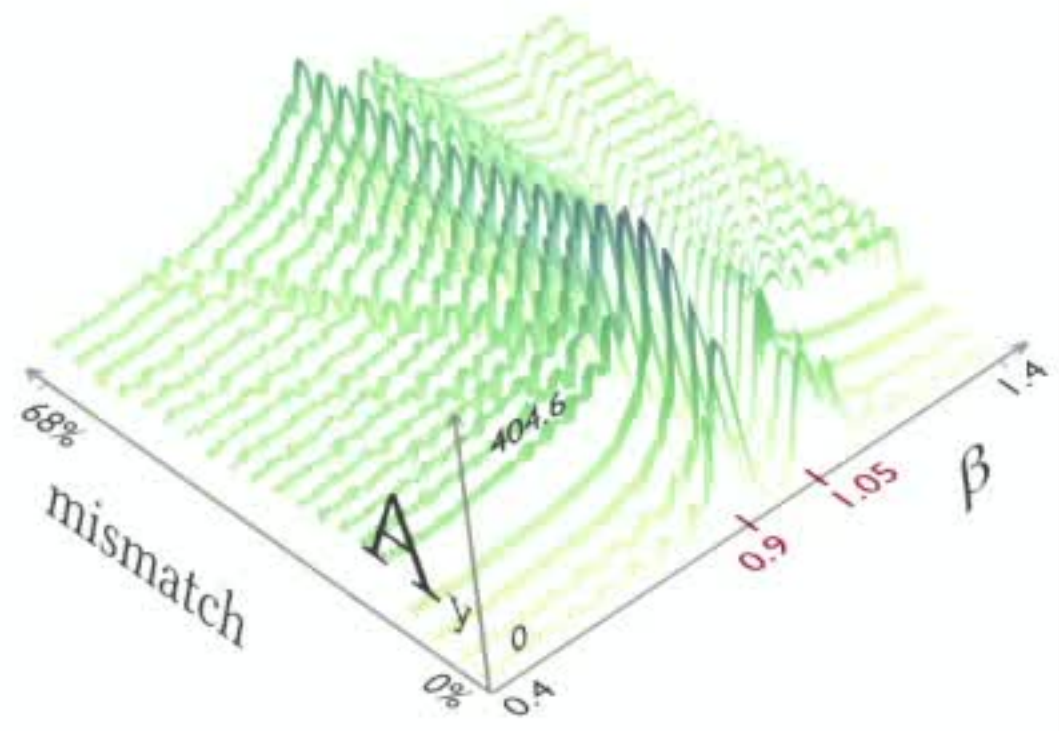
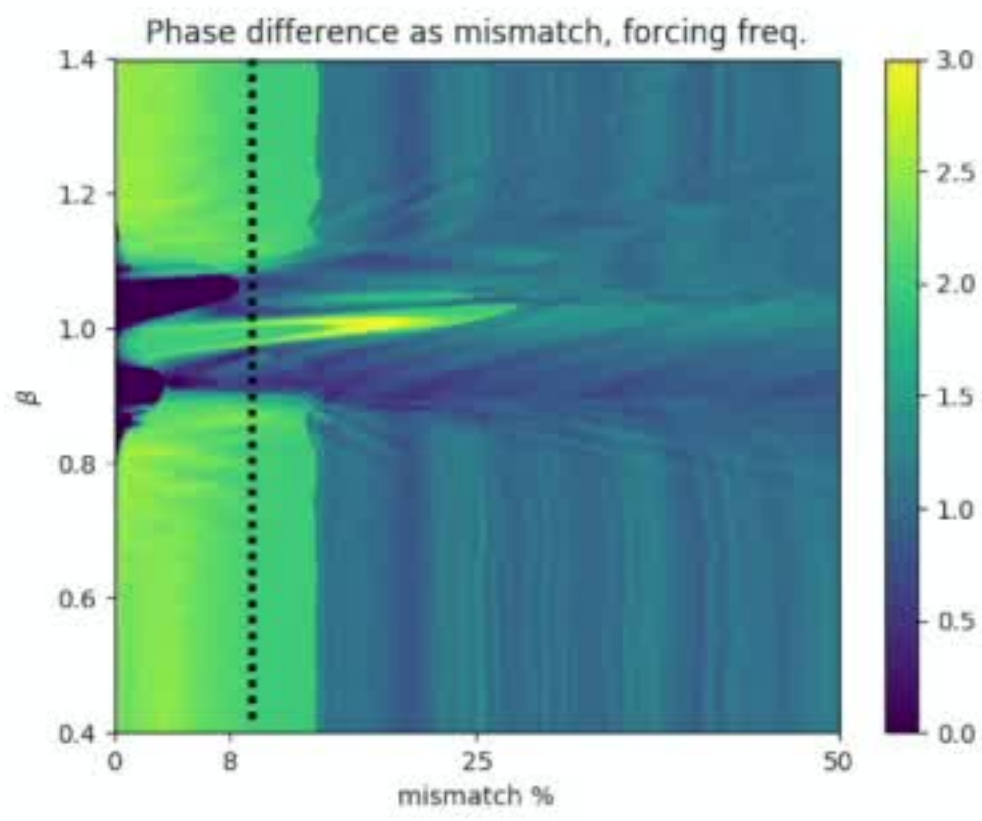


Slides

- 11
- 12
- 13
- 14
- 15
- 16
- 17
- 18
- 19 Slide 19

Normal Outline Notes Slide Sorter

Asymmetric design ("ugly" bridges) may mitigate destructive vibrations



Properties

Slide

- Format: Screen 16:9
- Orientation: Landscape
- Background: None
- Master Slide: Title, Content
- Master Background
- Master Objects

Layouts
