ShapeFit: Exact location recovery from corrupted pairwise directions

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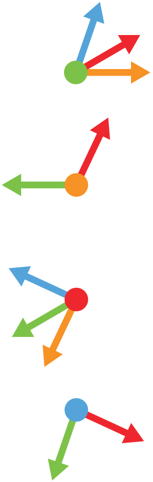
with Choongbum Lee, Vlad Voroninski

24 May 2016 – Funding: NSF, ONR

Location recovery from directions



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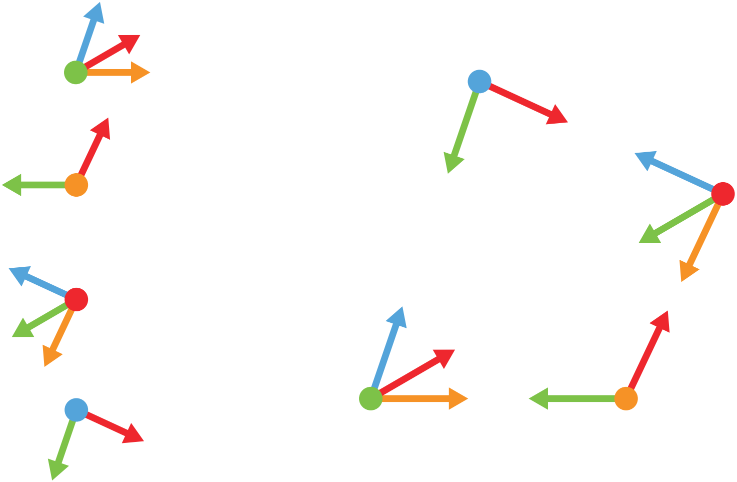


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Difficulty: Incorrect point-correspondences

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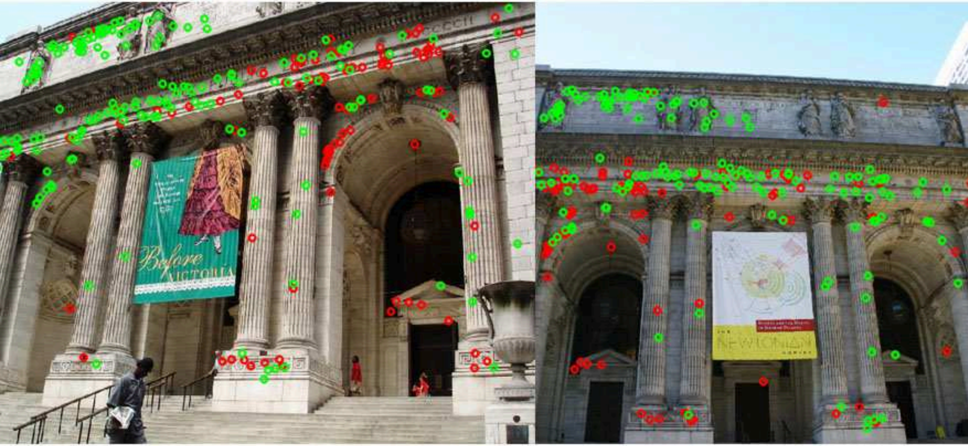
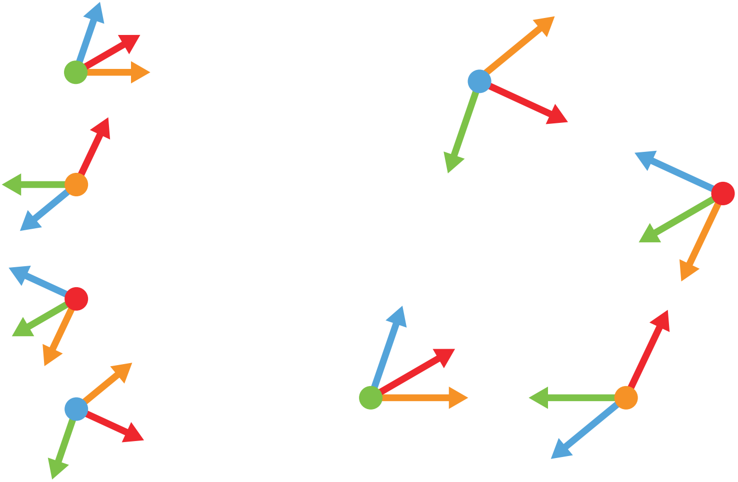


Location recovery from corrupted directions



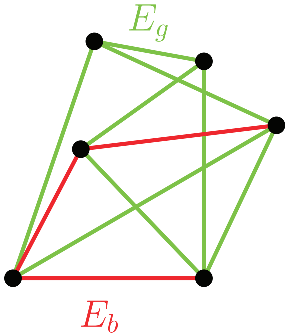
Formulation: Location recovery from directions

Let: $t_1 \dots t_n \in \mathbb{R}^3$

$G = ([n], E = E_g \sqcup E_b)$

$v_{ij} = \dfrac{t_i - t_j}{\|t_i - t_j\|_2}$ for $ij \in E_g$

$v_{ij} \in \mathcal{S}^2$ for $ij \in E_b$



Given: $G, \{v_{ij}\}$

Find: $\{t_i\}$ up to translation and scale

Algorithms for location recovery from directions

Not robust	Empirically robust	Provably robust
least squares	1dSfM	
ℓ_∞ methods	LUD	
spectral methods		
...		

Singer et al.; Kahl; Brand, Antone, Teller; Wilson, Snavely; Özyeşil and Singer

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ShapeFit

$$\begin{aligned} \underset{\{t_i\}}{\text{minimize}} \quad & \sum_{ij \in E} \|P_{v_{ij}^{\perp}}(t_i - t_j)\|_2 \\ \text{subject to} \quad & \sum_{ij \in E} \langle t_i - t_j, v_{ij} \rangle = 1, \quad \sum_{i \in [n]} t_i = 0 \end{aligned}$$

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– Robust to corruptions

– Recovery guarantee

– Reasonable on real data

– Fast implementation

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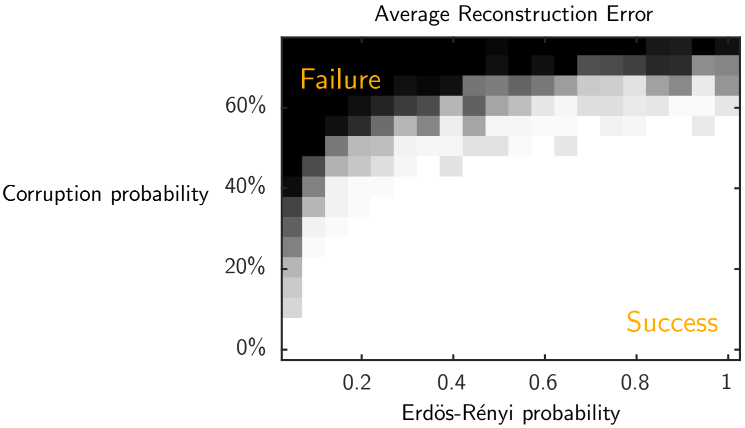
ShapeFit can succeed with 60% corruption on a random model



Shown for $\mathbb{R}^3$, Erdös-Rényi graph, $n = 200$, Gaussian locations

ShapeFit

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ShapeFit provably tolerates corruptions under a random data model

Let: $t_1 \dots t_n \sim \mathcal{N}(0, I_3)$

G be Erdös-Rényi with prob. p

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Theorem (Hand, Lee, Voroninski 2015)

Let $n \gtrsim 1$ and $p \gtrsim n^{-\frac{1}{3}}$. There is $\gamma = \Omega(p^5 / \log^3 n)$ such that whp:

If $\max_i \deg_b(i) \leq \gamma n$ *then ShapeFit's unique minimizer is exact.*

ShapeFit proof

$$\min \sum_{ij\in E} \|P_{v_{ij}^{\perp}}(t_i - t_j)\|_2 \text{ subject to } \sum_{ij\in E} \langle t_i - t_j, v_{ij}\rangle = 1, \sum_{i=1}^{n} t_i = 0$$

Proof:

$$\begin{aligned} \text{Constraint} &\Rightarrow \text{lengths scale differently} \\ &\Rightarrow \text{induced rotations on subgraph} \\ &\Rightarrow \text{induced rotations on good graph} \\ &\Rightarrow \text{increased objective} \end{aligned}$$

ShapeFit

$$\begin{aligned} \underset{\{t_i\}}{\text{minimize}} \quad & \sum_{ij \in E} \|P_{v_{ij}^{\perp}}(t_i - t_j)\|_2 \\ \text{subject to} \quad & \sum_{ij \in E} \langle t_i - t_j, v_{ij} \rangle = 1, \quad \sum_{i \in [n]} t_i = 0 \end{aligned}$$

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MAP DIVISION

Several methods have state-of-the-art median recovery errors

Median recovery error (m)

	ShapeFit	LUD	1d+Huber	1d + SF	1d+LUD
Ellis Island	30	**25**	40	29	25
NYC Library	2.5	2.9	**2.2**	2.4	2.8
Piazza Pop.	2.4	3.0	3.2	**1.7**	2.0
Metropolis	2.8	4.2	4.0	**2.4**	3.7
Montreal ND	1.6	1.2	**0.9**	1.5	1.1
Tow. London	3.3	5.6	3.5	**3.3**	4.3
Notre Dame	0.5	0.5	0.5	0.5	**0.5**
Alamo	0.9	0.9	0.8	**0.8**	0.9
Gendarmen.	35	29	37	**27**	27
Union Sq.	13	7.8	7.9	**7.4**	7.9
Vienna Cath.	19	6.0	**4.3**	7.6	5.8
Roman For.	18	7.6	**6.4**	19	7.7

LUD has state-of-the-art mean recovery errors

Mean recovery error (m)

	ShapeFit	LUD	1d+Huber
Ellis Island	442	**25**	1e6
NYC Library	3e3	**7.2**	995
Piazza Pop.	8.9	**6.2**	1e5
Metropolis	145	**15**	6e4
Montreal ND	3.1	**2.1**	4e4
Tow. London	99	**24**	2e5
Notre Dame	**1.5**	1.5	5e3
Alamo	3.4	**2.8**	8e3
Gendarmen.	266	**53**	2e5
Union Sq.	4e4	**13**	8e3
Vienna Cath.	2e3	**15**	2e5
Roman For.	661	**18**	6e4

ShapeFit can be solved faster than prior methods

Solution time (s)

	ShapeFit	LUD	1d+Huber
Ellis Island	**0.5**	6.1	8.8
NYC Library	**1.2**	6.5	38
Piazza Pop.	**0.4**	2.8	7.6
Metropolis	**0.9**	7.0	18
Montreal ND	**1.3**	13	115
Tow. London	**1.2**	6.8	142
Notre Dame	**2.9**	24	46
Alamo	**2.8**	18	199
Gendarmen.	**1.8**	16	24
Union Sq.	**1.6**	11	44
Vienna Cath.	**6.8**	29	201
Roman For.	**4.0**	24	82

ShapeFit performs reasonably with linear motion

Joint work with: Lee, Voroninski, Goldstein, Tsotsos, Soatto

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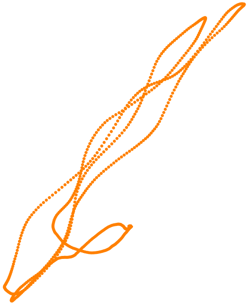
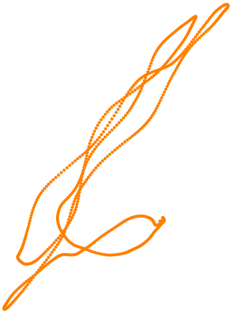


ShapeFit



Visual-Inertial Kalman Filter

Joint work with: Lee, Voroninski, Goldstein, Tsotsos, Soatto

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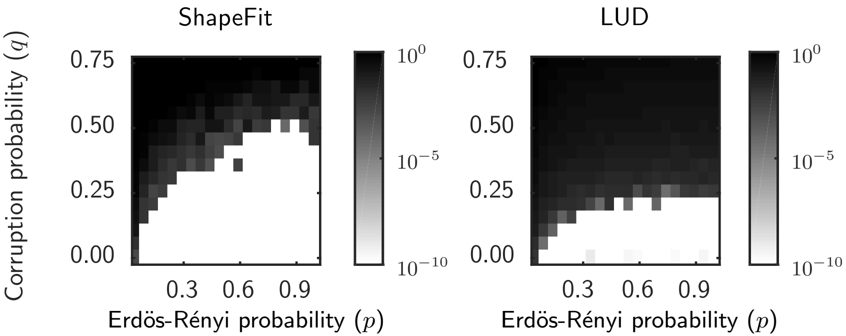
Comparison of ShapeFit and LUD in noiseless synthetic data



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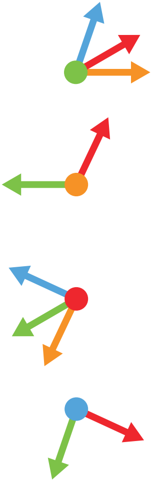
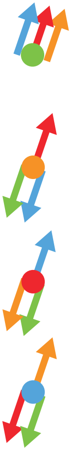
1dSfM filters outliers by inconsistent 1d projections



Wilson, Snavely

1dSfM filters outliers by inconsistent 1d projections



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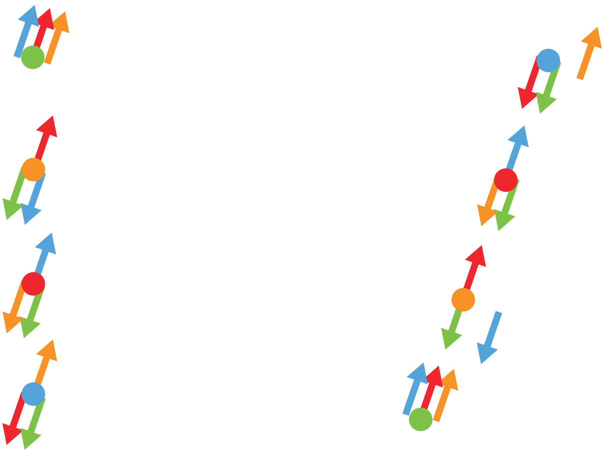


Wilson, Snavely

Problem Formulation with Noise

Let:

$$t_1 \dots t_n \in \mathbb{R}^3$$

$$G = ([n], E = E_g \sqcup E_b)$$

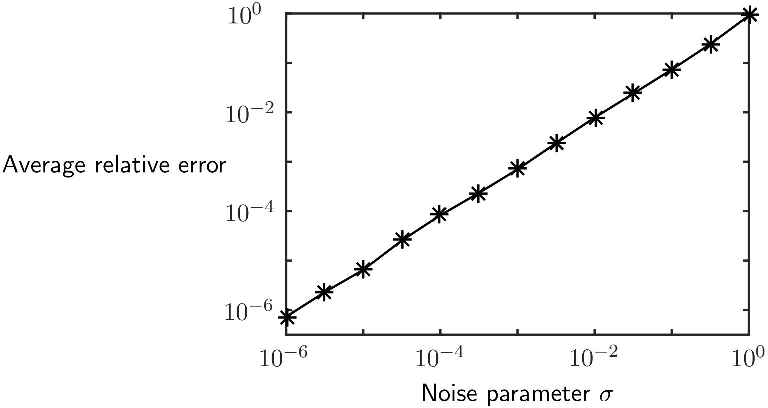
$$v_{ij} = \begin{cases} \left((t_i - t_j)^\wedge + \sigma z_{ij}\right)^\wedge & \text{if } ij \in E_g \\ z_{ij} & \text{if } ij \in E_b \end{cases}$$

$$z_{ij} \sim \mathrm{Unif}(\mathcal{S}^2)$$

Given: $G, \{v_{ij}\}$

Estimate: $\{t_i\}$ up to translation and scale

ShapeFit is empirically stable to noise



Shown for $n = 50$, Erdös-Rényi probability $= 1/2$, corruption probability $= 0.2$

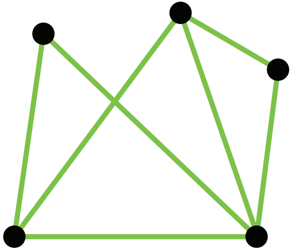
Triangles Inequality

Lemma

Let $d \geq 3$. If $\{t_i\}$ is c-well-distributed w.r.t. (x, y), then for all $h_x, h_y, h_1, \ldots, h_k \in \mathbb{R}^d$,

$$\sum_{i \in [k]} \|P_{(x-t_i)^\perp}(h_x - h_i)\|_2 + \|P_{(t_i-y)^\perp}(h_i - h_y)\|_2 \geq ck \cdot \|P_{(x-y)^\perp}(h_x - h_y)\|_2$$

Recovery from exact directions is possible if the graph is parallel rigid



Parallel rigid

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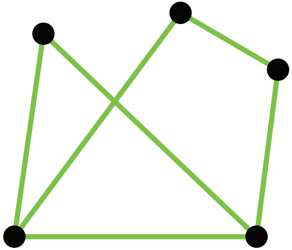

Not parallel rigid

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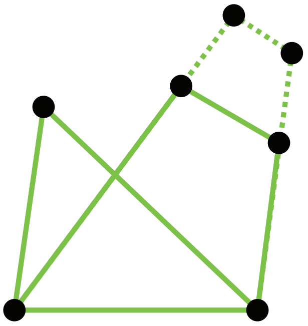
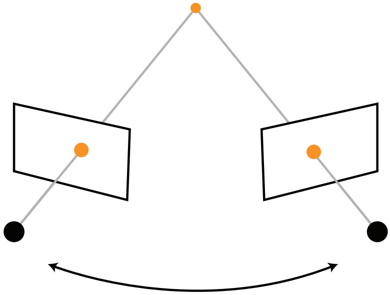


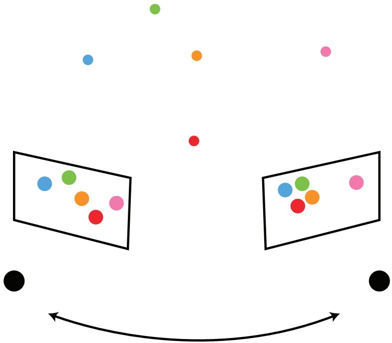
Not parallel rigid

Epipolar geometry



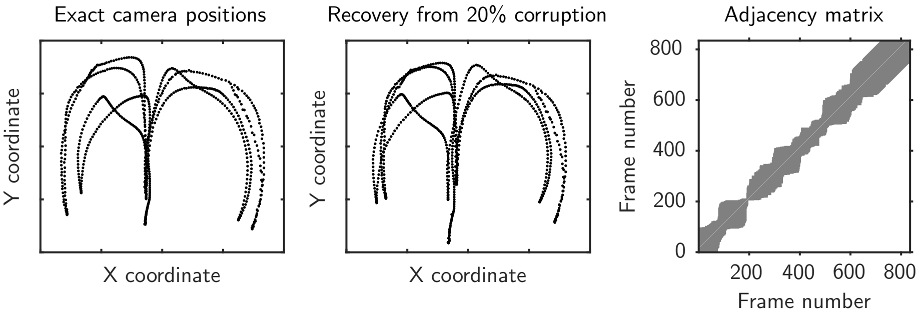
Rotation + Translation

Epipolar geometry:
5 point-correspondences allow relative pose recovery



Rotation + Translation

ShapeFit is fast enough for real time applications



ShapeFit can be iterated