Triggers of Rogue Waves in Deep Water Envelope Equations

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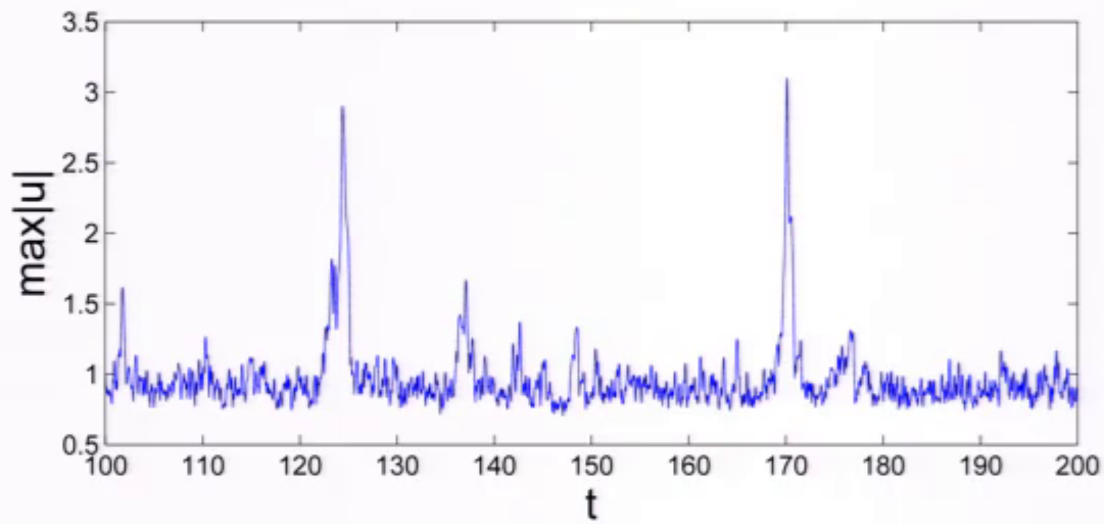
Extreme Events

- Dramatically atypical system responses
- Rare—live in the tails of the probability distribution
- Examples:
  1. Freak/Rogue Ocean Waves
  2. Financial market crashes
  3. Extreme weather events (hurricanes, droughts, earthquakes)



High Dimensional, Nonlinear, Noisy Systems

Why should you care?

1. Many systems demand such models, particularly when you want to represent extreme events

2. Nonlinearity can generate non-Gaussian, heavy-tailed statistics (extreme events thus more likely)

3. Combination of high dimensionality and nonlinearity make analysis challenging

Extreme Event Triggers

- We consider systems with intermittent, nonlinear, large magnitude events

- In some cases, it is possible to describe the ***triggers*** of these extreme events

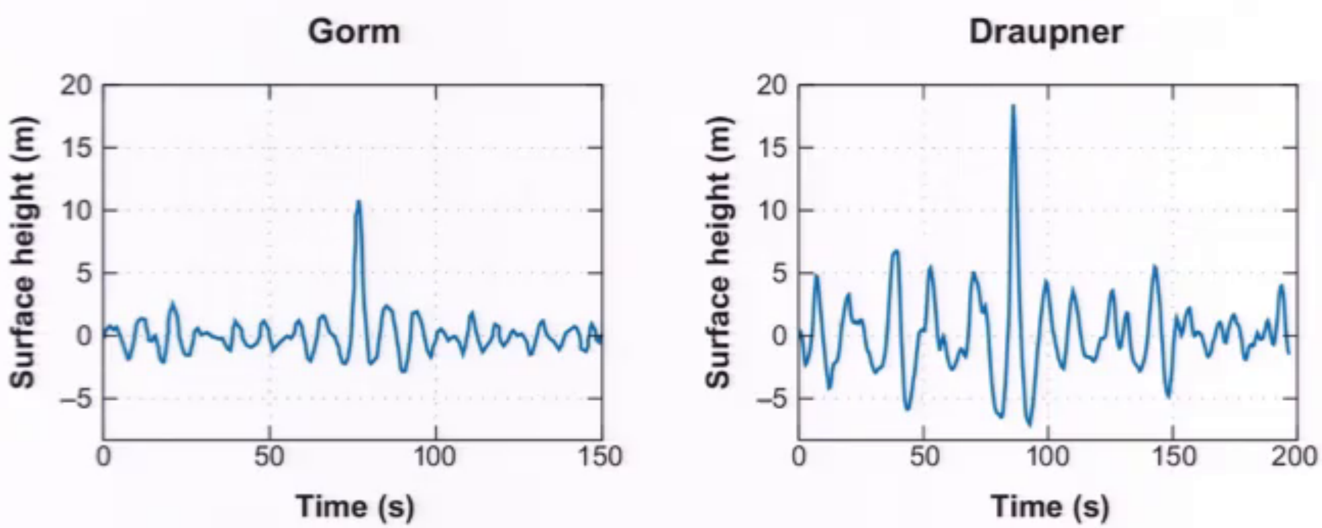
- We will show you how you can use this information to perform

  1. Prediction

  2. Density Estimation (of the state and functionals thereof)

Rogue/Freak Ocean Waves

- Waves whose height is extremely large for a given wave field
- Common definition: > 4 times the standard deviation
- Such waves have caused catastrophic damage to people, ships, and coastal structures



From Dysthe, Krogstad, Muller (2008)

Oceanic Rogue Waves

Rogue Waves – large elevation relative to a given sea state

Modified Nonlinear Schrodinger Equation

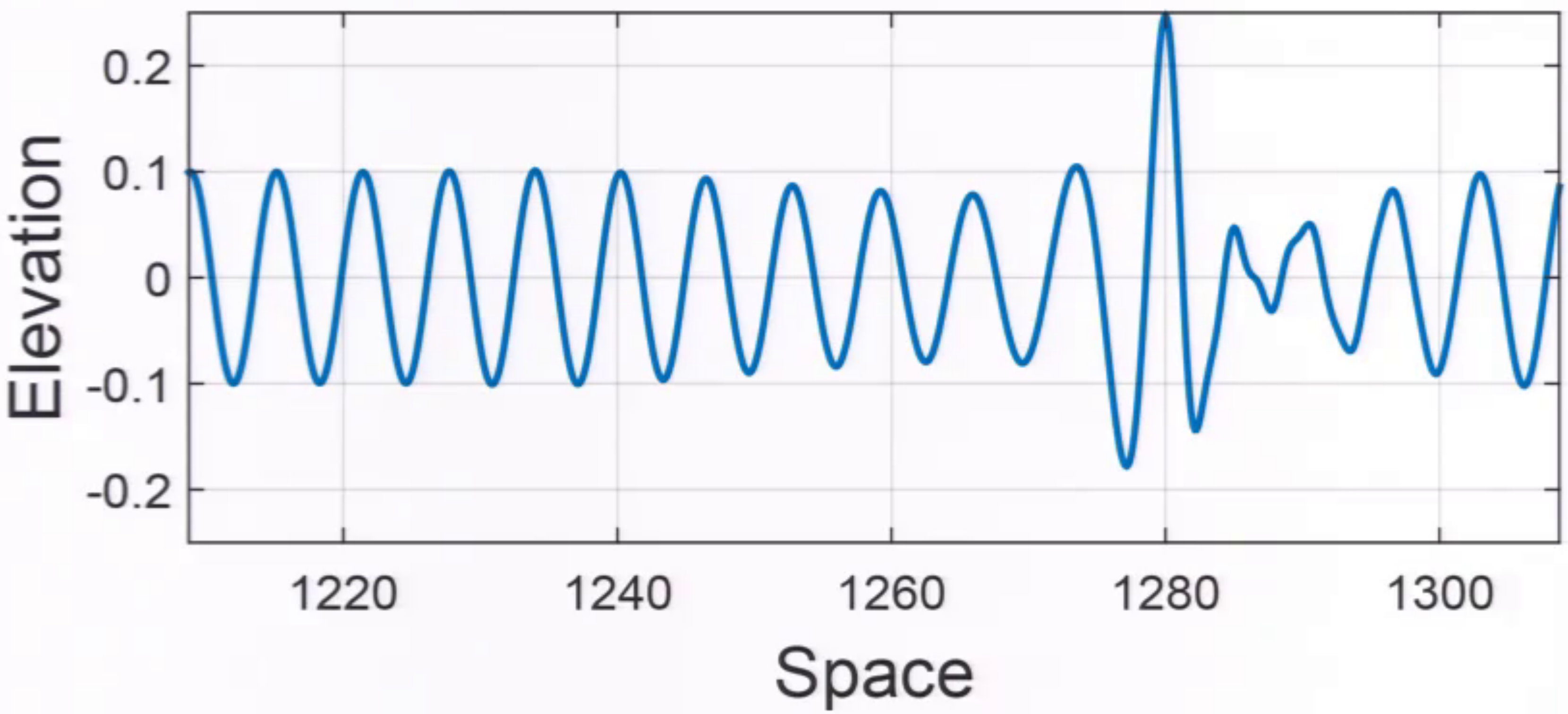
Dysthe derived closed equation for wave envelope u via perturbation method [Dysthe 1979]

Assumptions: Small steepness and slow varying u

$$0 = \frac{\partial u}{\partial t} + L(\partial_x, \partial_y) + \frac{i}{2}|u|^2 u + \frac{3}{2}|u|^2 \frac{\partial u}{\partial x} + \frac{1}{4}u^2 \frac{\partial u^*}{\partial x} + iu \frac{\partial \phi}{\partial x}\Big|_{z=0}$$

Note: Extreme waves are steep and vary quickly in *late* stages of their evolution, but less so in early stages. Envelope equations reproduce wave-tank extreme waves well [Chabchoub 2011]

Instabilities and Heavy Tails



Instabilities and Heavy Tails

- Real wave fields aren't infinitesimally perturbed plane waves—energy distributed over range of frequencies

- Ex: Gaussian Spectrum:

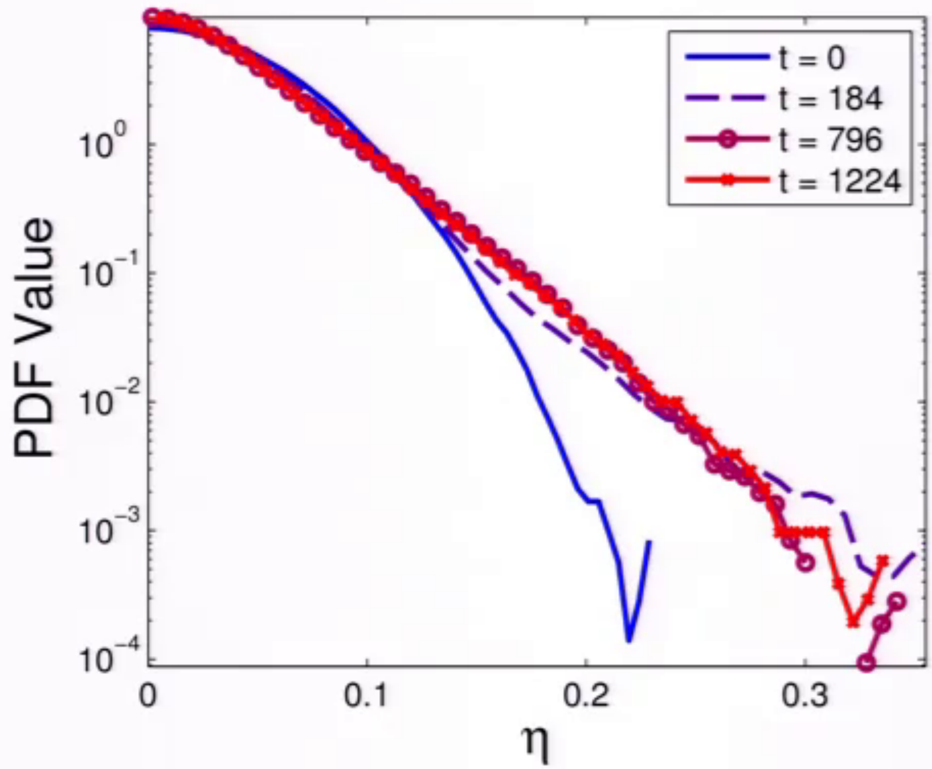
$$u(x,0) = \sum_{-N/2+1}^{N/2} \sqrt{2\Delta_k F(k\Delta_k)} e^{i(\omega_k x + \xi_k)}, \qquad F(k) = \frac{\epsilon^2}{\sigma\sqrt{2\pi}} e^{\frac{-k^2}{2\sigma^2}}$$

- ξ_k are random phases, meaning u(x,0) is normally distributed
- ε gives the magnitude of the envelope (wave steepness), σ determines the width of the power spectrum

Instabilities and Heavy Tails

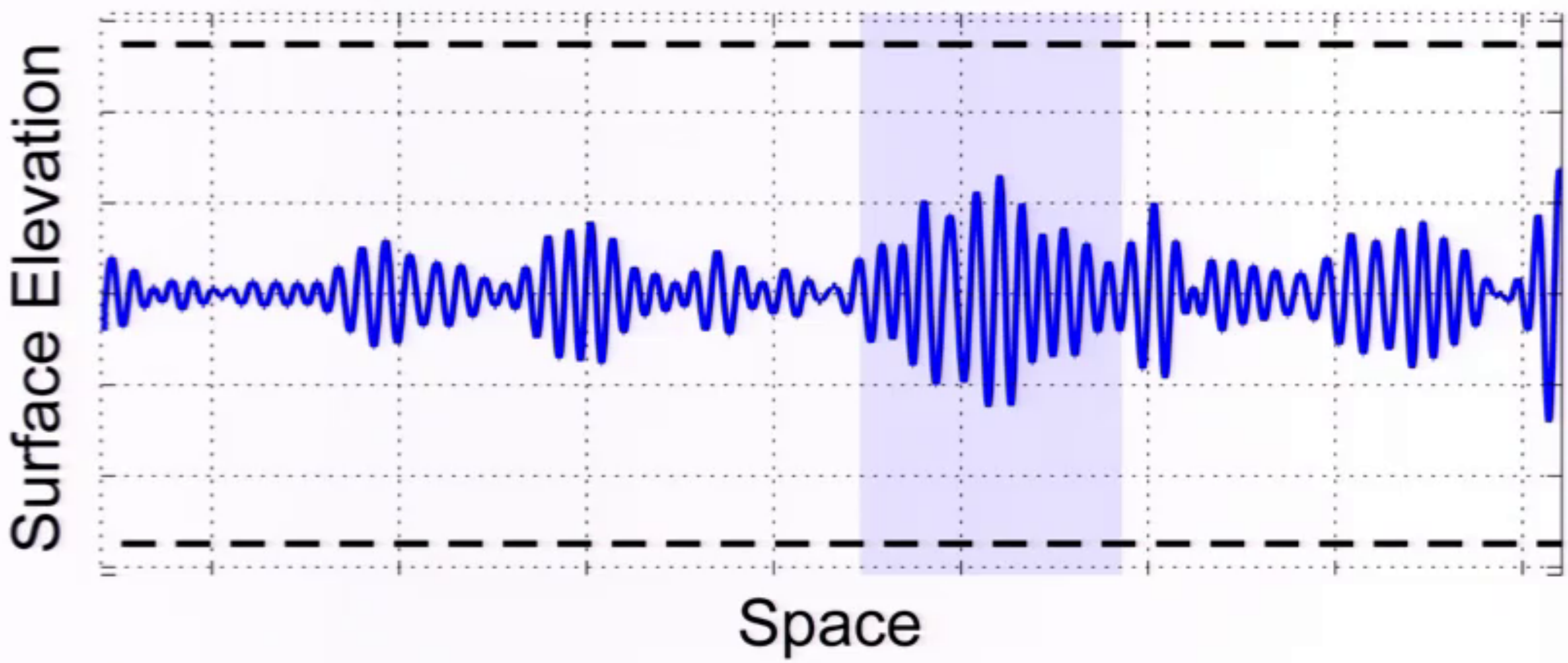
- Nonlinear interactions generate:
  1. Energy transfer between modes
  2. Non-stationary statistics—**heavy tails**

[Alber 1978, Janssen 2003]



Focusing Wave Groups Trigger Rogue Waves

Rogue waves are triggered by focusing of benign-looking wave groups



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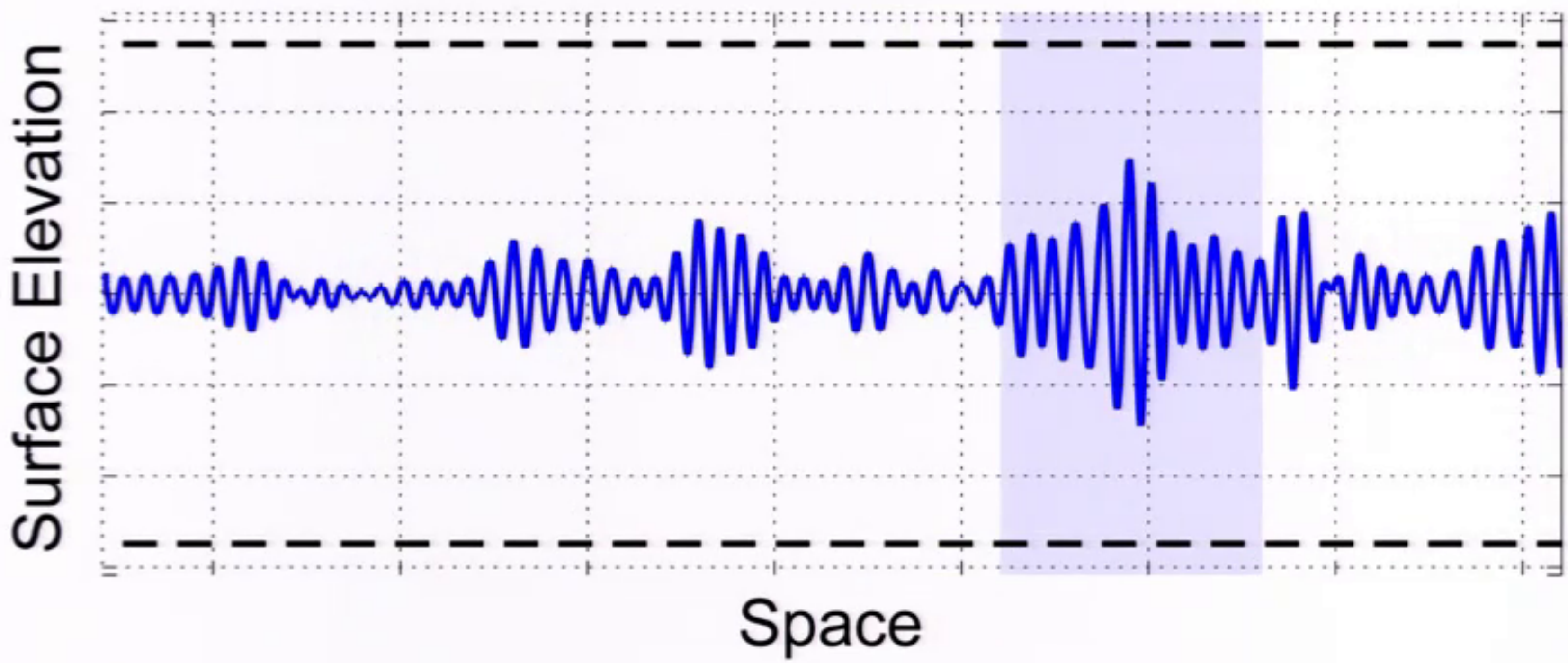

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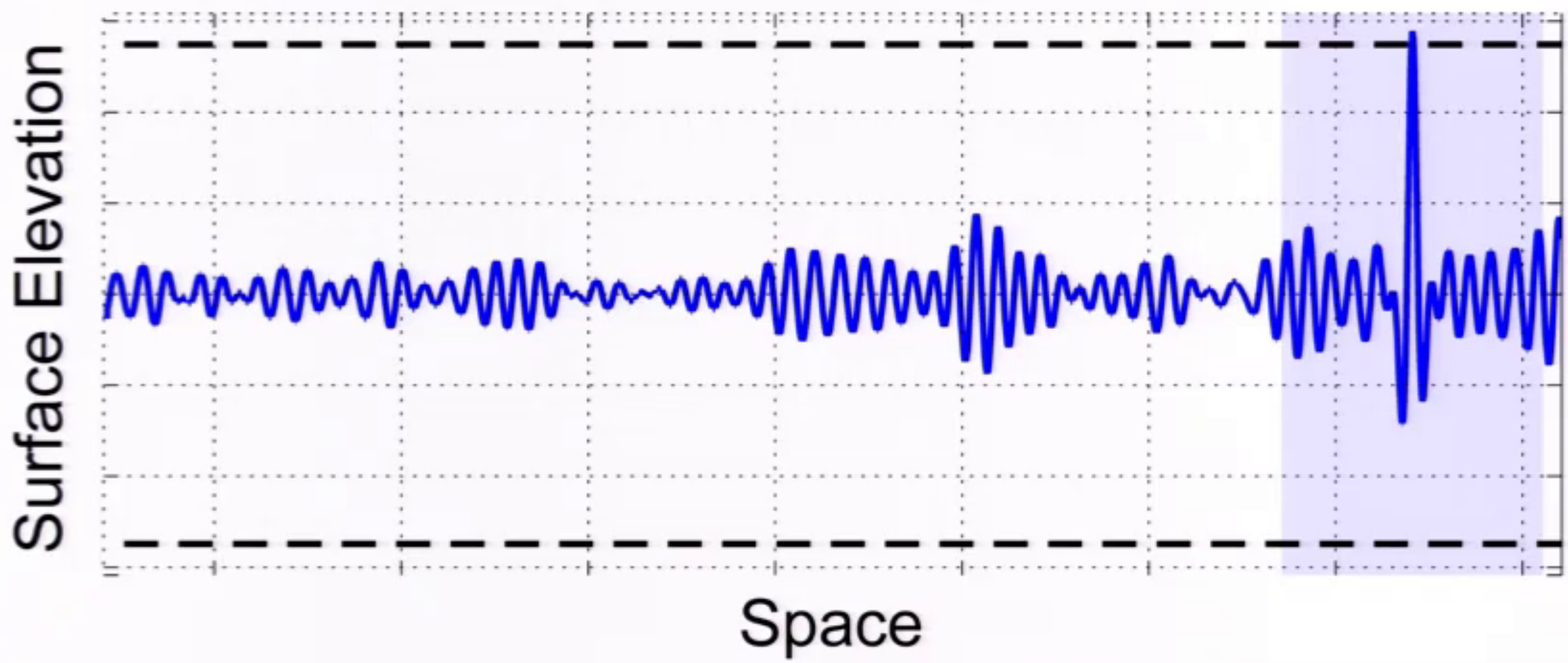
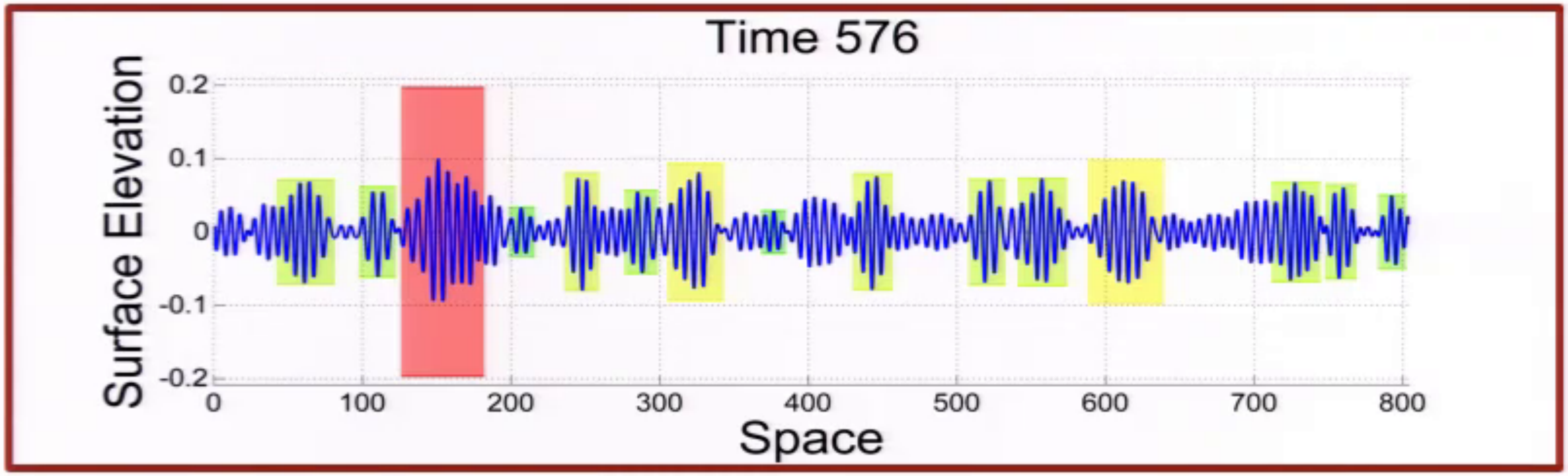


Our Simple Predictive Scheme

Our **computationally cheap, group-based** predictive scheme consists of two steps

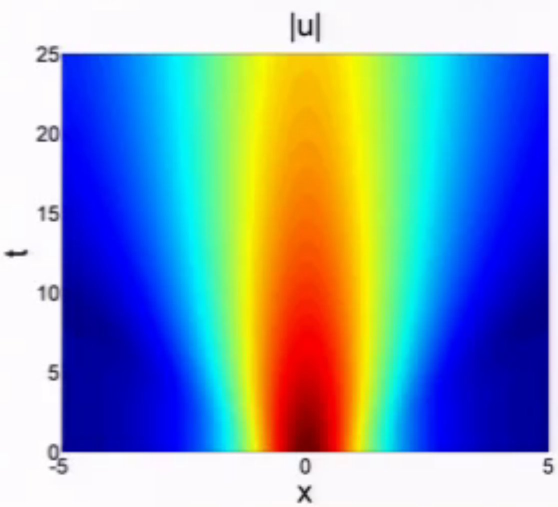
- **Step 1 (offline):** Quantify how a given wave group evolves as a function of its amplitude and length scale
- **Step 2 (online):** In an irregular wave field, pick out the groups. Use step 1 to predict future amplitude

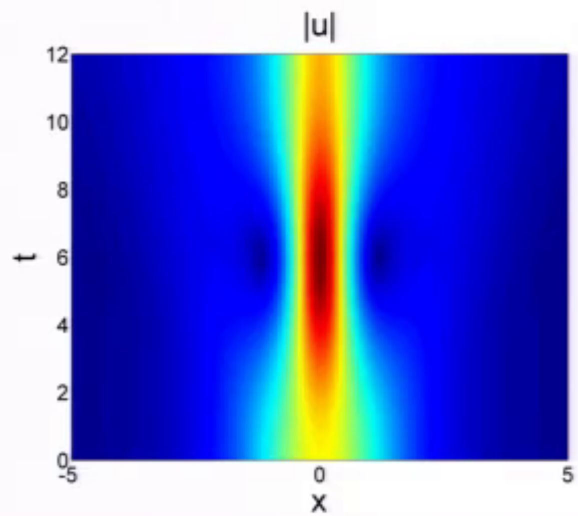


Localized Group Evolution

$$u(x,0) = A_0 \mathrm{sech}(x/L_0)$$

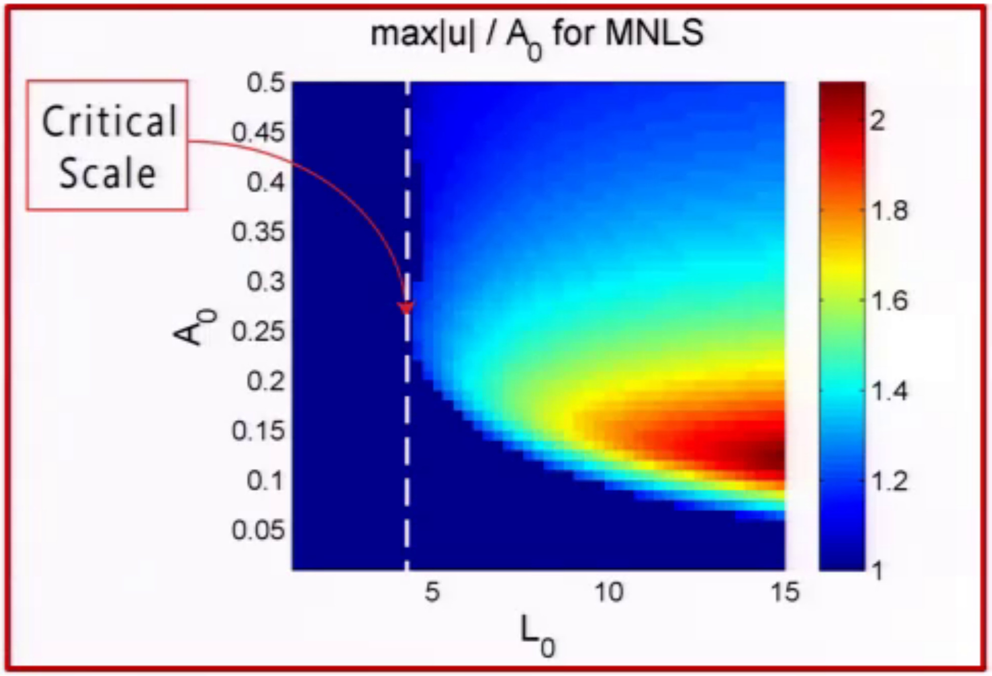
- For a given A_0 and L_0, how does this initial data evolve?
- Does group focus and increase in amplitude?
- If so, by how much? [Cousins, Sapsis 2015]





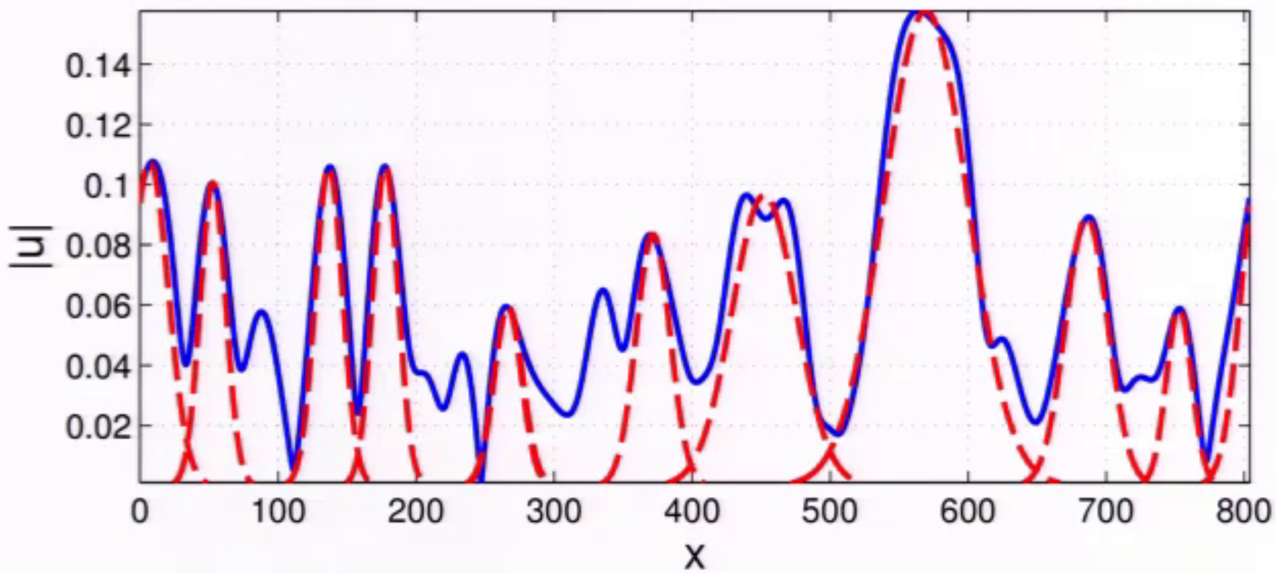
Localized Group Amplification Factors

Construct a map $u_{max}(A_0, L_0)$ using combination of offline numerical simulation and reduced order modeling



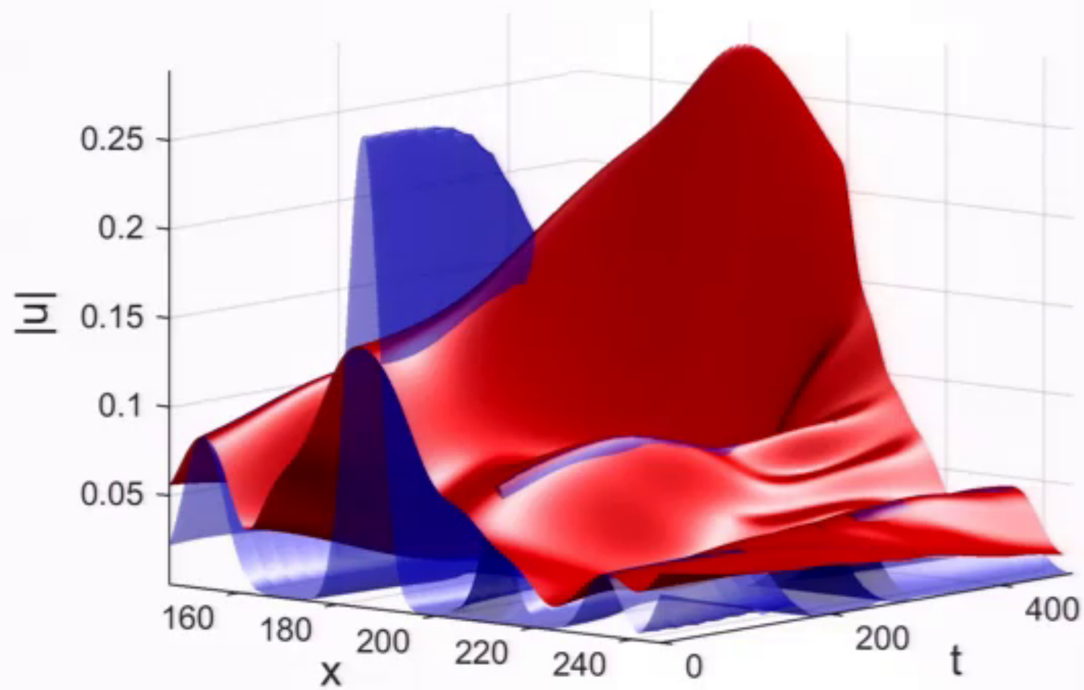
Group Detection

- Apple scale selection technique [Lindeberg 1998] to determine dominant groups in the wave field
- Gives locations, amplitude, and length scales of each group



Prediction: Simulated Data

- Tested our scheme on 100 simulations of MNLS with Gaussian spectra
- 336 rogue waves occurred, predicted all of them in advance
- Average warning time: 24 wave periods
- 91 false positives (21.3%)



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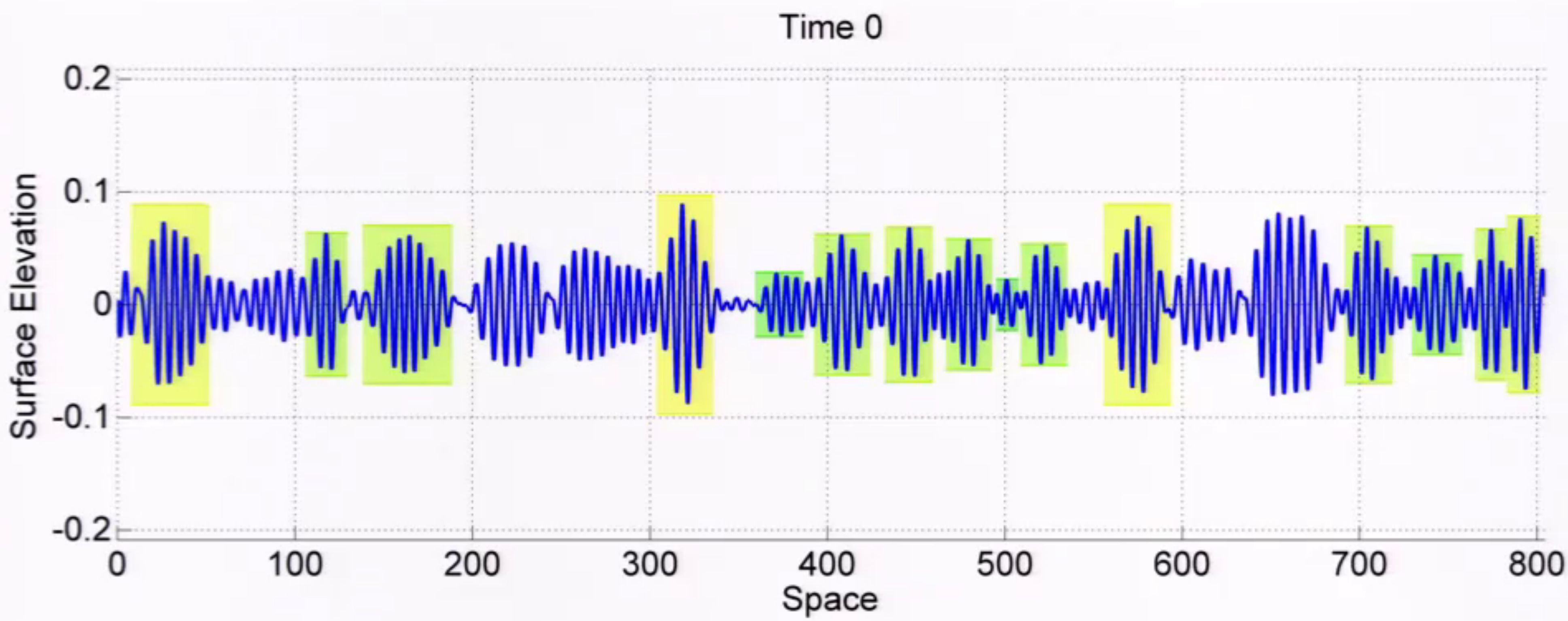
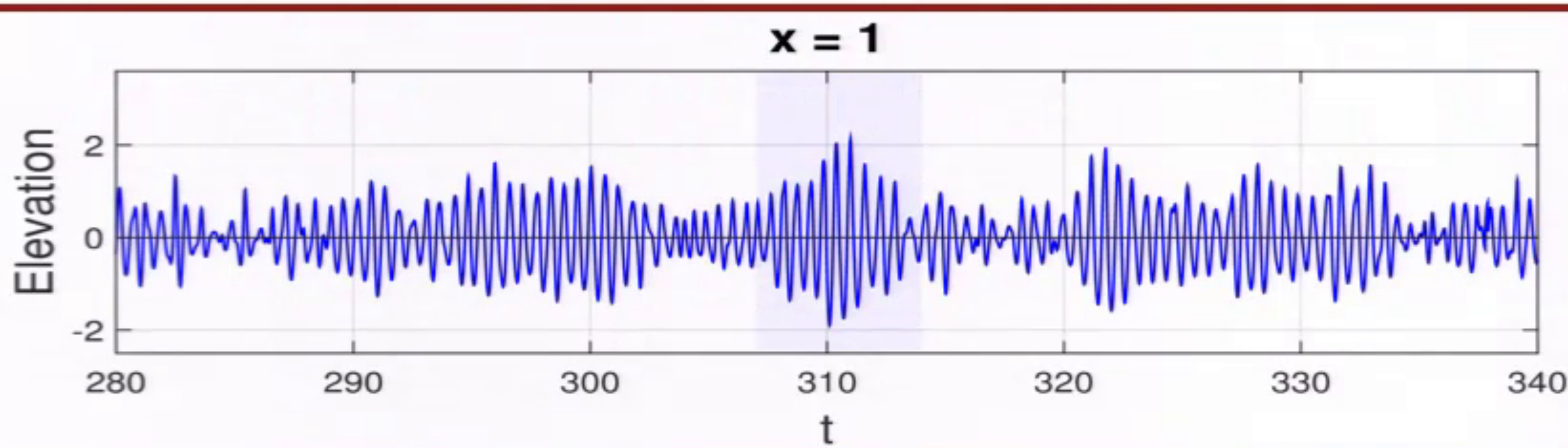


Prediction: Wave Tank Data

- We analyze data from an wave tank experiment
- Tank is 10 meters long, probes spaced 1 meter apart measure elevation time series
- Use measurements from one probe to cheaply predict downstream rogue wave



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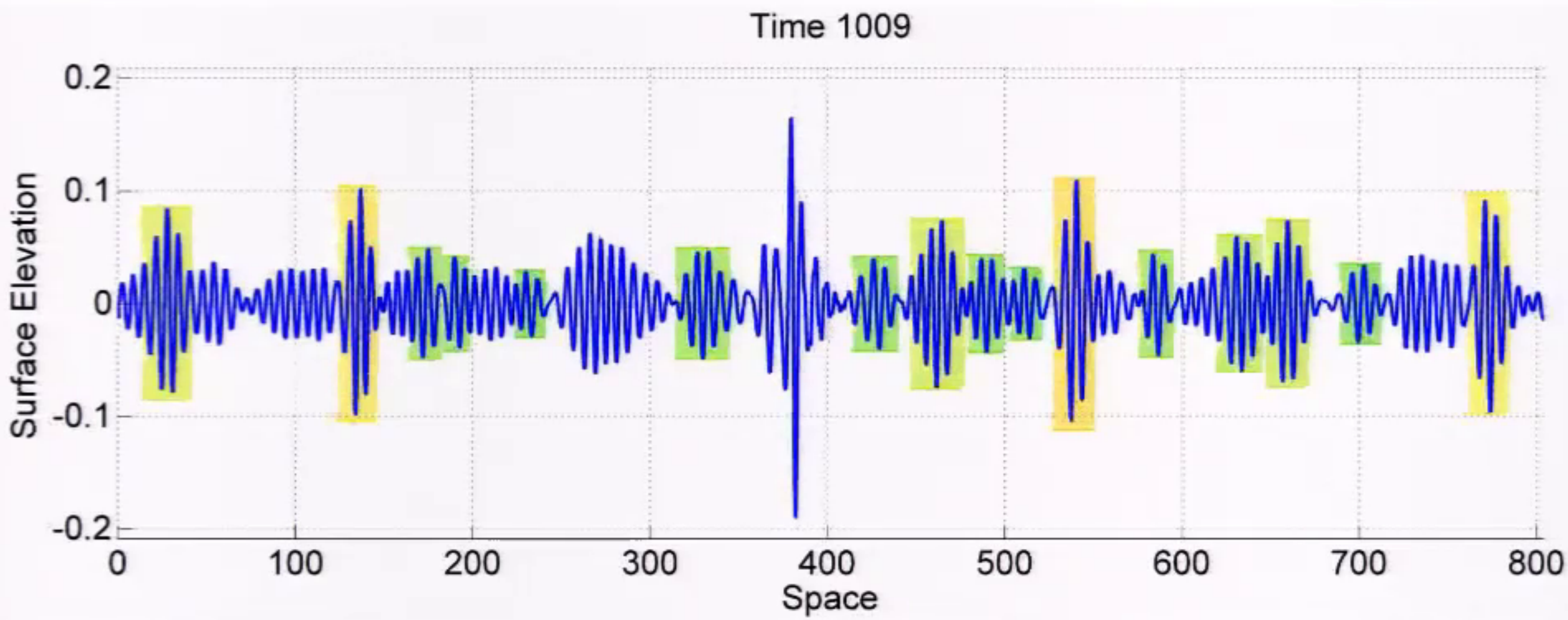

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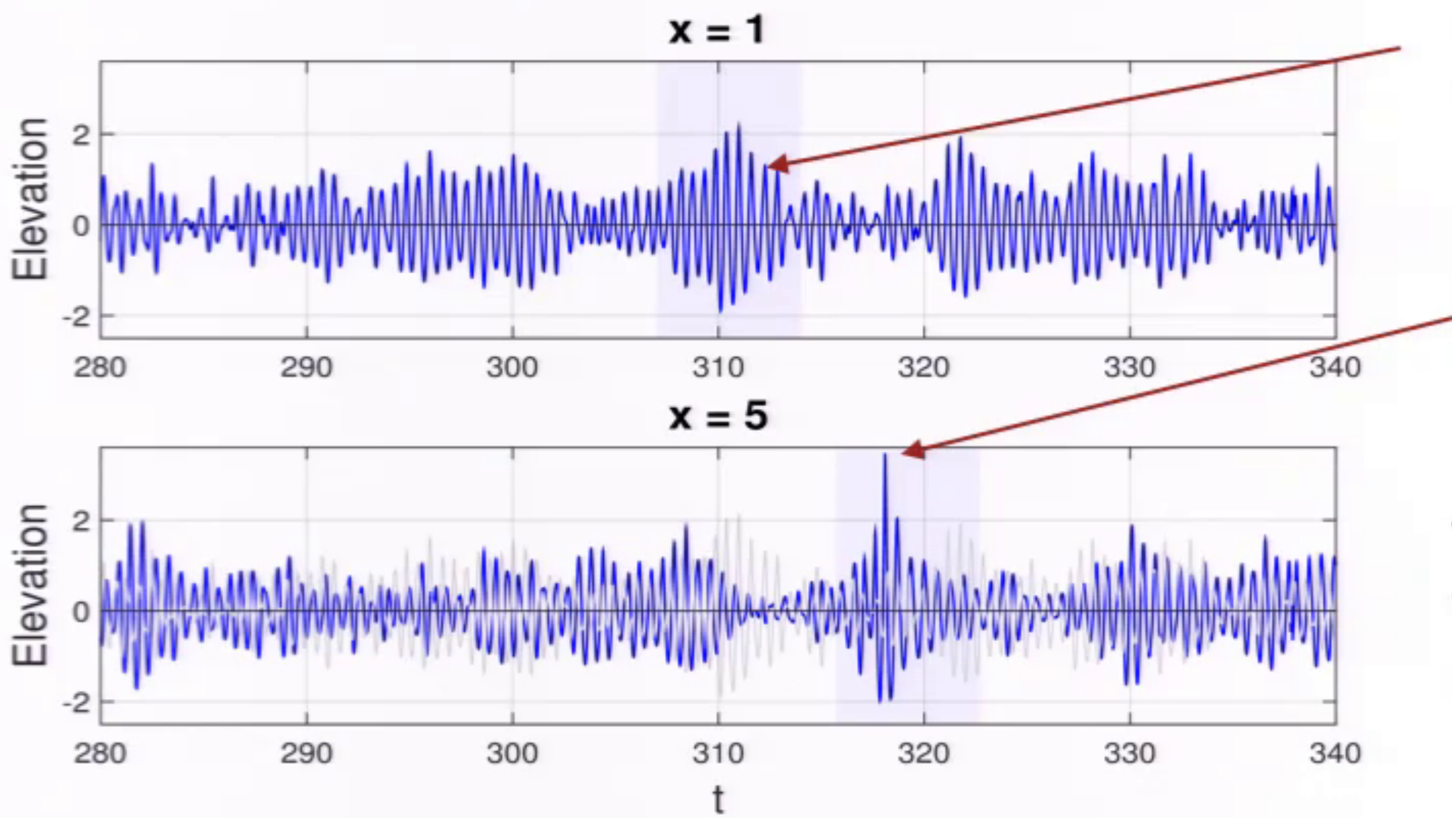


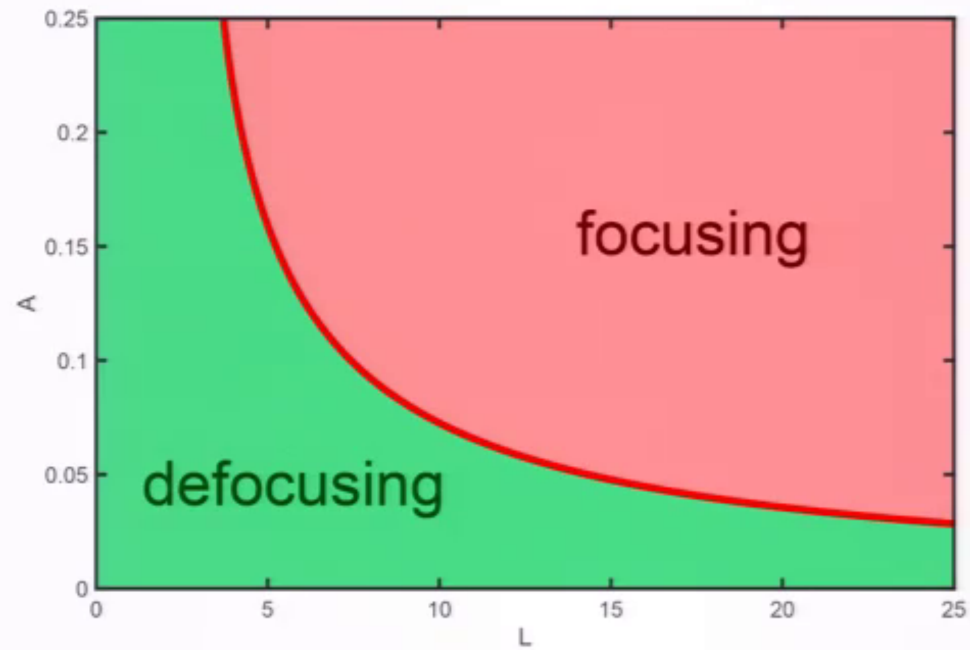
Predict future rogue wave here

Rogue wave occurs here

Warning Time: 8-9 temporal wave periods

Estimation of Heavy Tailed Statistics

- Understanding the trigger of rogue waves allows us to predict them in advance
- We can also use this characterization of the trigger to efficiently compute statistics
- Use idea from Mohamad, Sapsis (2015): partition phase space into a stable and unstable regime



Estimation of Heavy Tailed Statistics

- u = envelope of wave elevation, let $\mathrm{f(u)}$ be functional of the wave field (i.e. response of ocean structure)
- We decompose the probability measure of f as follows

$$\rho(f(u)) = \rho(f(u)|\text{Foc. Group})\rho(\text{Foc. Group}) + \rho(f(u)|\neg\text{Foc. Group})\rho(\neg\text{Foc. Group})$$

Nonlinear/Unstable Part

Linear/Stable Part

Evaluation of the Unstable Part

$$\rho(f(u)|\text{Foc. Group})\rho(\text{Foc. Group}) = \iint_{A,L\in\mathcal{F}} \rho(f(u)|A,L)\rho(A,L)dAdL$$