Mathematical Modeling: Changing the Landscape of the Mathematics Classroom

SIAM Applied Mathematics Education Conference

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What is Mathematical Modeling?

"Whether the problem is huge or little, the process of 'interaction' between the mathematics and real world is the same … This entire process is what is called *mathematical modeling*."

Henry Pollack (2012). *Mathematical Modeling Handbook: Introduction.* COMAP.

Available at http://www.comap.com/modelingHB/

Goals for the Session

- Talk about considerations as we think about incorporating math modeling into the middle and high school classrooms.
- Share my experiences working with teachers and students.
- Share resources and solicit ideas about how we can move forward, working together.

What do you picture when you think of the "landscape" of a math classroom?

Factors we might consider when we think about the landscape of our math classrooms.

- The make-up of the students - lots of factors…
- Class size
- Physical space
- Curriculum
- Accountability
- Access to technology
- How we (both teachers and students) spend our time in class
- How we spend our time outside of class

A Quote Dan Kennedy – First HS teacher to chair the AP Calculus Test Development Committee & Former MAA Board of Governors Member

"The rules of the game (the education game) are simple: we, the teachers, show them what to do and how to do it; we let them practice it for a while; and then we give them a test to see how closely they can match what we did. What we contribute to the game is called "teaching," what they contribute to the game is called "learning," and the game is won or lost for both of us on test day."

Some Concerns for High School and Middle School Teachers

- A wide range of levels of student preparation
- A range in learning styles
- Changes in curriculum standards
- Assessment – One size fits all
- Students' race to Calculus
- Access and equity

NCTM Position

Creating, supporting, and sustaining a culture of access and equity require being responsive to students' backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program and assessing its effectiveness.

Acknowledging and addressing factors that contribute to differential outcomes among groups of students are critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful.

Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.

These concerns are REAL

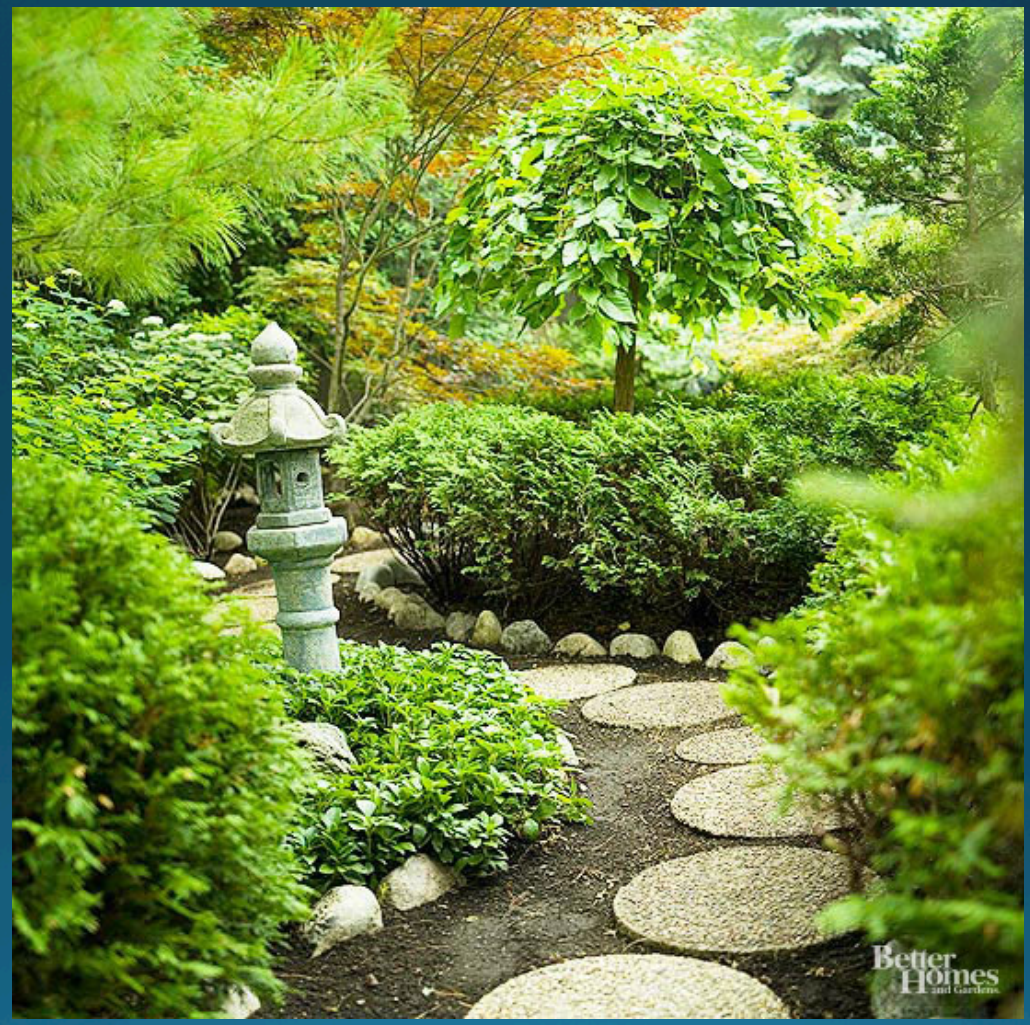
I am proposing that Mathematical Modeling can be incorporated into our high school classrooms and help address some of these concerns…

BUT

We'll need to change the landscape

from a place where teachers and students who are **familiar** and (some) **happy** with this…

To a place where teachers and students can become familiar with and see the value in this…




Better
Homes
and Gardens

Where Does Mathematical Modeling Fit In?

From Henry Pollack

> *"When you use mathematics to understand a situation in the real world, and then use it to take action or even to predict the future, both the real-world situation and the ensuing mathematics are taken seriously."*

Introduction: What is Mathematical Modeling
SIAM Math Modeling Handbook

My Experience "Fitting" Math Modeling in

- As applications of traditional mathematical topics
- Motivating the traditional mathematical topics through a real-world context
- Creating opportunities for students to work collaboratively to solve open-ended problems
- As a place where students can share ideas, make mistakes, struggle and produce solutions – In this process they both defend their reasoning and critique the reasoning of others.

Math Modeling in Specific Courses at NCSSM and Beyond…

- Precalculus/Algebra 2 (Math 3)
- Calculus (AP AB and BC)
- Modeling with Differential Equations
- Modeling Course
- Complex Systems Course

We believe that modeling opportunities can help our students become:

- Independent learners
- Better collaborators
- Creative problem solvers
- Better-prepared for life beyond high school

Challenges in Incorporating Modeling in the Curriculum

Modeling problems are “messy”.
How can we help students know where the mathematical topics are “situated” in the curriculum so that they don’t get lost in the messy, unordered landscape? How can we help them overcome the discomfort that can come with this messy territory?

Challenges of Teaching Modeling

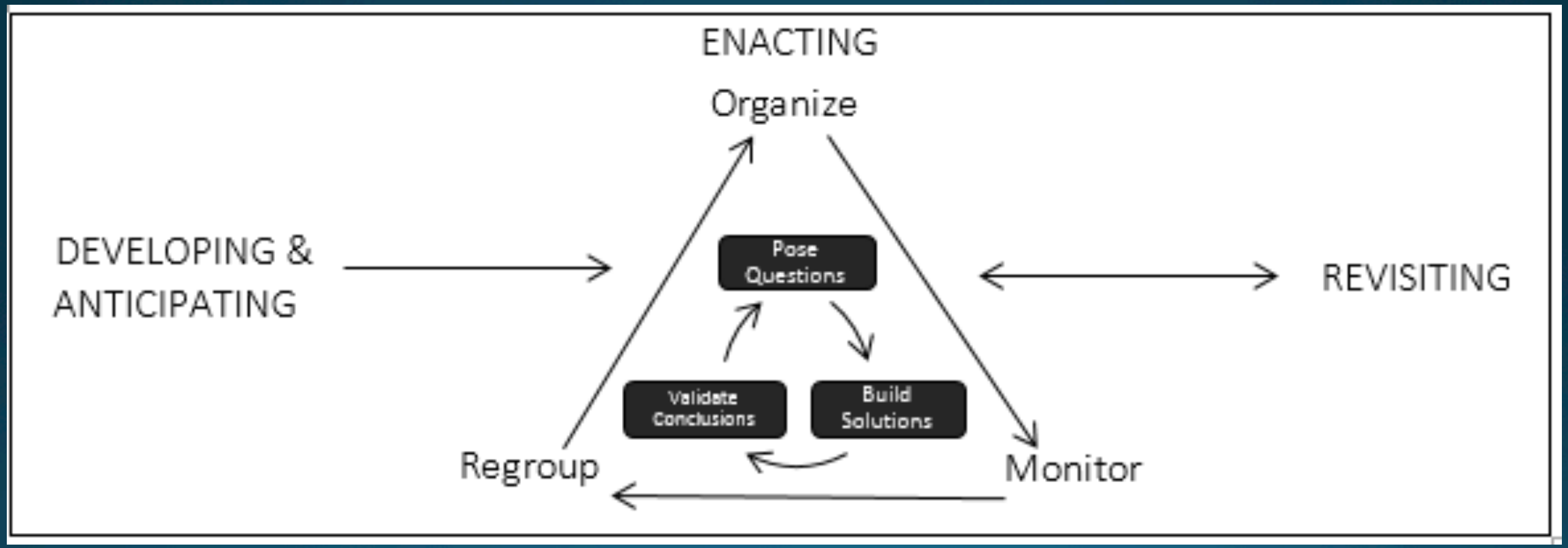
- There's not just one answer – so how do I know if my students are "right"?
- Students want to know if they're right, and if I don't tell them if they're right (or wrong), they get frustrated and don't think I am doing my job.
- How do I help students without taking the work away from them?
- How do I grade students on math modeling?

More questions

- How do I prepare for class?
- Where can I find good modeling tasks that are appropriate for my classes?
- How do I communicate with parents and administrators and get support from parents and administrators?

Many of the answers can be found in The GAIMME Report

Teacher Role



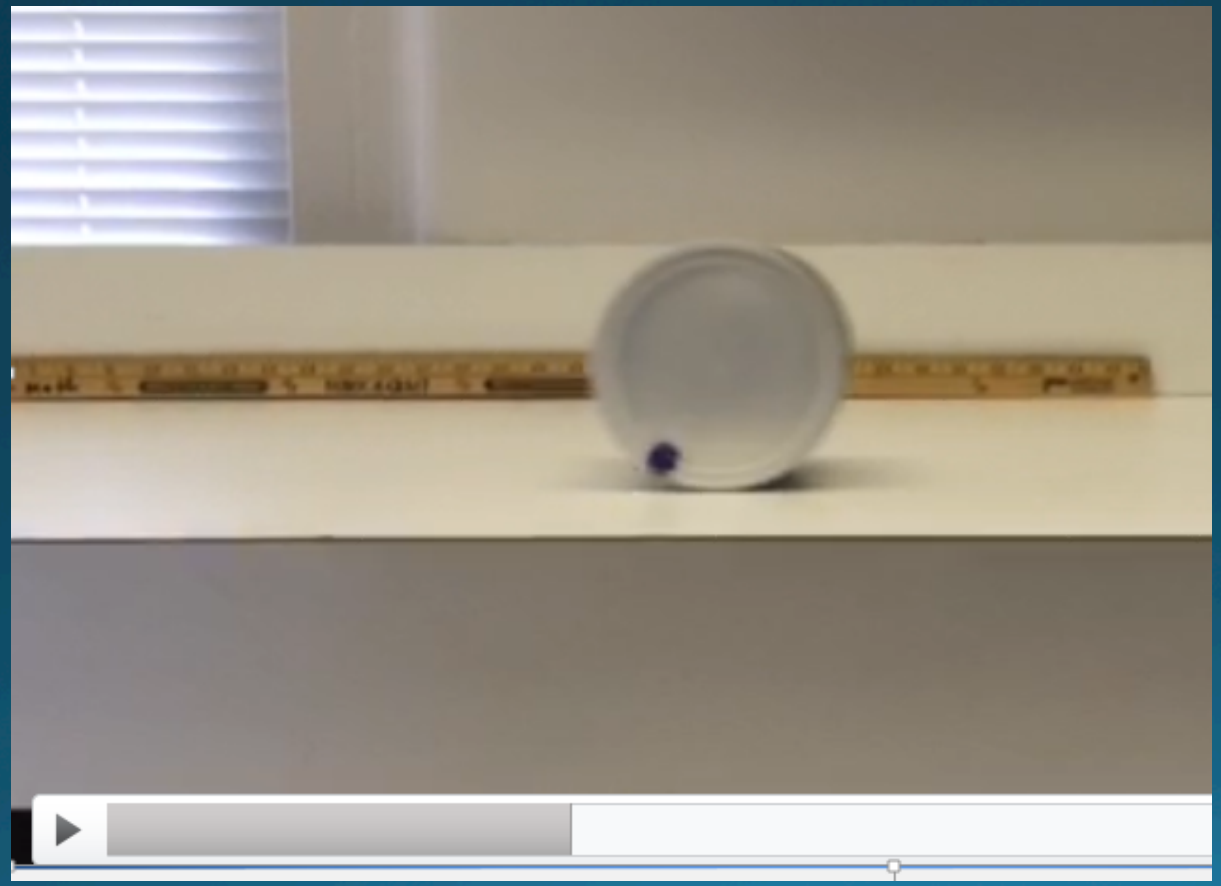
Carlson, Mary Alice, Wickstrom, Megan H., Burroughs, Elizabeth A., & Fulton, Elizabeth W. (2016)

Examples

- Rolling Can
- Driving for Gas

Rolling Can

Consider Rolling Can Video



Talk to a neighbor…

Where do you think this could fit in for a middle or high school class?

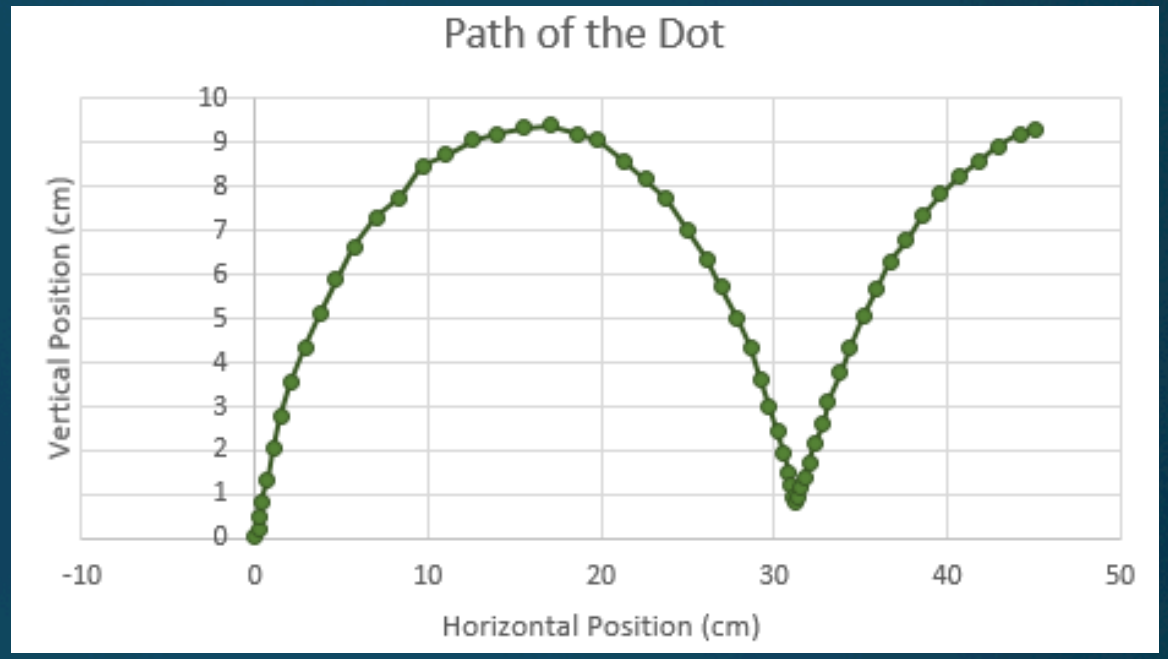
We ask students to consider the path of the dot.

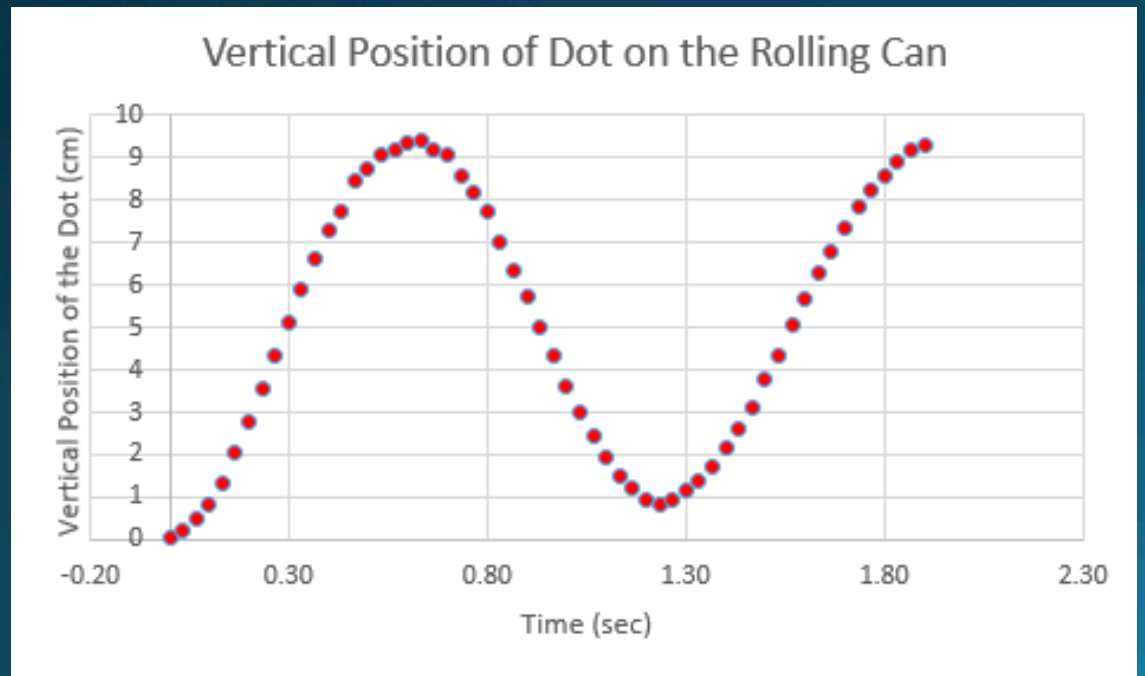
They make conjectures and then collect data from the video using LoggerPro.

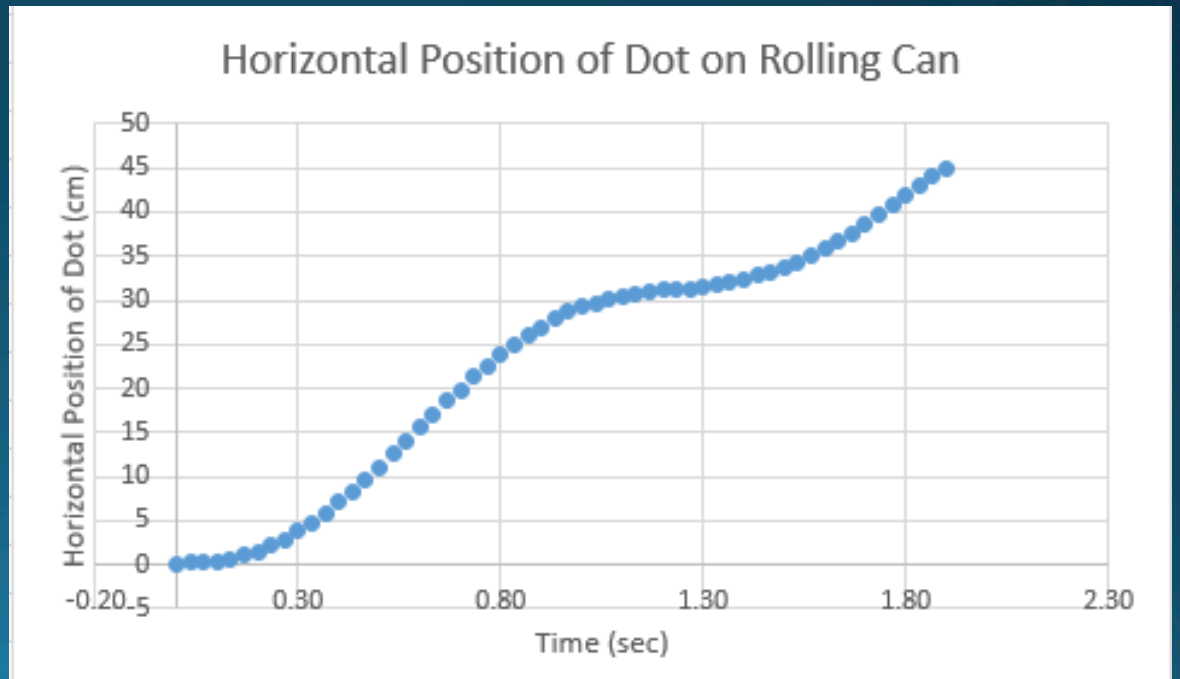
We ask them to create mathematical models for the horizontal and vertical positions of the dot.

As they create their models, we ask them to consider their simplifying assumptions and evaluate their models. This evaluation can include connecting the empirical models to theoretical model.

Sample Data


Path of the Dot
Vertical Position (cm)
Horizontal Position (cm)
10
9
8
7
6
5
4
3
2
1
0
-10
0
10
20
30
40
50


Vertical Position of Dot on the Rolling Can
Vertical Position of the Dot (cm)
Time (sec)
10
9
8
7
6
5
4
3
2
1
0
-0.20
0.30
0.80
1.30
1.80
2.30


Horizontal Position of Dot on Rolling Can
Horizontal Position of Dot (cm)
Time (sec)
50
45
40
35
30
25
20
15
10
5
0
-5
-0.20
0.30
0.80
1.30
1.80
2.30

This task fits into our curriculum

- But it extends the typical study of trigonometric functions and gives students a chance to create "new" mathematics.

- Commenting on the project, one student said that the project was "cool" because it made him think differently. He went on to express surprise at the idea of combining a linear and sine function in this way.

"I didn't even know that was a thing until this project."

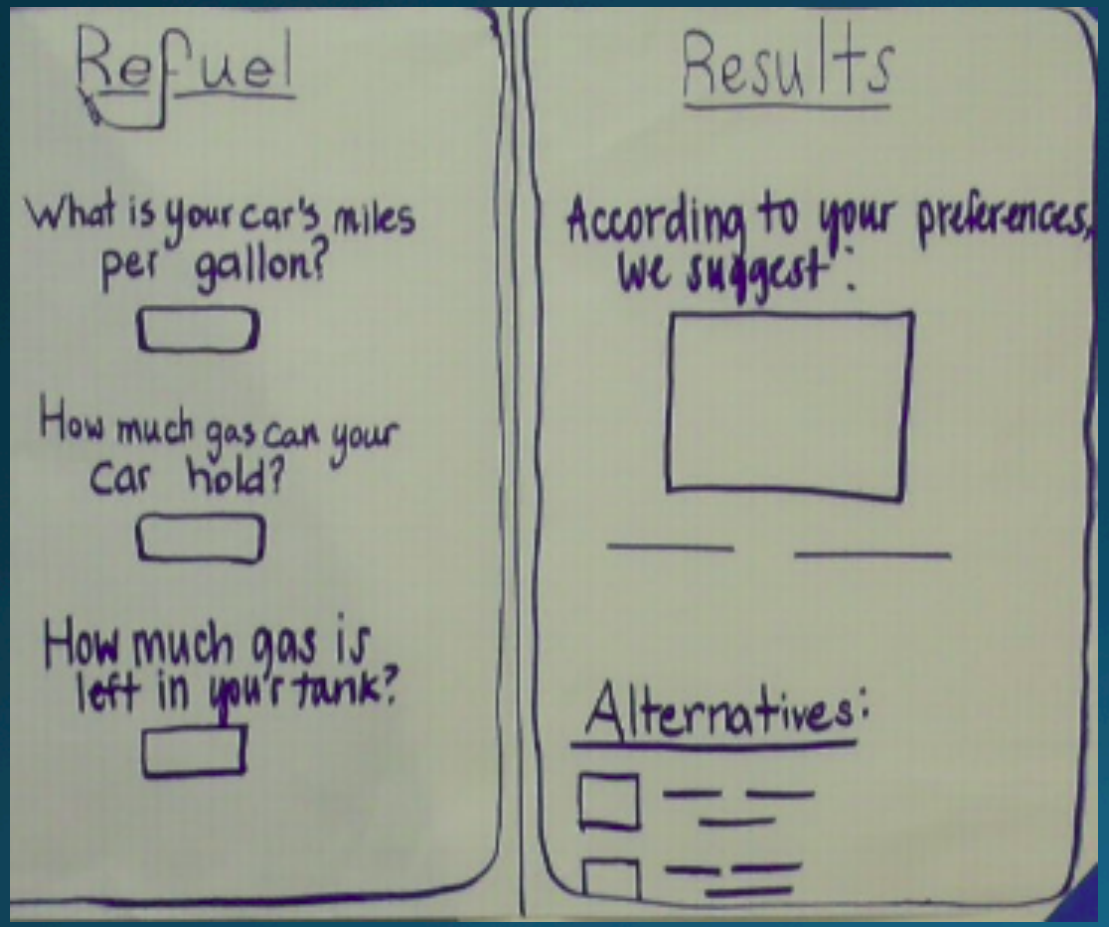
Driving for Gas

Most drivers have a 'usual' region in which they do most of their driving. However, gas prices may vary widely so that gas may be substantially cheaper somewhere other than within that usual region.

Would it be more economical to go to a station outside the usual region to buy gas? Thus, the general question we wish to address is, "How might we determine which gas station is the most cost-efficient?"

[GAIMME, p.183]

Driving for Gas – Student Work



price/mpg = price per mile

price per mile · total distance = distance price

distance price + (price of gas · gallons needed) = total price

d + (g · n) = total

Example:

Station A: on normal route, $3.50 per gallon

Station B: 5 miles away, $3.35 per gallon

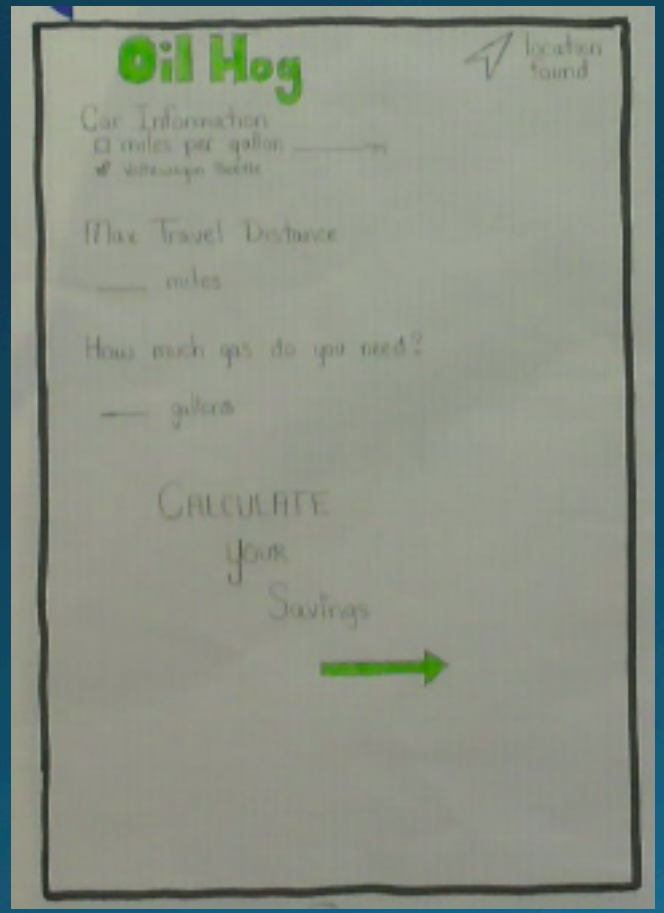
Car 1: 40 mpg, 10 gallon tank

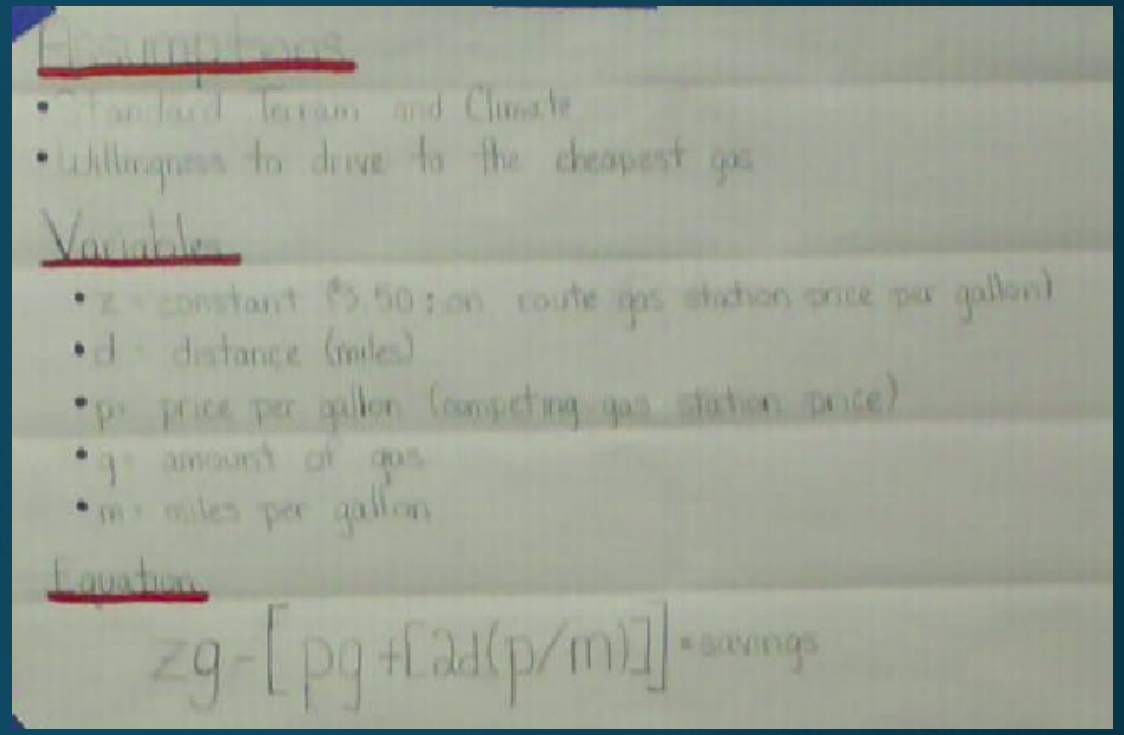
Car 2: 15 mpg, 10 gallon tank

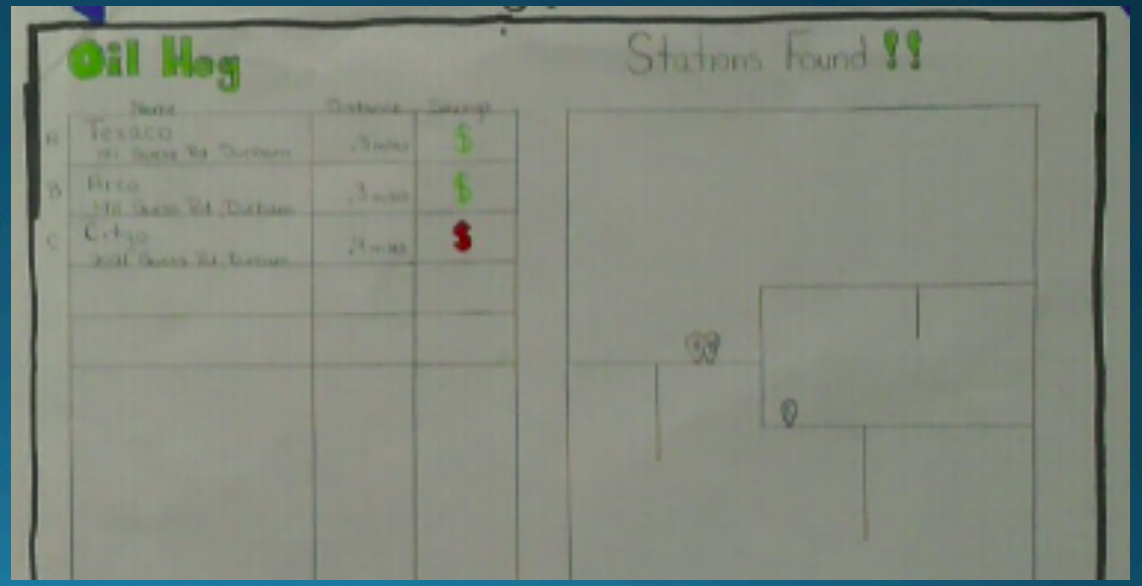
	Car 1	Car 2
Station A	$17.50	$17.60
Station B	$17.59	$13.98

-assumed each car had a ten gallon tank and needed to fill up half of a tank

Student Work







Modeling serves ALL students

- Share student reflections from NCSSM Summer Bridge students

- Math Modeling opportunities in HS

HiMCM and Moody's Mega Math Challenge

Student Reflections

- "Math for me was mostly focused on doing what the teacher wanted. Doing the first problem here opened my eyes to real math problems. While trying to solve the problem, I became thoroughly confused and frustrated. Solving it with my group **felt great** since it felt like we accomplished something."

- "This problem seemed impossible but with hard work we were finally able to solve it. It opened me up to new math."

More Student Reflections

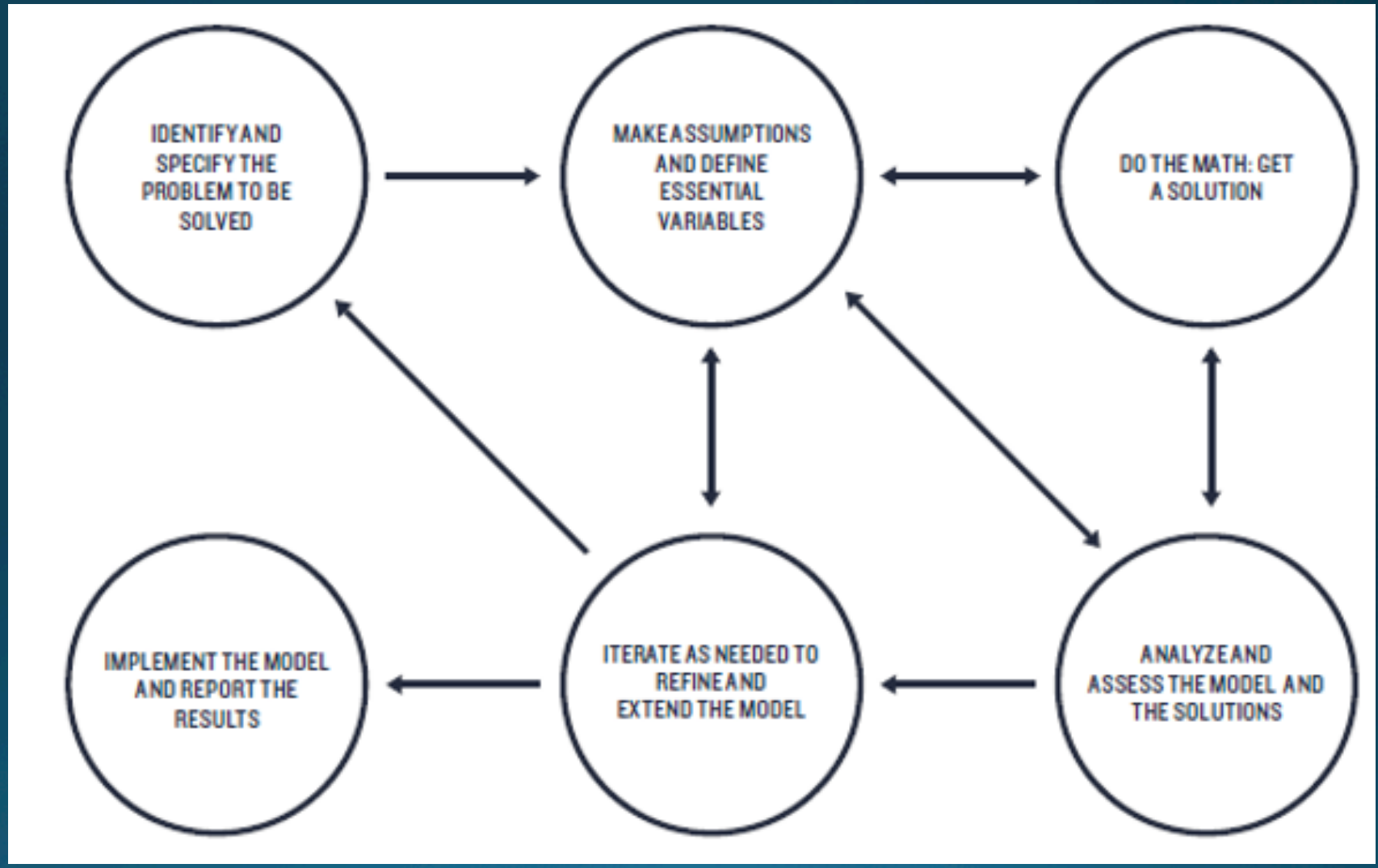
- "It helped me to think of solving in a different way, and I learned how to collaborate with my classmates through problem solving."

- "By doing this problem, I learned that if I kept at it I could solve it with an answer that worked if not the best answer. I will keep this lesson with me and apply it to my classes in the following years as well as the rest of my life."

How can Modeling Address the Teacher Concerns?

- A wide range of levels of student preparation
- A range in learning styles
- Changes in curriculum standards
- Assessment – One size fits all (?)
- Students' race to Calculus
- Access and equity

Allowing our students to engage in the modeling process can help us change the landscape of our classrooms.



GAIMME Report

Modeling Cycle

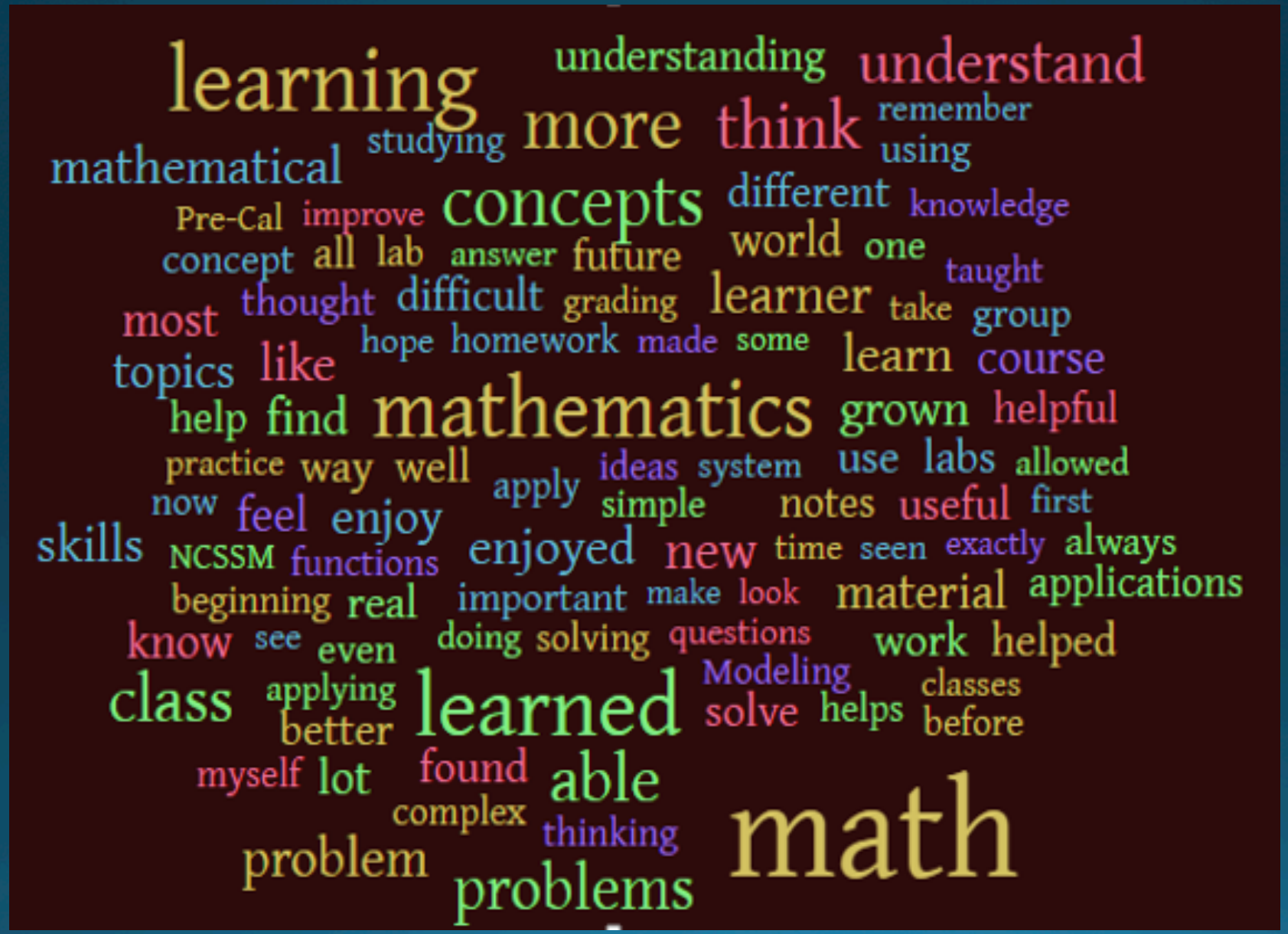
The cycle gives students a chance to make and change their assumptions, create and refine their models, evaluate the usefulness and accuracy of their models.

This process can lead to thoughtful consideration of mathematical topics and connections between those topics.

A chance for thoughtful reflection is NOT built into the curriculum.

This opportunity to reflect on their solutions and evaluate their own work is rare in the current landscape of the mathematics classroom. Instead students move (sometimes rapidly) from one topic to the next and they look to the teacher or the back of the book for affirmation of the right answer.

Precalculus Student Reflections Wordle

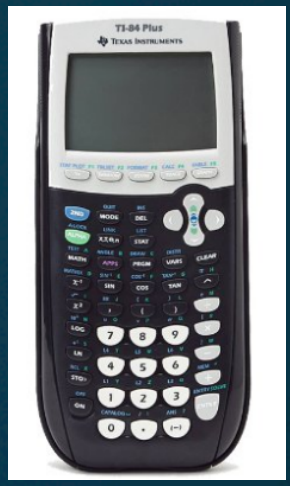


The Role of Technology

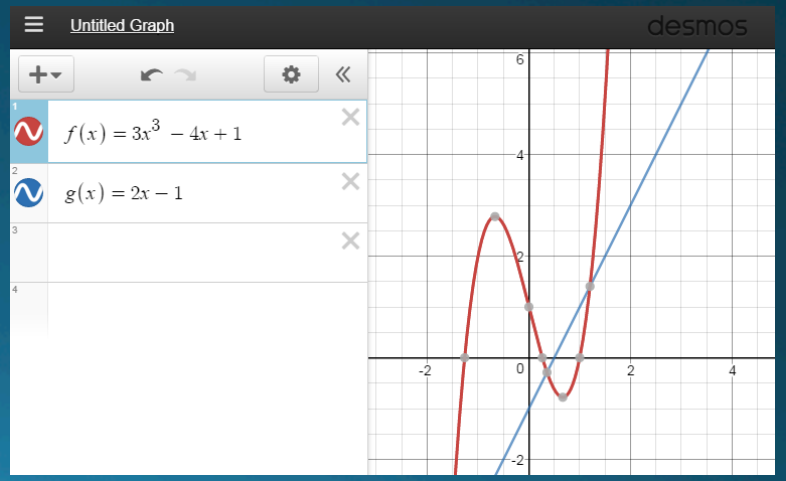
- Bivariate data analysis and function graphing tools

  Spreadsheets, Geogebra, TI Nspire, TI Graphing Calculator, Desmos,LoggerPro
- NCTM Core MathTools
- Fathom/JMP
- Website - GapMinder

The Reality

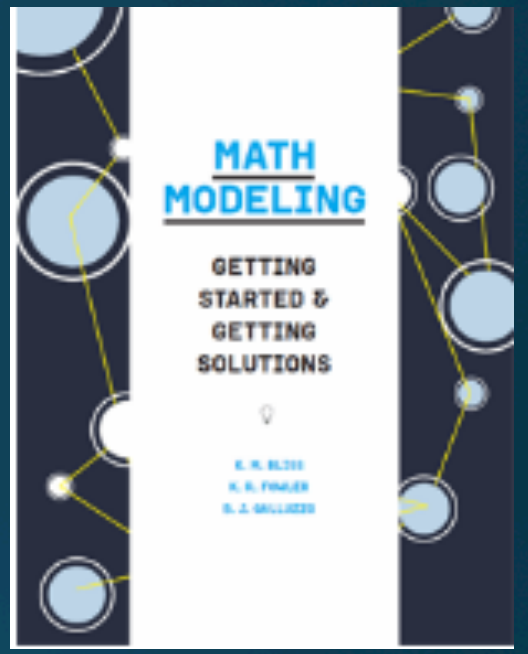


- Most high schools students have access to TI-84 graphing calculators in the classroom. Some middle school students have access to a graphing calculator.
- Bring your own devices (BYOD) policies are becoming somewhat more prevalent.
- Most teachers have access to a computer in their classes and a projector for demonstrations.
- Some students have access to computers or tablets.

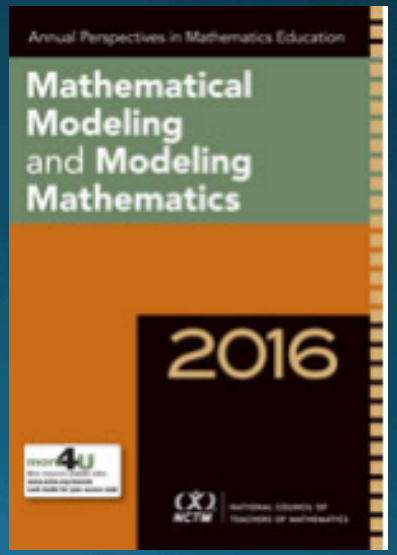


Teaching Resources

- GAIMME Report
- SIAM Math Modeling Handbook



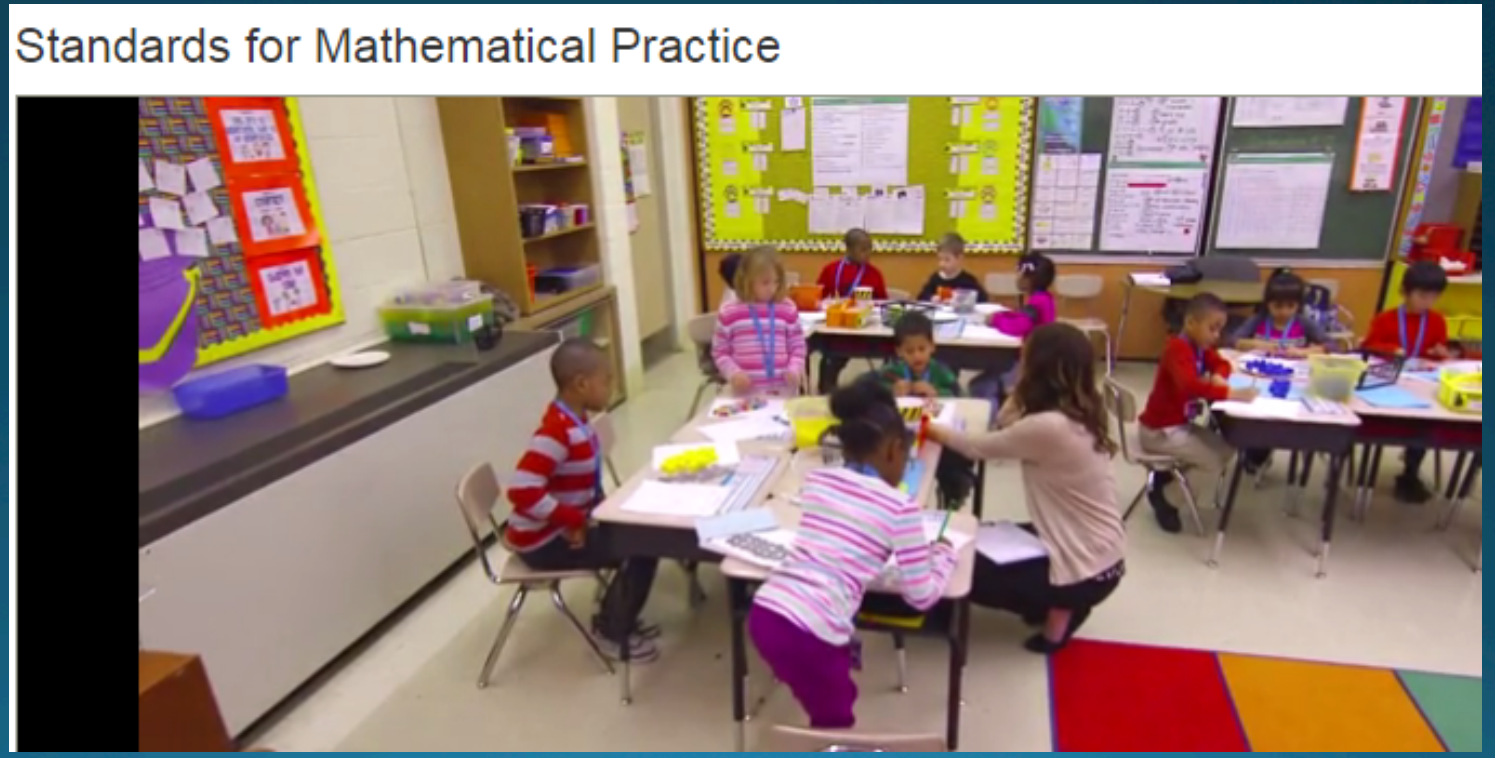
- *NCTM Annual Perspectives on Mathematics Education* 2016



Common Core State Standards

NCTM Video Series

http://www.nctm.org/Standards-and-Positions/Common-Core-State-Standards/Teaching-and-Learning-Mathematics-with-the-Common-Core/



NCSSM's Resources

NCSSM Math Faculty – Precalculus and Calculus textbooks, state, regional and national conferences such as NCTM, JMM, and SIAM, and summer workshops.

Website Materials and Conference

- Algebra 2

https://ncssm.instructure.com/courses/1065

- Post AP Calculus Projects

http://courses.ncssm.edu/math/apcalcprojects/

- Teaching Contemporary Mathematics Conference in Durham
  January 27 – 28, 2017

https://www.ncssm.edu/tcmconference

How can we move forward?

?

Conferences and Other Support

1. Anja Greer Math, Science and Technology Conference
   Phillips Exeter Academy,
   June 26 – June 30, 2017
   Exeter, New Hampshire
   https://www.exeter.edu/summer_programs/7325.aspx

2. Knowles Teaching Fellows Program (KSTF)
   http://kstf.org/

3. Duke Masters of Arts in Teaching
   https://educationprogram.duke.edu/graduate

References

- Carlson, Mary Alice, Wickstrom, Megan H., Burroughs, Elizabeth A., & Fulton, Elizabeth W. (2016). A Case for Mathematical Modeling in the Elementary Classroom. ***Annual Perspectives in Mathematics Education* (APME) 2016:** *Mathematical Modeling and Modeling Mathematics*, Reston, Va.: National Council of Teachers of Mathematics.

- *Com*mon Core State Standards Initiative. 2010. ***Common Core State Standards for Mathematics*.** Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. *http://www.corestandards.org/assets/CCSI_Math Standards.pdf.*

- Consortium of Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM). ***Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME) Report.*** Bedford, MA. and Philadelphia, PA., 2016.

References

- Hirsch, Christian R., ed. 2016. ***Annual Perspectives in Mathematics Education 2016: Mathematical Modeling and Modeling Mathematics***. Reston, VA: National Council of Teachers of Mathematics.

French Garden

- http://www.stylemotivation.com/landscape-design-french-garden

Japanese Garden

- http://www.bhg.com/gardening/design/styles/elements-of-a-japanese-garden/#page=2

Thank you!

We invite you to join us this afternoon in 2 sessions on mathematical modeling where you will get a chance to engage in modeling tasks and think about pedagogical questions as you do so.

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