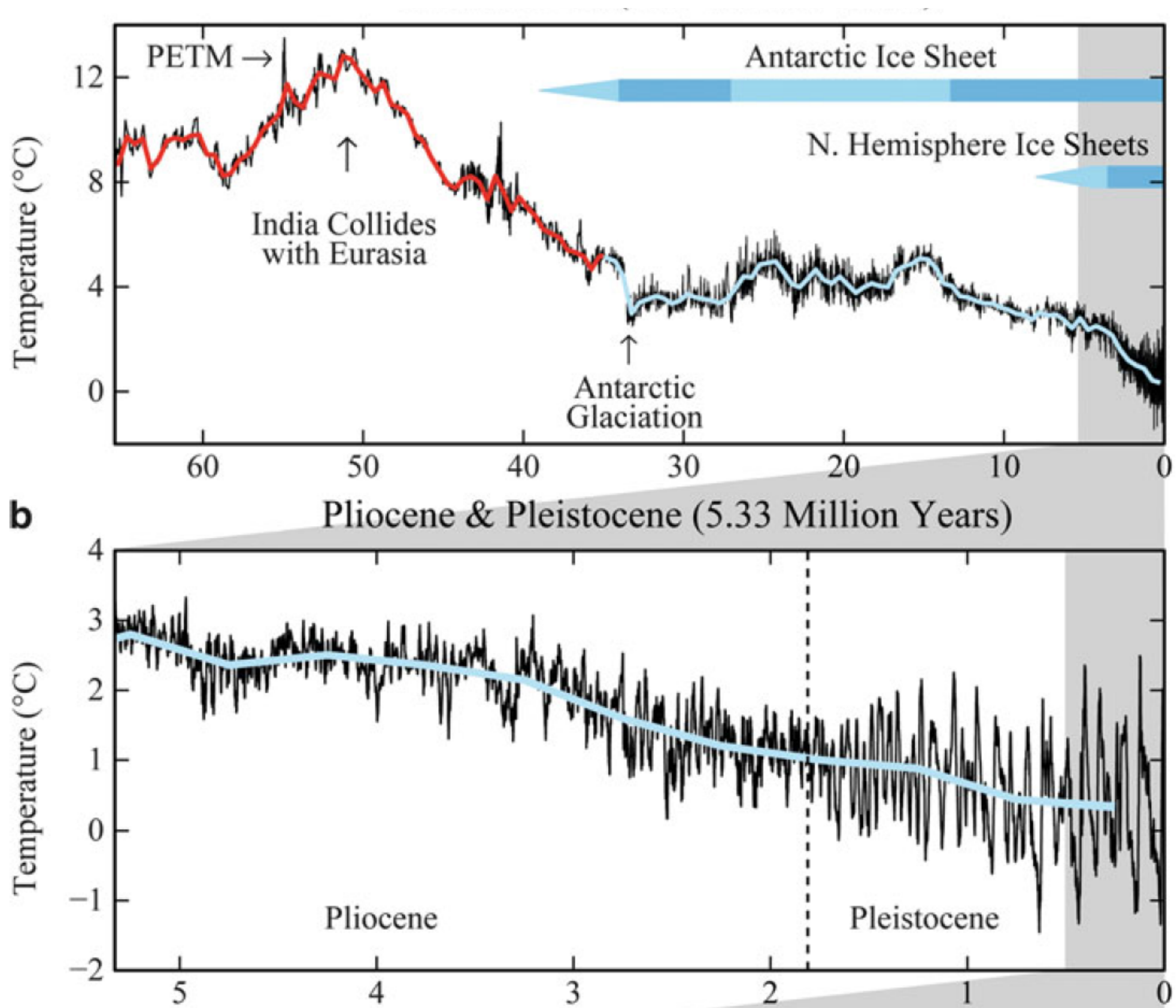
A Conceptual Glacial Cycle Model with Diffusive Heat Transport

Jim Walsh

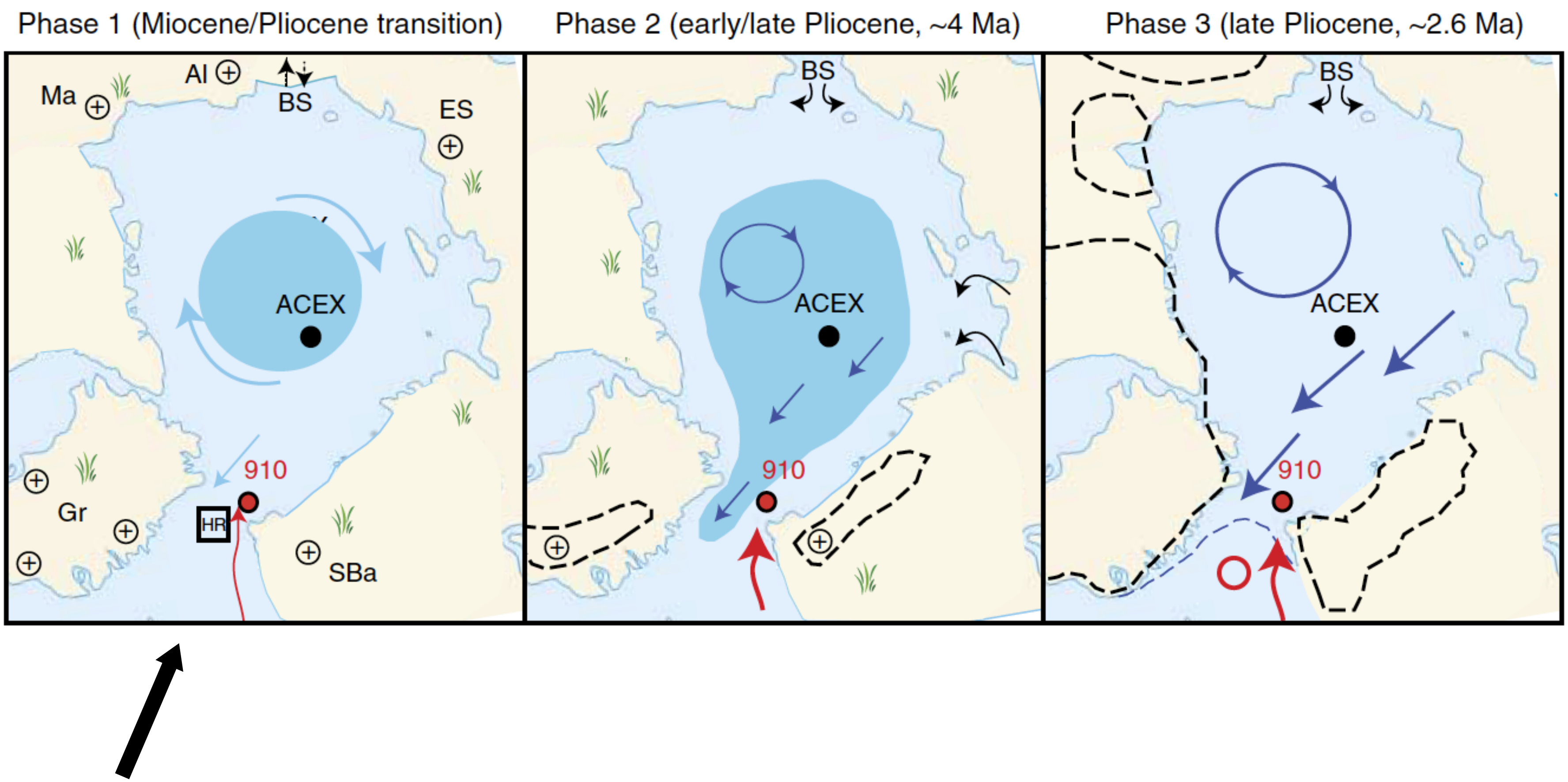
May 23, 2017

Joint work with E. Widiasih

Project Overview: Investigate the Pliocene-Pleistocene Transition (PPT) via conceptual mathematical models

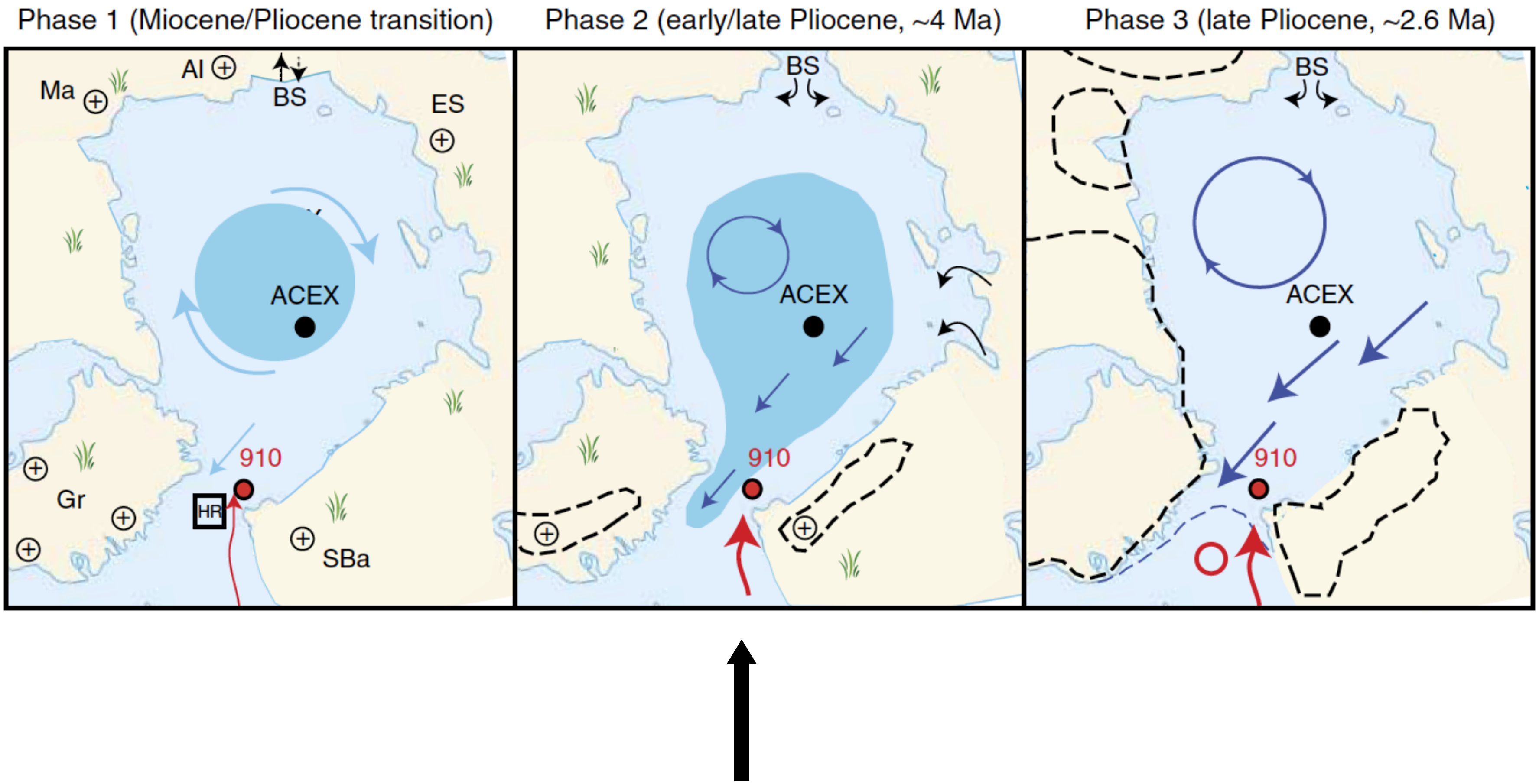


Hansen J.E., Sato M. (2012) Paleoclimate Implications for Human-Made Climate Change. In: Berger A., Mesinger F., Sijacki D. (eds) *Climate Change*. Springer.



Phase I (~5 Mya) Dense vegetation in high northern latitudes.
Arctic Ocean mainly ice-free or covered by first-year winter ice.

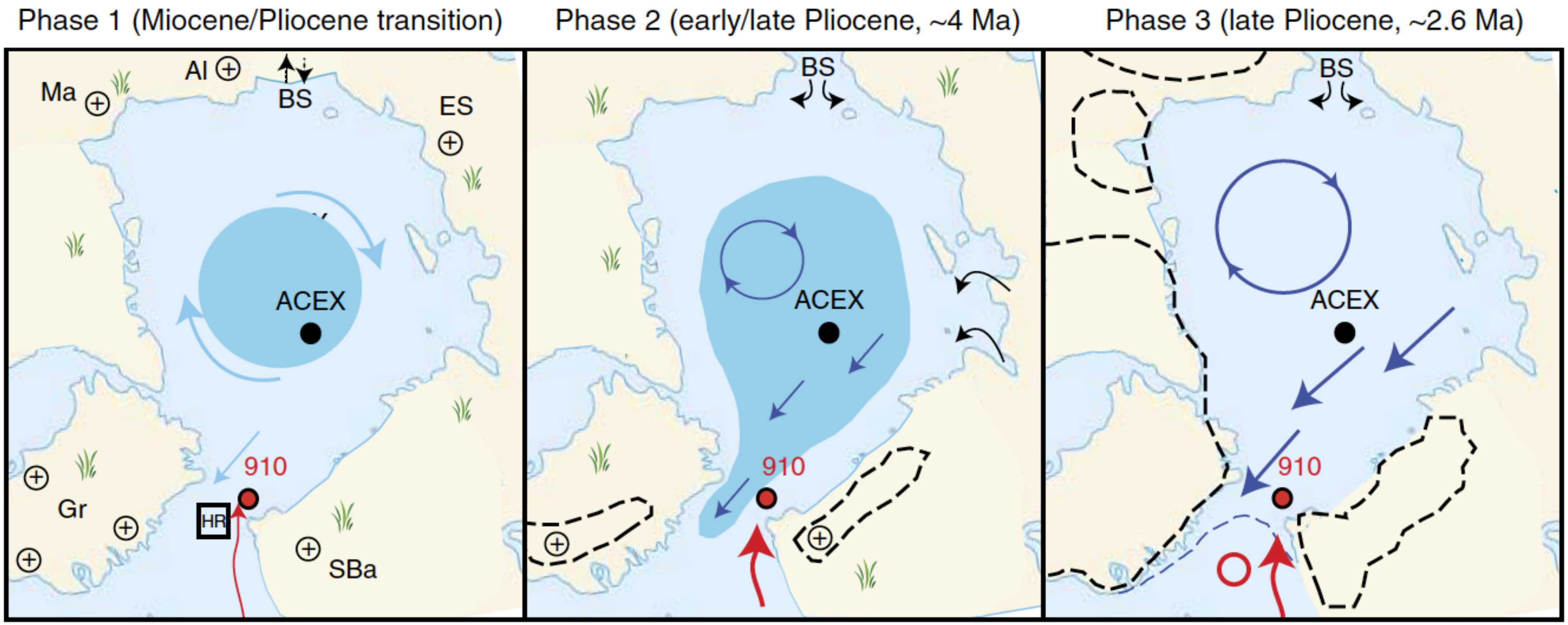
J. Knies et al. (2014). The emergence of modern sea ice cover in the Arctic Ocean. *Nature Communications*| DOI: 10.1038/ncomms6608



Phase II (~3.9 Mya) Arctic sea ice expanded to its modern summer limits.

(atmospheric CO_2 ≈ 400 ppm, 2-3^{o}C warmer than preindustrial)

J. Knies et al. (2014). The emergence of modern sea ice cover in the Arctic Ocean. *Nature Communications*| DOI: 10.1038/ncomms6608

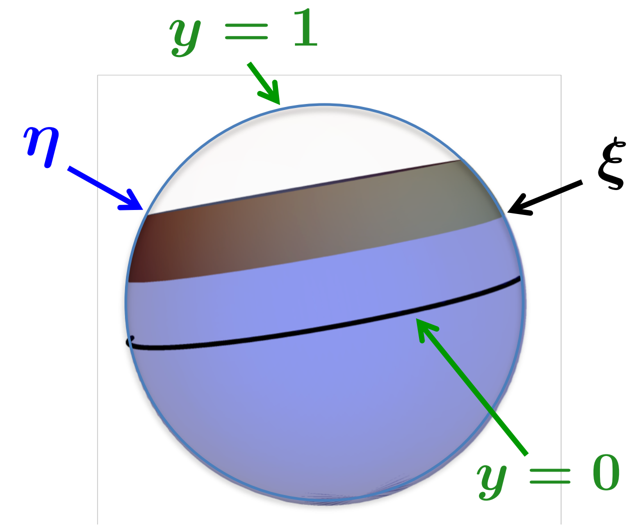


Phase III (~2.6 Mya) Arctic sea ice expanded to its modern winter limits.

J. Knies et al. (2014). The emergence of modern sea ice cover in the Arctic Ocean. *Nature Communications*| DOI: 10.1038/ncomms6608

The model

- $T(y,t)$ – zonal annual mean temperature
  at $y = \sin\theta$ (θ latitude)
- η – albedo line
- ξ – glacier's edge



$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + D\frac{\partial}{\partial y}(1-y^2)\frac{\partial T}{\partial y}$$

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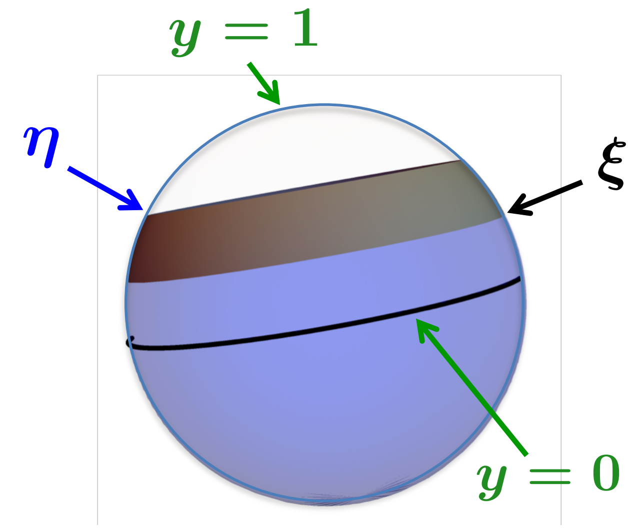
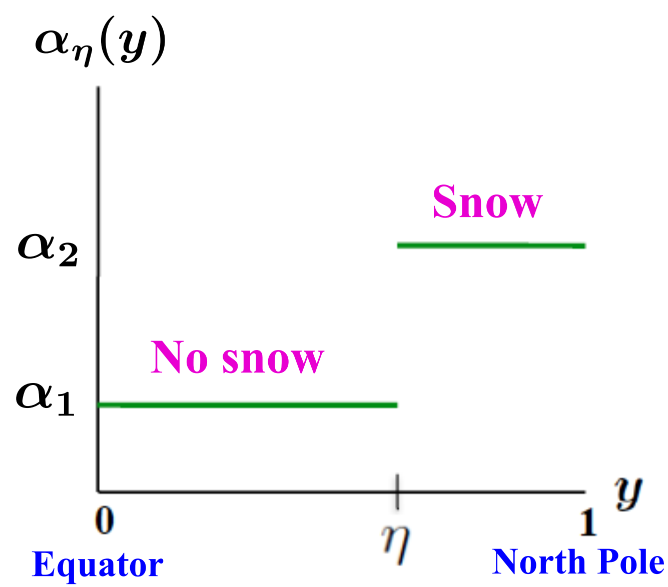


$$R\frac{\partial T}{\partial t} = Qs(y)(1-\alpha(y,\eta)) - (A+BT) + D\frac{\partial}{\partial y}(1-y^2)\frac{\partial T}{\partial y}$$

insolation

insolation distribution function

albedo



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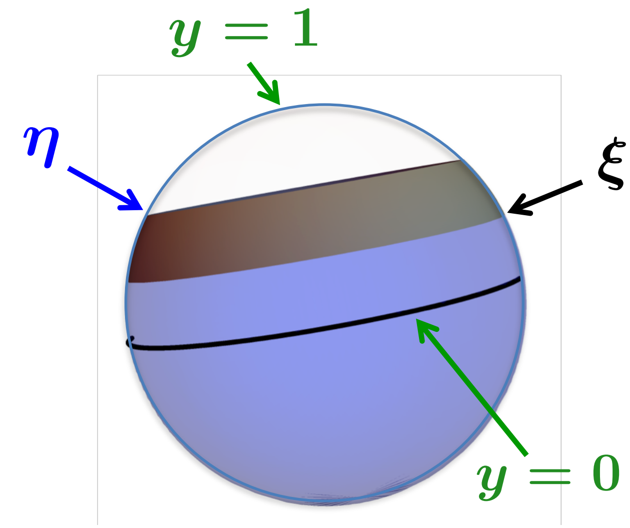

$$R\frac{\partial T}{\partial t} \quad = \quad Qs(y)(1-\alpha(y,\eta)) - (A+BT) + D\frac{\partial}{\partial y}(1-y^2)\frac{\partial T}{\partial y}$$

Outgoing longwave radiation

Diffusion coefficient

Meridional heat transport: diffusive process

The model

- $T(y,t)$ – zonal annual mean temperature at $y = \sin\theta$ (θ latitude)
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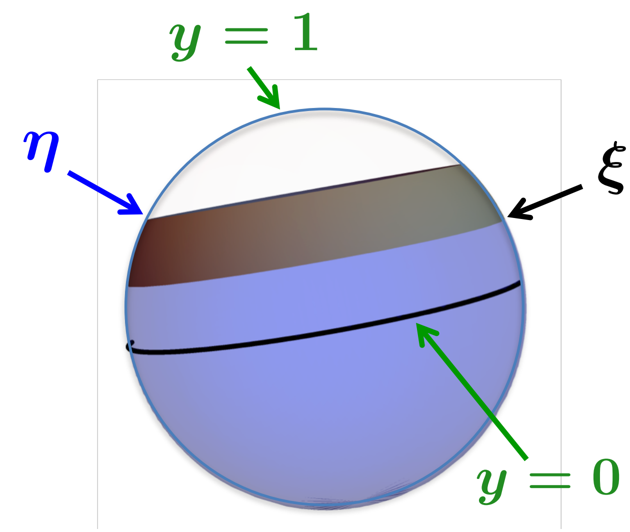
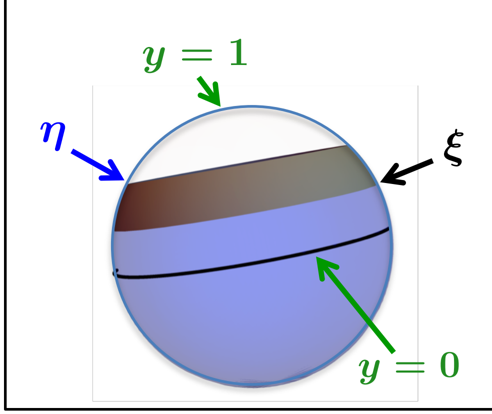


$$
\begin{aligned}
R\frac{\partial T}{\partial t} &= Qs(y)(1-\alpha(y,\eta)) - (A+BT) + D\frac{\partial}{\partial y}(1-y^2)\frac{\partial T}{\partial y} \\
\frac{d\eta}{dt} &= \rho(T(\eta,t) - T_c)
\end{aligned}
$$

Critical temperature

E. Widiasih, Dynamics of the Budyko energy balance model, *SIAM J. Appl. Dyn. Syst.* **12** (2013), 2068-2092.

The spectral method



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\frac{d\eta}{dt} &= \rho(T(\eta,t)-T_c)
\end{aligned}
$$

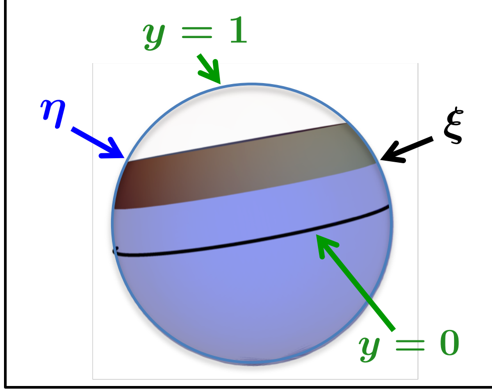
$$\boxed{T(y,t) = \sum_{n=0}^{N} T_n(t)p_n(y), \quad p_n(y) \text{ - } n\text{th } \underline{\textit{even}} \text{ Legendre polynomial}}$$

(expansions for functions $s(y)$ and $s(y)\alpha(y,\eta)$ as well)

H. Kaper and H. Engler, *Mathematics and Climate*, SIAM (2013)

Model equations

$$T(y,t) = \sum_{n=0}^{N} T_n(t) p_n(y)$$



$$\begin{cases} \dfrac{dT_n}{dt} = -\gamma_n (T_n - f_n(\eta)), \; n = 0, 1, .., N \\[2ex] \dfrac{d\eta}{dt} = \rho \left(\displaystyle\sum_{n=0}^{N} T_n p_n(\eta) \; - \; T_c \right) \end{cases}$$

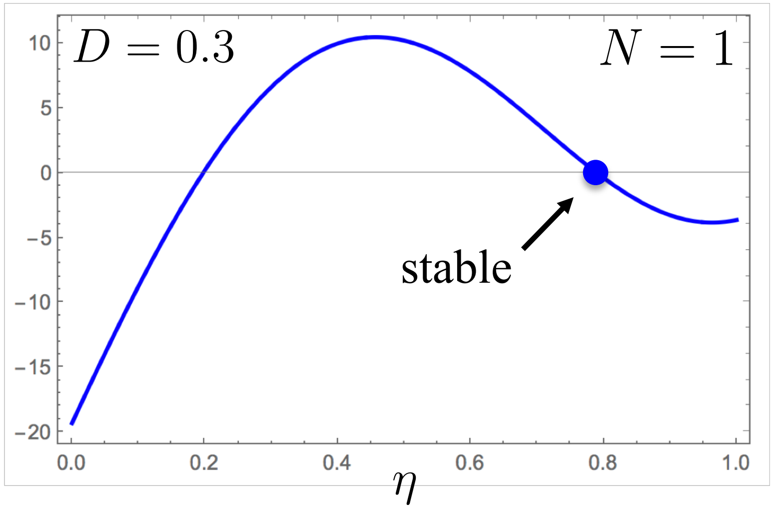
- Attracting curve of rest points Λ when $\rho = 0$
- Λ perturbs to an attracting invariant manifold, and system can be approximated by

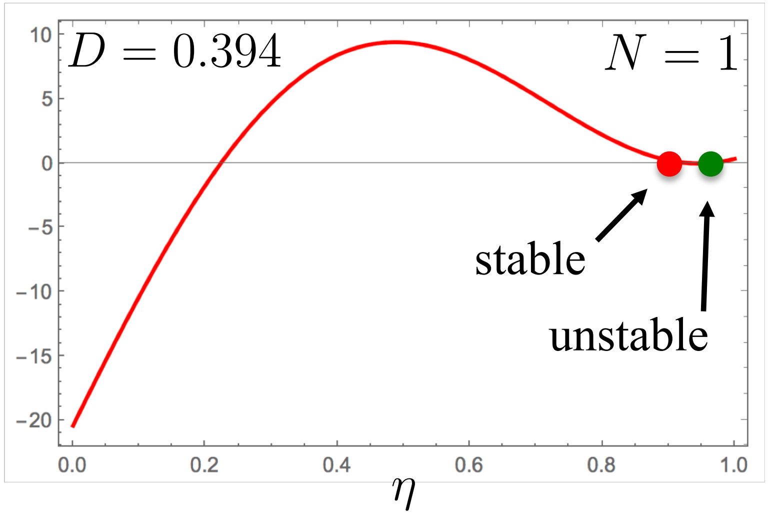
$$\boxed{\frac{d\eta}{dt} = \rho \left(\sum_{n=0}^{N} f_n(\eta) p_n(\eta) \; - \; T_c \right) \equiv \rho \, h(\eta, D)}$$

Plots of $\boldsymbol{h(\eta, D)}$

$$\frac{d\eta}{dt} = \rho\, h(\eta, D)$$

$$T(y,t) = \sum_{n=0}^{N} T_n(t) p_n(y)$$

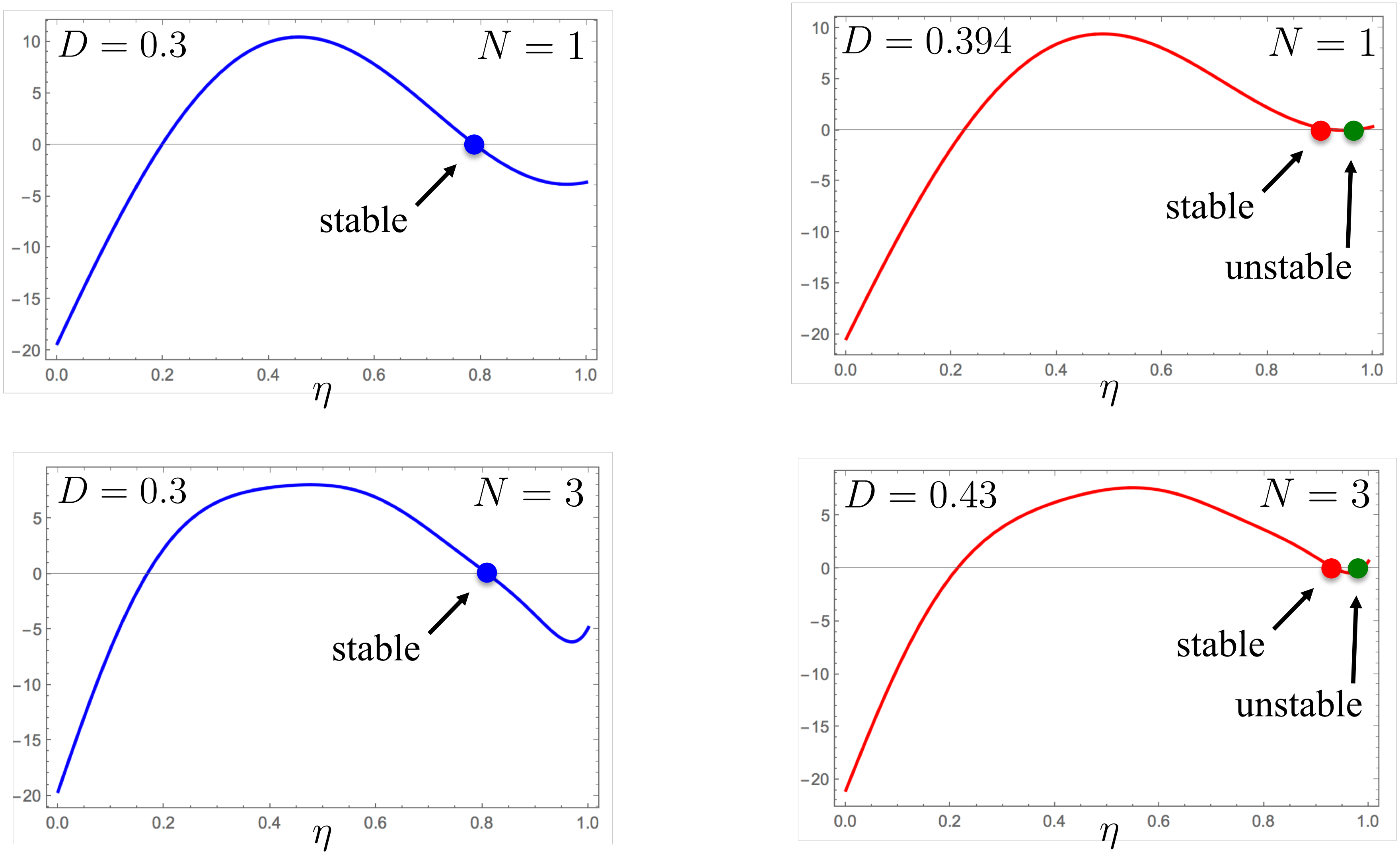




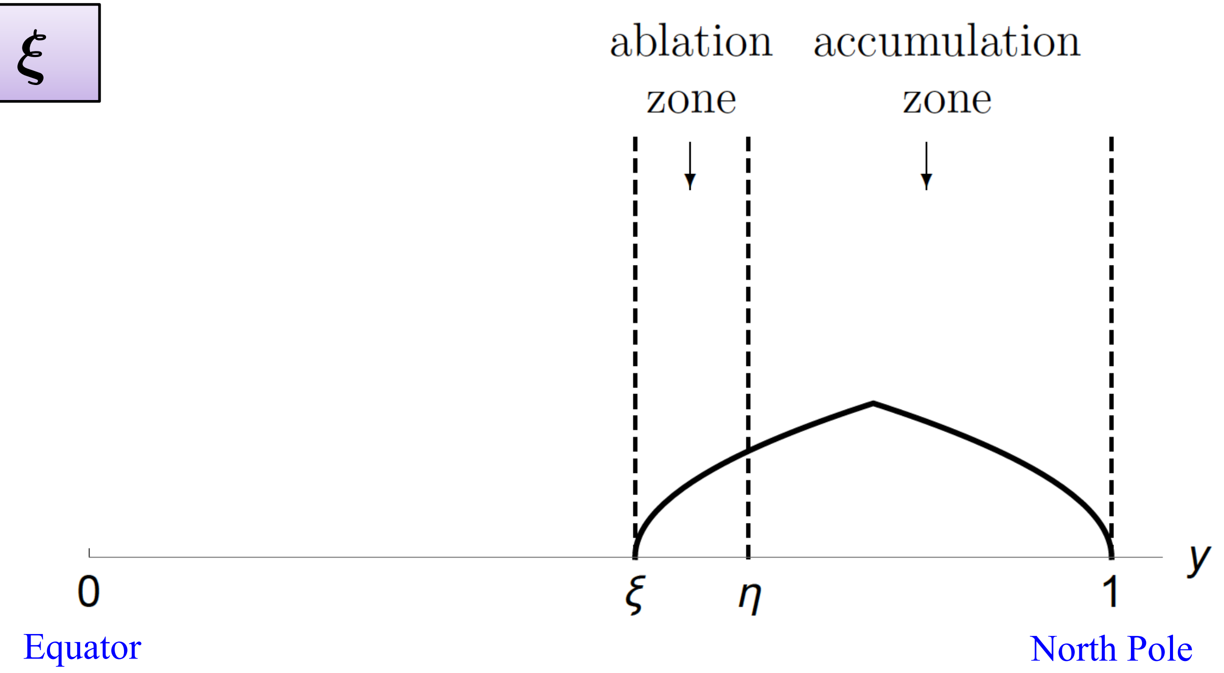
Plots of $\boldsymbol{h(\eta, D)}$

$$\frac{d\eta}{dt} = \rho\, h(\eta, D)$$

$$T(y,t) = \sum_{n=0}^{N} T_n(t) p_n(y)$$



Mass balance: ξ



$$\begin{cases} \dfrac{d\eta}{dt} = \rho\, h(\eta, D) \\ \dfrac{d\xi}{dt} = \epsilon(b(\eta - \xi) - a(1 - \eta)) \end{cases}$$

Ablation rate

Accumulation rate

Two states: Flip-flop

Glacial state

- Ablation rate b_G is smaller
- Diffusion coeff. D_G is smaller

$$\begin{cases} \dfrac{d\eta}{dt} = \rho\, h(\eta, D_G) \\ \dfrac{d\xi}{dt} = \epsilon(b_G(\eta - \xi) - a(1-\eta)) \end{cases}$$

Interglacial state

- Ablation rate b_I is larger
- Diffusion coeff. D_I is larger

$$\begin{cases} \dfrac{d\eta}{dt} = \rho\, h(\eta, D_I) \\ \dfrac{d\xi}{dt} = \epsilon(b_I(\eta - \xi) - a(1-\eta)) \end{cases}$$

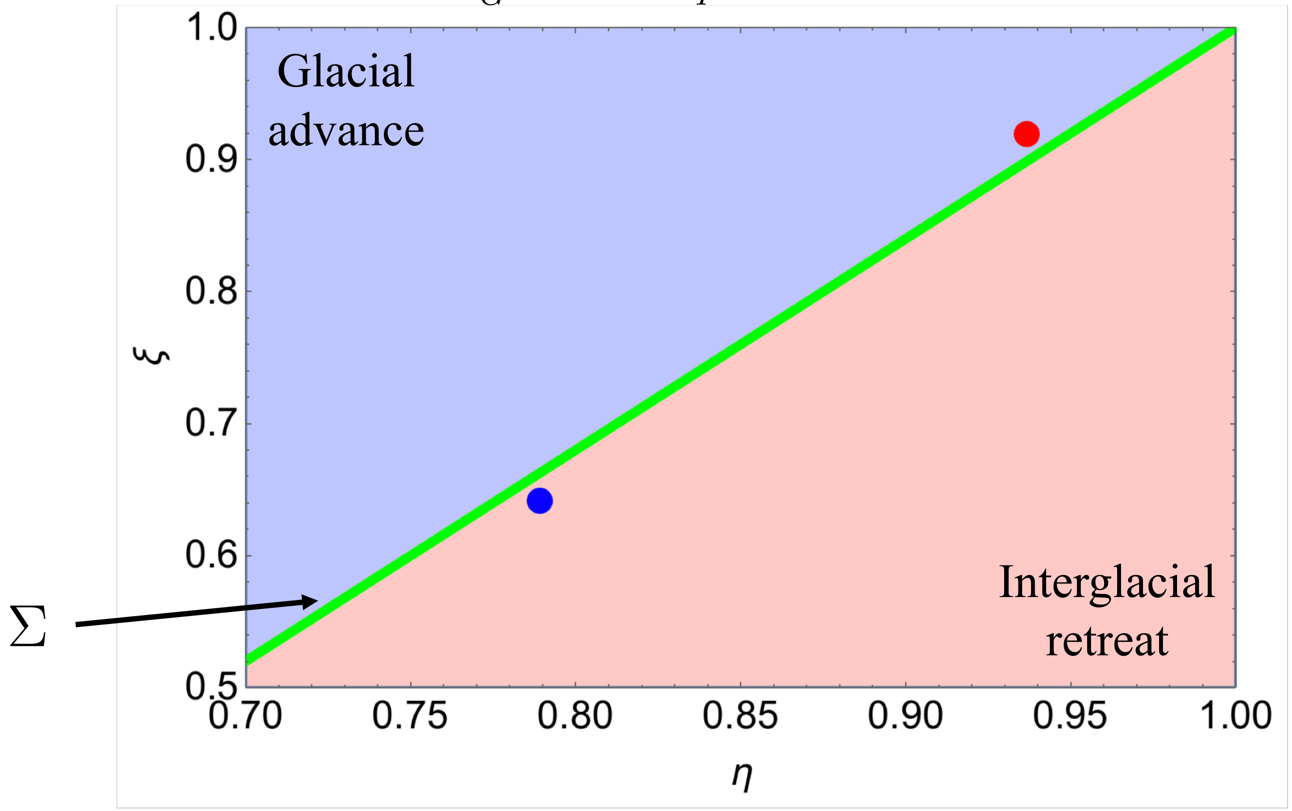
Switching boundary $\Sigma = \{(\eta, \xi) : b(\eta - \xi) - a(1-\eta) = 0\}$

critical ablation rate $b \in (b_G, b_I)$

$$\begin{cases} \dfrac{d\eta}{dt} = \rho\, h(\eta, D_G) \\ \dfrac{d\xi}{dt} = \epsilon(b_G(\eta - \xi) - a(1-\eta)) \end{cases}$$

$$\begin{cases} \dfrac{d\eta}{dt} = \rho\, h(\eta, D_I) \\ \dfrac{d\xi}{dt} = \epsilon(b_I(\eta - \xi) - a(1-\eta)) \end{cases}$$

$D_G = 0.3 \quad D_I = 0.394 \quad N = 1$



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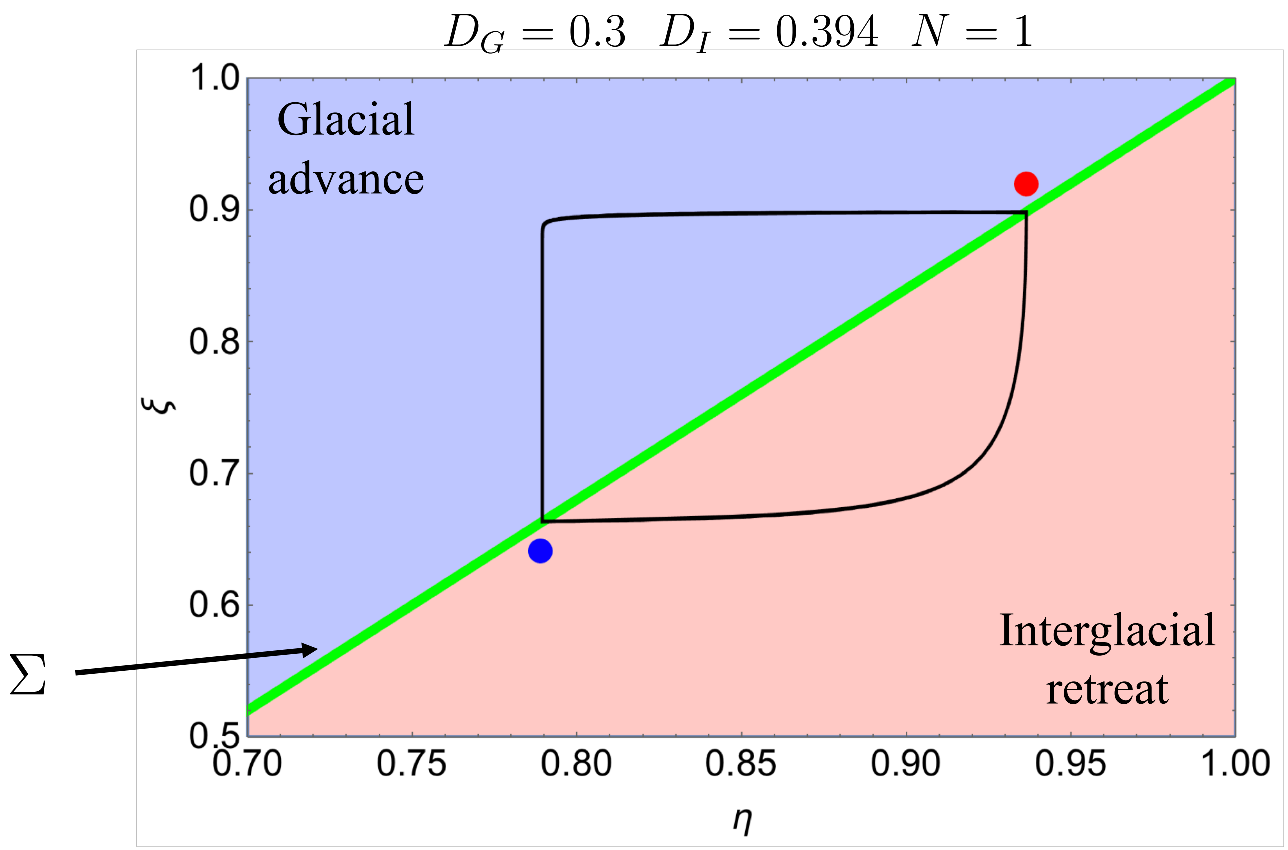

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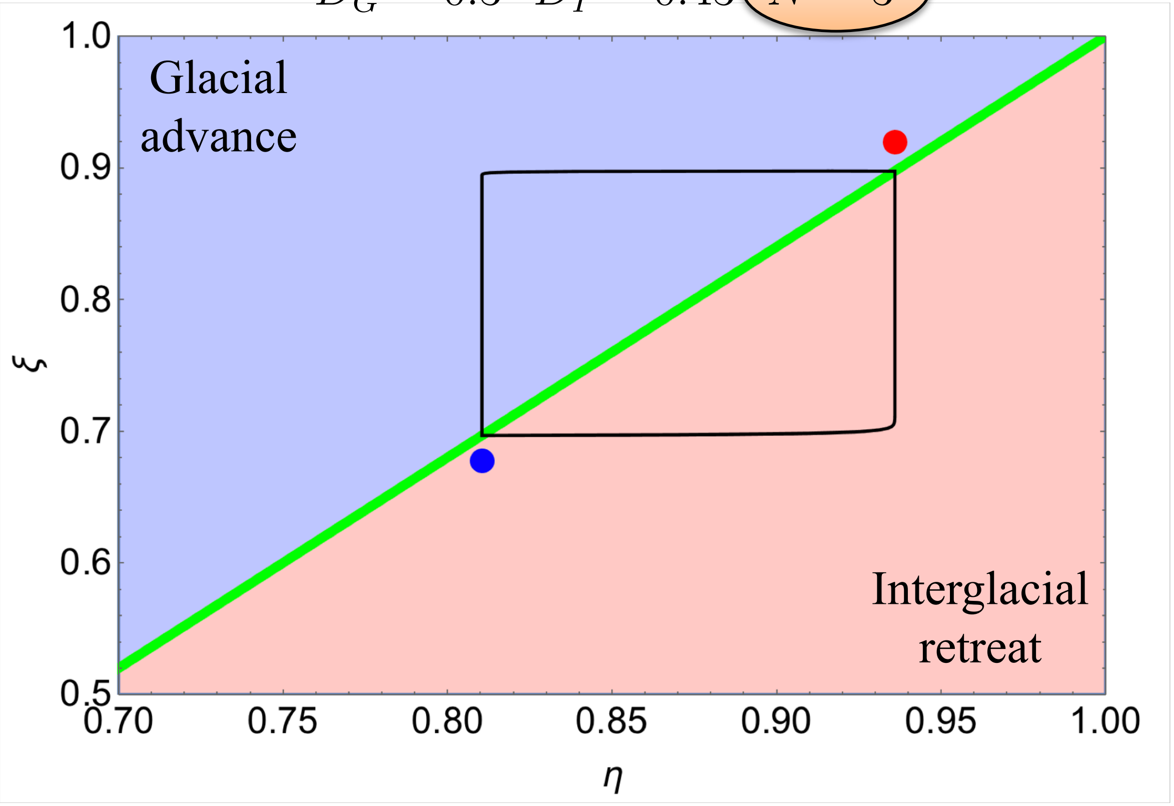
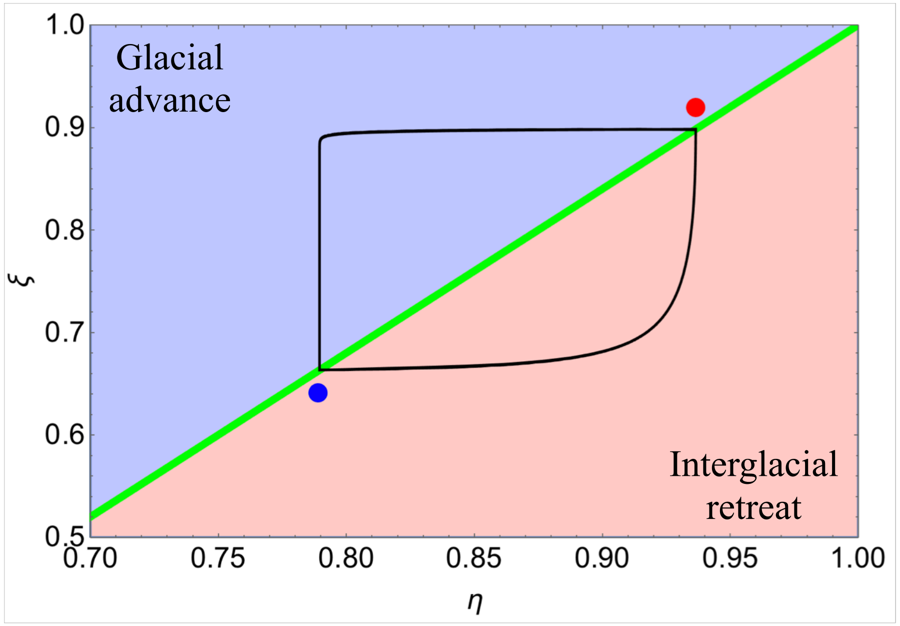
$D_G = 0.3 \quad D_I = 0.43 \quad N = 3$

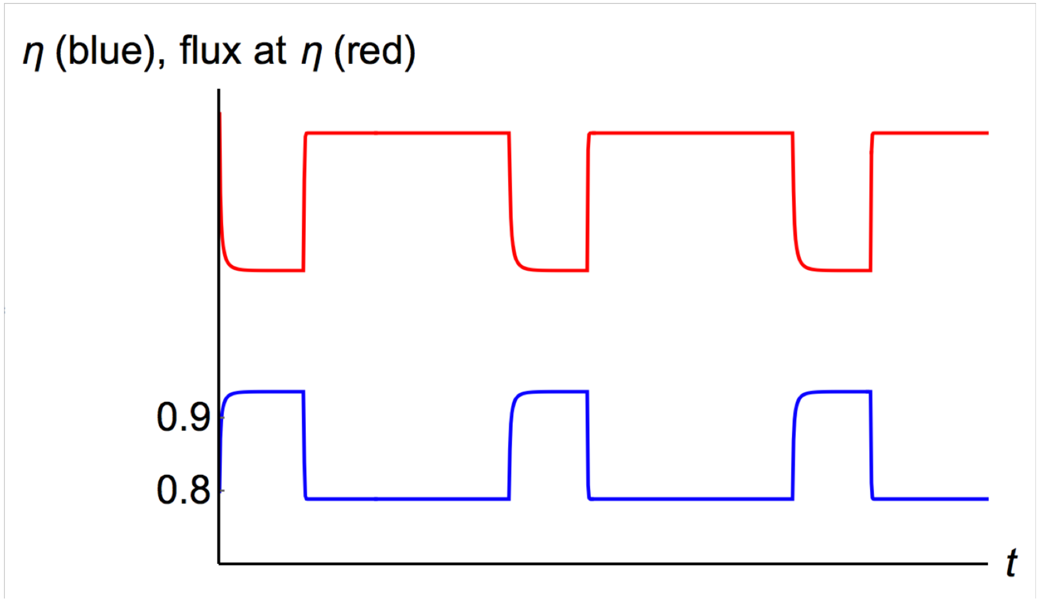


Phase III

$\exists$ unique (nonsmooth) limit cycle for ϵ suff. small

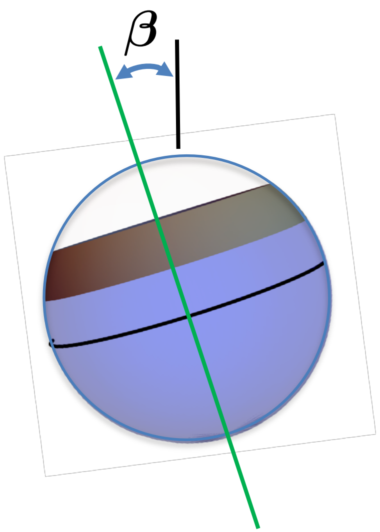
Flux at η

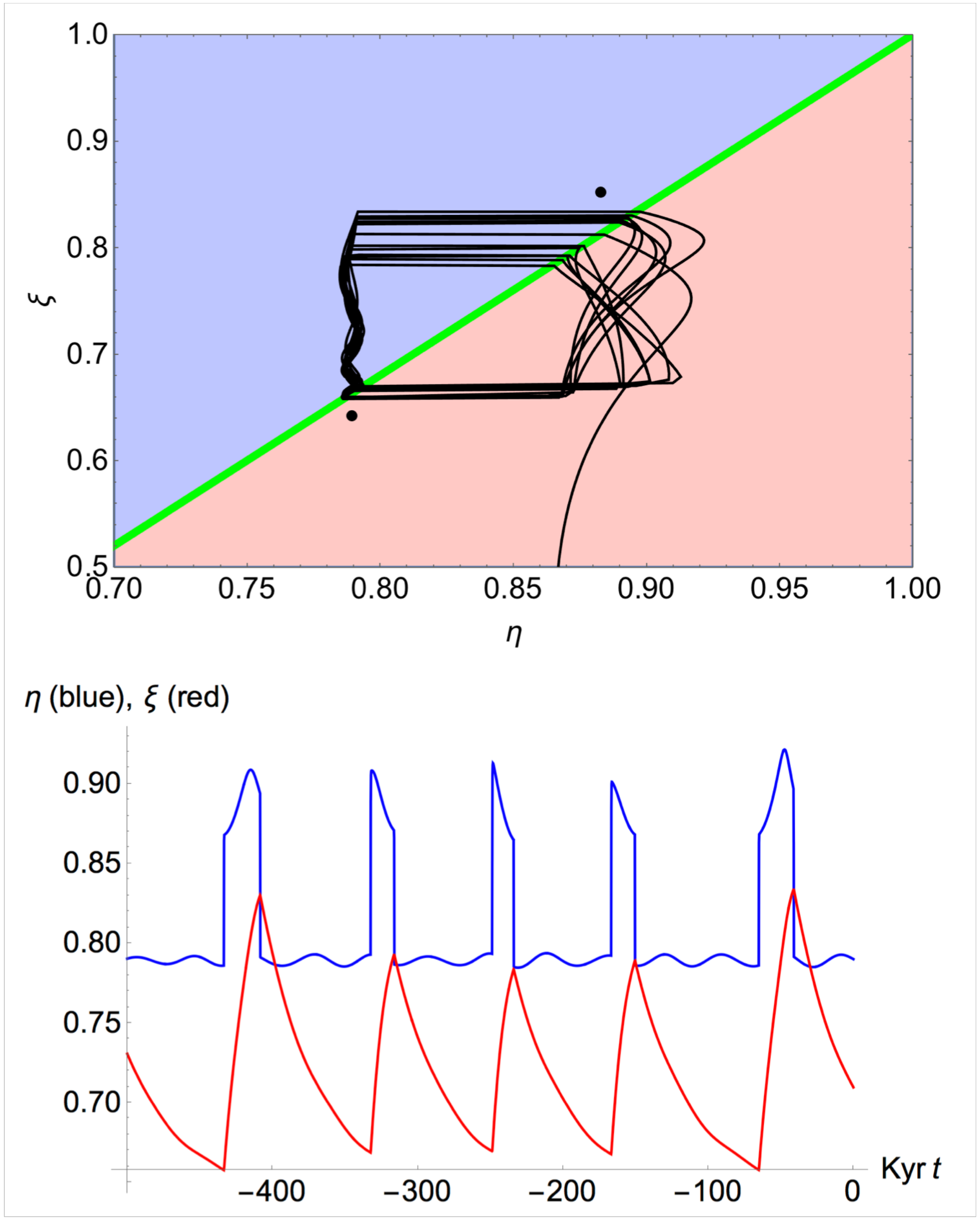




⬅ Watts (scaled)

Obliquity forcing





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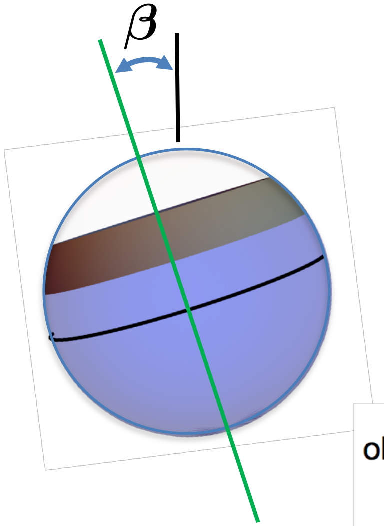
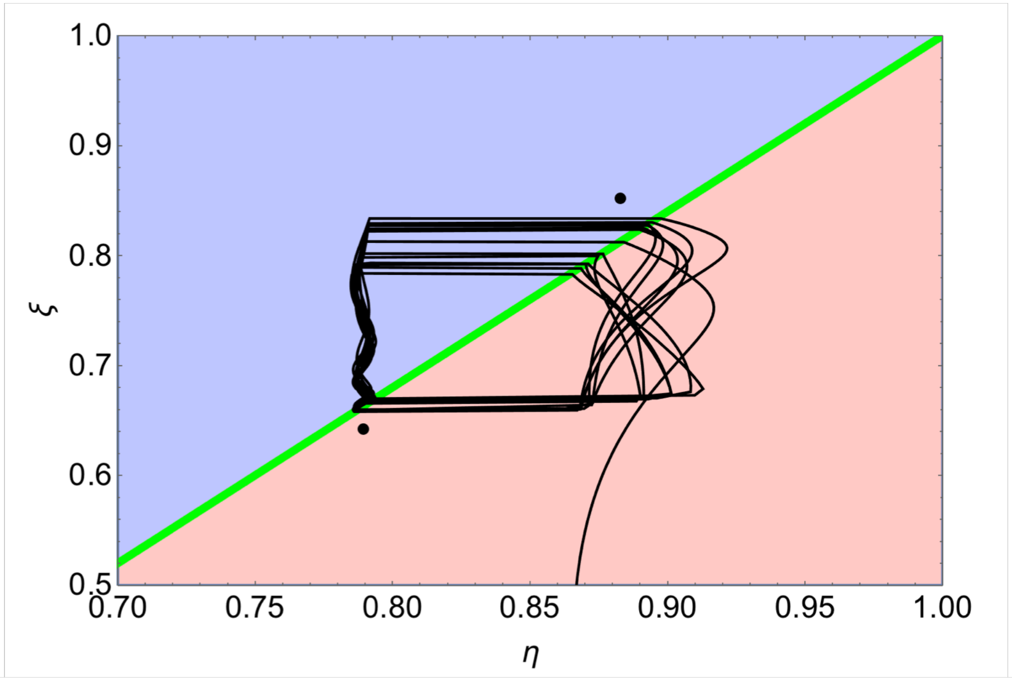
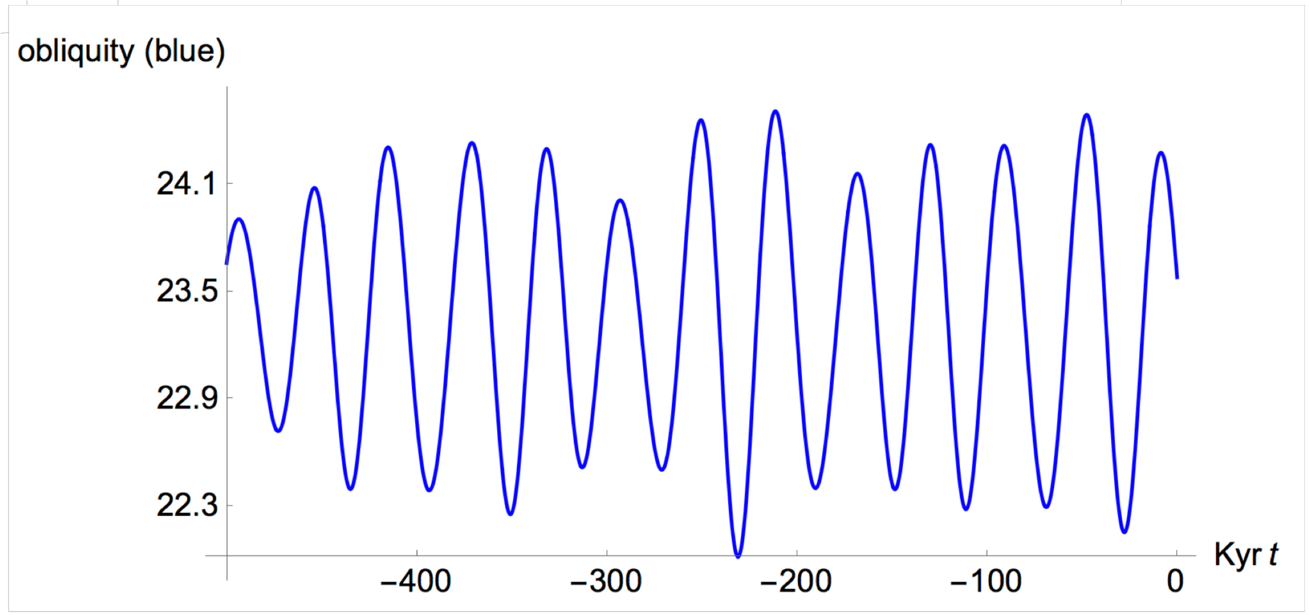
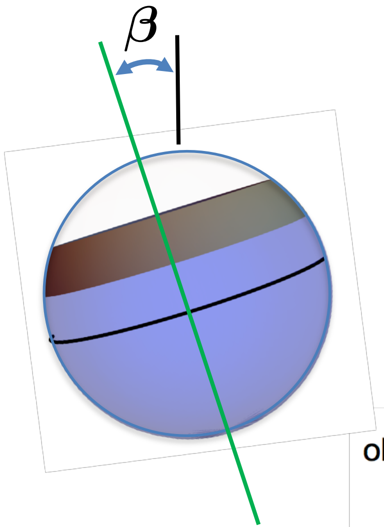
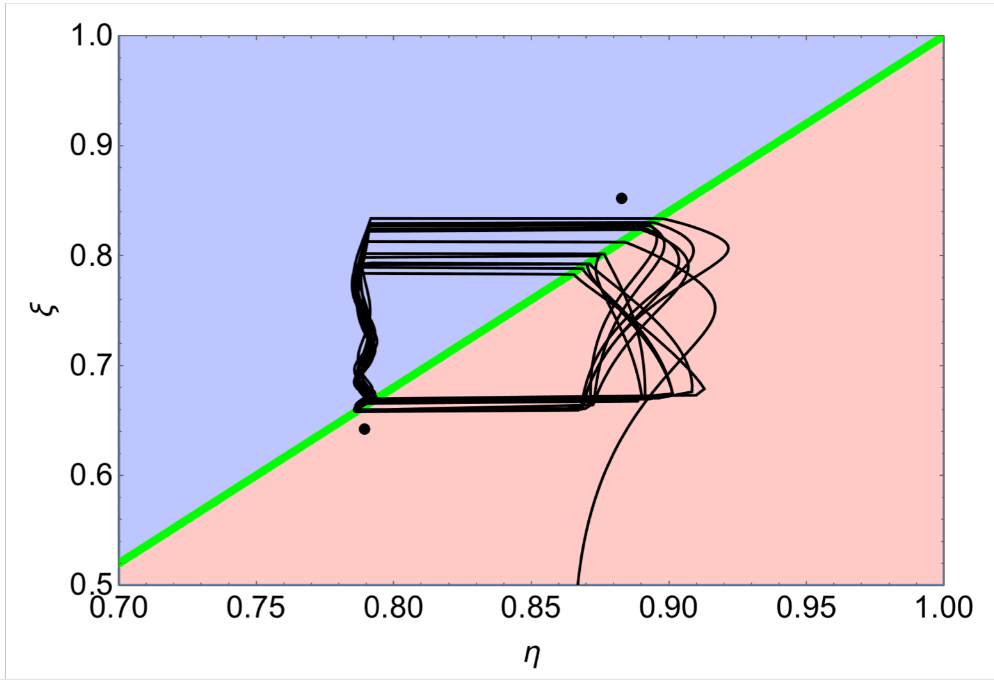


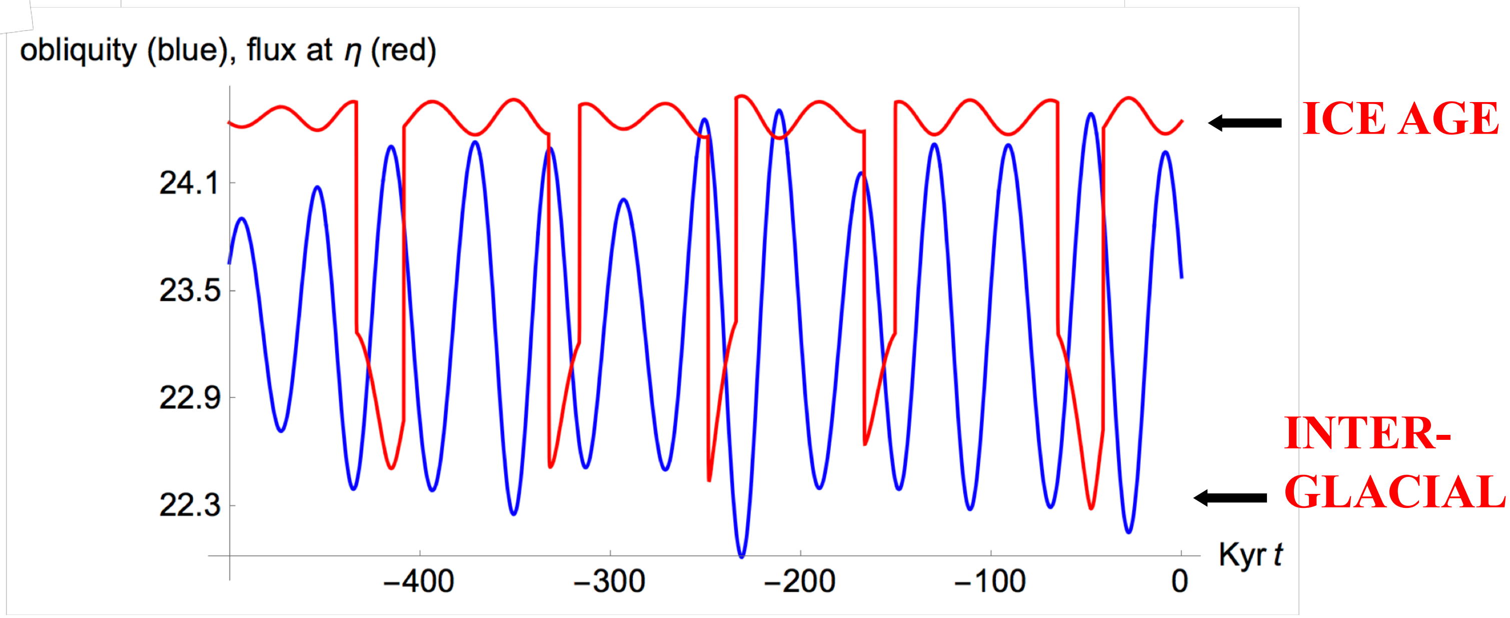




Obliquity forcing







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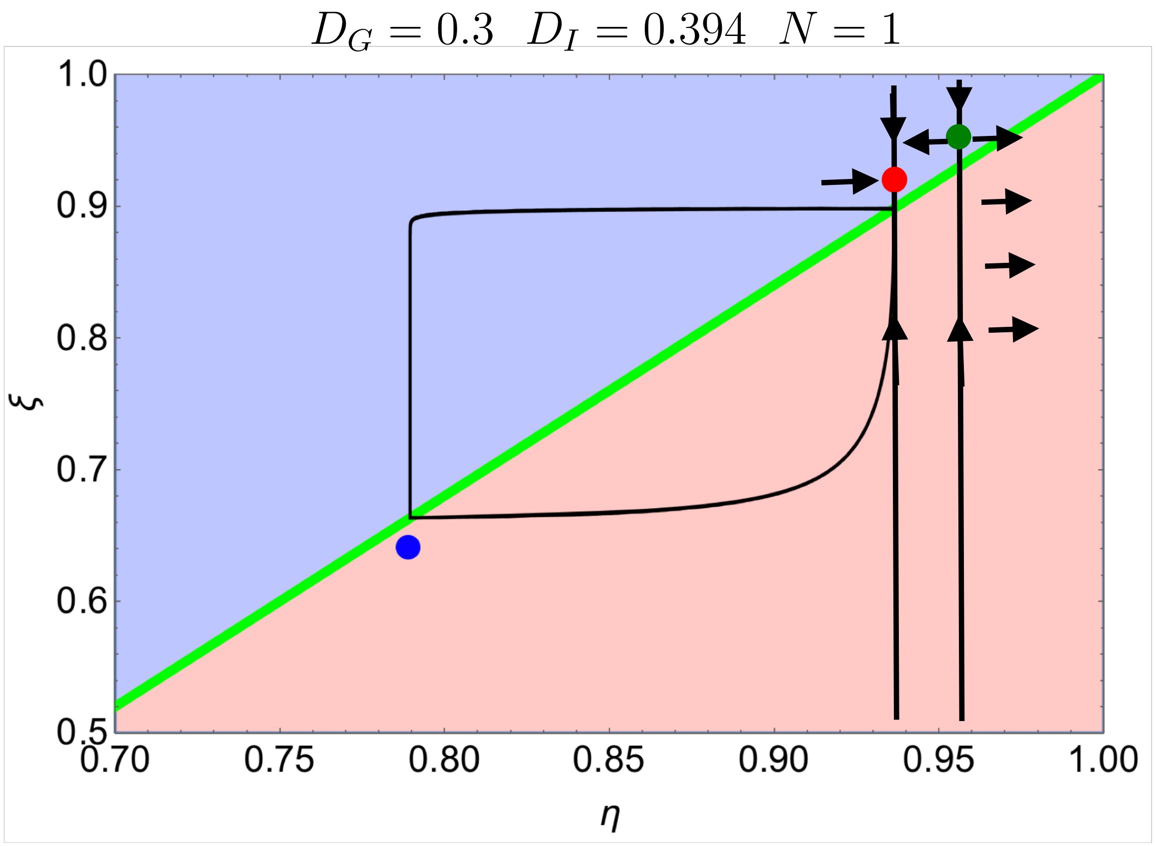

Obliquity forcing
Decrease CO_2?
Phase II?

$$\begin{cases} \dfrac{d\eta}{dt} = \rho\, h(\eta, D_G) \\ \dfrac{d\xi}{dt} = \epsilon(b_G(\eta - \xi) - a(1-\eta)) \end{cases}$$

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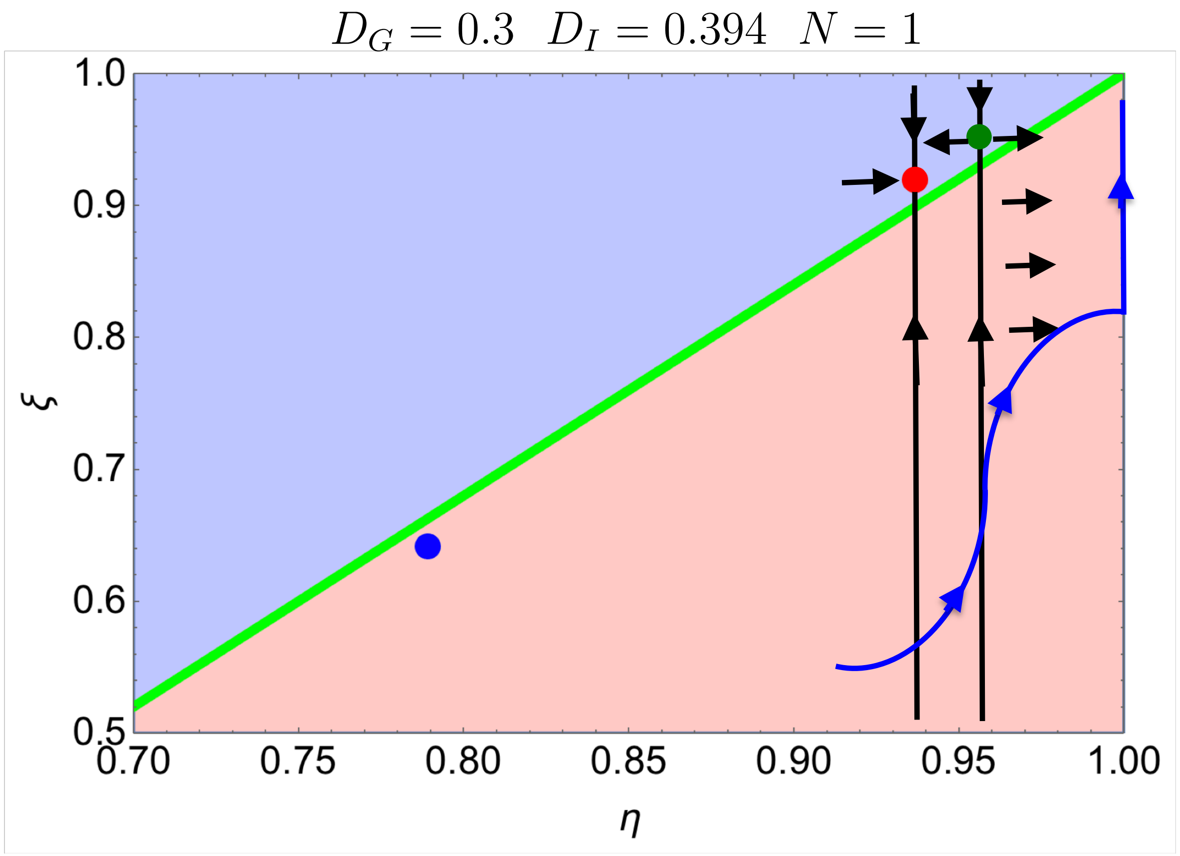

Obliquity forcing?
Decrease CO_2?
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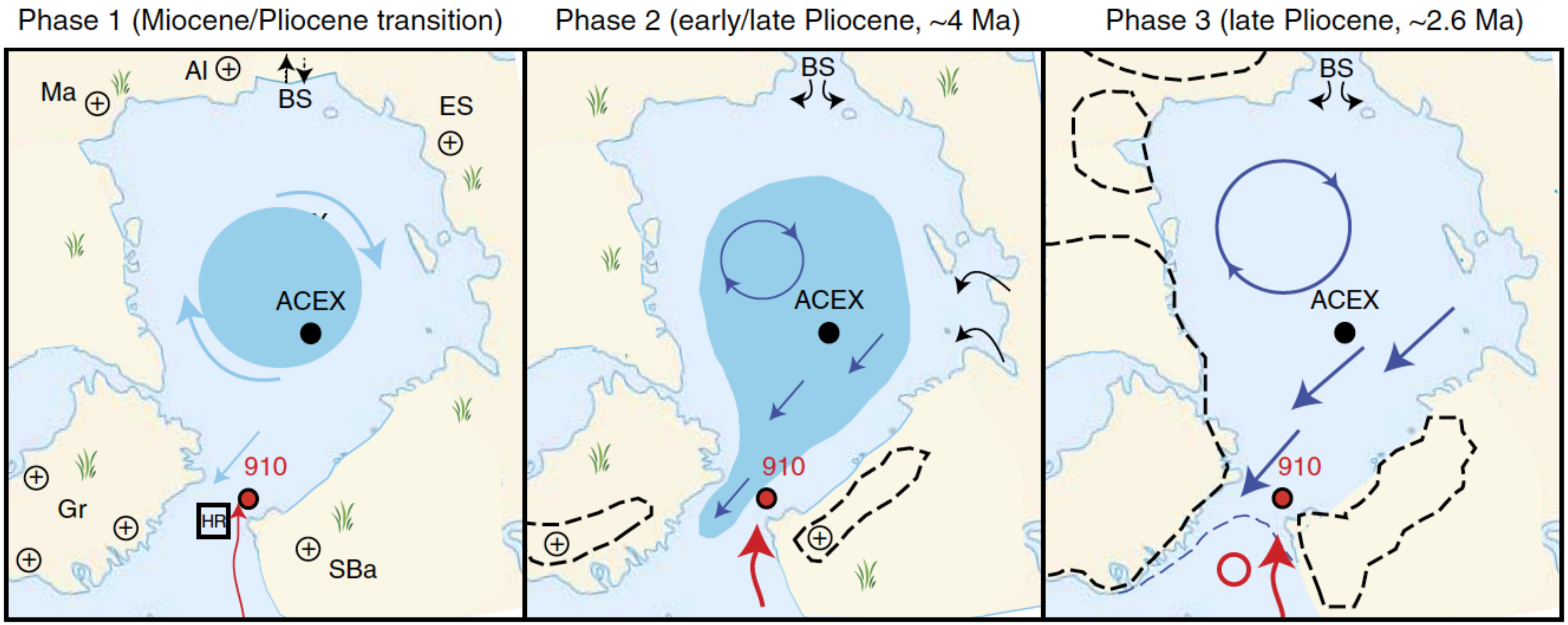
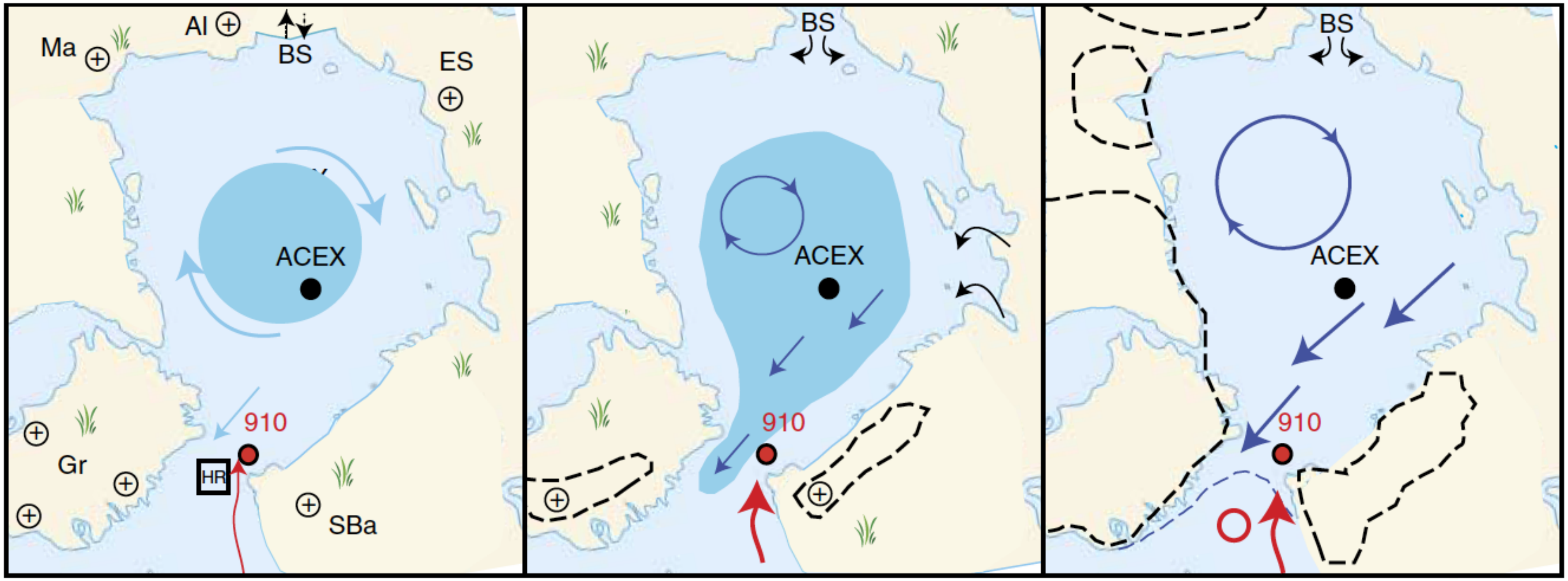


Data indicates: "threshold behavior," an "abrupt transition" in the intensification of Northern Hemishphere ice sheet growth

J. Knies et al. (2014). The emergence of modern sea ice cover in the Arctic Ocean. *Nature Communications*| DOI: 10.1038/ncomms6608

Phase 1 (Miocene/Pliocene transition) Phase 2 (early/late Pliocene, ~4 Ma) Phase 3 (late Pliocene, ~2.6 Ma)



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Thank you!

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