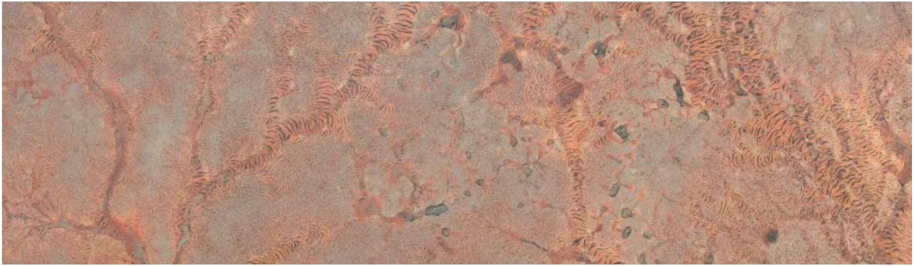
# Water transport in models of dryland vegetation patterns

Punit Gandhi

Mathematical Biosciences Institute, Ohio State University, Columbus, OH 43210

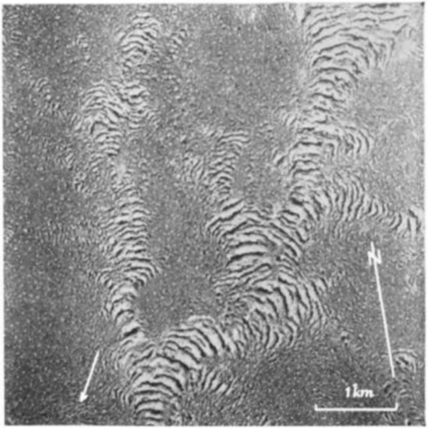
May 20, 2019



# “Landscape Scale” → “Pattern Scale”

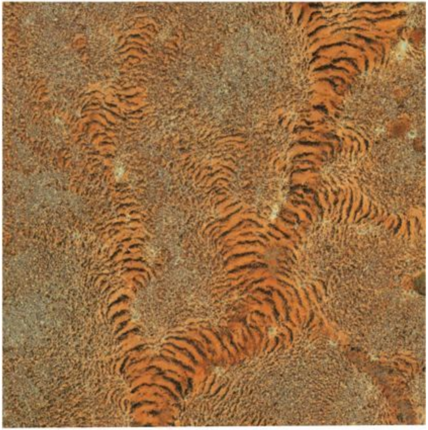
British Somaliland, 1945 Ethiopia, 2011

a




Mar. 1945

b



Dec. 2011

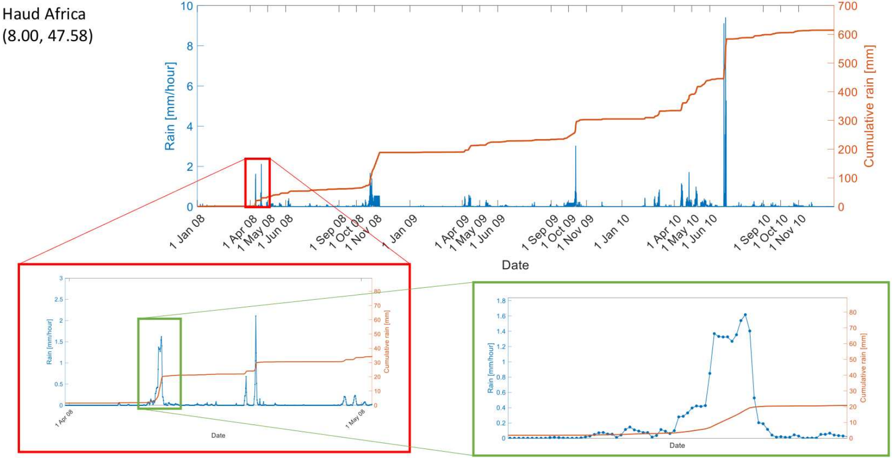
Macfadyen, *Geograph. J.*, **116**(4) (1950). Gowda, Iams, Silber, *Scientific Reports*, **8**(1) (2018).

# “Pattern Scale” → “Plant Scale”

Boaler & Hodge, *Ecology J.*, **52**(3) (1964).

# Infrequent and upredictable water input

## Rainfall data: 2008-2010




Compiled by S. Bonetti

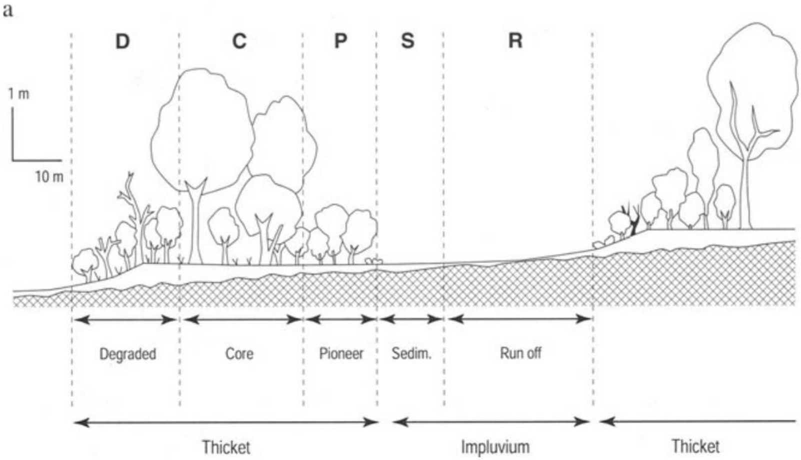



Figure 1.3a. Schematic diagram of a typical transect through the tiger bush in Niger. (Adapted from Thiéry, d'Herbès, and Valentin 1995; Hiernaux and Gérard 1999.)

J.-M. d'Herbès et al. *Banded vegetation patterning in arid and semiarid environments*. Springer, New York (2001).

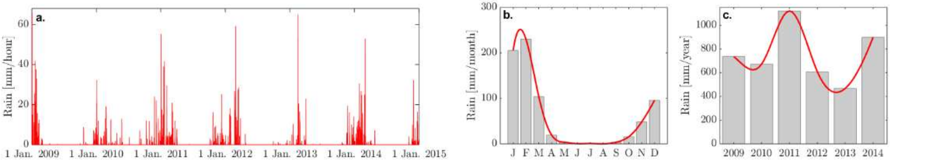
# Processes: Different timescales and different locations

**Timescales for water and biomass**

- Minutes-Hours: rain events, surface water transport, infiltration
- Weeks-Months: evaportranspiration, plant growth and death
- Years-Decades: vegetation colonization

**Locations for hydrological processes**

- Surface: rain, transport, -infiltration
- Soil: +infiltration, evapotranspiration




Stewart Plains, Australia: Beringer et al. *Biogeosciences* (2016).

# Conceptual modeling with a focus on timescales

**Goals**

- Better use data to constrain model parameters.
- Build a modeling framework that can incorporate stochasticity from infrequent and largely unpredictable water input.
- Keep the model simple enough for analysis (or at least simple enough for numerical simulations)

Sara Bonetti
(ETH Zurich)

Amilcare Porporato
(Princeton)

Sarah Iams
(Harvard)

Mary Silber
(U Chicago)

# Presentation Outline

# A simple reaction-advection-diffusion model

Klausmeier, *Science*, **284** (1999)

## Klausmeier Model

Biomass:
$$\frac{\partial B}{\partial T} = \underbrace{-MB}_{\text{mortality}} + \underbrace{JRWB^2}_{\text{growth}} + \underbrace{D\nabla^2 B}_{\text{dispersal}}$$

Water:
$$\frac{\partial W}{\partial T} = \underbrace{A}_{\text{precipitation}} \underbrace{-LW - RWB^2}_{\text{evapotranspiration}} + \underbrace{V\frac{\partial W}{\partial X}}_{\text{transport}}$$

## Klausmeier's Parameters

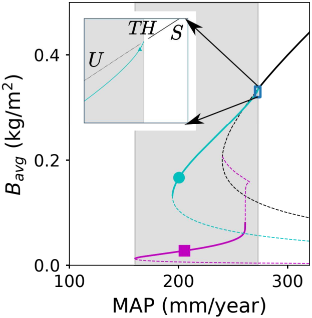
$J$=0.003 kg Dry Mass/kg $H_2O$ (maximum water use efficiency)
$M$=1.8 $yr^{-1}$ (carbon costs required to maintain leaves)

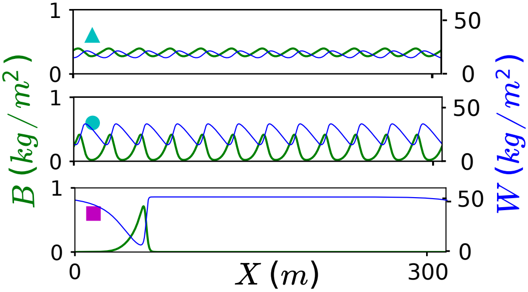
Mauchamp et al, *Ecol. Model.*, **71** (1994).

$L$=4 $yr^{-1}$ (observed soil water + predicted bare soil state)
$R$ = 100 mm $H_2O$/yr/(kg Dry Mass)$^2$ (observed biomass + predicted vegetated state)
$V$, $D$ Fit to match observed pattern wavelength and migration speed

# Predictions about hydrology

**Klausmeier Model**




Water is maximum in bare soil between vegetation bands

# Modeling infiltration of surface water into soil

Klausmeier "Water" $\rightarrow$ Surface water + Soil water

Rietkerk et al, *Am. Nat.* **160** (2002). See also: Gilad et al, *PRL* **93** (2004).

$$\text{Biomass}: \quad \frac{\partial B}{\partial T} = -\underbrace{mB}_{\text{mortality}} + \underbrace{c\mathcal{G}(W)B}_{\text{growth}} + \underbrace{D_b \nabla^2 B}_{\text{seed dispersal}}$$

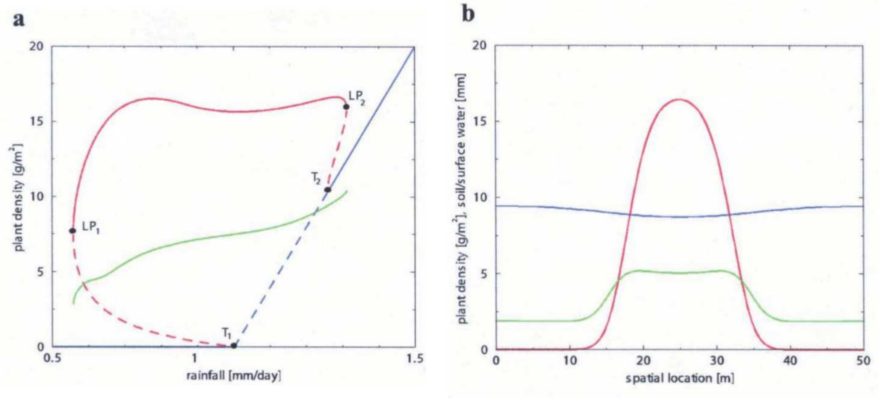
$$\text{Soil water}: \quad \frac{\partial W}{\partial T} = \underbrace{\mathcal{I}(B)H}_{\text{infiltration}} - \underbrace{rW - \mathcal{G}(W)B}_{\text{evapotranspiration}} + \underbrace{D_w \nabla^2 W}_{\text{soil diffusion}}$$

$$\text{Surface water}: \quad \frac{\partial H}{\partial T} = \underbrace{P}_{\text{precipitation}} - \underbrace{\mathcal{I}(B)H}_{\text{infiltration}} + \underbrace{V_h \frac{\partial H}{\partial X}}_{\text{surface transport}}$$

$$\mathcal{G}(W) = g\frac{W}{W+k}, \qquad \mathcal{I} = \alpha \frac{B+qf}{B+q}.$$

# Predictions about hydrology

## Rietkerk Model




Rietkerk et al, *Am. Nat.* **160** (2002)

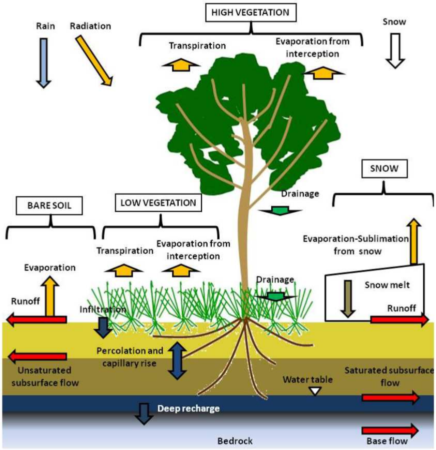
Soil water is maximum in vegetation band

Surface water is minimum in vegetation band

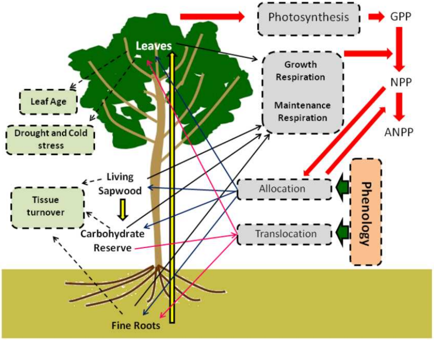
other phase possible in 3-field models: Kinast et al, *PRL* **112** (2014)

# A mechanistic ecohydrological model

"Tethys-Chloris" (T&C) Model




Hydrology and energy balance




Carbon fluxes

Fatichi et al., *J. Adv. Model. Earth Syst.* **4** M05002 (2012)

# A mechanistic ecohydrological model

## "Tethys-Chloris" (T&C) Model

**Table 2.** A List of Principal Fluxes and States Simulated by "Tethys-Chloris"

| Variable | Description | Units |
|---|---|---|
| | *Energy and Mass Fluxes* | |
| $R_{abs}$ | Absorbed shortwave radiation | $[W\ m^{-2}]$ |
| $L_{abs}$ | Absorbed longwave radiation | $[W\ m^{-2}]$ |
| $PAR_{abs}$ | Absorbed *Photosynthetically Active Radiation* | $[W\ m^{-2}]$ |
| $R_n$ | Net radiation | $[W\ m^{-2}]$ |
| $H$ | Sensible heat | $[W\ m^{-2}]$ |
| $\lambda E$ | Latent heat | $[W\ m^{-2}]$ |
| $G$ | Ground heat flux | $[W\ m^{-2}]$ |
| $Q_v$ | Incoming heat with precipitation | $[W\ m^{-2}]$ |
| $T_s$ | Prognostic surface temperature | $[^{\circ}C]$ |
| $T_d$ | Ground temperature at damping depth | $[^{\circ}C]$ |
| $T_{H,\ or\ L,}$ | Transpiration flux from vegetation | $[mm\ h^{-1}]$ |
| $E_g$ | Evaporation flux from ground under vegetation | $[mm\ h^{-1}]$ |
| $E_{bare}$ | Evaporation flux from bare soil | $[mm\ h^{-1}]$ |
| $E_{wat}$ | Evaporation flux from water surface | $[mm\ h^{-1}]$ |
| $E_{sno,f\ or\ s}$ | Evaporation/sublimation flux from snow in open surface and snow under vegetation | $[mm\ h^{-1}]$ |
| $E_{In,H,\ or\ L,}$ | Evaporation flux from intercepted water in canopy | $[mm\ h^{-1}]$ |
| $\alpha_{soil,\Lambda}$ | Albedo of ground | $[-]$ |
| $\alpha_{sno}$ | Albedo of snow | $[-]$ |
| $\alpha_{wat,\Lambda}$ | Water albedo | $[-]$ |
| $\epsilon_s$ | Emissivity of a generic surface | $[-]$ |
| $\alpha_s$ | Absorptivity of a generic surface | $[-]$ |
| $P_{r,liq}$ | Liquid (rain) precipitation | $[mm\ h^{-1}]$ |
| $P_{r,sno}$ | Solid (snow) precipitation | $[mm\ h^{-1}]$ |
| | *Turbulent Regime/Resistances* | |
| $r_a$ | Aerodynamic resistance | $[s\ m^{-1}]$ |
| $r_a'$ | Undercanopy resistance | $[s\ m^{-1}]$ |
| $r_b$ | Leaf boundary layer resistance | $[s\ m^{-1}]$ |
| $r_{soil}$ | Soil resistance | $[s\ m^{-1}]$ |
| $r_s$ | Stomatal resistance | $[s\ m^{-1}]$ |
| $d$ | Zero-plane displacement height | $[m]$ |
| $z_{om}$ | Roughness length for momentum | $[m]$ |
| $z_{oh}$ | Roughness length for heat flux | $[m]$ |
| $z_{ow}$ | Roughness length for water vapor | $[m]$ |
| $\alpha$ | Attenuation coefficient for undercanopy resistance | $[-]$ |
| $\alpha'$ | Attenuation coefficient for leaf boundary layer conductance | $[-]$ |
| $\grave{\alpha}$ | Humidity equilibrium value for soil water content | $[-]$ |
| | *Biochemical Model of Photosynthesis* | |
| $A_{nC}$ | Net assimilation rate | $[\mu mol\ CO_2\ s^{-1}m^{-2}]$ |
| $R_{dC}$ | Dark respiration | $[\mu mol\ CO_2\ s^{-1}m^{-2}]$ |
| $A_C$ | Gross photosynthetic rate | $[\mu mol\ CO_2\ s^{-1}m^{-2}]$ |
| $V_{max}$ | Canopy maximum Rubisco capacity at 25°C | $[\mu mol\ CO_2\ s^{-1}m^{-2}]$ |
| $J_{max}$ | Canopy maximum electron transport capacity at 25°C | $[\mu mol\ Eq\ s^{-1}\ m^{-2}]$ |
| $V_m$ | Maximum Rubisco capacity at canopy scale after accounting for temperature dependence | $[\mu mol\ CO_2\ s^{-1}m^{-2}]$ |
| $J_m$ | Maximum electron transport capacity at canopy scale after accounting for temperature dependence | $[\mu mol\ Eq\ s^{-1}\ m^{-2}]$ |
| $J^L_{max}$ | Maximum electron transport capacity at 25°C at leaf scale | $[\mu mol\ Eq\ s^{-1}\ m^{-2}]$ |
| $\theta_R$ | Soil water content available to roots | $[-]$ |
| $c_i$ | Partial pressure of intercellular $CO_2$ | $[Pa]$ |
| | *Snow Hydrology* | |
| $C_{sno}$ | Boolean operator reflecting presence or absence of snow | [0/1] |
| $S_{WE}$ | Snow water equivalent of ground snowpack | $[mm]$ |
| $In_{S_{WE}}$ | Snow water equivalent of intercepted snow in high-vegetation layer | $[W\ m^{-2}]$ |
| $dQ$ | Net energy flux input to snowpack | $[W\ m^{-2}]$ |
| $S_m$ | Snowmelt rate | $[mm]$ |
| $Q_{fm}$ | Heat release from melting (negative) or freezing (positive) of liquid water content held by snow | $[W\ m^{-2}]$ |
| $U_{In_{S_{WE}}}$ | Unloading of intercepted snow | $[mm]$ |
| $In^N_{S_{WE}}$ | Intercepted fresh snow | $[mm]$ |
| $E_{In_{S_{WE}}}$ | Sublimation/evaporation from intercepted snow | $[mm\ h^{-1}]$ |
| $d_{w,sno}$ | Fraction of canopy covered by snow | $[-]$ |
| $W_r$ | Water released from snowpack | $[mm]$ |
| $Sp_{wc}$ | Water content in snowpack | $[mm]$ |
| $S_{dep}$ | Snow depth | $[m]$ |
| $\rho^{new}_{sno}$ | Density of fresh snow | $[kg\ m^{-3}]$ |
| $\rho_{sno}$ | Snow density of ground snowpack | $[kg\ m^{-3}]$ |
| $Sp^M_{wc}$ | Maximum water holding capacity of snowpack | $[mm]$ |

**Table 2.** Continued

| Variable | Description | Units |
|---|---|---|
| | *Canopy Interception* | |
| $C_{fol}$ | Fraction of PFT area occupied by leaves and stems projected in vertical direction | $[m^2$ *vegetated area* $m^{-2}$ *PFT area*$]$ |
| $In$ | Intercepted water stored in canopy | $[mm]$ |
| $Dr_{H,\ or\ L,}$ | Total drainage from a vegetation layer | $[mm\ h^{-1}]$ |
| $Dr_d$ | Rate of dripping from canopy | $[mm\ h^{-1}]$ |
| $Dr_s$ | Canopy drainage at saturation | $[mm\ h^{-1}]$ |
| | *Subsurface Water Dynamics* | |
| $q_{ins}$ | Total rate of influx of water to soil surface | $[mm\ h^{-1}]$ |
| $q_{runon}$ | Runon flux rate | $[mm\ h^{-1}]$ |
| $I^C_f$ | Infiltration capacity rate | $[mm\ h^{-1}]$ |
| $I_f$ | Actual infiltration rate | $[mm\ h^{-1}]$ |
| $R_H$ | Rate of infiltration excess runoff | $[mm\ h^{-1}]$ |
| $R_D$ | Rate of saturation excess runoff | $[mm\ h^{-1}]$ |
| $Q_{l,in}$ | Incoming subsurface lateral flux rate | $[mm\ h^{-1}]$ |
| $Q_{l,out}$ | Outgoing subsurface lateral flux rate | $[mm\ h^{-1}]$ |
| $L_{kb}$ | Leakage between vadose zone and bedrock, recharge to deep aquifers | $[mm\ h^{-1}]$ |
| $\rho_{cr}$ | Bulk density of seal | $[kg\ m^{-3}]$ |
| $E_K$ | Rainfall cumulative kinetic energy | $[J\ mm^{-2}]$ |
| $\theta$ | Volumetric soil water content | $[-]$ |
| $\Psi(\theta)$ | Soil water potential | $[mm]$ or $[MPa]$ |
| $K_v(\theta)$ | Unsaturated hydraulic conductivity | $[mm\ h^{-1}]$ |
| $Z_{wt}$ | Water table depth | $[mm]$ |
| | *Surface Water Dynamics* | |
| $R_{tot}$ | Overland runoff depth | $[mm]$ |
| $R_{ch}$ | Channel runoff depth | $[mm]$ |
| $y$ | Overland flow depth | $[mm]$ |
| $y_{ch}$ | Channel flow depth | $[m]$ |
| $U$ | Overland flow velocity | $[m\ s^{-1}]$ |
| $U_{ch}$ | Channel flow velocity | $[m\ s^{-1}]$ |
| $\overrightarrow{R_{tot}}$ | Routed fraction of $R_{tot}$ | $[mm]$ |
| $\overrightarrow{R_{ch}}$ | Routed fraction of $R_{ch}$ | $[mm]$ |
| $Q_{ov}$ | Overland discharge | $[mm\ h^{-1}]$ |
| $Q_{ch}$ | Channel discharge | $[mm\ h^{-1}]$ |
| | *Vegetation Productivity* | |
| $C_{leaf}$ | Green aboveground biomass (leaves or grass) carbon pool | $[gC\ m^{-2}\ PFT]$ |
| $C_{sapw}$ | Living sapwood carbon pool | $[gC\ m^{-2}\ PFT]$ |
| $C_{root}$ | Fine roots carbon pool | $[gC\ m^{-2}\ PFT]$ |
| $C_{hydr}$ | Carbohydrate reserve carbon pool | $[gC\ m^{-2}\ PFT]$ |
| $NPP$ | Net Primary Production | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $GPP$ | Gross Primary Production | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $ANPP$ | Above-ground Net Primary Production | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_A$ | Autotrophic respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_g$ | Growth respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_m$ | Maintenance respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_{mS}$ | Living sapwood maintenance respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_{mR}$ | Fine root maintenance respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_{mH}$ | Carbohydrate reserve maintenance respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $R_{mF}$ | Foliage maintenance respiration | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $N_s$ | Living sapwood carbon-nitrogen C:N mass ratio | $[gC\ gN^{-1}]$ |
| $N_r$ | Fine root carbon-nitrogen C:N mass ratio | $[gC\ gN^{-1}]$ |
| | *Carbon Allocation and Turnover* | |
| $f_l$ | Allocation fraction to green aboveground biomass | $[-]$ |
| $f_s$ | Allocation fraction to living sapwood | $[-]$ |
| $f_r$ | Allocation fraction to fine roots | $[-]$ |
| $f_h$ | Allocation fraction to carbohydrate reserves | $[-]$ |
| $f_f$ | Allocation fraction to fruit and flowers | $[-]$ |
| $T_{rC}$ | Carbohydrate translocation rate | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $S_{sapw}$ | Rate of turnover of sapwood to heartwood biomass | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $S_{root}$ | Tissue turnover of fine root biomass to litter | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| $S_{leaf}$ | Tissue turnover of green aboveground biomass to litter | $[gC\ m^{-2}\ PFT\ day^{-1}]$ |
| | *Vegetation Phenology* | |
| $LAI$ | Leaf area index | $[m^2$ *leaf area* $m^{-2}$ *ground area*$]$ |
| $A_{gL}$ | Leaf age | $[day]$ |
| $\Phi$ | Phenology state | [1,....,4] |
| $N_{LAI}$ | New leaf area onset over a time step $dt$ | $[m^2$ *leaf area* $m^{-2}$ *PFT area*$]$ |

**Table 1.** Meteorological Input Used for Forcing "Tethys-Chloris"

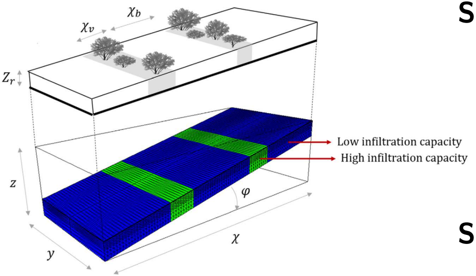
| Variable | Description | Units |
|---|---|---|
| $R_{dir,\Lambda}$ | Incoming direct beam shortwave radiation | $[W\ m^{-2}]$ |
| $R_{dif,\Lambda}$ | Incoming diffuse shortwave radiation | $[W\ m^{-2}]$ |
| $PAR_B$ | Incoming direct beam *Photosynthetically Active Radiation* | $[W\ m^{-2}]$ |
| $PAR_D$ | Incoming diffuse *Photosynthetically Active Radiation* | $[W\ m^{-2}]$ |
| $P_r$ | Precipitation | $[mm\ h^{-1}]$ |
| $T_a$ | Air temperature at a reference height | [°C] |
| $e_a$ | Vapor pressure at a reference height | $[Pa]$ |
| $N$ | Cloud cover | $[-]$ |
| $P_{atm}$ | Atmospheric pressure | $[Pa]$ |
| $W_s$ | Wind speed at a reference height | $[m\ s^{-1}]$ |
| $c_a$ | Atmospheric $CO_2$ concentration | $[Pa]$ |

Fatichi et al., *J. Adv. Model. Earth Syst.* **4** M05002 (2012)

# Detailed simulations of banded patterns based on T&C

## Simplifying Assumptions

- spatially static carbon
- Single vegetation layer
- Depth-averaged soil moisture




## Idealized Hillslope

- Size: 1 km (by 2 m)
- Slope: 0.5 - 5 %
- Vegetation band width: 16 m
- Vegetation band period: 62 m

## Simulation Time Steps

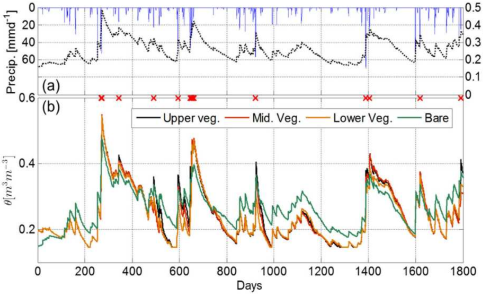
- Vegetation dynamics: daily
- Energy fluxes: hourly
- Soil crust formation: 5 min
- Hydrology: ∼ sec - 5 min

**Simulation time:** <20 years

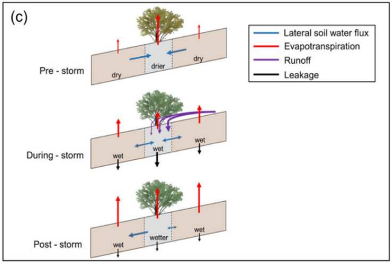
Paschalis et al., *Water Resour. Res.* **52** 2259–2278 (2016)

# A detailed simulation of banded patterns based on T&C

## Time series for 5 years of simulation

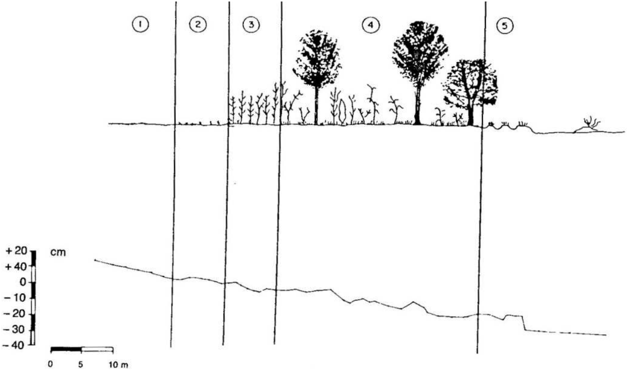



Mechanistic explanation of water fluxes

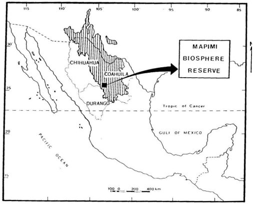



Paschalis et al., *Water Resour. Res.* **52** 2259–2278 (2016)

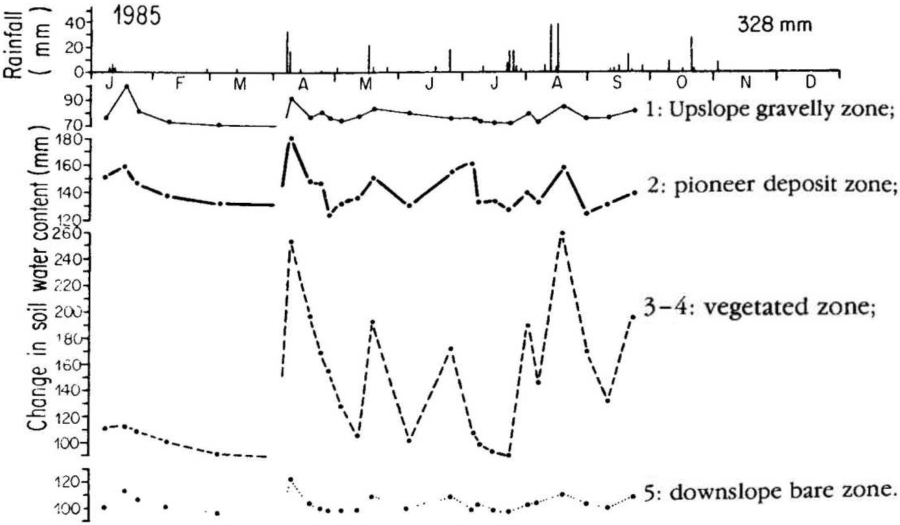
# Time-resolved soil moisture data is limited




Cornet et al, "Dynamics of striped vegetation patterns and water balance in the Chihuahuan Desert." (1988)



# Time-resolved soil moisture data is limited




Cornet et al, "Dynamics of striped vegetation patterns and water balance in the Chihuahuan Desert." (1988).

# Presentation Outline

# Build off of existing modeling frameworks

Rietkerk et al., *Am. Nat.*, **160** (2002). See also: Gilad et al., *PRL*, **93** (2004).

$$\text{Biomass}: \quad \frac{\partial B}{\partial T} = \underbrace{-MB}_{\text{mortality}} + \underbrace{C_g \Gamma s B(1 - B/K_B)}_{\text{growth}} + \underbrace{D_B \nabla^2 B}_{\text{seed dispersal}}$$

$$\text{Soil moisture}: \quad \phi Z_r \frac{\partial s}{\partial T} = \underbrace{\mathcal{I}(H, s, B)}_{\text{infiltration}} - \underbrace{(L_{ev} + \Gamma B)s}_{\text{evapotranspiration}} - \underbrace{A_L s e^{\beta_L (s-1)}}_{\text{leakage}}$$

$$\text{Surface water}: \quad \frac{\partial H}{\partial T} = \underbrace{P(T)}_{\text{precipitation}} - \underbrace{\mathcal{I}(H, s, B)}_{\text{infiltration}} + \underbrace{K_w \frac{\partial}{\partial X}\left(\frac{\sqrt{|\nabla \zeta|}\ H^{\delta}}{1 + NB}\right)}_{\text{surface transport}}$$

$$\mathcal{I}(H, s, B) = K_{sat}\left(\frac{B + fQ}{B + Q}\right)\left(\frac{H}{H + A_I}\right)(1 - s)^{\beta_I}$$

# Parameter values based on processes being modeled

**Typical values for states:**

- Biomass: $B \sim 0.4 - 2$ kg/m$^2$ for vegetation
- Soil moisture: $S \sim 0.2 - 0.5$ during rainy season
- Surface water: $H \sim 1 - 5$ cm during rain events

**Typical values for process being modeled:**

- Saturated hydraulic conductivity $\sim 20 - 200$ cm/day
- Maximum transpiration $\sim 0.4$ (cm/day)/(kg/m$^2$)
- Manning roughness coefficient $\sim 0.01 - 0.1$ s/m$^{1/3}$
- Bare soil surface water flow speed $\sim 1$ m/s

**"Effective" processes in model:**

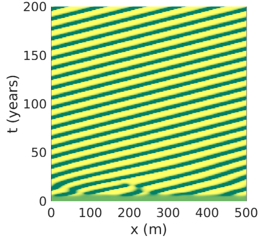
- Biomass transport
- Infiltration (soil moisture/biomass dependence)

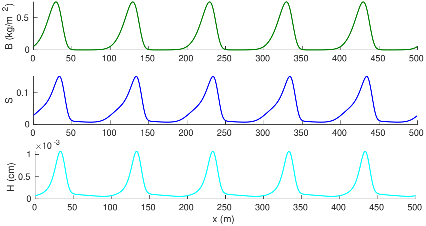
Rodriguez-Iturbe & Porporato. *Ecohydrology of Water-Controlled Ecosystems* (2005)

# Constant and seasonal rain with 80 mm/year of rainfall

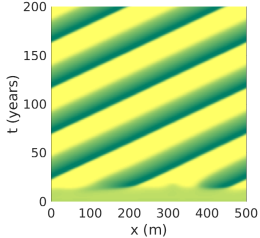
$$P(T) = P_0 C_\alpha \operatorname{sech}^2 \left(\alpha \cos(\pi T / T_{year})\right)$$

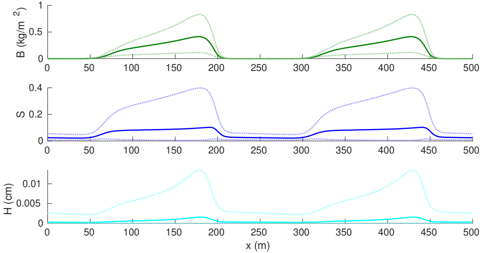
## constant rain $(\alpha = 0)$




## 3 month rainy season $(\alpha = 5)$



# Separation of scales suggests fast/slow model

**Dimensionless equations with** $\epsilon = \frac{Infiltration\,Timescale}{Growth\,Timescale} \sim 10^{-3}$

$$b_t = \epsilon\big(db_{xx} - \mu b + sb(1-b)\big)$$
$$s_t = \alpha_I \iota - \epsilon\gamma(\sigma + b)\, s - \ell_L$$
$$h_t = p(t) - \iota + \partial_x\left(\nu h^{\delta}\right)$$

$$\iota = \left(\frac{b+qf}{b+q}\right)\left(\frac{h}{h+1}\right)(1-s)^{\beta_I}$$
$$\nu = (1+\rho b)^{-1}$$
$$\ell_L = \alpha_L s e^{\beta_L(1-s)}$$

**Alternative approach: fast/slow system**

Fast System, $\epsilon = 0$
(minutes - hours)

$$b_t = 0$$
$$s_t = \alpha_I \iota - \ell_L$$
$$h_t = p(t) - \iota + \partial_x\left(\nu h^{\delta}\right)$$

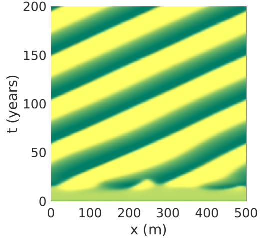
Slow System, $h = 0$, $\tau = \epsilon t$
(weeks - months)

$$b_\tau = db_{xx} - \mu b + sb(1-b)$$
$$s_\tau = -\gamma(\sigma + b)\, s$$
$$h = 0$$

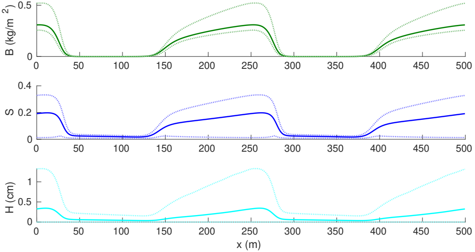
# Presentation Outline

# Simulation of fast/slow model with 80 mm/year rainfall

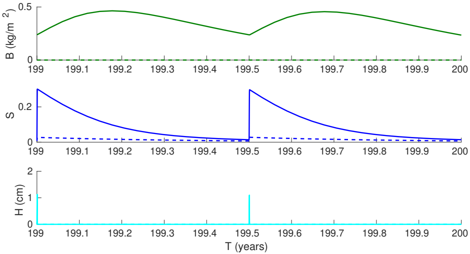
## two 6-hour rain events

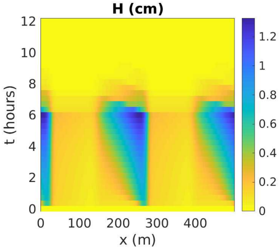



Spatial profile: annual avg. (solid), max/min (dotted)




Time series in vegetation band (solid), in bare soil (dashed)

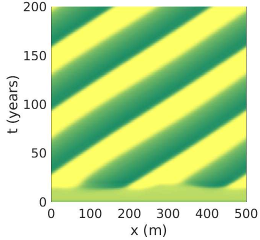


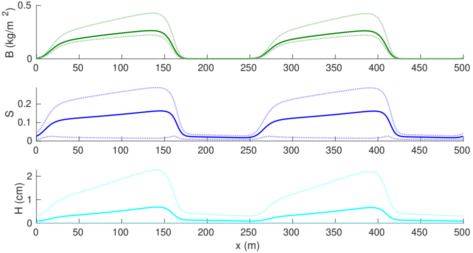



# Rain intensity for 80 mm/year of rainfall

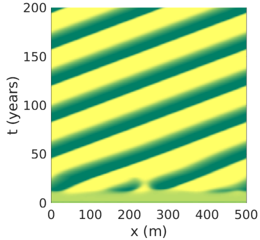
## two 3-hour rain events

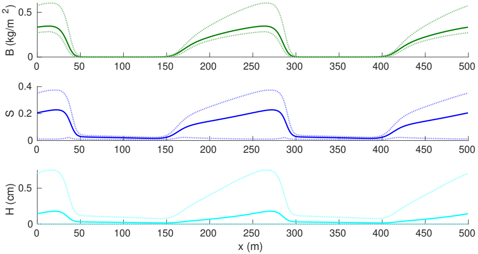
Spatial profile: annual avg. (solid), max/min (dotted)




## two 12-hour rain events

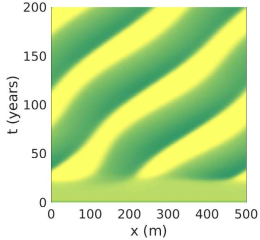


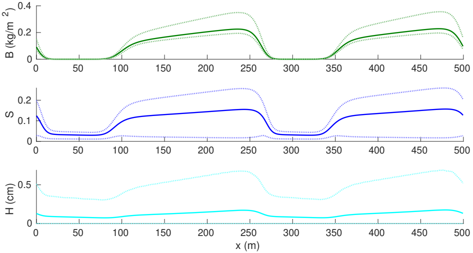



# boimass feedback on infiltration and surface transport

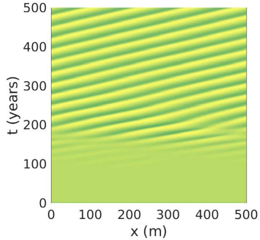
## weak surf. feedback ($N = 5$)

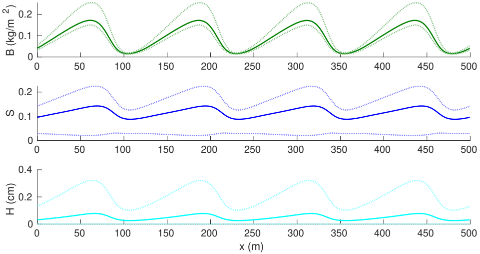
Spatial profile: annual avg. (solid), max/min (dotted)




## no infl. feedback ($f = 1$)



# Approximations in fast system for computational speed up

**Rain:** $s \to s + \theta[b, s, h_0]$

**(slow) time evolution until next rain:**

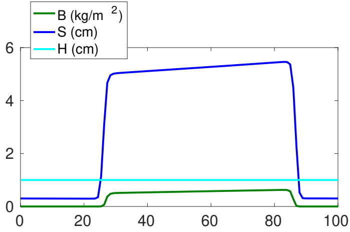
$$b_\tau = d b_{xx} - \mu b + s b (1 - b)$$
$$s_\tau = -\gamma\, (\sigma + b)\, s$$

Merlin Pelz

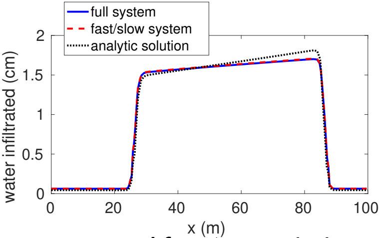
(TU Munich)

## Use fast system to update soil moisture profile after rain

assume delta function rain event, $\delta = 1$ and $h, s \ll 1 \to$ Analytic solution for $\theta[b, s, h_0]$



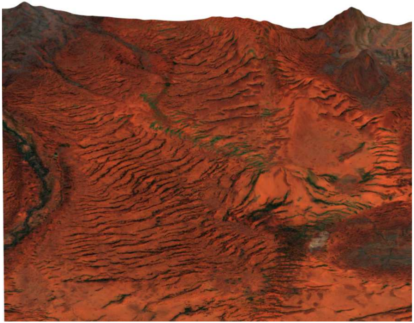

Initial condition




After 1 cm rain impulse

# Outlook

- Target: Resolve timescales of underlying processes
- Additional features needed: runoff, biomass processes,. . .
- Fast/slow system: Stochasticity, approximate analytic solutions, relation to “full” model



Mathematical Biosciences Institute

The Mathematical Biosciences Institute receives major funding from the National Science Foundation under grant DMS 1440386 and is supported by The Ohio State University.