An Embedded Discontinuity Model for the Simulation of Fracture Deformations

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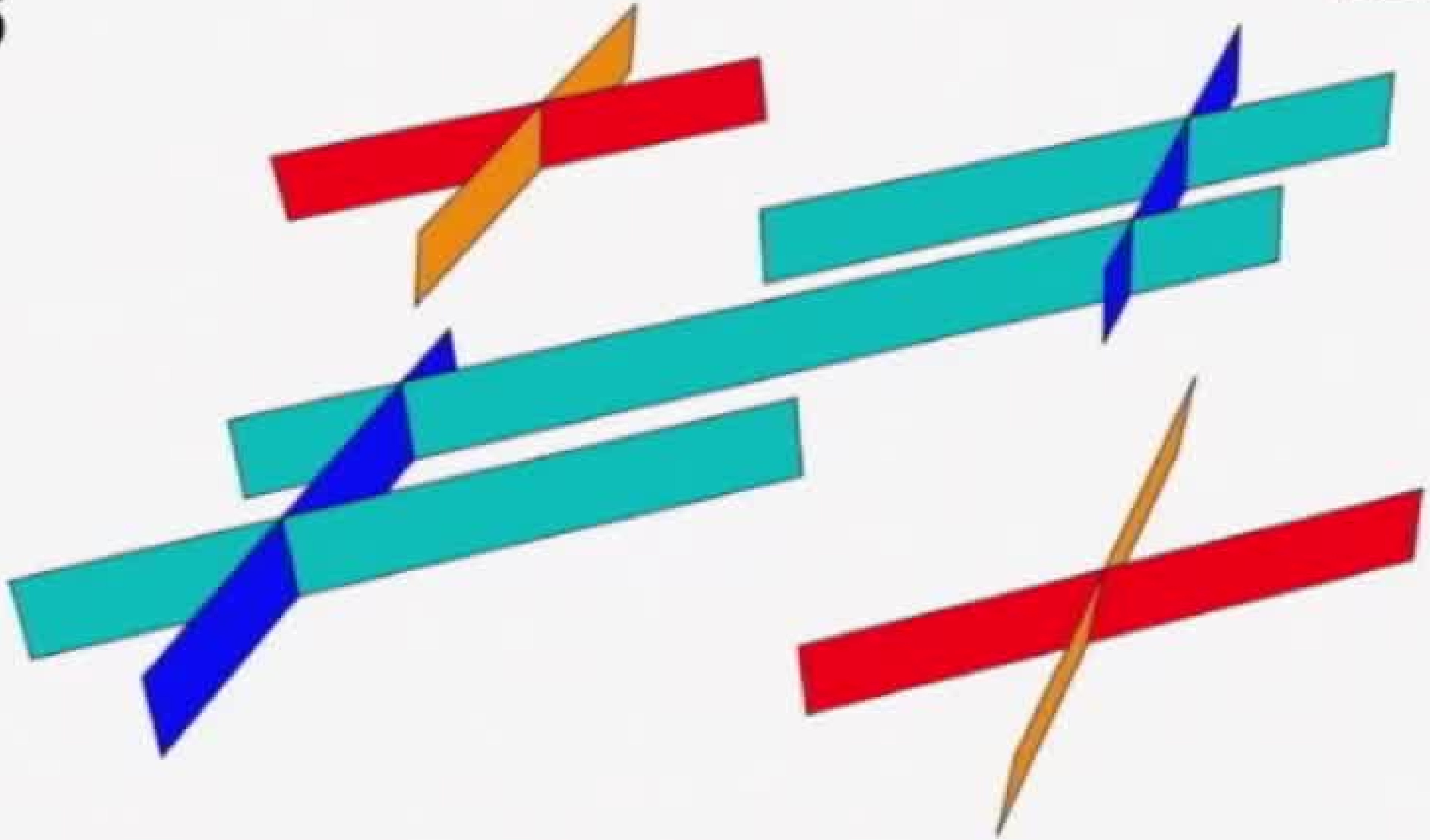

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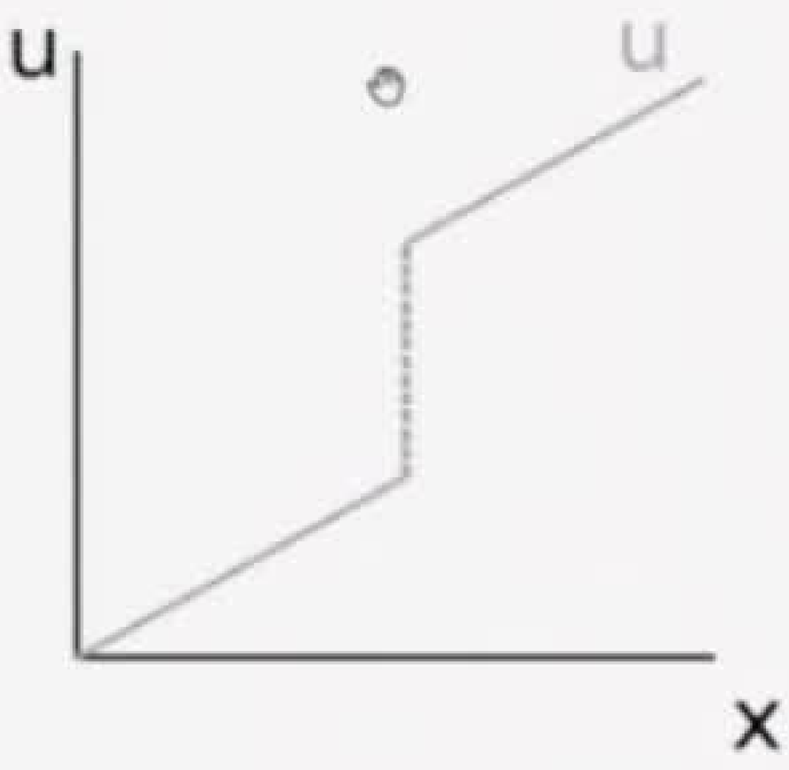
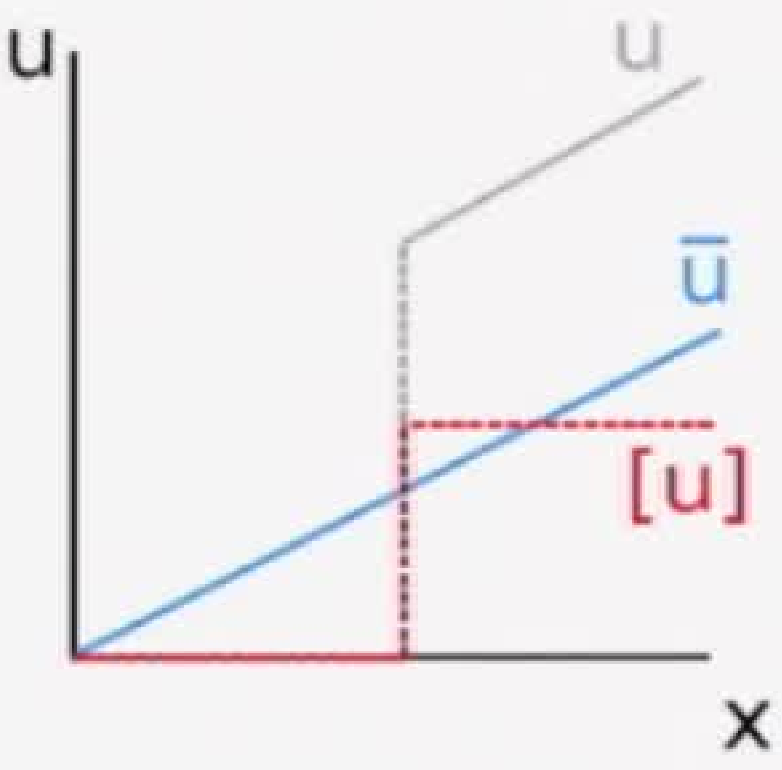
Houson, TX, 2019

Displacement Enrichment



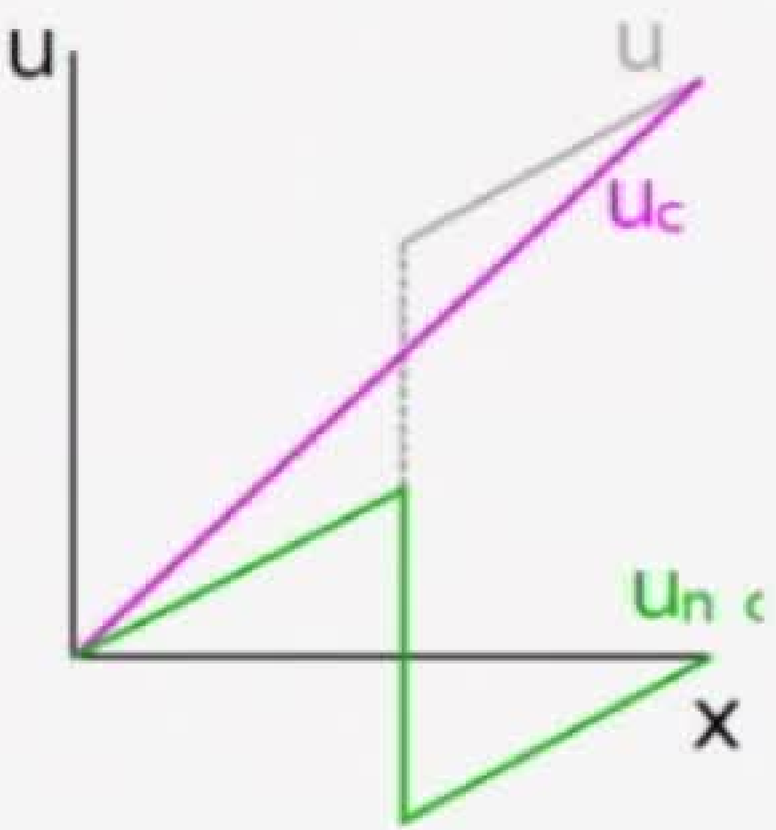
Discontinuous displacement



Decomposition:

$$u = \overline{u} + [u]H_\Gamma$$

H_Γ – unit step function

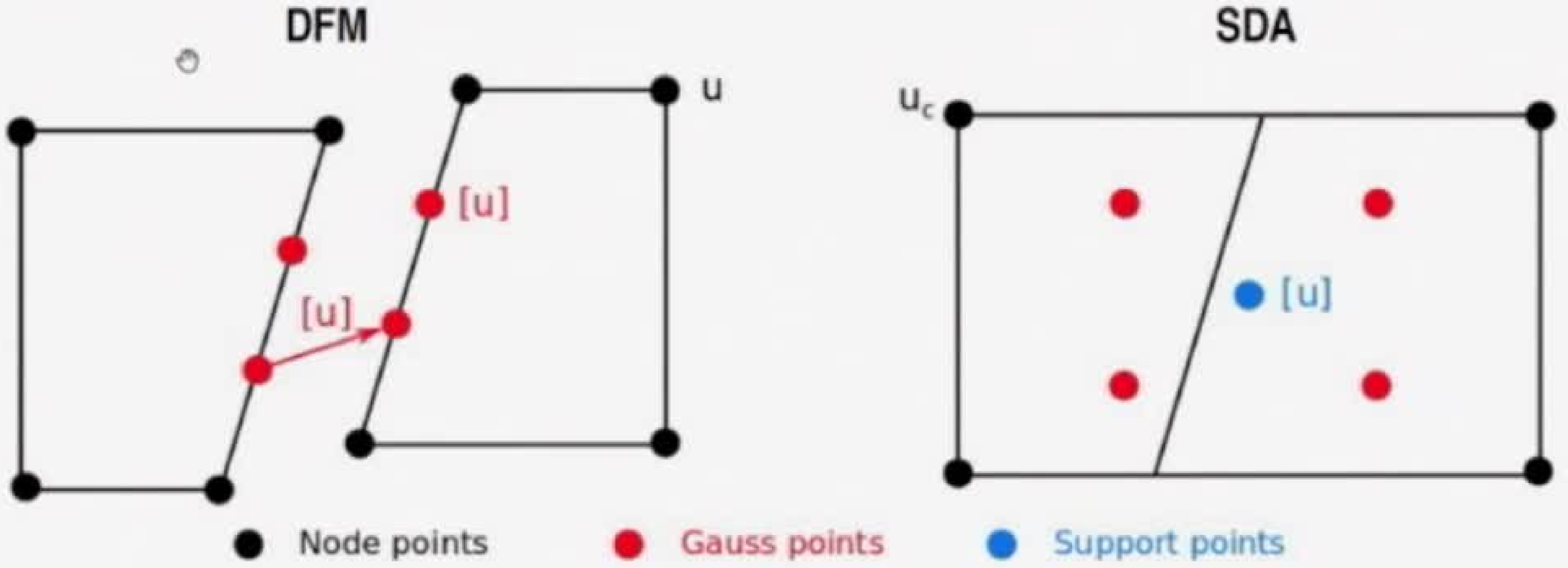


Regularization:

$$u = \underbrace{\overline{u} + f[u]}_{u_c} + \underbrace{(H_\Gamma - f)[u]}_{u_{nc}}$$

u_c : conforming displacement
u_{nc}: non-conforming displacement
f : level set function

Discretization



Global variable: displacement u
Local variables: jump [u]

Jump [u] obtained from displacement by interpolation

Cannot interpolate to obtain [u]

Analogy with Plasticity

System of plasticity:

$$\begin{cases} \dot{\sigma} = \mathbf{C} : \left(\dot{\varepsilon} - \dot{\varepsilon^p}\right) \\ \dot{\varepsilon^p} = \lambda \dfrac{\partial G}{\partial \sigma} \\ \dot{q} = -\lambda H \dfrac{\partial G}{\partial \sigma^\Gamma} \\ F(\sigma, q) = 0 \end{cases}$$

Plasticity		SDA
←	Stress-strain equation	→
←	Plastic strain evolution	→
←	Softening/hardening	→
←	Flow rule	→
	Stress continuity	→

SDA system:

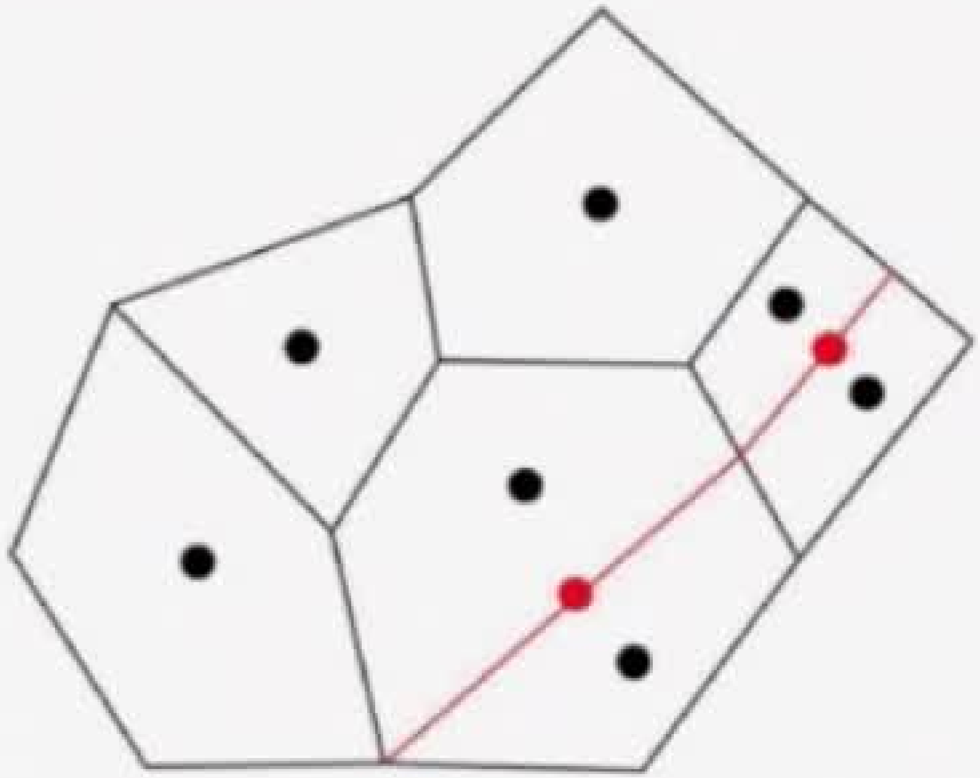
$$\begin{cases} \dot{\sigma} = \mathbf{C} : \left[\nabla^s \dot{u}_c - \left([\dot{u}] \otimes \nabla f\right)^s\right] \\ [\dot{u}] = \lambda_\Gamma \dfrac{\partial G}{\partial t} \\ \dot{q} = -\lambda_\Gamma H_\Gamma \dfrac{\partial G}{\partial q} \\ F(t, q) = 0 \\ t = < \sigma > \cdot n \end{cases}$$

σ : stress
C : elastic stress-strain tensor
ε : total strain
ε^p : plastic strain
λ : Lagrange multiplier
q : internal state variable (cohesion)
H : hardening modulus
G, F : plastic potential and flow rule

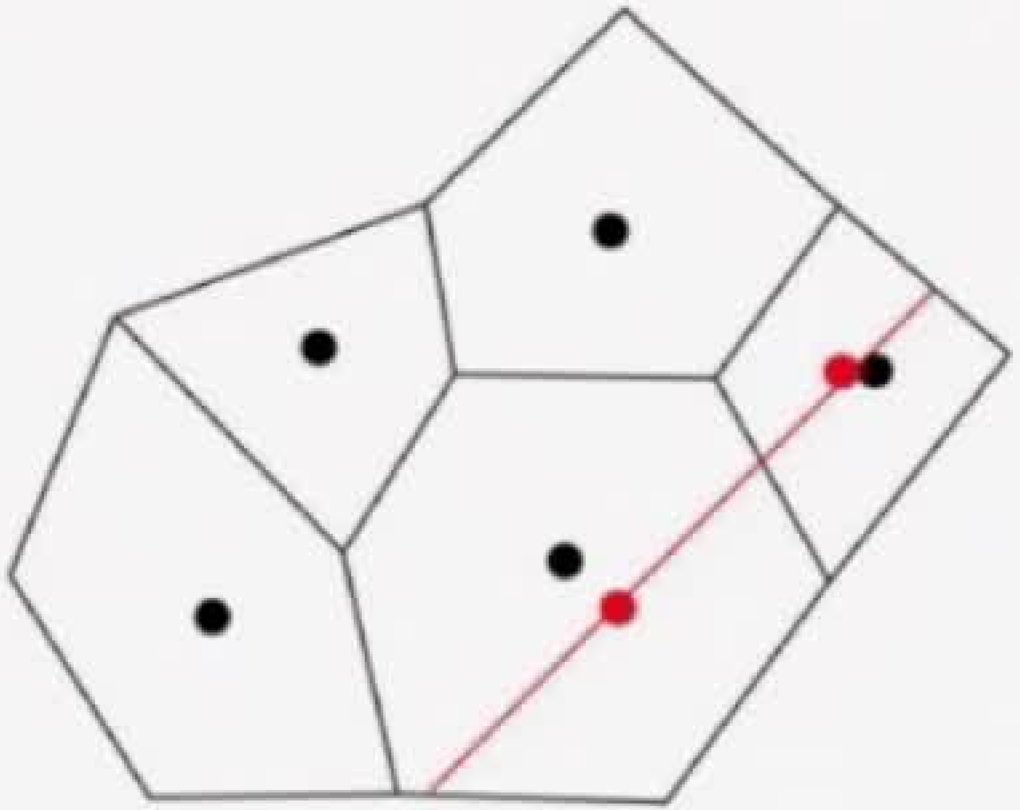
t : traction of fracture surface
u_c : conforming displacement
[u] : jump
f : level set
n : fracture normal
λ_Γ : Lagrange multiplier
H_Γ : Hardening modulus

Embedded Fractures: Fluid Flow

Discrete Fractures (DFM)



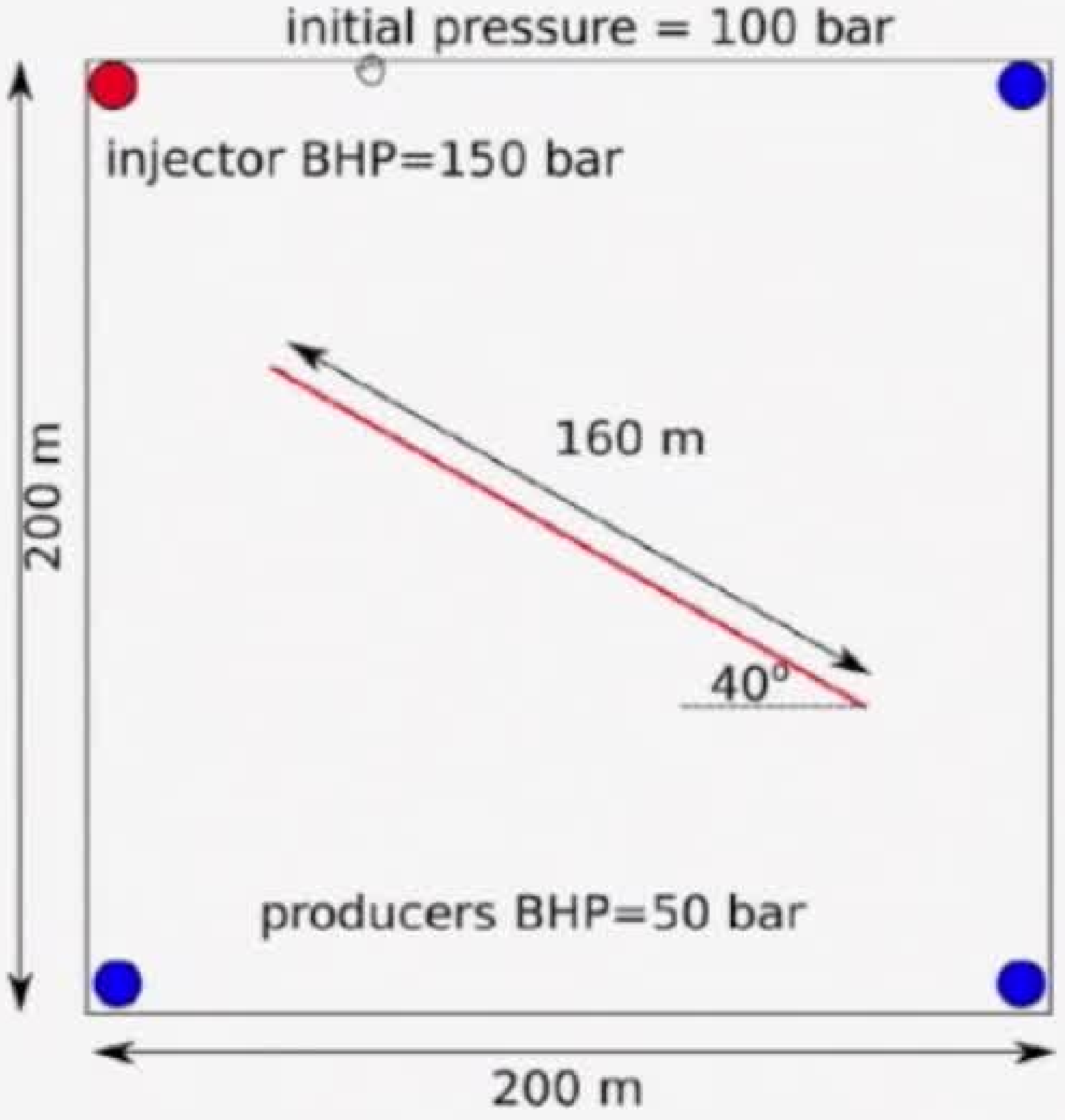
Embedded Fractures (EDFM)



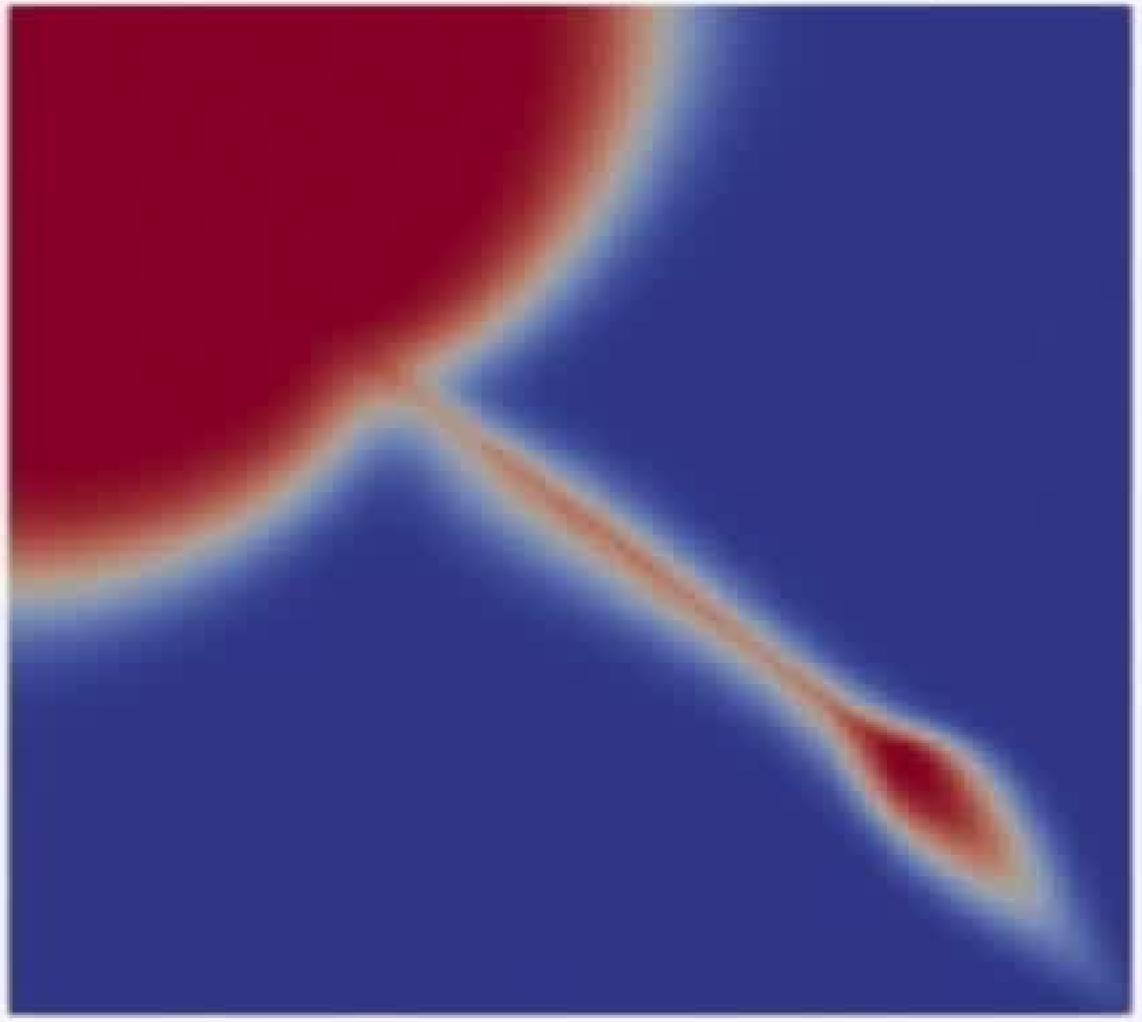
EDFM Matrix-Fracture Transmissibility:

$$T_{MF} = \frac{2Akn}{< d >}$$

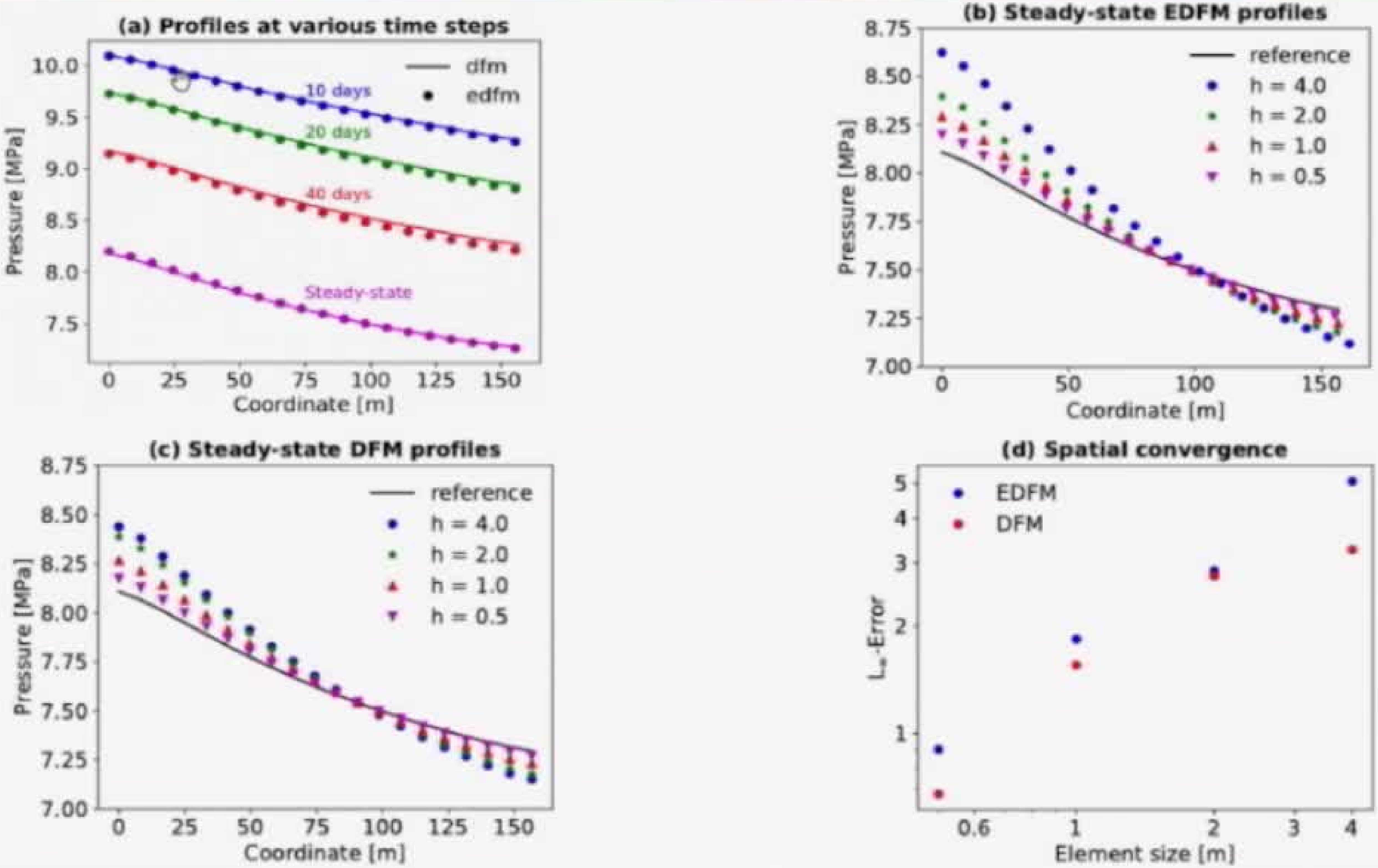
Test Problem: Single-Phase Fluid Flow



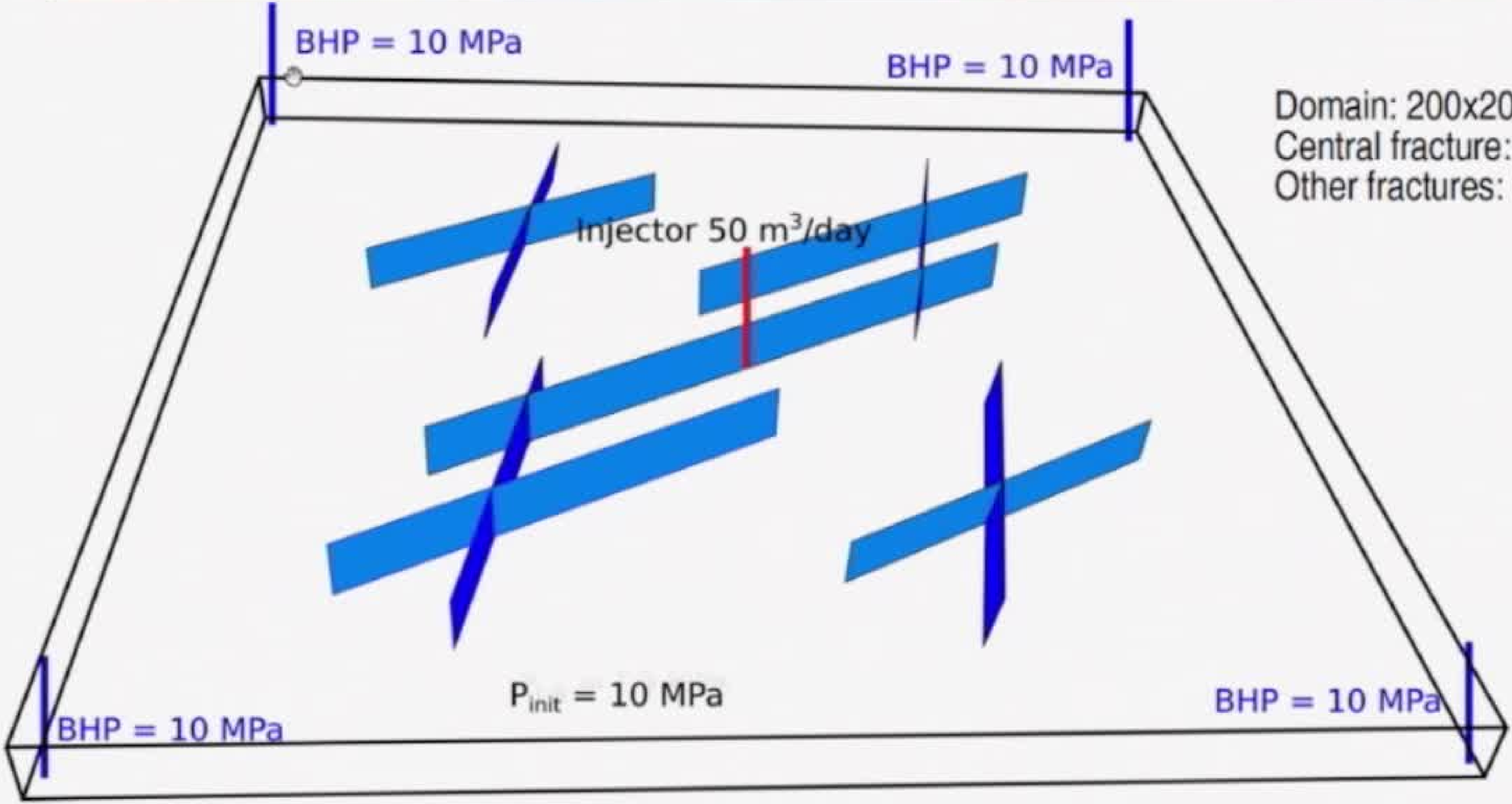
Saturation Profile at 100 days



Test Problem: Single-Phase Fluid Flow (cont.)

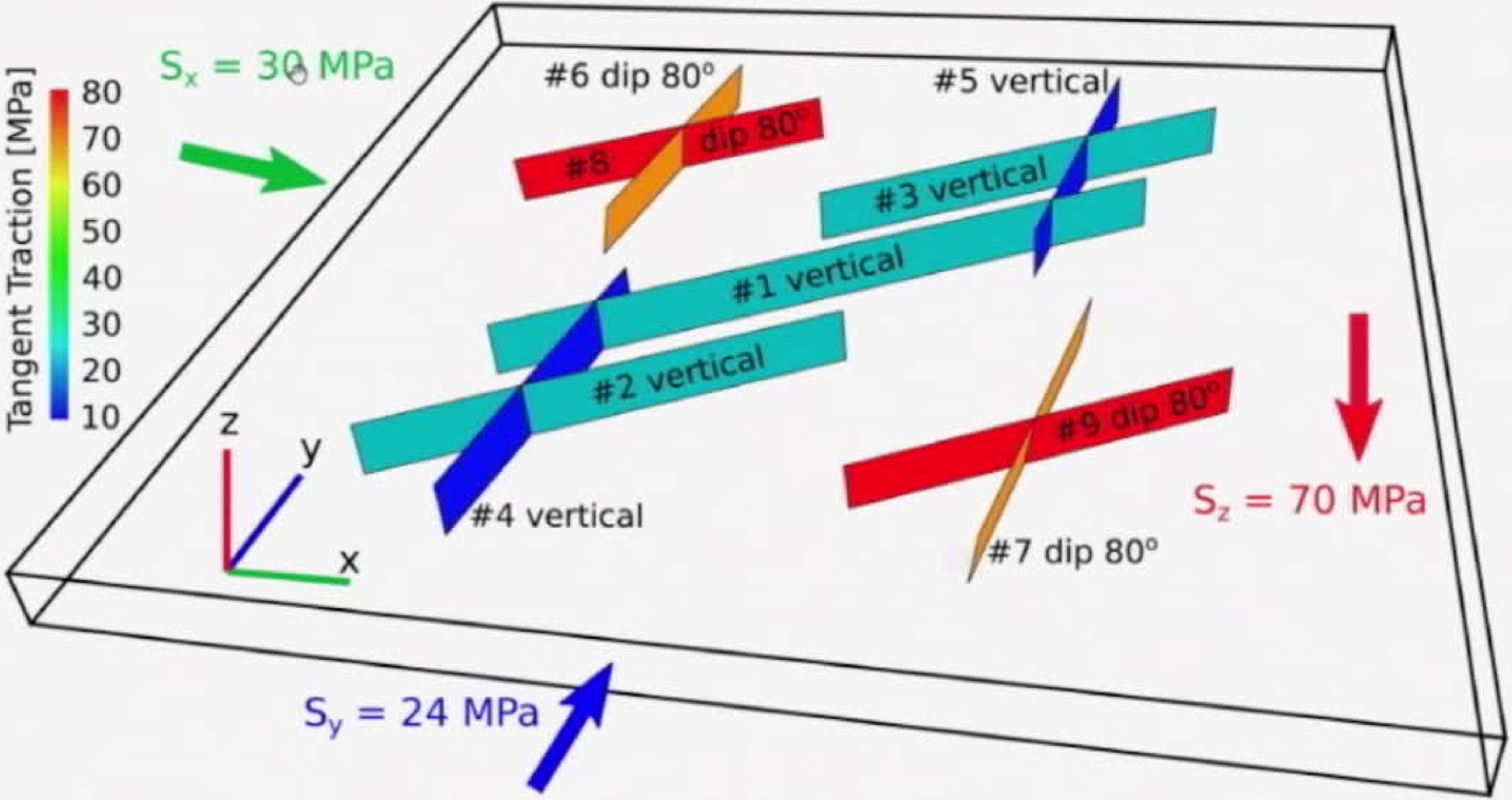


Simulation Domain: Wells



Domain: 200x200x100 m^3
Central fracture: 120 m
Other fractures: 60 m

Initial Stress State

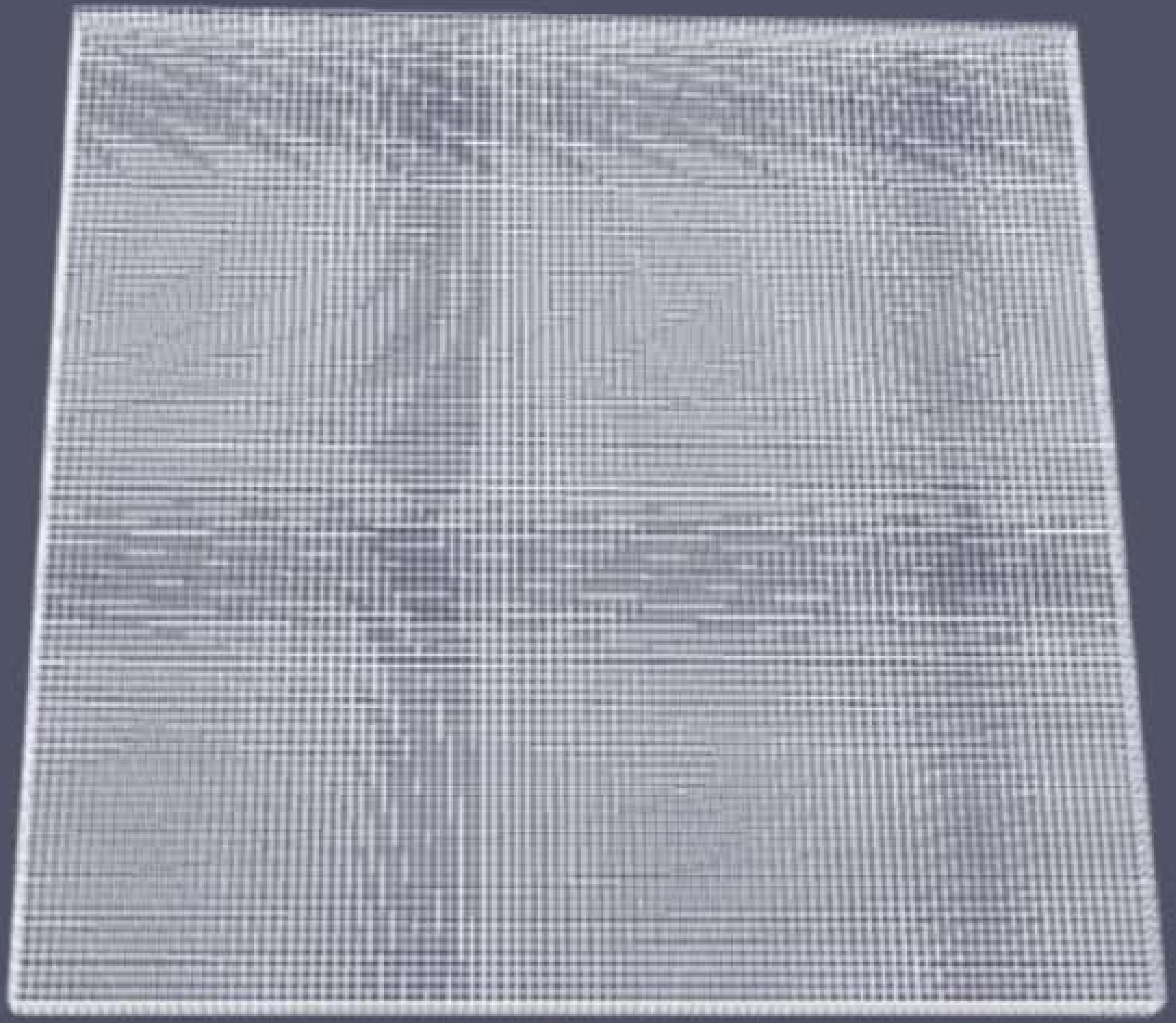


Gridding

DFM

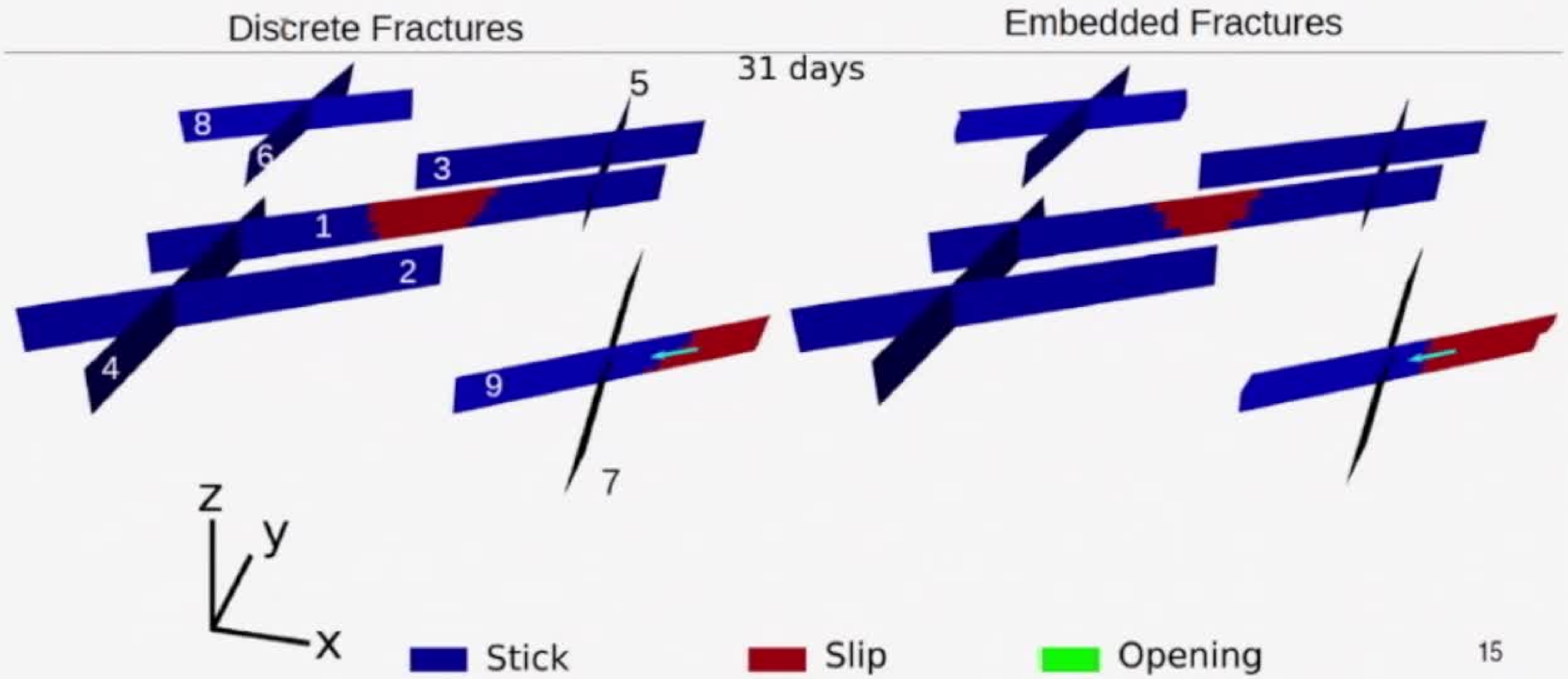
59032 prisms
2.2 m near fractures
10 m at boundaries

EDFM

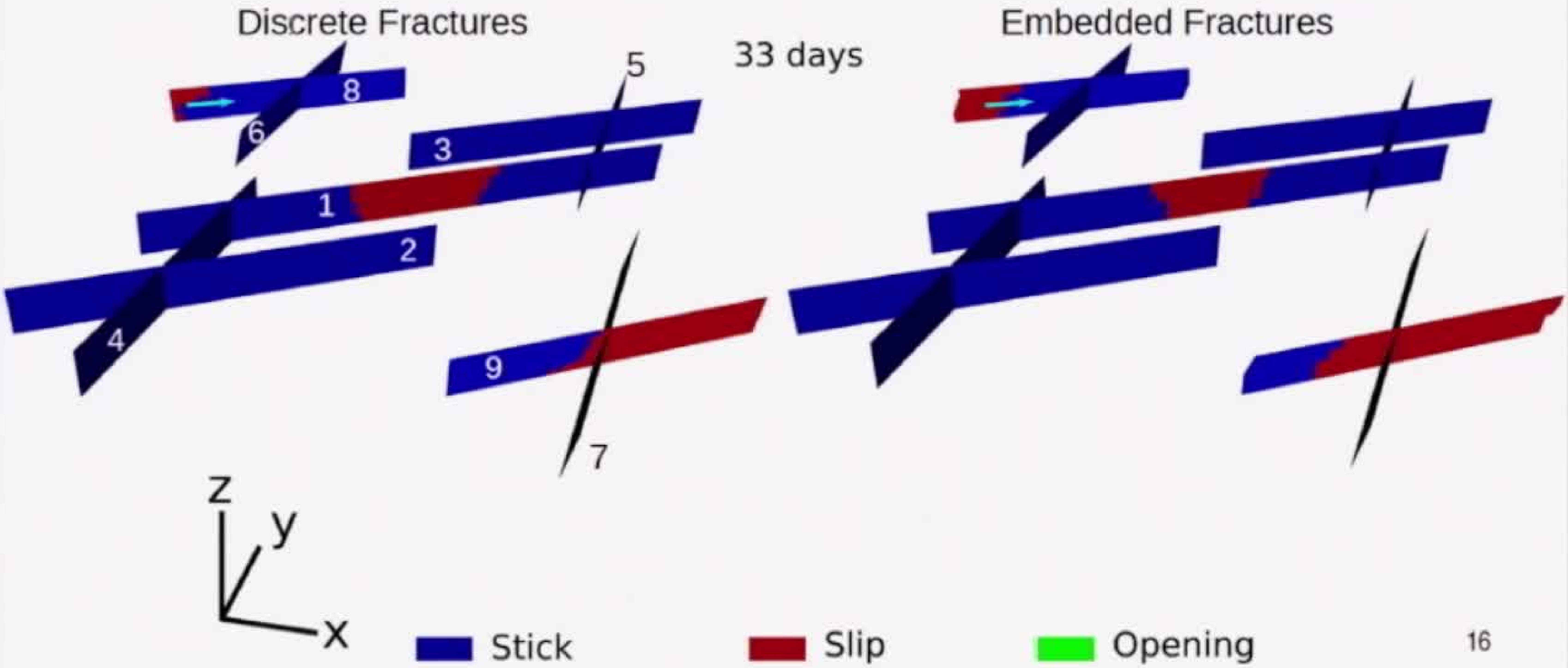


40804 hexahedrons
2.0 m

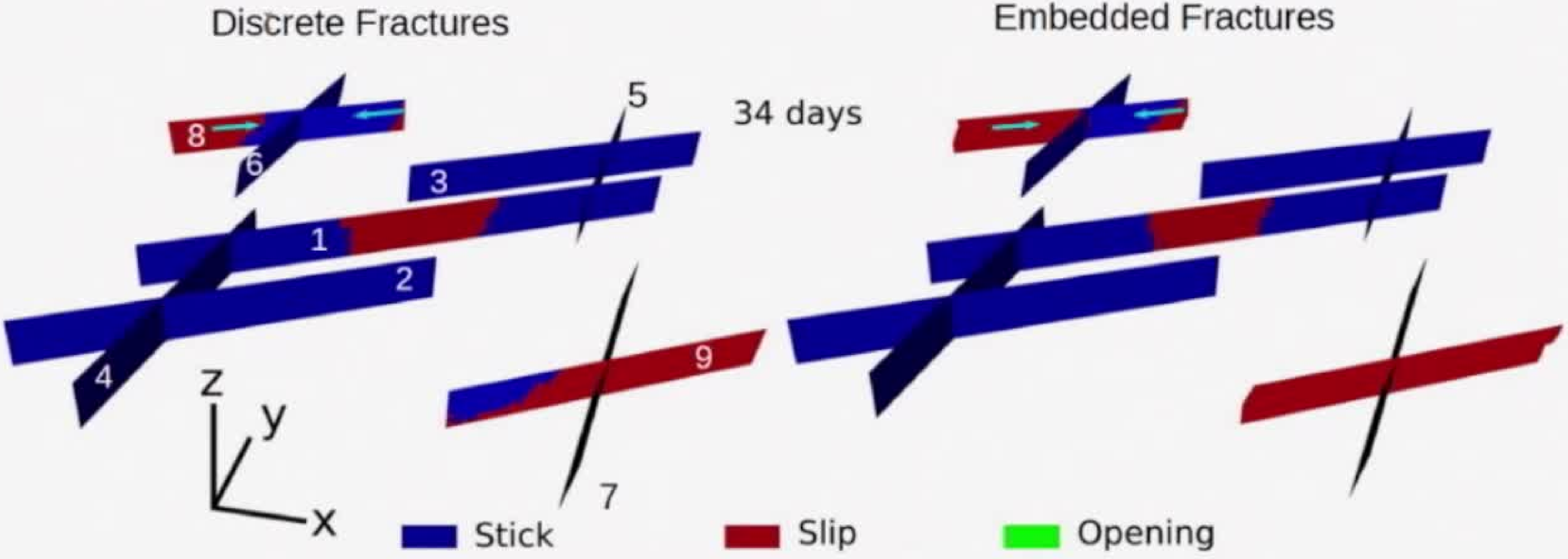
Fracture Reactivation: 31 days



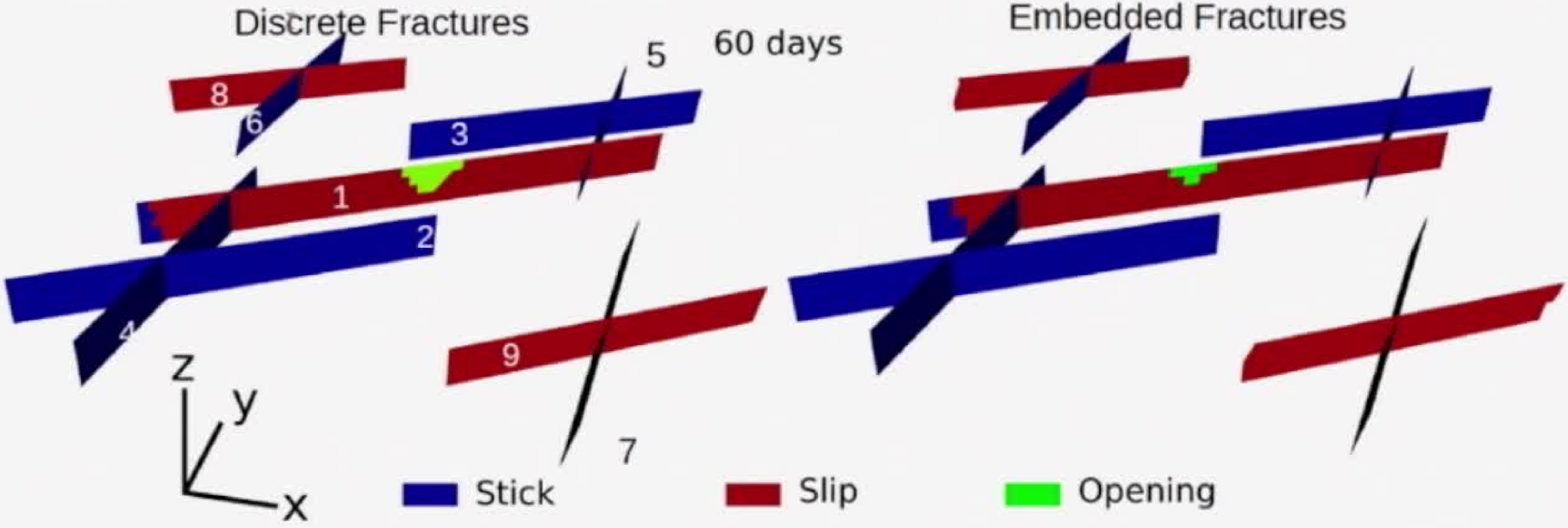
Fracture Reactivation: 33 days



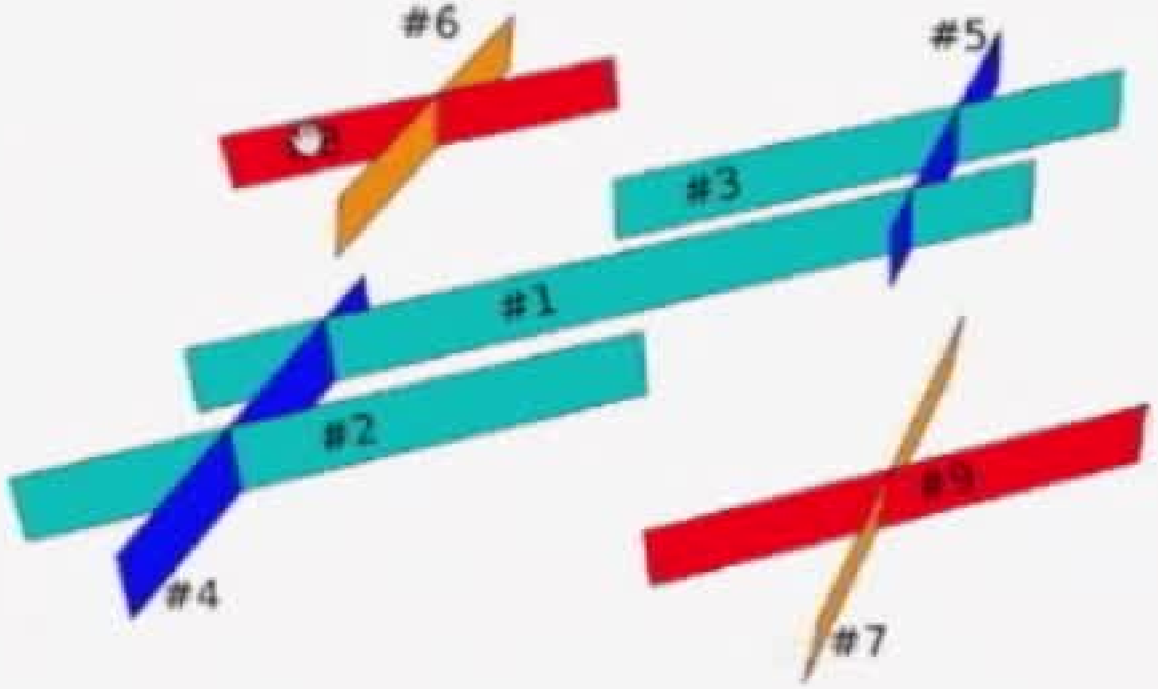
Fracture Reactivation: 34 days

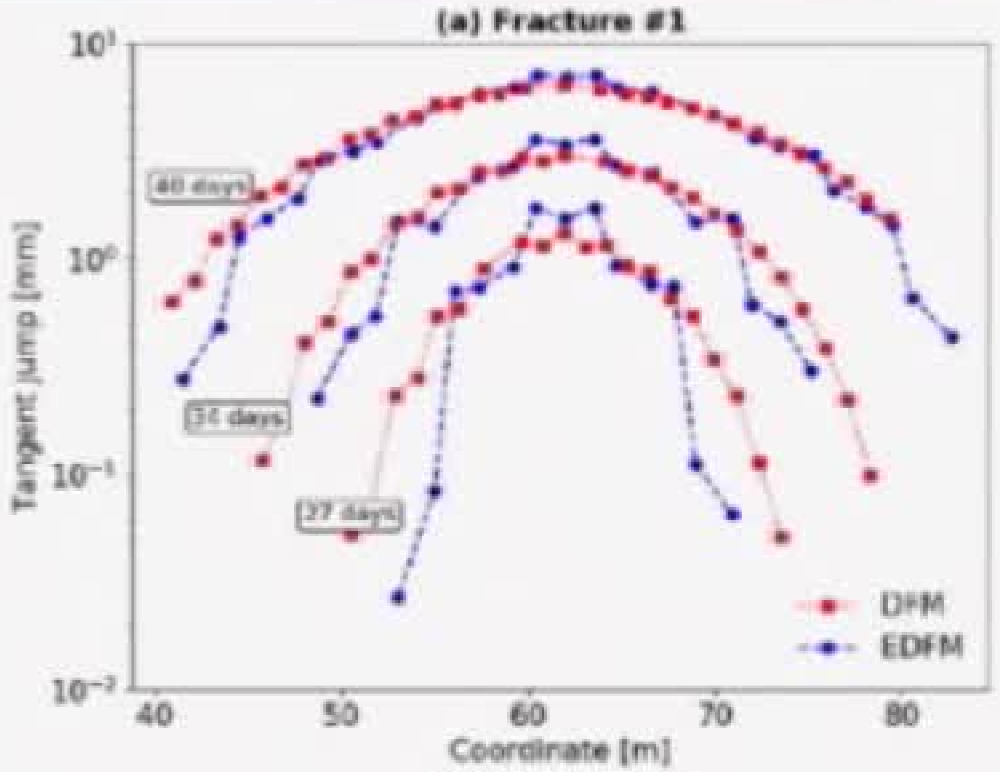


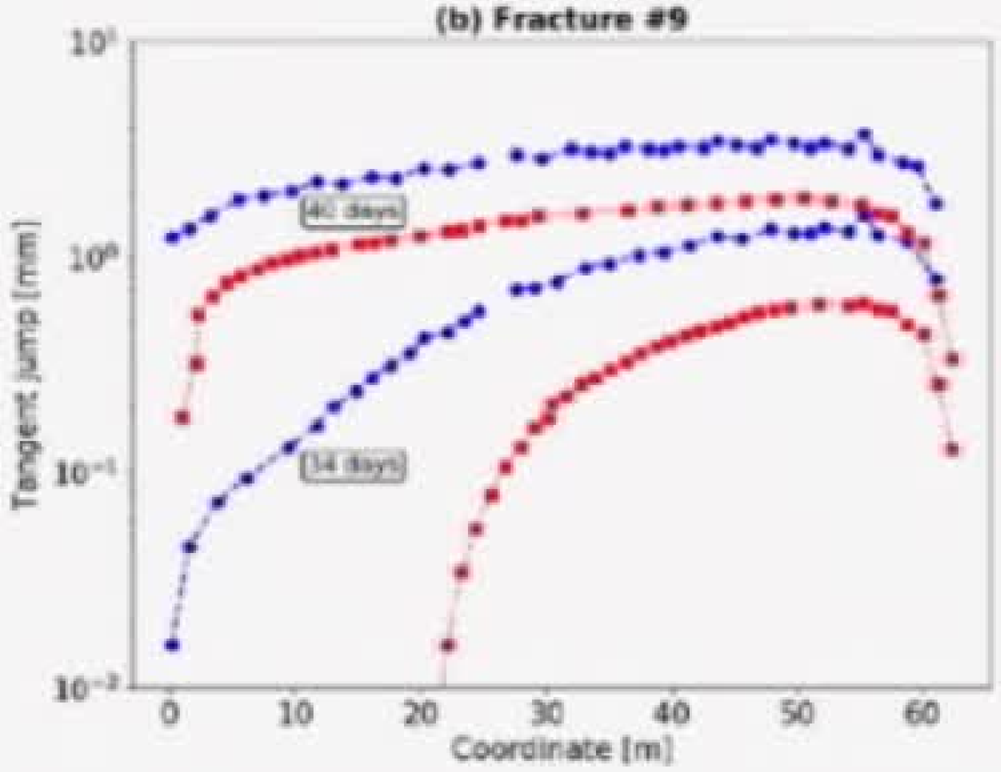
Fracture Opening: 60 days

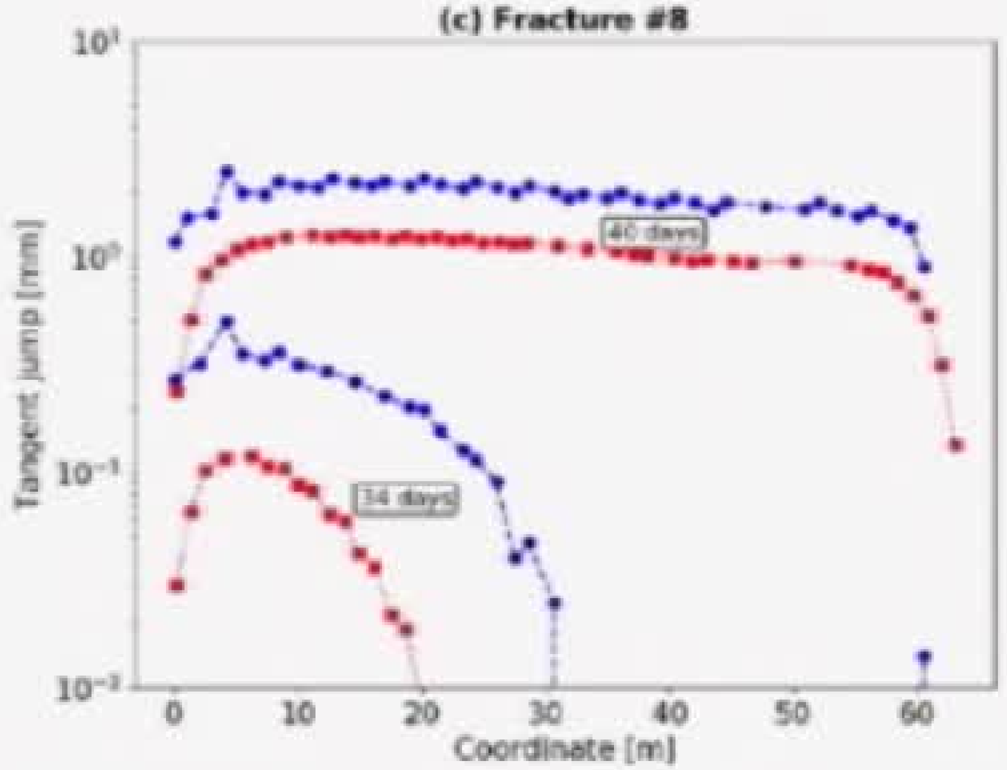


Fracture Jump Comparison









Summary

- EDFM overestimates slip whereas DFM underestimates slip
- Both EDFM and DFM exhibit super-linear convergence
- Using EDFM on non-conforming grids results in non-smooth fracture jump profiles
- EDFM and DFM give similar results even in complex coupled cases

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Thank You!