EVANESCENT WANDERING TRAPS:
Can a target survive?

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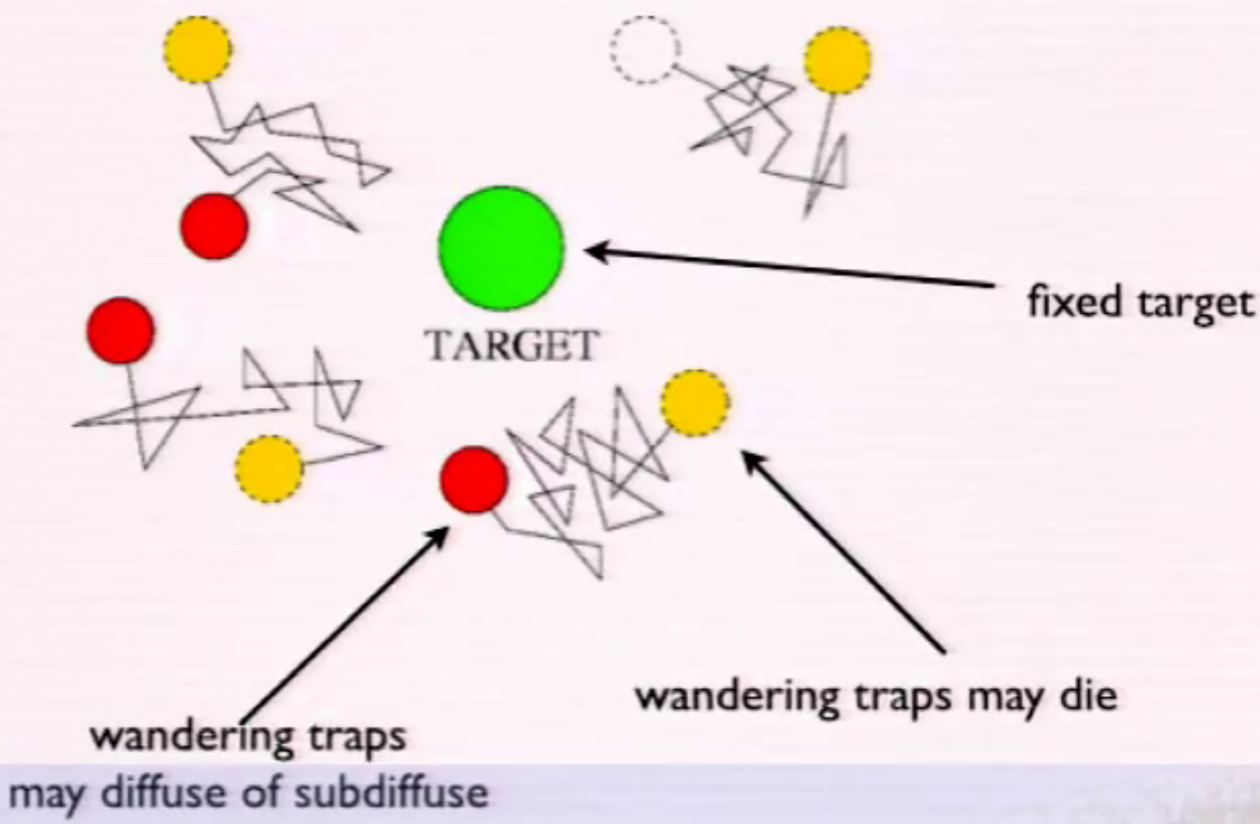
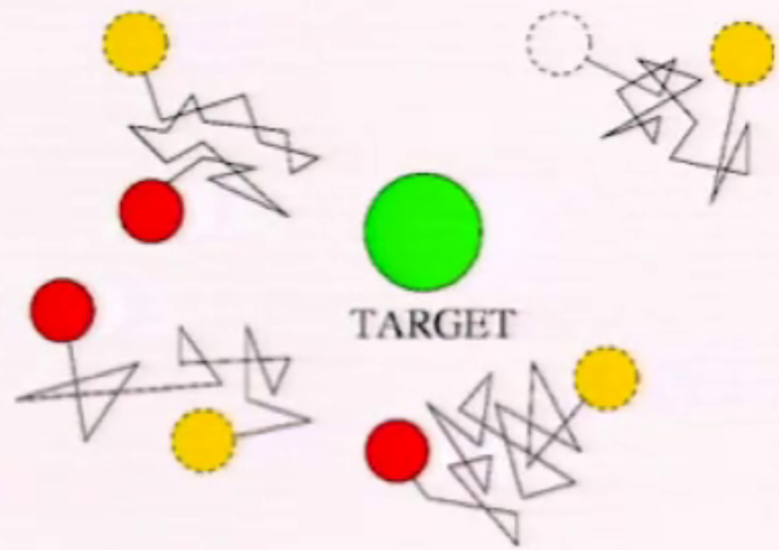


The question

Survival probability of the target?



The answer depends on

Is target FIXED or does it move?

Dimension d=1, 2, 3

How traps move diffusive or subdiffusive

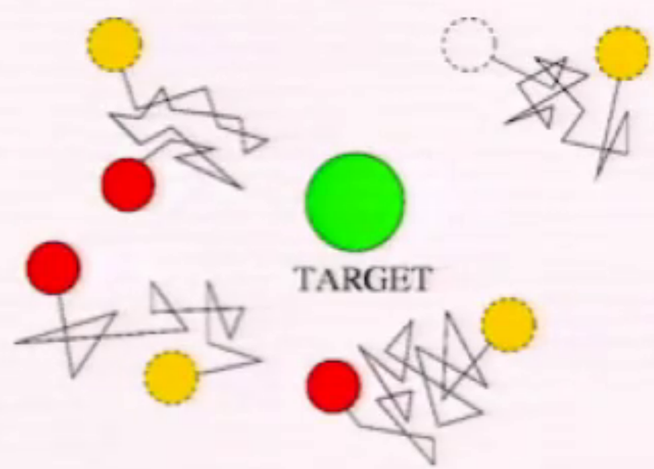
How traps die evanescence

Lesson when there is one target and many traps

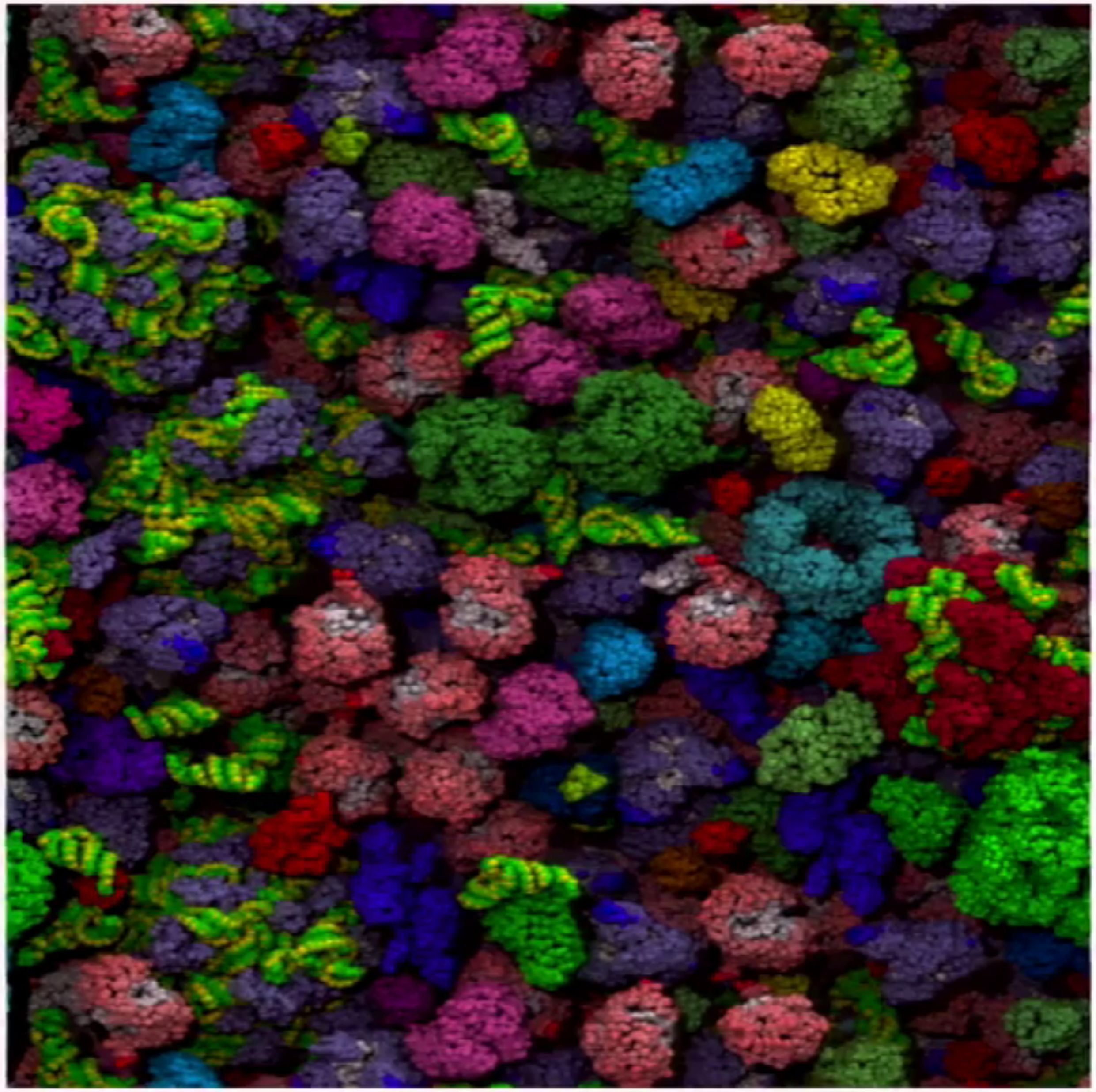
If the traps live forever, the target certainly dies whether the traps diffuse or subdiffuse

If the traps die sufficiently rapidly, the target may survive whether the traps diffuse or subdiffuse

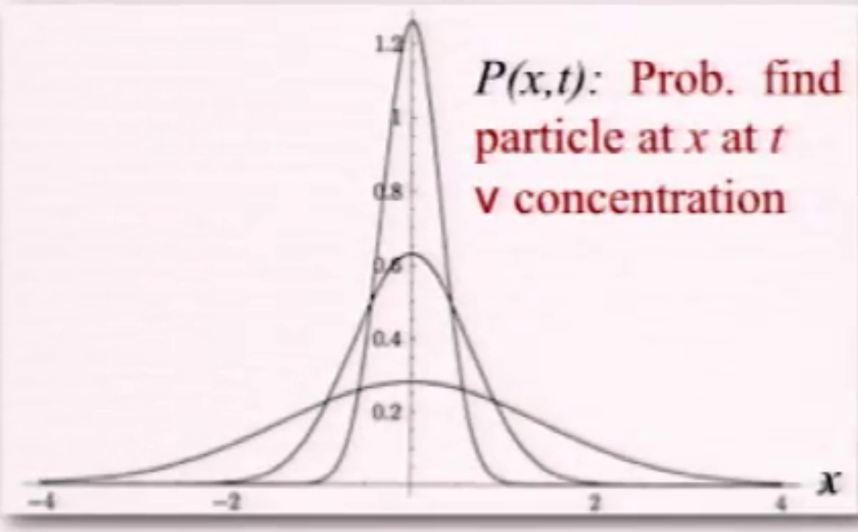
How fast the target dies in those cases where it dies is affected by diffusion vs subdiffusion



Macromolecular crowding in living cells (from R. Metzler)



SR McGuffee & AH Elcock, PLoS Comp Biol (2010)



Diffusion

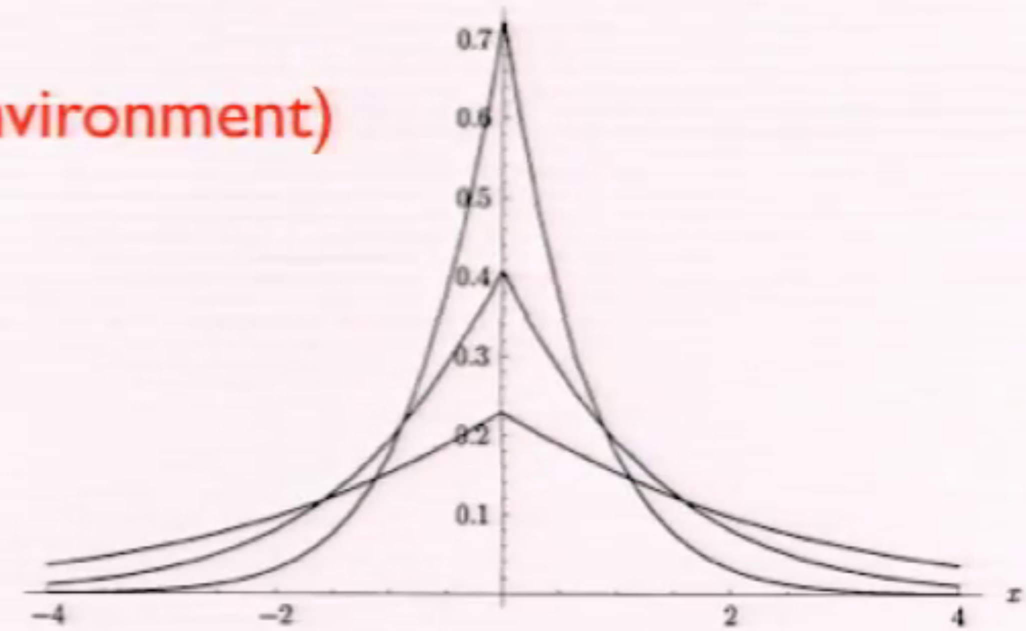
$$\frac{\partial P(x,t)}{\partial t} = D\,\frac{\partial^2 P(x,t)}{\partial x^2}$$

$$\langle x^2 \rangle \sim t$$

Subiffusion (e.g. crowded environment)

$$\langle x^2 \rangle \sim t^{\gamma} \qquad 0 < \gamma < 1$$

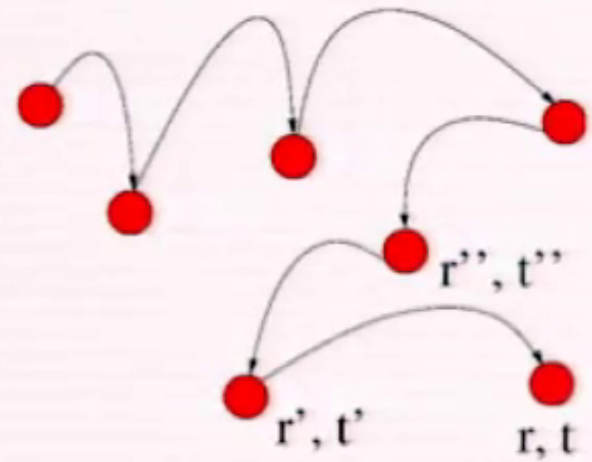
Equation: see below



One approach: CTRW $\rightarrow$ Fractional diffusion equation

Long-tailed waiting time distribution, jump length distribution of finite variance

First: NO evanescence, ONE trap



$w(r,t|r_0,0) =$ Probability that the trap is at r at time t if it was at r_0 at time t_0

$$\frac{\partial w(r,t|r_0,0)}{\partial t} = K_\gamma \ {}_0D_t^{1-\gamma} \, \nabla_r^2 w(r,t|r_0,0)$$

where $${}_0D_t^{1-\gamma} f(r,t) = \frac{1}{\Gamma(\gamma)} \frac{\partial}{\partial t} \int_0^t dt' \frac{f(r,t')}{(t-t')^{1-\gamma}}$$

Memory!

When a trap reaches the target, both die

SINGLE TRAP

$d = 1$ Target dies as $t^{-\gamma/2}$

$d = 2$ Target dies as $1/\ln(t)$

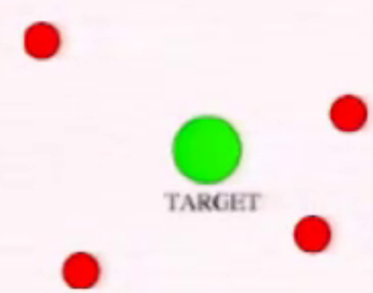
$d = 3$ Target lives

MANY TRAPS

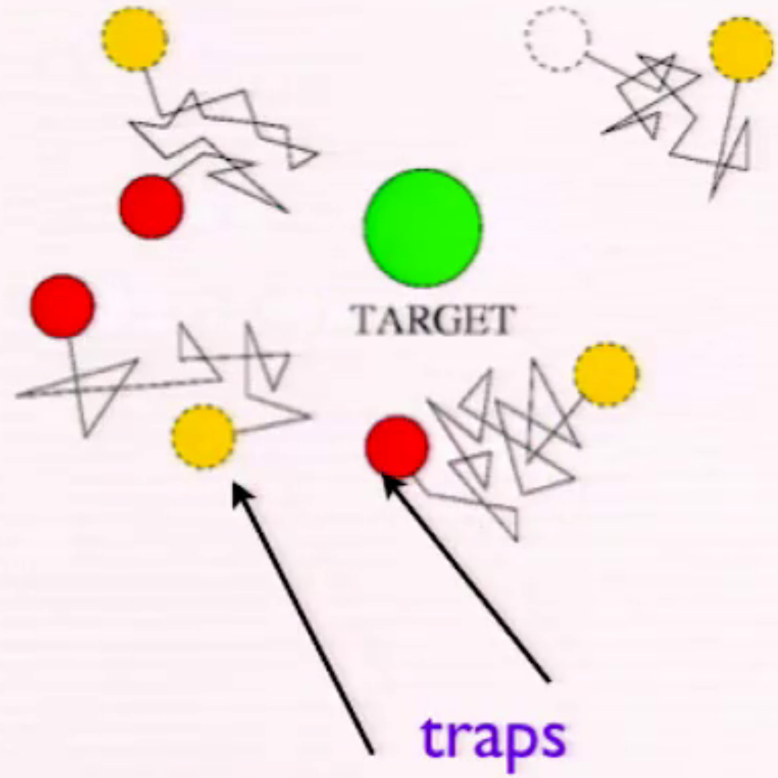
$d = 1$ Target dies as $\exp[-\rho_0 R t^{\gamma/2}]$

$d = 2$ Target dies as $\exp[-\rho_0 R^2 t^{\gamma} / \ln(t)$

$d = 3$ Target dies as $\exp[-\rho_0 R^3 t^{\gamma}]$



Now: EVANESCENT traps



Density $\rho(t), \ \ \rho(0) = 1$

Exponential evanescence

$$\rho(t) = \rho_0 \exp(-\lambda t)$$

Power law evanescence

$$\rho(t) = \frac{\rho_0}{(1+\lambda t)^\beta}$$

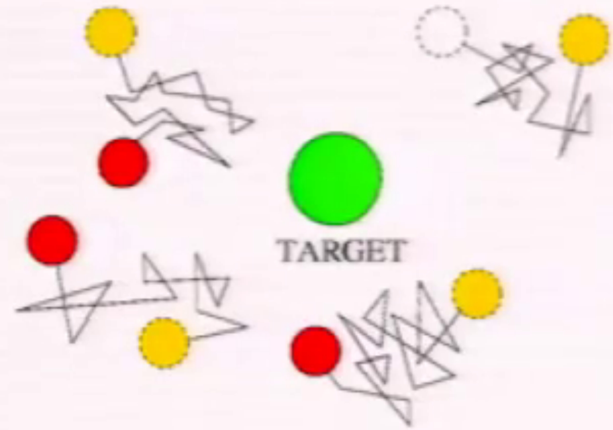
EVANESCENT TRAPS

$$\dot{\rho}(t) = -\lambda(t)\rho(t)$$

In our model, trap evanescence process is independent of trap reaction with target

How to combine with fractional diffusion equation?

Can not just add!!



SINGLE TRAP

$$\frac{\partial w(r,t|r_0,0)}{\partial t} = \frac{\rho(t)}{\rho_0} K_{\gamma\,0} D_t^{1-\gamma} \frac{\rho_0}{\rho(t)} \nabla_r^2 w(r,t|r_0,0) + \frac{\dot{\rho}(t)}{\rho(t)} w(r,t|r_0,0)$$

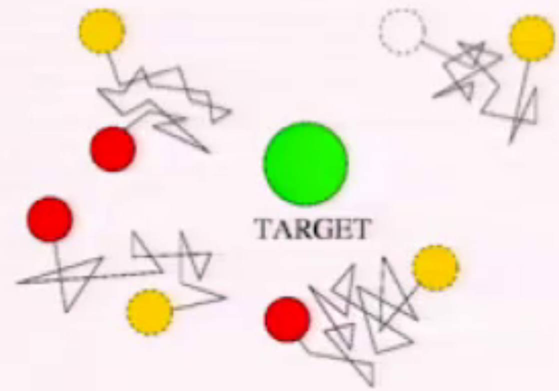
As before $Q_1^*(r_0,t;R) = \int dr\, w(r,t|r_0;0)$

But now survival probability of the target and the trap are no longer the same

$$Q_{1,T}^*(r,t;R) = Q_1^*(r,t;R) - \int_0^t dt'\, Q_1^*(r,t';R) \frac{\dot{\rho}(t')}{\rho(t')}$$

MANY TRAPS

If traps can die, what is the survival probability of the target ?



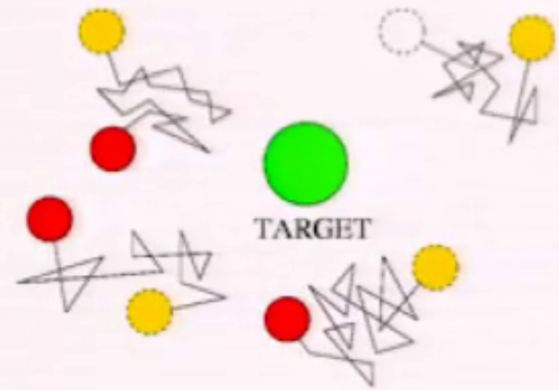
$$Q_T^*(t;R) = \exp\left[-\rho_0 R^d \sigma^*(t,R)\right]$$

$\sigma^*(t,R)$ depends on the rate of evanescence

Exponential evanescence $\rho(t) = \rho_0 \exp(-\lambda t)$

The survival probability is finite in all dimensions for one or even for many traps!

MANY TRAPS



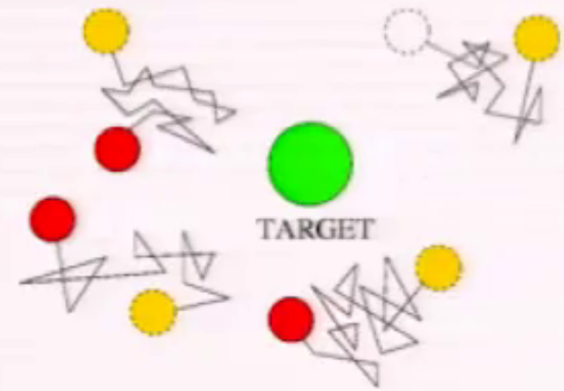
Survival probability of target

Exponential evanescence

The survival probability of the target is finite in all dimensions for one or even for many traps!

- The decay of the survival probability to the final state is faster as dimensionality increases
- The decay of the survival probability to the final state is slowed by subdiffusion
- The final survival probability is increased by subdiffusion
- The limit of no evanescence is singular

MANY TRAPS



If traps can die, what is the survival probability of the target ?

$$Q_T^*(t;R) = \exp\left[-\rho_0 R^d \sigma^*(t,R)\right]$$

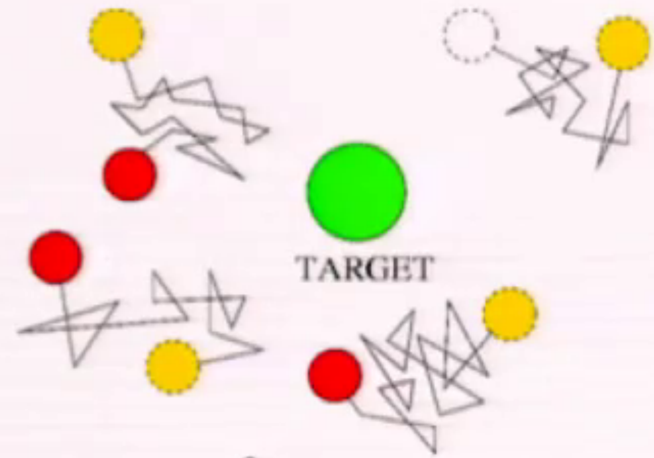
$\sigma^*(t,R)$ depends on the rate of evanescence

Power law evanescence $\rho(t) = \dfrac{\rho_0}{(1+\lambda t)^\beta}$

More intricate dependence on parameters and dimension

MANY TRAPS

If traps can die, what is the survival probability of the target ?



Power law evanescence → $$\rho(t) = \frac{\rho_0}{(1+\lambda t)^{\beta}}$$

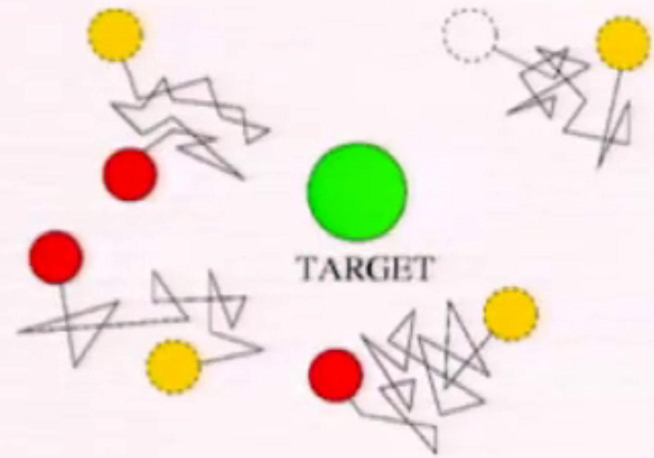
$d = 1$

Survival probability of target is finite if $\beta > \gamma/2$

Otherwise survival probability of target goes to zero

MANY TRAPS

If traps can die, what is the survival probability of the target ?



Power law evanescence

$$\rho(t) = \frac{\rho_0}{(1+\lambda t)^\beta}$$

$d = 1$ Survival probability of target is finite if $\beta > \gamma/2$

$d = 2, 3$ Survival probability of target is finite if $\beta > \gamma$

Otherwise survival probability of target goes to zero

Lesson when there is one target and many traps

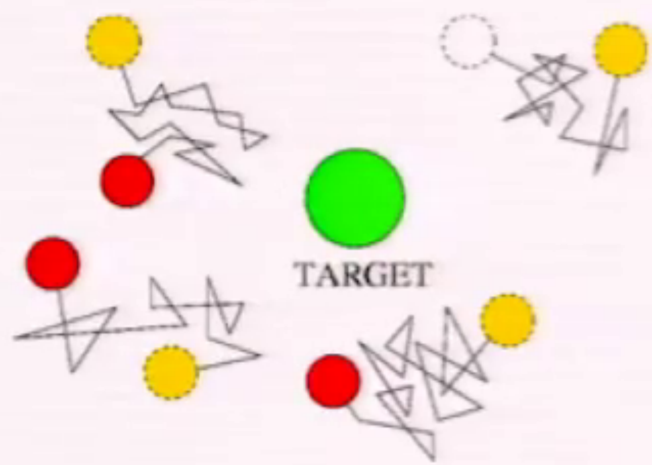
If the traps live forever, the target certainly dies whether the traps diffuse or subdiffuse

If the traps die sufficiently rapidly, the target may survive whether the traps diffuse or subdiffuse

How fast the target dies in those cases where it dies is affected by diffusion vs subdiffusion

More detailed summary

All very logical!



Without evanescence

1. Single trap: subdiffusive character only matters in d=1. Target dies for sure but it takes longer with subdiffusion than ordinary diffusion.

2. Single trap when d=2: target dies for sure, marginally slowed down with subdiffusion

3. Single trap when d=3: target may survive! Subdiffusion does not affect survival probability

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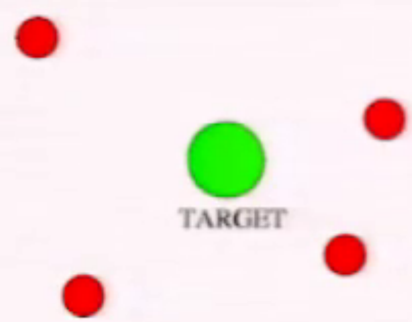
2. Single trap when d=2: target dies for sure, marginally slowed down with subdiffusion

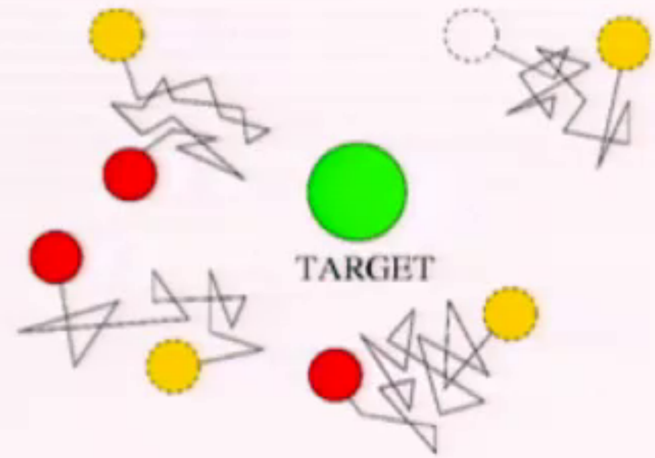
3. Single trap when d=3: target may survive! Subdiffusion does not affect survival probability

4. Many traps: survival probability of target goes to zero in all dimensions, more slowly with subdiffusing than with diffusing traps.



With evanescence

Completely modifies behavior



Exponential evanescence

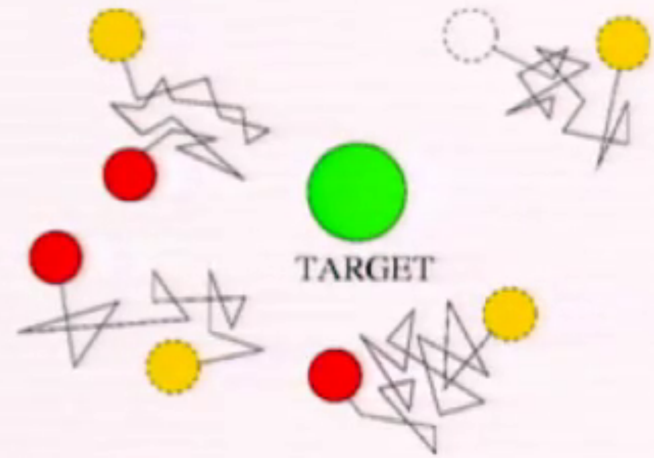
1. Single trap: target may survive in all dimensions

2. Many traps: target may still survive in all dimensions

Asymptotic survival probability increased by subdiffusion (traps die before getting to target) but approach to asymptotia is slower

With evanescence

Completely modifies behavior



Power law evanescence

1. Single trap: target may survive in all dimensions only if power law decay is sufficiently rapid

2. Many traps: target may still survive in all dimensions if power law decay is sufficiently rapid

Asymptotic survival probability increased by subdiffusion (traps die before getting to target) but approach to asymptotia is slower

A totally different approach to the same results:

There is a well-known relation between first passage times and distinct number of sites visited as a function of time for DIFFUSION problems

We showed that this relation also holds for SUBDIFFUSION

We calculated distinct number of sites visited for SUBDIFFUSION