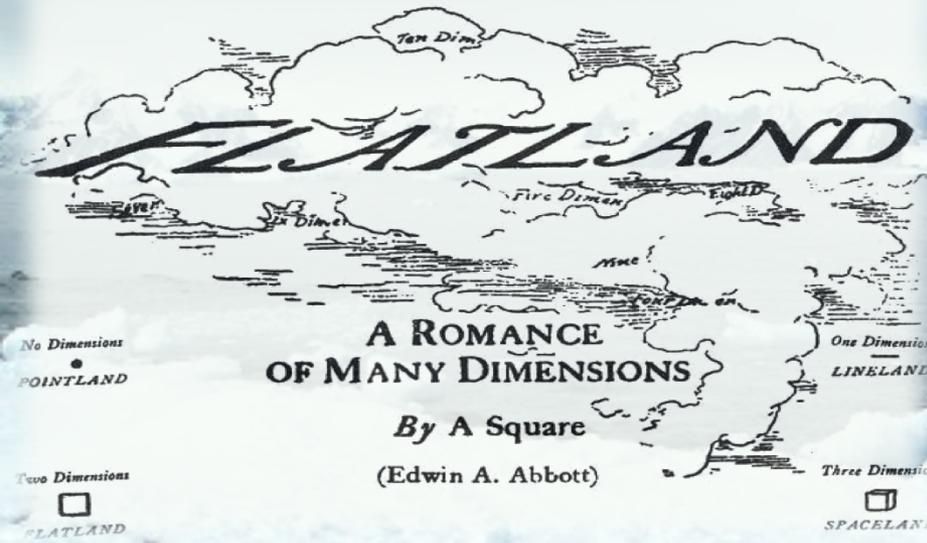


"O day and night, but this is wondrous strange"

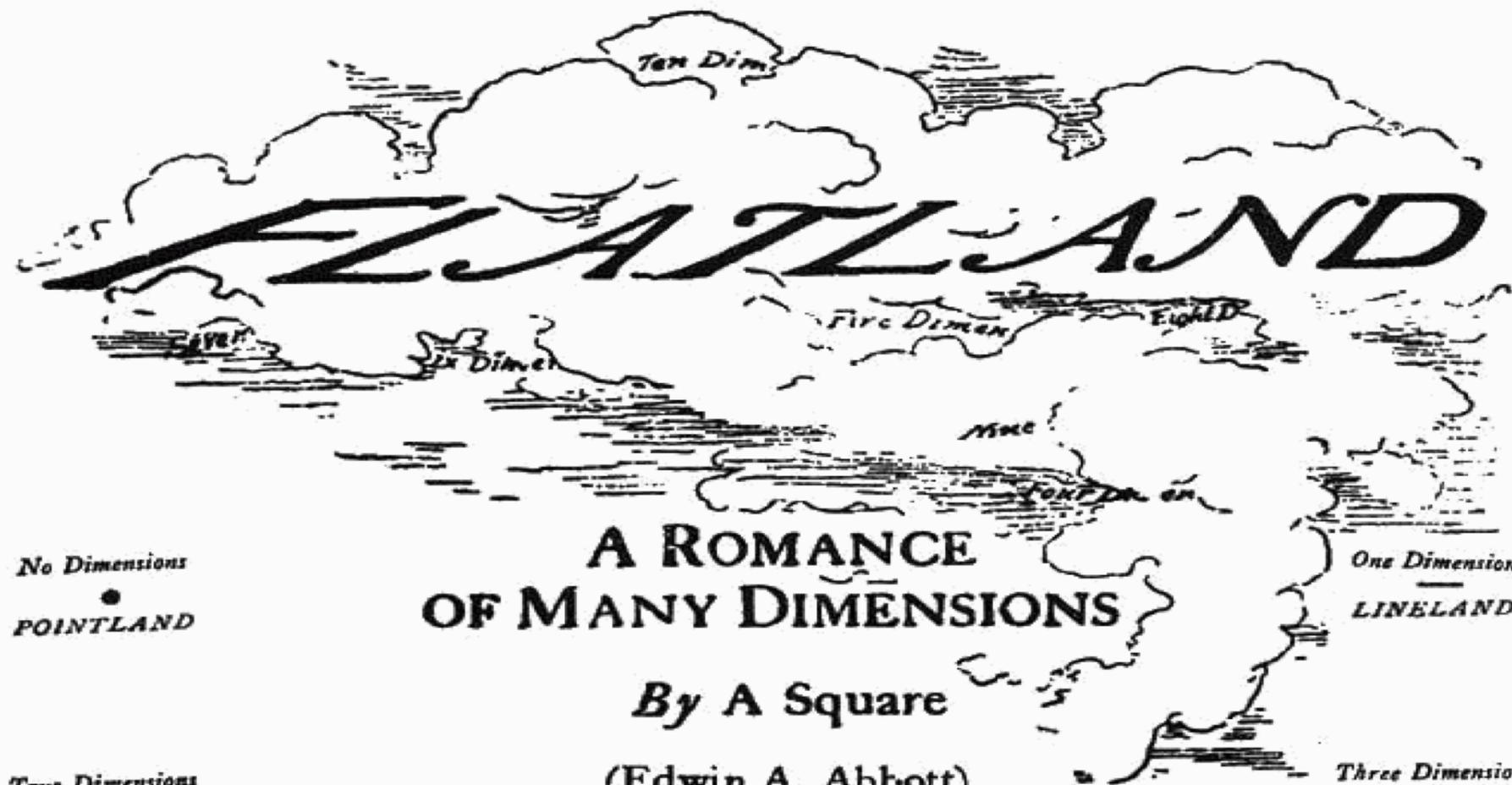


# From Flatland to Our Land

A mathematician's journey through our changing planet

Emily Shuckburgh @emilyshuckburgh  
British Antarctic Survey

*"O day and night, but this is wondrous strange"*



No Dimensions  
●  
POINTLAND

Two Dimensions  
□  
FLATLAND

# A ROMANCE OF MANY DIMENSIONS

*By A Square*  
(Edwin A. Abbott)

One Dimension  
—  
LINELAND

Three Dimensions  
▢  
SPACELAND

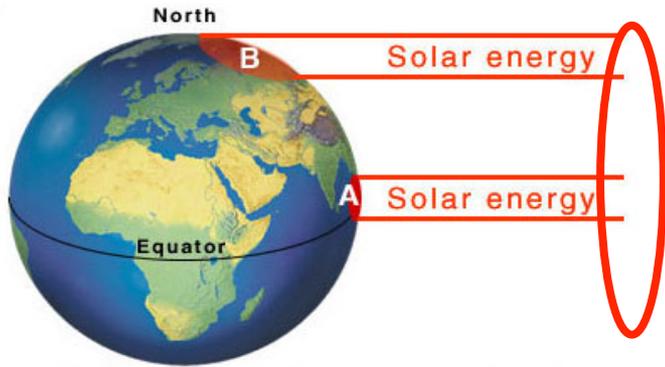
## Joseph Fourier, 1827: Mémoire sur les températures du globe terrestre et des espaces planétaires



He said he hoped:

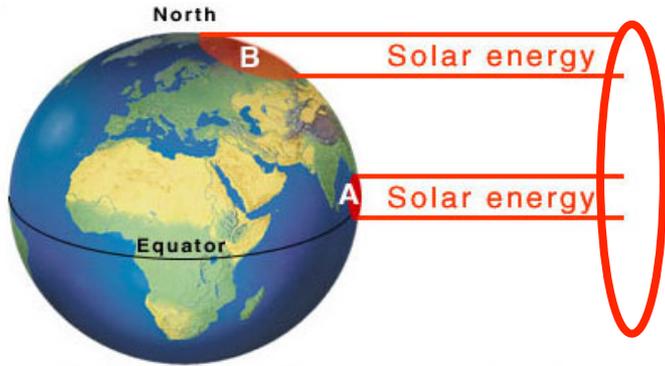
**“geometers would not only go on in their researches into questions of calculus, but they would consider... the question of terrestrial temperature, one of the most important and most difficult of all of natural philosophy”.**

# What determines the Earth's temperature?

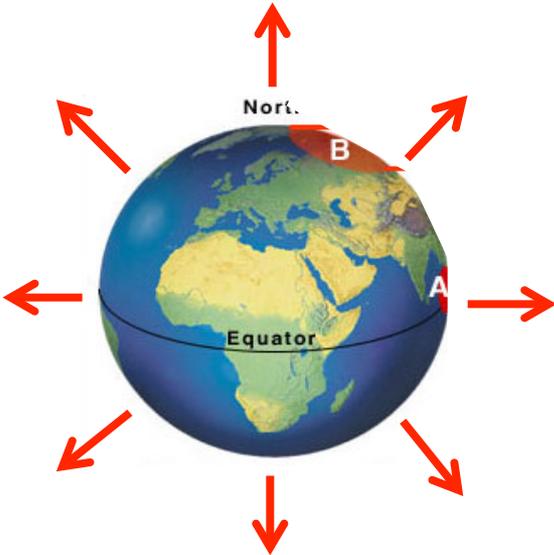


- Incident solar flux  $F = 1370 \text{ W/m}^2$
- Cross-sectional area  $\pi a^2$
- 30% reflected (albedo  $\alpha = 0.3$ )
- **incoming power** =  $(1 - \alpha)F\pi a^2$

# What determines the Earth's temperature?



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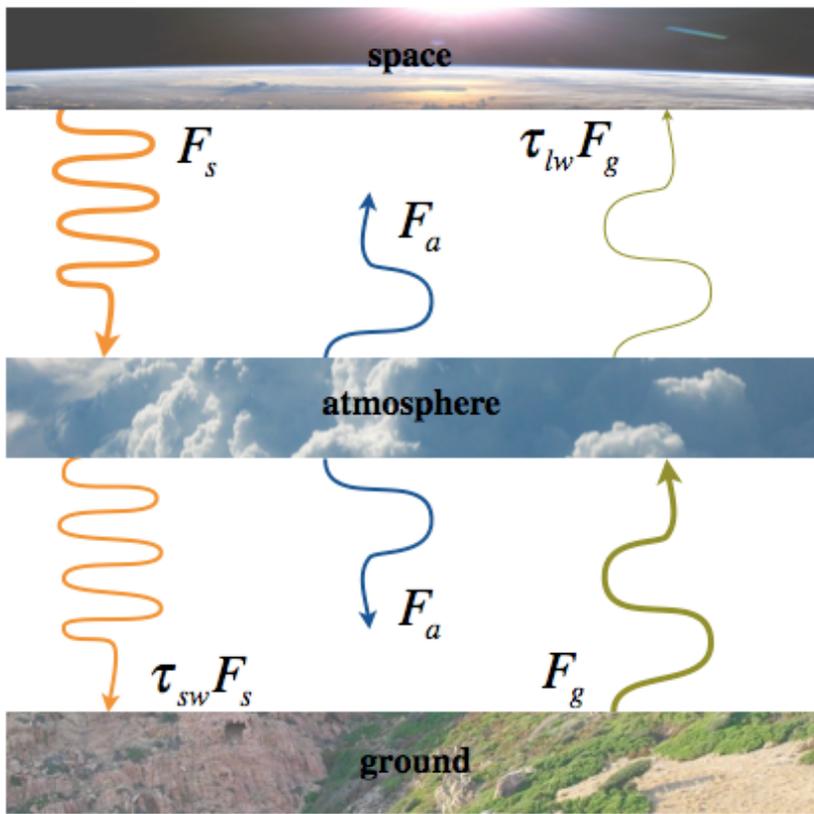


- Assume Earth is **black body** at  $T_{bb}$
- Emits from surface following Stefan-Boltzmann law: **outgoing power** =  $4\pi a^2\sigma T_{bb}^4$

incoming = outgoing

$$T_{bb} = 255\text{K}$$

**observed surface temperature**  $\sim 288\text{K}$

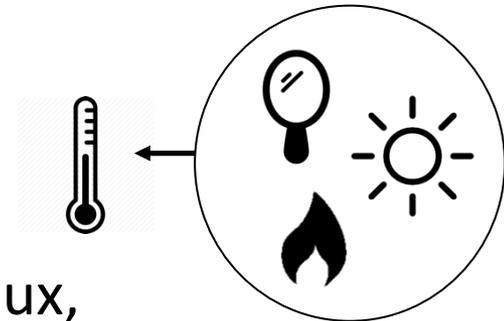


Atmosphere transmits larger fraction of **shortwave** ( $\tau_{sw} = 0.9$ ) than **longwave** ( $\tau_{lw} = 0.2$ ) radiation

$$T_g = \left( \frac{(1 - \alpha)(1 + \tau_{sw})}{4\sigma(1 + \tau_{lw})} F \right)^{1/4}$$

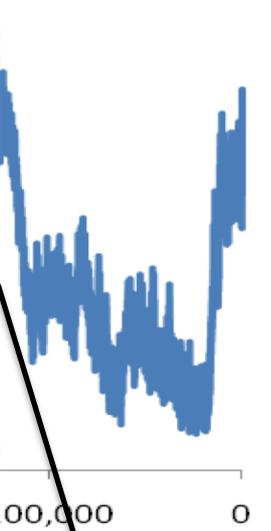
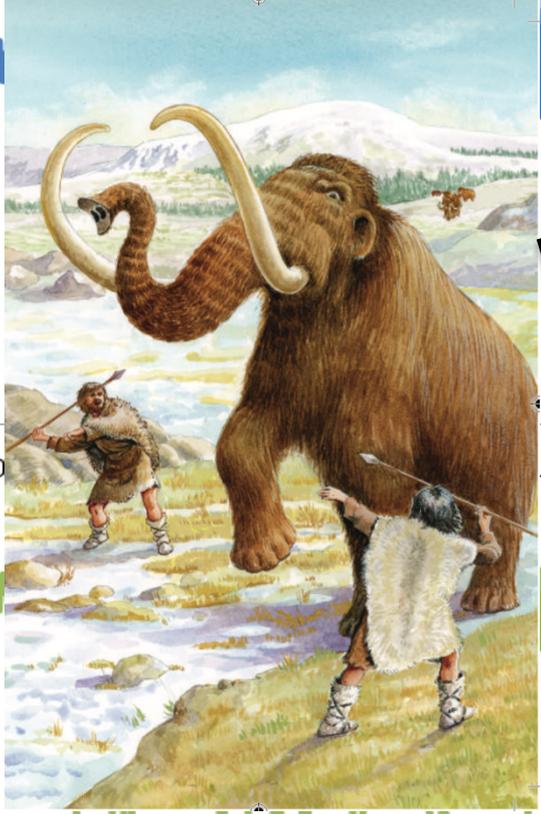
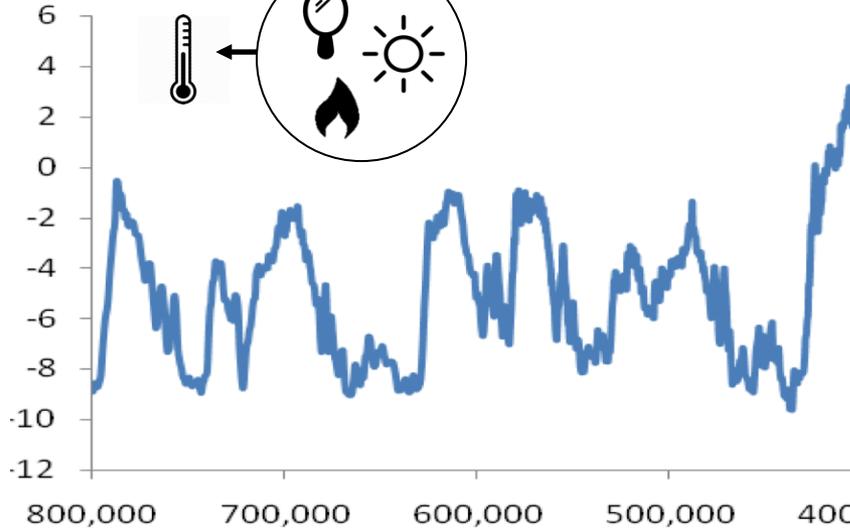
$$T_g = 286K$$

observed surface temperature  $\sim 288K$

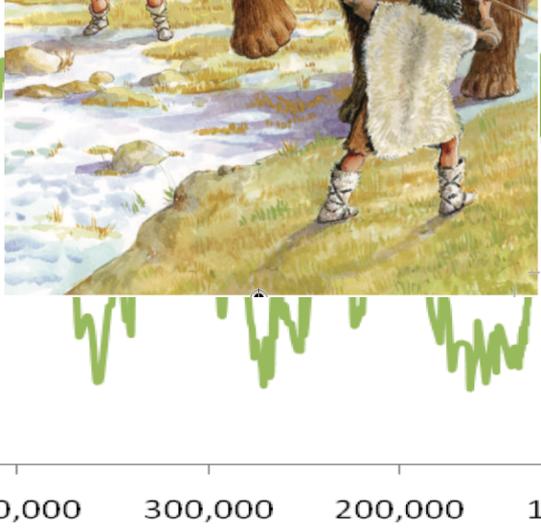
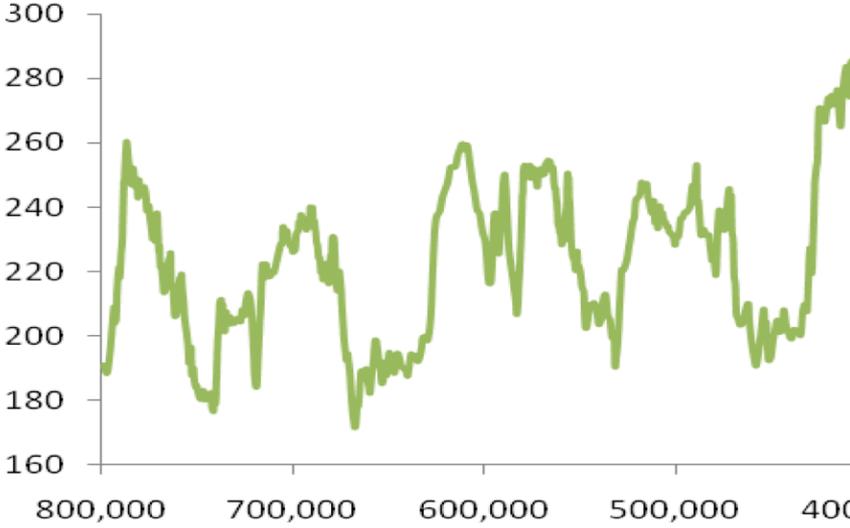


Temperature can change by changing solar flux, fraction of radiation transmitted or reflectivity.

Temperature relative to present (°C)



CO<sub>2</sub> concentration (ppmv)

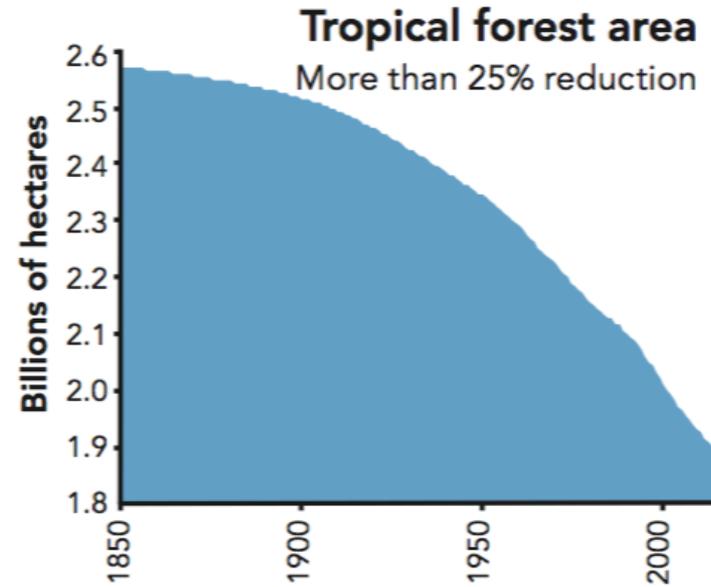
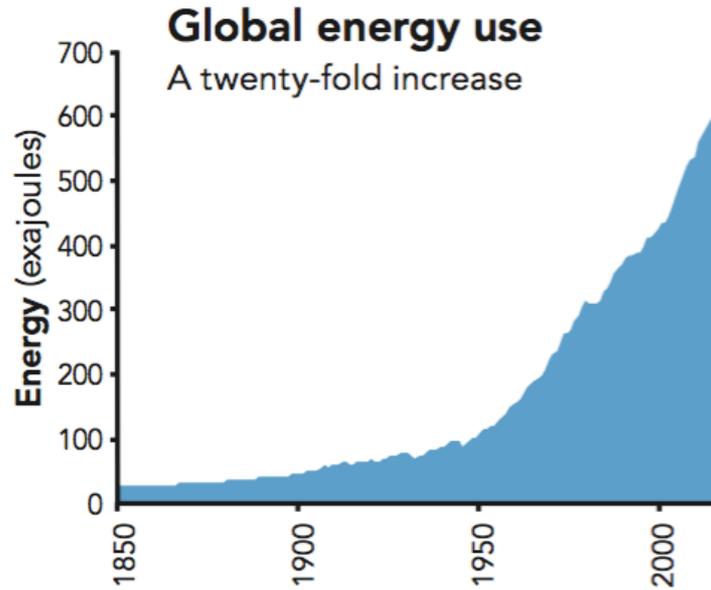
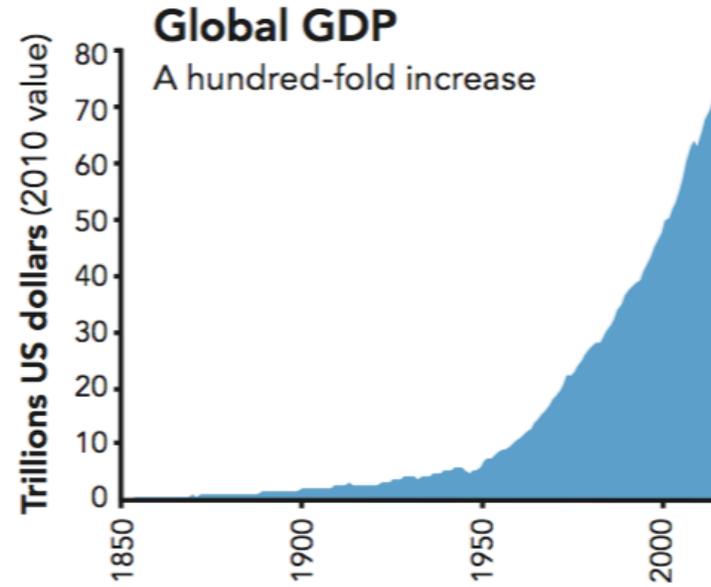
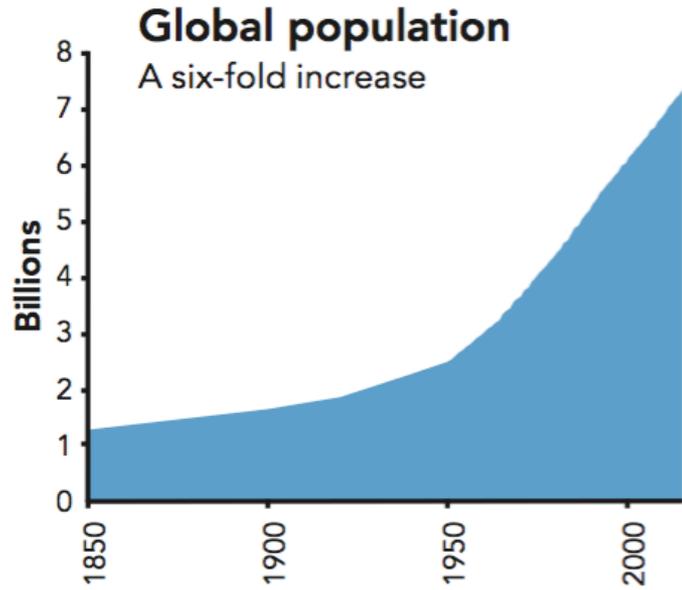


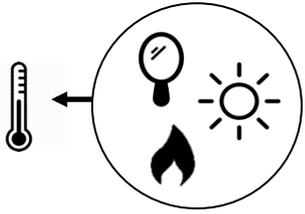
800,000 years ago

time

present



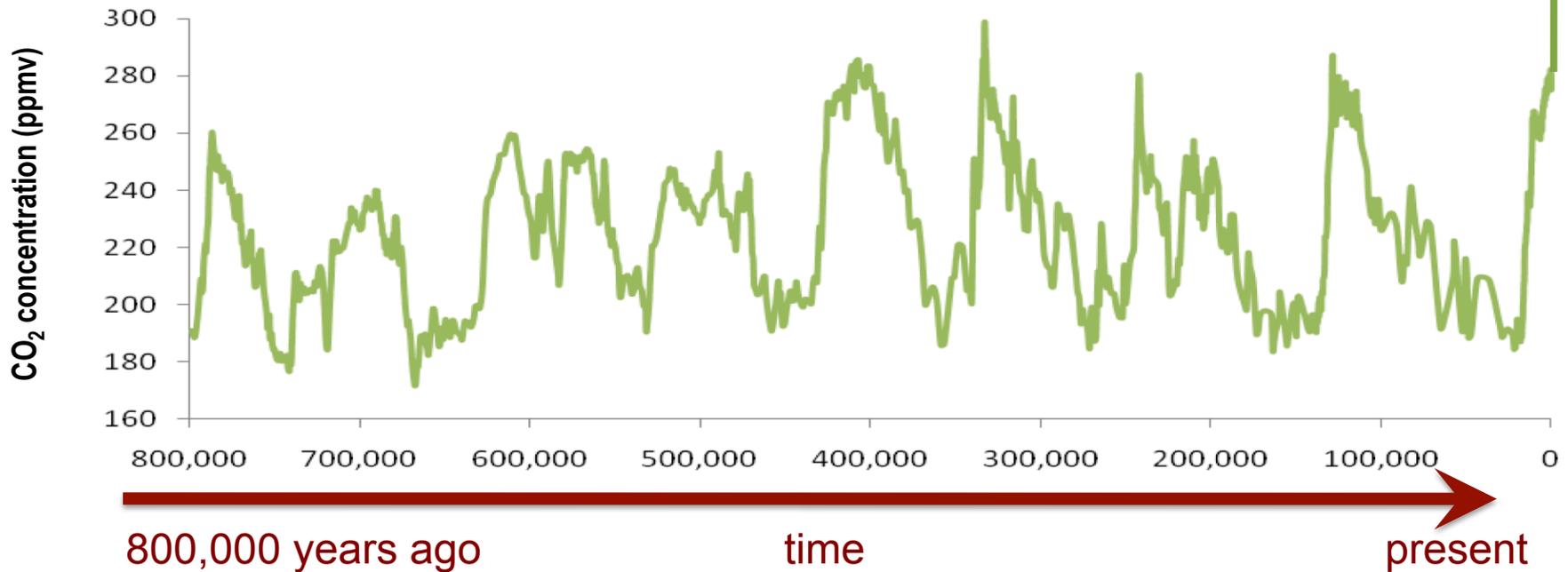




More greenhouse gases means smaller fraction of **longwave** radiation transmitted

405 ppmv 

45% increase



"All the News  
That's Fit to Print"

# The New York Times

National Edition

Northern California: Cloudy. Rain. Mountain snow showers. Highs in 30s to 50s. Heavy rain and mountain snow tonight. Lows in teens to 40s. Weather map appears on Page A28.

VOL. CLXVI . . . No. 57,482

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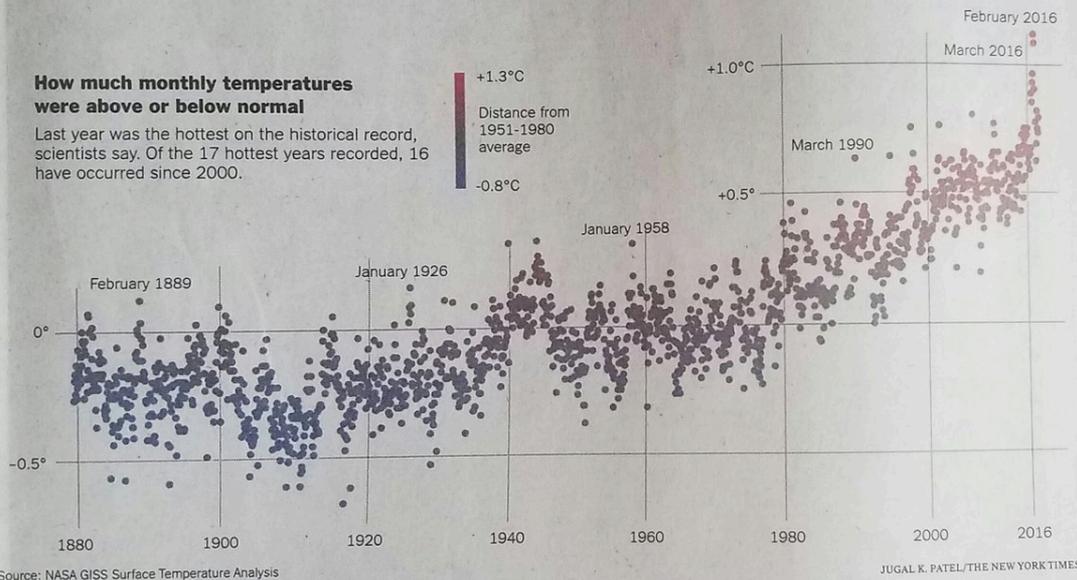
THURSDAY, JANUARY 19, 2017

Printed in California

\$2.50

## How much monthly temperatures were above or below normal

Last year was the hottest on the historical record, scientists say. Of the 17 hottest years recorded, 16 have occurred since 2000.



## FOR THIRD YEAR, THE EARTH IN 2016 HIT RECORD HEAT

### Threat to Society and Nature Is Rising — Scale of Shift Startles Scientists

By JUSTIN GILLIS

Marking another milestone for a changing planet, scientists reported on Wednesday that the Earth reached its highest temperature on record in 2016, trouncing a record set only a year earlier, which beat one set in 2014. It is the first time in the modern era of global warming data that temperatures have blown past the previous record three years in a row.

The findings come two days before the inauguration of an American president who has called global warming a Chinese plot and vowed to roll back his predecessor's efforts to cut emissions of heat-trapping gases.

In reality, the Earth is heating up, a point long beyond serious scientific dispute, but one becoming more evident as the records keep falling. Temperatures are heading toward levels that many experts believe will pose a profound threat to both the natural world and to human civilization.

In 2015 and 2016, the planetary warming was intensified by the weather pattern known as El Niño, in which the Pacific Ocean released a huge burst of energy and water vapor into the atmosphere.

gases.

"A single warm year is something of a curiosity," said Deke Arndt, chief of global climate monitoring for the National Oceanic and Atmospheric Administration. "It's really the trend, and the fact that we're punching at the ceiling every year now, that is the real indicator that we're undergoing big changes."

The heat extremes were especially pervasive in the Arctic, with temperatures in the fall running 20 to 30 degrees Fahrenheit above normal across large stretches of the Arctic Ocean. Sea ice in that region has been in precipitous decline for years, and Arctic communities are already wrestling with enormous problems, such as rapid coastal erosion, caused by the changing climate.

"What's going on in the Arctic is really very impressive; this year was ridiculously off the chart," said Gavin A. Schmidt, head of the Goddard Institute for Space Studies in Manhattan, a unit of the National Aeronautics and Space Administration that tracks global temperatures.

But Arctic people were hardly alone in feeling the heat. Drought

## In Farewell, Obama Sets Red Lines That Would Pull Him Back

By MICHAEL D. SHEAR  
and PETER BAKER

WASHINGTON — When President Obama arrived in office eight years ago, the departing President George W. Bush essentially withdrew from public life, declaring that his successor "deserves my silence." It was an approach that Mr. Obama greatly appreciated but does not intend to follow.

At the final news conference of his presidency, Mr. Obama made clear on Wednesday that he finds

that normal functioning of politics and certain issues or certain moments where I think our core values may be at stake," Mr. Obama told reporters in the White House briefing room.

Mr. Obama continued: "I put in that category if I saw systematic discrimination being ratified in some fashion. I put in that category explicit or functional obstacles to people being able to vote, to exercise their franchise. I'd put in that category institutional efforts to silence dissent on the press. And

### Pledge to Re-enter Fray When 'Core Values May Be at Stake'

country."

All of his red lines seemed to refer to positions taken in the past by Mr. Trump, foreshadowing the possibility of a periodic clash of ideas over the next four years be-

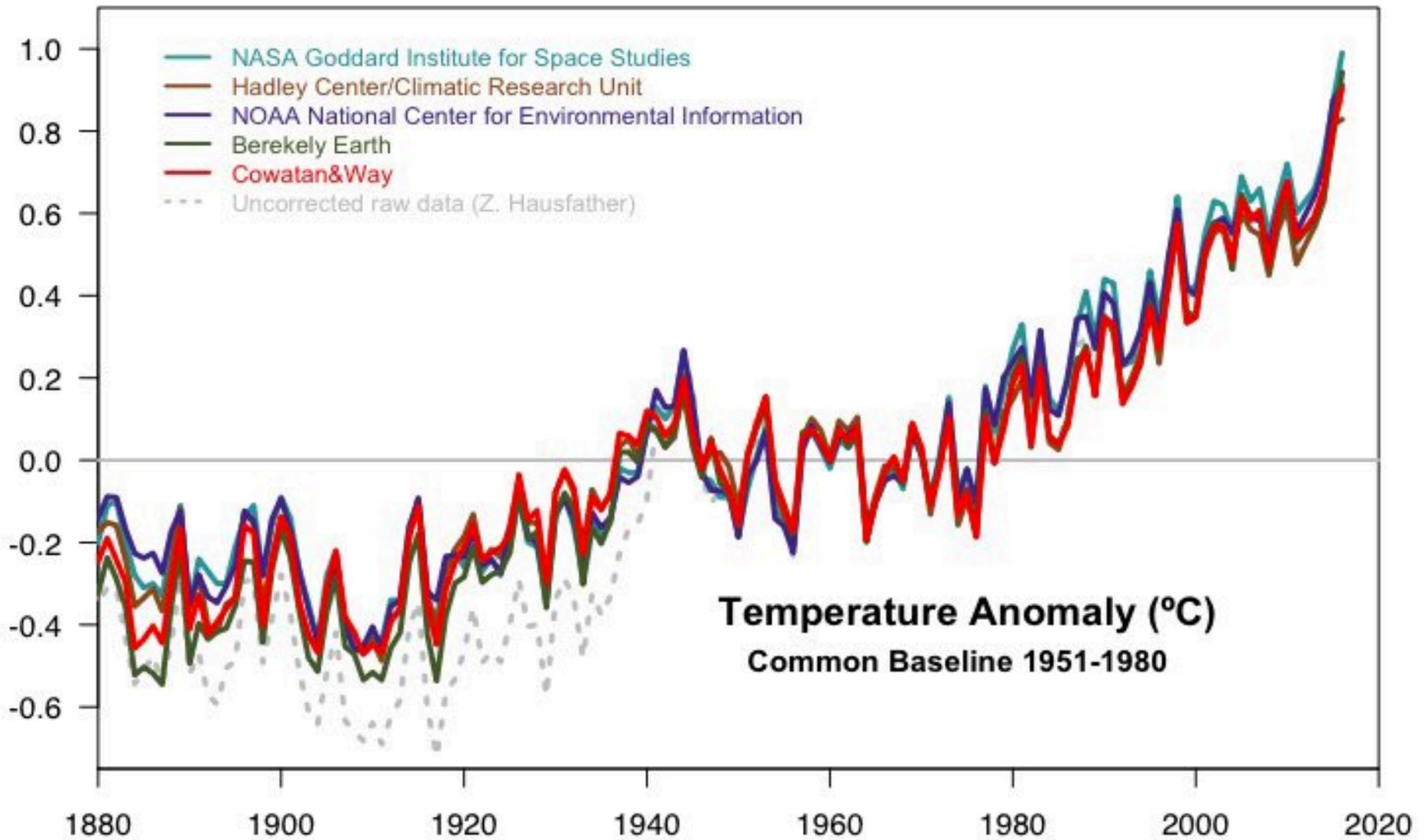
office since Woodrow Wilson.

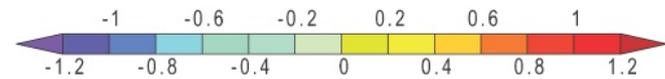
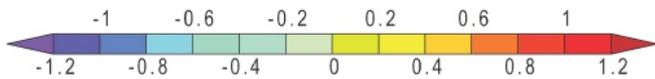
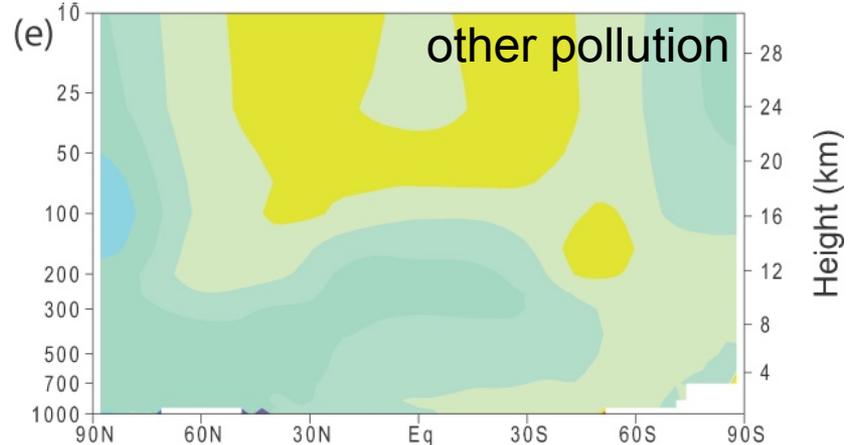
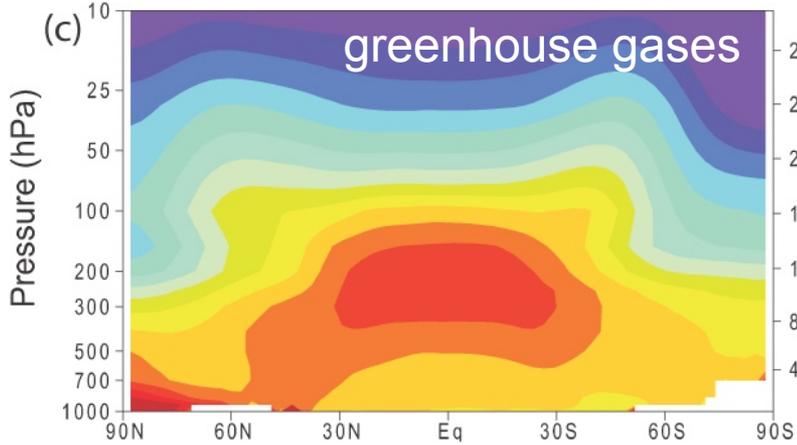
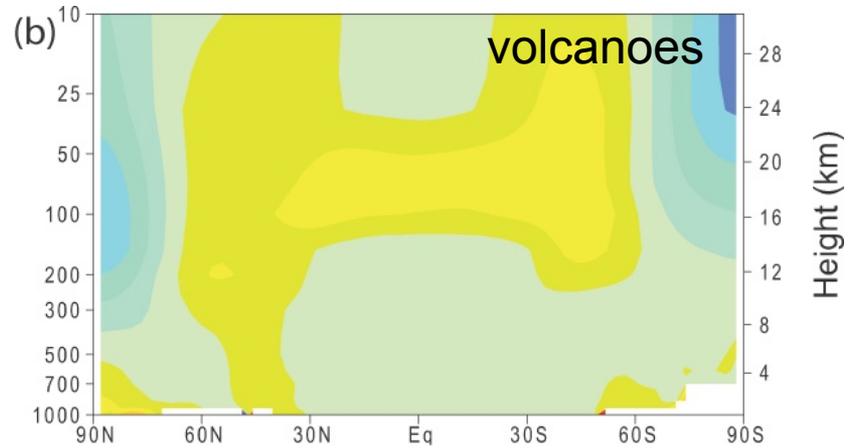
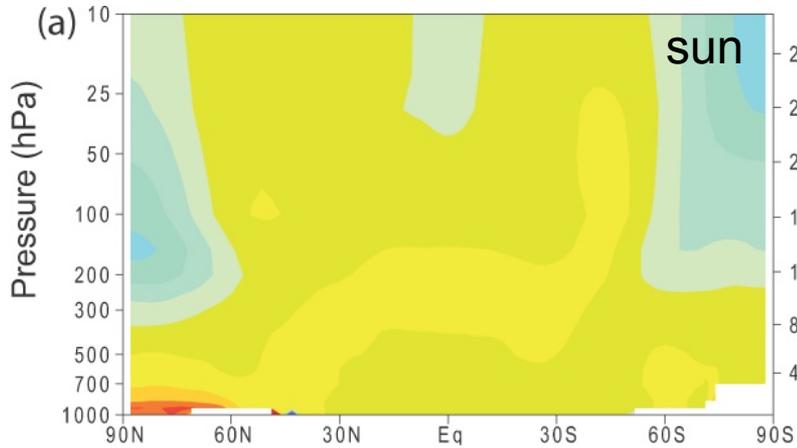
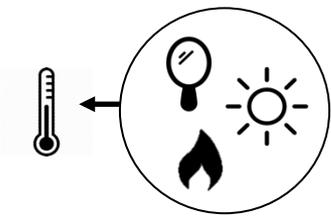
Mr. Obama did say he was looking forward to some quiet time and does not plan to stay involved in the hurly-burly of politics. He has told advisers and friends that he wants to be careful not to become such a regular public critic of Mr. Trump that he alienates the mercurial new president.

Since the election, the departing president has tried to forge a relationship of sorts with his successor and hopes to keep lines of communication open to privately

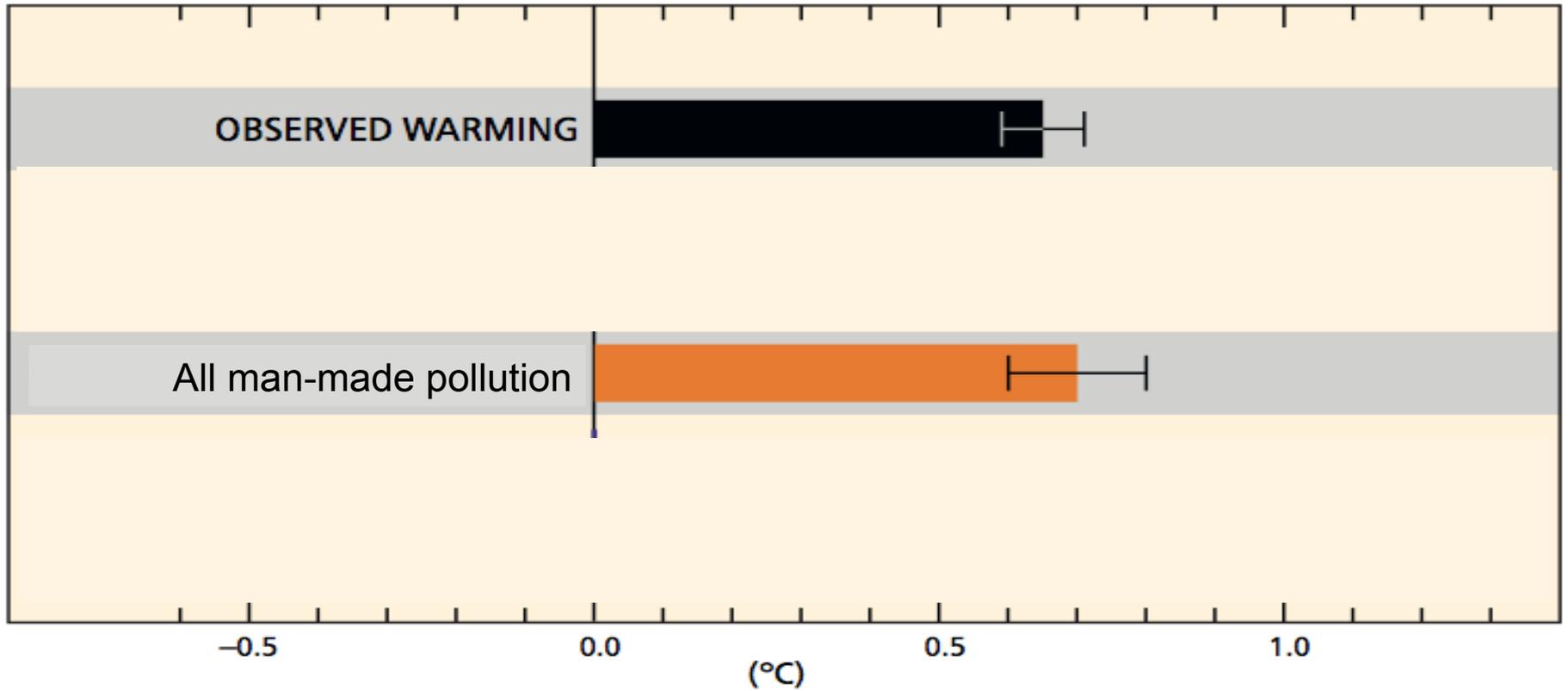
Mary Ellen Thomas, Nunavut Research Institute: *“It is as if a friend that we could trust is suddenly acting strangely”*



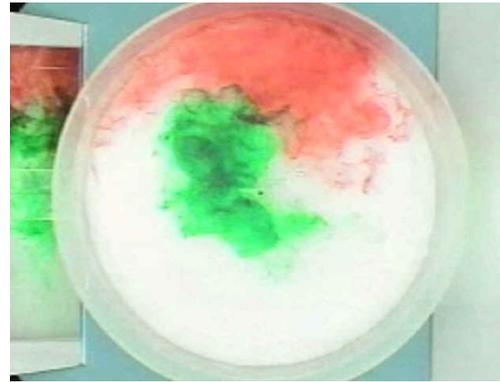




# Contributions to observed surface temperature change over the period 1951–2010



# Fluid on a rotating sphere



non-rotating



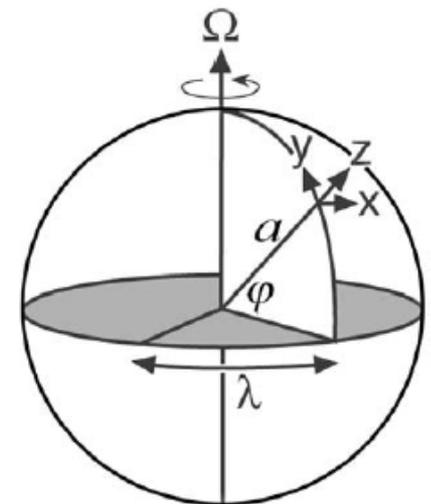
rotating

## Equations of motion

Newton's 2nd law: 
$$\frac{D\mathbf{u}}{Dt} + \frac{1}{\rho} \nabla p + g\hat{\mathbf{z}} = \mathcal{F}$$

Rotating frame: 
$$\mathbf{u}_{inertial} = \mathbf{u}_{rotating} + \boldsymbol{\Omega} \times \mathbf{r}$$

Sphere: 
$$\boldsymbol{\Omega} = (0, \Omega \cos \varphi, \Omega \sin \varphi) \quad f = 2\Omega \sin \varphi$$



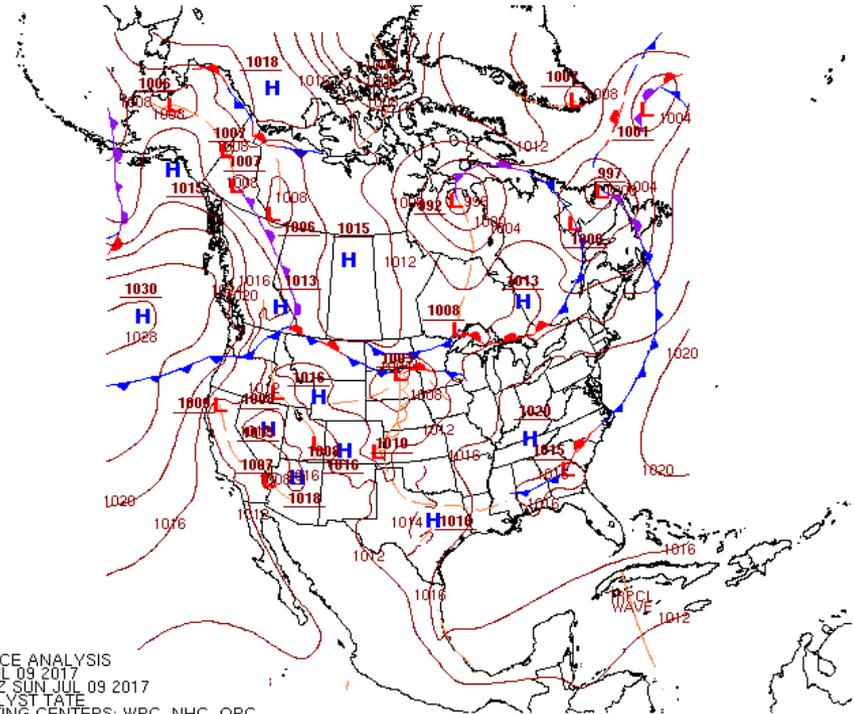
Equations in  
weather and  
climate  
models

$$\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial x} - fv = \mathcal{F}_x$$

$$\frac{Dv}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial y} + fu = \mathcal{F}_y$$

$$\frac{1}{\rho} \frac{\partial p}{\partial z} + g = 0$$

“Geostrophic balance”

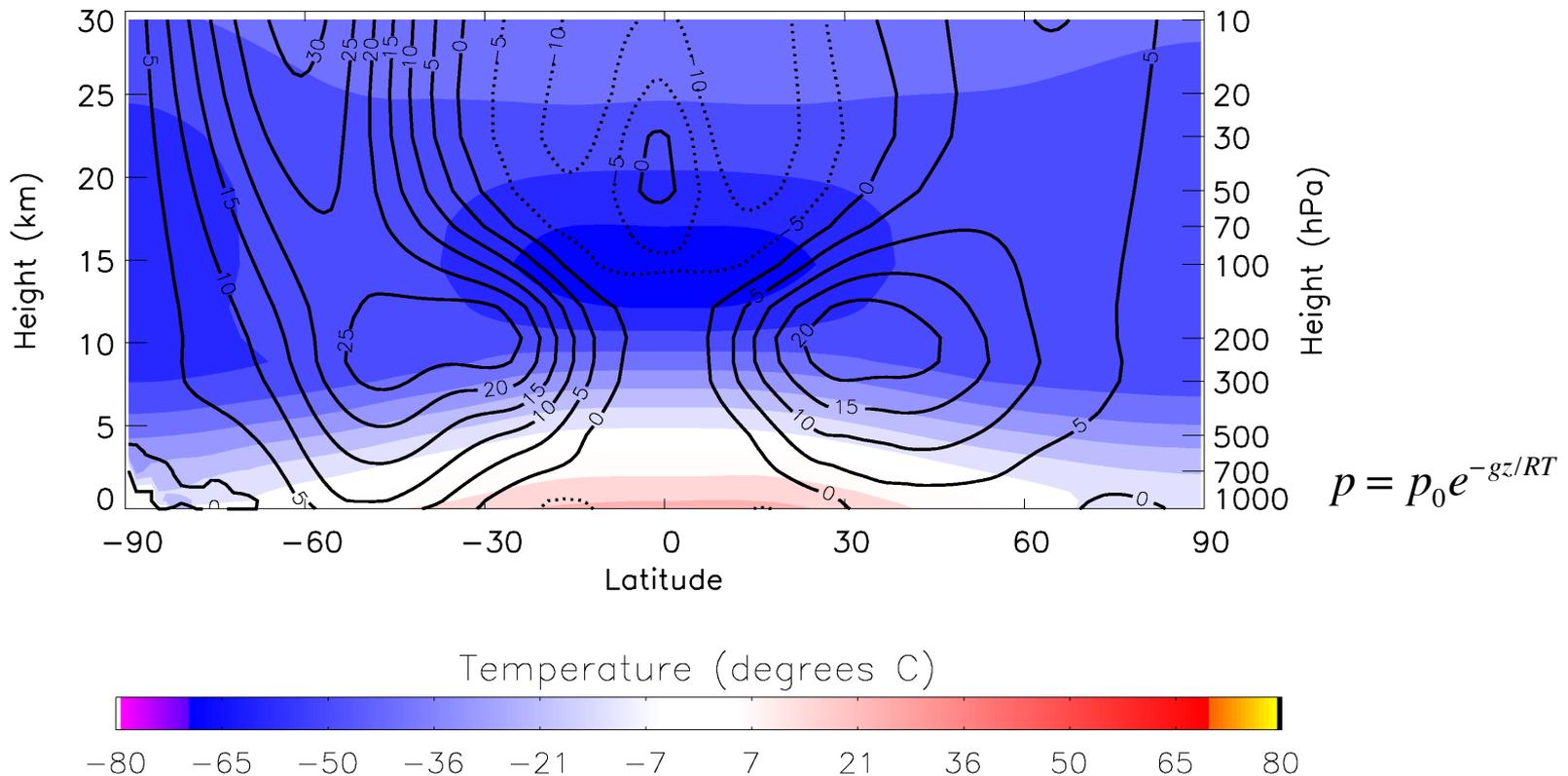


2100Z SURFACE ANALYSIS  
DATE: SUN JUL 09 2017  
ISSUED: 2234Z SUN JUL 09 2017  
BY WPC ANALYST TATE  
COLLABORATING CENTERS: WPC, NHC, OPC

Atmosphere is an  
ideal gas:  $\frac{p}{\rho} = RT$

$$\frac{\partial u}{\partial p} = \frac{R}{f\rho} \frac{\partial T}{\partial y}$$

“Thermal wind”



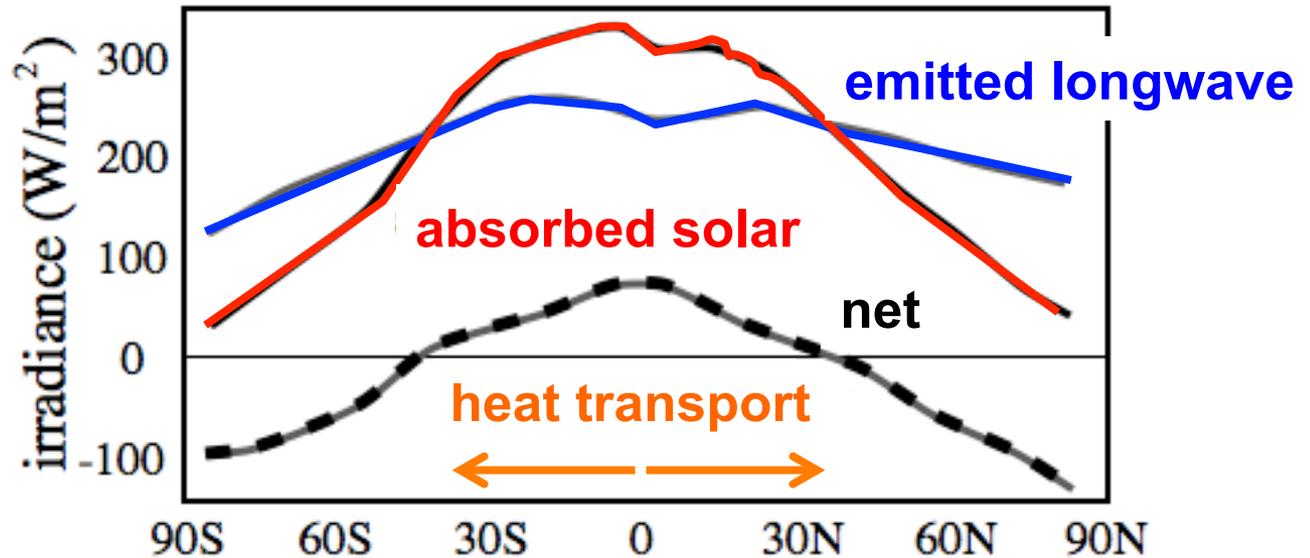
$u = \bar{u} + u'$  mean flow + "eddies"

$v = \bar{v} + v'$

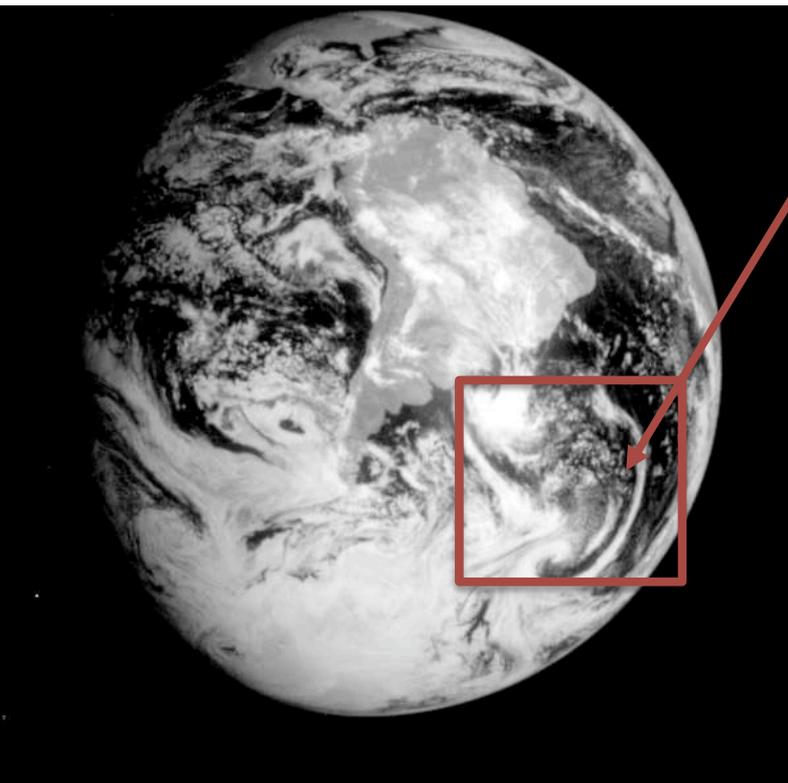
$$\frac{\partial \bar{u}}{\partial t} - f \bar{v} = -\frac{\partial}{\partial y} \overline{u'v'} + \overline{\mathcal{F}_x}$$

eddy fluxes can drive mean flows

observed annual mean radiation



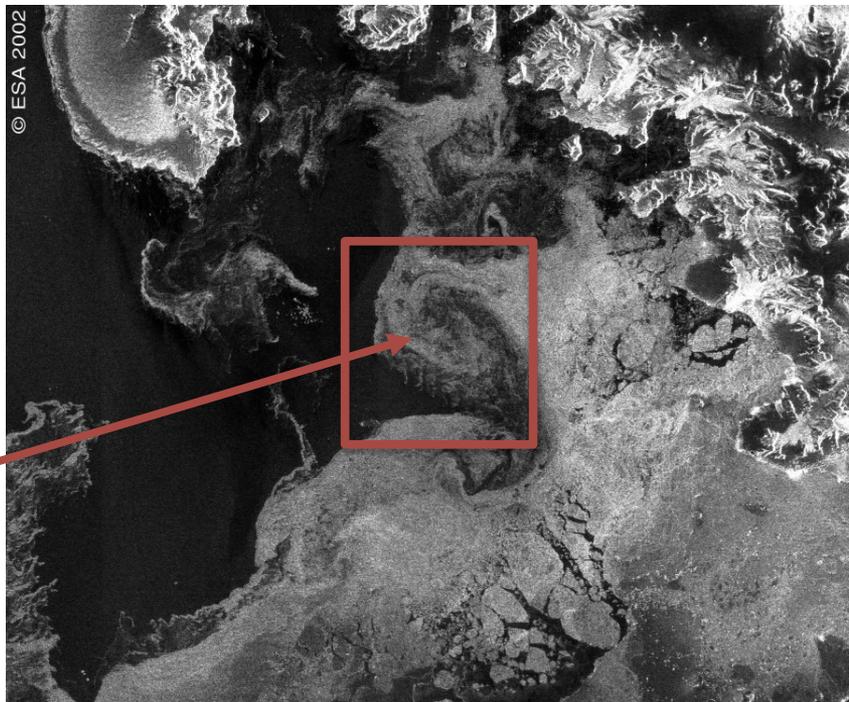
# Eddies in the atmosphere and ocean



Eddies in atmosphere of size  $\sim 1000$  km

This near-infrared photograph of the Earth was taken by the Galileo spacecraft

ENVISAT/ASAR image shows the marginal ice zone outside the ice shelf at Marguerite Bay on the west side of the Antarctic Peninsula.



Eddies in ocean size  $\sim 25$  km

(40 times smaller than atmosphere & usually parameterized in climate models)

Atmosphere  
**16**  
billion tonnes CO<sub>2</sub>

**9.5**  
billion tonnes CO<sub>2</sub>

Ocean sink

30% carbon emissions have gone into ocean

Fossil fuels & industry

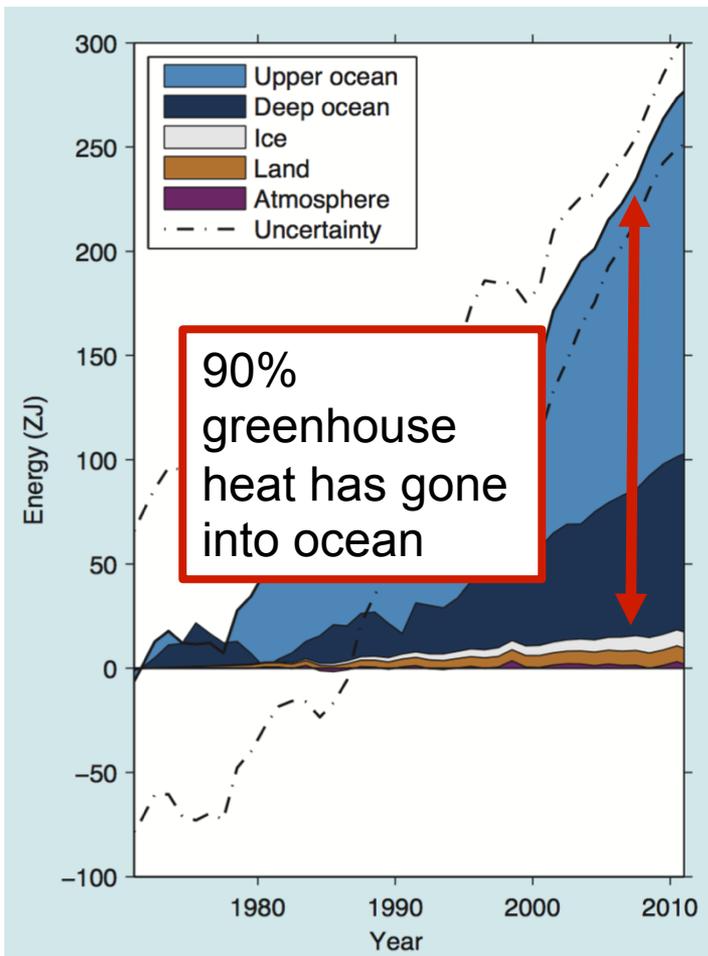
**33**  
billion tonnes CO<sub>2</sub>

**10.9**  
billion tonnes CO<sub>2</sub>

Land sink

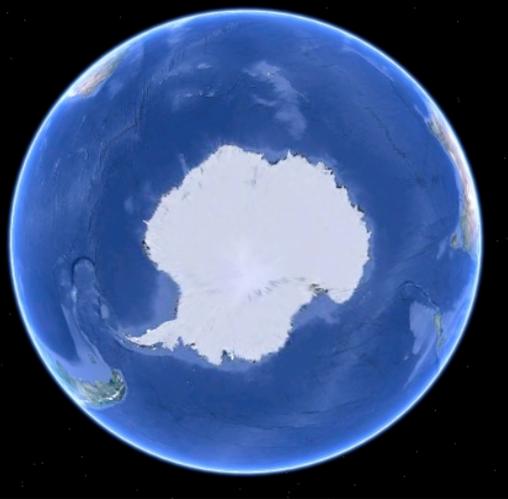
Deforestation and land-use change

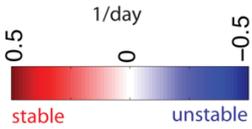
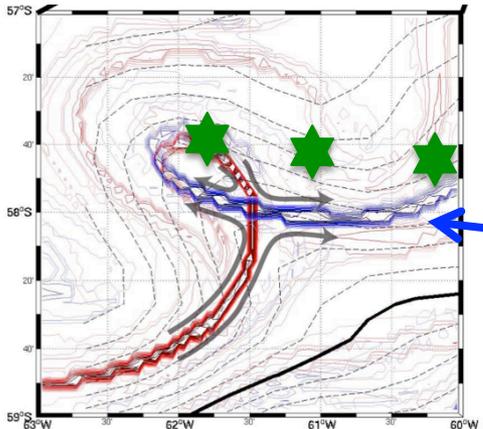
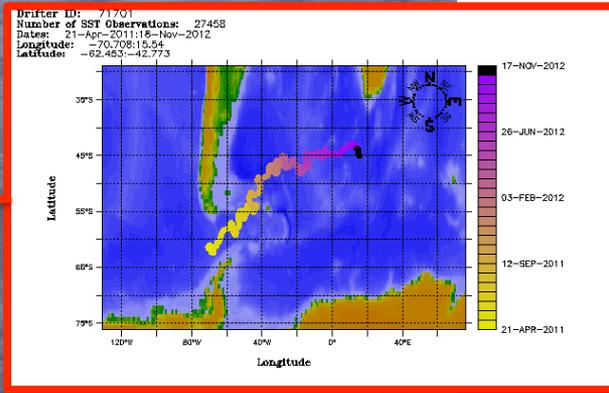
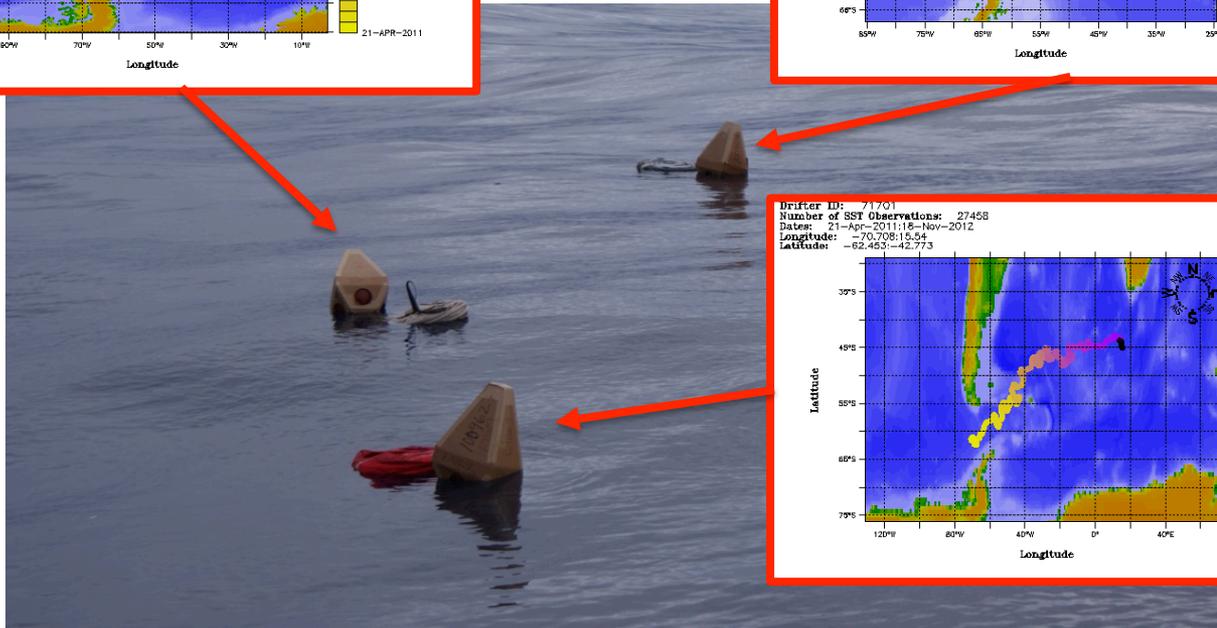
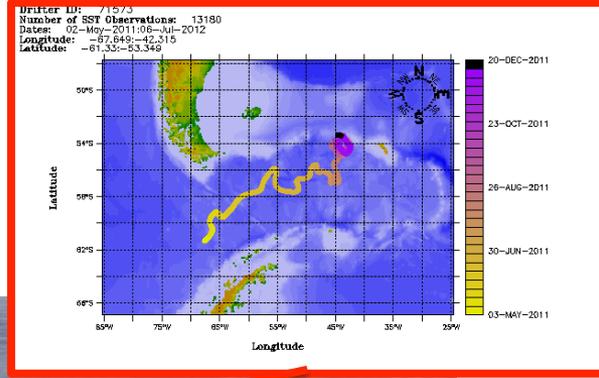
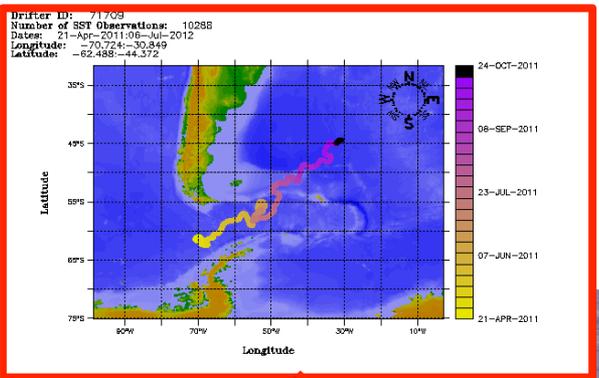
**3.4**  
billion tonnes CO<sub>2</sub>



WG1 Box 3.1

Southern Ocean estimated to be responsible for about 40% of ocean carbon uptake and 75% of ocean heat uptake. Why? Eddies.

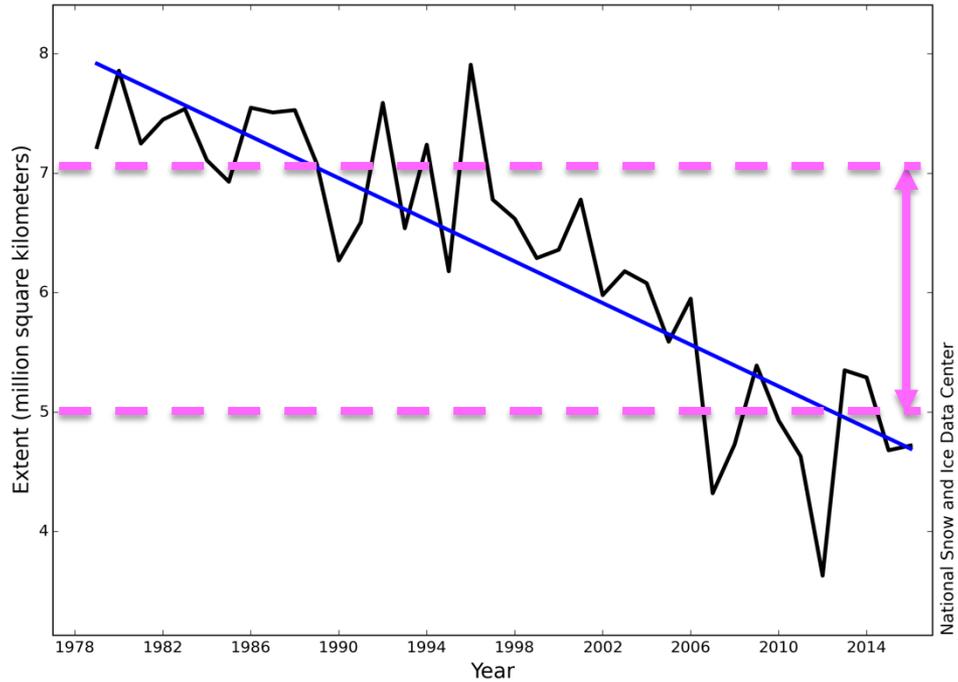




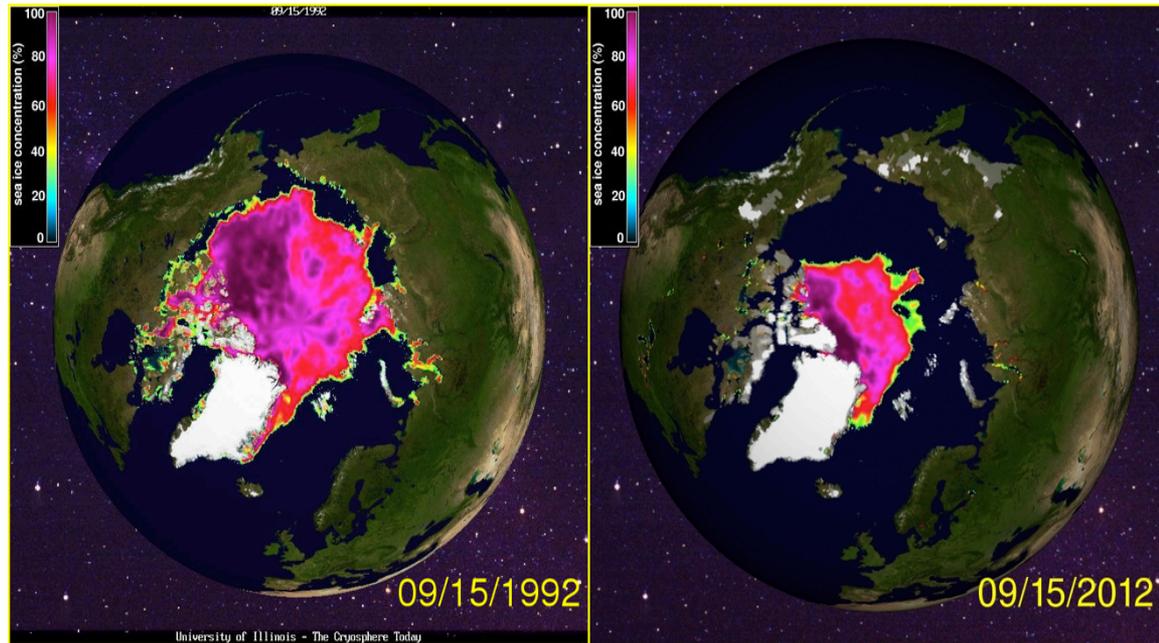
Ridges in backward-time finite-size Lyapunov exponents

**Release floats  $\leq 10$  km apart along altimetry-estimated unstable manifold, close to hyperbolic point**

# Average Monthly Arctic Sea Ice Extent September 1979 - 2016



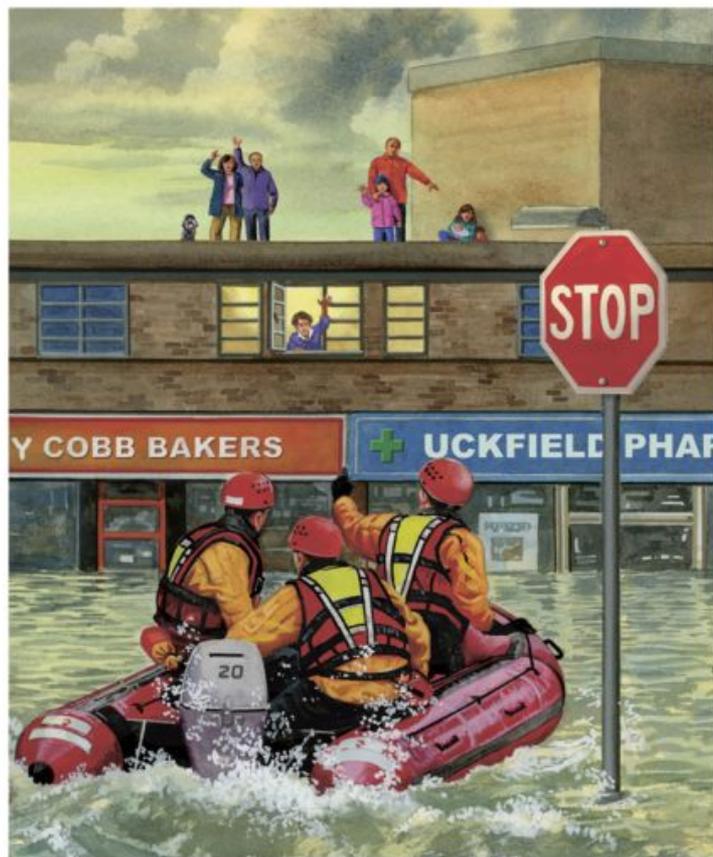
2m km<sup>2</sup> = California, New Mexico, Arizona, Nevada & Texas



# Climate Change



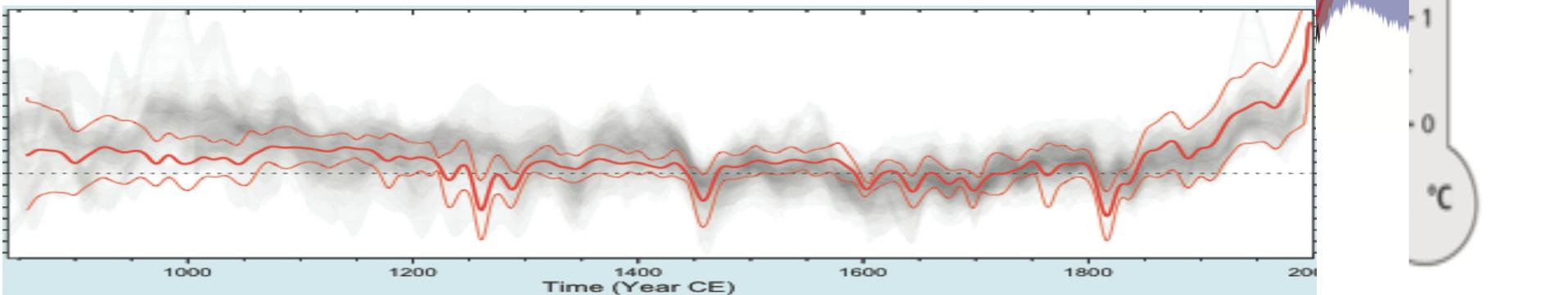
HRH The Prince of Wales  
Tony Juniper  
Emily Shuckburgh



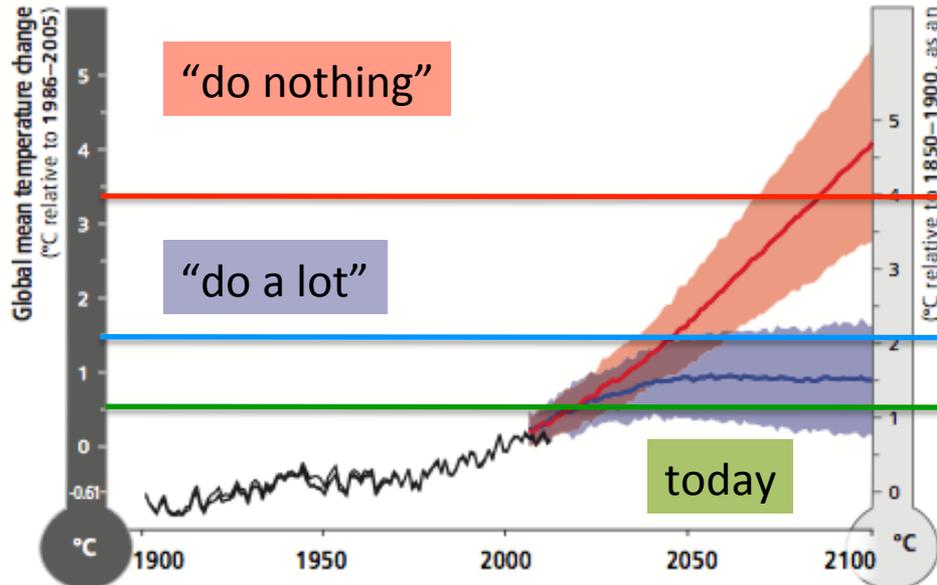
Projected temperature change to 2100 under two possible futures:

“do nothing”

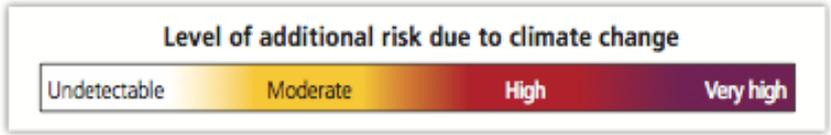
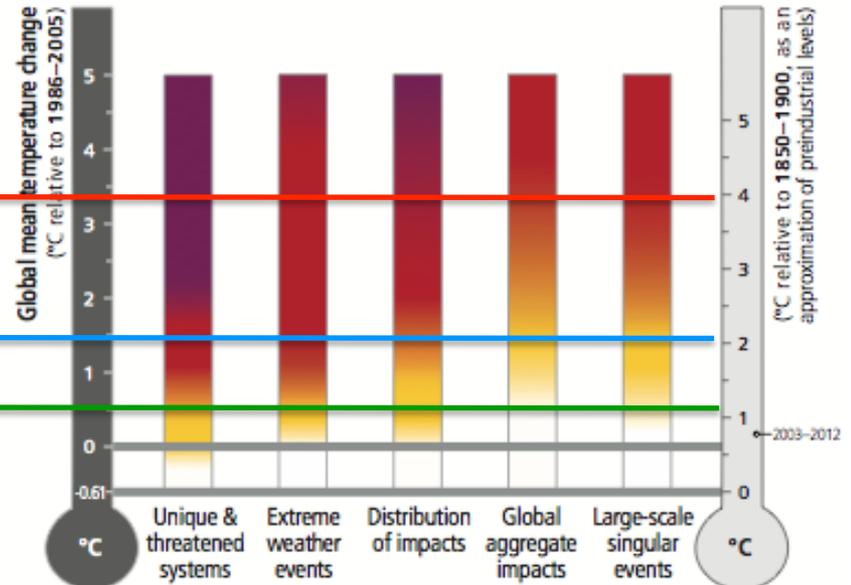
“do a lot”

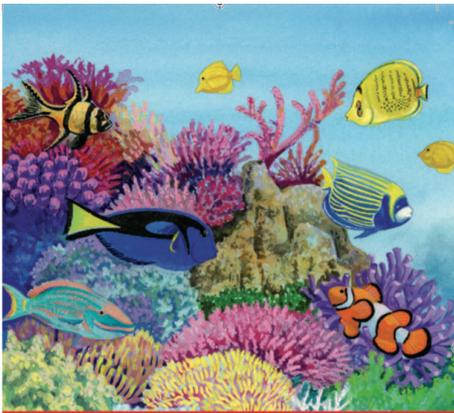


# Risks of future climate change

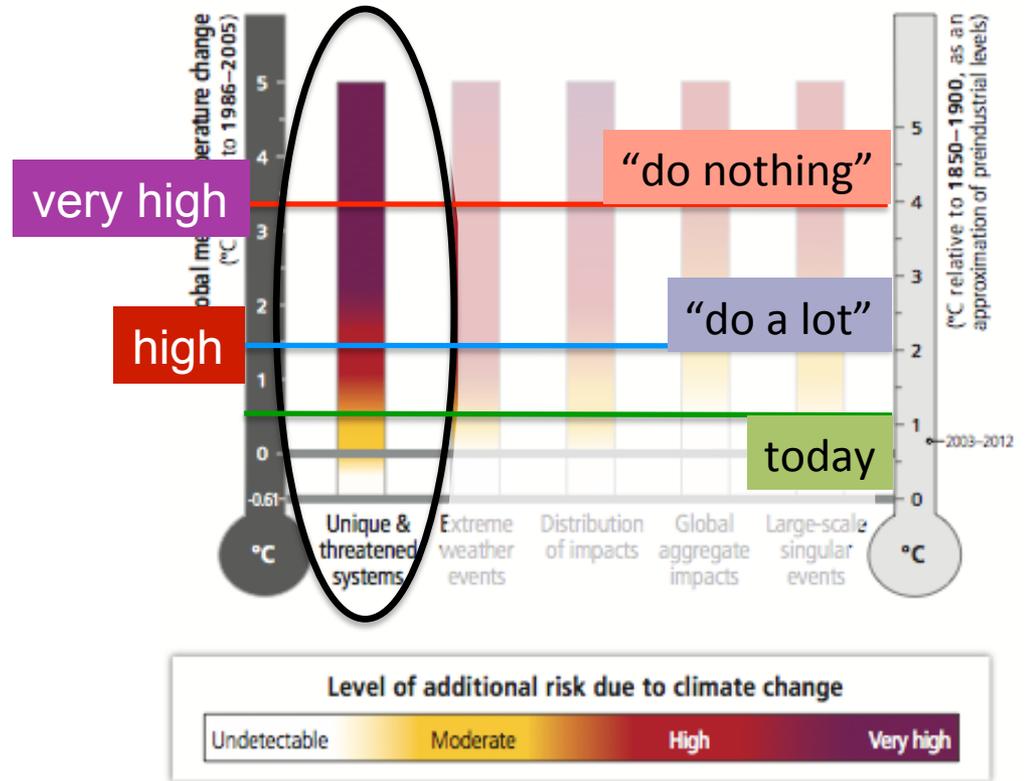


- Observed
- RCP8.5 (a high-emission scenario)
- Overlap
- RCP2.6 (a low-emission mitigation scenario)

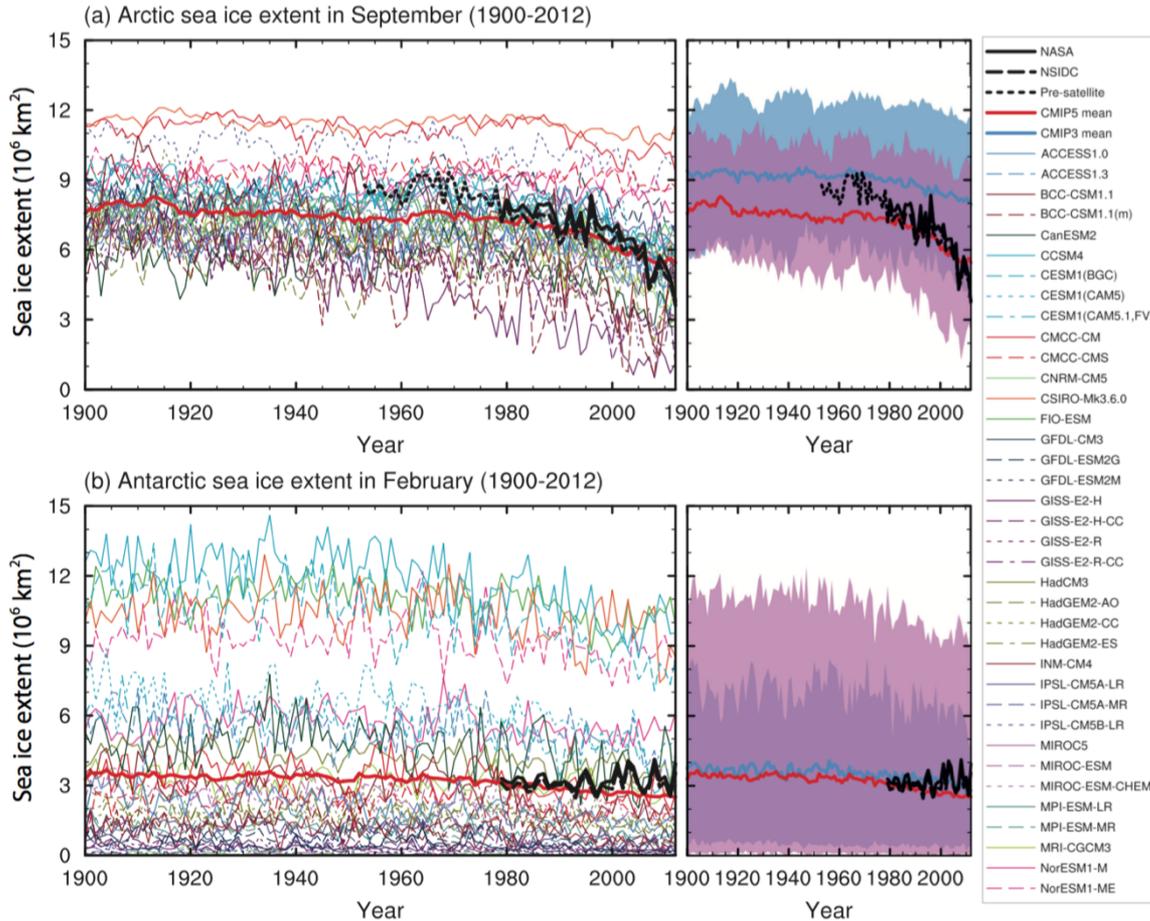




## Unique & threatened systems



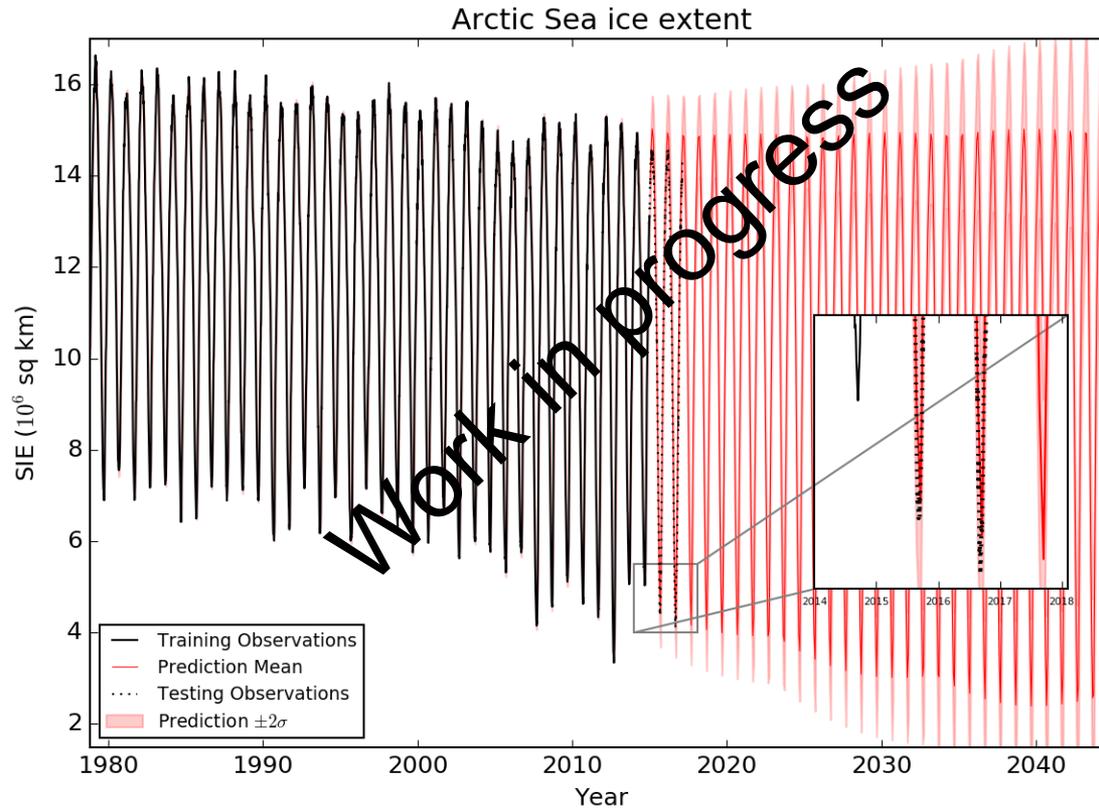
# Predicting future sea ice



Depending on its morphology & microstructure, ice may behave as an elastic, brittle, viscoelastic or quasi-liquid material.

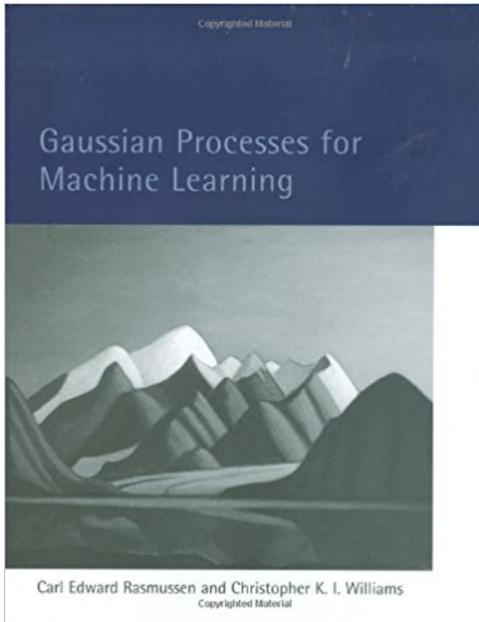
Sea-ice consists of solid fresh-water ice, liquid salty brine, gas inclusion & possibly some other components, which makes it difficult to describe.

# Machine Learning tools (Gaussian Processes) using observational data



with Scott Hosking

Include: robust information from climate models for future (e.g. global average temperature), information on long-term variability from other datasets (e.g. North Atlantic)



### 5.4 Model Selection for GP Regression

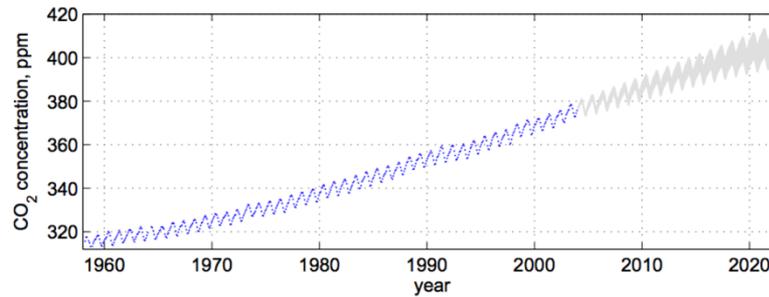
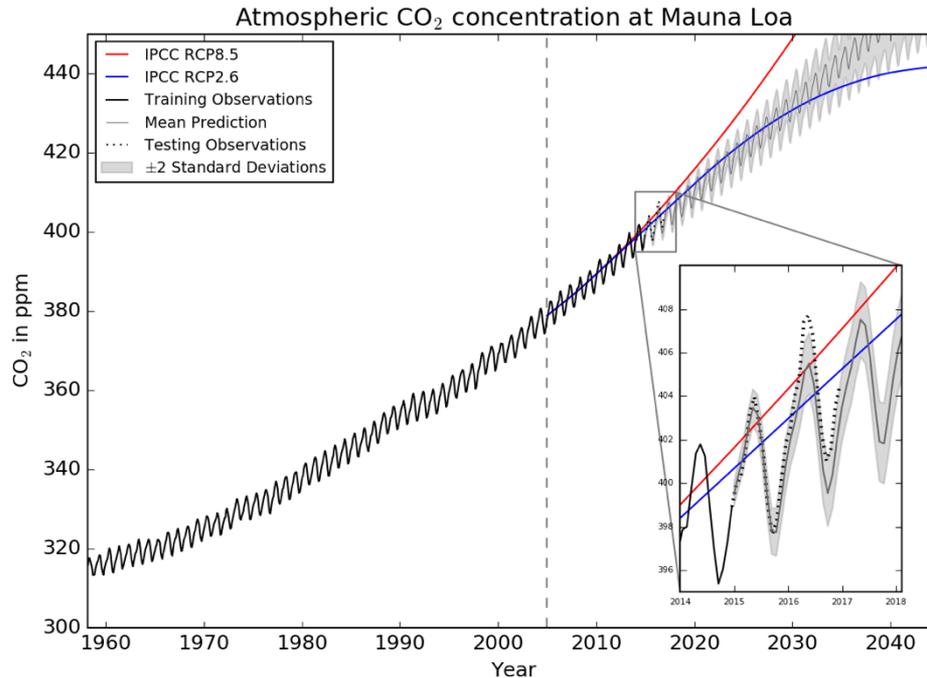
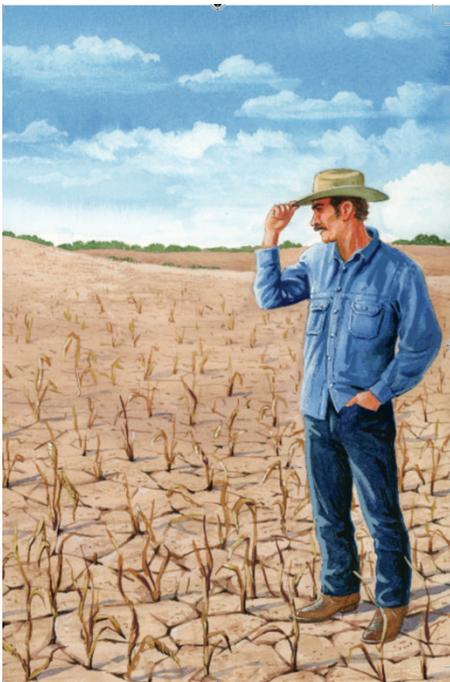
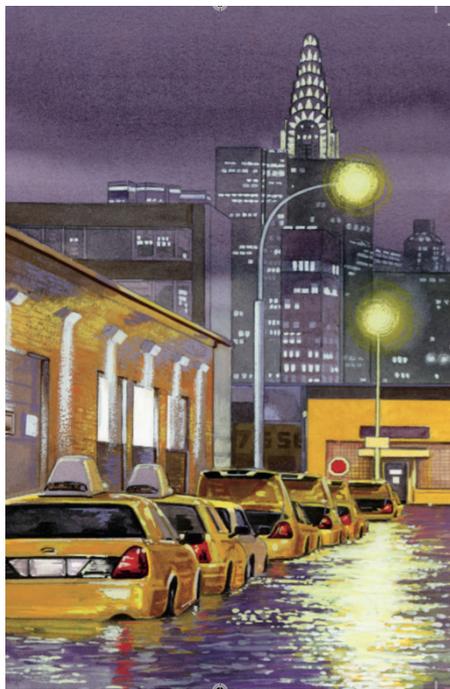


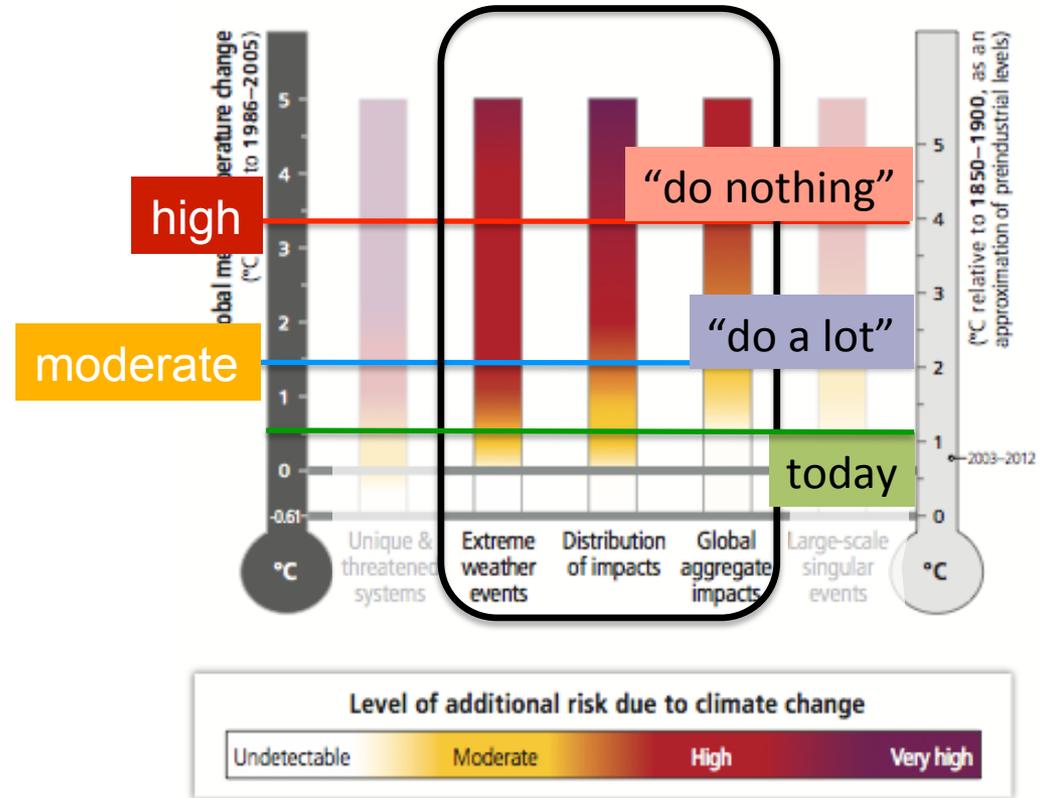
Figure 5.6: The 545 observations of monthly averages of the atmospheric concentration of CO<sub>2</sub> made between 1958 and the end of 2003, together with 95% predictive confidence region for a Gaussian process regression model, 20 years into the future. Rising trend and seasonal variations are clearly visible. Note also that the confidence interval gets wider the further the predictions are extrapolated.



Include information about the future



# Extreme weather, distribution & global aggregate of impacts



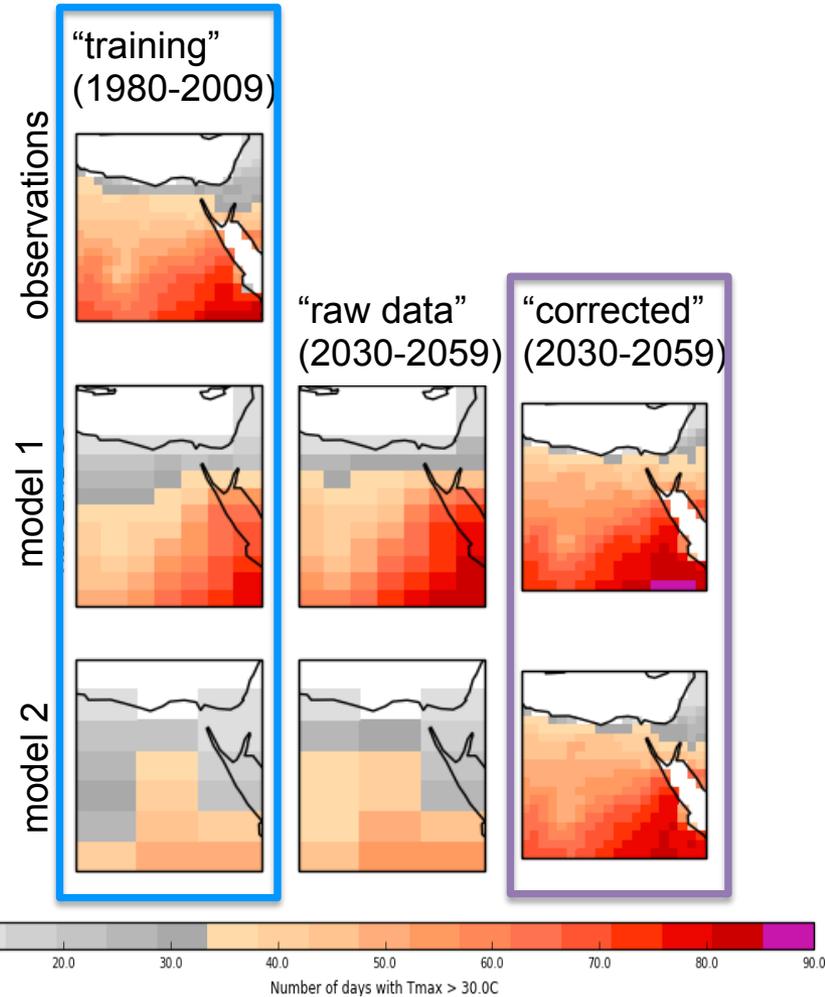
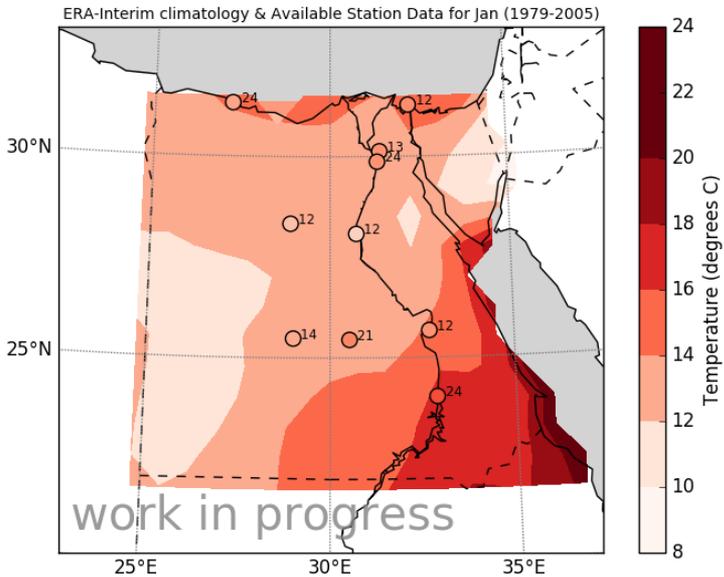
# Egyptian heatwave, Aug 2015: more than 90 deaths

70% more likely due to climate change

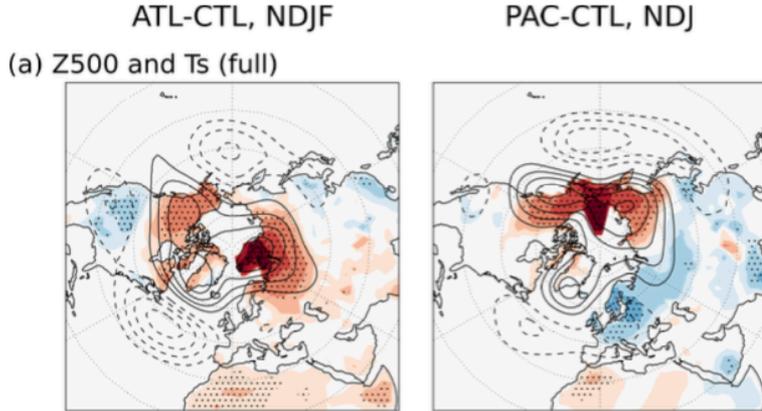
Mitchell, 2016



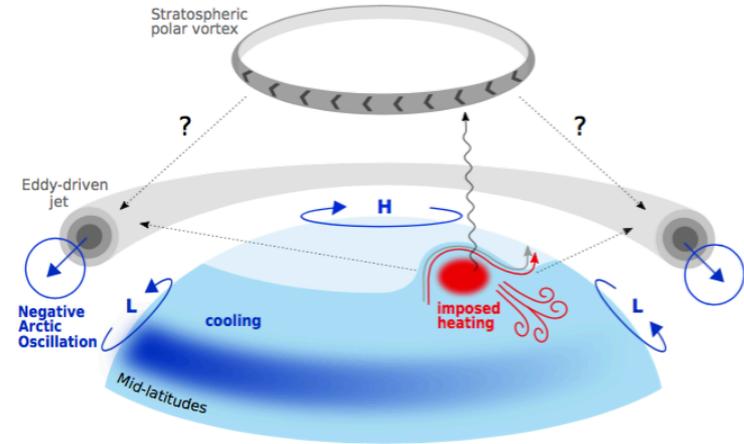
DEVELOPMENT FINANCE: future need for air conditioning in Cairo & impact on power network?



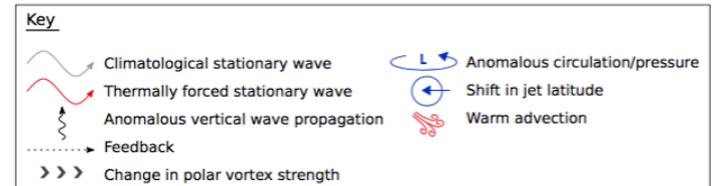
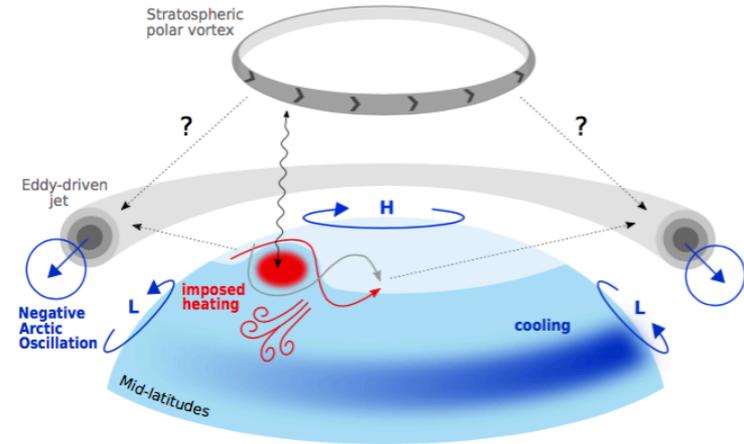
with Scott Hosking



(a) Sea-ice loss in Atlantic sector of Arctic



(b) Sea-ice loss in Pacific sector of Arctic



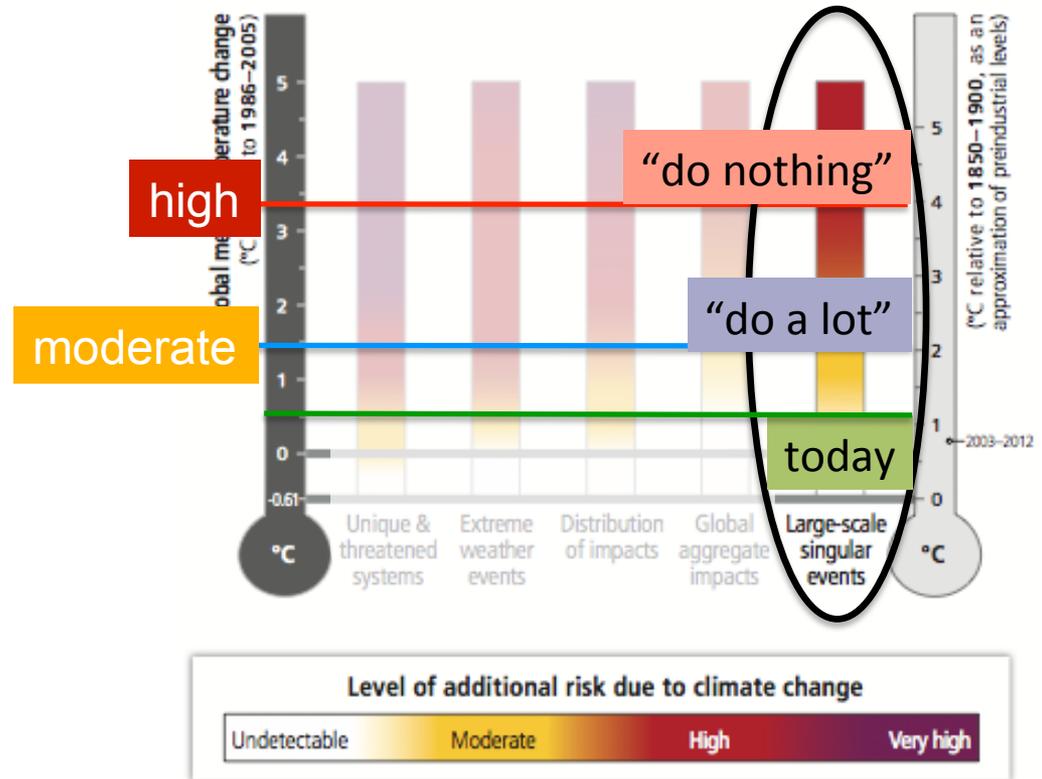
Modelling studies used to identify dynamical pathways of response to Arctic sea ice loss

Indicates that it can impact weather patterns across Europe, Asia and North America

with Christine McKenna, Tom Bracegirdle

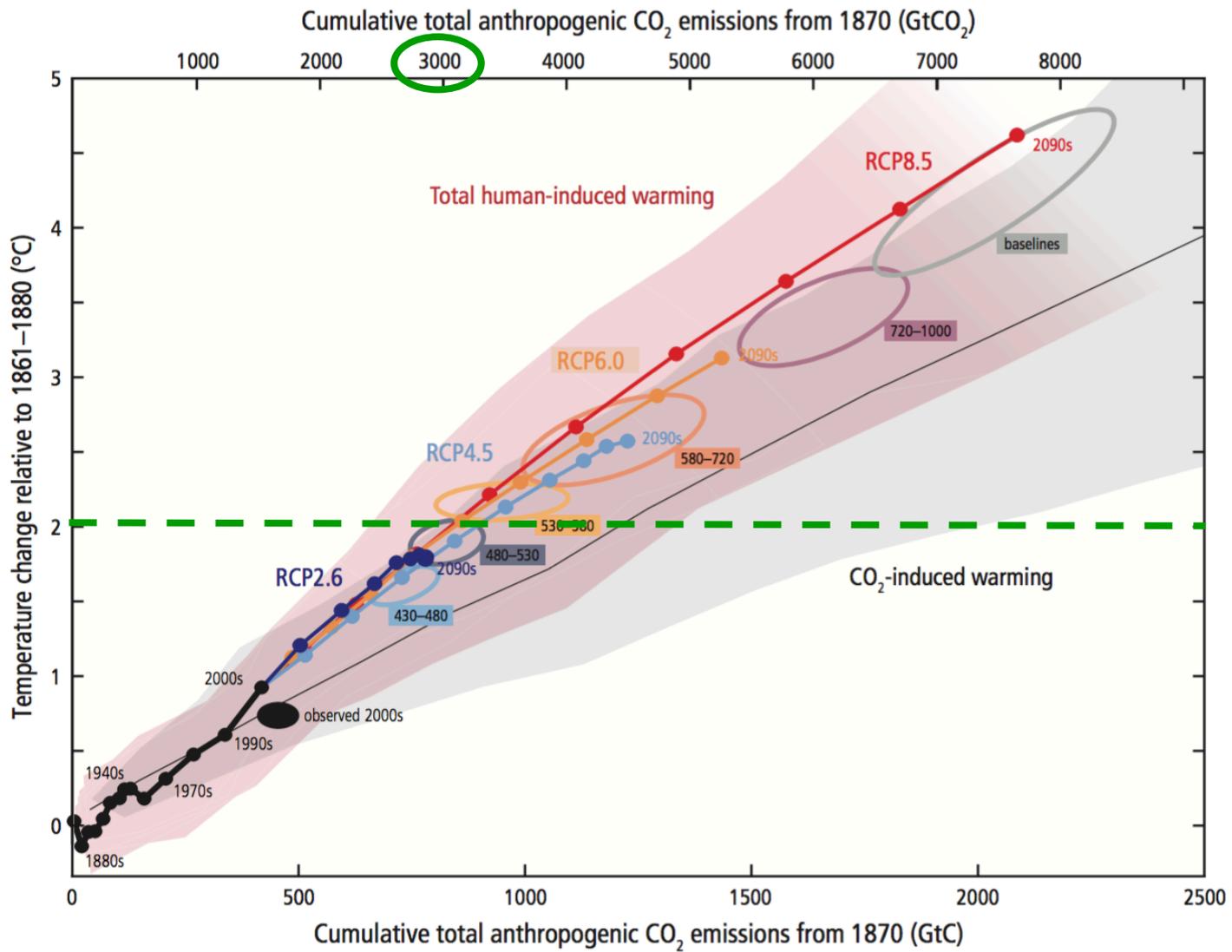


## Large-scale singular events

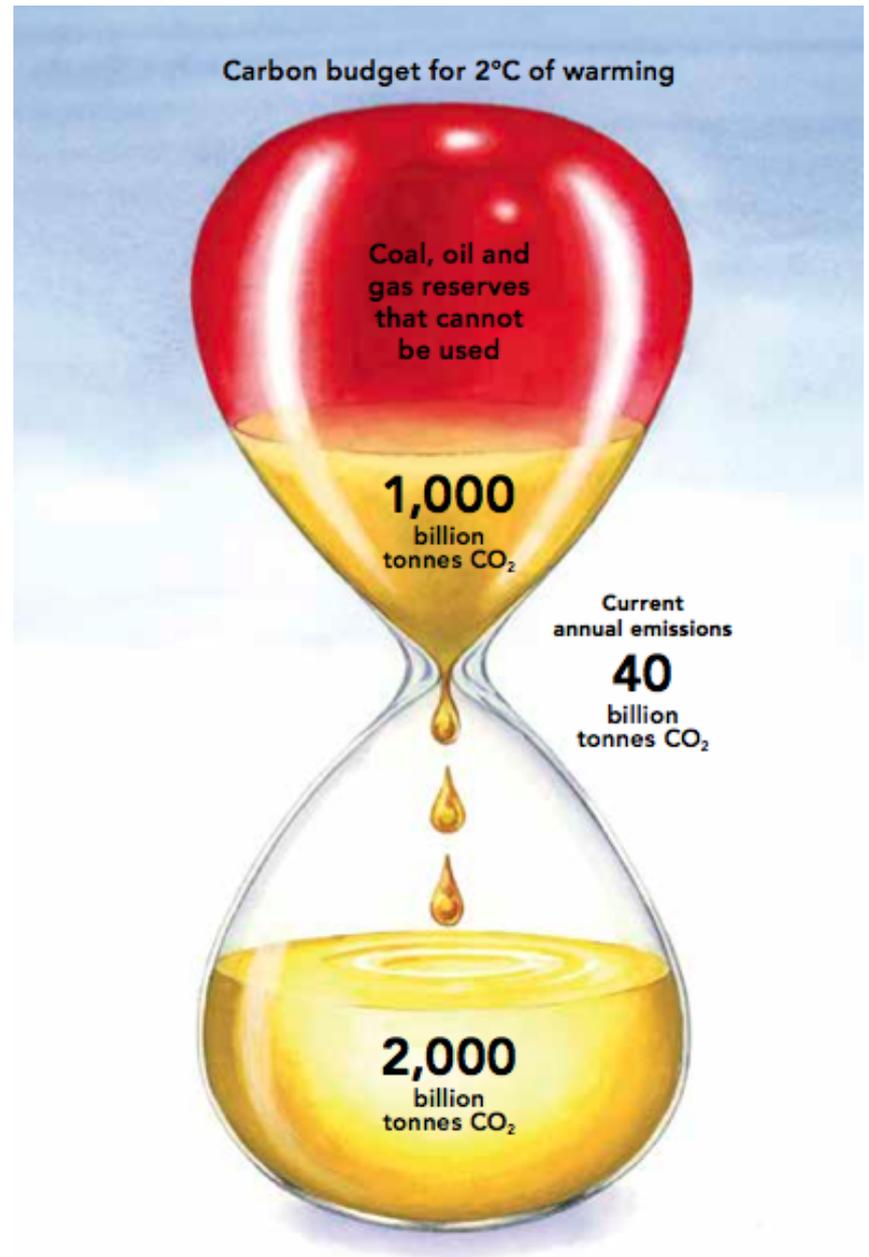
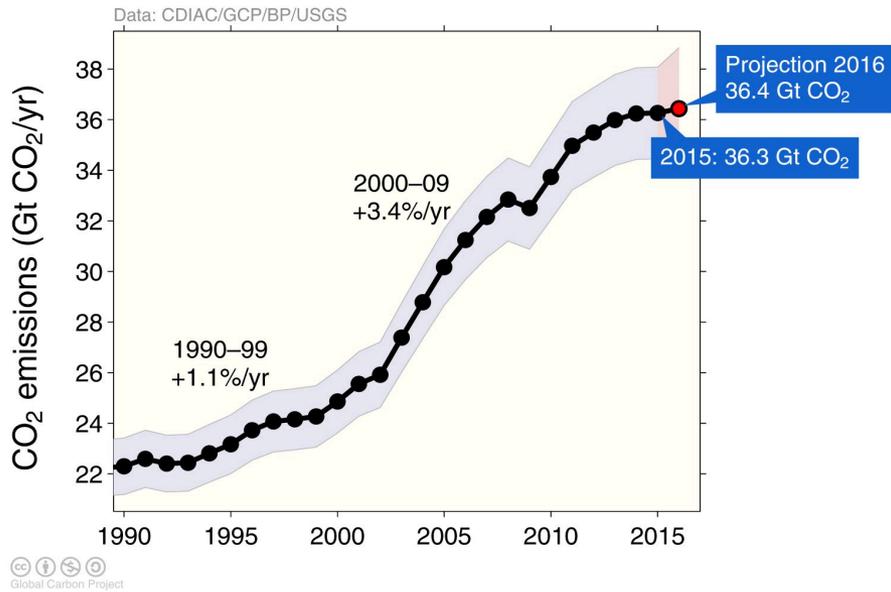


WG2 Box SPM.1

e.g. Feldmann & Levermann, 2015



SYR Fig 2.3



For likely chance of staying below 2°C have about 30 years emissions left at current level

[Estimates of \$20 trillion put on “unburnable” carbon]



Our understanding of our climate is rooted in mathematics

Climate change is one of the greatest challenges of our time

The application of mathematical ideas & tools is driving forward our knowledge of our changing climate and the risks posed to society and the natural world, and is guiding our response to this global threat

It is a clear demonstration of the power of mathematics to address the world's most pressing issues