Incorporating Stochastic Leadtimes into the Guaranteed Service Model of Safety Stock Optimization

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A Quote to Remember

- “If we have to take the final result from your model and still multiply it by a fudge factor, we won’t use it…”
- “…we could just save ourselves the time and effort by winging it right from the start.”

Truly, and completely, solving the problem is our mission

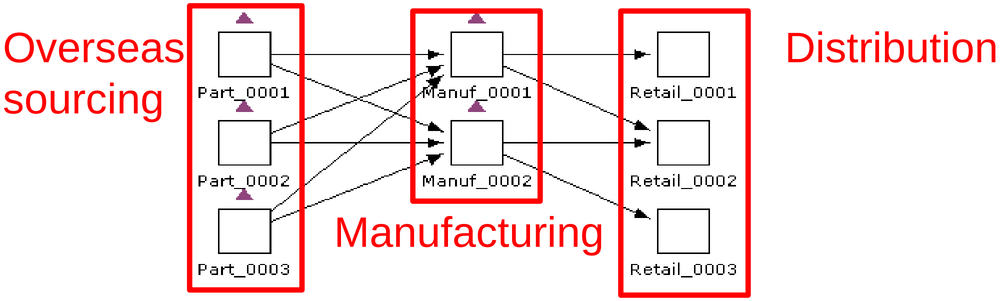
Translating This Quote to Practice

- The answer we produce has to be the answer that feeds the company's execution system
- We need to account for all the major factors that a planner considers when setting inventory levels
- Our solution approach needs to be understandable to smart planners

Agenda

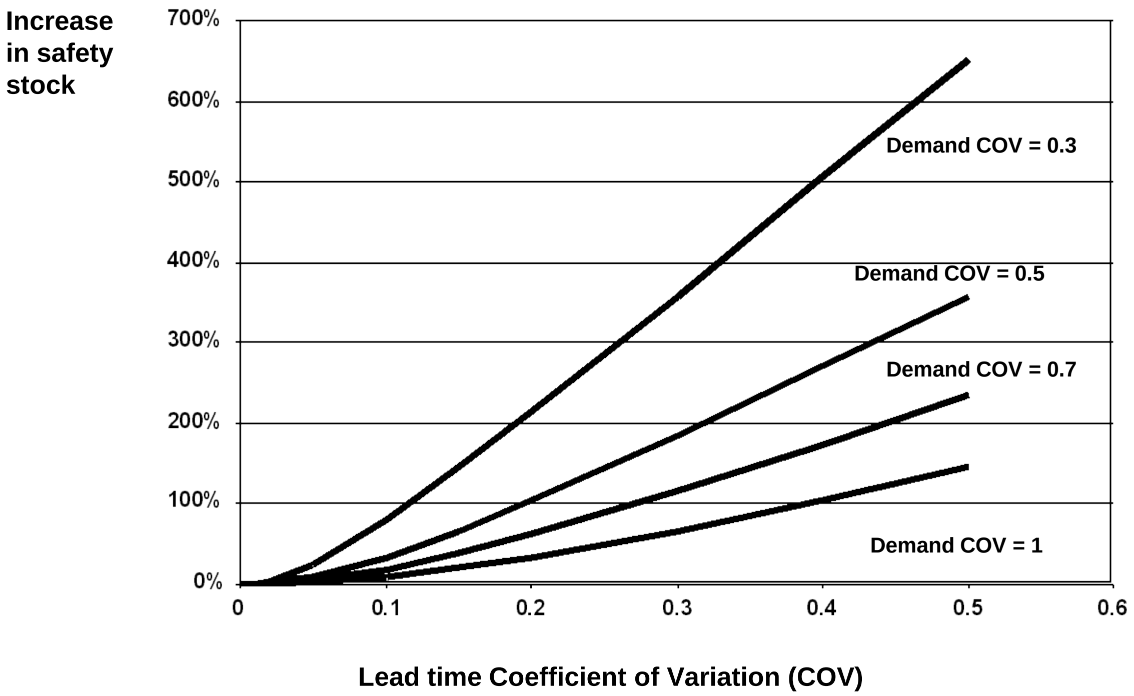
- Variability primer
- Guaranteed service model overview
- Enhancement for stochastic leadtimes
- Example
- Conclusion

Examples of Leadtime Variability



- Overseas sourcing creates long and discrete variable lead times
  - 30 days 70% of the time, 60 days 20% of time, 180 days 10%
  - This is extreme variability
- Manufacturing experiences queuing and schedule adherence issues
  - Normally distributed with mean 7 and standard deviation 3
  - We often have the best data on this variability
- Distribution and transportation creates variability of plus or minus a day
  - 2 days 95% of the time, 1 day 1% of the time, 3 days 4%
  - This variability occurs at the product's most expensive point
  - Buffering an extra day's worth of demand at retail is nontrivial

Leadtime Variability Significantly Increases Required Safety Stock



- Leadtime variability helps explains why planners require so much inventory in practice

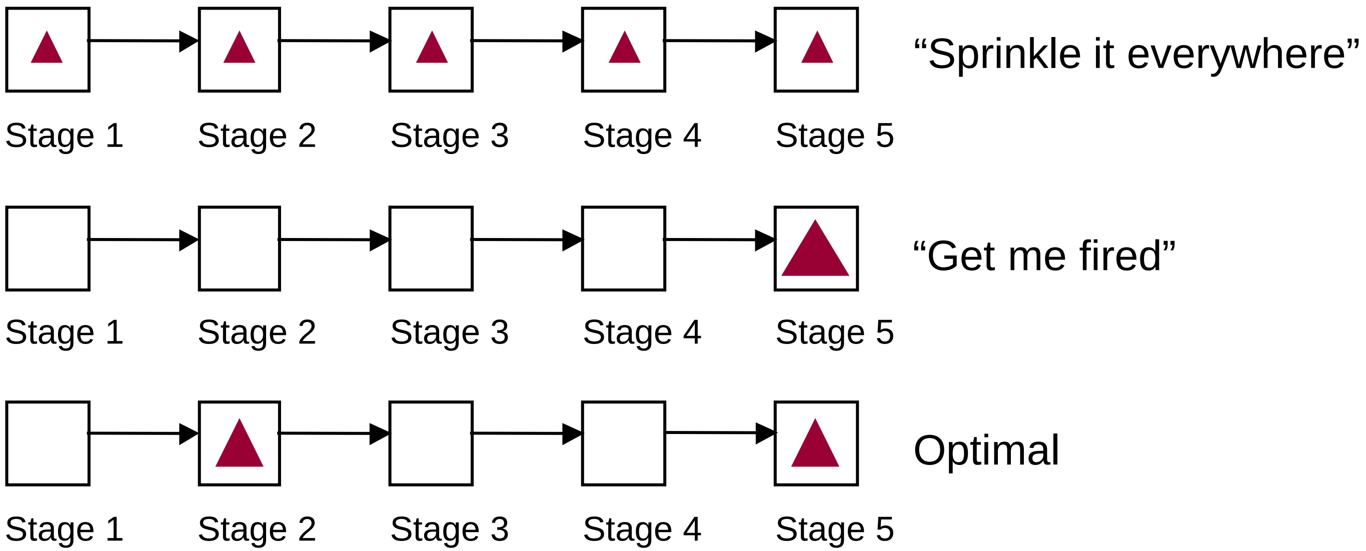
Guaranteed Service Model for Inventory Optimization

- A stage is a SKU-location

$$\tau = \text{net replenishment time}$$

$$= SI + T - S \qquad \text{where } SI = \max\left(S_j : (j,i) \in A\right)$$

- Three potential inventory policies for serial supply chain



Guaranteed Service Model for Inventory Optimization

- Net replenishment time drives model
- Inventory on-hand can be a complex function of net replenishment time

$$E\left[I\right] = \max\left\{D(\tau)\right\} - E\left[D(\tau)\right]$$

The GS Framework

- The math programming formulation

$$
\begin{aligned}
\text{Min} \quad & \sum_{i=1}^{N} c_i(SI_i, S_i) & \\
\text{s.t.} \quad & S_i - SI_i \le T_i & \text{for all } j = 1,\ldots,N \\
& S_j - SI_i \le 0 & \text{for all } (i, j) \in A \\
& S_i \le s_i & \text{for all end-customer stages } i \\
& S_i, SI_i \ge 0, \text{integer} & \text{for all } i = 1,\ldots,N
\end{aligned}
$$

- Cost, and inventory, functions can have arbitrary structure so long as they only depend on service times

What Makes Multi-Echelon Hard

1. Demand propagation
   - How demand transmits between different stages in the supply chain.
2. Service time optimization
   - How changing the net replenishment time at one stage impacts other stages in the supply chain
3. Cost propagation
   - How cost accrues across the supply chain impacts the best locations to deploy inventory

Authors’ Published Work Extending GS Theory

- Humair, S. and S. P. Willems, “Optimizing Strategic Safety Stock Placement in General Acyclic Networks,” *Operations Research*, May/June 2011, Vol. 59, No. 3 pp 781-787.
- Neale, J. J. and S. P. Willems, “Managing Inventory in Supply Chains with Nonstationary Demand,” *Interfaces*, Sept-Oct 2009, Vol. 39, No. 5. pp. 388-399.
- Graves, S. C. and S. P. Willems, “Strategic Inventory Placement in Supply Chains: Non-Stationary Demand,” *Manufacturing & Service Operations Management*, Spr 2008, Vol. 10, No. 2, pp. 278-287.
- Bossert, J. M. and S. P. Willems, “A Periodic Review Modeling Approach for Guaranteed Service Supply Chains,” *Interfaces*, Sept-Oct 2007, Vol. 37, No. 5, pp. 420-435.
- Humair, S. and S. P. Willems, “Optimizing Strategic Safety Stock Placement in Supply Chains with Clusters of Commonality,” *Operations Research*, July-Aug 2006, Vol. 54, No. 4, pp. 725-742.
- Graves, S. C. and S. P. Willems, "Optimizing Strategic Safety Stock Placement in Supply Chains," *Manufacturing & Service Operations Management*, Winter 2000, Vol. 2, No. 1, pp. 68-83.

Authors' Published Work Extending GS Practice

- Wieland, B., P. Mastrantonio, S. P. Willems, and K. G. Kempf, "Optimizing Inventory Levels within Intel's Channel Supply Demand Operations," February 2011, 19 pages; accepted February 2011, to appear in *Interfaces*.
- Farasynm I., S. Humair, J. I. Kahn, J. J. Neale, O. Rosen, J. D. Ruark, W. Tarlton, W. Van de Velde, G. Wegryn, and S. P. Willems, "Inventory Optimization at Procter & Gamble: Achieving Real Benefits through User Adoption of Inventory Tools," *Interfaces*, Jan-Feb 2011, Vol. 41, No. 1, pp. 66-78.
- Manary, M. P. and S. P. Willems, "Correcting Heterogeneous and Biased Forecast Error at Intel for Supply Chain Optimization," *Interfaces*, Sept-Oct 2009, Vol. 39, No. 5. pp. 415-427.
- Manary, M. P. and S. P. Willems, "Setting Safety Stock Targets at Intel in the Presence of Forecast Bias," *Interfaces*, Mar-Apr 2008, Vol. 38, No. 2, pp. 112-122.
- Willems, S. P., "Real-World Multi-Echelon Supply Chains Used for Inventory Optimization," *Manufacturing & Service Operations Management*, Winter 2008, Vol. 10, No. 1, pp. 19-23.
- Billington, C., G. Callioni, B. Crane, J. D. Ruark, J. Unruh Rapp, T. White, and S. P. Willems, "Accelerating the Profitability of Hewlett-Packard's Supply Chains," *Interfaces*, Jan-Feb 2004, Vol. 34, No. 1, pp. 59-72.

Contributions in Extending GS Model to include Stochastic Leadtimes

- Methodological contributions
  - Characterizing stocks and defining a new type of stock
  - Optimizing non-concave non-differentiable cost functions
- Practical contribution
  - Characterizing stochastic lead times for large-scale, automated, repeated calculations to input into companies' planning processes

Methodological Contribution: Characterizing Stocks

- Common models for characterizing safety stocks assume 0 quoted service time
- With non-zero service times, replenishments can arrive earlier or later than expected, therefore
  - Safety stocks need to include only replenishments that arrive as expected or later than expected

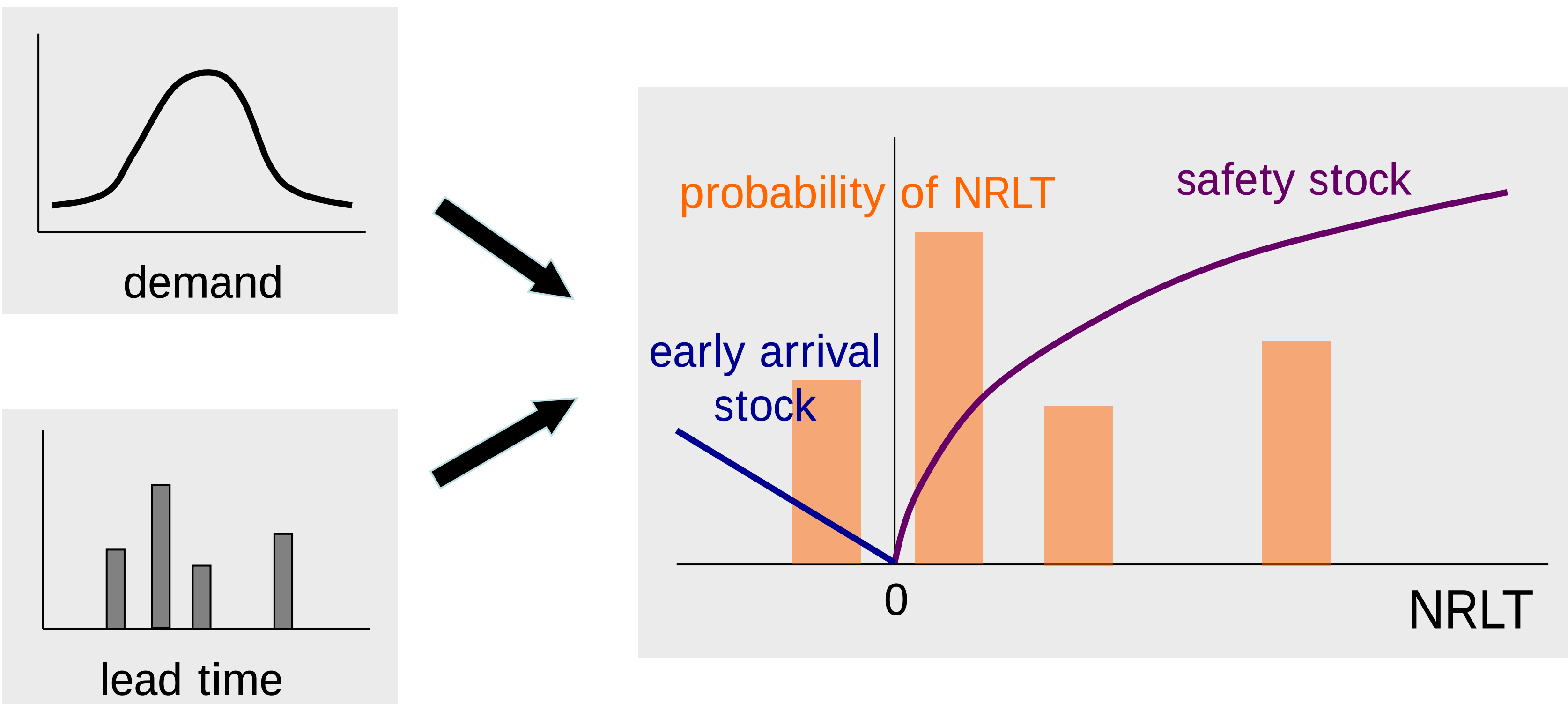
$$SFTY_k(SI_k, S_k) = z_k \sqrt{Q([S_k - SI_k]^+)(\sigma_k^d)^2 + (\mu_k^d)^2 R([S_k - SI_k]^+)}$$

  - We define a new type of stock carried by stages, “Early Arrival Stock”, due to replenishments that arrive earlier than expected

$$EARLY_k(SI_k, S_k) = \mu_k^d \left(Q([S_k - SI_k]^+) - \mu_k^l + [S_k - SI_k]^+ \right)$$

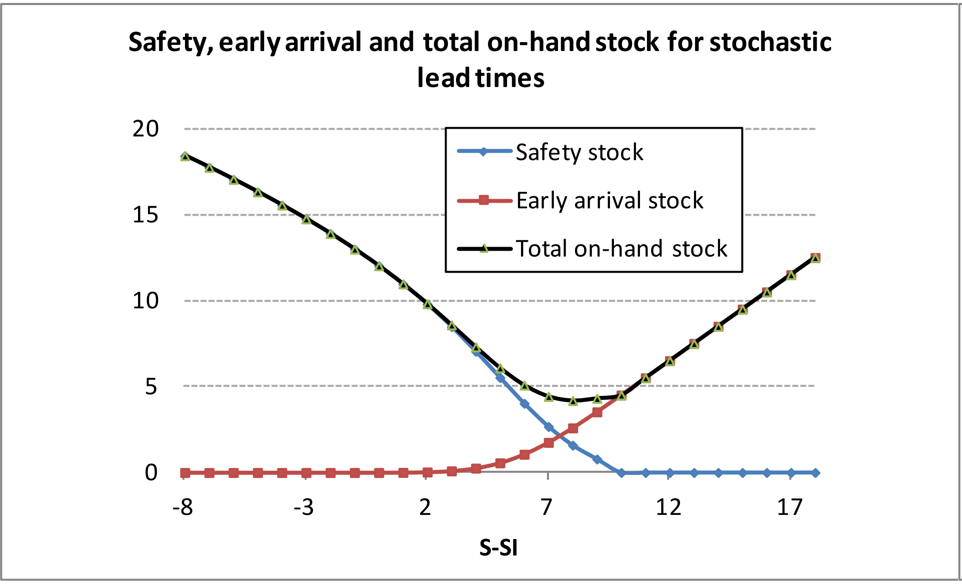
Methodological Contribution: Characterizing Stocks

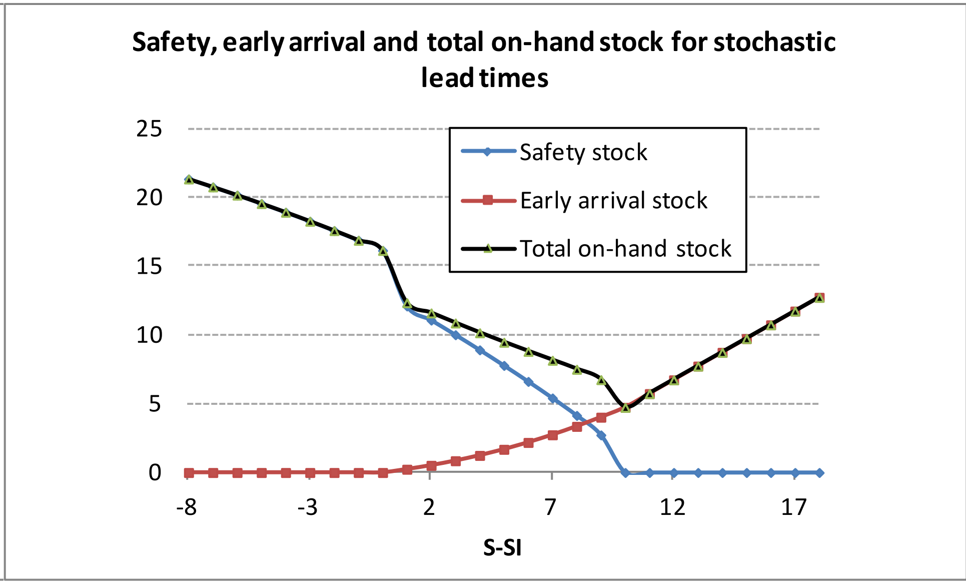
- Stylized representation of safety and early arrival stocks



Methodological Contribution: Optimizing with Stochastic Lead times

- With stochastic leadtimes, the cost function may be:
  - Non-concave
  - Non-differentiable





- We developed an algorithm to handle these cost functions with general network topologies

Companies Using Our Model of Stochastic Leadtimes

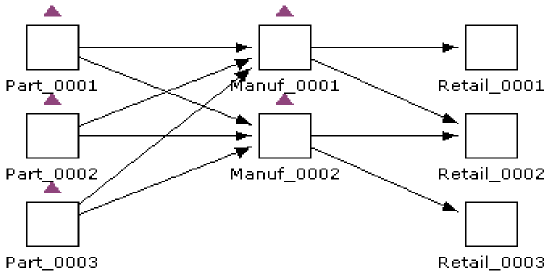
Where are the Big Wins from Multi-Echelon Inventory Optimization?

- Optimizing (and then continually reoptimizing) inventory levels as part of the ongoing planning process
  - Raising inventory levels for some SKUs, reducing inventory levels for many SKUs
  - Taking into account time, variability and service when setting inventory targets
- Quantifying supply chain analyses that could be not be done before
  - Implementing postponement
  - Reallocating consignment inventories
  - Reconciling inventory practices across geographies
  - Making channel inventory more revenue-producing

Numerical Test of Stochastic Leadtimes Model

- We draw our examples from:
  - Willems, S. P., “Real-World Multi-Echelon Supply Chains Used for Inventory Optimization,” Manufacturing & Service Operations Management, Winter 2008, Vol. 10, No. 1, pp. 19-23.
  - 26 of the 38 chains employ stochastic leadtimes
- Compare our approach to two heuristics
  - Fix stage’s leadtime to average value
  - Fix stage’s leadtime to maximum leadtime

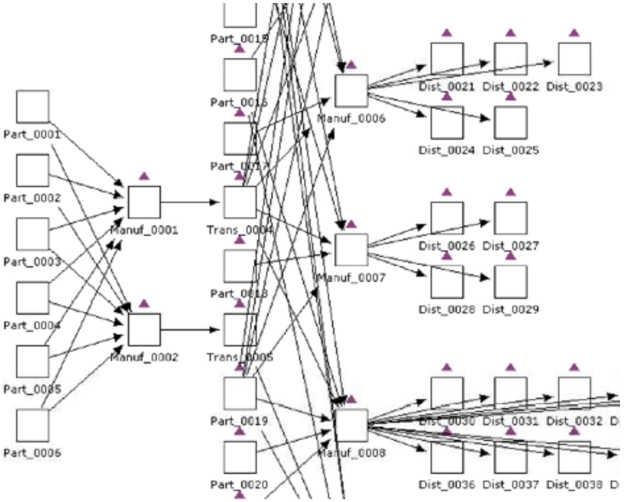
Chain 01 Analysis



- Only one stage employs a variable leadtime
- Highlights the impact of leadtime variability on safety stock
- Heuristics under and overestimate total inventory investment by 18% and 4%, respectively.

Stock level	Stochastic	Fix at Mean	Fix at Max
Safety	7,724	319	427
Pipeline	11,704	11,704	20,900
Total	19,428	12,023	21,327

Chain 12 Analysis



- 88 stages and 107 arcs
  - 16 Normally-distributed leadtimes
  - 12 discrete stochastic leadtimes
- Heuristics under and overestimate total inventory investment by 3% and 2%, respectively.
- Number of stocking locations increases from 75 to 80

Stock level	Stochastic	Fix at Mean	Fix at Max
Safety	1,532,450	1,255,293	1,268,673
Early Arrival	88,514		
Pipeline	9,921,038	9,921,038	10,545,649
Total	11,542,002	11,176,331	11,814,322

Practical Contribution: Characterizing Stochastic Leadtimes

- Few companies have leadtime data stored in centralized ERP systems
  - Planners often employ rules of thumb (10% COV everywhere)
  - Advanced planners calculate values using Excel
- Leadtime distribution must be calculated from transactional data
  - Implementation challenges include product versus SKU variability, frequency of leadtime calculation
- Implemented a system to obtain leadtime distribution from transactional data

Conclusion

- We have extended the guaranteed service model for safety stock optimization to accommodate stochastic leadtimes
  - Practical contribution includes automating leadtime characterization for large-scale systems
  - Methodological contribution includes modifying traditional safety stock formula to include early arrival stock
- This is the most-used advanced functionality in our software, making model's results match existing supply chain conditions