Advances on the Control of Nonlinear Network Dynamics

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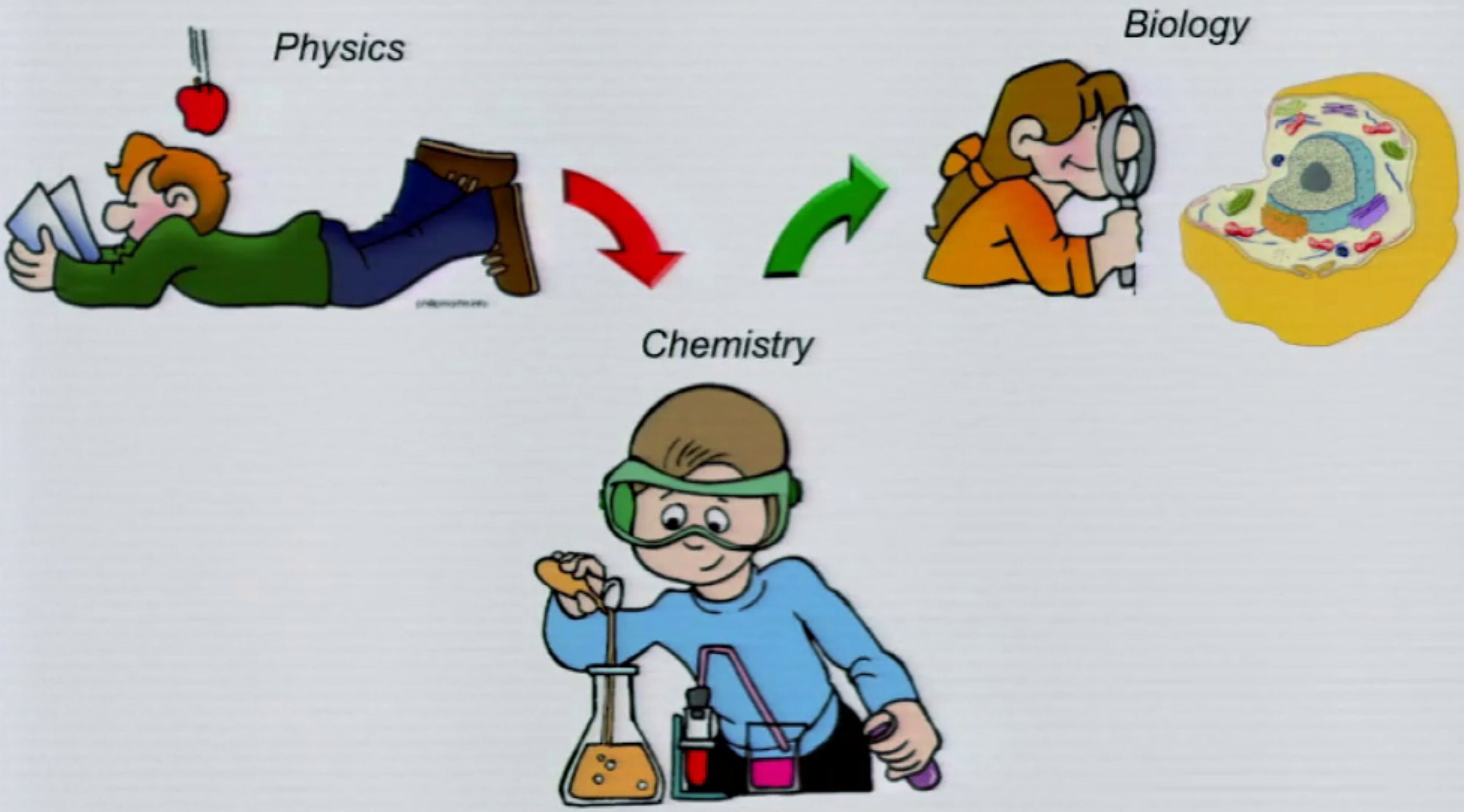
Failure of the “Everything is Known” View

An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis... for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

— Pierre Laplace, A Philosophical Essay on Probabilities

Natural Sciences



Adapted from Phillip Martin Clip Art

Definitions I

Complex system: a system that 1) is made up of a large number of interacting component parts and 2) exhibits collective dynamical behavior that cannot be inferred from the behavior of the parts themselves.

That is, the interactions between the component parts can be as important as the parts themselves in determining the collective dynamical behavior.

It is all about interactions

Diamond and Graphite

Definitions I

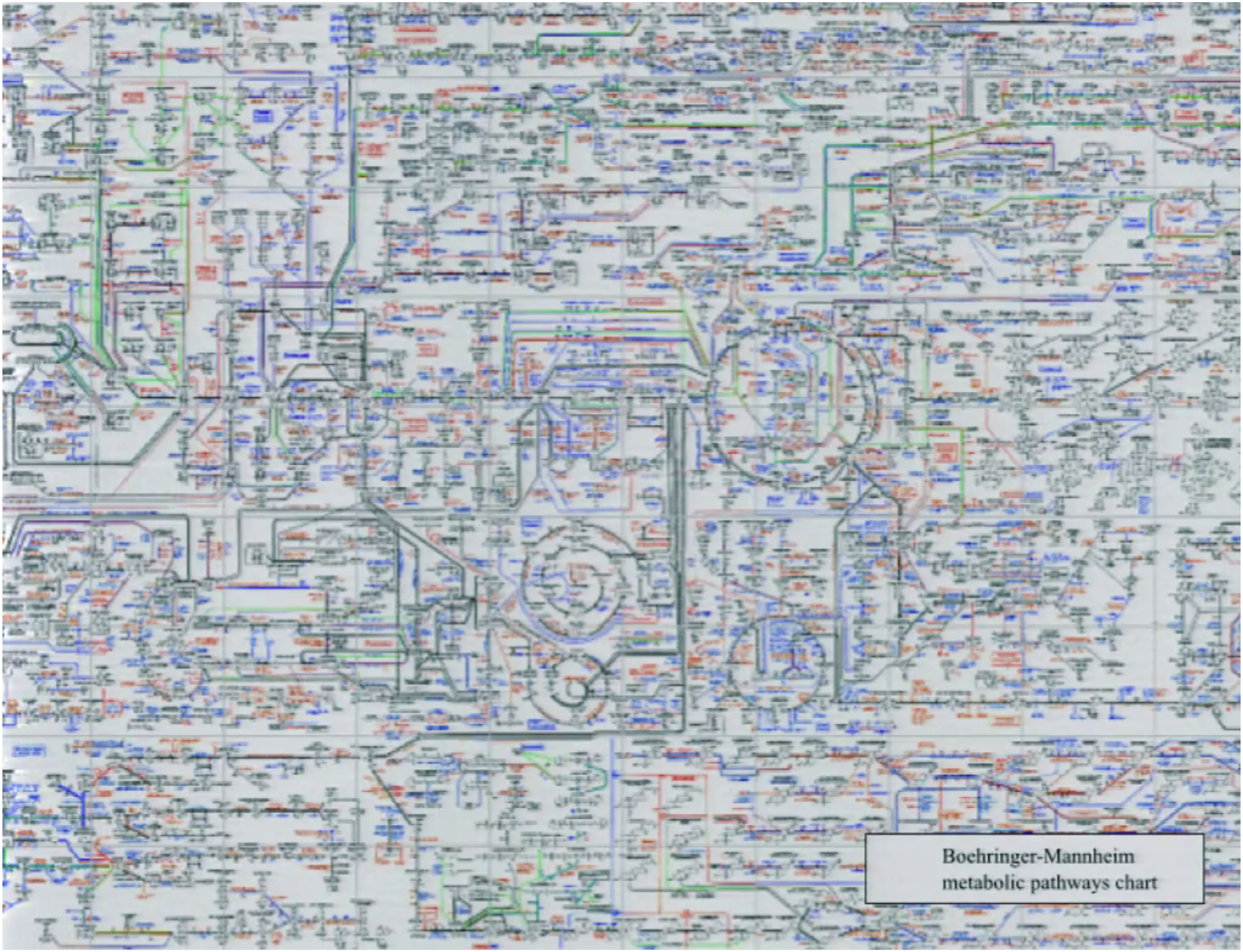
Complex system: a system that 1) is made up of a large number of interacting component parts *and* 2) exhibits collective dynamical behavior that cannot be inferred from the behavior of the parts themselves.

That is, the interactions between the component parts can be as important as the parts themselves in determining the collective dynamical behavior.

Therefore, complex systems naturally lend themselves to be modeled as networks of interactions between the component parts of the system.

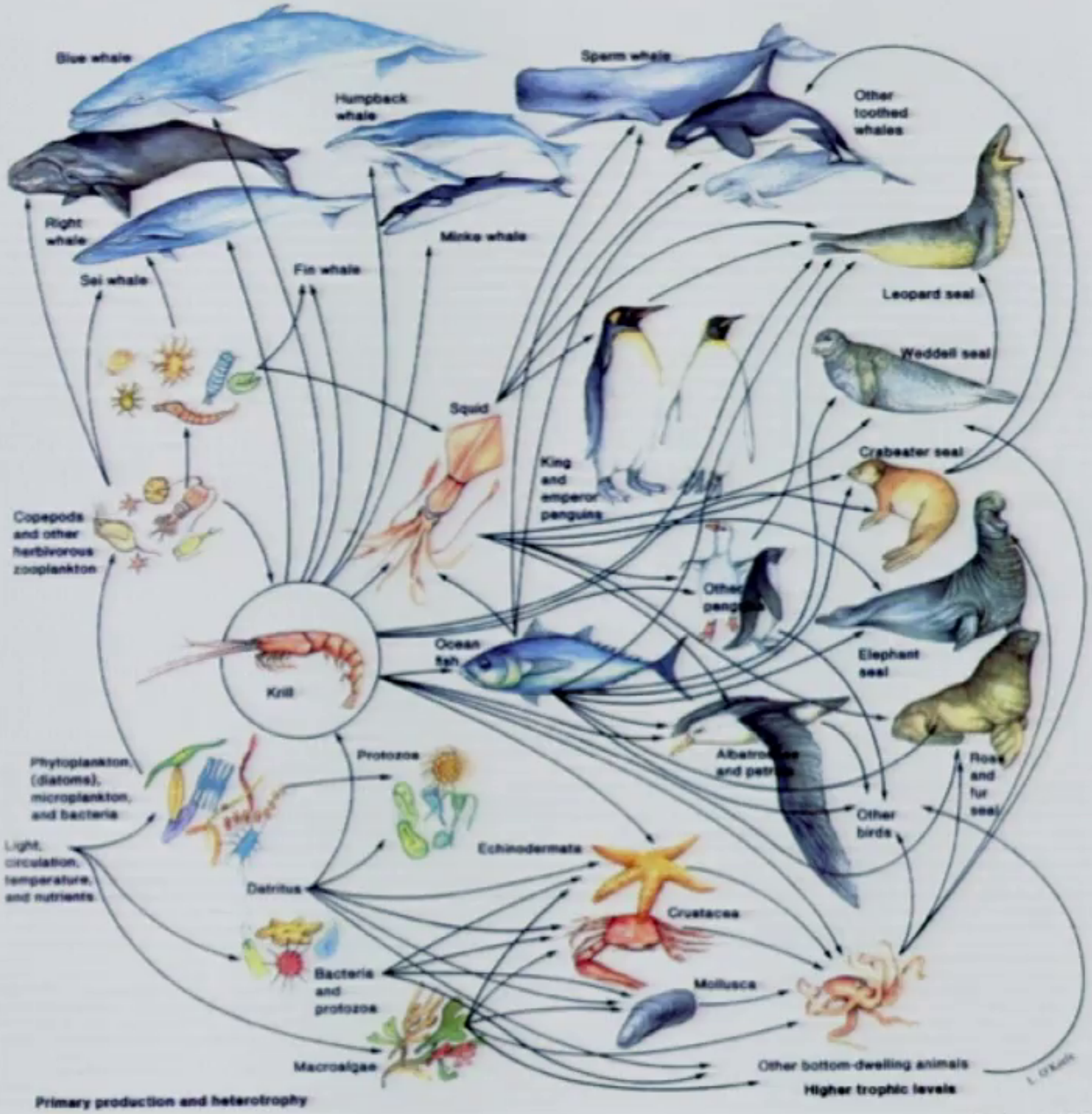
The control of such systems can in principle be based on controlling their dynamical units *or* the interactions between them.

That complex systems require different approaches and lead to different behavior has long been recognized in physics.
See e.g.: P.W. Anderson, "More is Different". Science (1972).



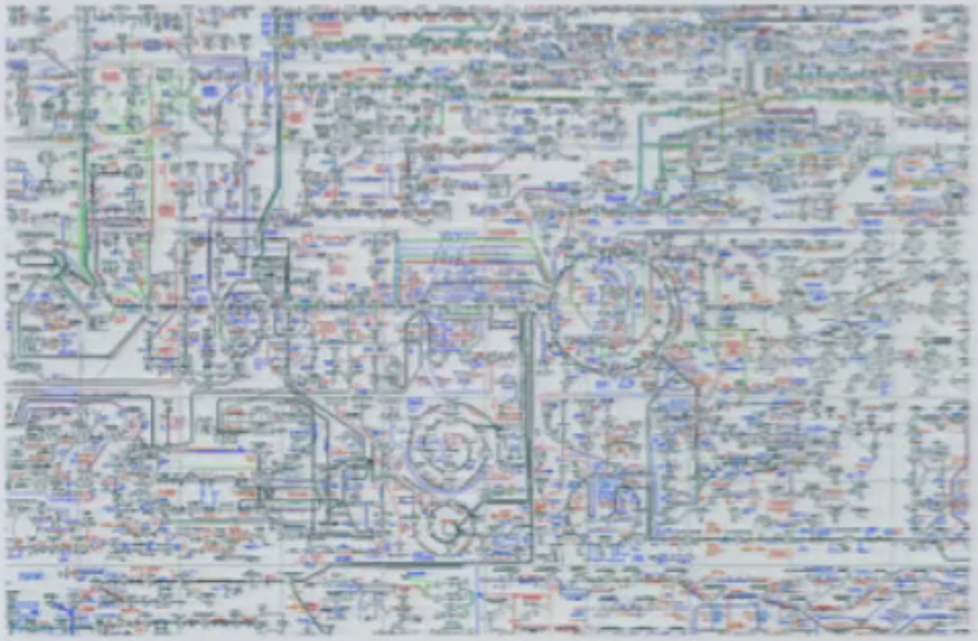
Boehringer-Mannheim metabolic pathways chart

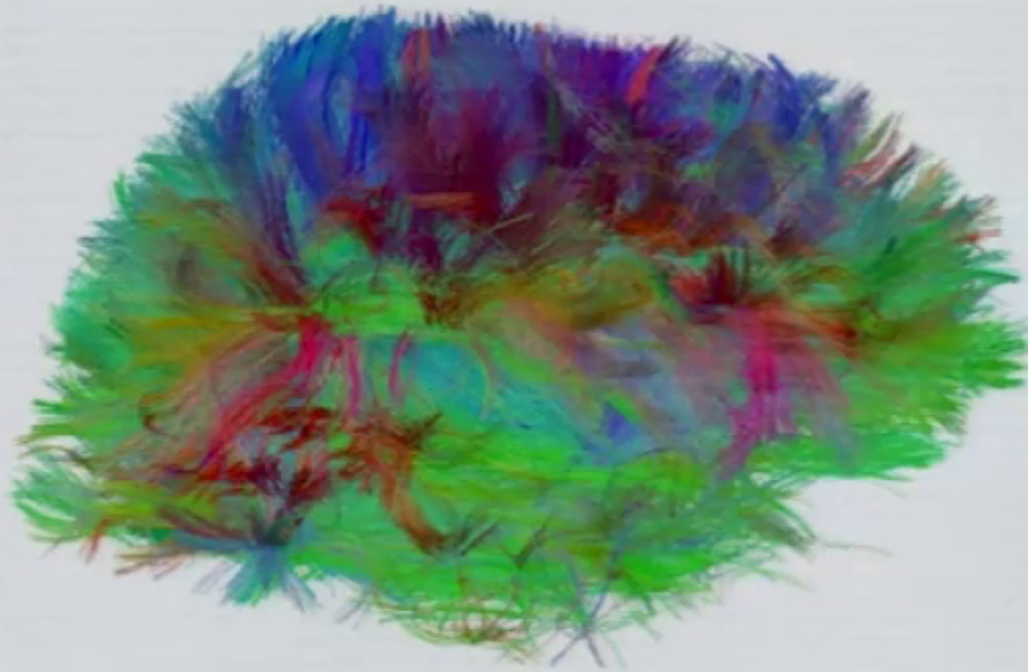

V. J. Wedeen/Harvard

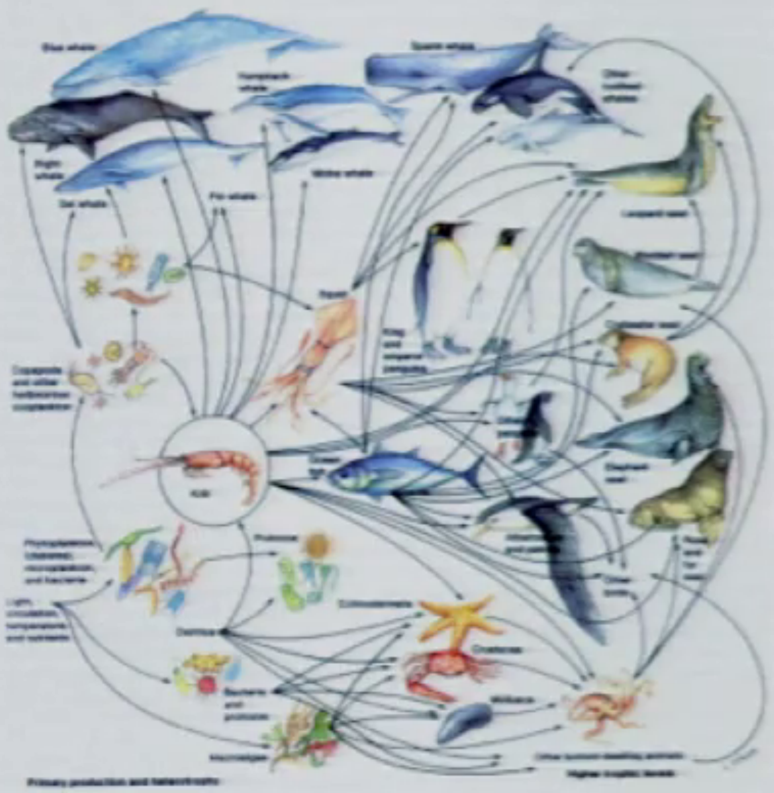


S.A. Miller & P. Harley

NASA







Dynamical Properties of Real Networks

(i) the dynamics is nonlinear

(ii) the system has multiple stable states (or attractors)

(iii) the system is described by a large number of dynamical variables

(iv) there are constraints on the physically feasible control interventions

(v) there might be noise and parameter uncertainty

(vi) decentralized (hence suboptimal) response to perturbations

Definitions II

Controllability: concerns the property of being able to steer the system from an arbitrary given initial state to an arbitrary given target state in finite time.

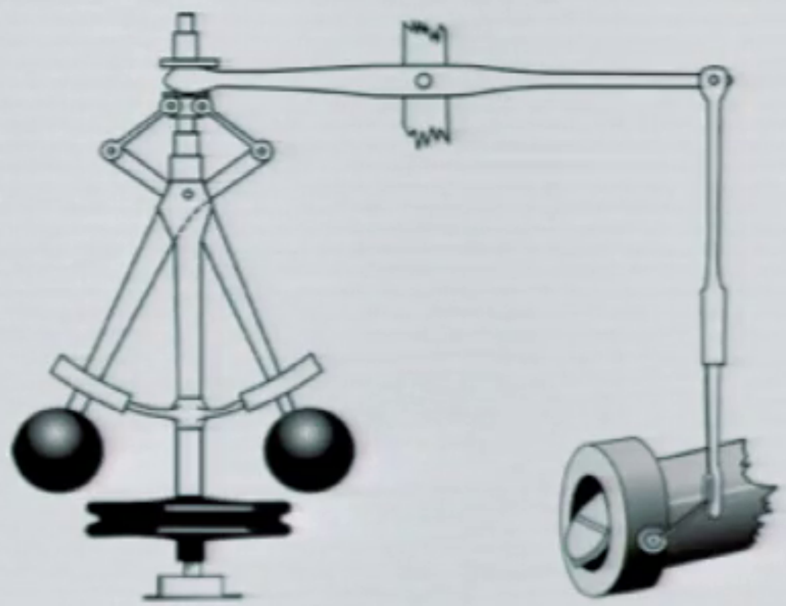
Control: more qualitative than controllability, may concern the problem of driving the system from specific initial to specific final states, the stabilization of a state of interest, or other actuated changes to the dynamics.

We focus mainly on the second, which is the most relevant for the network systems we consider, but with the goal of systematically exploring all relevant possibilities.

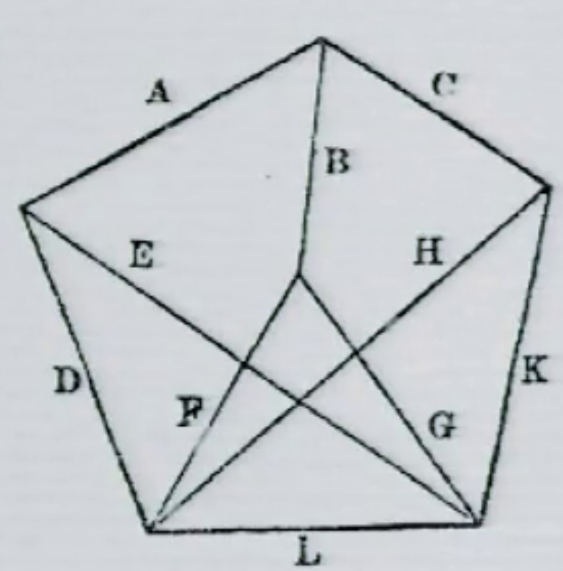
Talk: Control of network dynamics

Control Theory

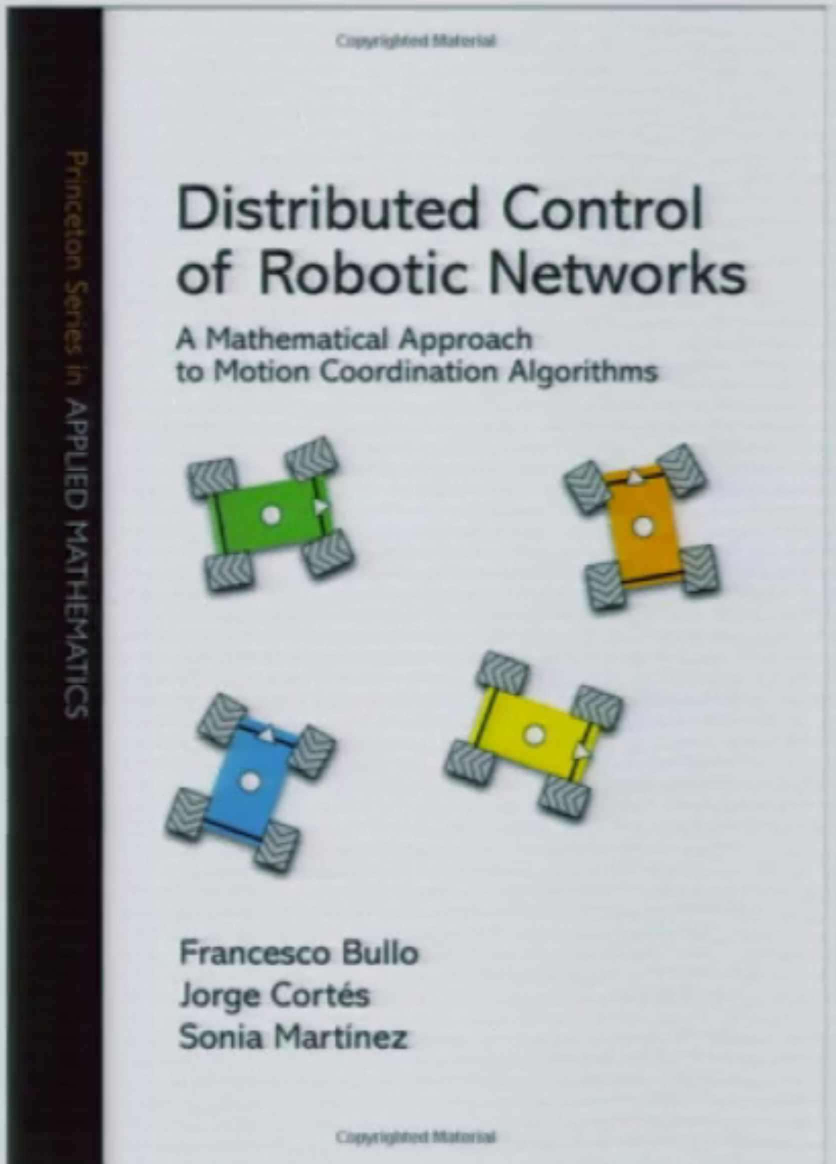
Maxwell, J.C. On Governors. Proceedings of the Royal Society of London 16, 270 (1868).

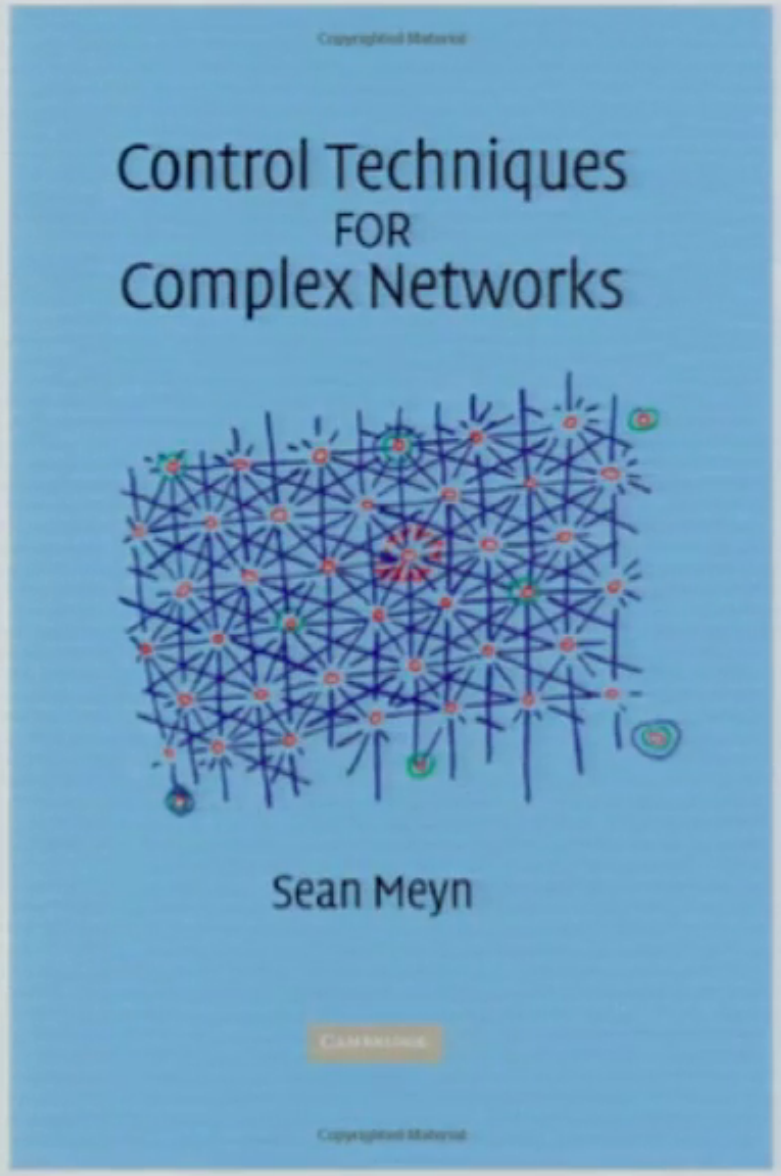


Maxwell, J.C.
On Reciprocal Figures and Diagrams of Forces.
Philosophical Magazine 27, 250 (1864).



Control Theory





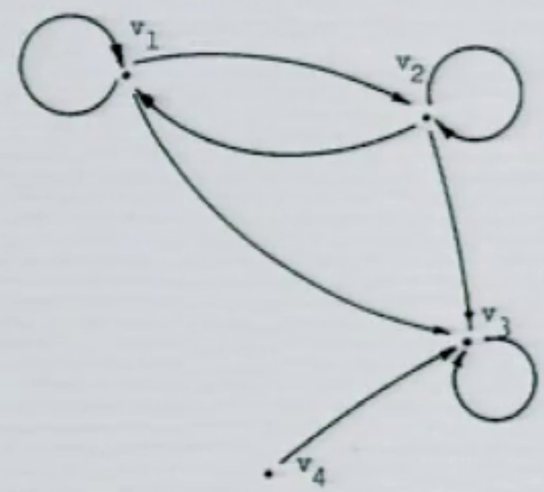
Control Theory

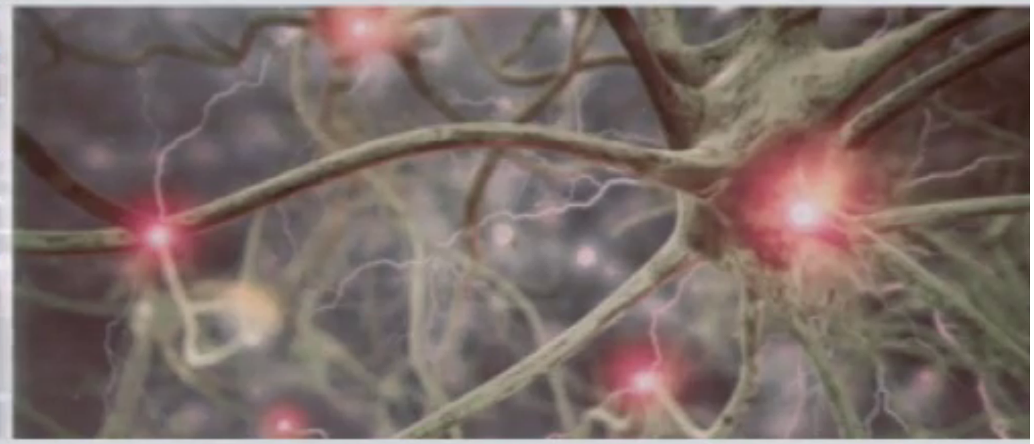
IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-19, NO. 3, JUNE 1974 201

Structural Controllability

CHING-TAI LIN, MEMBER, IEEE

Abstract—The new concepts of "structure" and "structural controllability" for a linear time-invariant control system (described by a pair (A,b)) are defined and studied. The physical justification of these concepts and examples are also given.



IEEE Transactions on Control of Network Systems


The Challenge

… to account for the following properties at the same time:

(i) the dynamics is nonlinear

(ii) the system has multiple stable states (or attractors)

(iii) the system is described by a large number of dynamical variables

(iv) there are constraints on the physically feasible control interventions
(e.g., constraints on the nodes that can be manipulated)

(v) there might be noise and parameter uncertainty

(vi) decentralized (hence suboptimal) response to perturbations

That is, the problem is the lack of scalability of most existing methods for high-dimensional nonlinear systems exhibiting phase-space phenomena.

Entirely Determined by Network Structure

Why Your Friends Have More Friends than You Do
Scott L. Feld, Am. J. Soc. 1991

mean number of friends of friends ≥ mean number of friends

Epidemic control by acquaintance immunization

Cohen, Havlin, ben-Avraham, Phys. Rev. Lett. 2003

Early detection of contagious outbreaks

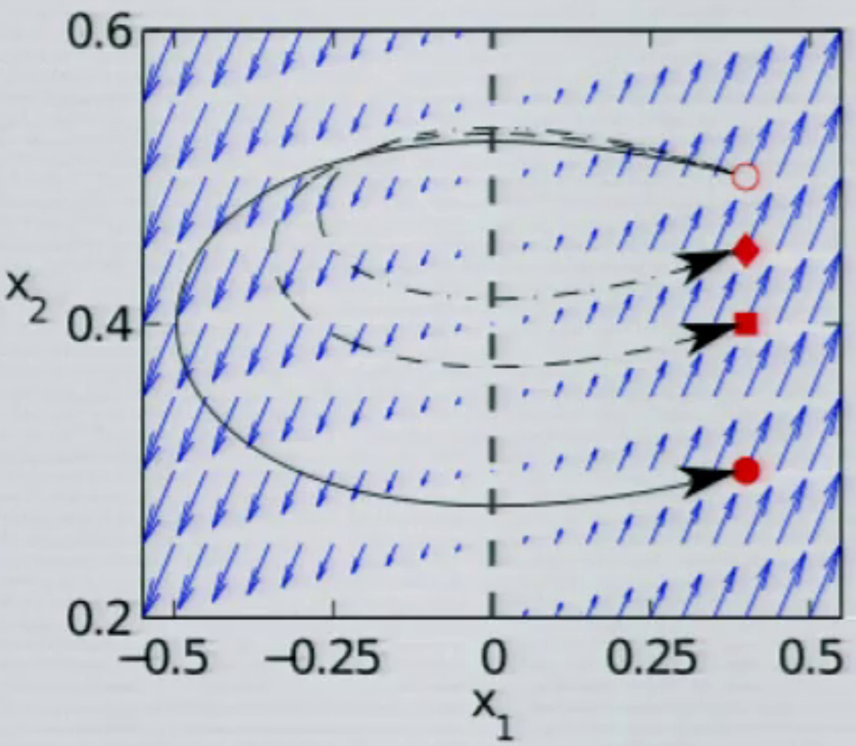
Christakis, Fowler, PLoS One 2010

Linear Network Systems

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$

Kalman's controllability matrix:

$$K = \left[B\ AB \cdots A^{n-1}B\right]$$



$$\dot{x}_1 = x_1 + u_1(t)$$
$$\dot{x}_2 = x_1$$

Control trajectories are nonlocal!

Sun and Motter, PRL 2013

Linear Network Systems

- Condition number of the controllability Gramian increases as the length of the control trajectory increases.

- Need of a controllability criterion that accounts not only for the existence but also for the actual computability of the control interventions.

  The system is controllable in practice if and only if the controllability Gramian has full numerical rank.

- Linear controllability fails in nonlinear systems even locally because control trajectories generally go outside the "linear" region.

- When sufficiently high dimensional, even linear systems are difficult to control.

Sun and Motter, PRL 2013

Control of Nonlinear Network Systems

First we can assume the most favorable conditions: the system is described by deterministic ODEs and we know the equations as well the parameters.

Then we can relax each of these assumptions (for example, include stochasticity, parameter uncertainty, spatial dependences, etc) and study how the methods have to be modified.

Example Problem

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x};\boldsymbol{\beta})$$

$$\begin{aligned}
\text{state: } & \mathbf{x} \in D \subset \mathbb{R}^m \\
\text{parameters: } & \boldsymbol{\beta} \in S \subset \mathbb{R}^M \\
\text{vector field: } & \mathbf{F} : D \times S \to \mathbb{R}^m \\
\text{attraction basin: } & \Omega_{\boldsymbol{\beta}}(A) \subset D \\
& (\mathbf{x}_0 \in \Omega_{\boldsymbol{\beta}}(A) \text{ iff } \boldsymbol{\phi}_{\boldsymbol{\beta}}(t,\mathbf{x}_0) \to A \text{ as } t \to +\infty)
\end{aligned}$$

Given $\mathbf{x}_0 \notin \Omega_{\boldsymbol{\beta}}(A)$, find $\Delta\mathbf{x}_0^A$ such that

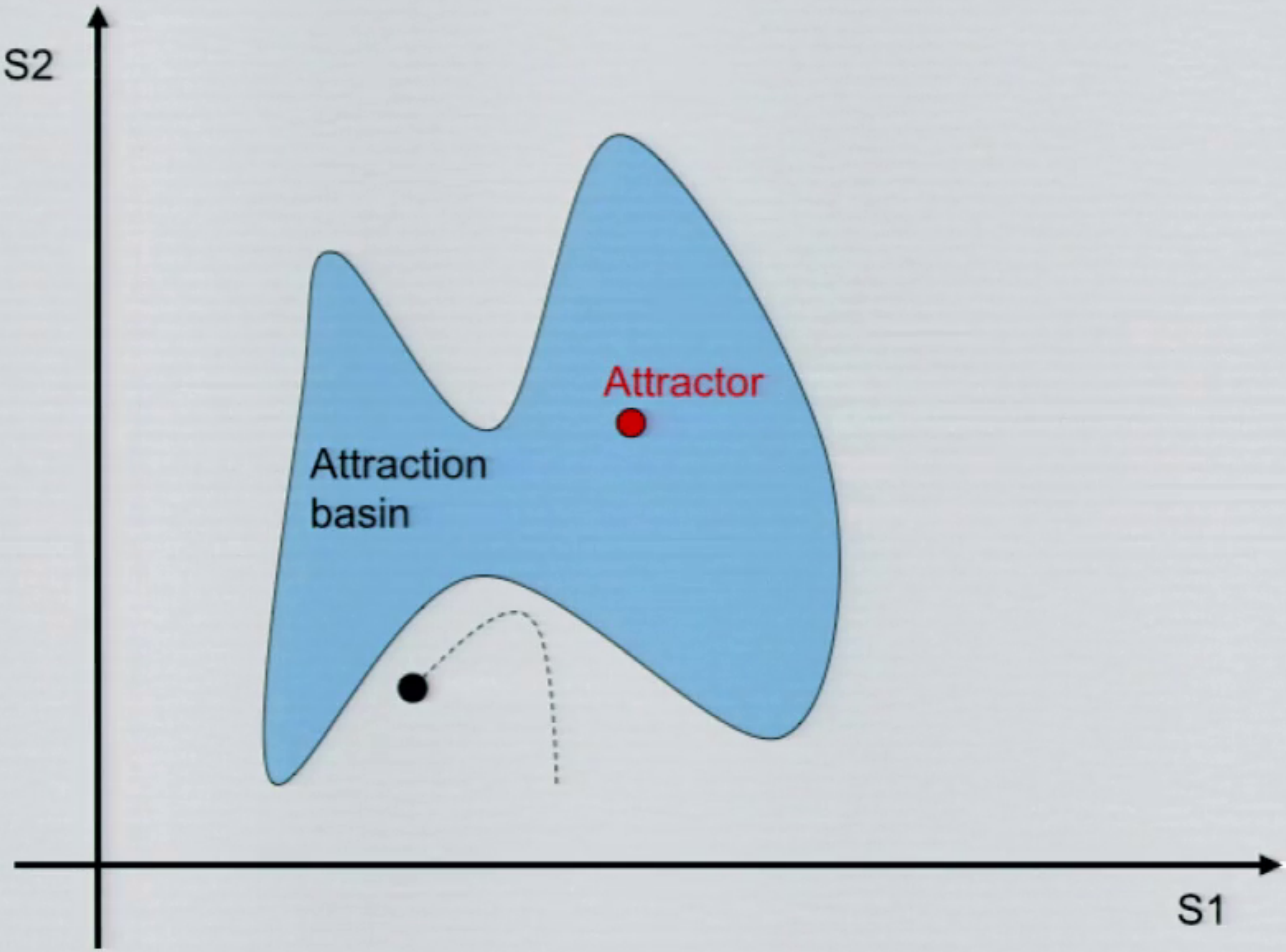
$$\begin{aligned}
& \mathbf{x}_0' \equiv \mathbf{x}_0 + \Delta\mathbf{x}_0^A \in \Omega_{\boldsymbol{\beta}}(A) \\
& g_i^x(\mathbf{x}_0', \mathbf{x}_0) \le 0, \quad i = 1, ..., p \\
& h_j^x(\mathbf{x}_0', \mathbf{x}_0) = 0, \quad j = 1, ..., q
\end{aligned}$$

Given $\mathbf{x}_0 \notin \Omega_{\boldsymbol{\beta}}(A)$, find $\boldsymbol{\beta}'$ under the constraints

$$\begin{aligned}
& g_i^{\beta}(\boldsymbol{\beta}', \boldsymbol{\beta}) \le 0, \quad i = 1, ..., P \\
& h_j^{\beta}(\boldsymbol{\beta}', \boldsymbol{\beta}) = 0, \quad j = 1, ..., Q
\end{aligned}$$

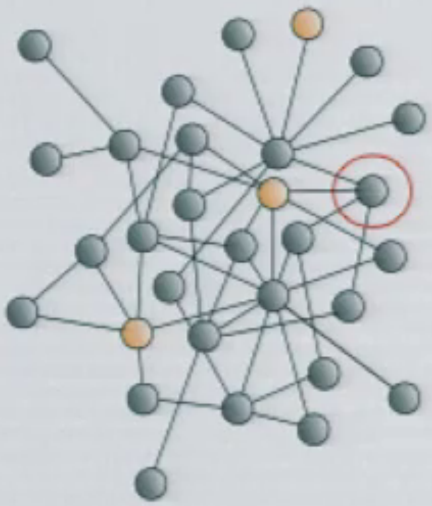
such that $\mathbf{x}_0 \in \Omega_{\boldsymbol{\beta}'}(A')$

Cornelius, Kath, Motter, Nature Comm 2013


S2
Attractor
Attraction
basin
S1

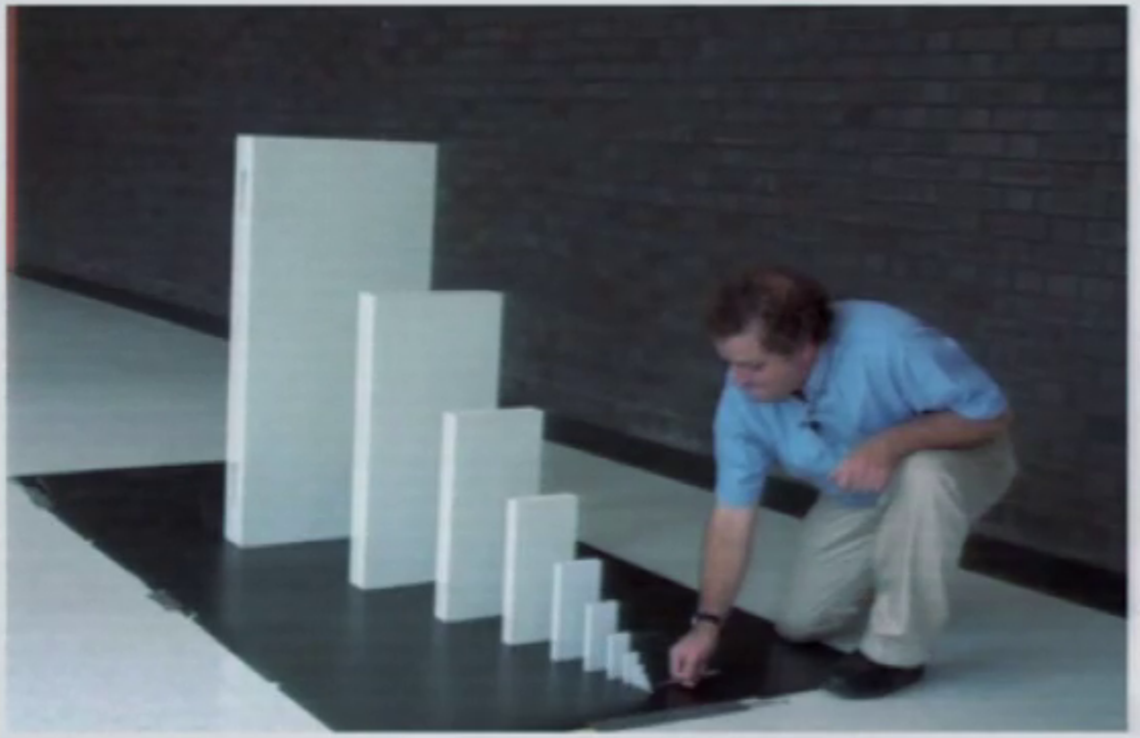
Nonlinear Deterministic Dynamics

Control of Cascading Processes



Cornelius, Kath, Motter, Nature Comm 2013

Amplification Effect



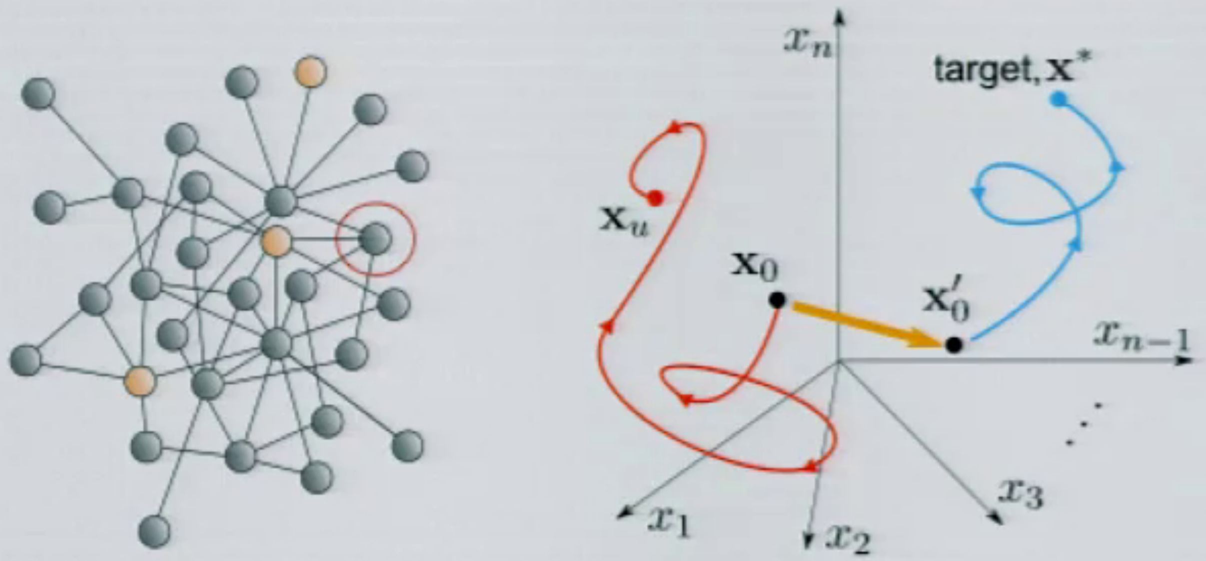
Stephen Morris

Amplification Effect

Stephen Morris

Nonlinear Deterministic Dynamics

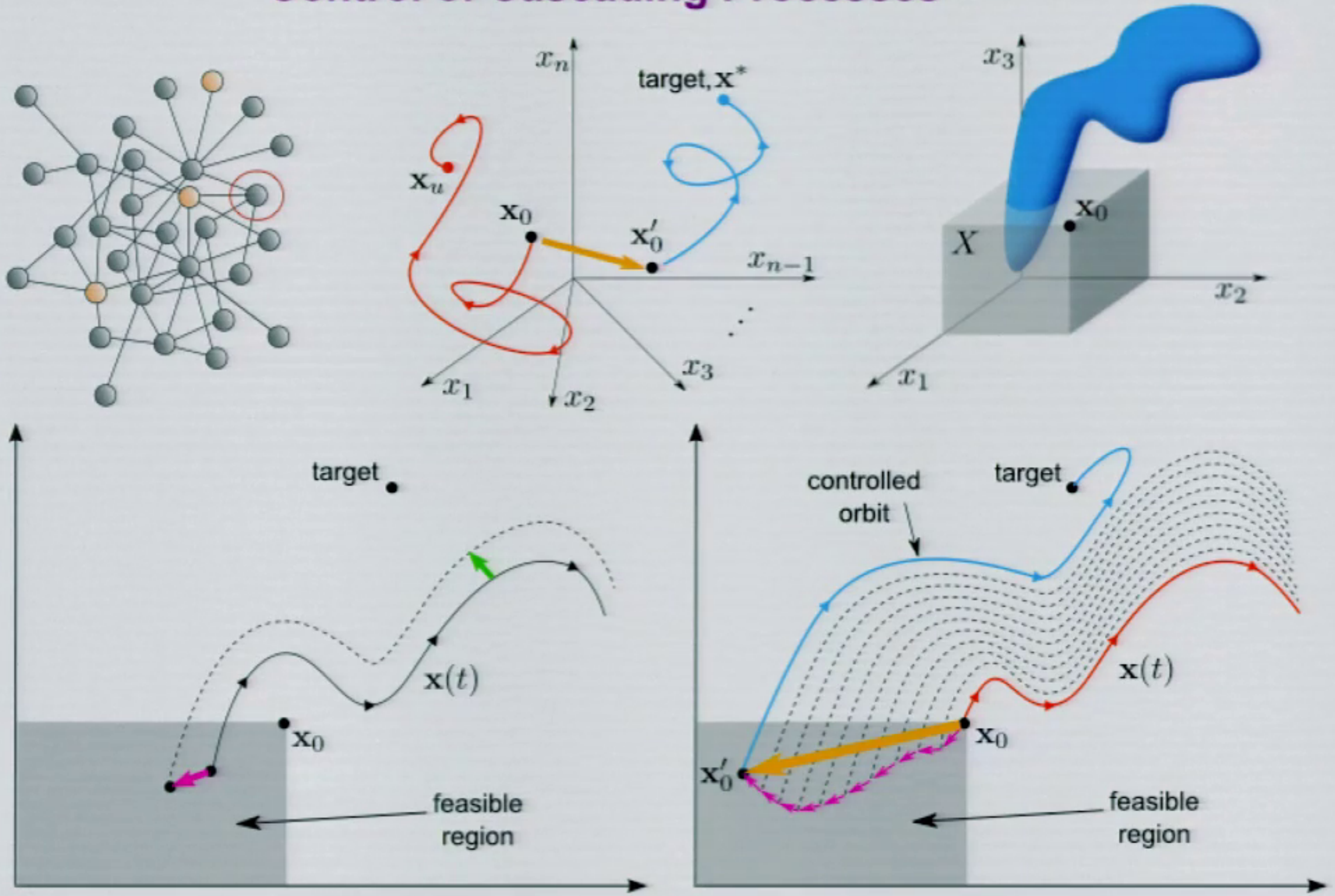
Control of Cascading Processes



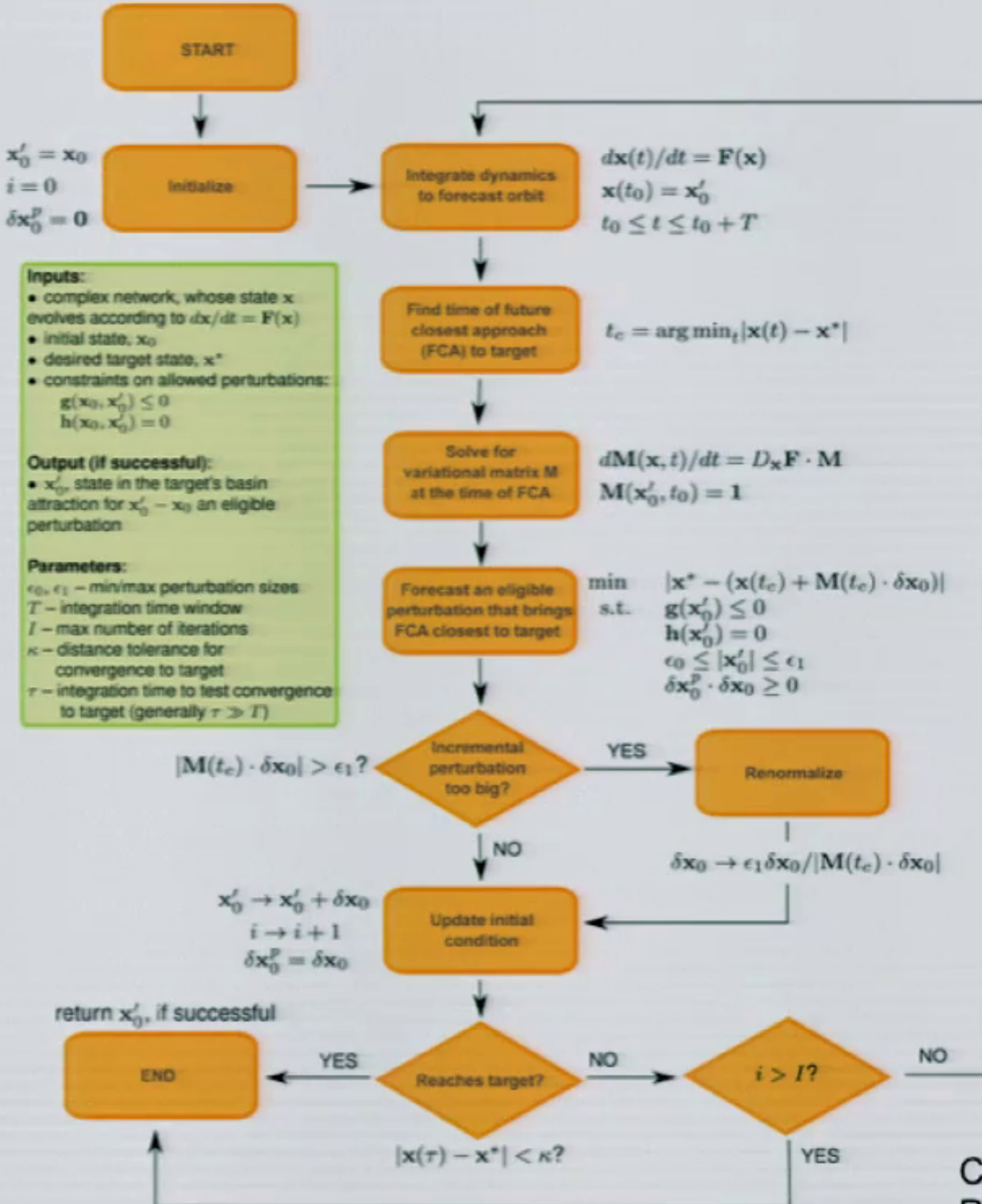
Cornelius, Kath, Motter, Nature Comm 2013

Nonlinear Deterministic Dynamics

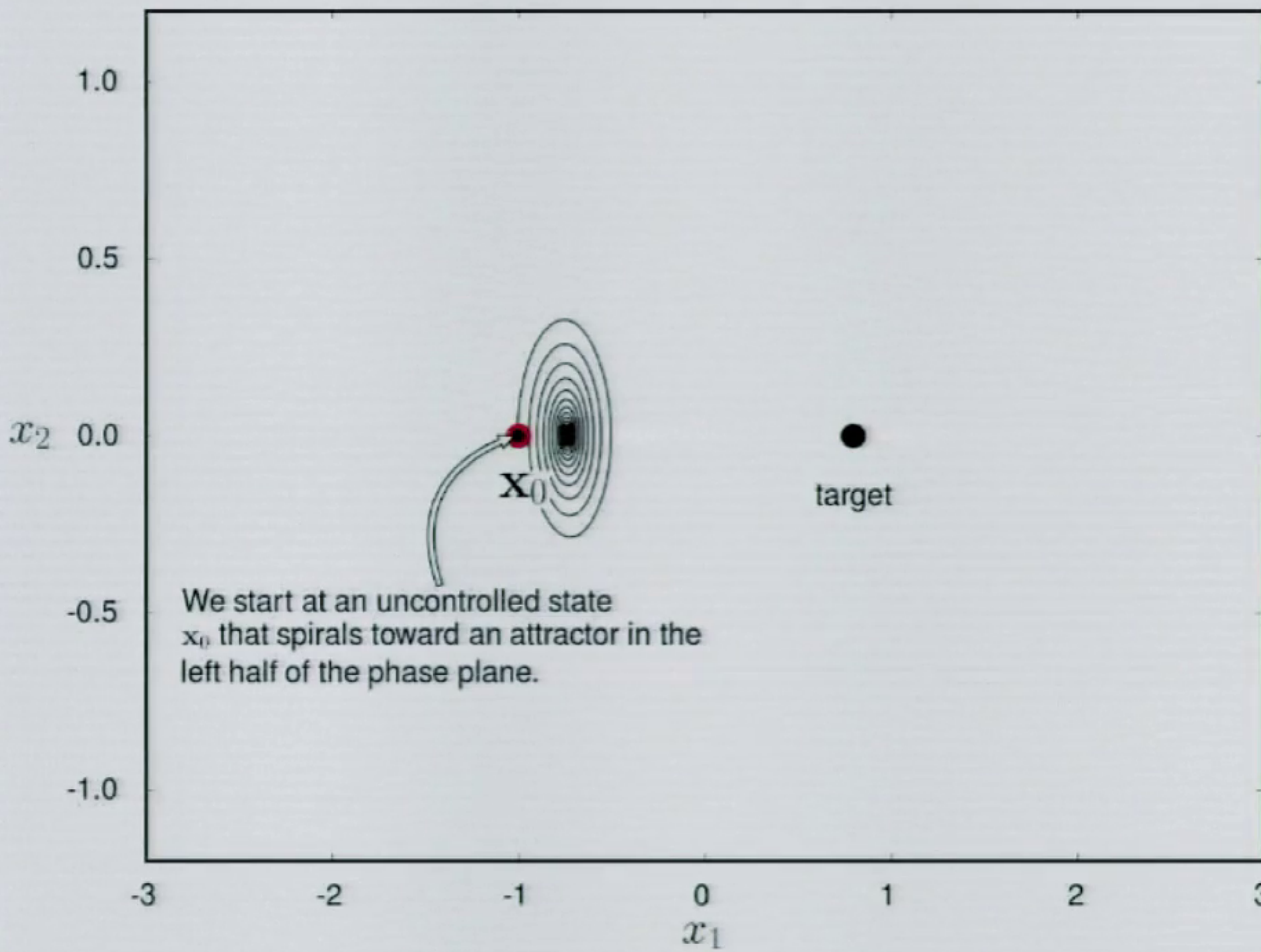
Control of Cascading Processes

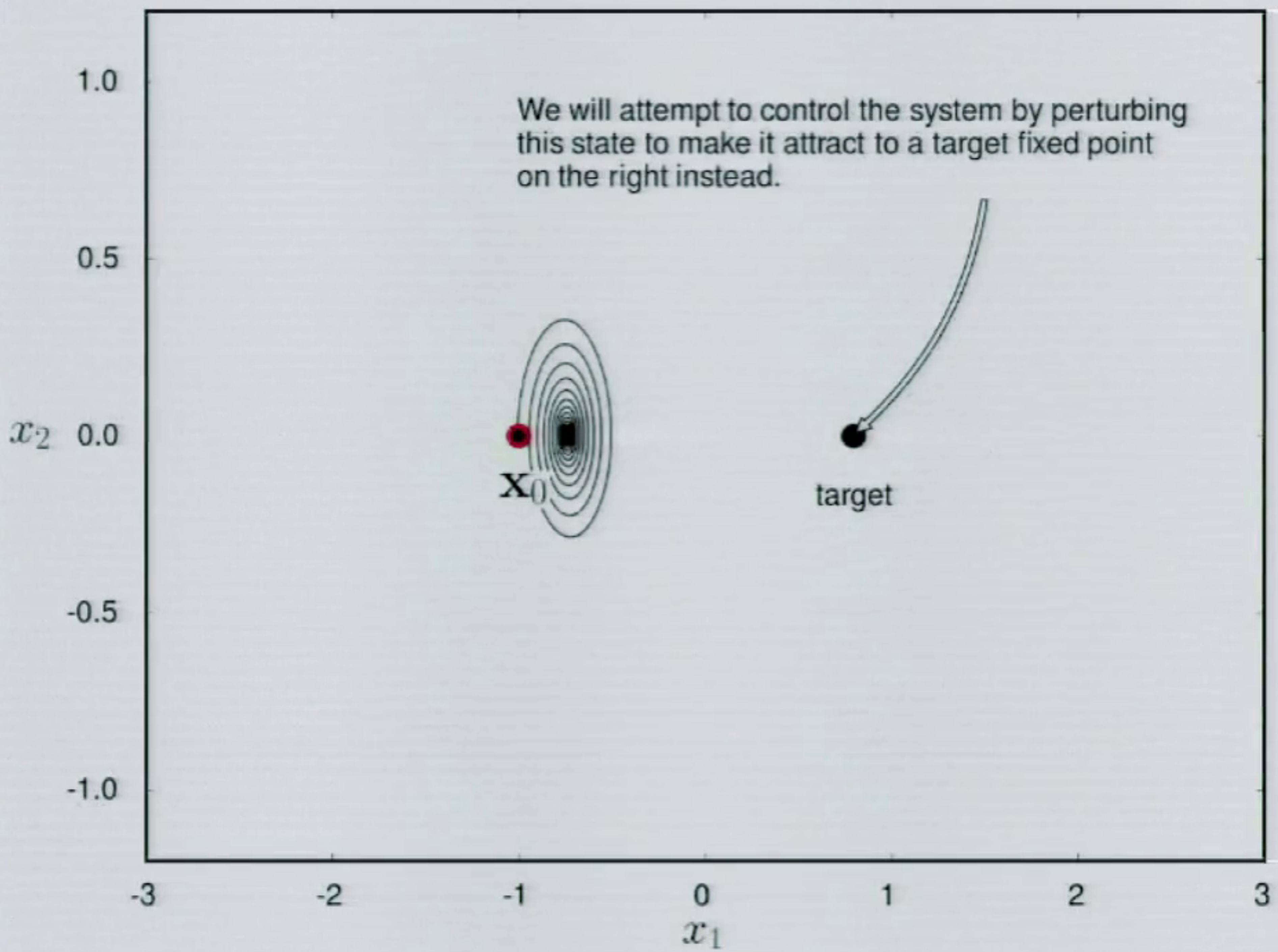


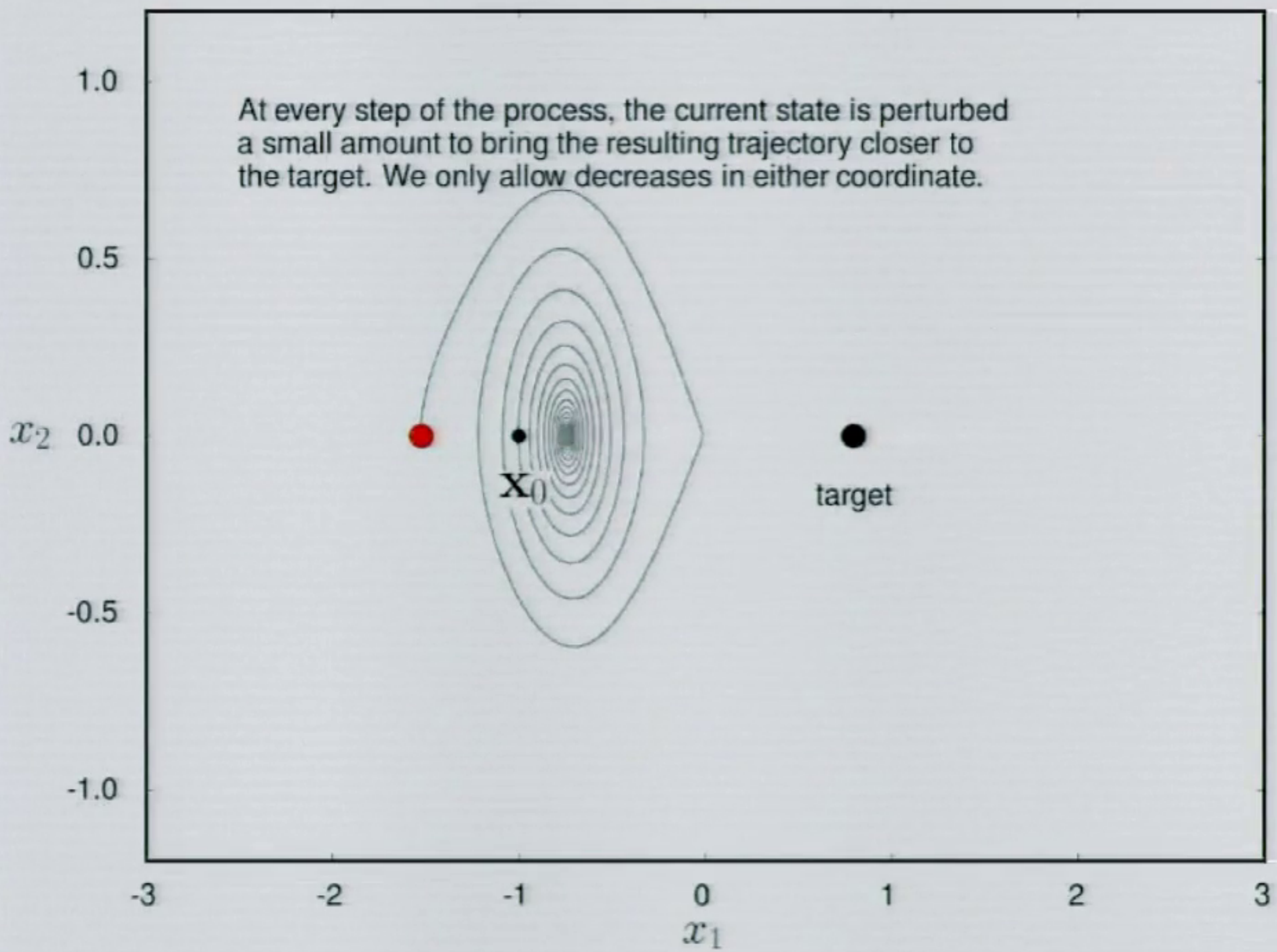
Cornelius, Kath, Motter, Nature Comm 2013

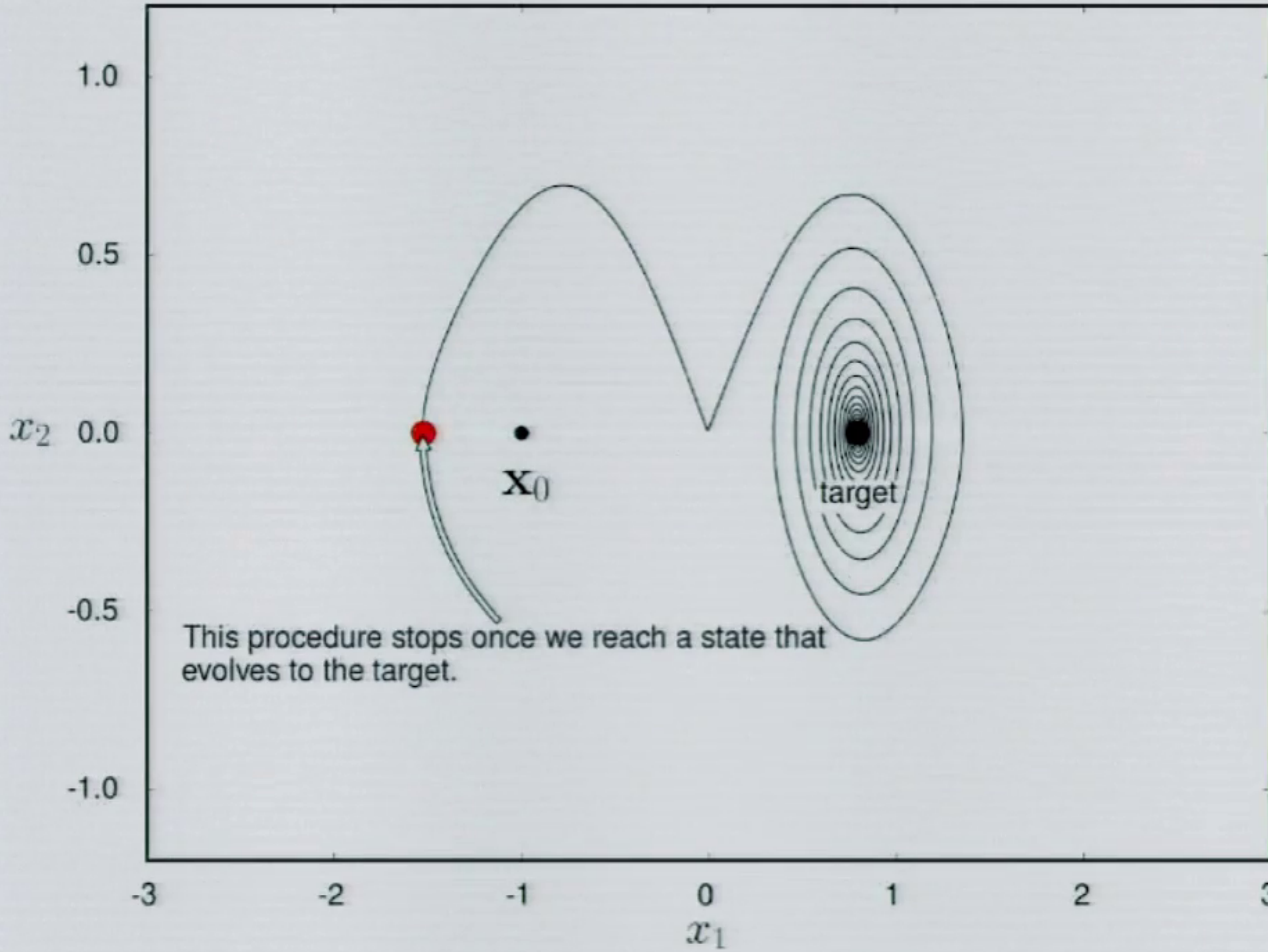


Cornelius and Motter, Protocol Exchange 2013

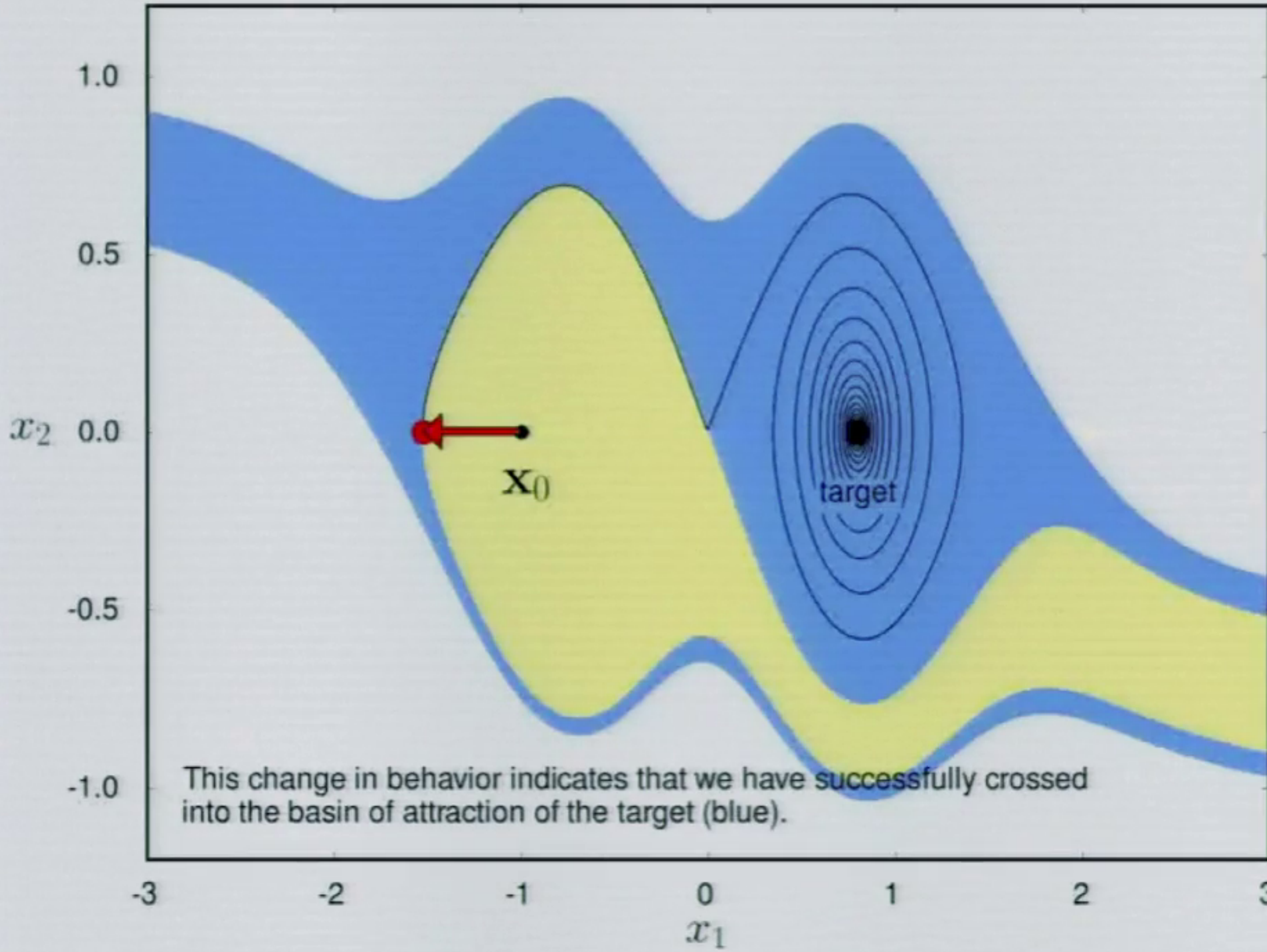

1.0
0.5
x_2 0.0
-0.5
-1.0
-3
-2
-1
0
1
2
3
x_1
$\mathbf{x}_0$
target
We start at an uncontrolled state x_0 that spirals toward an attractor in the left half of the phase plane.


We will attempt to control the system by perturbing this state to make it attract to a target fixed point on the right instead.
target
$\mathbf{x}_0$
x_1
x_2
1.0
0.5
0.0
-0.5
-1.0
-3
-2
-1
0
1
2
3

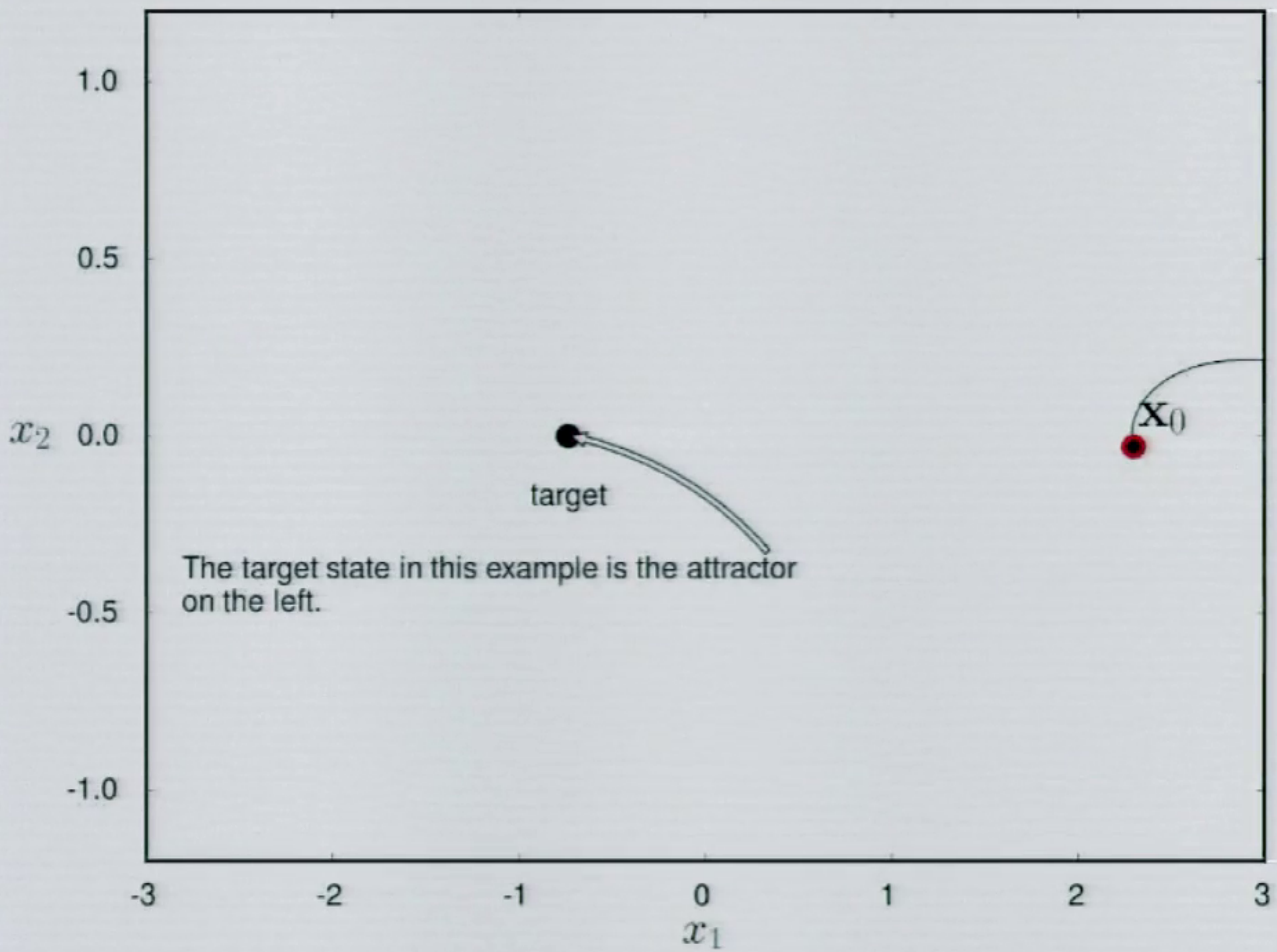

At every step of the process, the current state is perturbed a small amount to bring the resulting trajectory closer to the target. We only allow decreases in either coordinate.
1.0
0.5
0.0
-0.5
-1.0
x_2
$\mathbf{x}_0$
target
-3
-2
-1
0
1
2
3
x_1

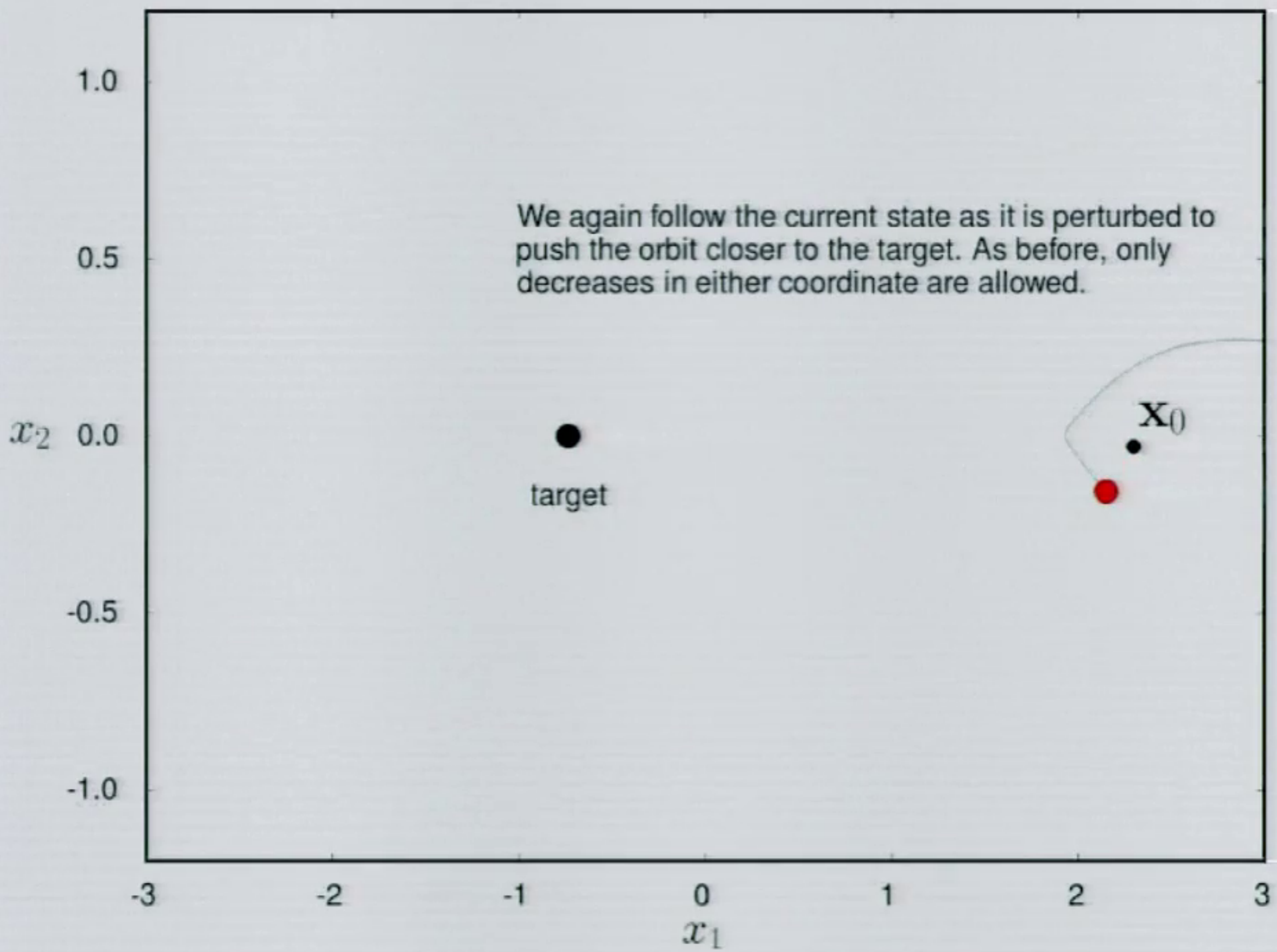


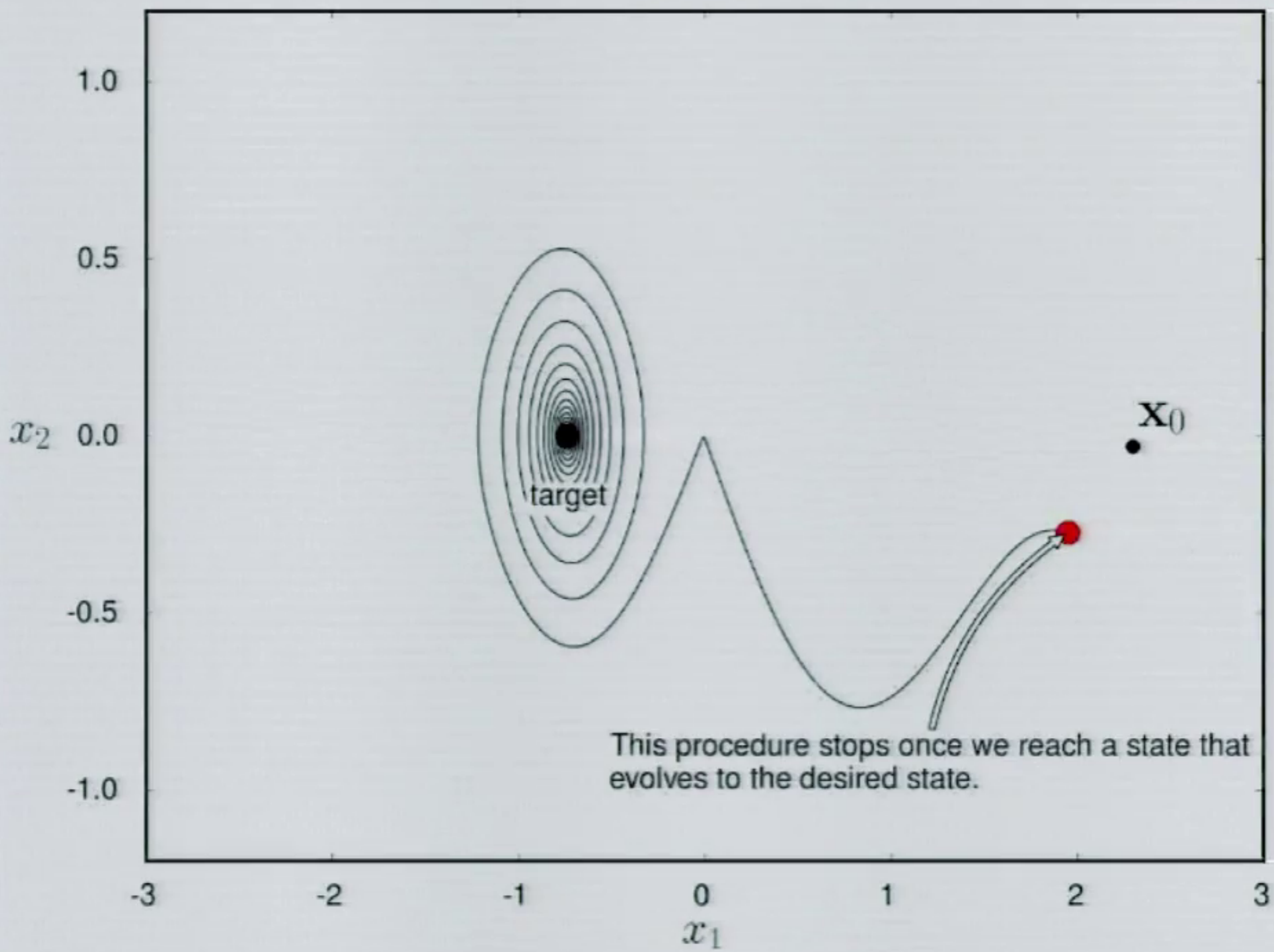
1.0
0.5
x_2 0.0
-0.5
-1.0
-3
-2
-1
0
1
2
3
x_1
$\mathbf{x}_0$
target
This procedure stops once we reach a state that evolves to the target.



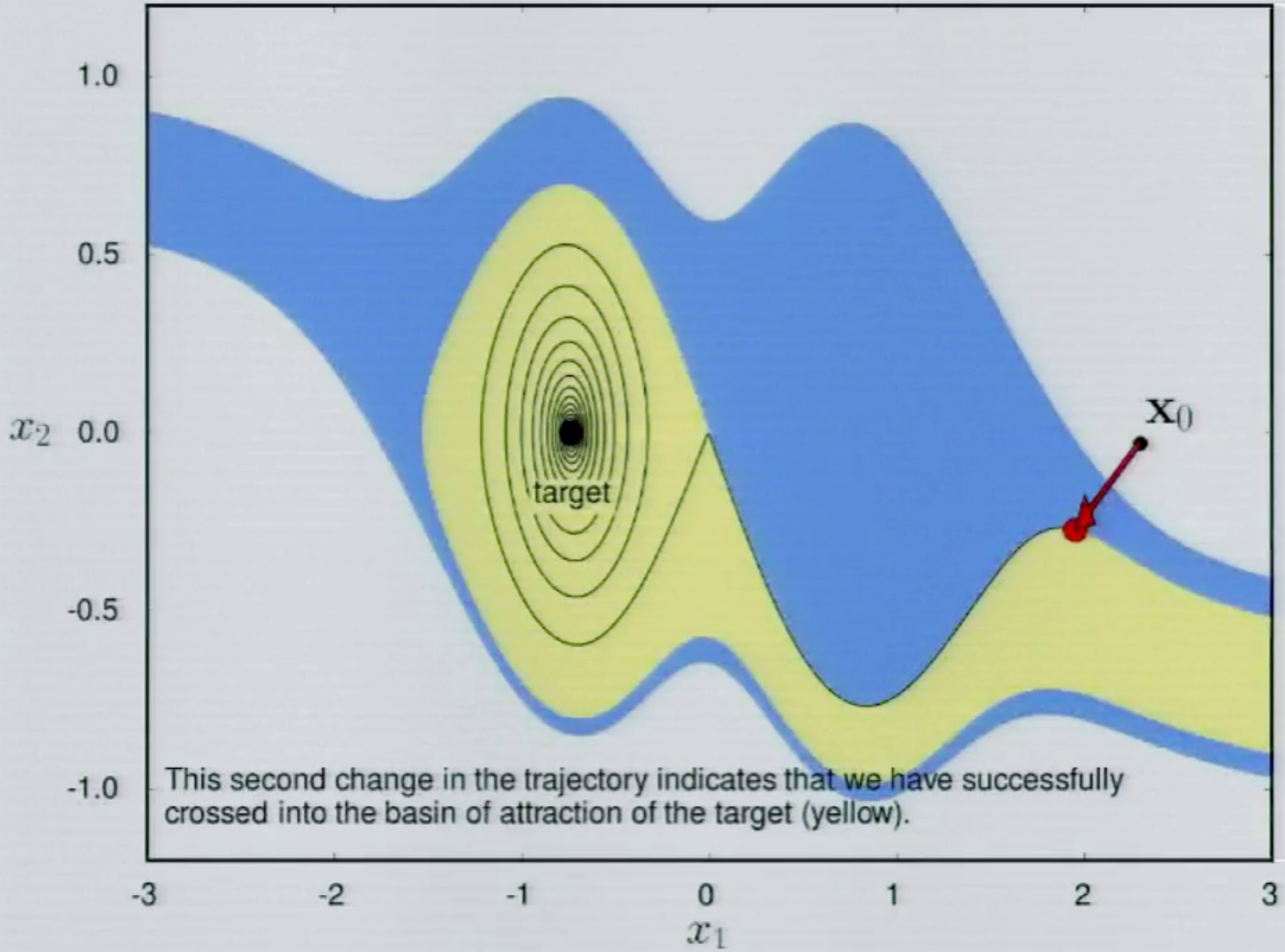
1.0
0.5
0.0
-0.5
-1.0
x_2
$\mathbf{x}_0$
target
This change in behavior indicates that we have successfully crossed into the basin of attraction of the target (blue).
-3
-2
-1
0
1
2
3
x_1


1.0
0.5
x_2 0.0
-0.5
-1.0
-3
-2
-1
0
1
2
3
x_1
$\mathbf{x}_0$
target
The target state in this example is the attractor on the left.

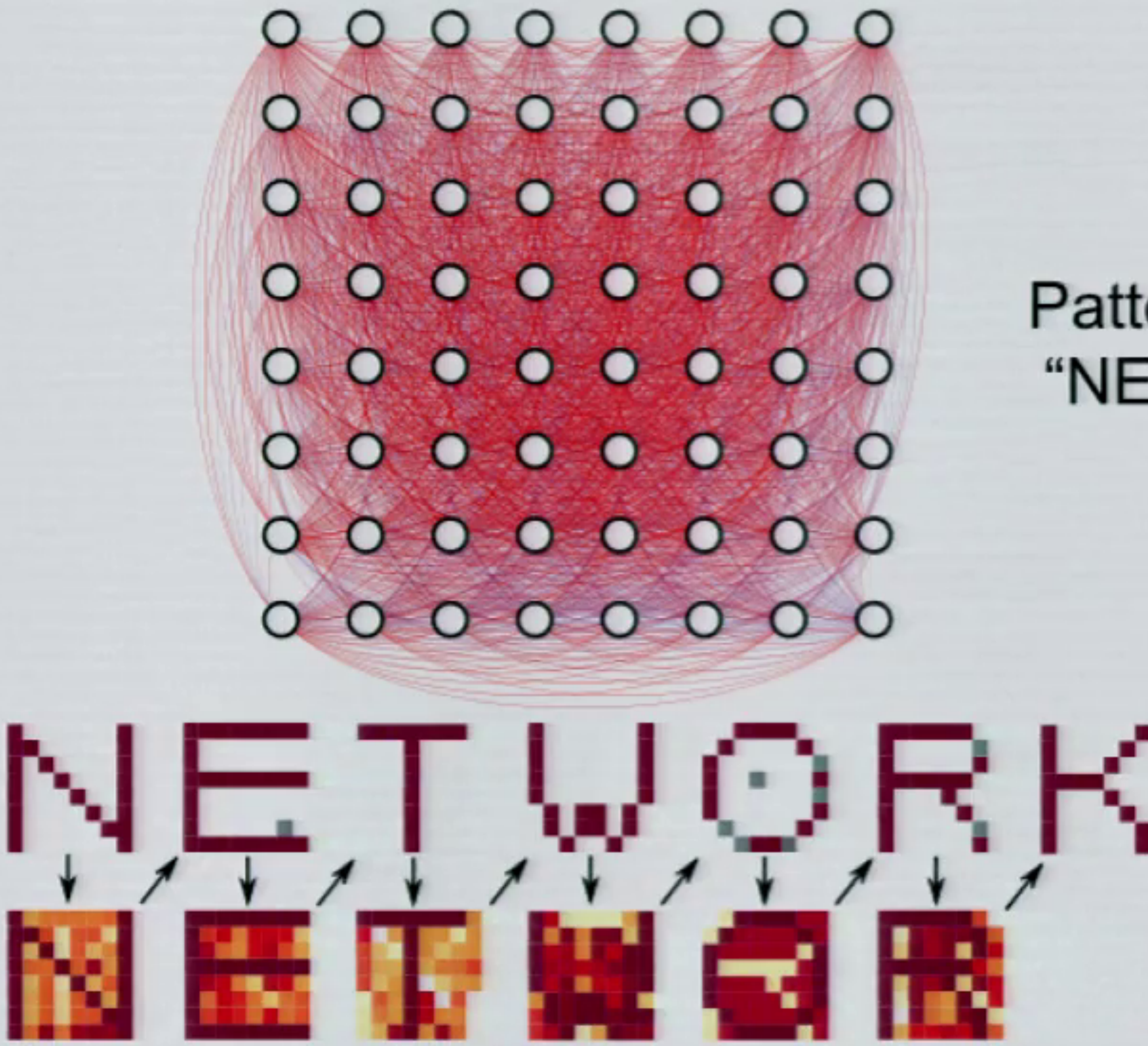

We again follow the current state as it is perturbed to push the orbit closer to the target. As before, only decreases in either coordinate are allowed.
1.0
0.5
0.0
-0.5
-1.0
x_2
$\mathbf{x}_0$
target
-3
-2
-1
0
1
2
3
x_1



1.0
0.5
x_2 0.0
-0.5
-1.0
-3
-2
-1
0
1
2
3
x_1
target
$\mathbf{x}_0$
This procedure stops once we reach a state that evolves to the desired state.


1.0
0.5
x_2 0.0
-0.5
-1.0
-3
-2
-1
0
1
2
3
x_1
target
$\mathbf{x}_0$
This second change in the trajectory indicates that we have successfully crossed into the basin of attraction of the target (yellow).

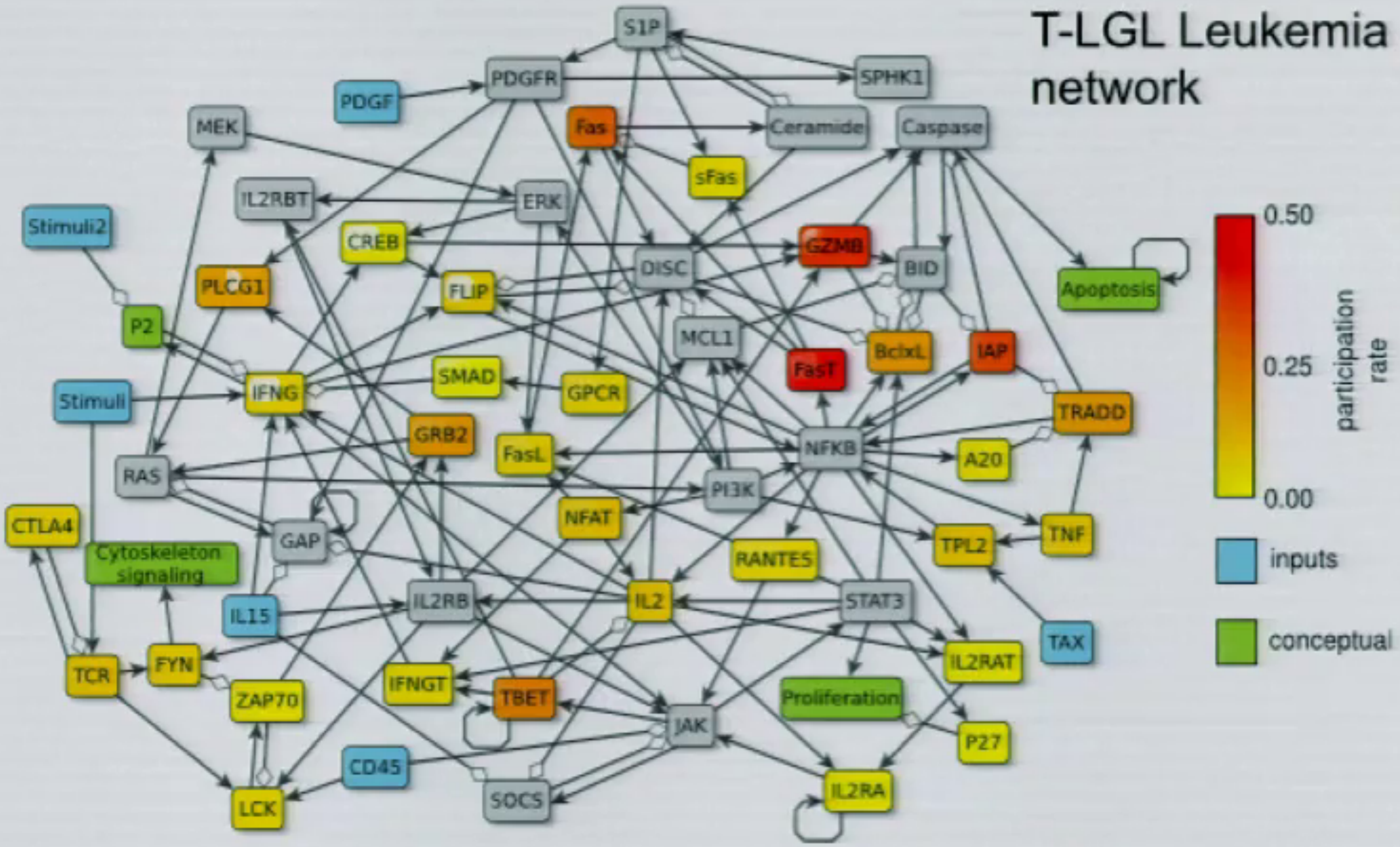
Application to Associative-Memory Networks



Pattern stored
"NETWORK"

Cornelius, Kath, Motter, Nature Comm 2013

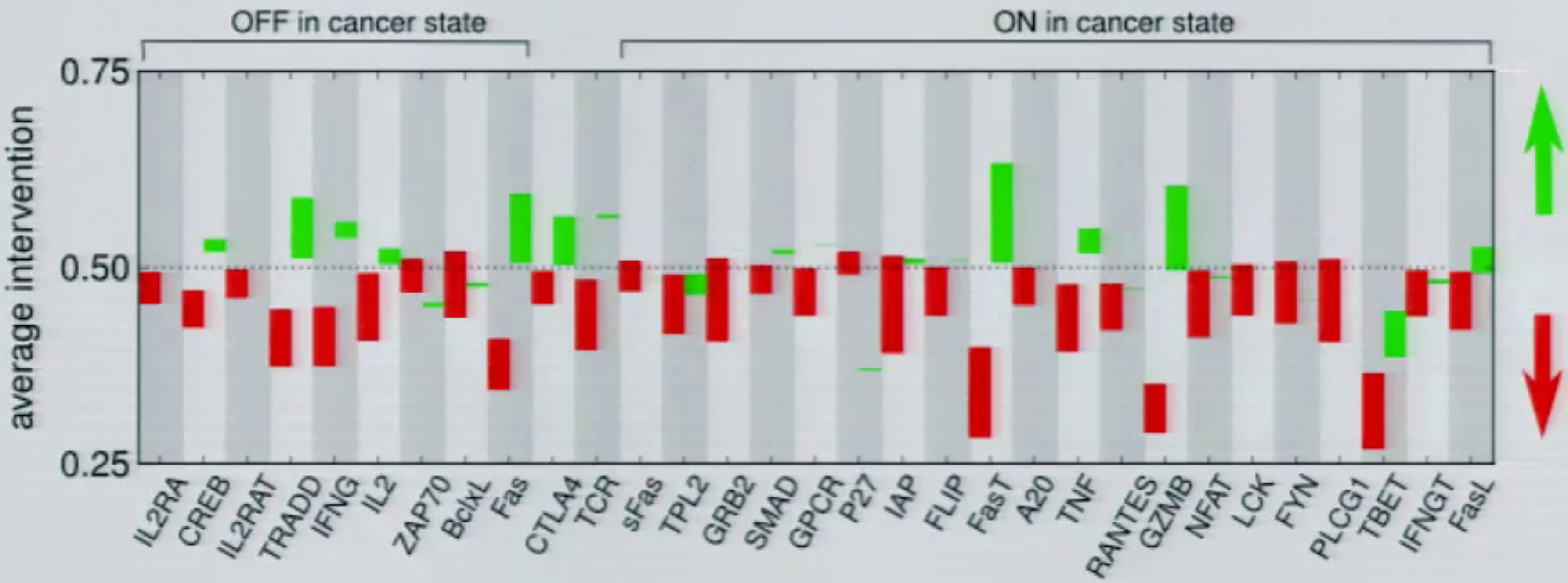
Application to the Identification of Therapeutic Interventions



T-LGL Leukemia signaling network

Cornelius, Kath, Motter, Nature Comm 2013

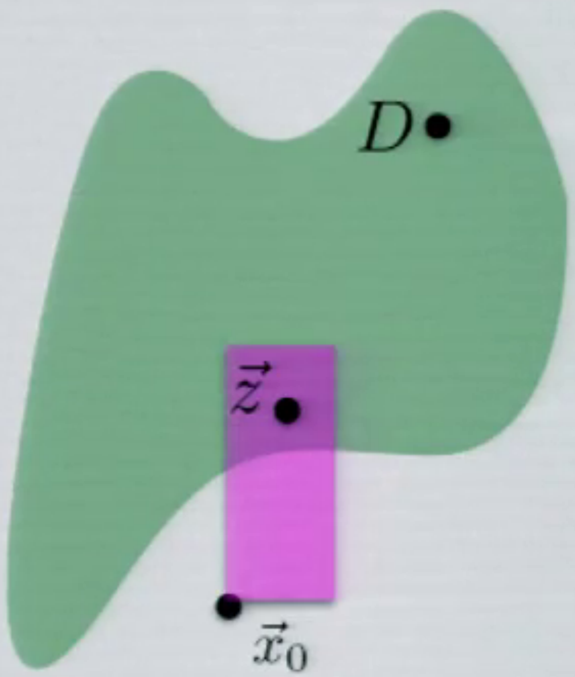
Application to the Identification of Therapeutic Interventions

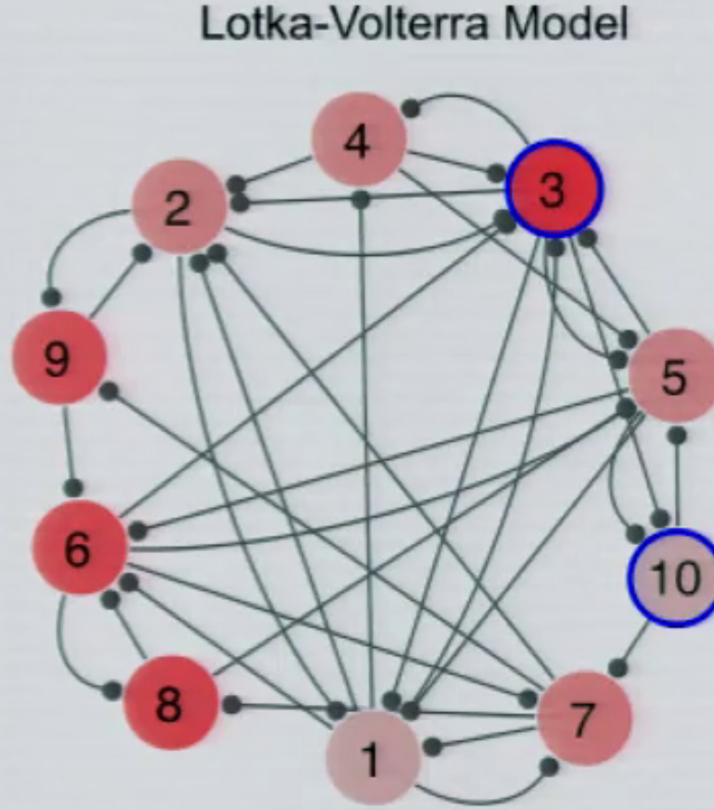


Size and orientation of interventions

Cornelius, Kath, Motter, Nature Comm 2013

Scheduling of Interventions

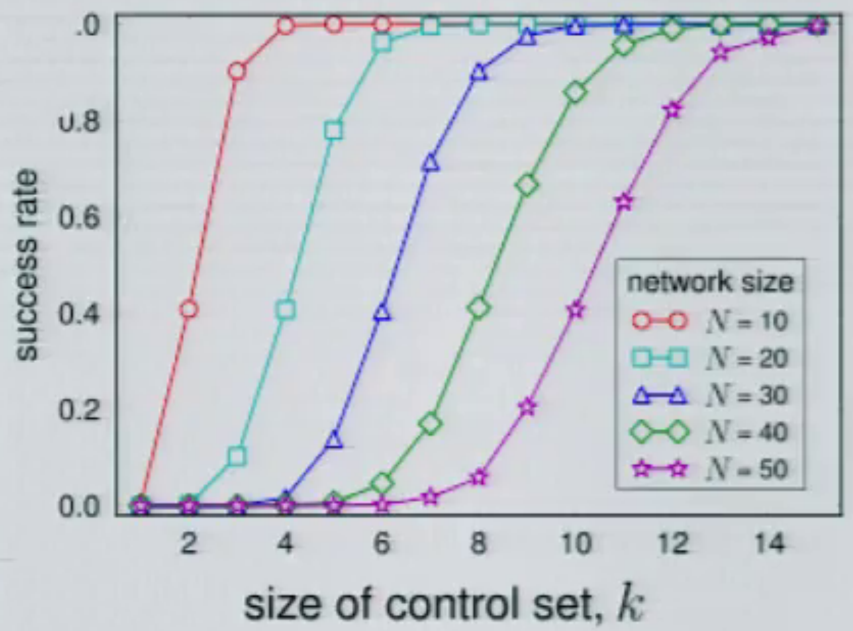


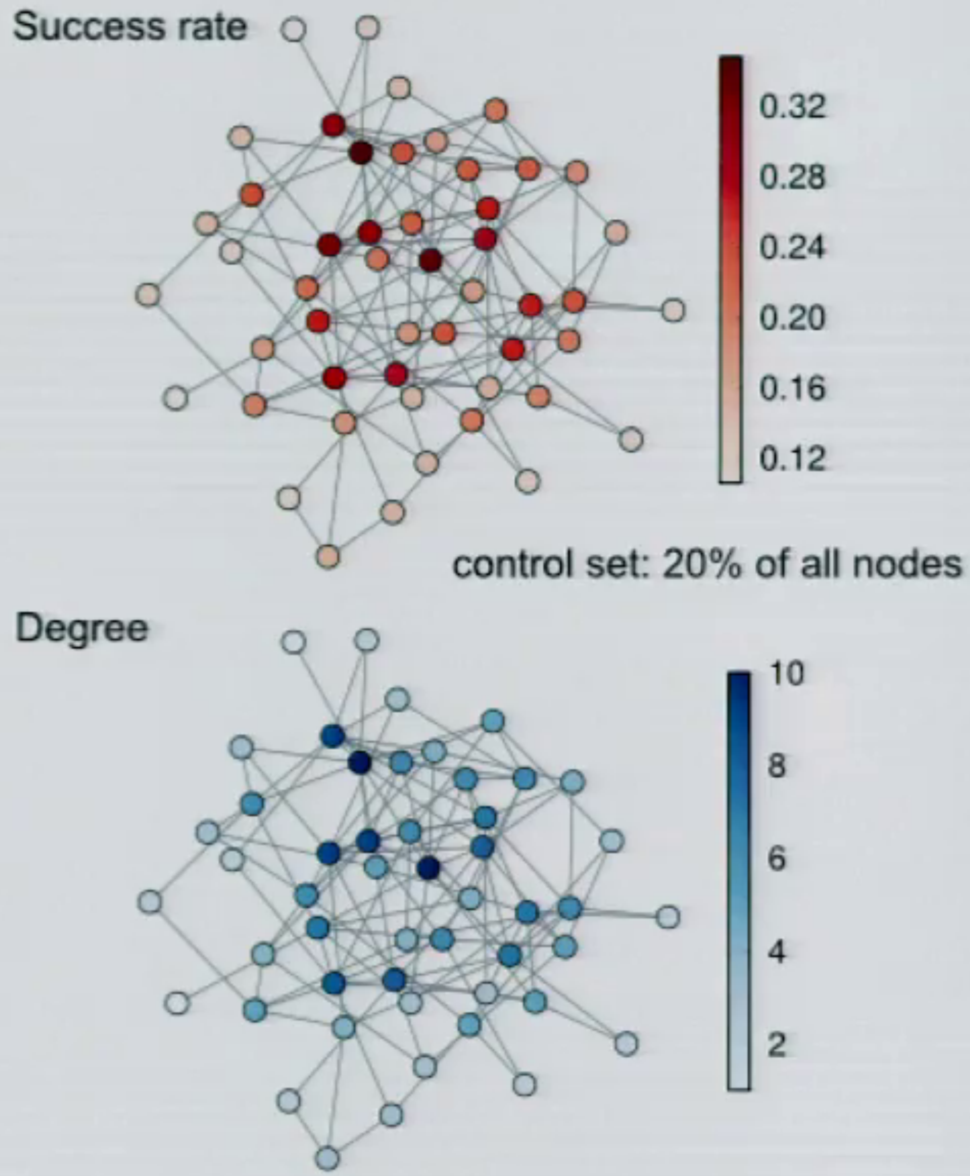


Sun, Sahasrabudhe, Motter 2015

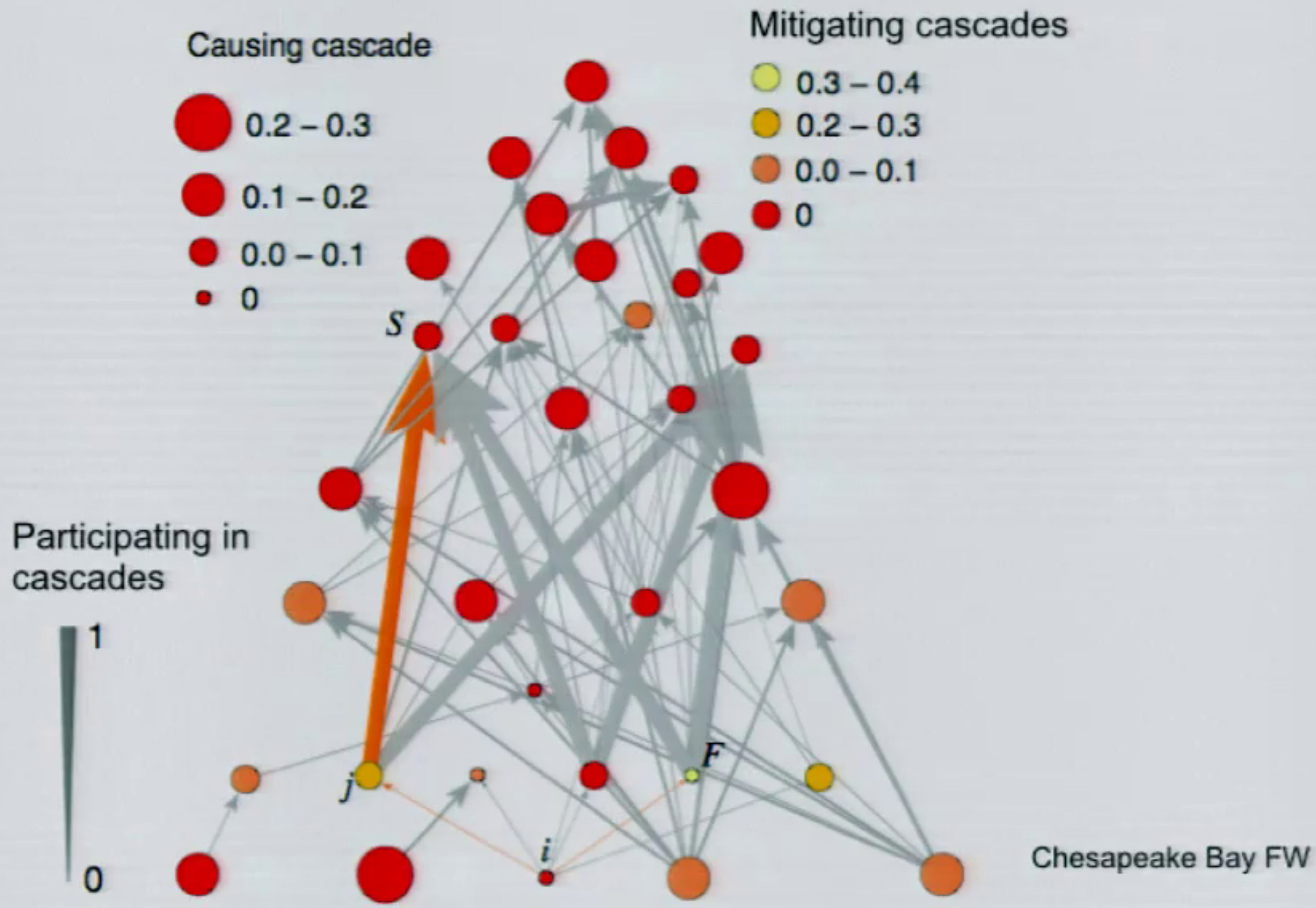
Relations with Network Structure

Randomly selected nodes





Mitigation of extinctions in food webs



Sahasrabudhe, Motter, Nature Comm 2011

Extensions

Continuous interventions, e.g., formulated in terms of optimal control

Closed-loop control benefiting for real-time feedback

Scenarios in which the state of the system is only partially known

Aleksandar Haber

Model-independent approaches, based entirely on data

Thomas Wytock

Control based on parameter manipulation

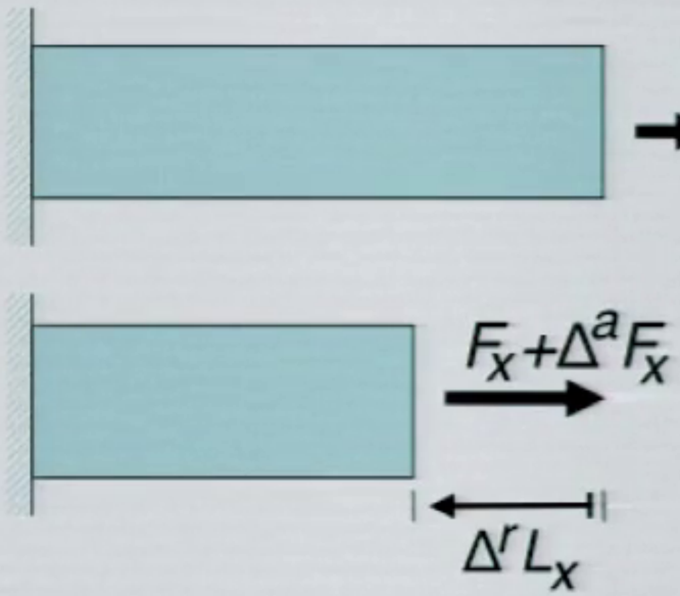
Ferenc Molnar: NS15 Sun 14:30-15:00 PM

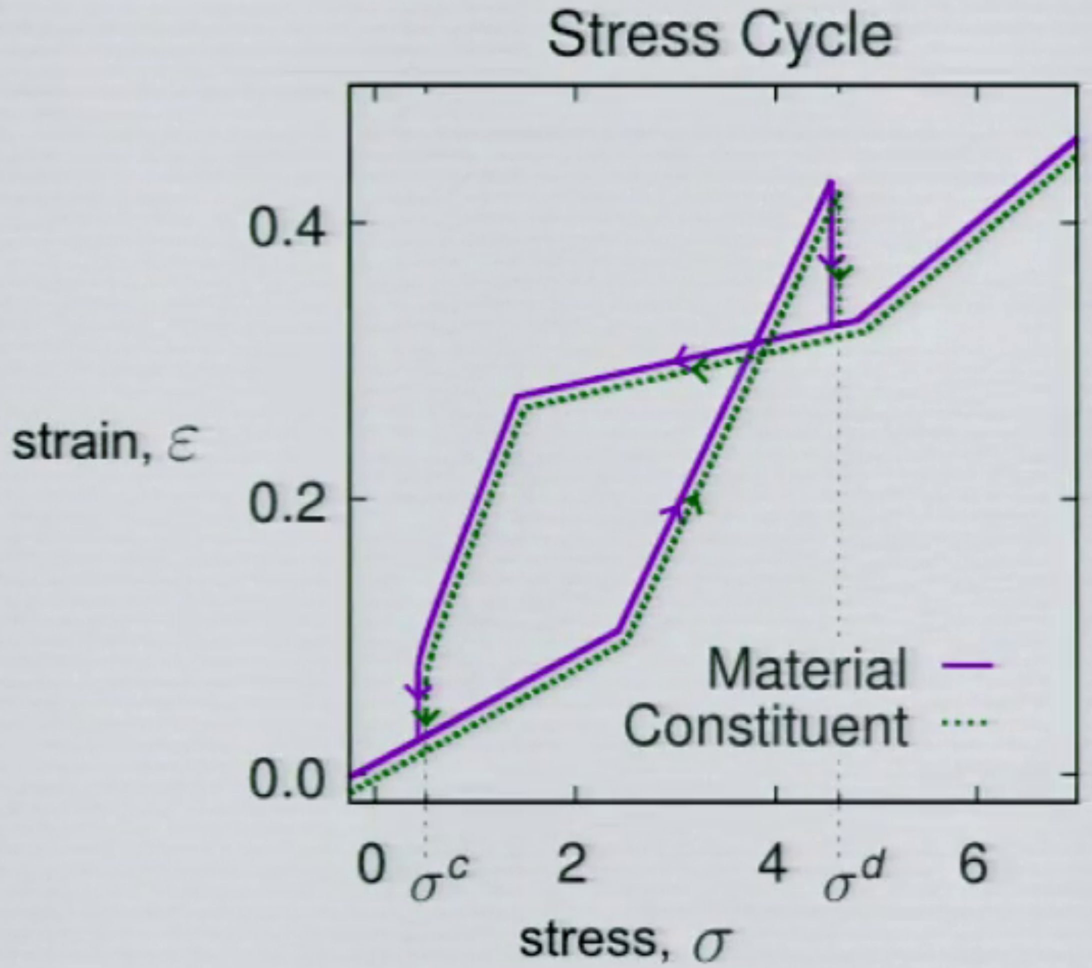
Control of noise-induced transitions

Daniel Wells: CP16 Mon 3:45-4:00 PM

Control "by Design"

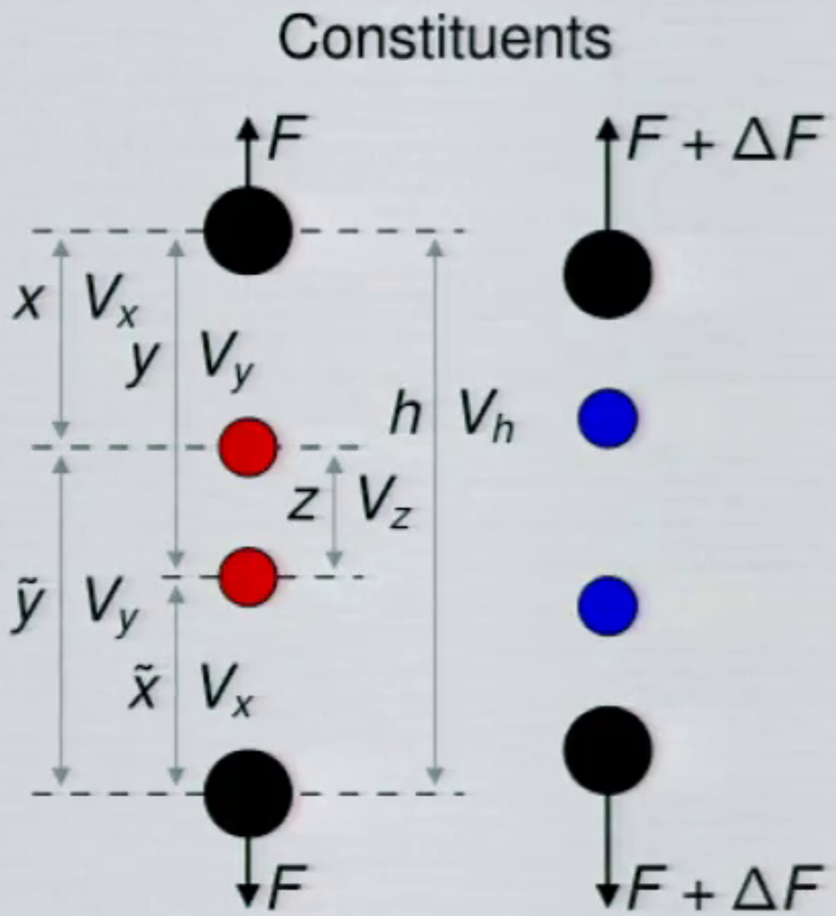
The example of negative compressibility transitions





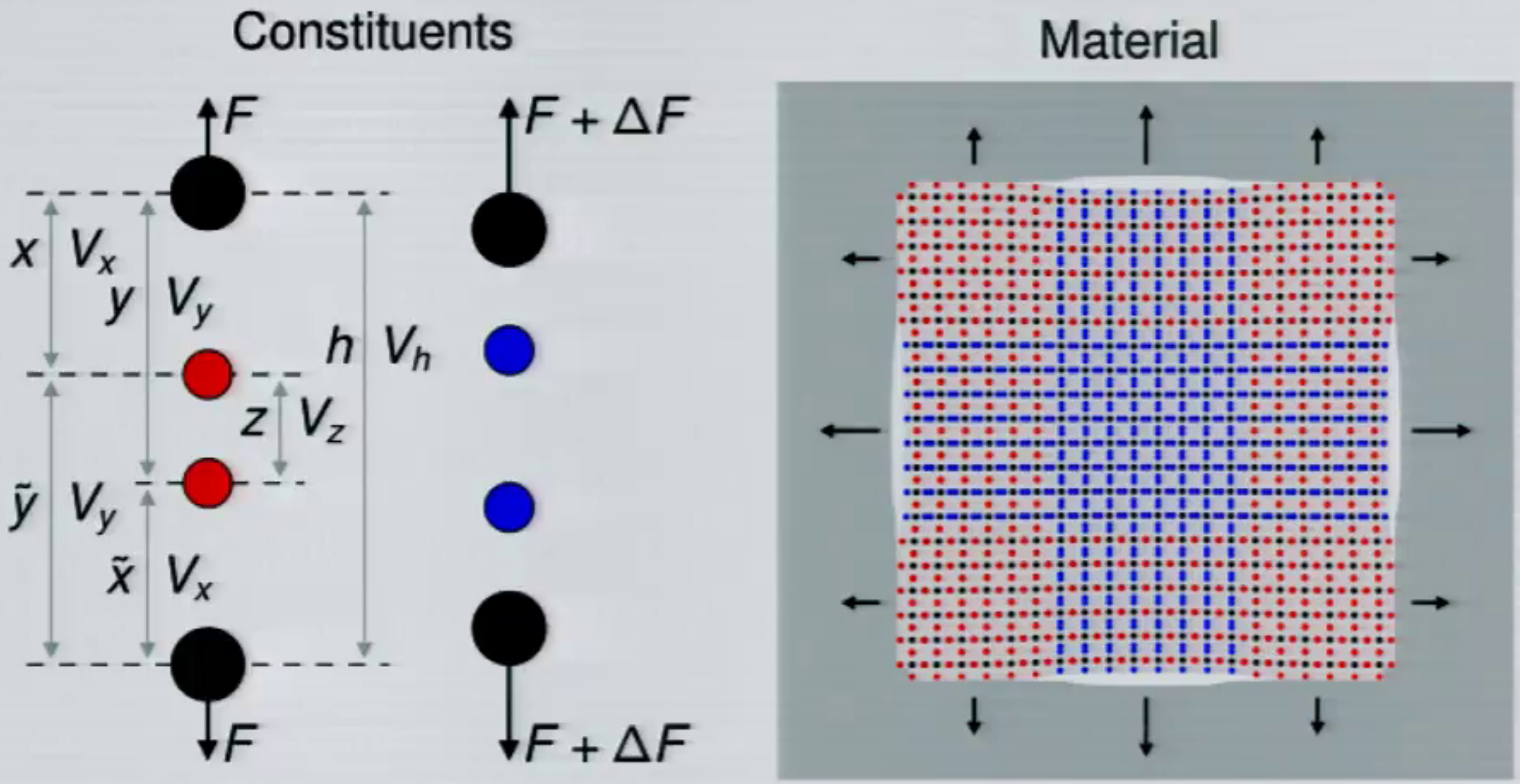
Nicolaou and Motter, Nature Materials 2012

Negative compressibility transitions

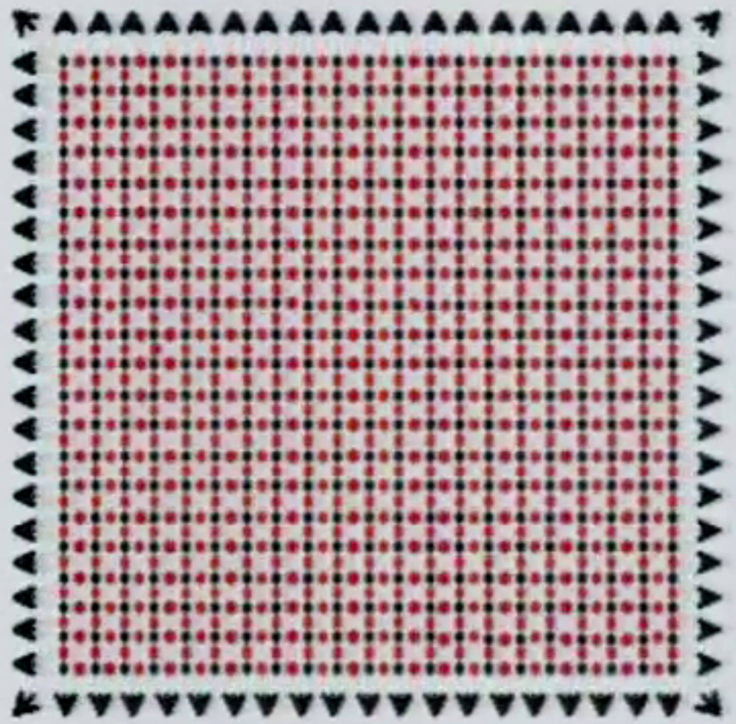


Nicolaou and Motter, Nature Materials 2012

Negative compressibility transitions



Nicolaou and Motter, Nature Materials 2012

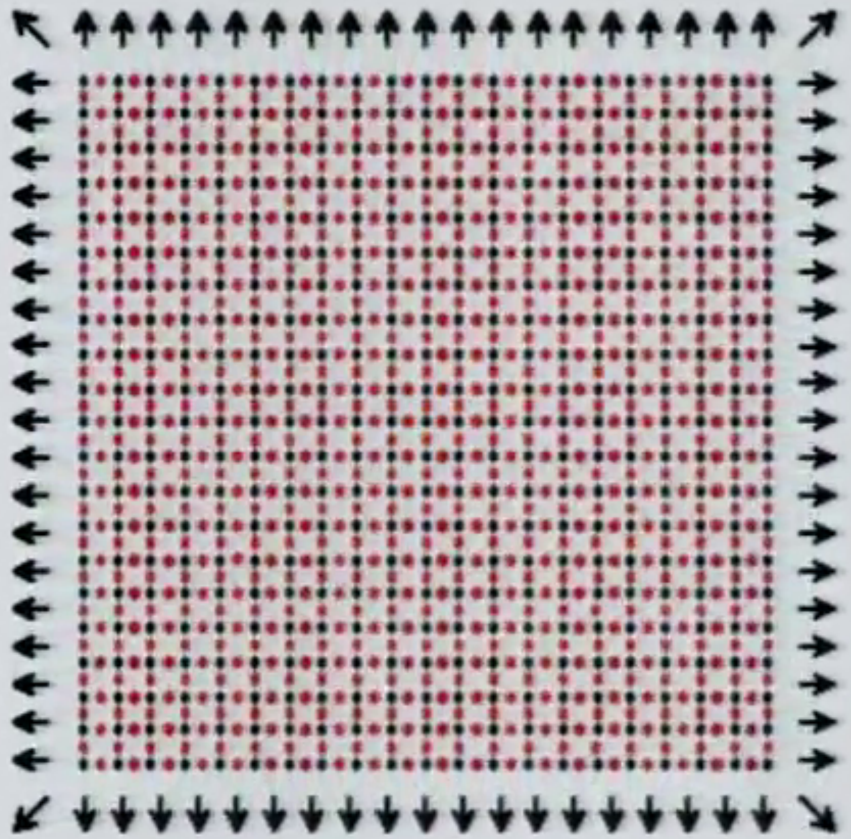


20×20 unit cells

temperature $T = 10^{-4} T_{0}$

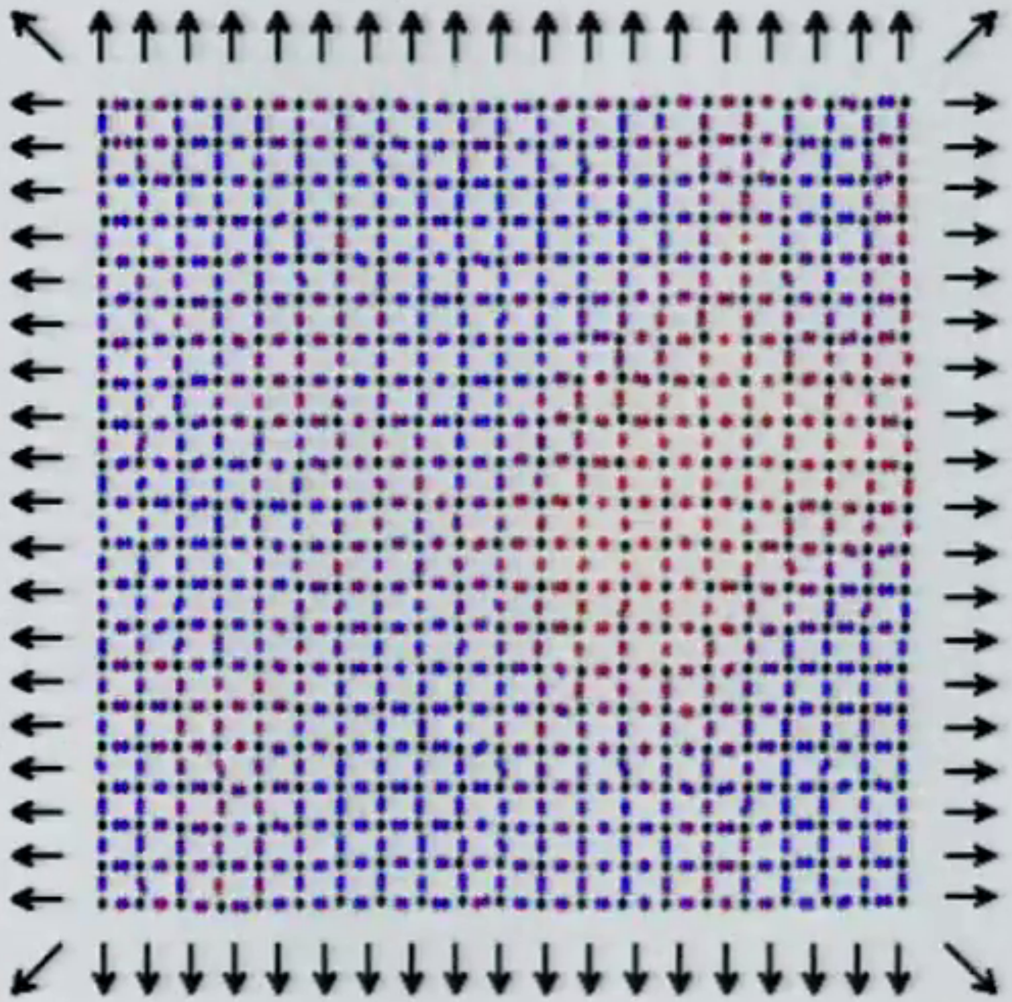
cycle time $\tau = 10^{1.5} \tau_{V}$

Nicolaou and Motter, Nature Materials 2012



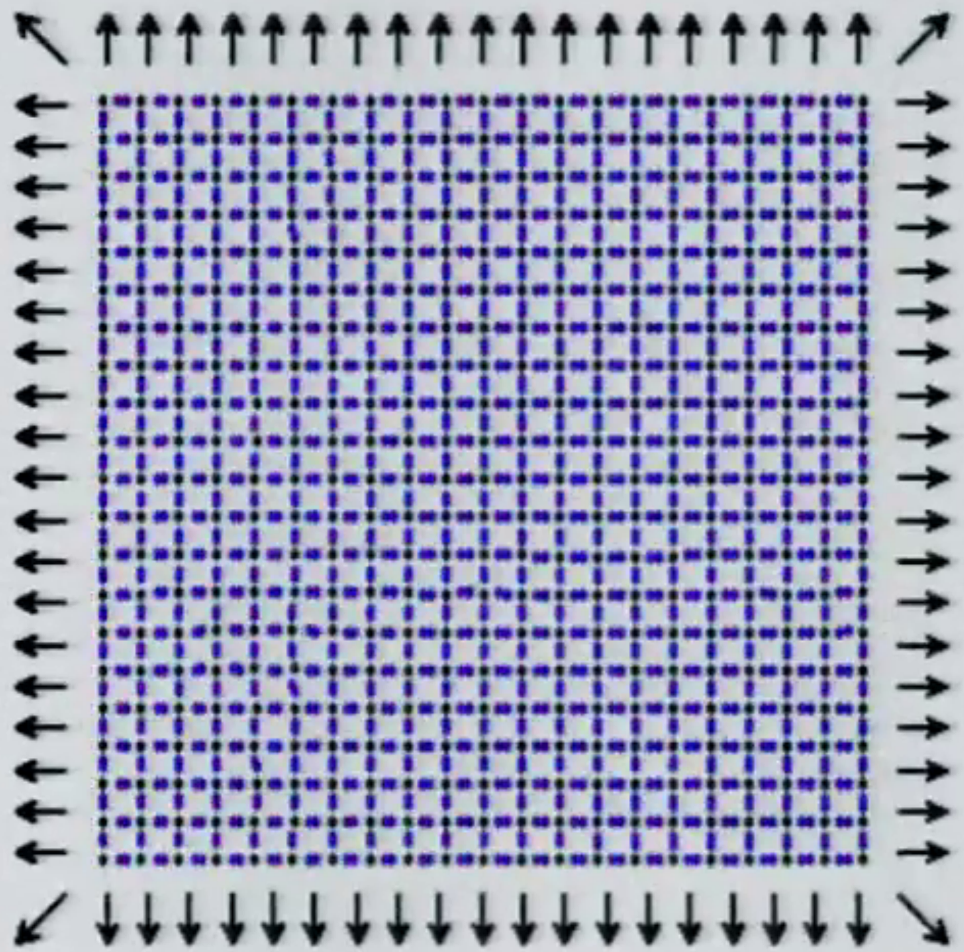
20×20 unit cells
temperature $T = 10^{-4}T_0$
cycle time $\tau = 10^{1.5}\tau_V$

Nicolaou and Motter, Nature Materials 2012



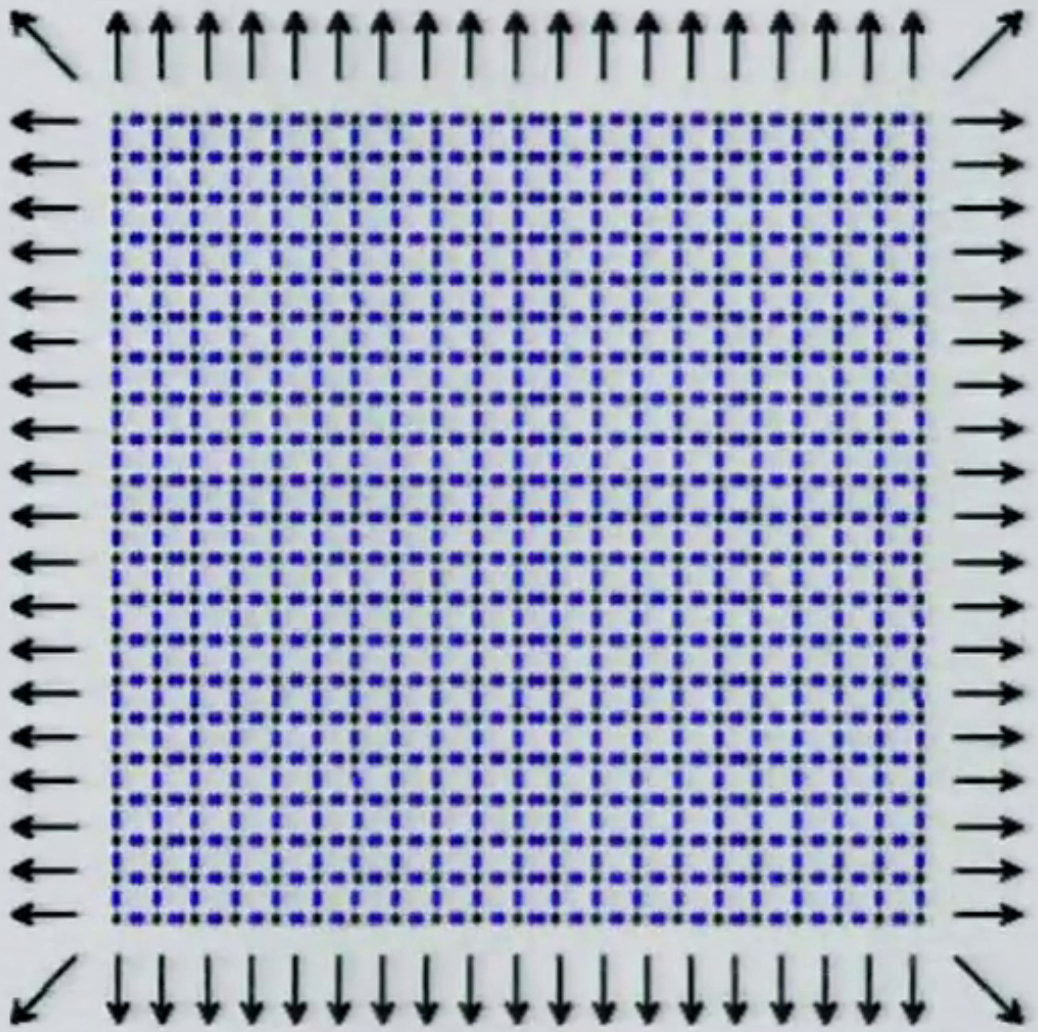
20×20 unit cells
temperature $T = 10^{-4}T_0$
cycle time $\tau = 10^{1.5}\tau_V$

Nicolaou and Motter, Nature Materials 2012



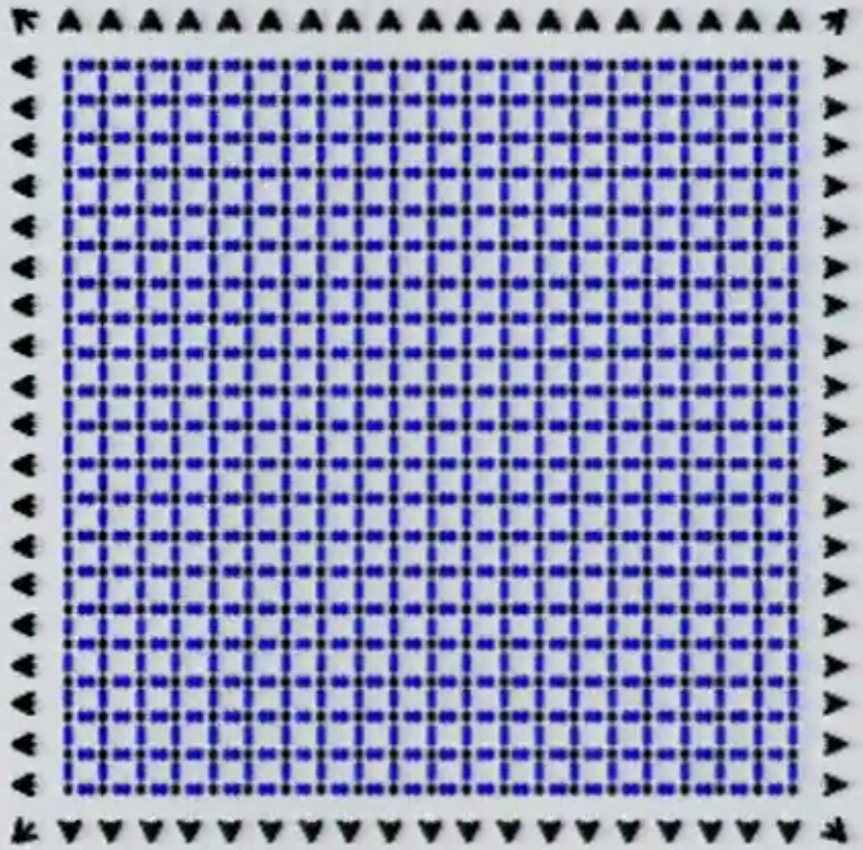
20×20 unit cells
temperature $T = 10^{-4}T_0$
cycle time $\tau = 10^{1.5}\tau_V$

Nicolaou and Motter, Nature Materials 2012



20×20 unit cells
temperature $T = 10^{-4} T_{0}$
cycle time $\tau = 10^{1.5} \tau_{V}$

Nicolaou and Motter, Nature Materials 2012



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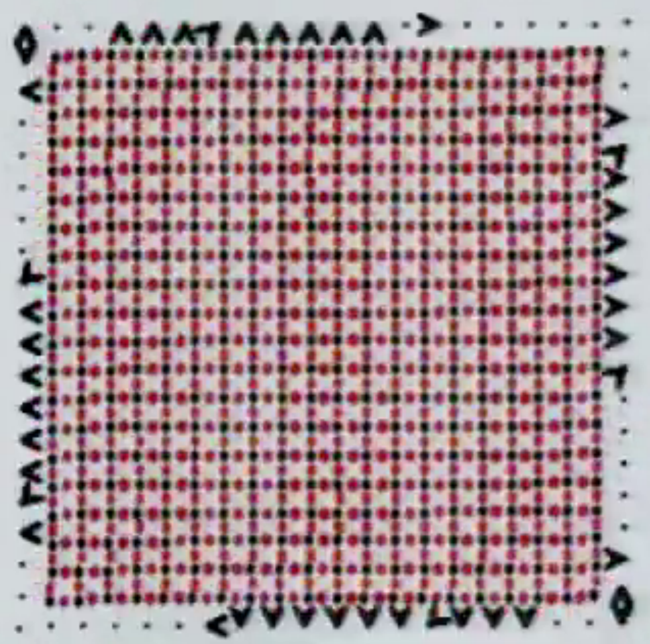

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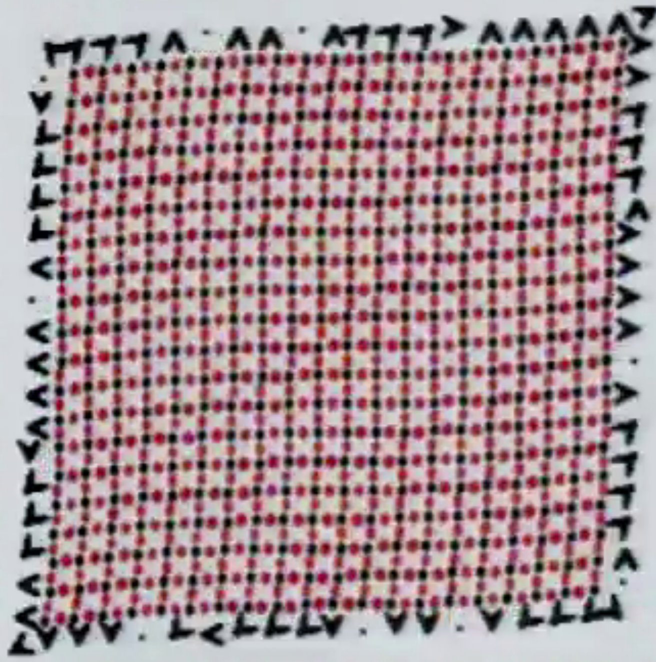

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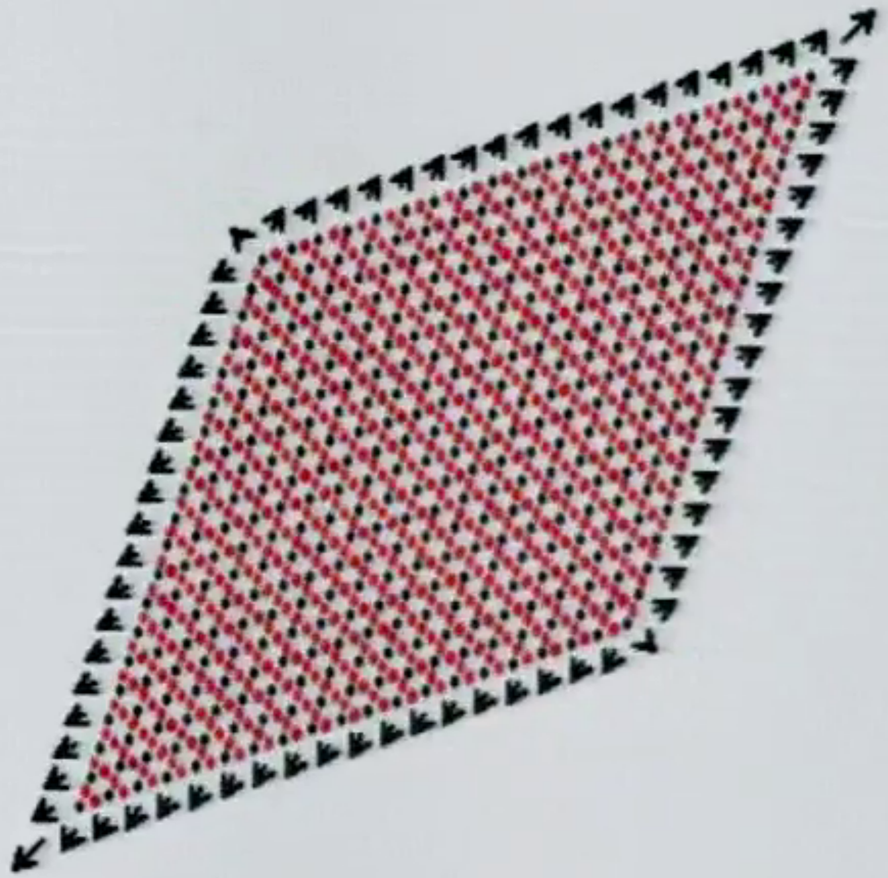

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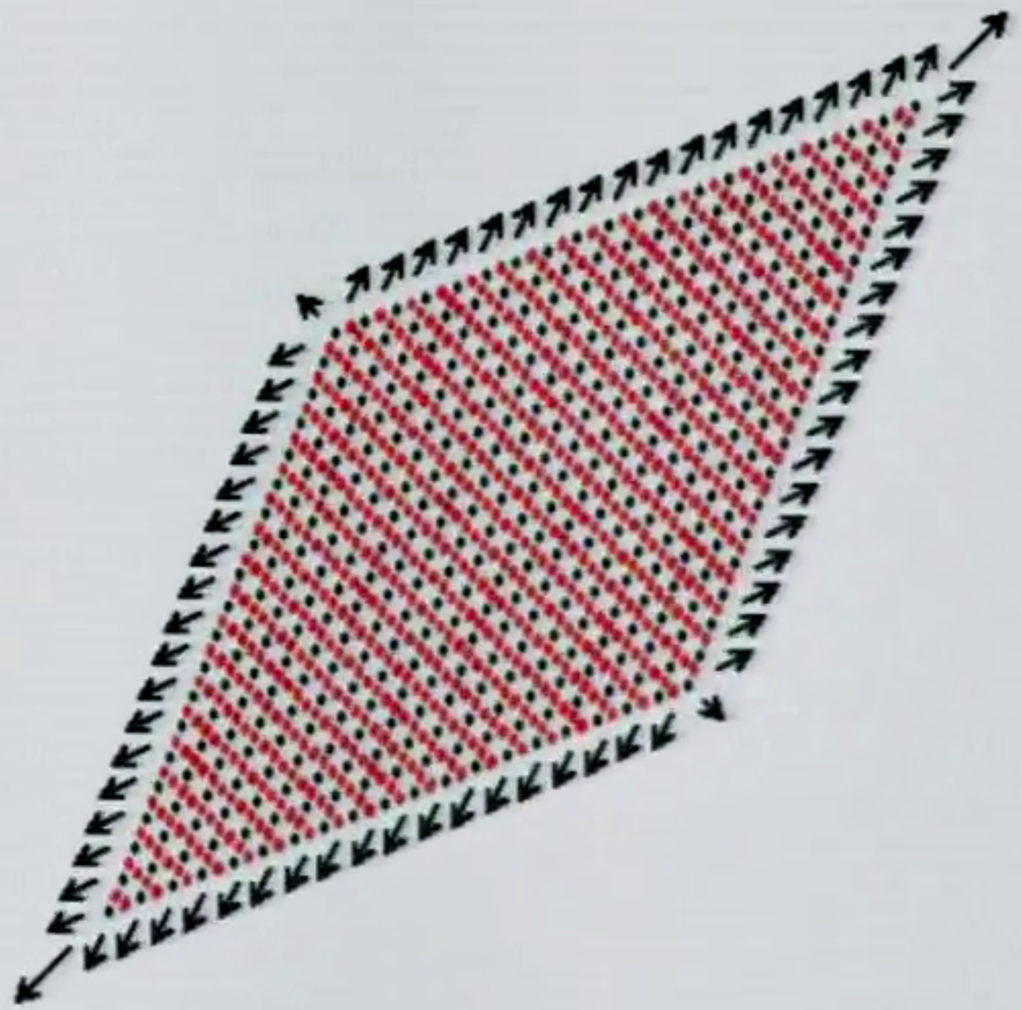

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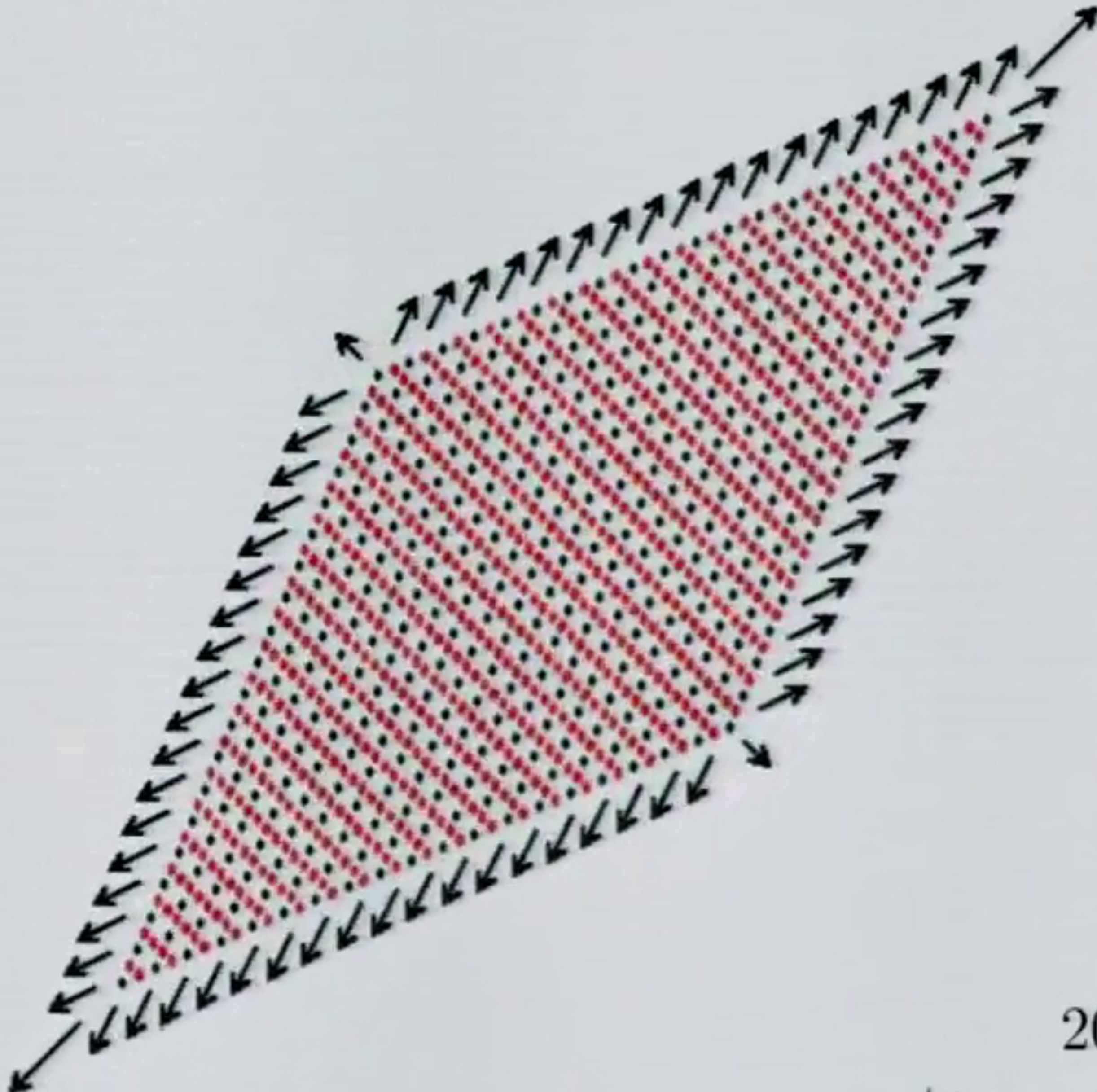

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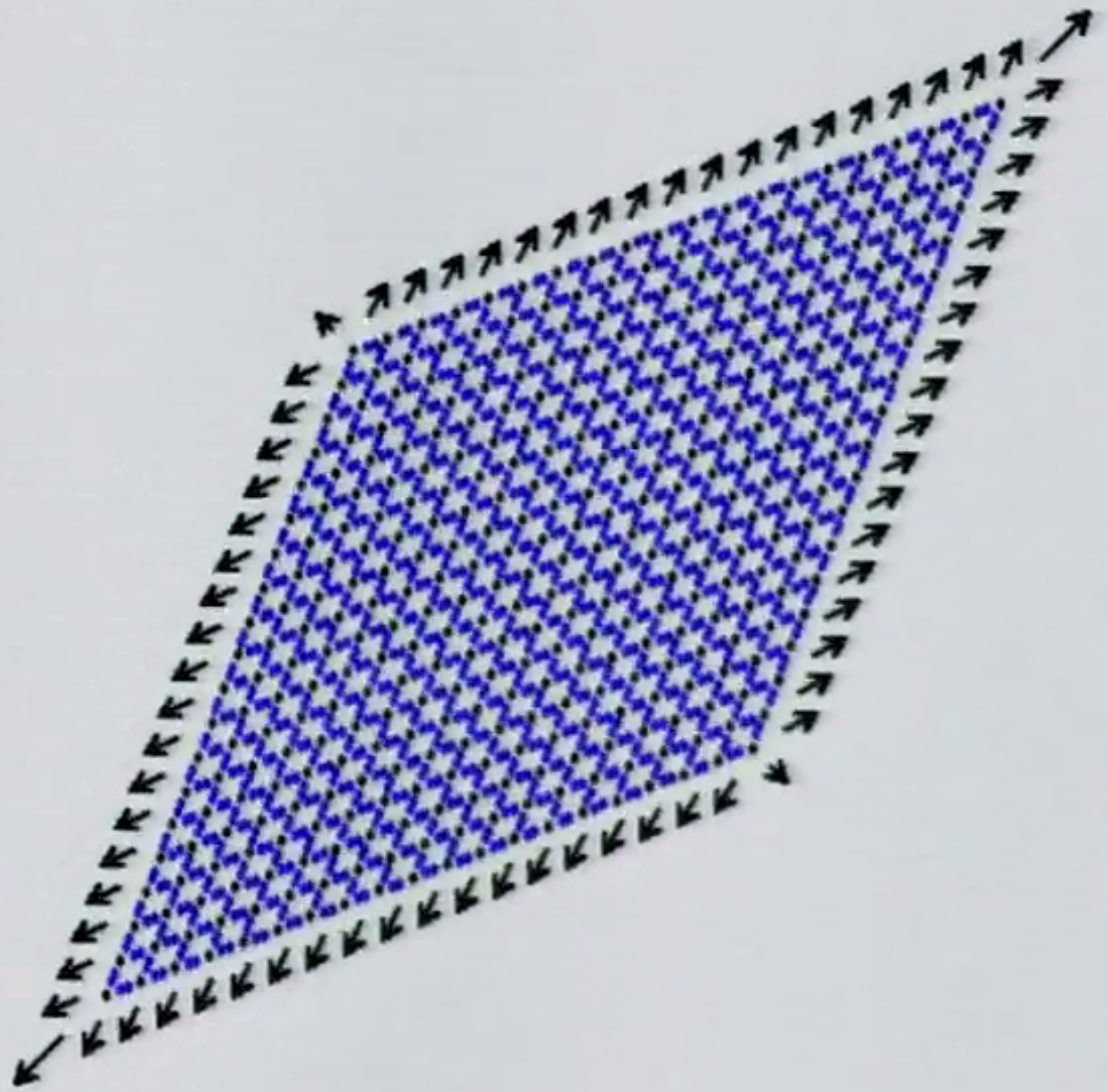

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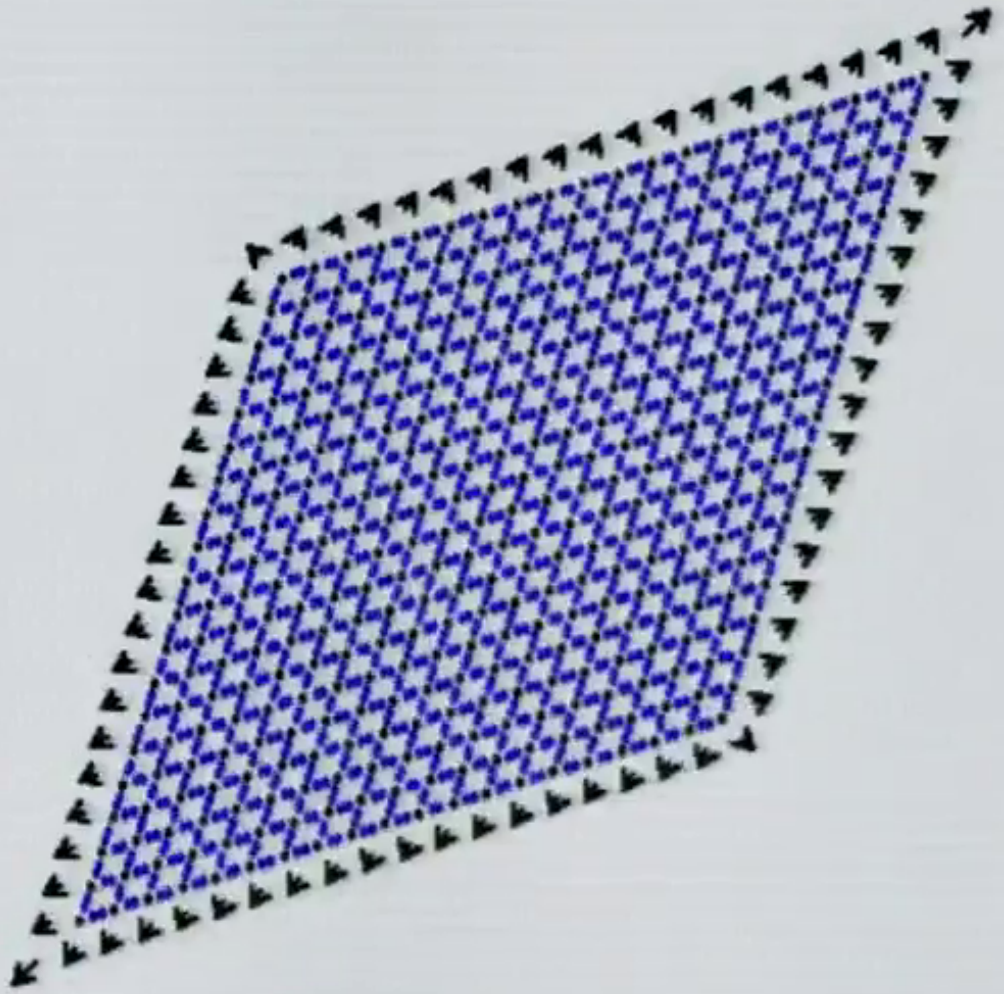

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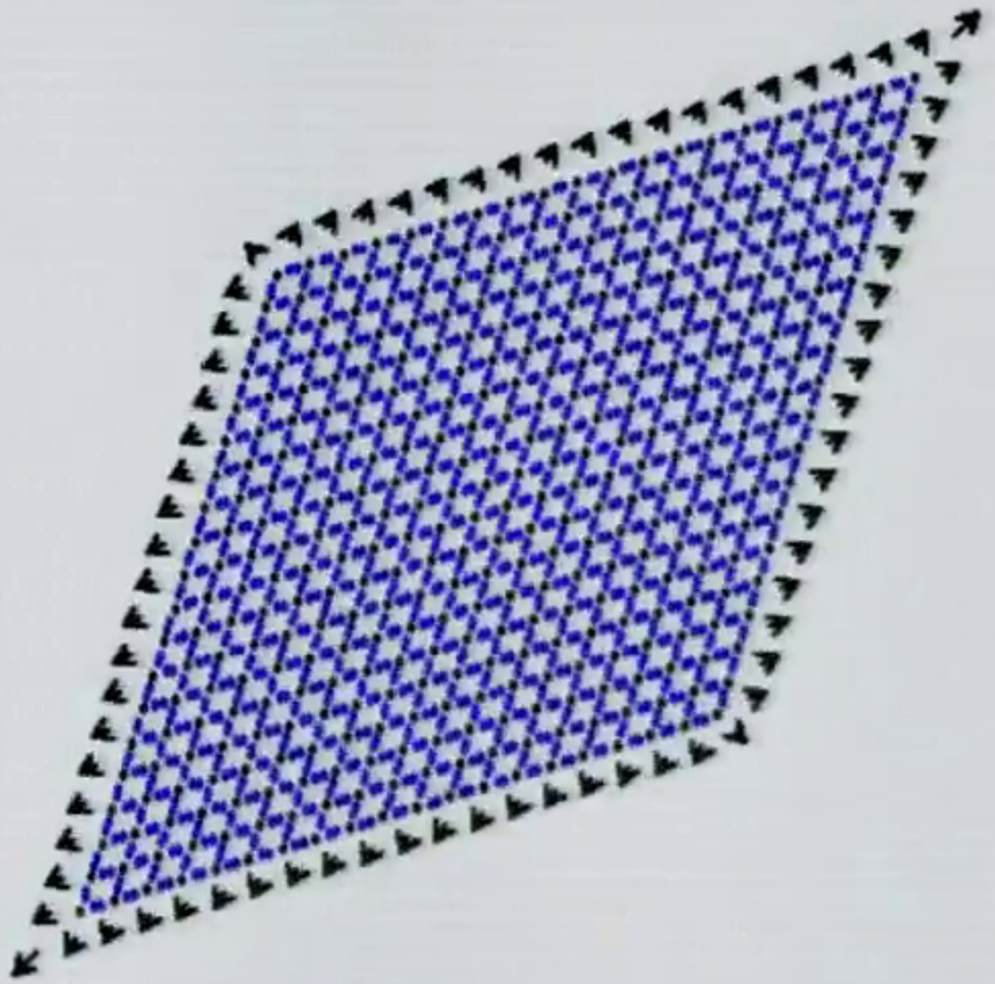

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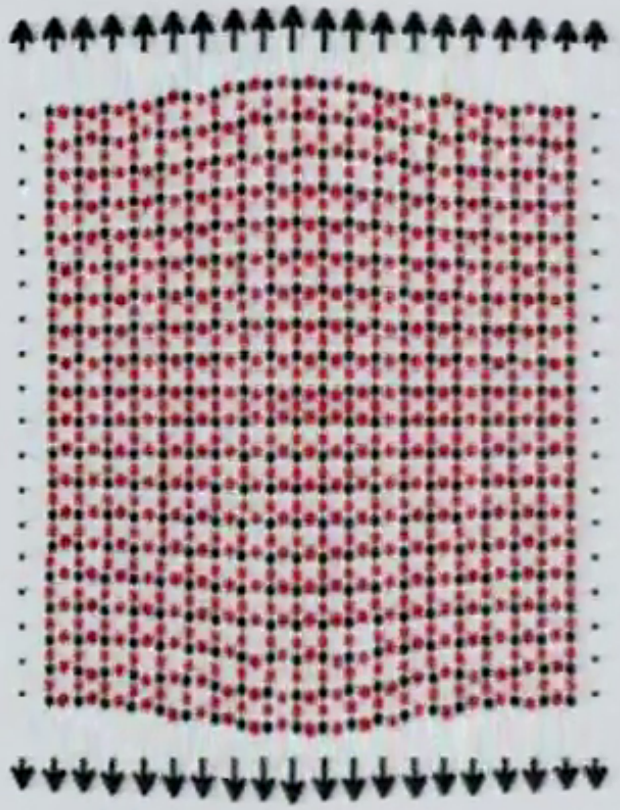

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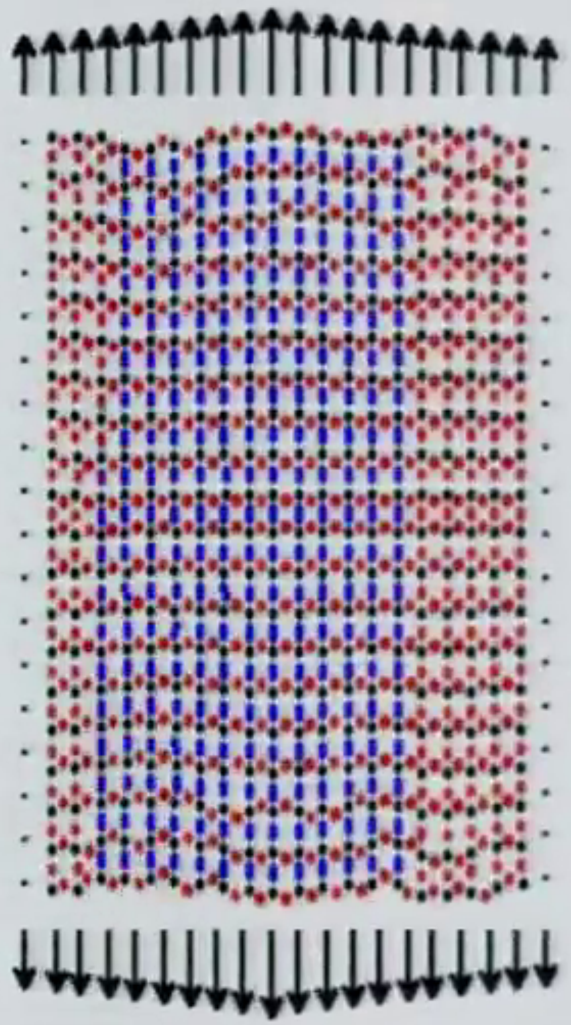

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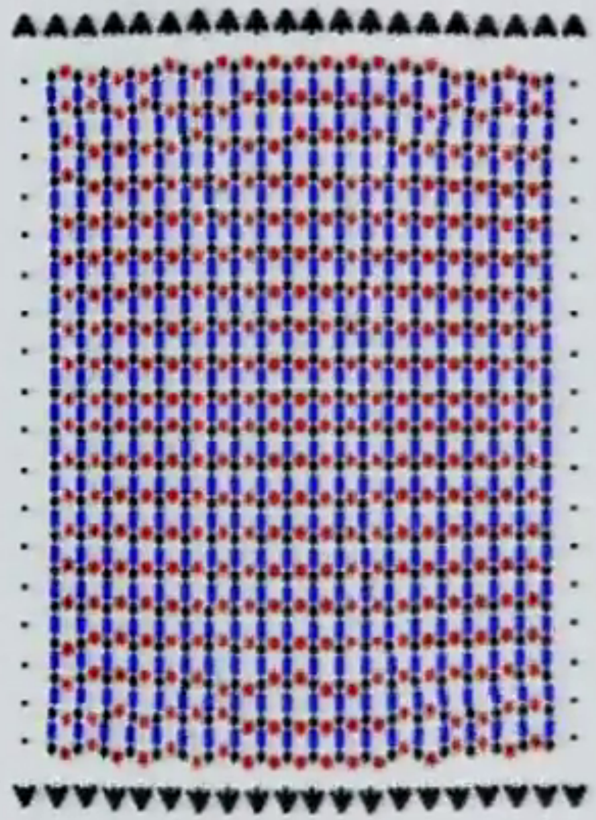

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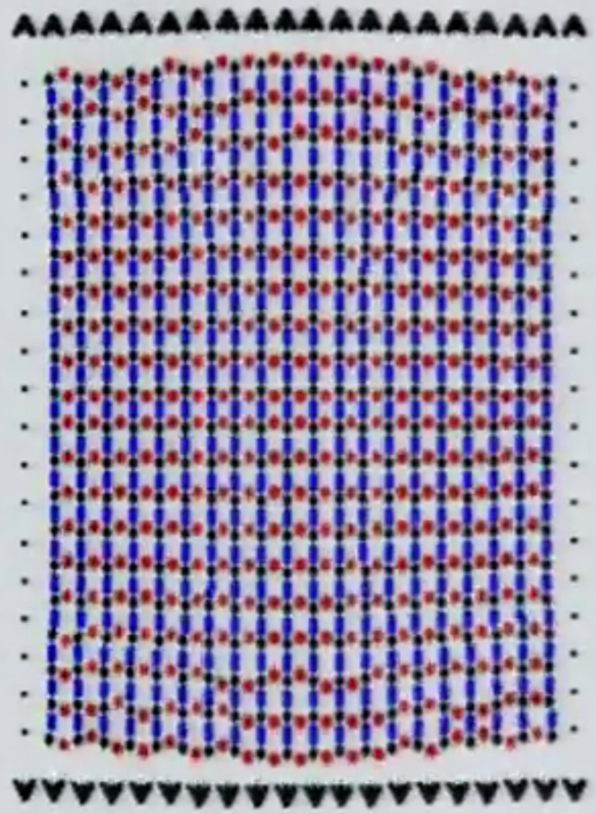

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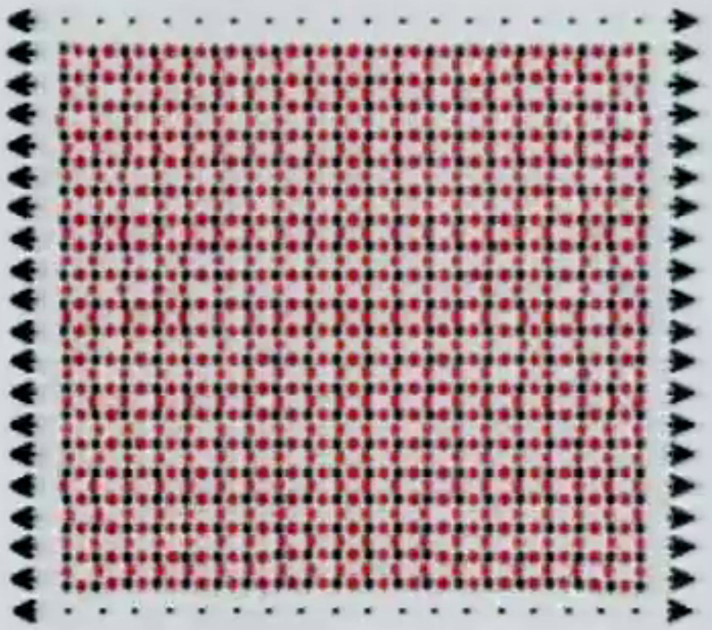

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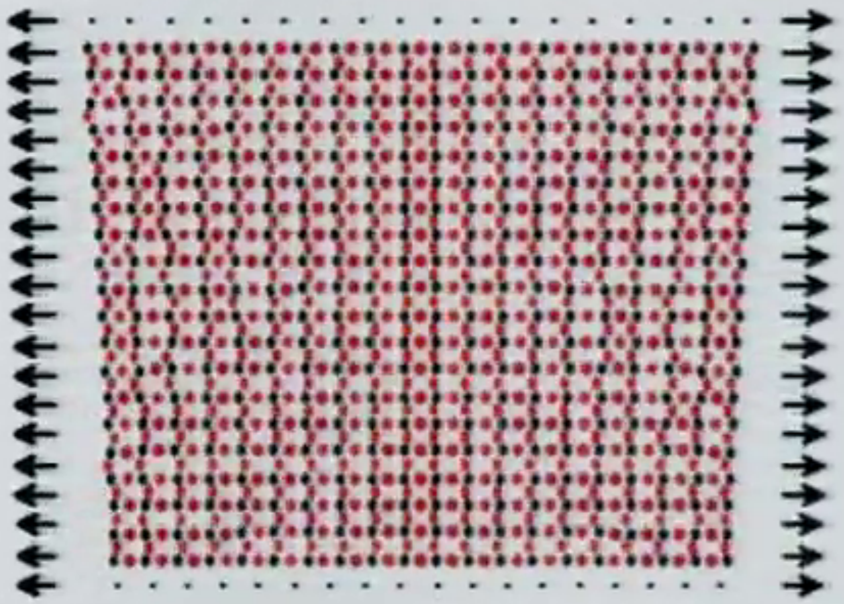

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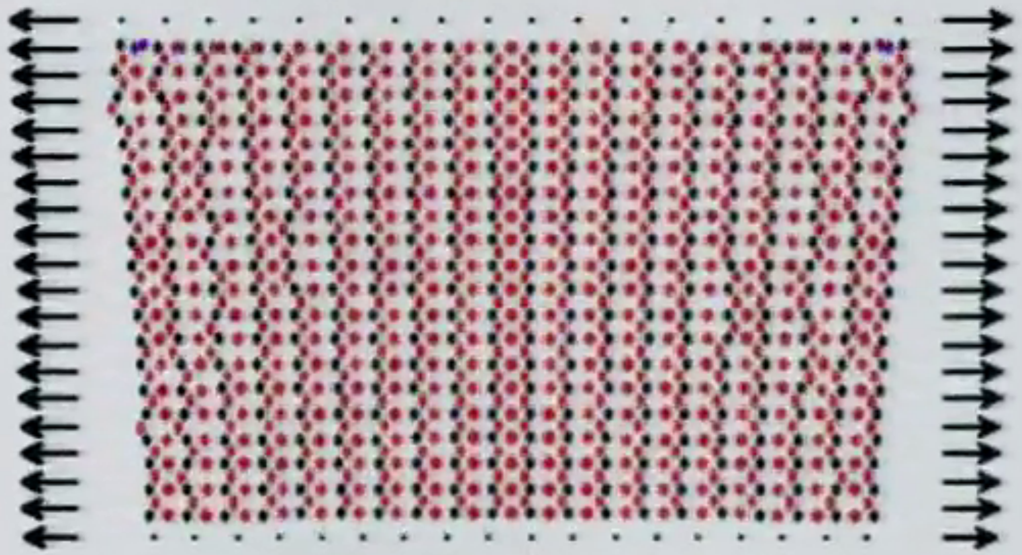

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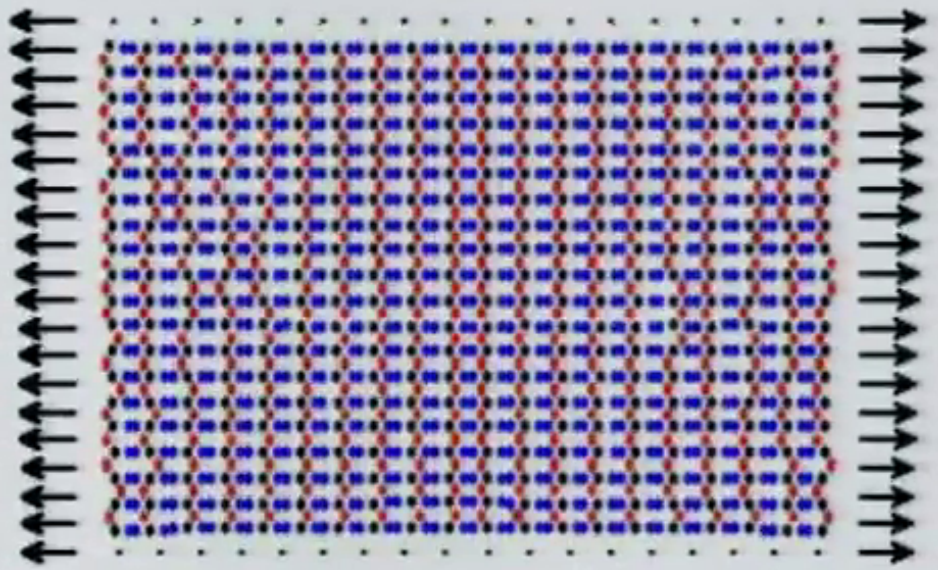

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