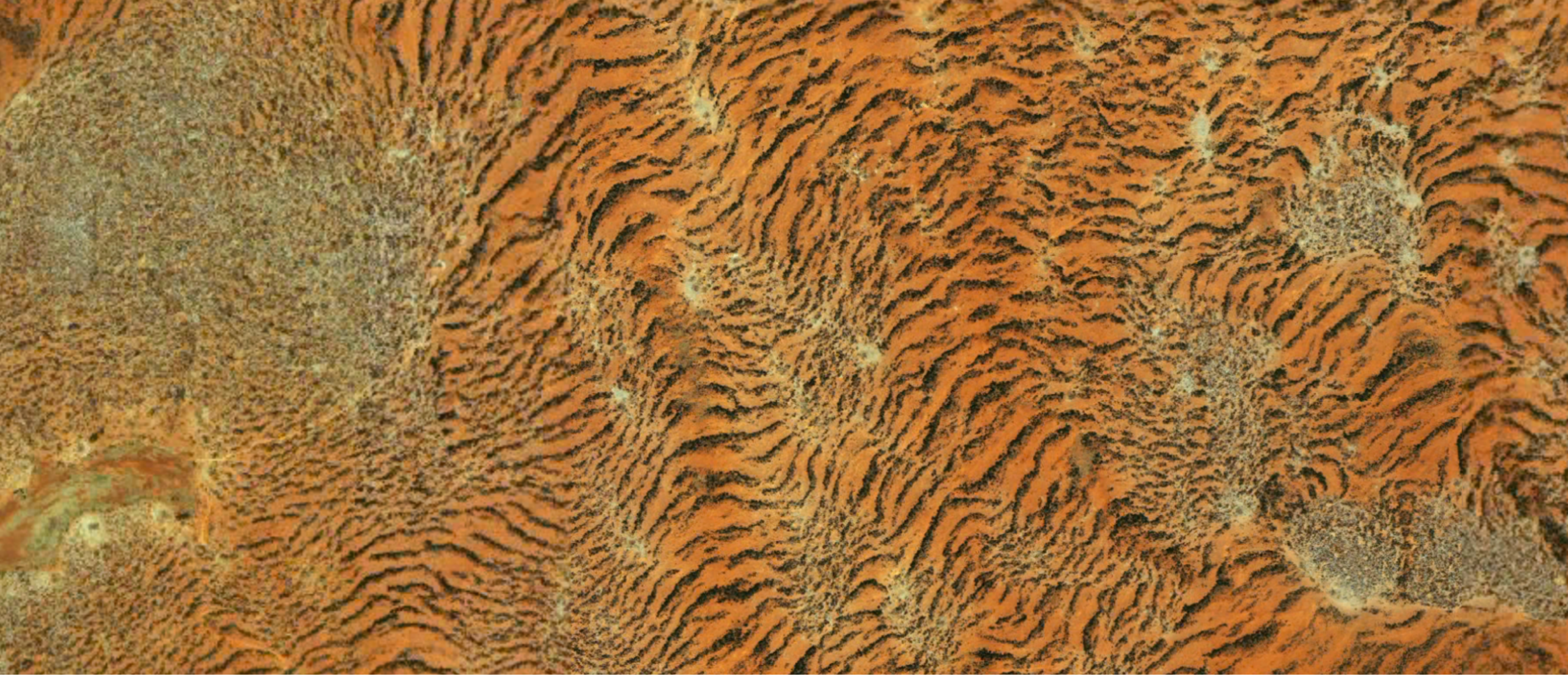


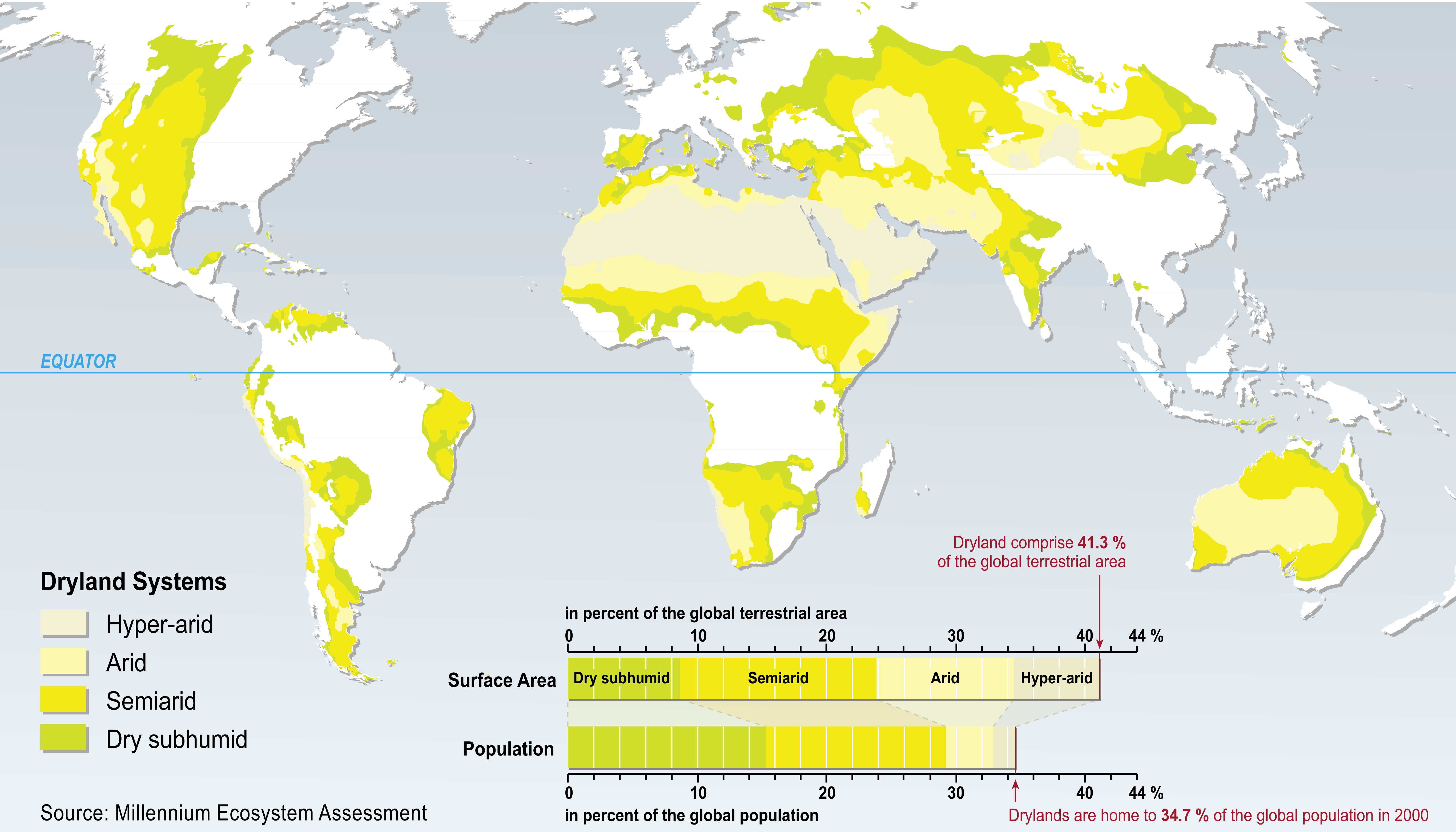
Pattern Formation in the Drylands:

Vegetation Patterns in Mathematical Models & in Satellite Images of the Horn of Africa

Mary Silber
Committee on Computational and Applied Mathematics
+ Dept. of Statistics, University of Chicago

Drylands: water-controlled ecosystems

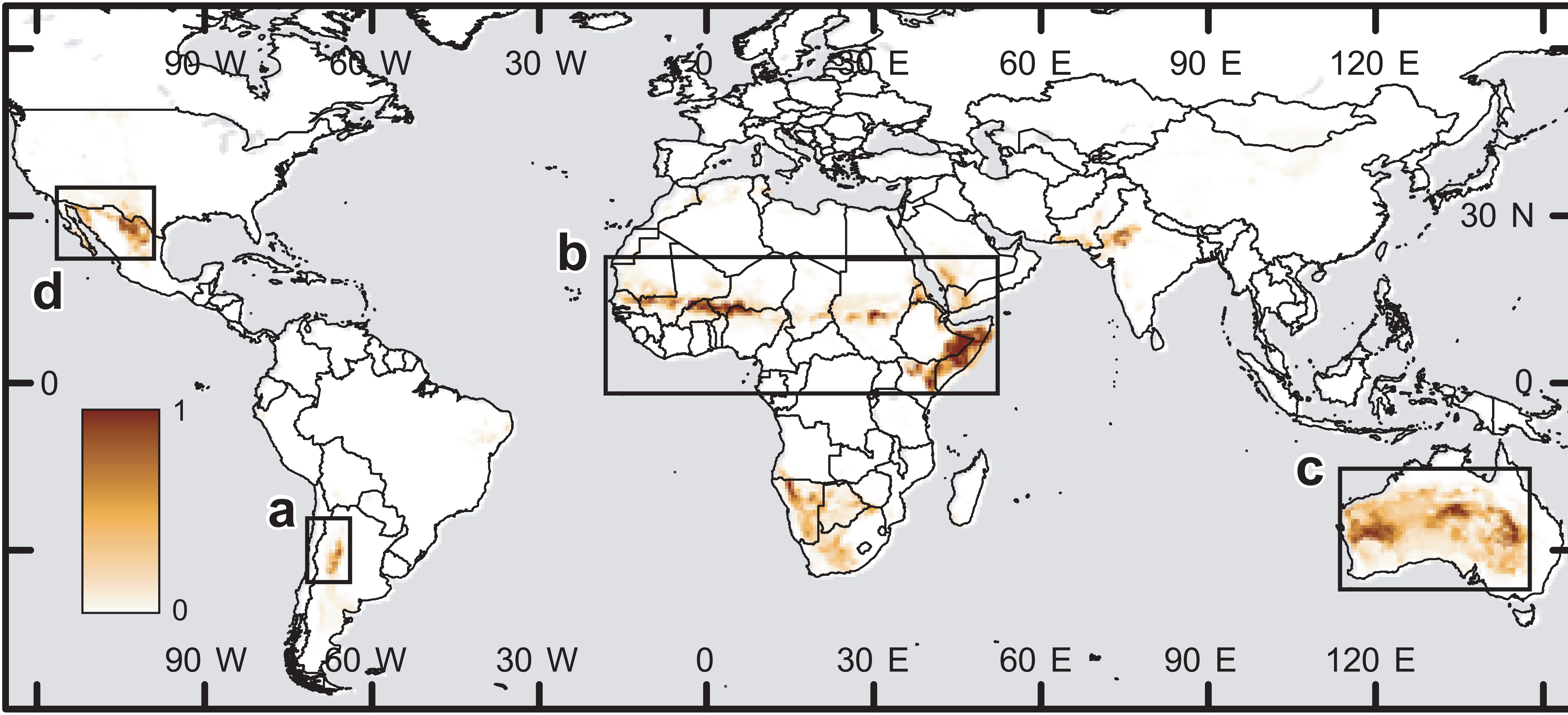
with infrequent, discrete, and largely unpredictable water inputs.



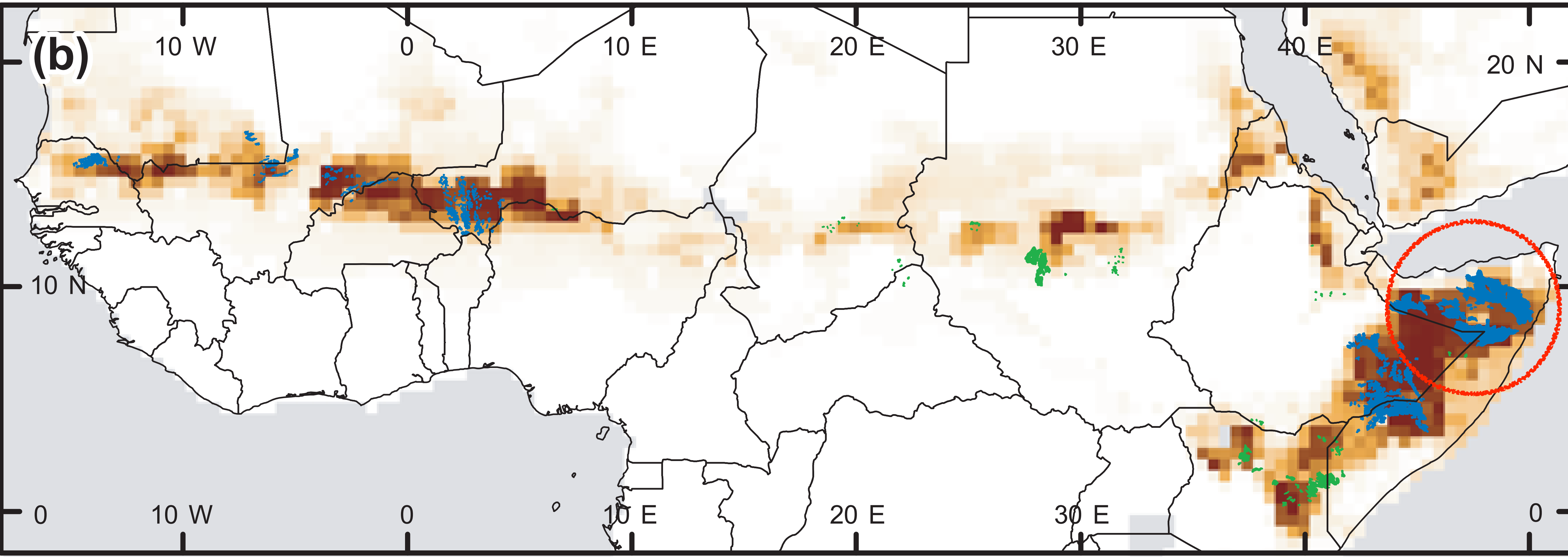
Millennium Ecosystem Assessment (2005)

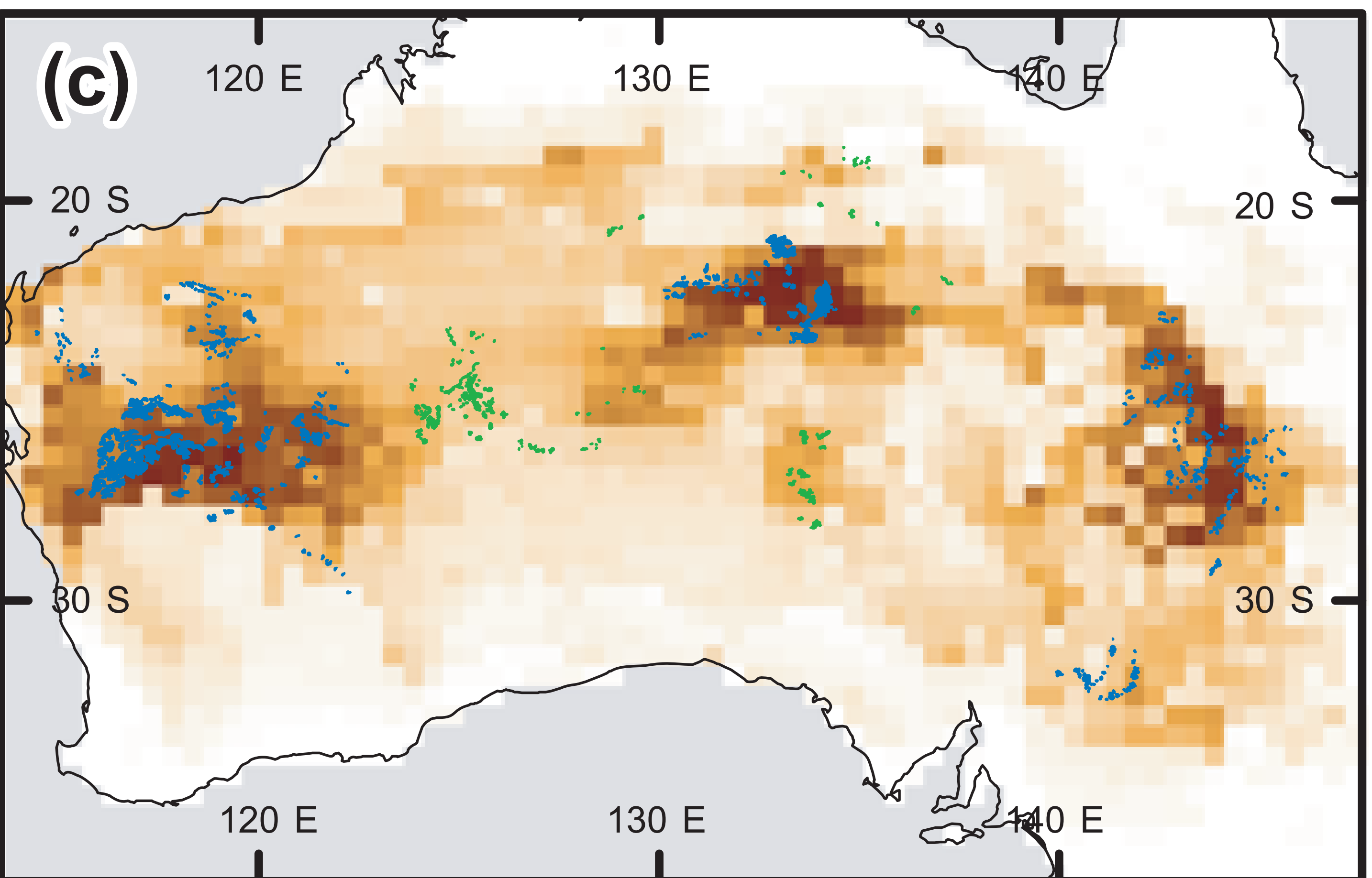
The global biogeography of semi-arid periodic vegetation patterns (2008)

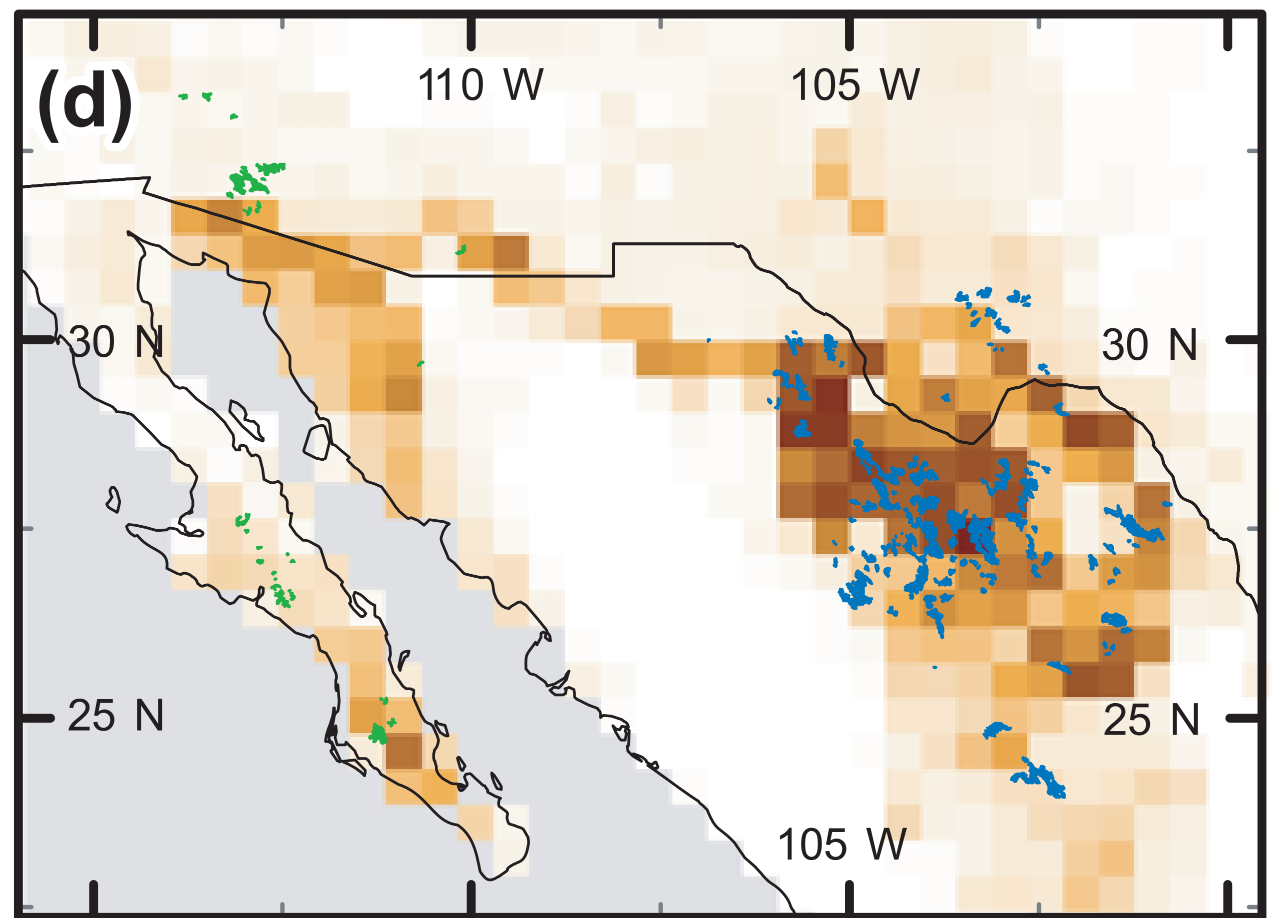
Vincent Deblauwe[1*], Nicolas Barbier[2], Pierre Couteron[3], Olivier Lejeune[4] and Jan Bogaert[1]



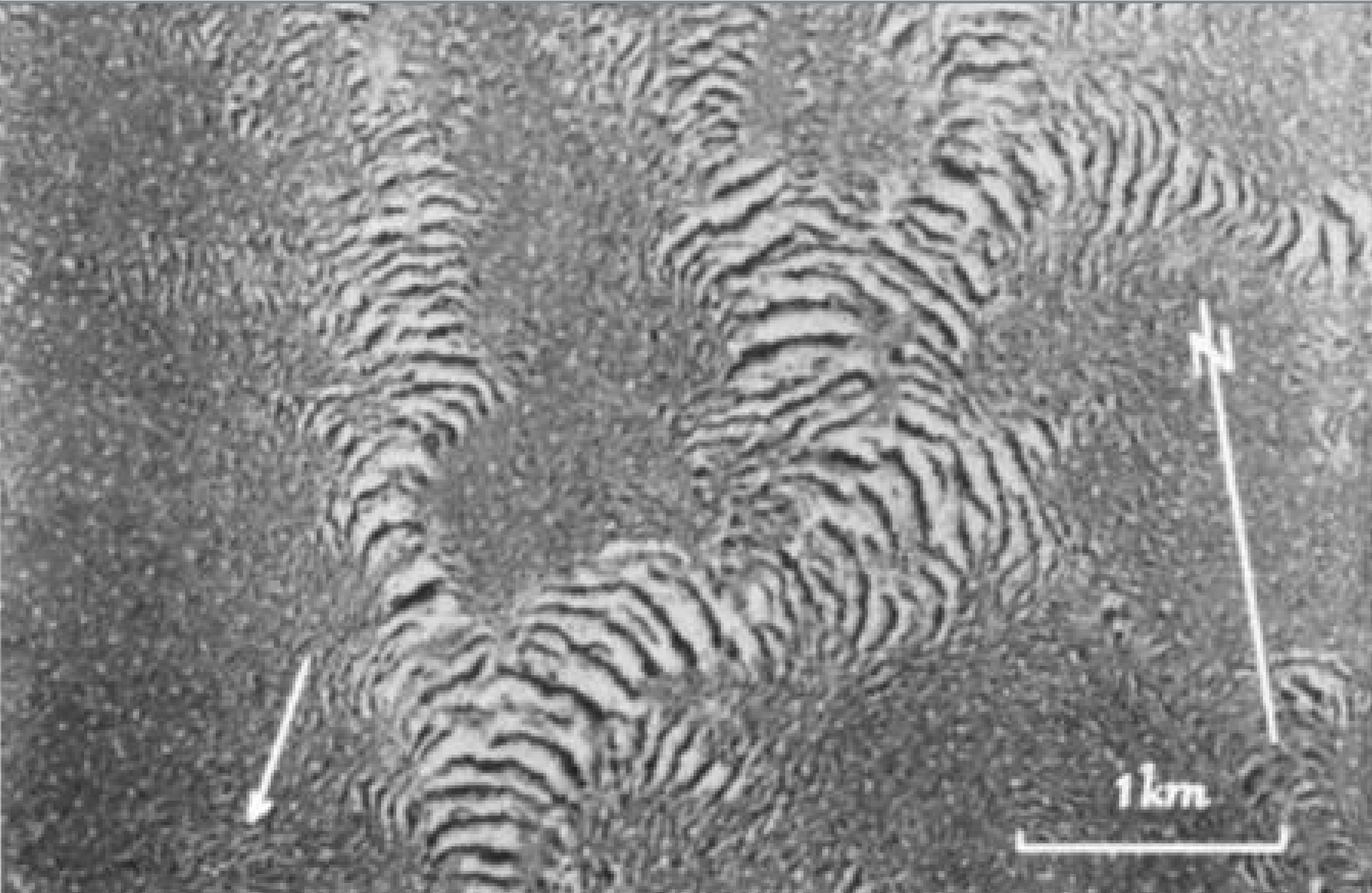
Deblauwe, et al. (2008)





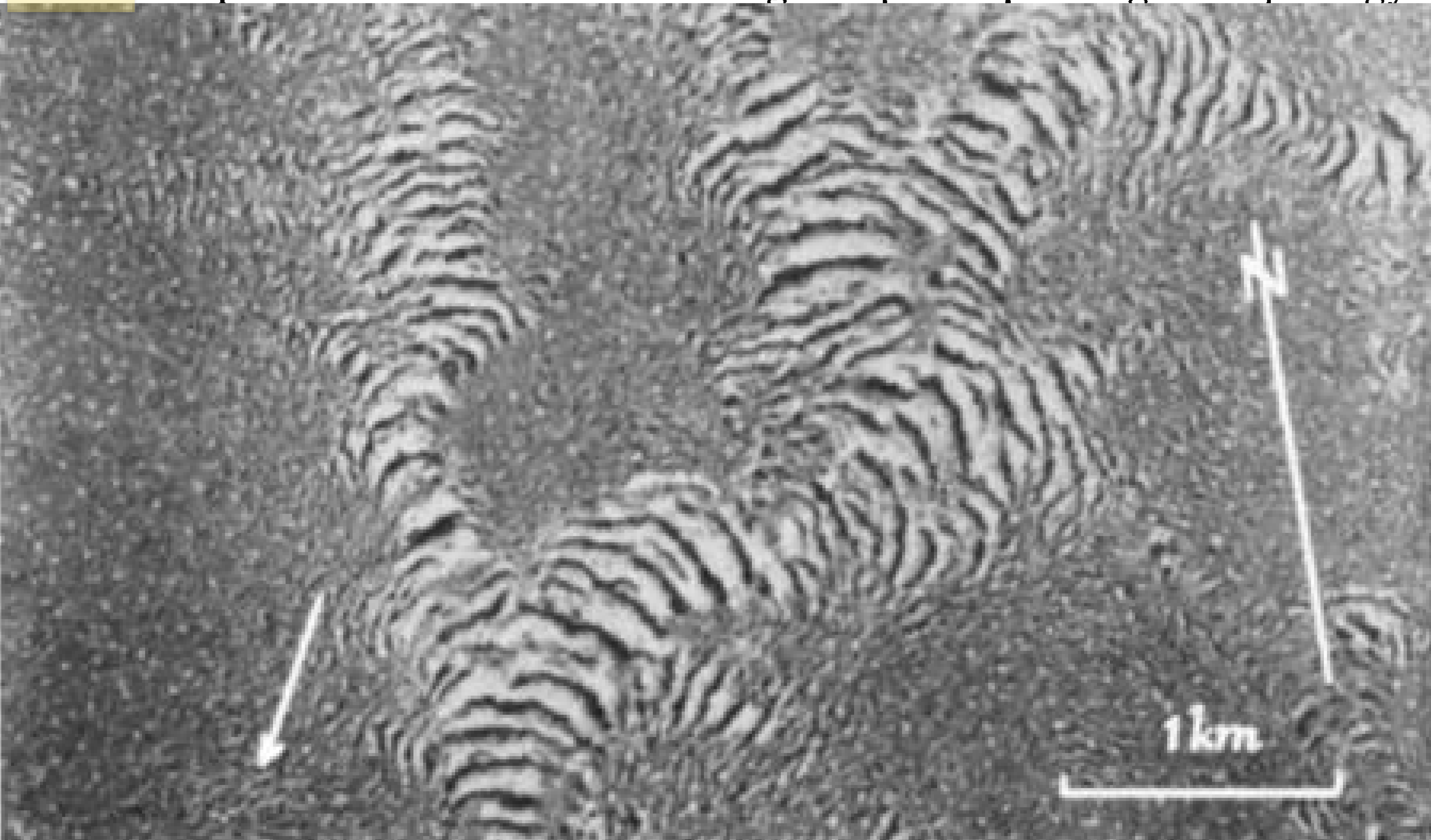


Vegetation Patterns in the Semi-Desert Plains of British Somaliland
Author(s): W. A. Macfadyen
Source: *The Geographical Journal*, Vol. 116, No. 4/6 (Oct. - Dec., 1950), pp. 199-211




4

as I believe, must be investigated by physics and mathematics; and the whole matter must be studied on air photographs, since on the ground it proved difficult to recognize the patterns at all. While the superficial deposits are of importance, the underlying solid geology seems to have no particular significance except in its influence in moulding the pre-requisite geomorphology.



What this talk is *not* about:

fairy circles - with and without termites
(e.g. Bonachela et al.; Getzin et al.; Zelnik et al.;)

scale-free patch size distributions
(e.g. Kefi et al.; Manor et al.; Scanlon et al.)

multi-scale modeling & vegetation patterns
(e.g. Pringle & Tarnita; Tarnita et al.; Vincenot et al.)

localized patterns, homoclinic snaking, vegetation front propagation, colonization vs. Turing
(e.g. Bel et al.; Dawes & Williams; Lejeune et al.; Meron et al.; Sewalt & Doelman; Sherratt; Zelnik et al.)

ramped precipitation, rate-induced transitions
(e.g. Chen et al.; Sherratt & Lord; Siteur et al.)

stochastic/seasonal precipitation
(e.g. D'Odorico et al.; Guttal & Jayaprakash; Rodriguez-Iturbe et al.)

feedbacks to climate system - land/atmosphere boundary condition is very important
(e.g. Baudena et al.; Konings et al.; Pielke et al; Rietkerk et al.+++++)

What this talk is about:

I. *Vegetation Patterns in Mathematical Models*

Bifurcation theoretic framing of an early warning sign scenario
(with K.Gowda and H.Riecke)

Robustness within a simple reaction-diffusion model framework.
(*with K.Gowda, S. Iams, and Y.Chen)*

II. *Vegetation Patterns in Satellite Images*

Persistence of patterns over past 60 years, subtle dynamics,
& not so subtle human impacts.
(see talk by K.Gowda, Tuesday MS93)

Topographic influence on patterns, more than just "upslope migration".
(see poster by L. Werner and P. Gandhi, Wednesday PP2)

	"WETLANDS" (Pattern Formation in Fluids)	"DRYLANDS" (Pattern Formation in the Environment)
Equations	Navier-Stokes+BCs	models exist, but not validated due to lack of experiments
Parameters	often excellent specs	Some inferred at order of magnitude level; some constrained to match phenomena; some models have a lot
Time-scales	minutes - "PhD-scale"	decades-centuries
Spatial-scales	cm scale - "table-top"	50m-"landscape scale"
Symmetries	excellent approximation in controlled experiments	opportunity presented by heterogeneities?
Mechanisms	well developed and validated understanding of pattern formation mechanisms	generic mechanisms invoked

Why study dryland patterns?

- Patterns are so Earthy and beautiful.
- Challenging applied direction for a “mature field” of pattern formation. (See Doelman plenary on Thursday!)
- Occur in ecosystems vulnerable to desertification, meant to feed a third of the world population! Is there useful information in the patterns? Any “early warning signs”?

Vegetation Patterns in Mathematical Models

MATHEMATICAL MODELS OF VEGETATION PATTERN FORMATION IN ECOHYDROLOGY

F. Borgogno,[1] P. D'Odorico,[2] F. Laio,[1] and L. Ridolfi[1]

Reviews of Geophysics, 47, RG1005 / 2009

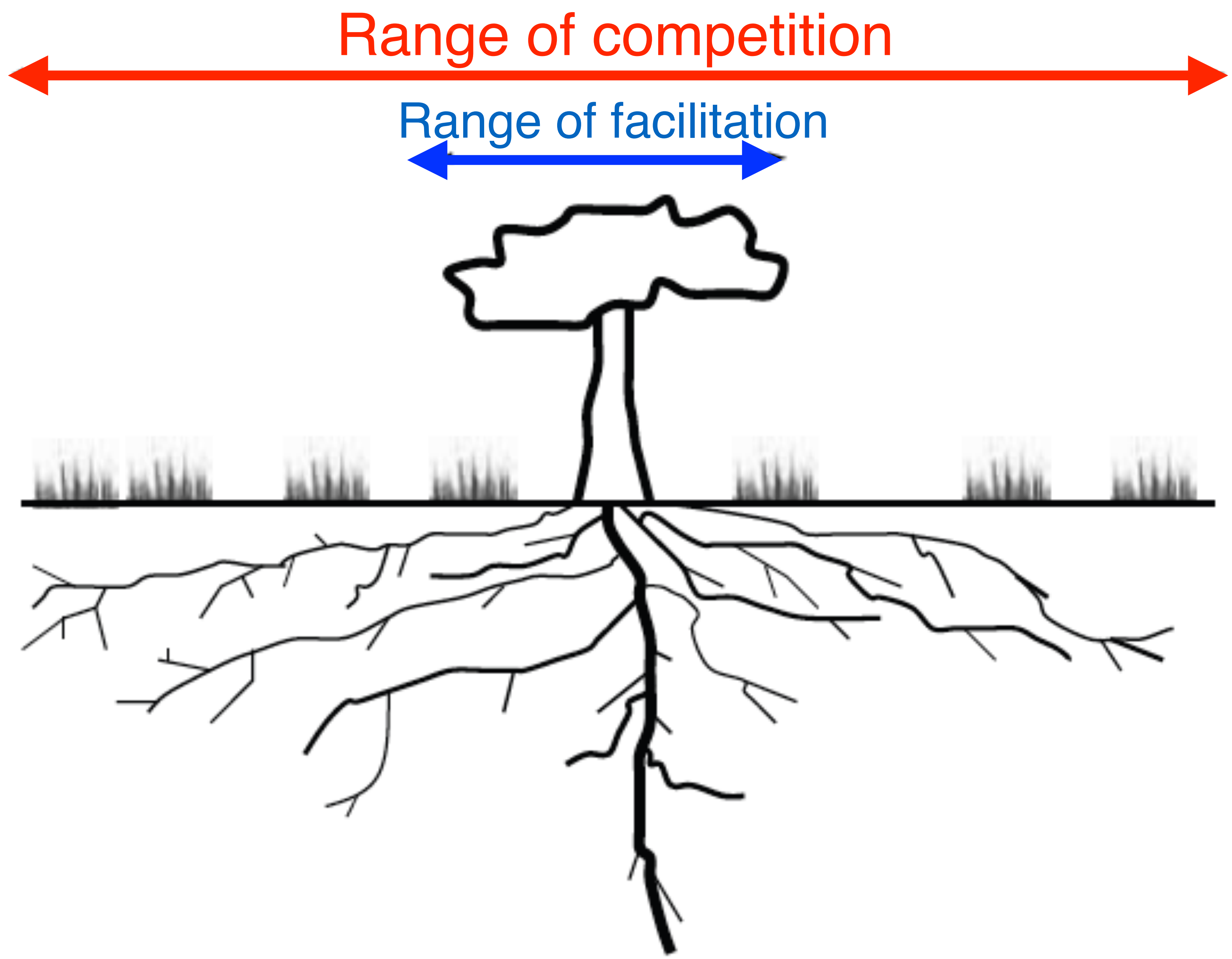


Figure 8. Visualization of the positive and negative interactions typical of a tree.

THE CHEMICAL BASIS OF MORPHOGENESIS

BY A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

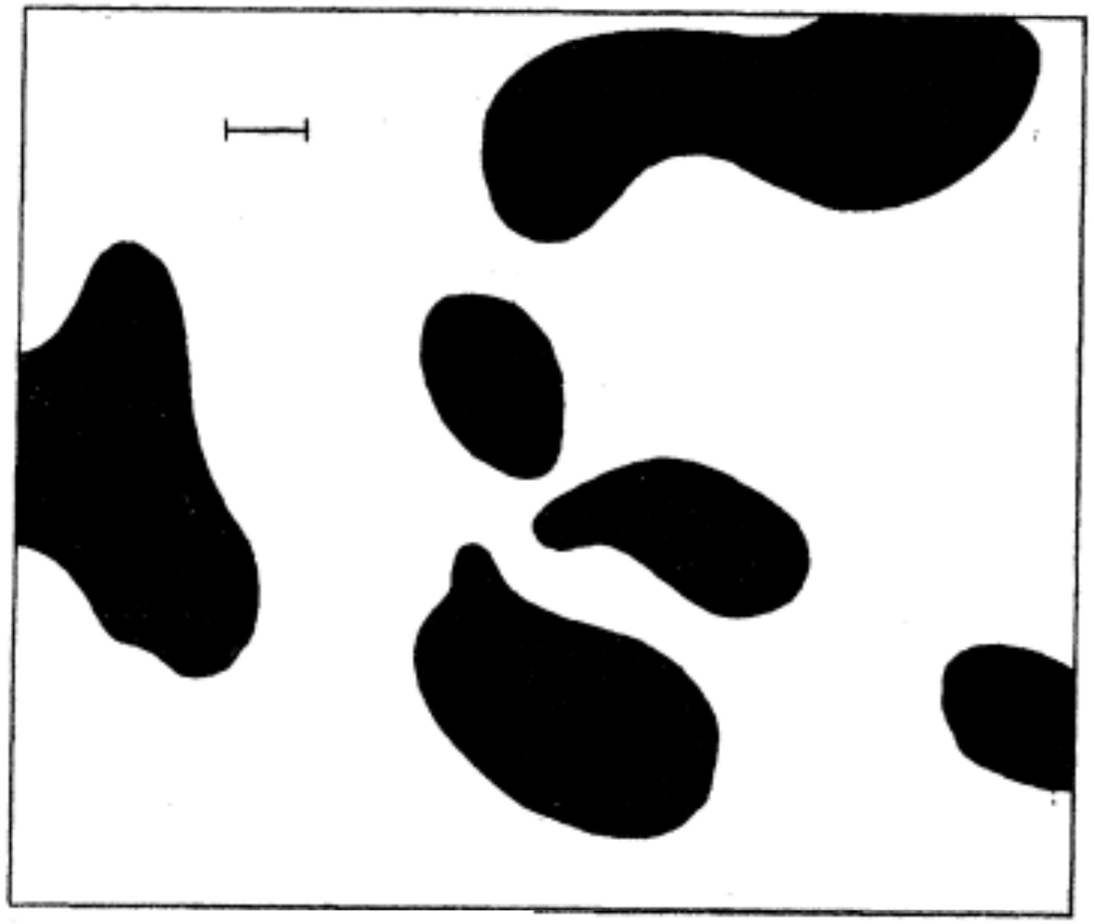


FIGURE 2. An example of a 'dappled' pattern

- "Activator-Inhibitor" reaction-diffusion systems
- "Activator" diffusion $\ll$ "inhibitor" diffusion
- Symmetry-breaking instability result of diffusion
- Pattern length scale not set by geometry

"…framework seductively versatile and generalizable", Pringle & Tarnita, Ann. Rev. of Entomology 2017.

$$\frac{\partial b}{\partial t} = \text{growth}(b, w) - \text{death}(b, w) - \text{dispersal}(b)$$

$$\frac{\partial w}{\partial t} = \text{precipitation} - \text{evapotranspiration}(b, w) - \text{water transport}(w)$$

biomass density (*b*)

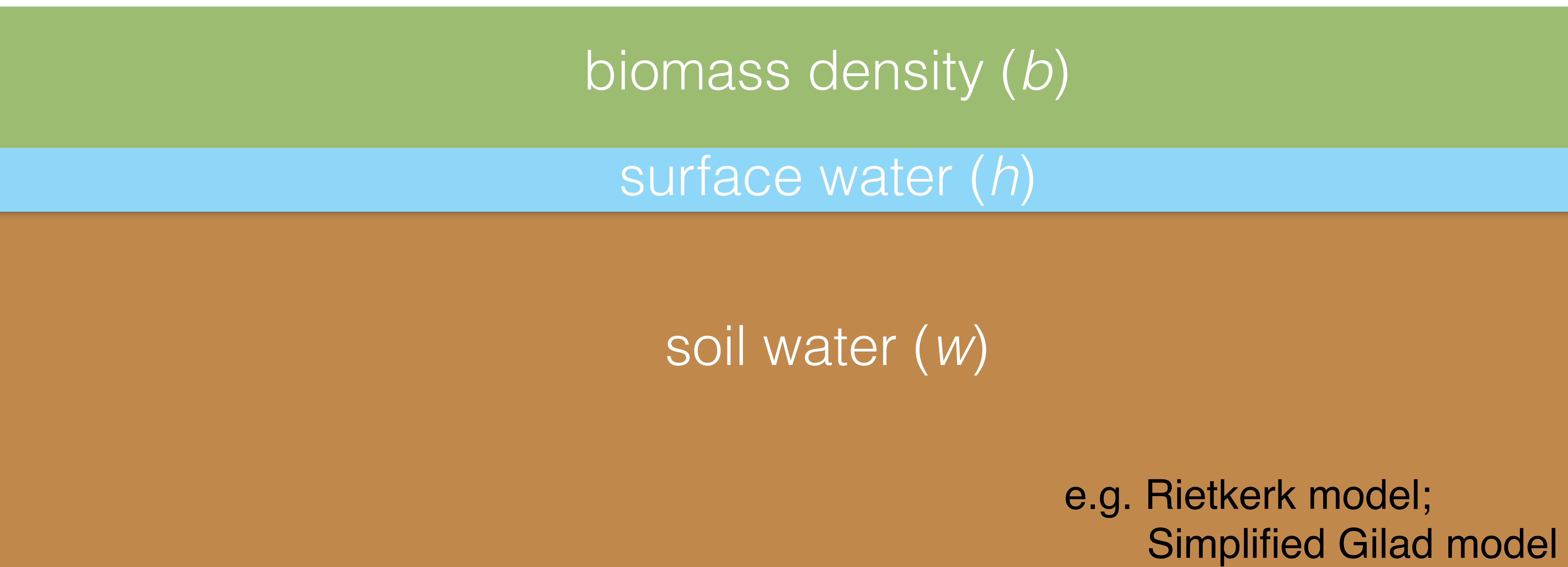
water density (*w*)

e.g. Klausmeier model;
Meron/von Hardenberg model

$$\frac{\partial b}{\partial t} = \text{growth}(b, w) - \text{death}(b, w) - \text{dispersal}(b)$$

$$\frac{\partial h}{\partial t} = \text{precipitation} - \text{infiltration}(b, h) - \text{surface water runoff}(h)$$

$$\frac{\partial w}{\partial t} = -\text{evapotranspiration}(b, w) + \text{infiltration}(b, h) - \text{soil water tranport}(w)$$

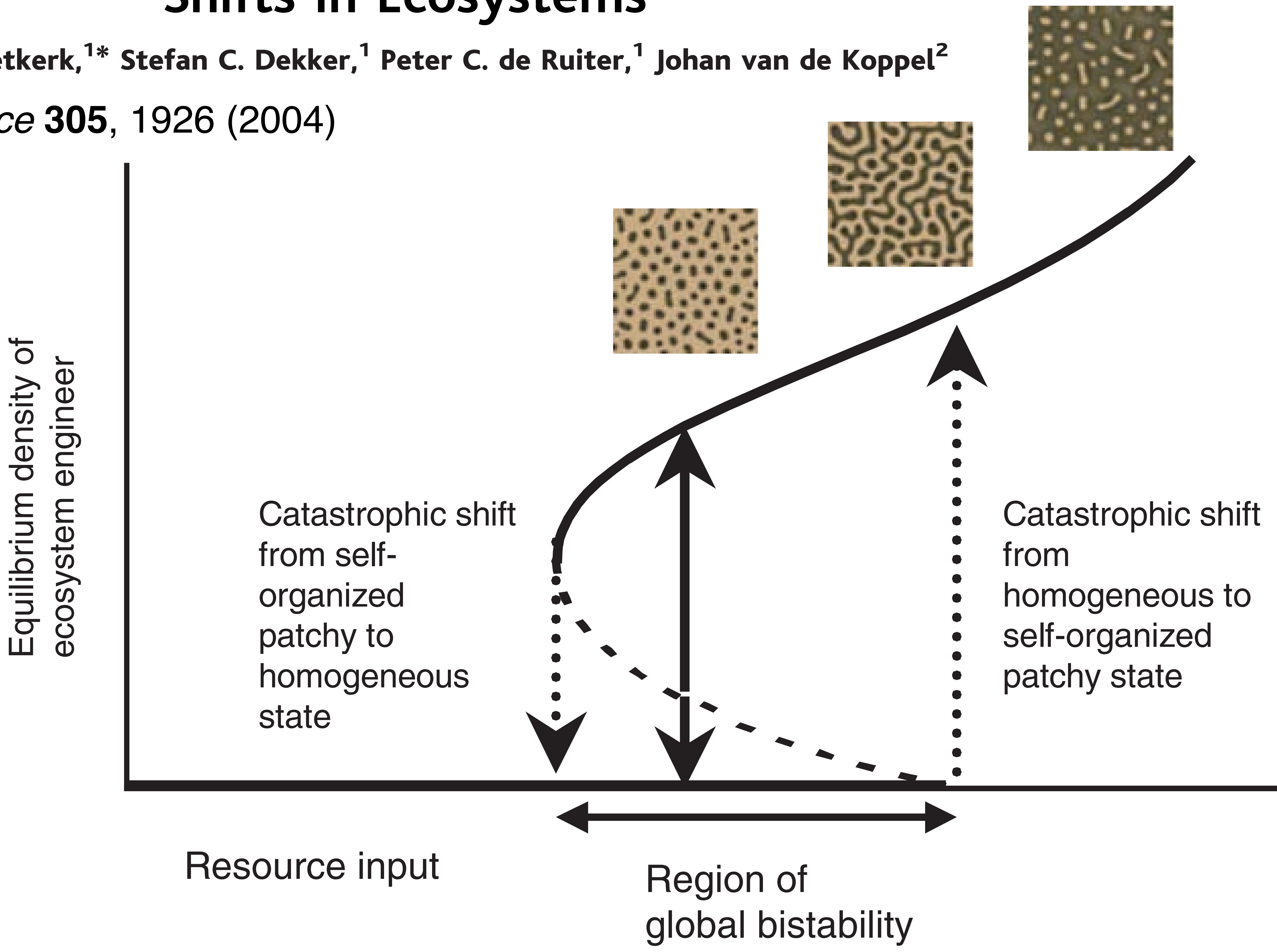


e.g. Rietkerk model;
Simplified Gilad model

Self-Organized Patchiness and Catastrophic Shifts in Ecosystems

Max Rietkerk,[1*] Stefan C. Dekker,[1] Peter C. de Ruiter,[1] Johan van de Koppel[2]

Science **305**, 1926 (2004)

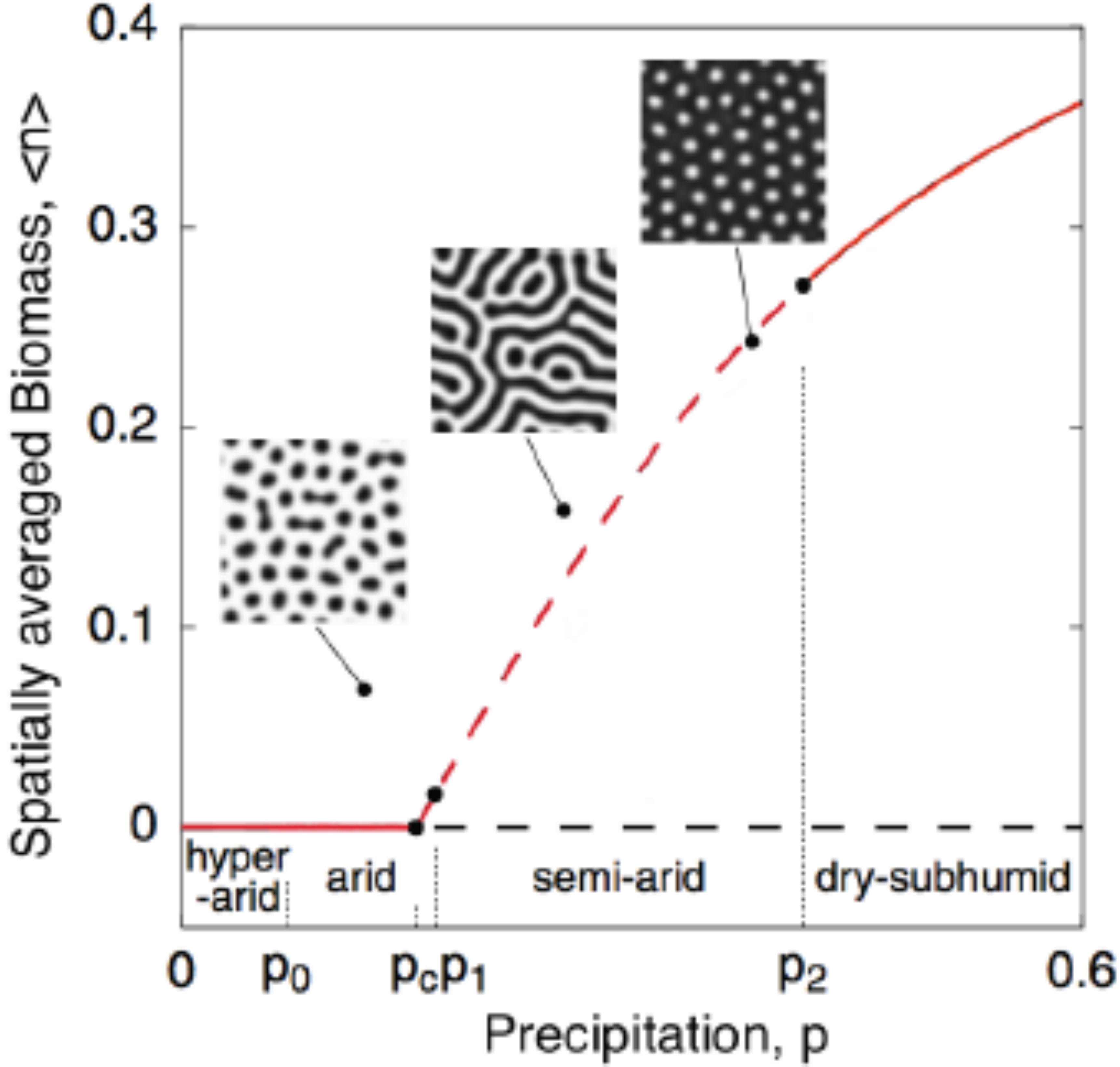


Vegetation Pattern Models: Turing mechanism

Diversity of Vegetation Patterns and Desertification

J. von Hardenberg,[1,4] E. Meron,[1,3] M. Shachak,[2] and Y. Zarmi[1,3]

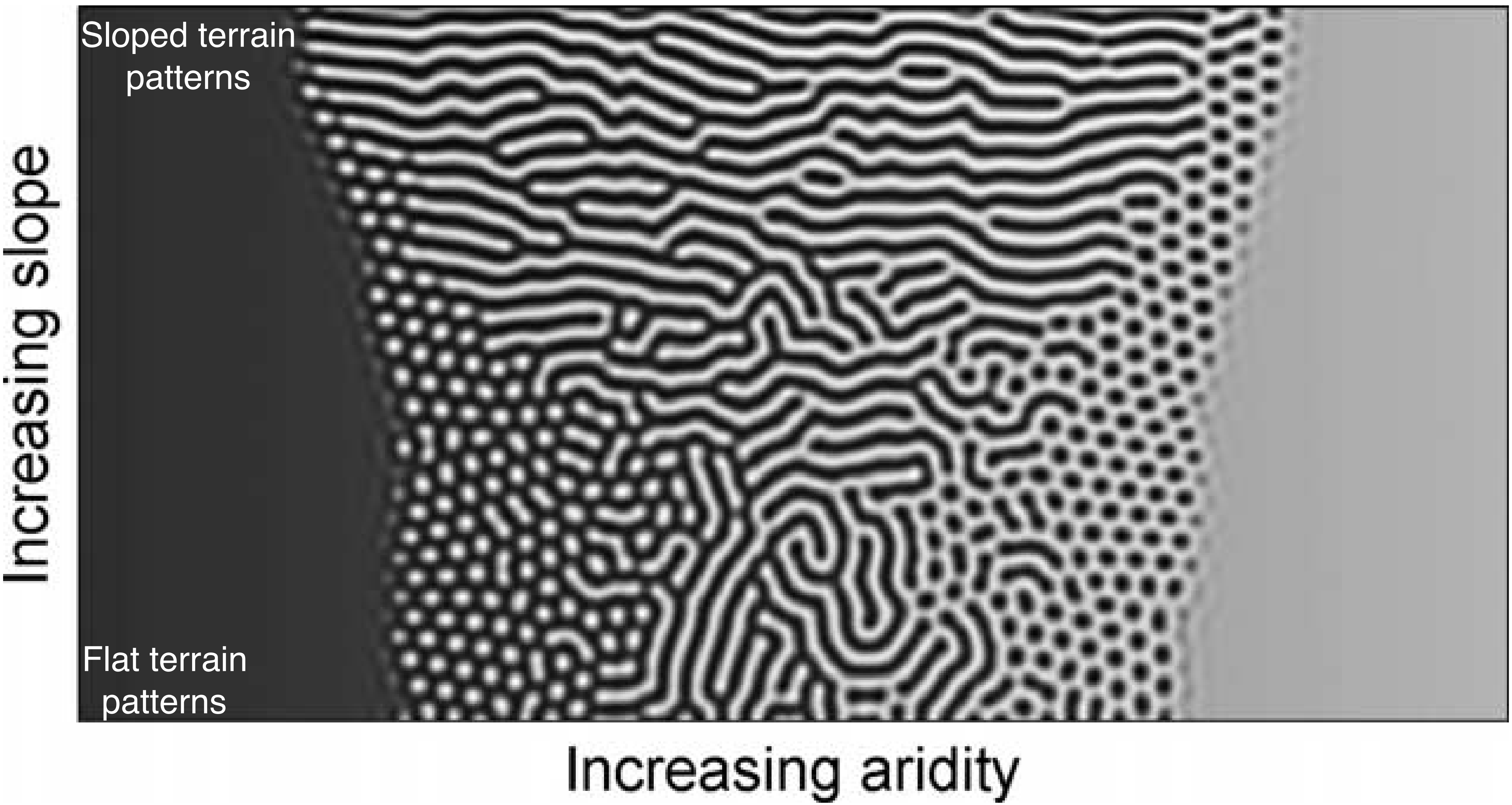
Phys. Rev. Lett. (2001)



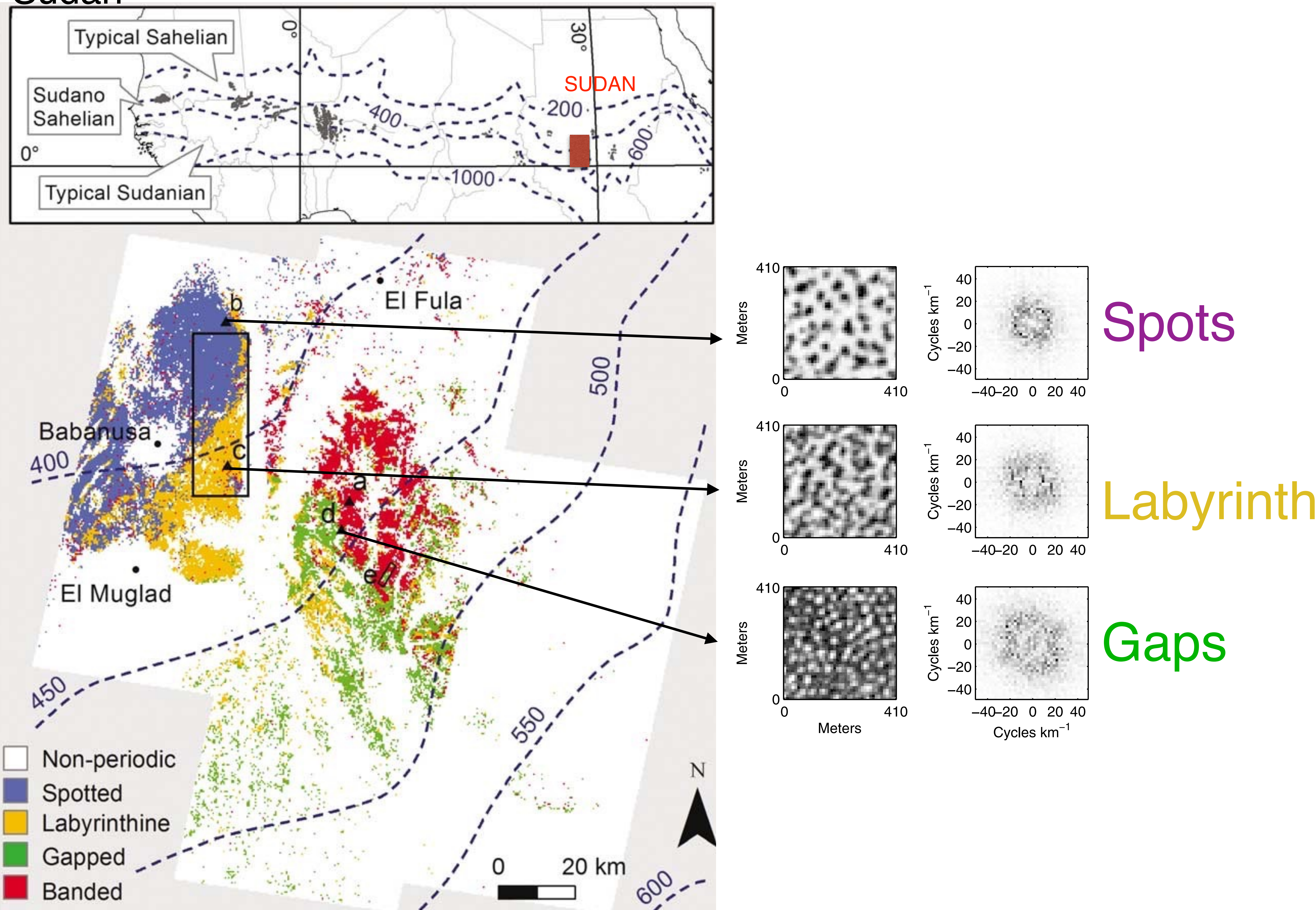
Environmental modulation of self-organized periodic vegetation patterns in Sudan

Vincent Deblauwe, Pierre Couteron, Olivier Lejeune, Jan Bogaert and Nicolas Barbier

Ecography 34: 990–1001, 2011



Deblauwe et al. (2011)
Sudan

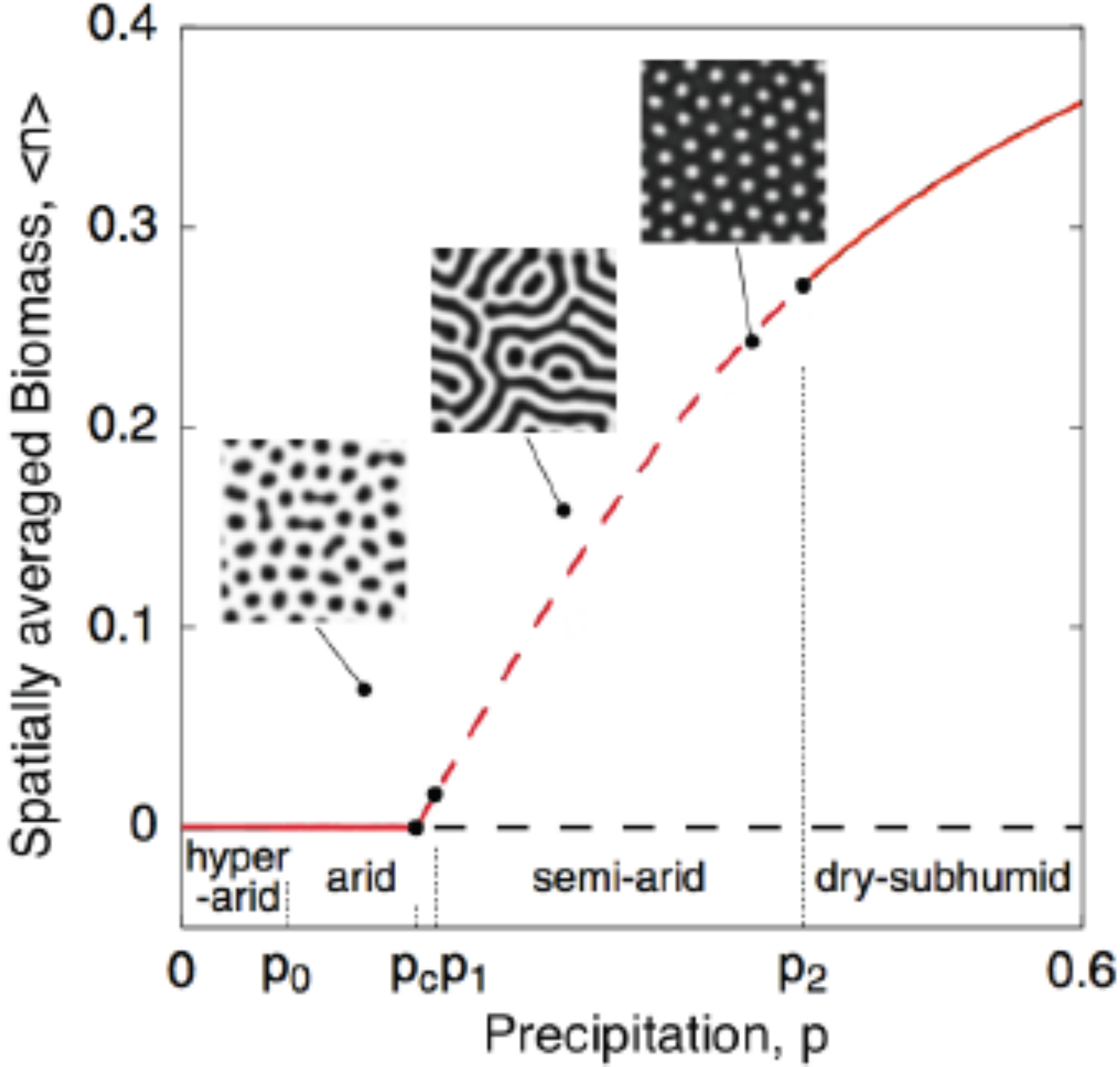


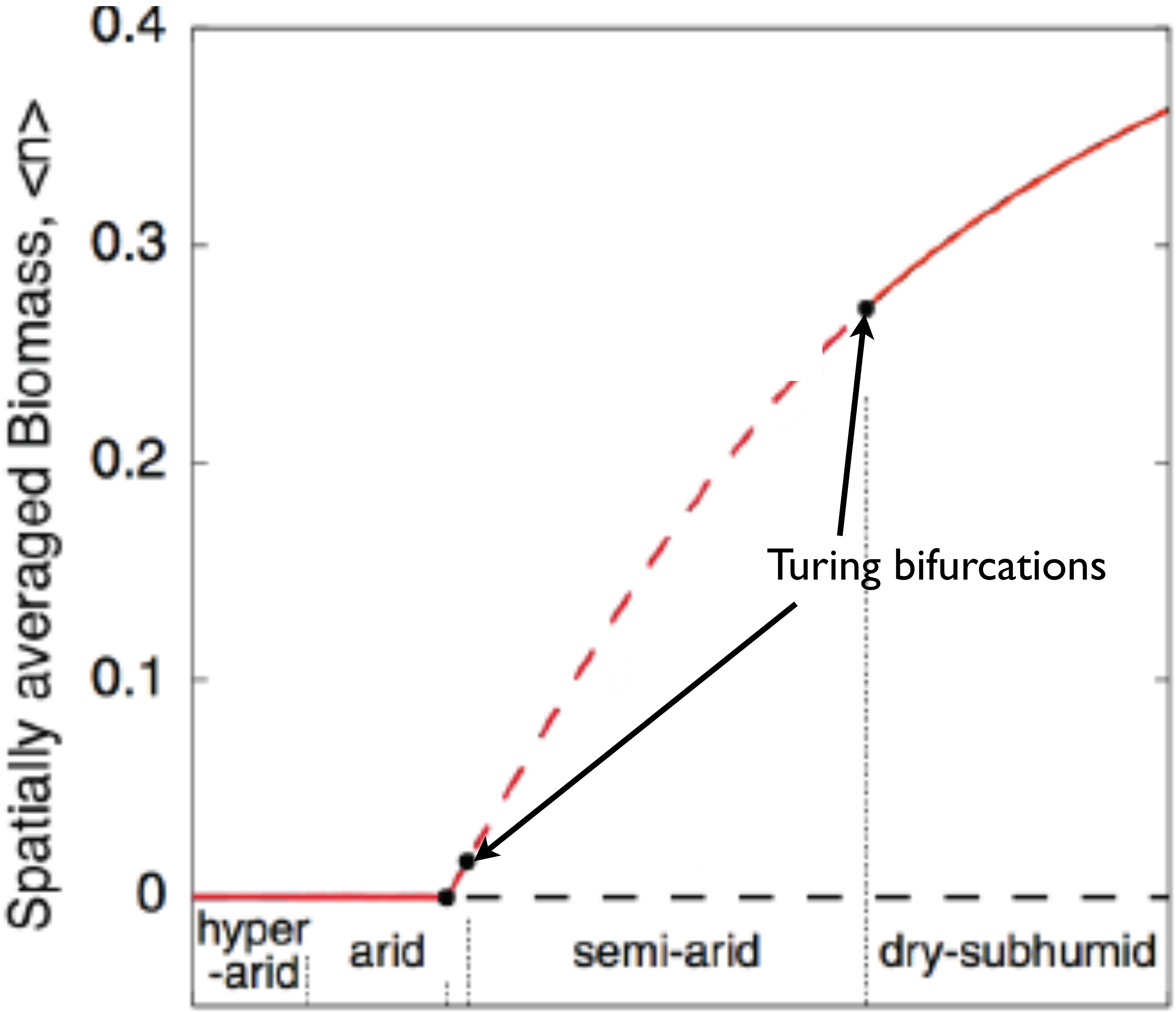
Vegetation Pattern Models: Turing mechanism

Diversity of Vegetation Patterns and Desertification

J. von Hardenberg,[1,4] E. Meron,[1,3] M. Shachak,[2] and Y. Zarmi[1,3]

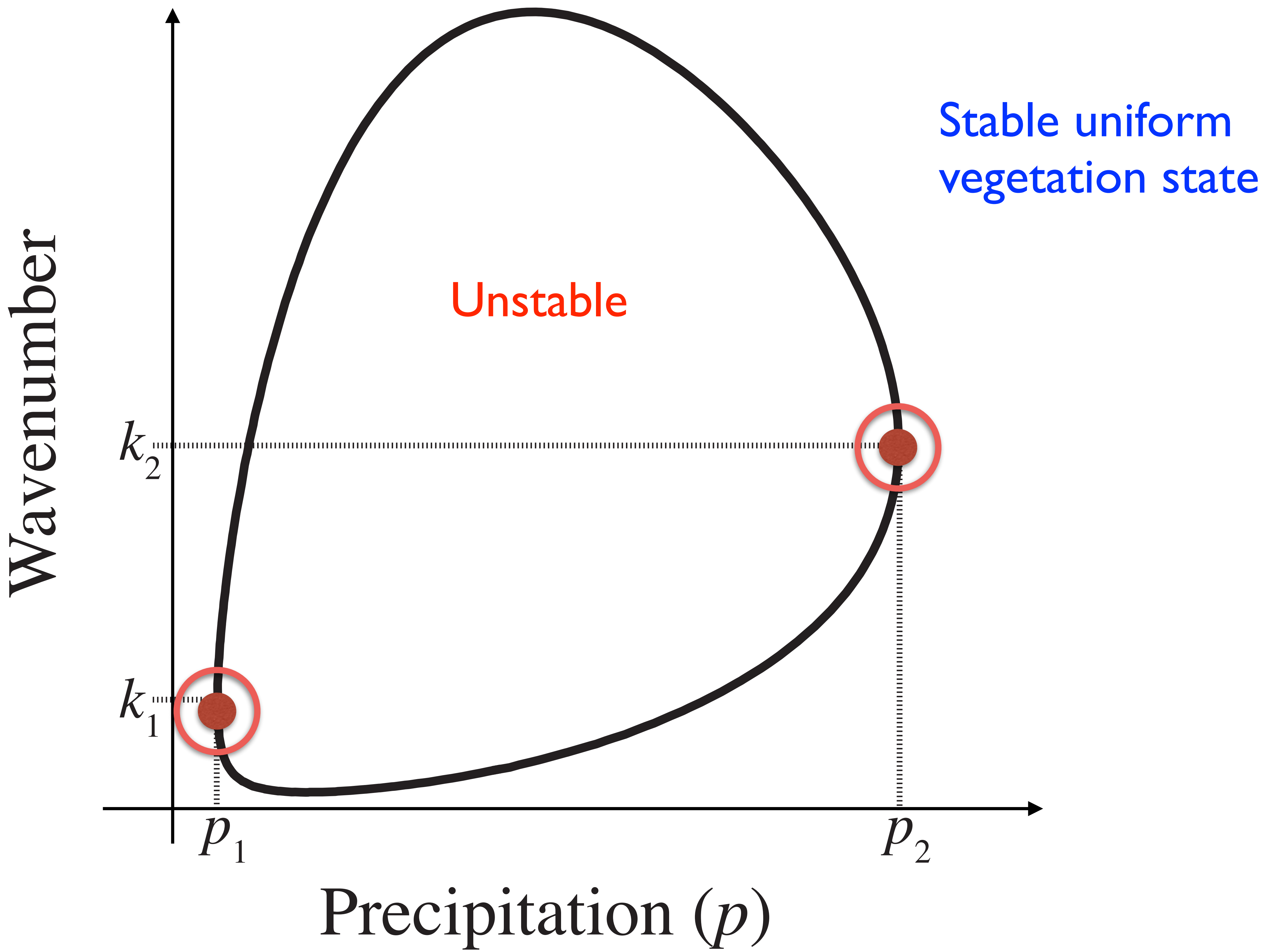
Phys. Rev. Lett. (2001)





uniform $\leftrightarrow$ patterns $\leftrightarrow$ uniform

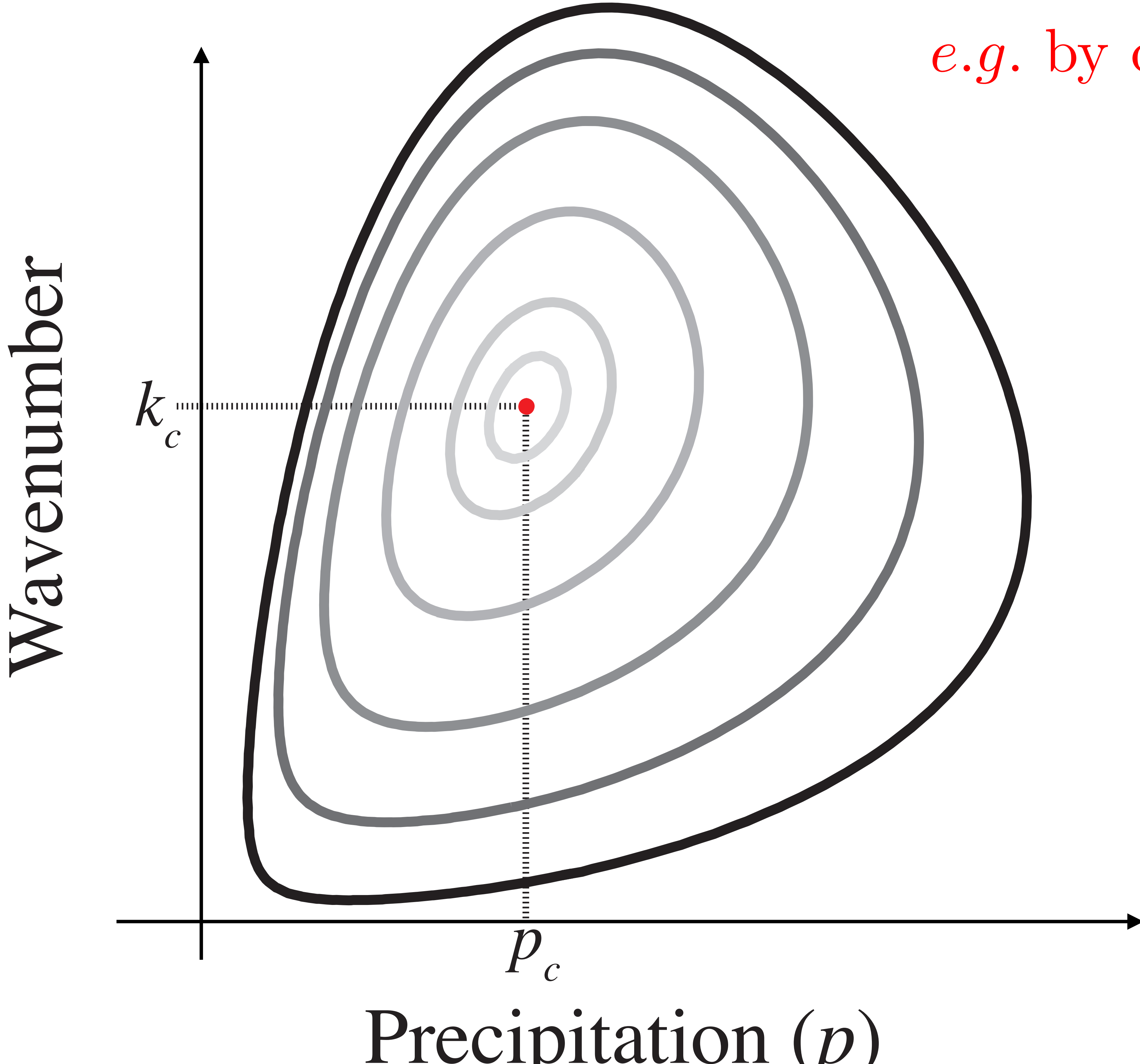
Bubble of linear instability to Fourier mode perturbations: "Turing Bubble"



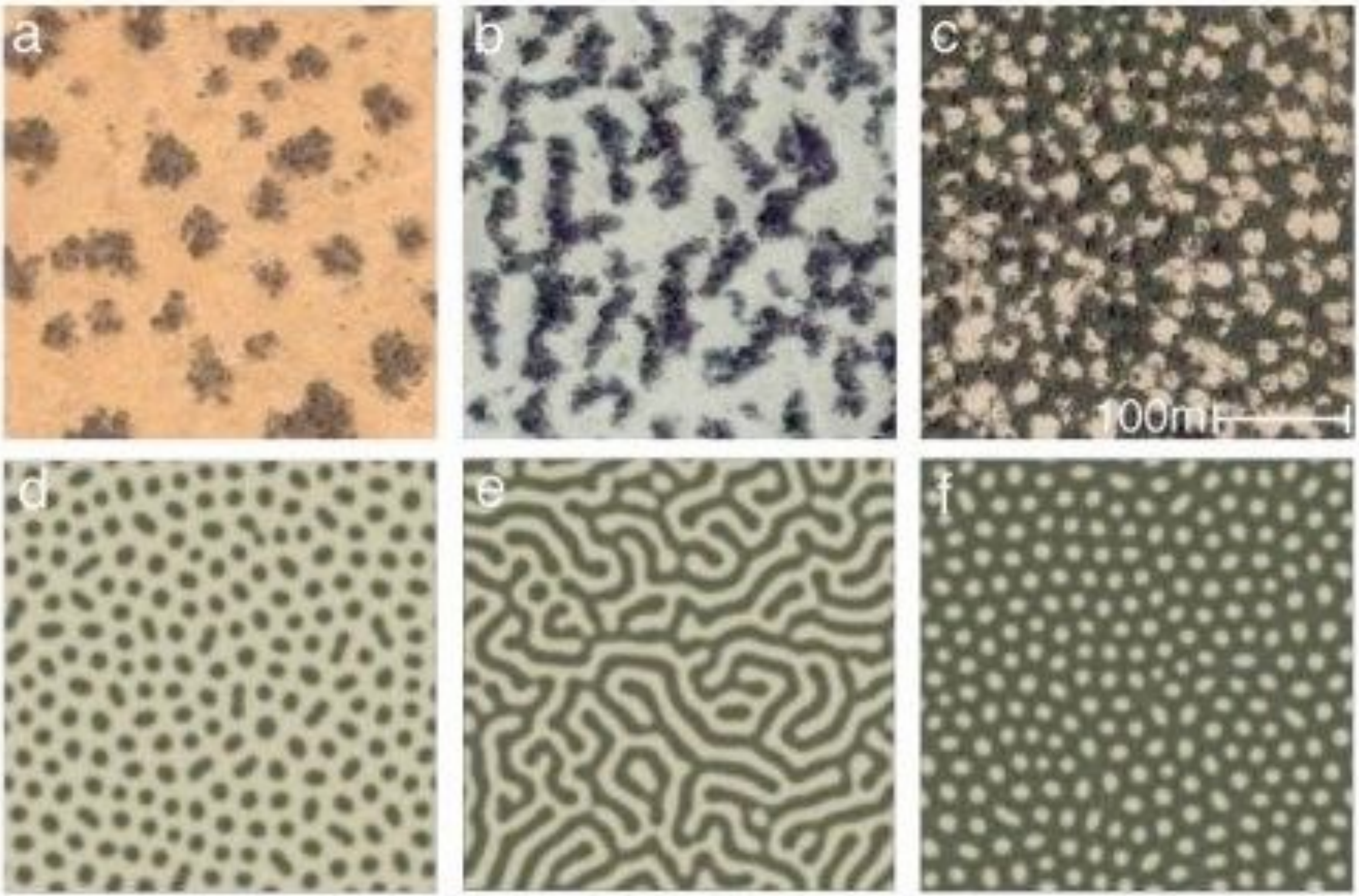
"Degenerate Turing Bubble"

(work with K. Gowda and H. Riecke, PRE 2014)

$e.g.$ by decreasing $\frac{D_h}{D_b}$



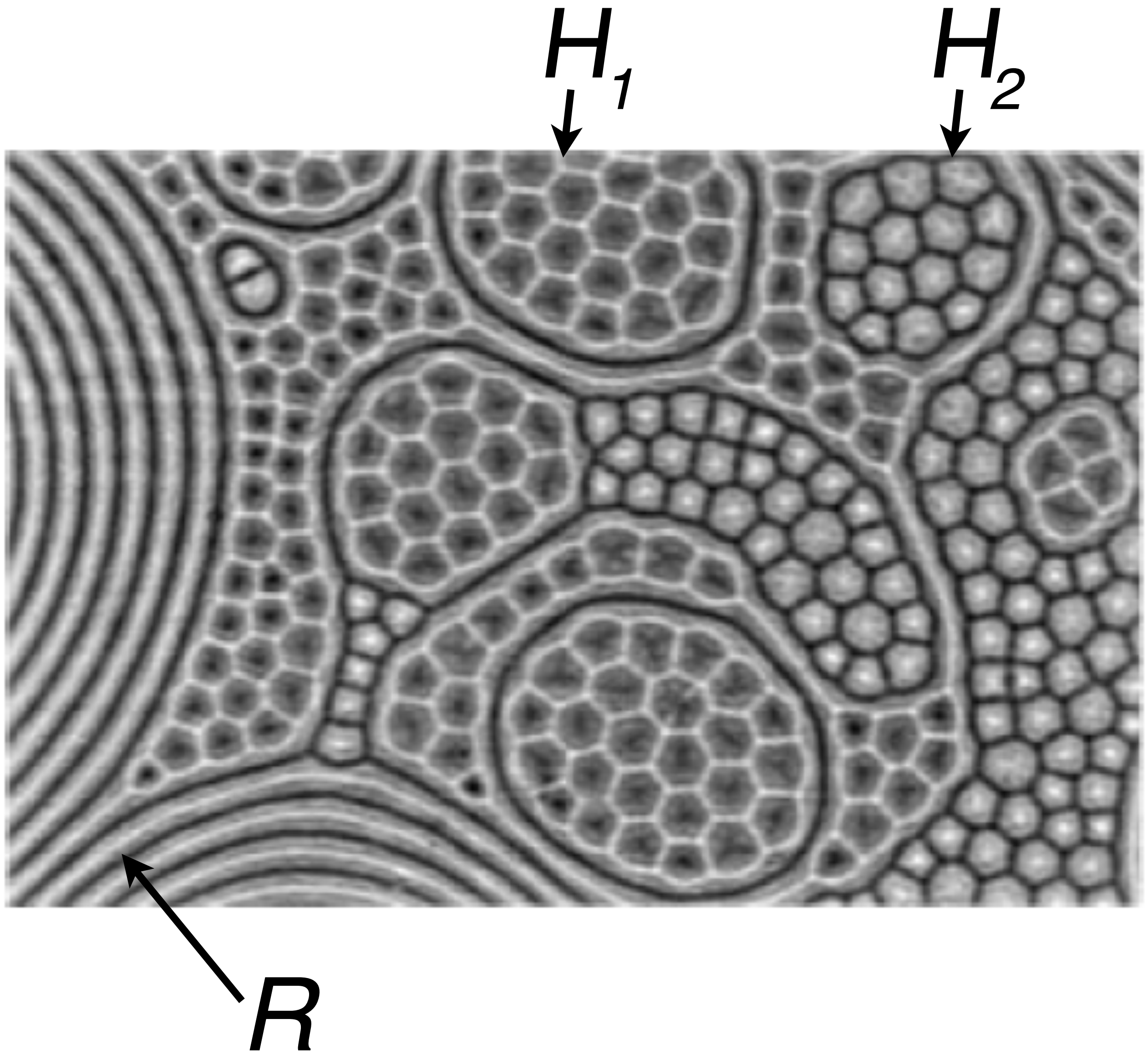
“Spots” “Labyrinths” “Gaps”

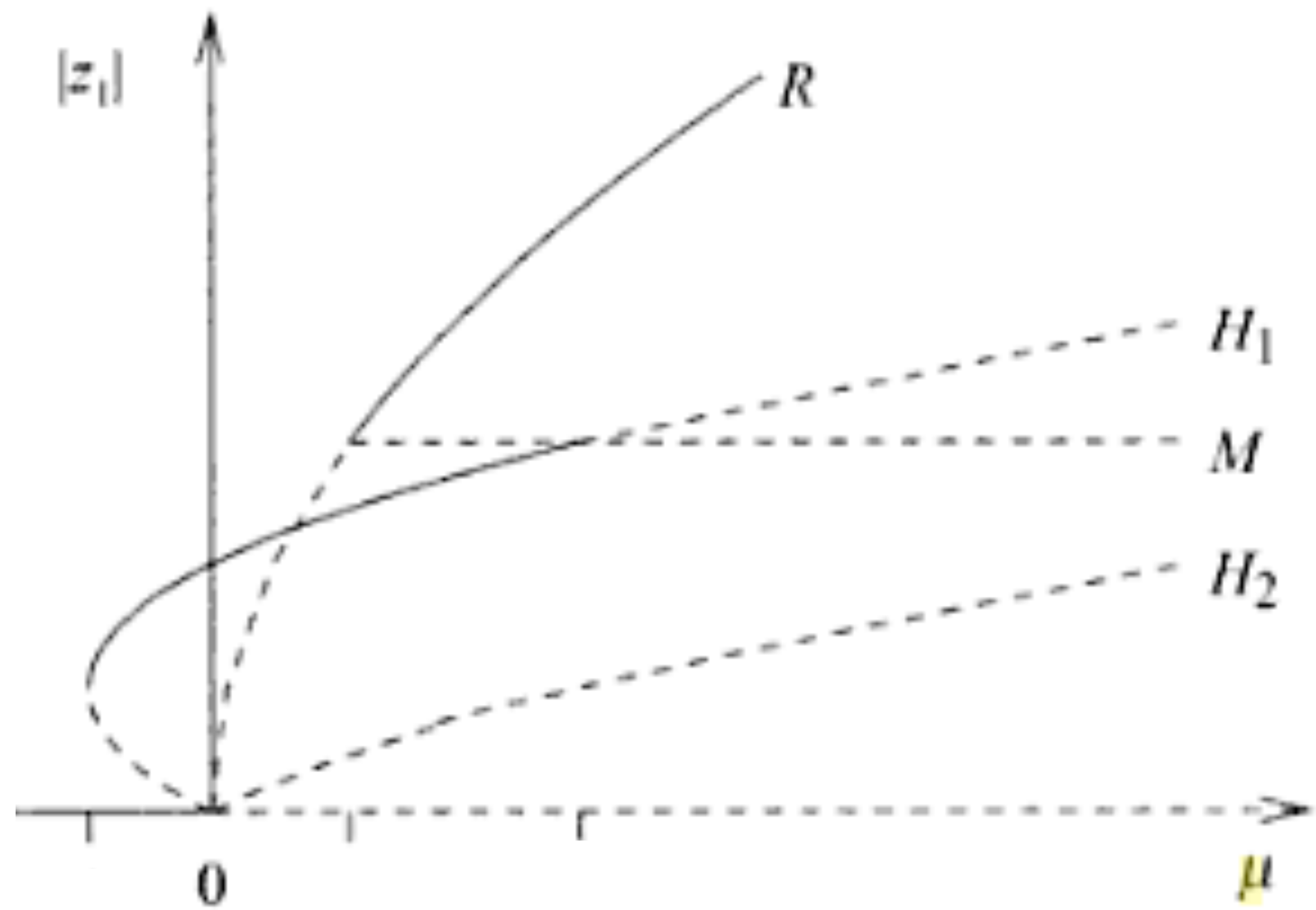


“Up-hexagons” Distorted “Stripes” “Down-hexagons”

Assenheimer & Steinberg PRL (1996)

Fig. from R.B.Hoyle “Pattern Formation:…”(2006)



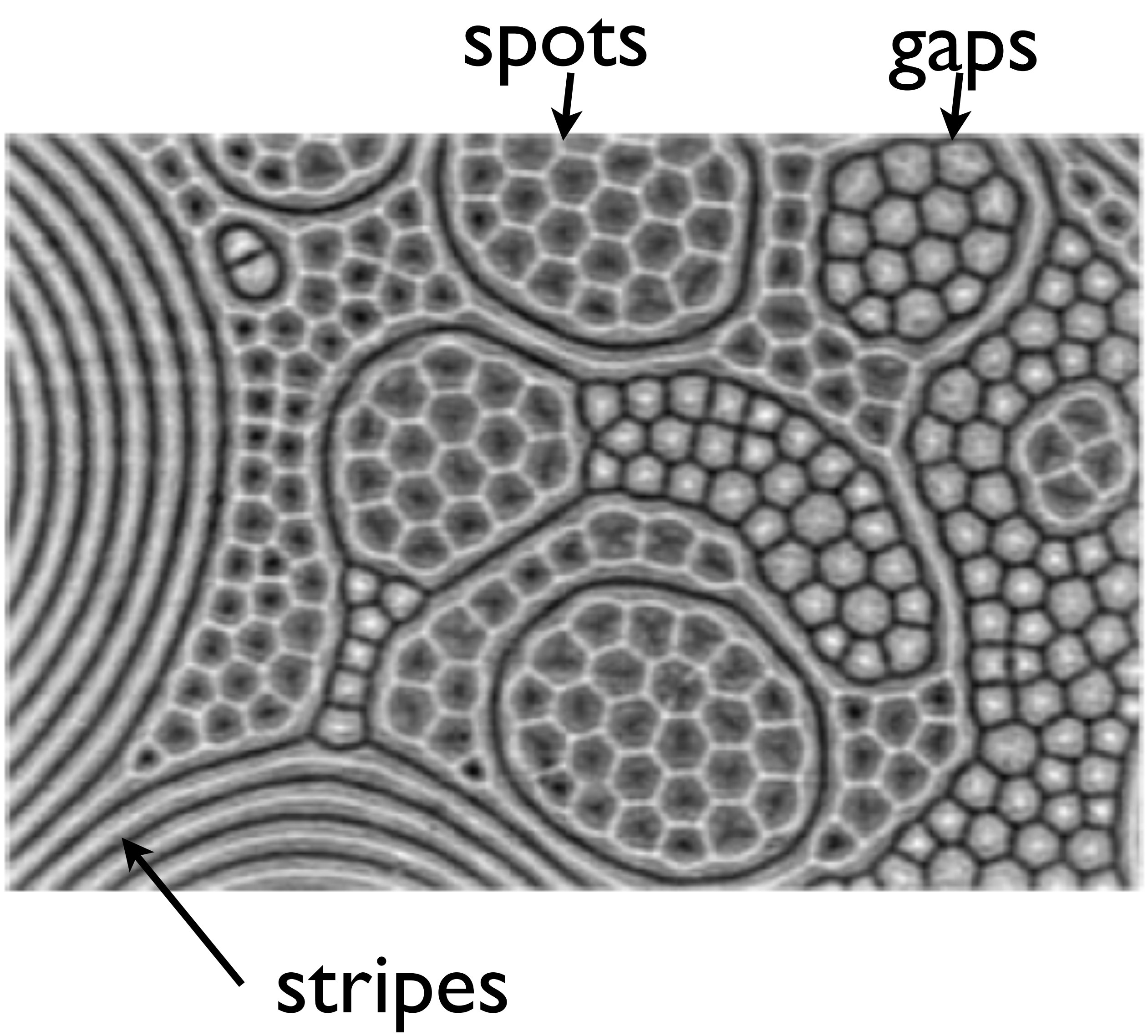


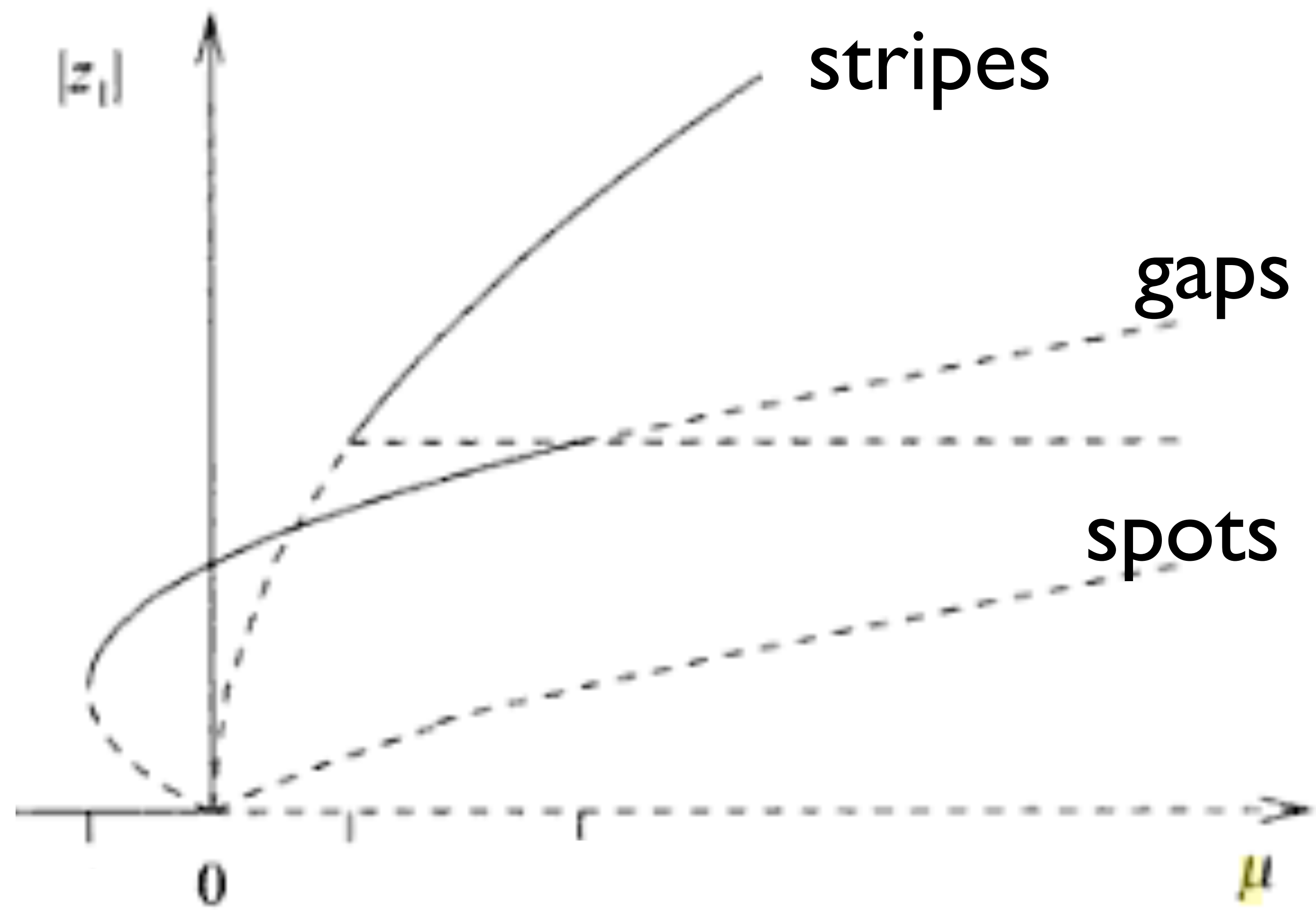
$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

$$\dot{z}_2 = \mu z_2 + a\bar{z}_1\bar{z}_3 - b|z_2|^2 z_2 - c\left(|z_1|^2 + |z_3|^2\right) z_2 + \ldots$$

$$\dot{z}_3 = \mu z_3 + a\bar{z}_1\bar{z}_2 - b|z_3|^2 z_3 - c\left(|z_1|^2 + |z_2|^2\right) z_3 + \ldots$$

Assenheimer & Steinberg PRL (1996)

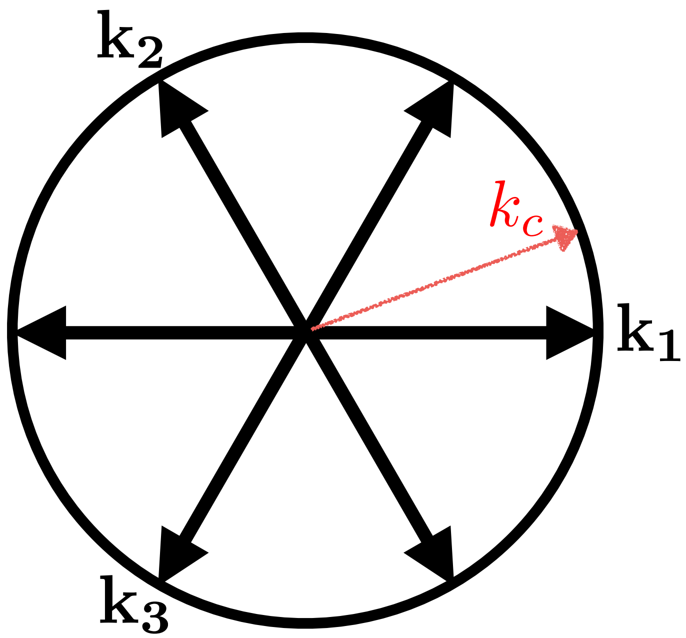




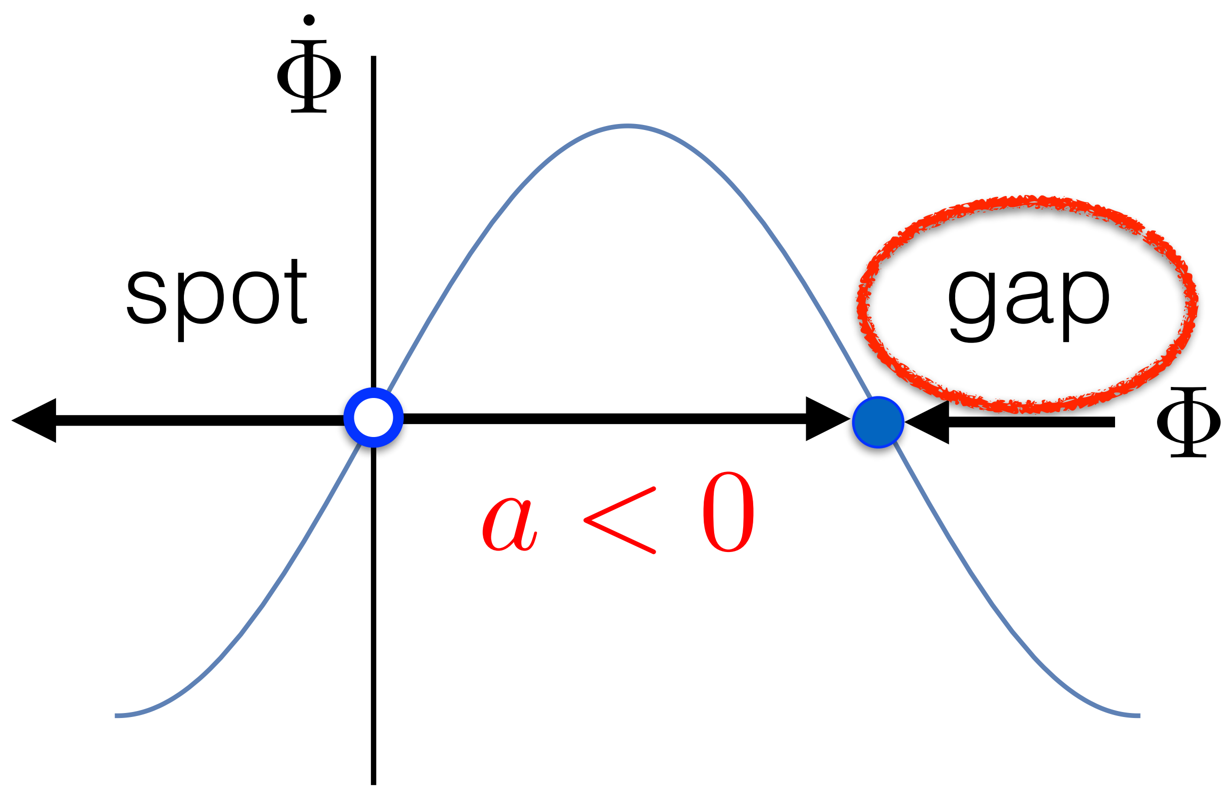
$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

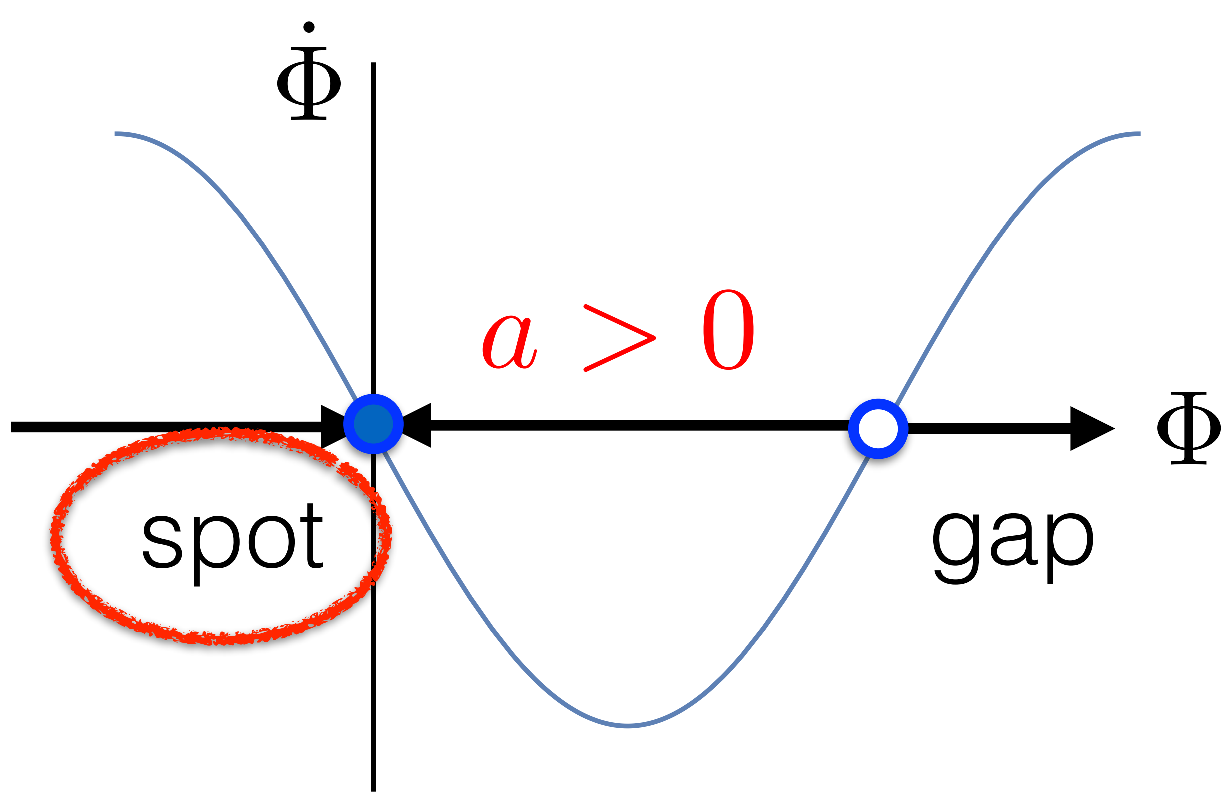
$$\dot{z}_2 = \mu z_2 + a\bar{z}_1\bar{z}_3 - b|z_2|^2 z_2 - c\left(|z_1|^2 + |z_3|^2\right) z_2 + \ldots$$

$$\dot{z}_3 = \mu z_3 + a\bar{z}_1\bar{z}_2 - b|z_3|^2 z_3 - c\left(|z_1|^2 + |z_2|^2\right) z_3 + \ldots$$

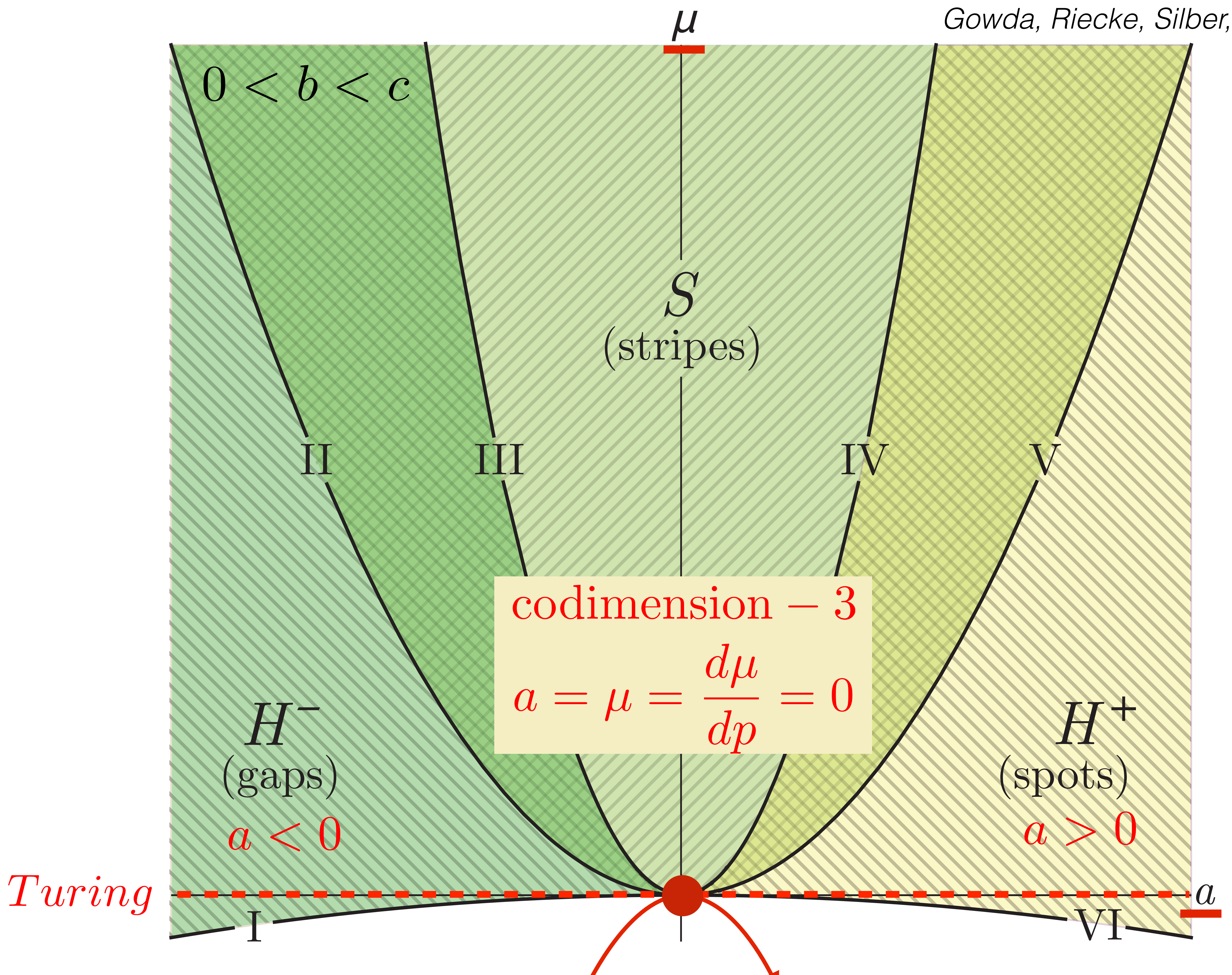


$$\dot{z}_j = \lambda z_j + a\overline{z}_{j+1}\overline{z}_{j-1} + \cdots$$ } near Turing bifurcation pt. ($\lambda = 0$)



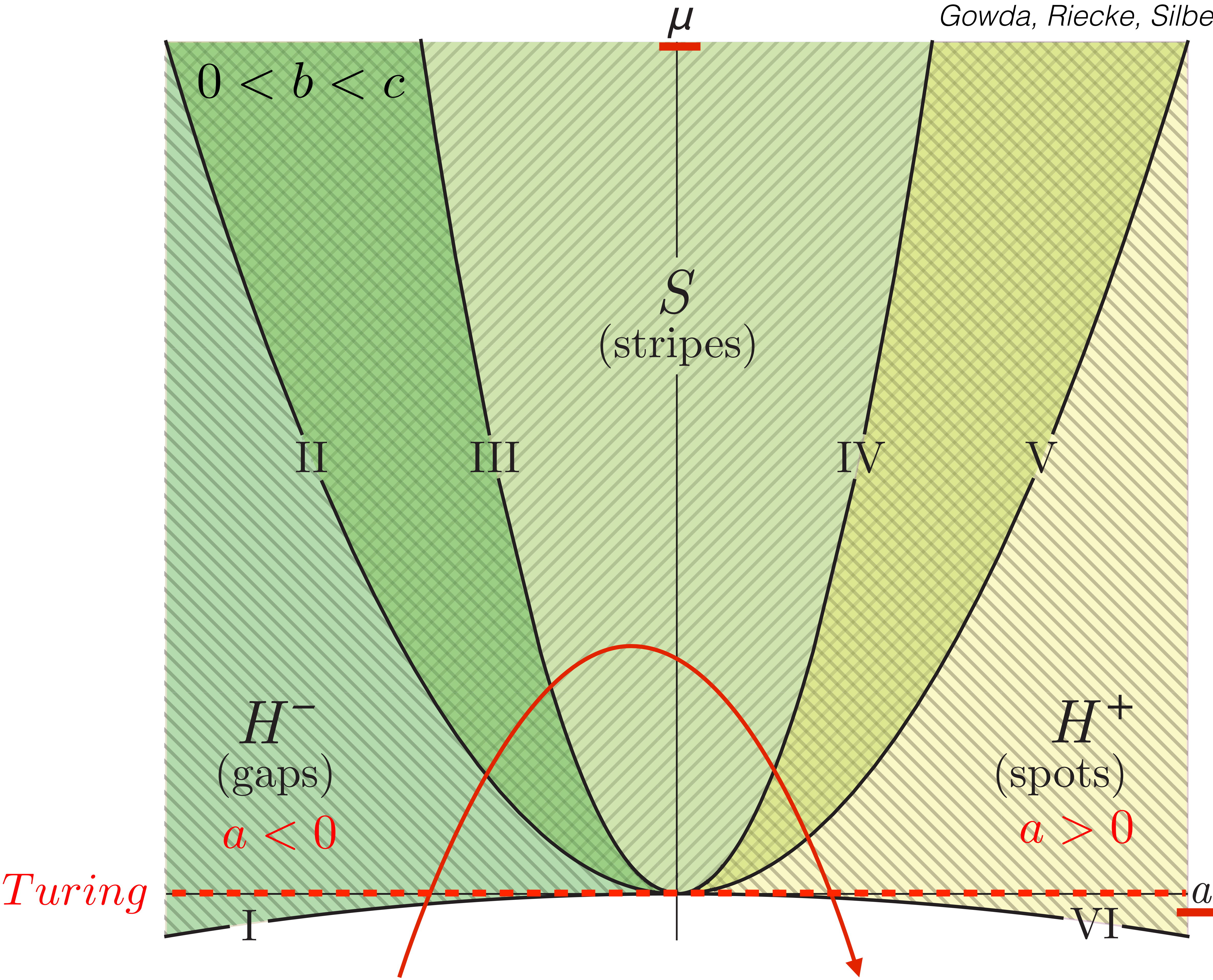


Gowda, Riecke, Silber, PRE (2014)



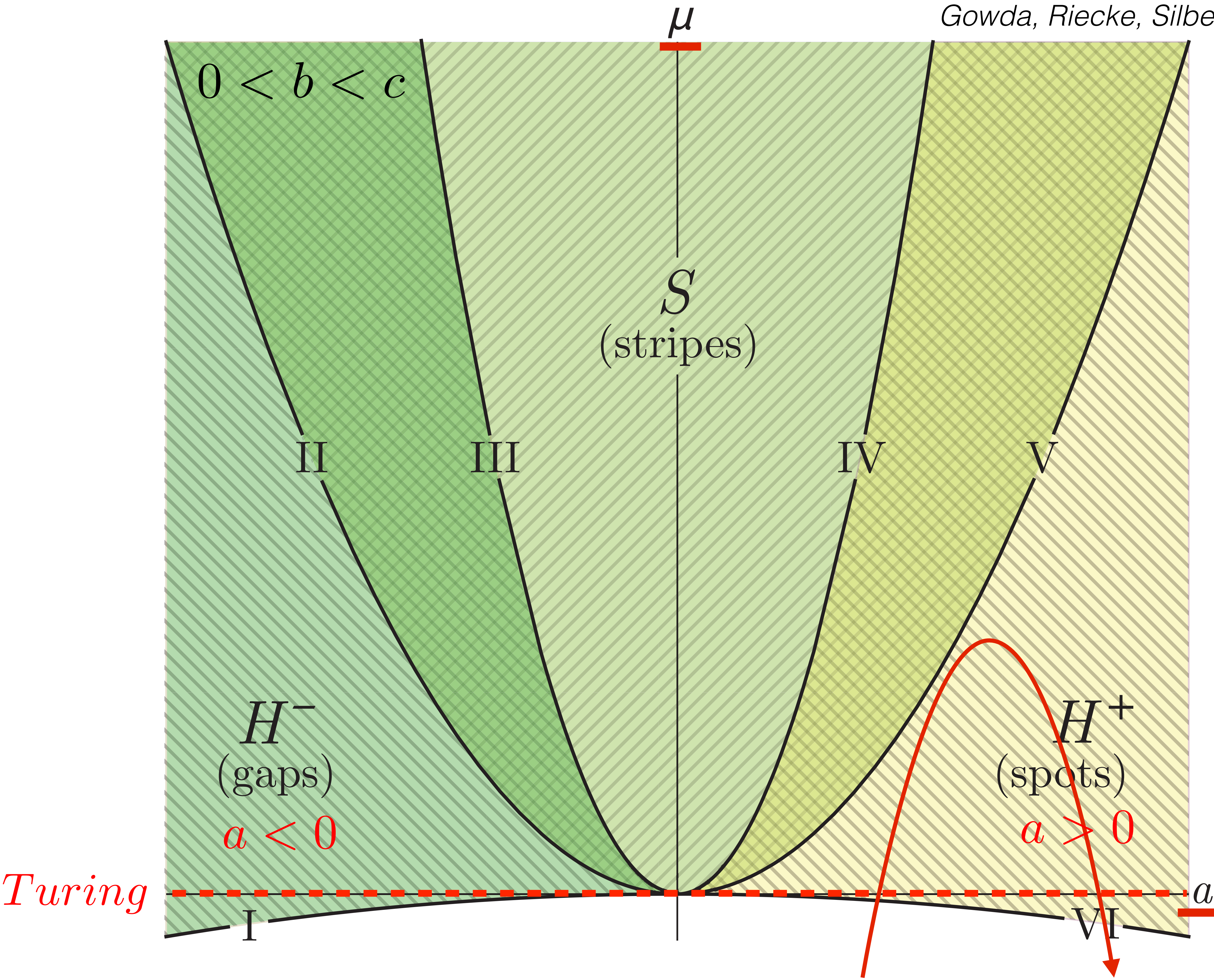
$$\dot{z}_1 = \mu z_1 + a\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

Gowda, Riecke, Silber, PRE (2014)



$$\dot{z}_1 = \underline{\mu} z_1 + \underline{a}\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

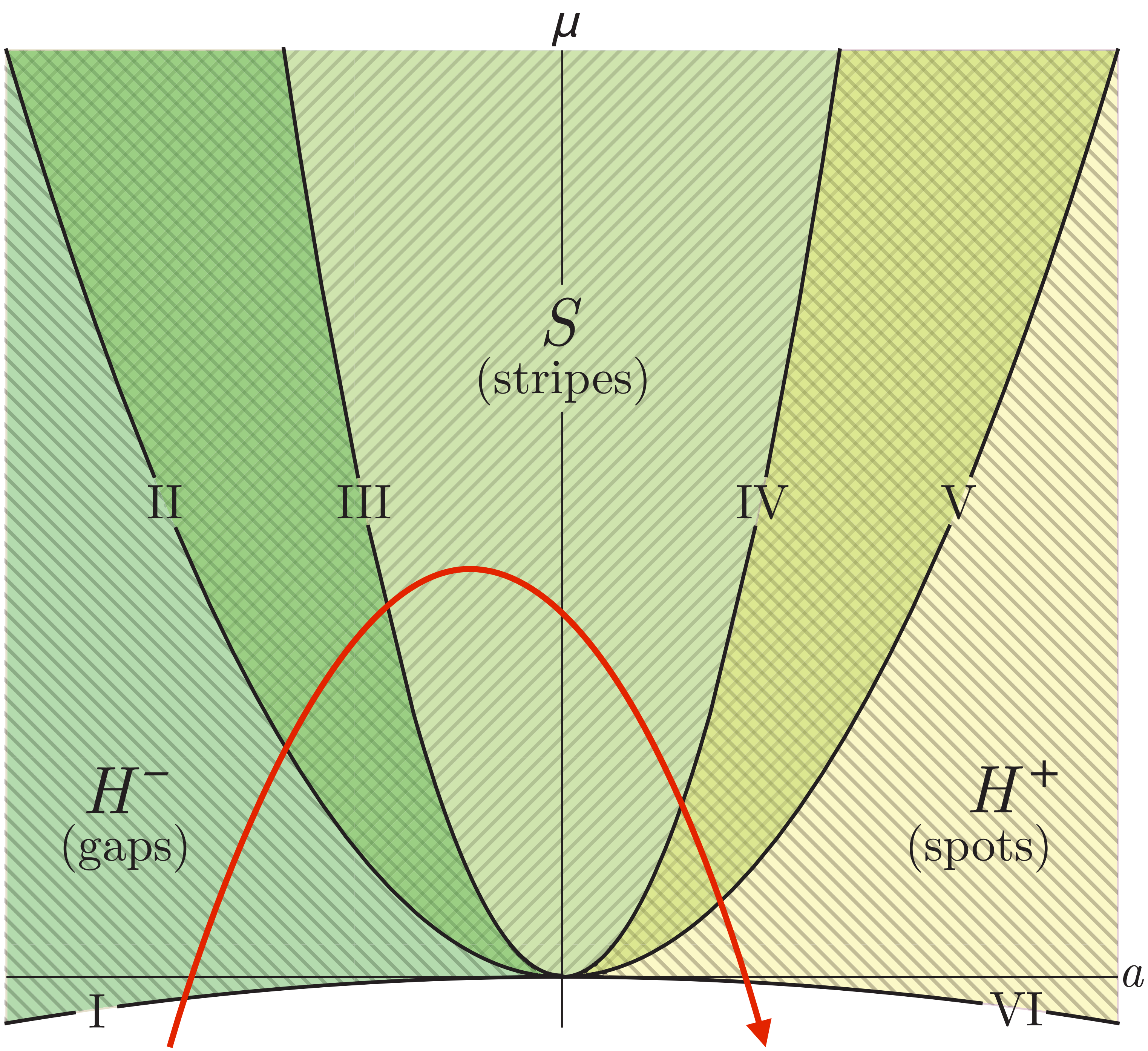
Gowda, Riecke, Silber, PRE (2014)

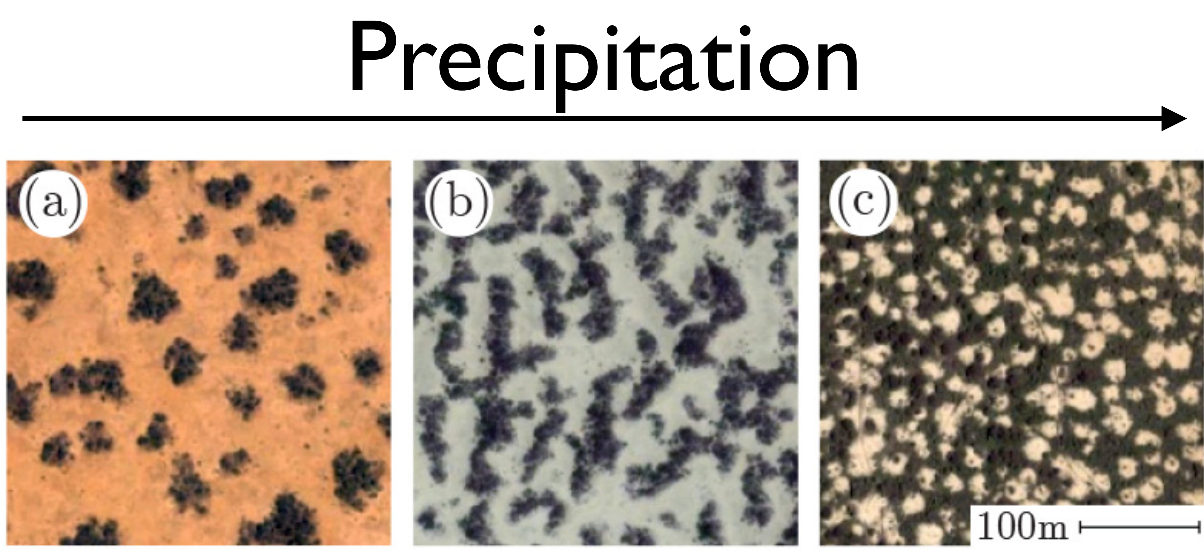


$$\dot{z}_1 = \underline{\mu} z_1 + \underline{a}\bar{z}_2\bar{z}_3 - b|z_1|^2 z_1 - c\left(|z_2|^2 + |z_3|^2\right) z_1 + \ldots$$

Proposal:

Quadratic coefficient changing sign from negative to positive, with decreasing precipitation serves as a proxy for the "standard sequence":





Therein lies the ecology?

PROCEEDINGS A

rspa.royalsocietypublishing.org

Assessing the robustness of spatial pattern sequences in a dryland vegetation model

(2016)

Karna Gowda
(Northwestern)

Yuxin Chen
(Northwestern)

Sarah Iams
(Harvard)

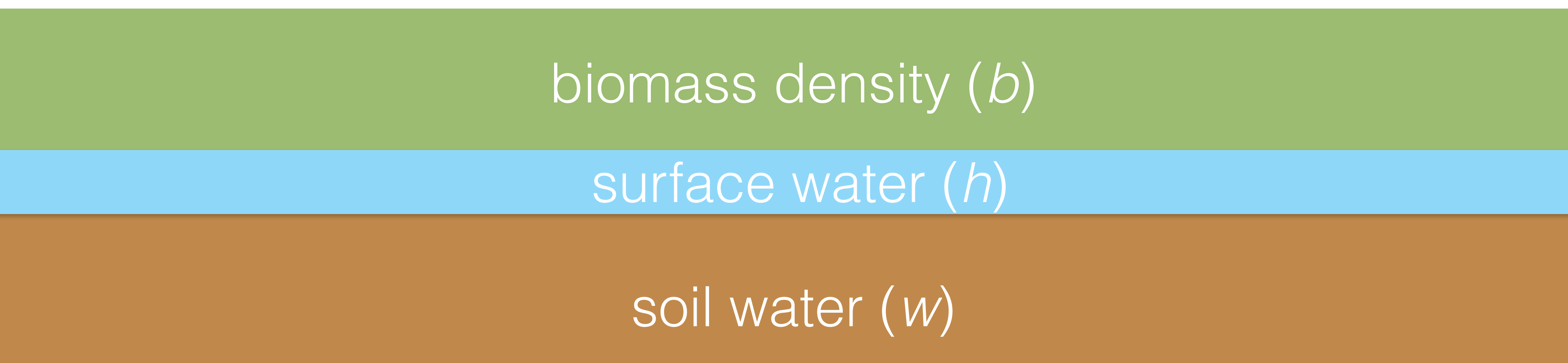
Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat.* **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1} b}_{growth} + \underbrace{\nabla^2 b}_{dispersal} ,$$

$$\partial_t h = \underbrace{p}_{precip.} - \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} + \underbrace{D_h \nabla^2 h}_{diffusion} .$$

$D_h \gg 1$

$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1} b}_{transp.} + \underbrace{D_w \nabla^2 w}_{diffusion} ,$$



Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat*. **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1}b}_{growth} + \underbrace{\nabla^2 b}_{dispersal},$$

p = bifurcation parameter

$$\partial_t h = \underbrace{p}_{precip.} - \underbrace{\alpha \frac{b+f}{b+1}h}_{infil.} + \underbrace{D_h \nabla^2 h}_{diffusion}.$$

$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1}h}_{infil.} - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1}b}_{transp.} + \underbrace{D_w \nabla^2 w}_{diffusion},$$

biomass density (*b*)

surface water (*h*)

soil water (*w*)

Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat.* **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1} b}_{growth} + \underbrace{\nabla^2 b}_{dispersal} ,$$

$$\partial_t h = \underbrace{p}_{precip.} - \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} + \underbrace{D_h \nabla^2 h}_{diffusion} .$$

$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1} b}_{transp.} + \underbrace{D_w \nabla^2 w}_{diffusion} ,$$

Growth rate

$$G(w) = \frac{w}{w+1}$$

biomass density (*b*)

surface water (*h*)

soil water (*w*)

Rietkerk M, Boerlijst MC, van Langevelde F, HilleRisLambers R, van de Koppel J, Kumar L, Prins HH, de Roos AM. 2002 Self-organization of vegetation in arid ecosystems. *Am. Nat.* **160**,

$$\partial_t b = -\underbrace{\mu b}_{mort.} + \underbrace{\frac{w}{w+1} b}_{growth} + \underbrace{\nabla^2 b}_{dispersal} ,$$

$$\partial_t h = \underbrace{p}_{precip.} - \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} + \underbrace{D_h \nabla^2 h}_{diffusion} .$$

$$\partial_t w = \underbrace{\alpha \frac{b+f}{b+1} h}_{infil.} - \underbrace{\nu w}_{evap.} - \underbrace{\gamma \frac{w}{w+1} b}_{transp.} + \underbrace{D_w \nabla^2 w}_{diffusion} ,$$

Infiltration rate

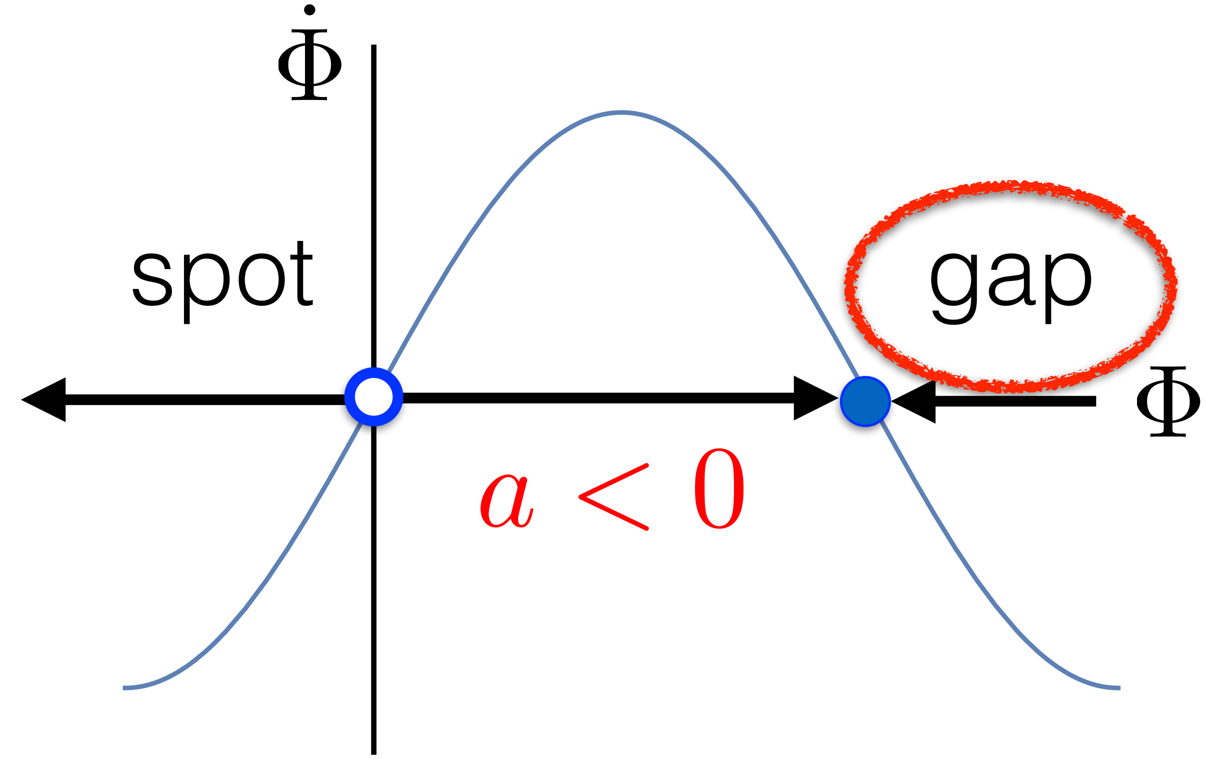
$$I(b) = \frac{b+f}{b+1}$$

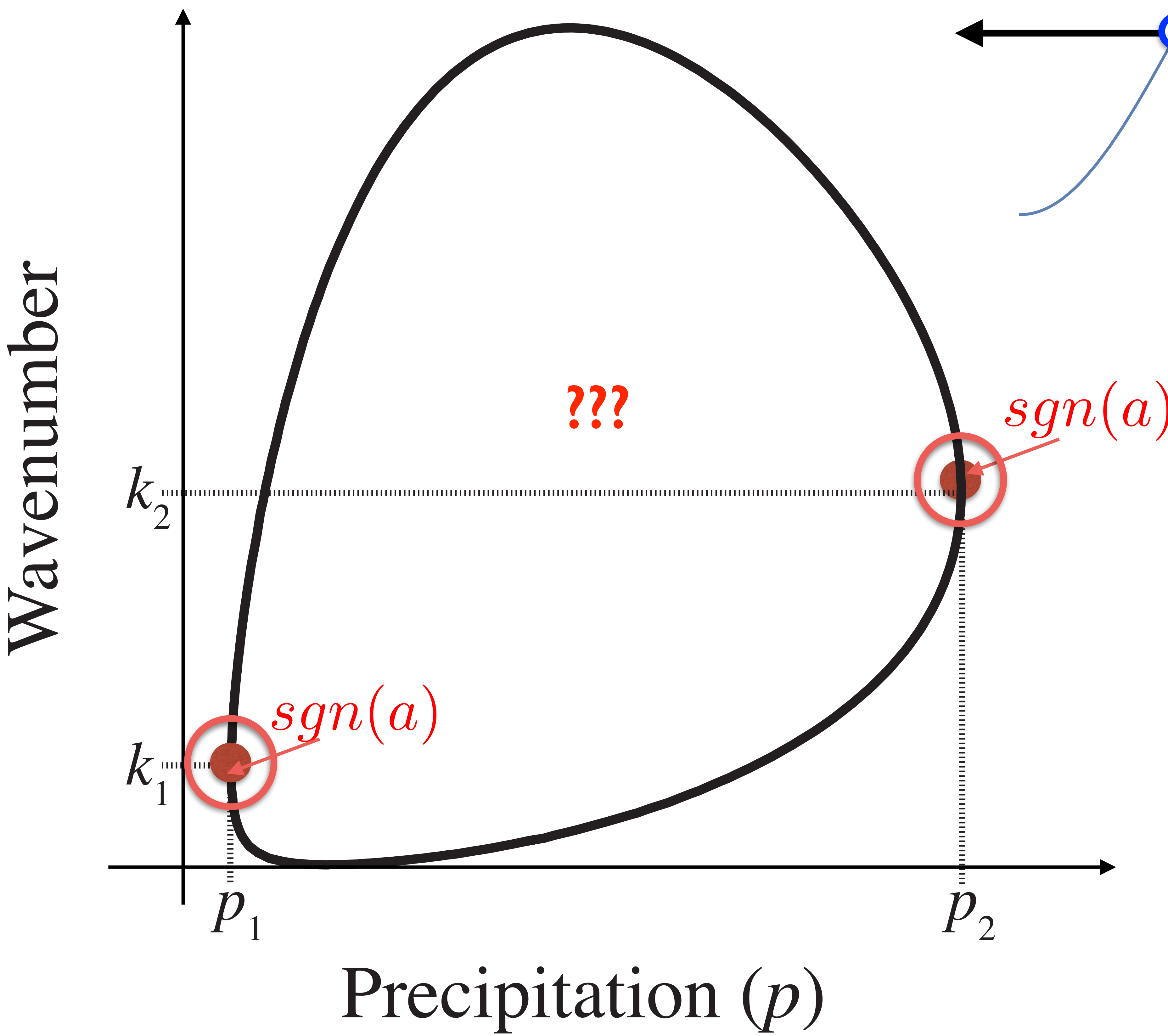
biomass density (*b*)

surface water (*h*)

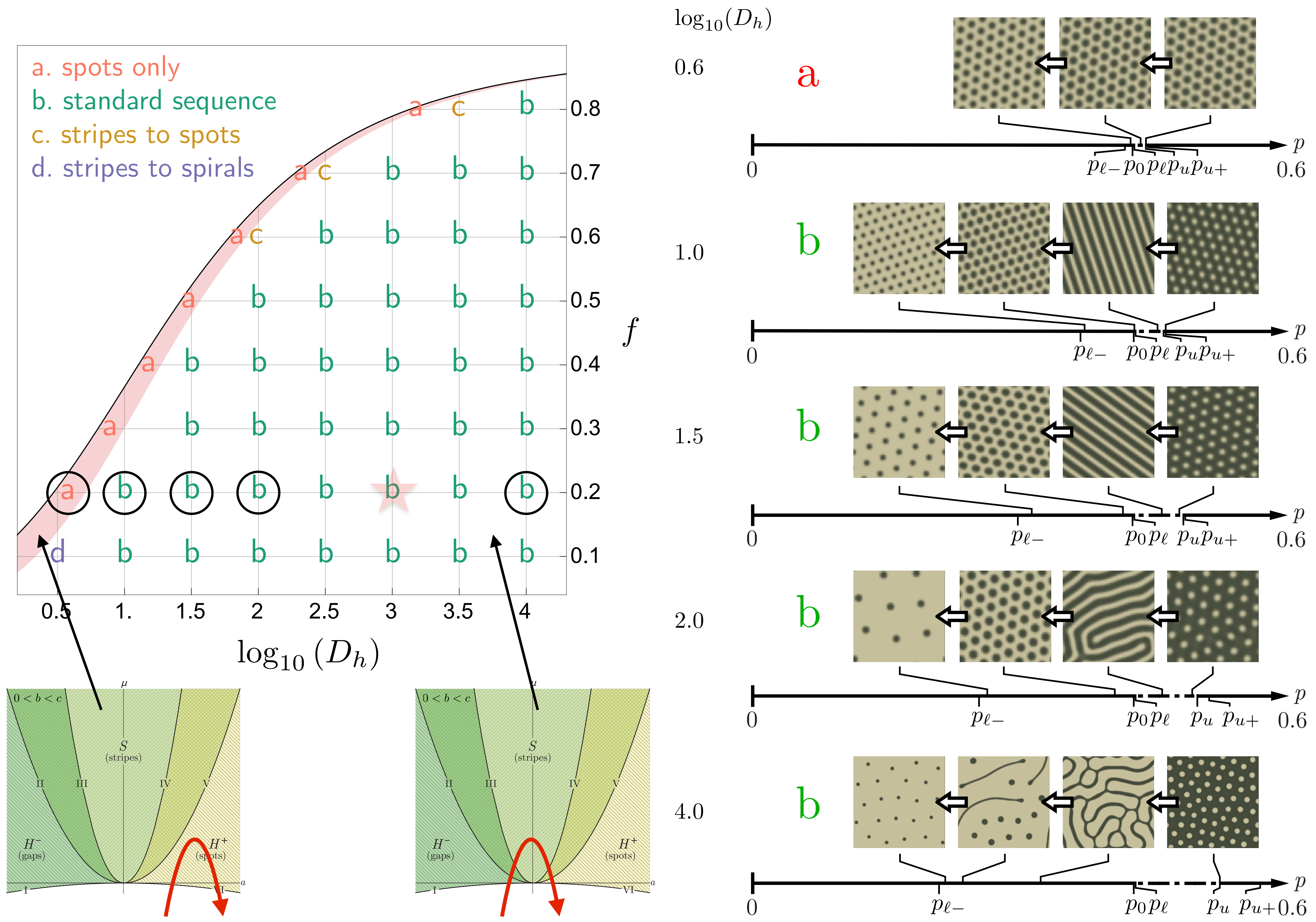
soil water (*w*)

$$\dot{z}_j = \lambda z_j + a\bar{z}_{j+1}\bar{z}_{j-1} + \cdots$$



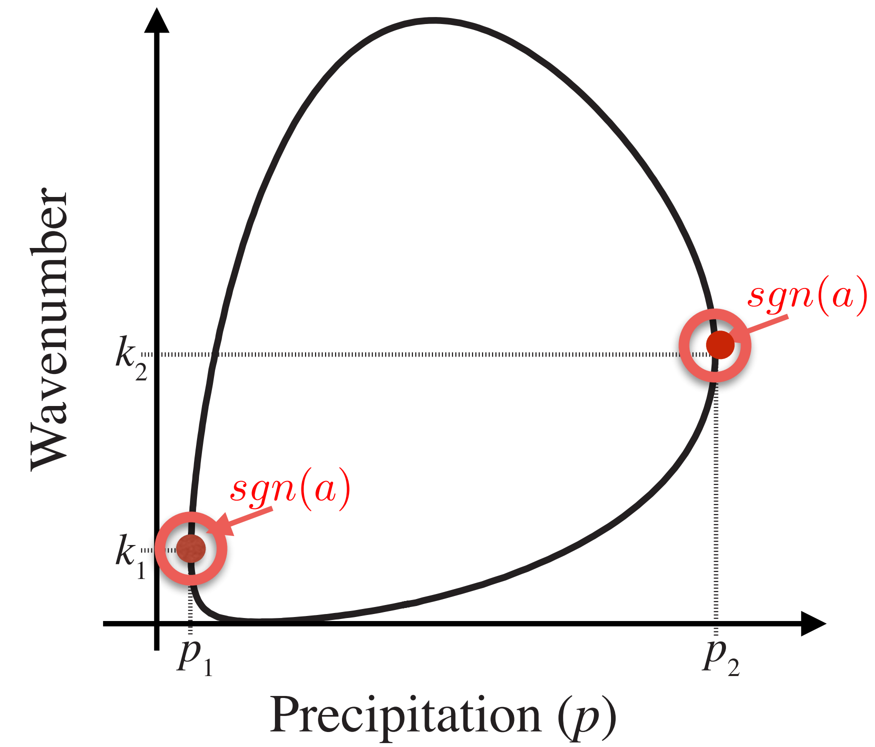


Gowda, Chen, Iams, Silber, Proc. R. Soc. A (2016)

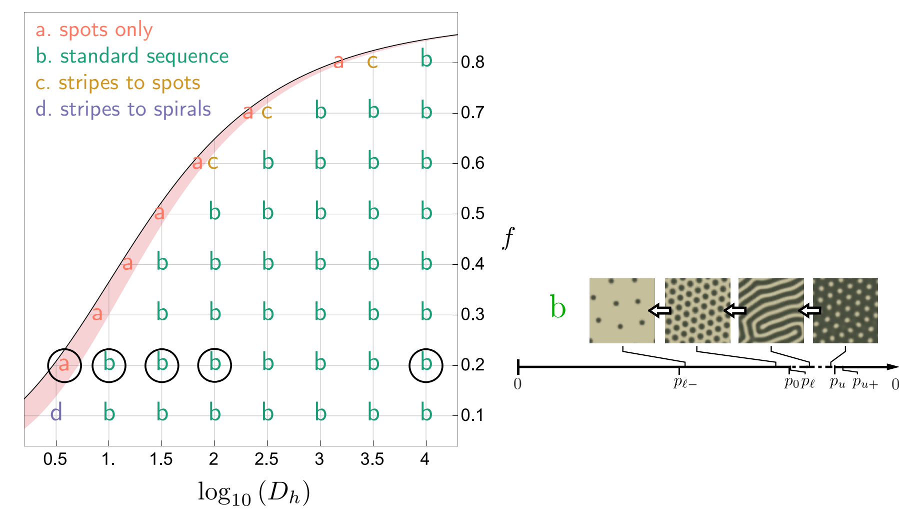


Part I Summary

"Degenerate Turing Bubble" suggests "nonlinear proxy" for "standard sequence"



Numerical simulations to test proxy's skill



Analytic expression for proxy derived from model provides ecological model insights

$$a_\ell = C_\ell G'(w_0) + \mathcal{O}(\epsilon)$$

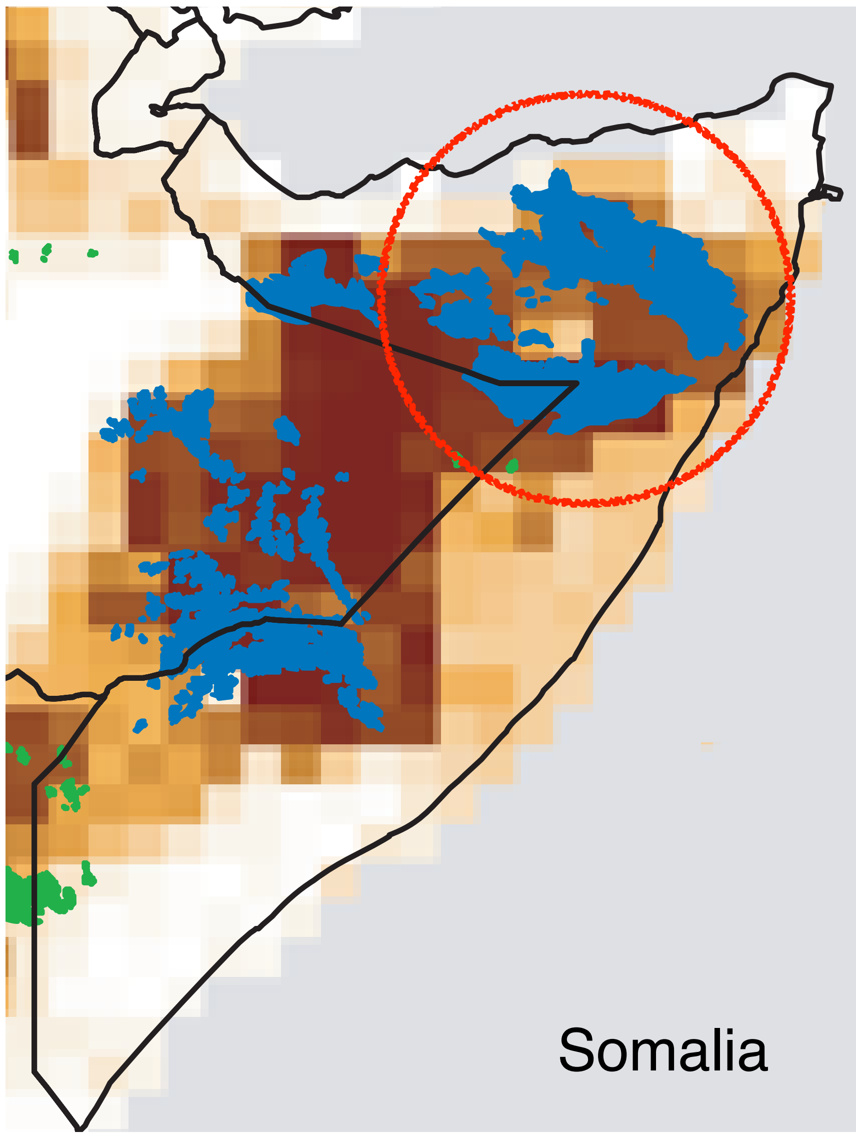
$$a_u = C_u I''(b_0) + \mathcal{O}(\sqrt{\epsilon})$$

$$\epsilon \sim 1/D_h, \; C_\ell, \; C_u > 0$$

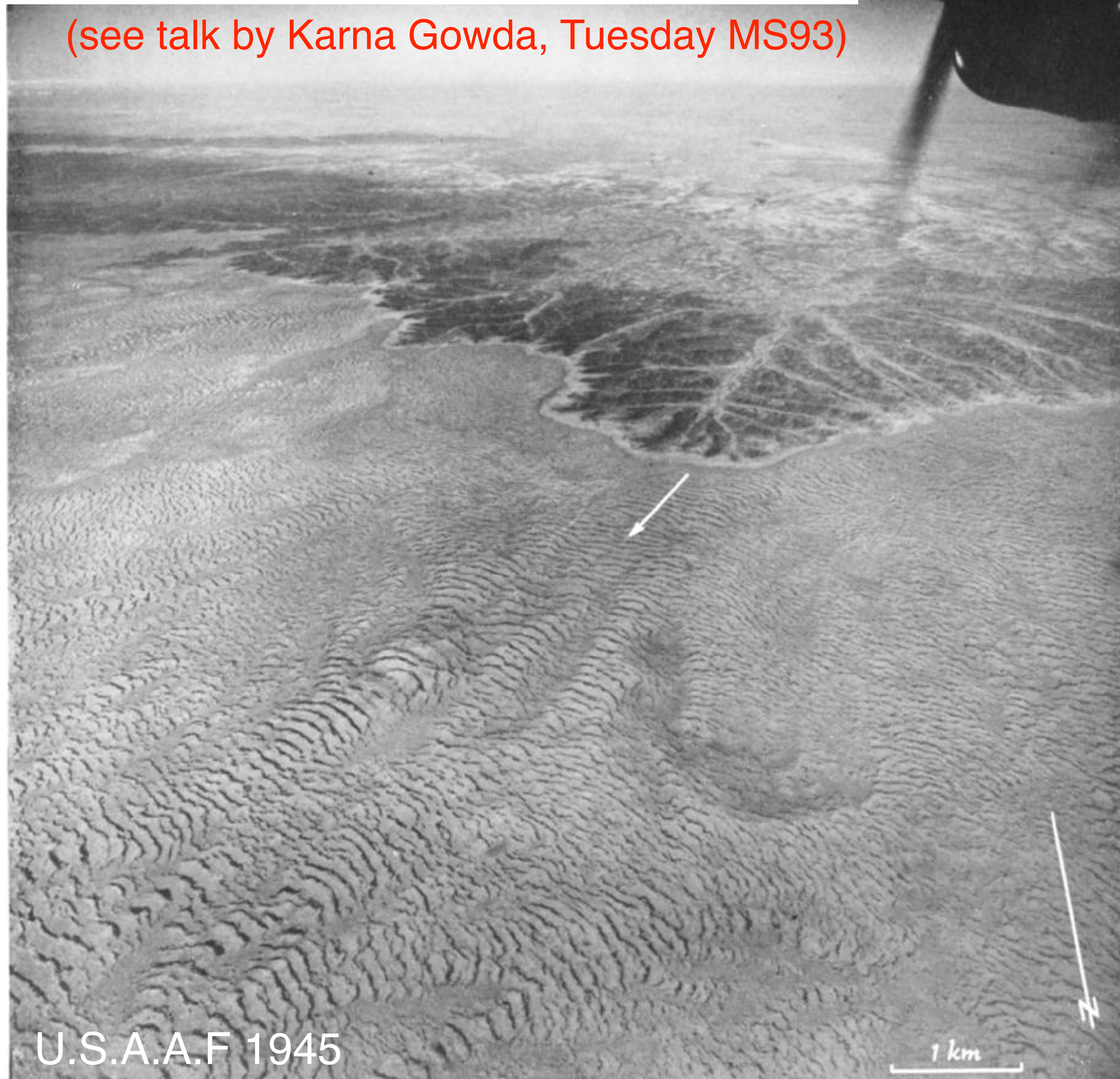
II. *Vegetation Patterns in the Horn of Africa*

with Karna Gowda (Northwestern) and Sarah Iams (Harvard)

(see talk by Karna Gowda, Tuesday MS93)



Deblauwe, et al. (2008)



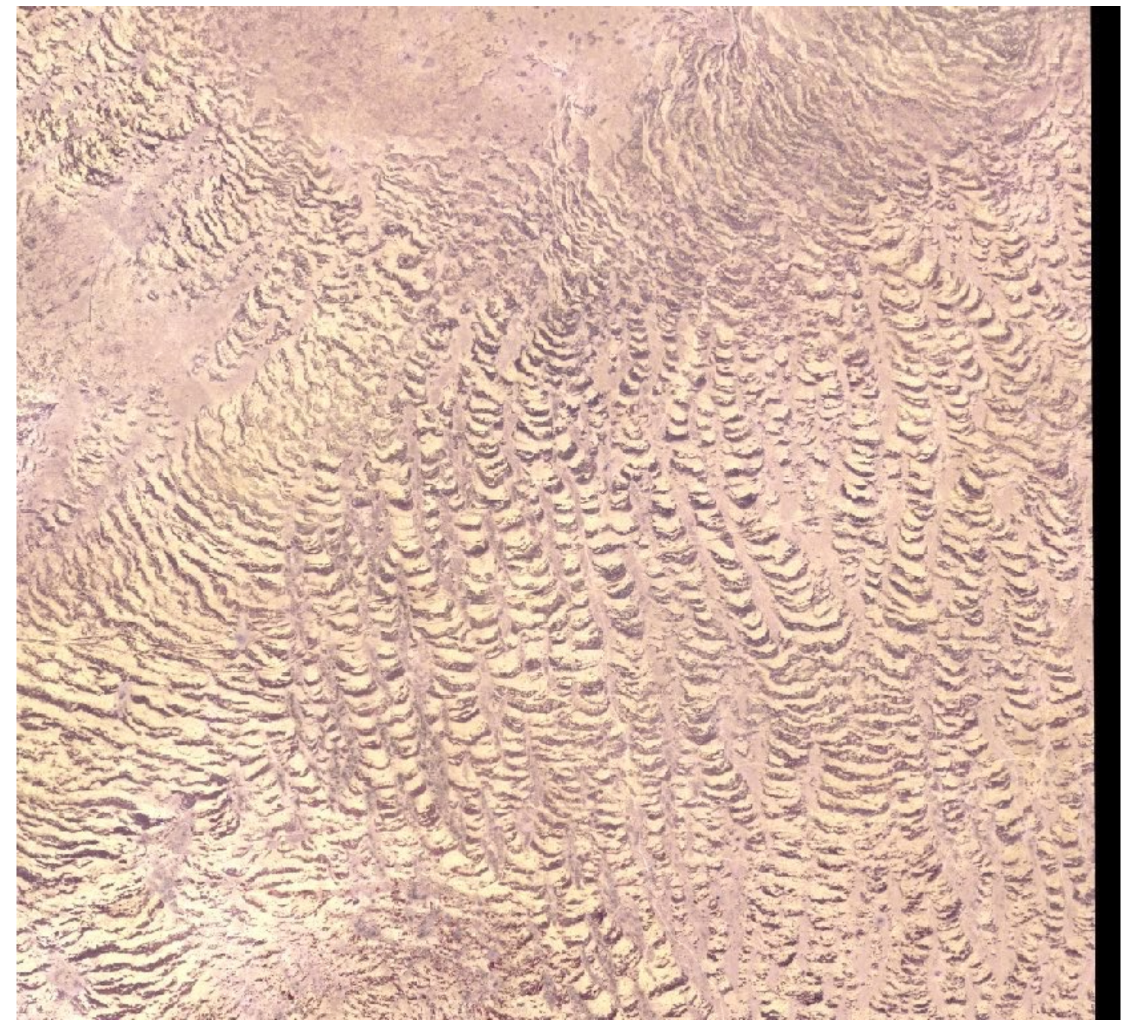
Macfadyen (1950)

Slide from Karna Gowda, a modern day explorer

British Somaliland, circa 1950

Ethiopia, circa 2015

We compare aerial imagery from the 50s, 60s, and modern satellites.



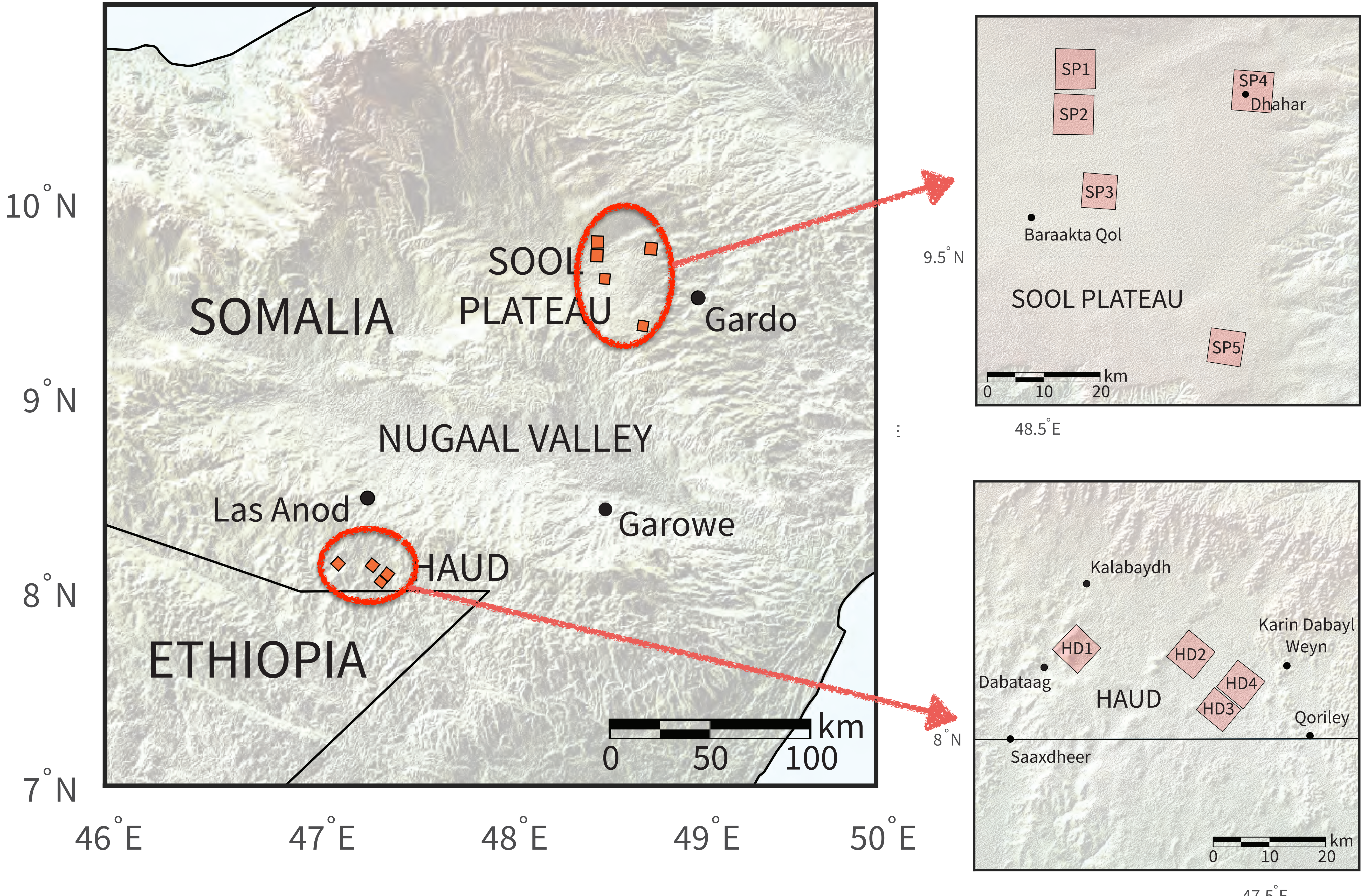
Aerial survey photography taken over banded regions in Somalia (1952) +spy satellite photos (1967)

- ~2 m/pixel resolution
- 1 channel (grayscale)
- Aligned via control points (ArcGIS)

Modern satellite imagery (2004-2016) thanks to DigitalGlobe Foundation

- 0.5-2.4 m/pixel
- 4-8 channels
- Can compute vegetation indices (NDVI, SAVI)

2 regions, 9 photographs, $\sim 50\ km^2$ each



Gowda, Iams, Silber (2017)

Related Aerial/Satellite Image Studies

(some of our inspiration!)

Niger:

Valentin & d'Herbès (1999)
Wu, Thurow & Whisenant (2000)
Barbier, Couteron, Lejoly, Deblauwe, Lejeune (2006)

Sudan:

Deblauwe, Couteron, Lejeune, Bogaert, & Barbier (2011)

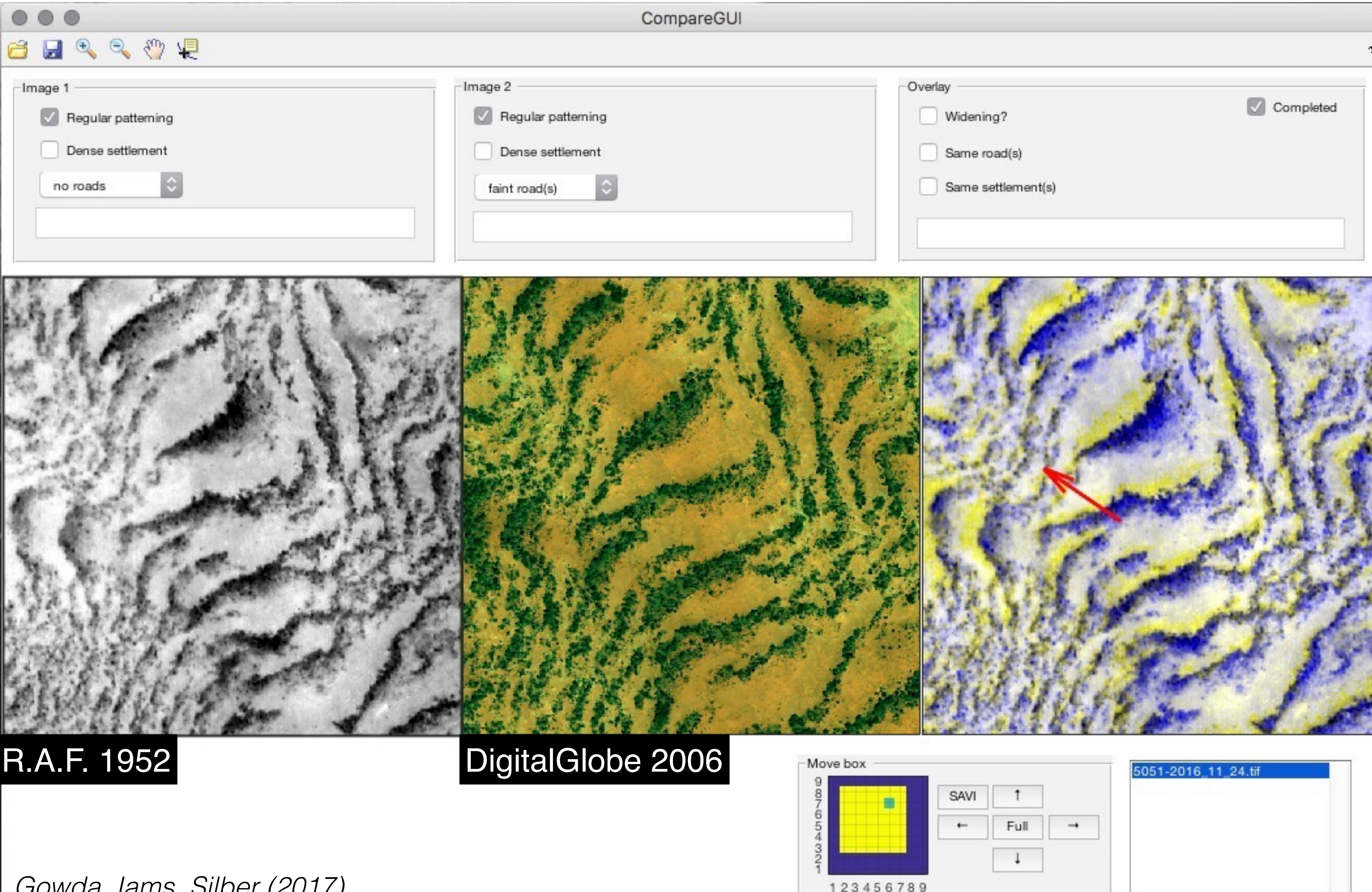
Australia, Morocco, **Somalia**, Texas:

Deblauwe, Couteron, Bogaert, & Barbier (2012)

Texas:

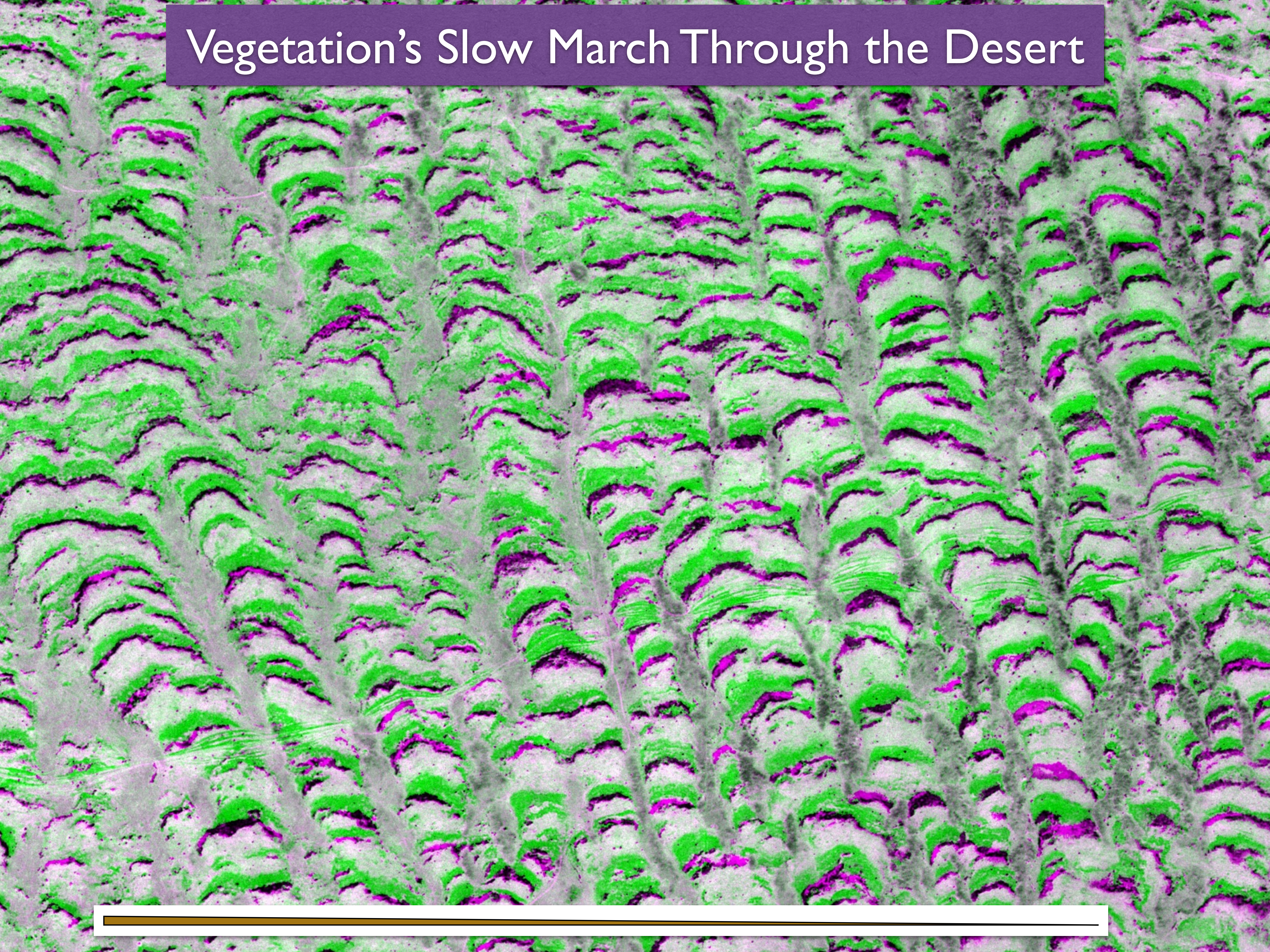
Penny, Daniels, Thompson (2013)

Karna Gowda's GUI



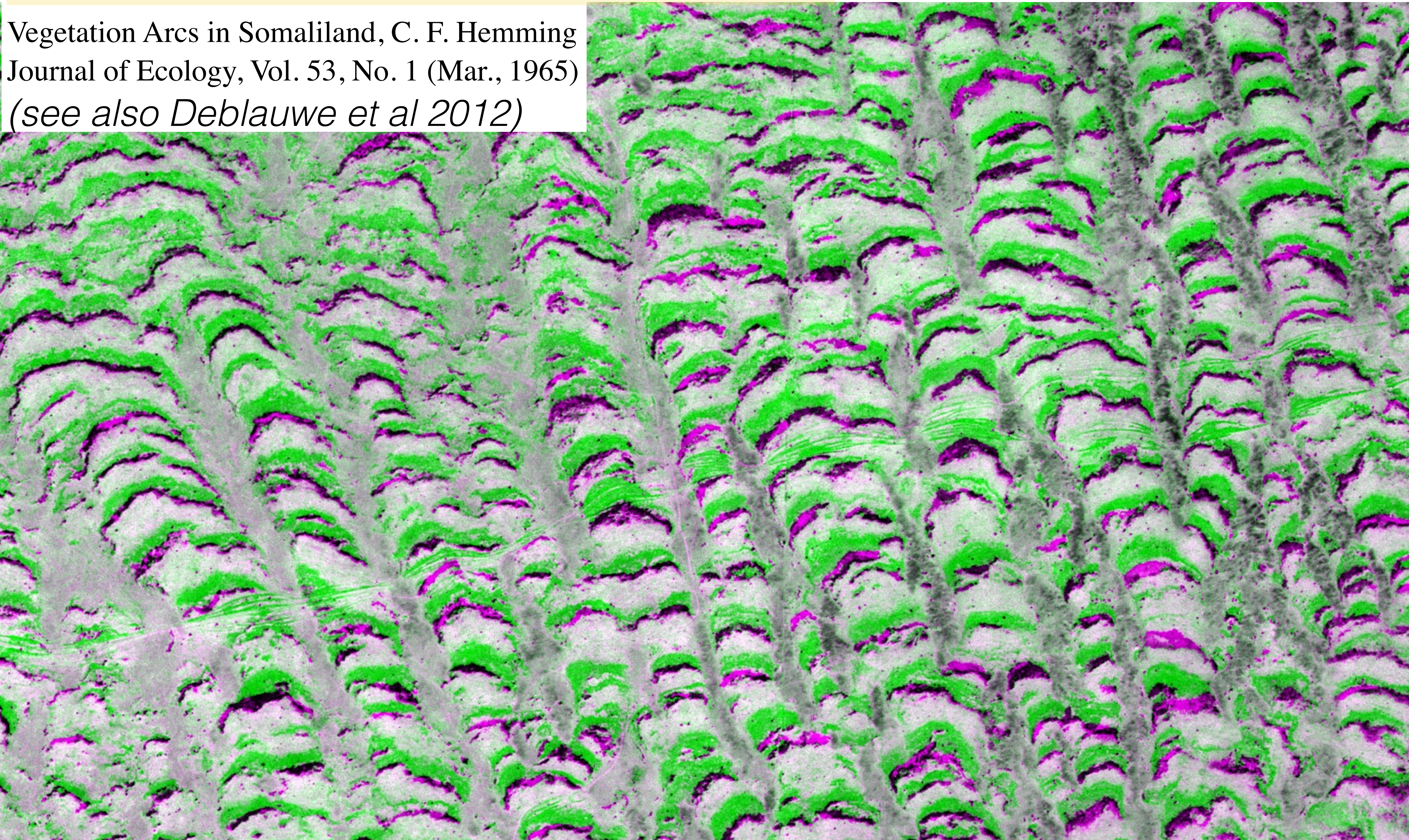
Gowda, Iams, Silber (2017)

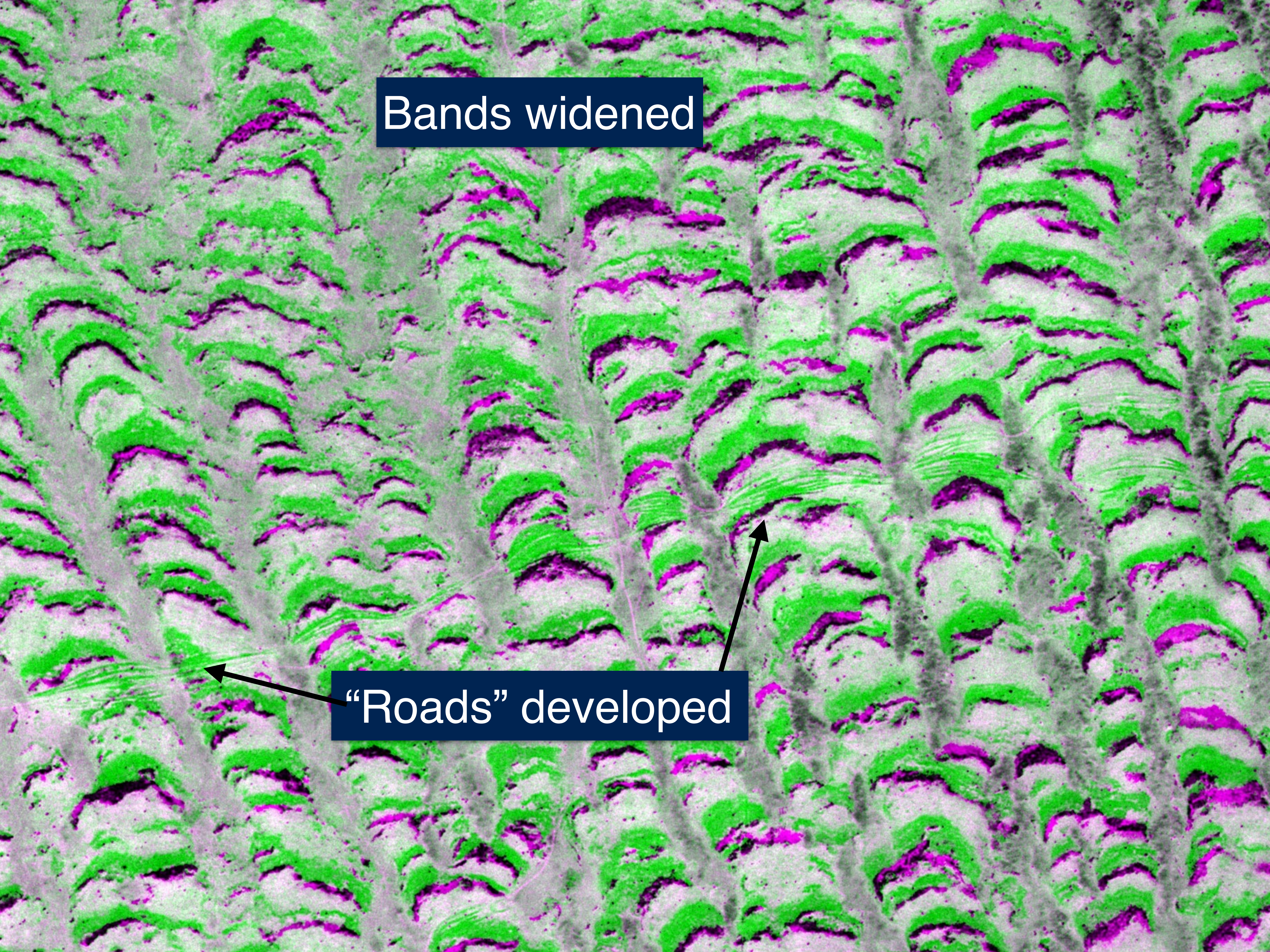
Vegetation’s Slow March Through the Desert



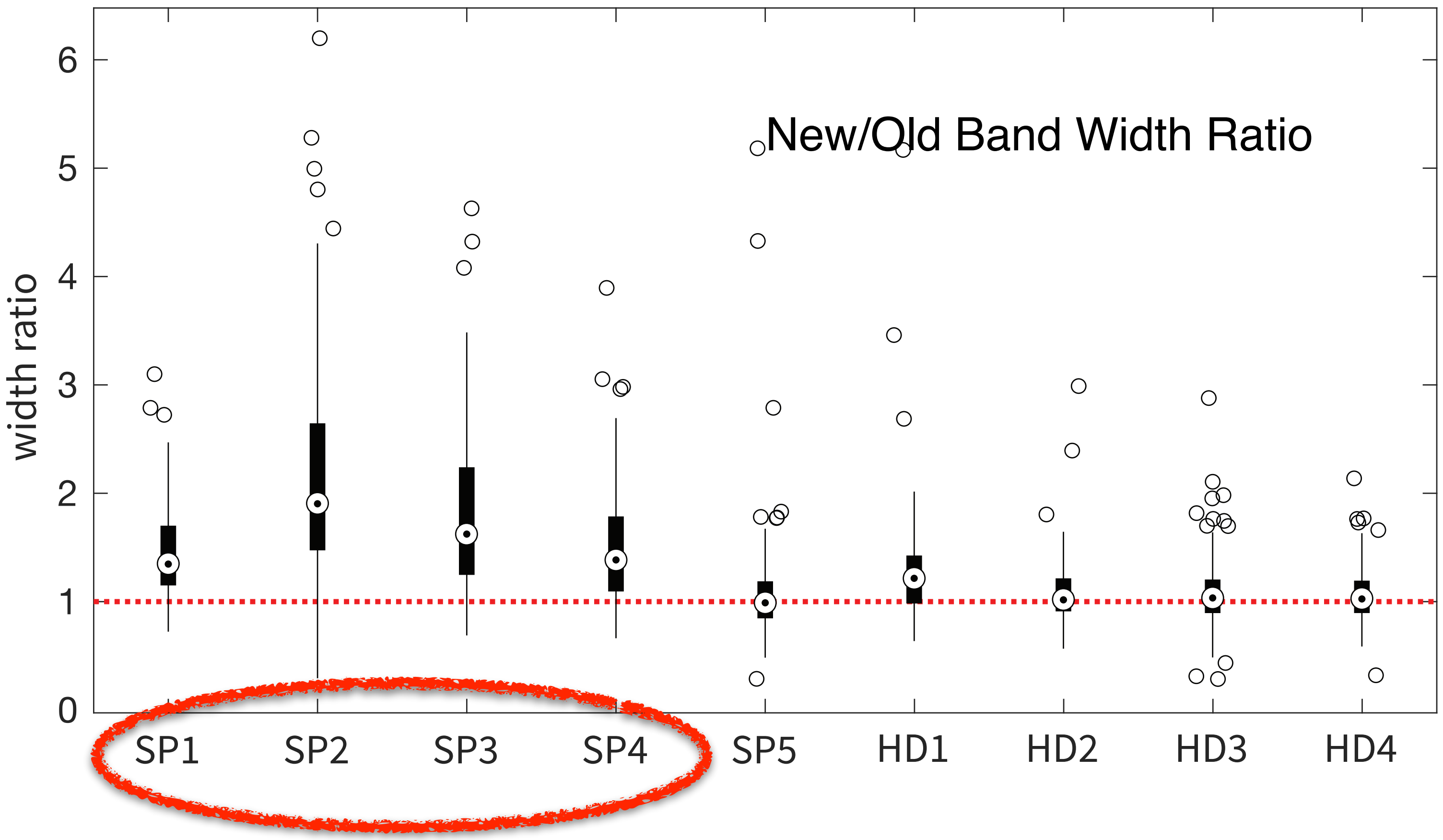
The rate of migration of vegetation arcs is of interest. As far as is known no measurements have been made, but having looked at many advancing upper edges it is estimated that up-flow colonization may occur at an average rate of 6–12 in. (15–30 cm) per year. The small arc surveyed in detail was about 60 ft (18 m) wide and might therefore take between 60 and 120 years to move one arc's width and abandon any trees now living

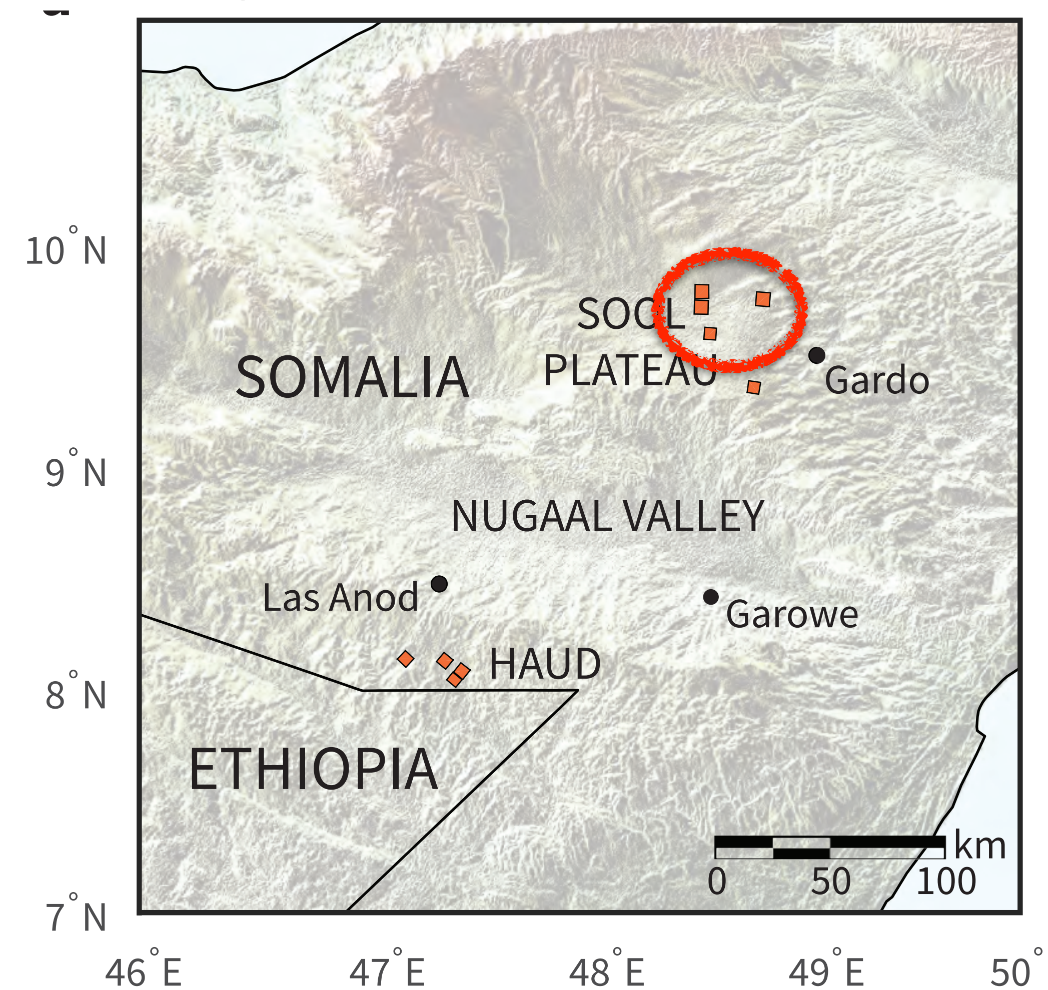
Vegetation Arcs in Somaliland, C. F. Hemming
Journal of Ecology, Vol. 53, No. 1 (Mar., 1965)
(see also Deblauwe et al 2012)

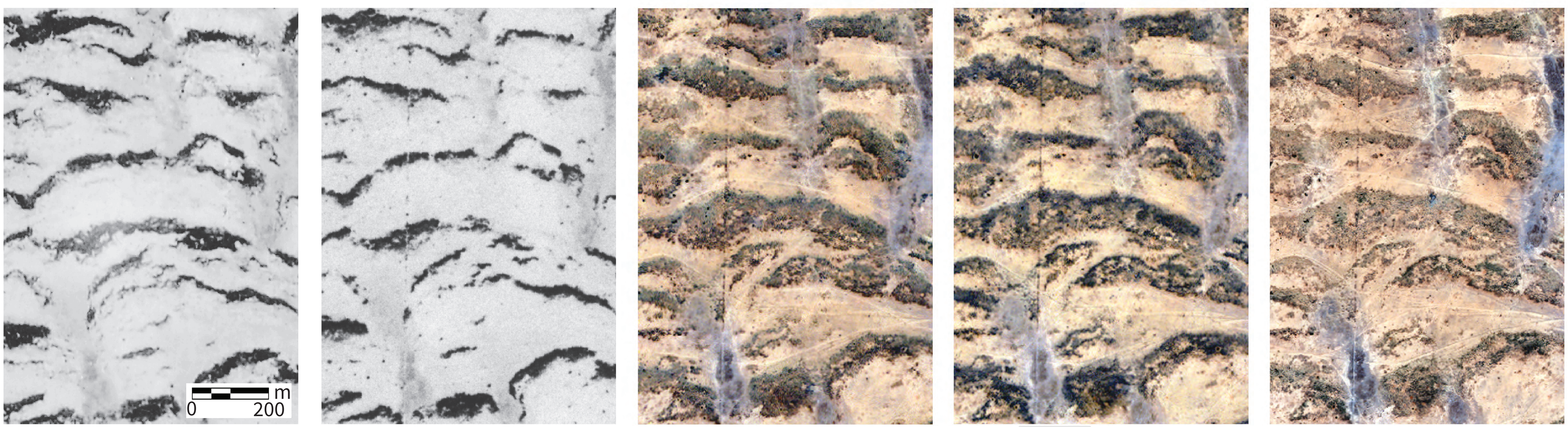



Bands widened
“Roads” developed

Vegetation Band Widening?

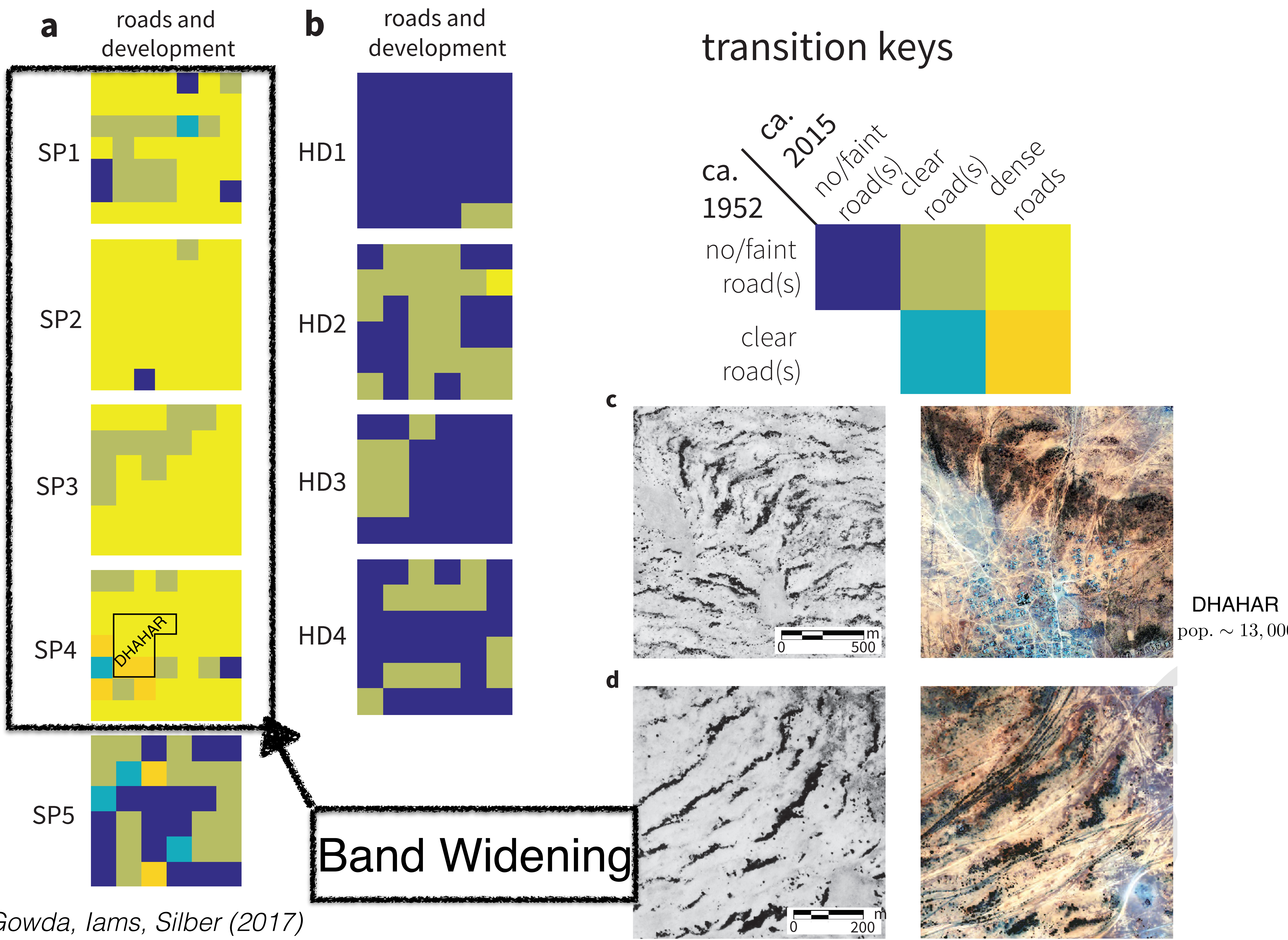




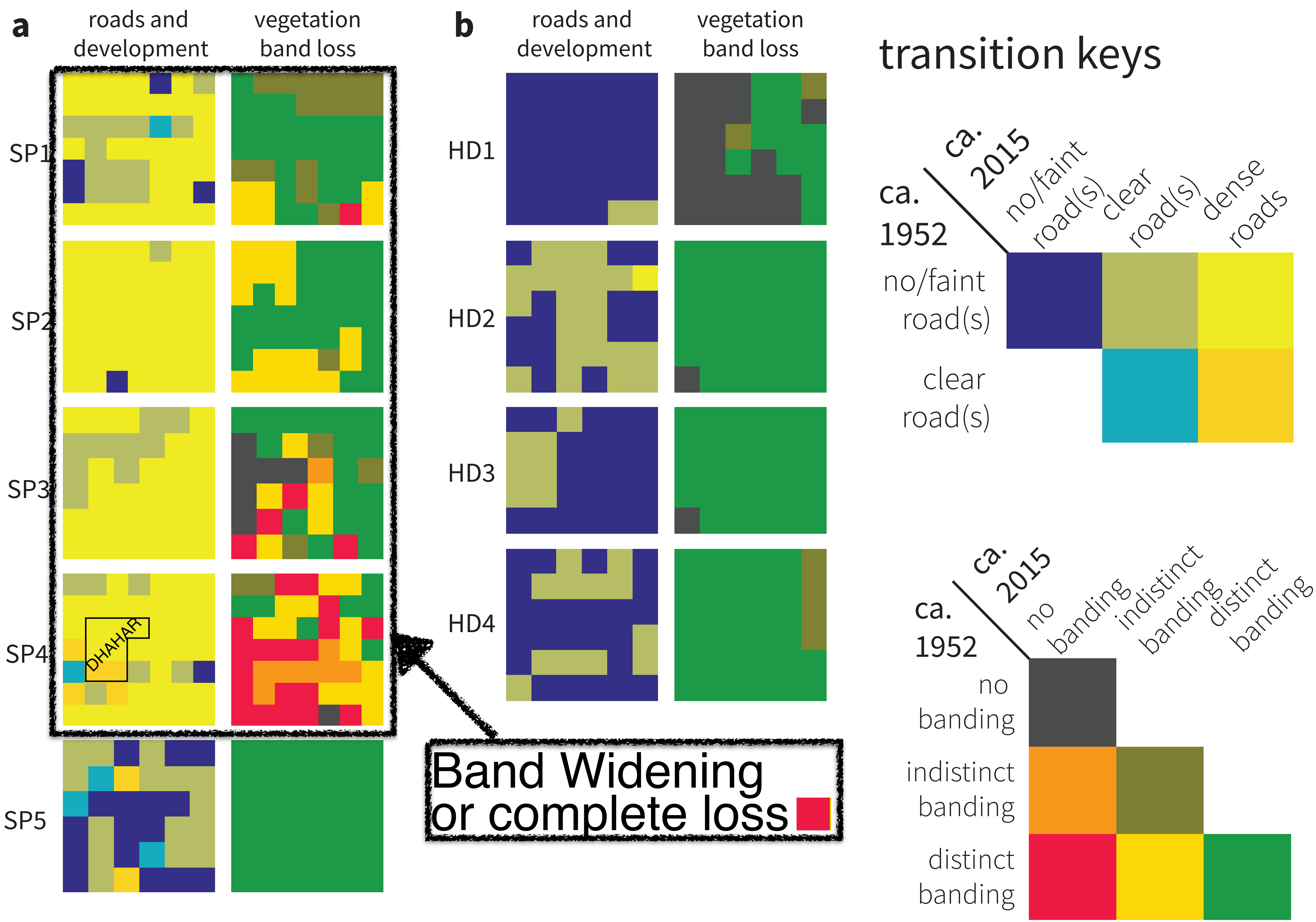


Feb. 1952 Dec. 1967 Jun. 2004 Mar. 2006 Dec. 2011

Gowda, Iams, Silber (2017)

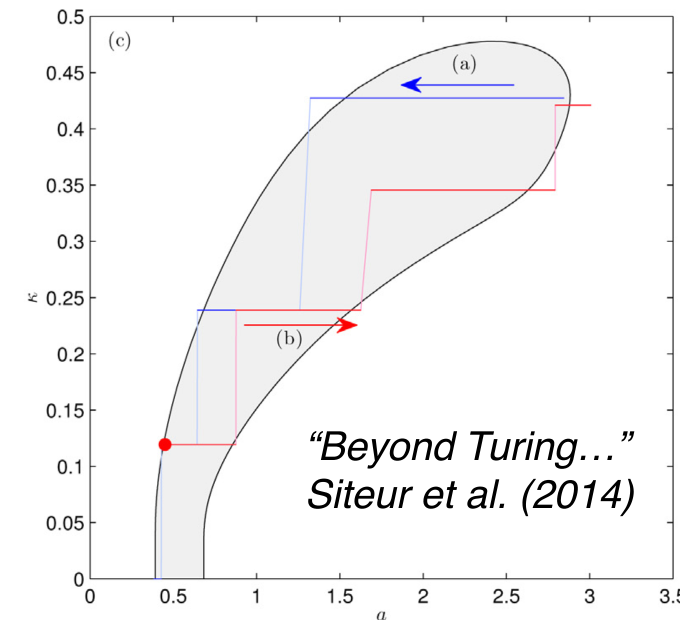


Gowda, Iams, Silber (2017)



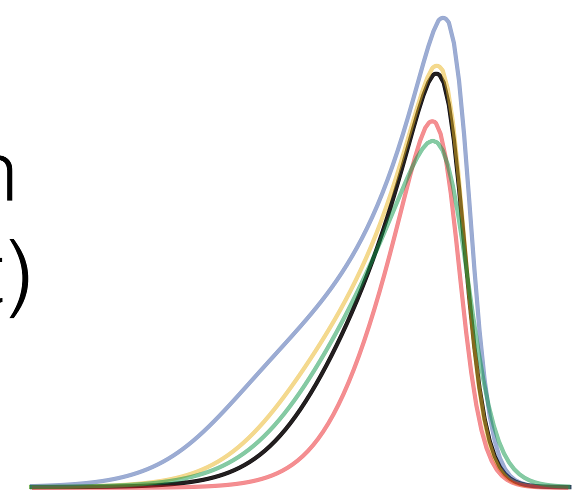
Gowda, Iams, Silber (2017)

Summary Related to Observations



Wavelength change (coarsening) not observed & hard to observe - bands may be quite resilient. (See works by Doelman and co., Sherratt and co.)

Changes to band-interband ratio (vegetation “pulse profile”), migration speed easier to monitor on modern satellite timescale of decades. (also noted by Sherratt)



Sool Plateau: band-interband ratio *increased* in regions of *increased* human impacts (based on “road proxy”) — *not* what we’d expect based on models… (See works of Sherratt and co, Meron and co.)

Due to change in vegetation composition? Increased seed dispersal? Or some form of degradation/human impacts not captured by models?

Desertification

"land degradation in arid, semi-arid and dry sub-humid regions resulting from various factors, including climatic variations and human activities." (United Nations)

We used roads/tracks as a convenient proxy for human activity; easy to detect in the satellite images.

And they may be more than just a proxy - they may be directly influencing the ecohydrology, as described in Hemming (1966).

Vegetation arcs are found in areas without any incised drainage pattern, though they may adjoin such areas. This indicates that the rain water is absorbed either where it falls, or where it arrives after non-erosive sheet-flow.

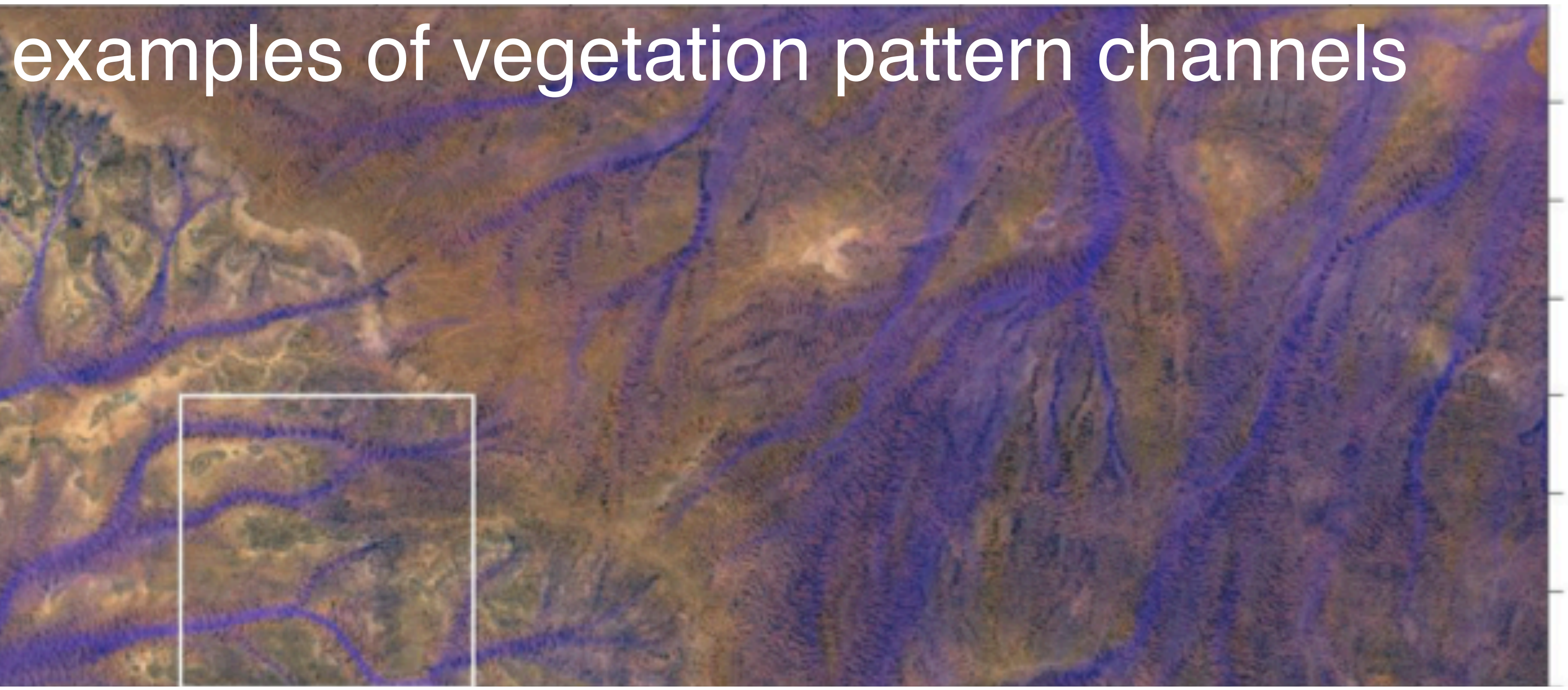
Hemming, Journal of Ecology (1965)

Topographic Influences on Patterns

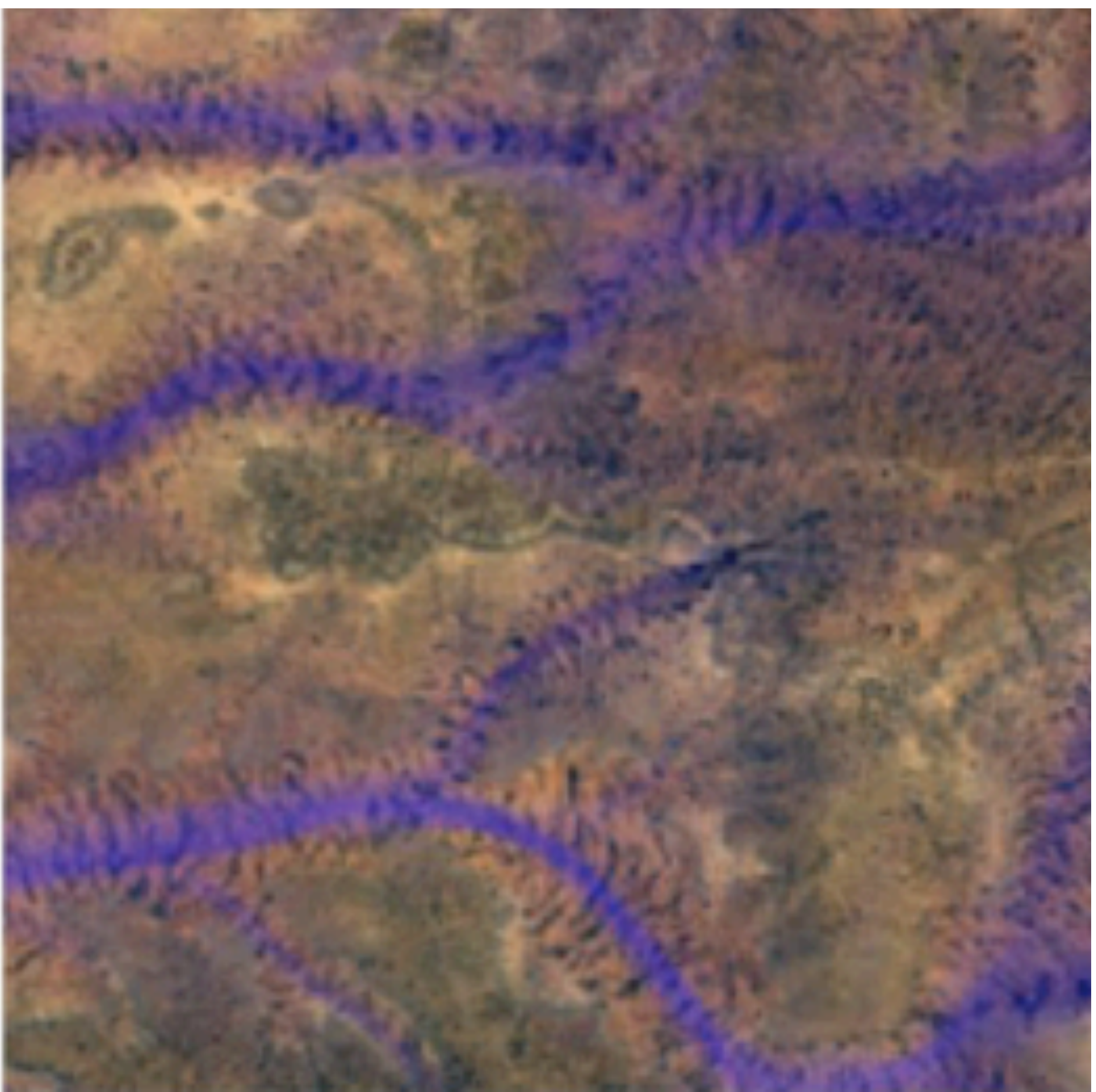
More than just banding & upslope migration

work in progress: Arcing & Channels

2016 Summer Undergraduate Project @ Macalester College
Macalester (Jake Ramthun, Elle Weeks, Prof. Chad Topaz)
+ Harvey Mudd (Jordan Haack, Dan Schmidt, Gavin Zhang, Prof. Andy Bernoff)
+Sarah Iams (Harvard)+Karna Gowda & Yuxin Chen (Northwestern)



"Flow" computed using Matlab Topotoolbox flowacc_IM



"Flow" as proxy for a spatially varying effective "precipitation"?

Images From Elle Weeks, Macalester College Math student

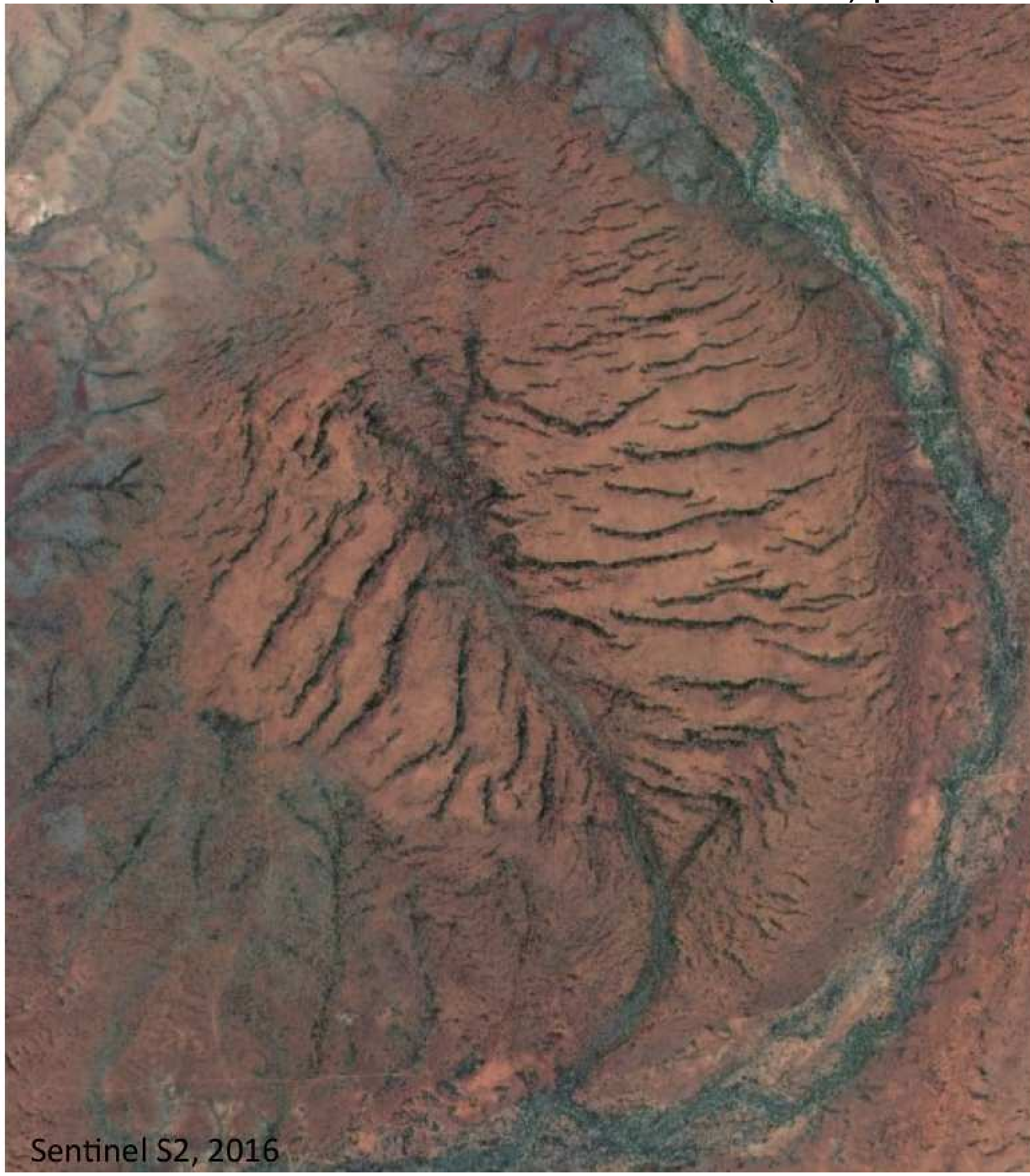
work in progress with
Lucien Werner and Punit Gandhi
(see poster, Wednesday PP2)

Caltech PhD student (starting Fall 2017)

Mathematical Biosciences Institute (MBI) postdoc

“The important point is that the arcs are invariably oriented so that their chords are at right angles to the direction of drainage, and that they are convex upslope.” Macfadyen (1950)

Macfadyen Rendering of Vegetation Arcs, Somalia

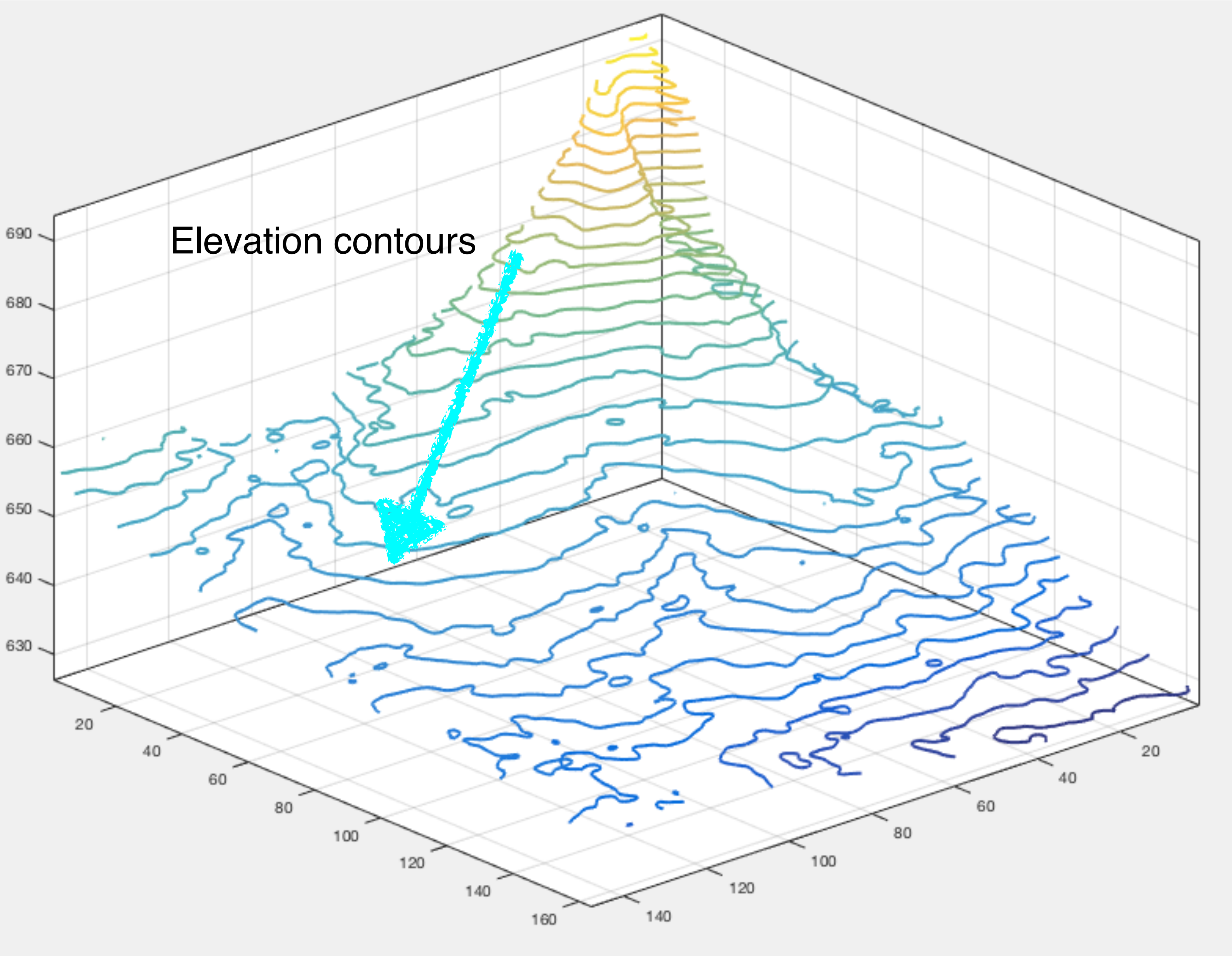


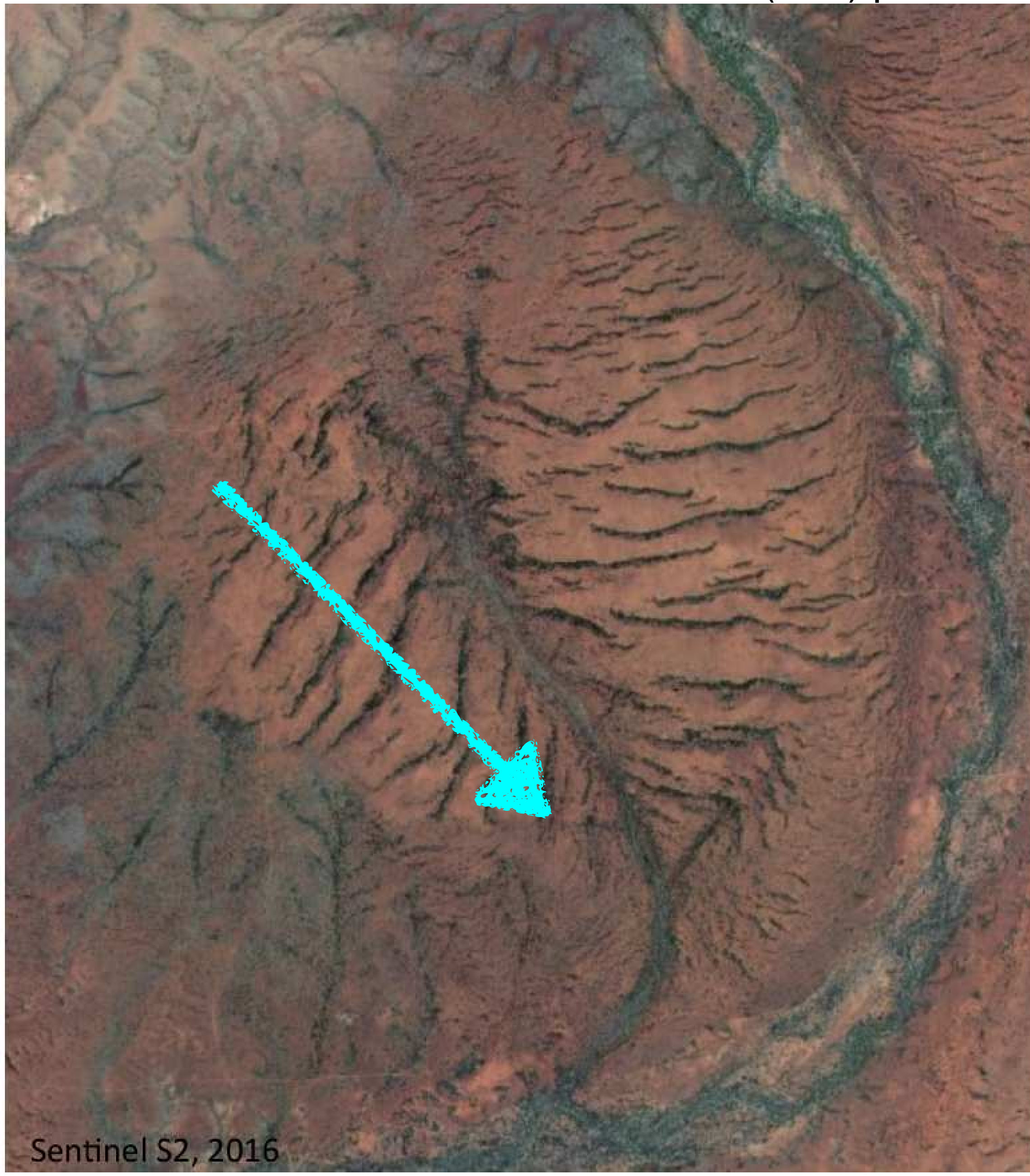
Vegetation Arcs, Western Australia

Caltech ACM PhD student (starting Fall 2017)

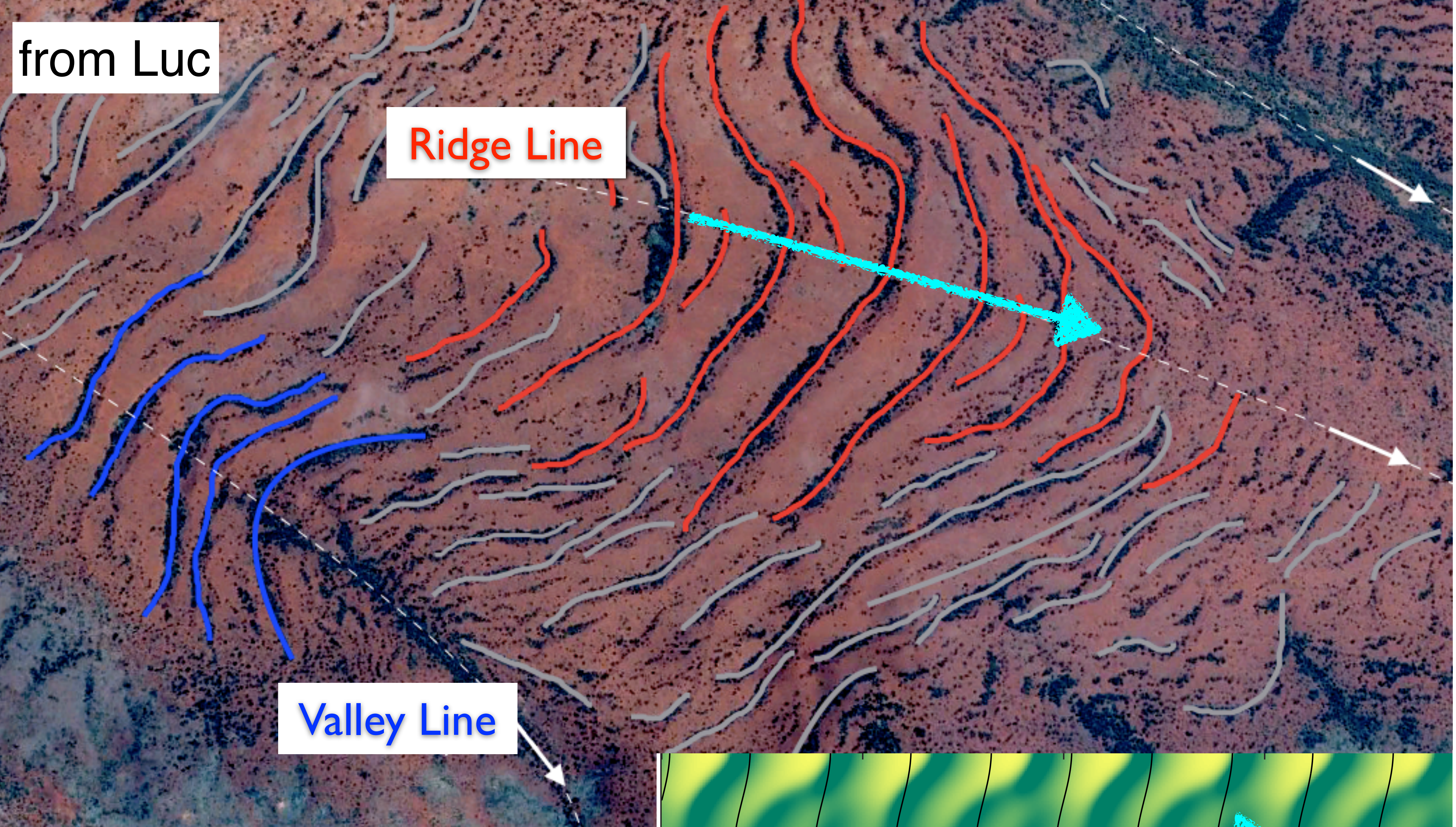
work in progress with
Lucien Werner and Punit Gandhi
(see poster, Wednesday PP2)

Mathematical Biosciences Institute (MBI) postdoc





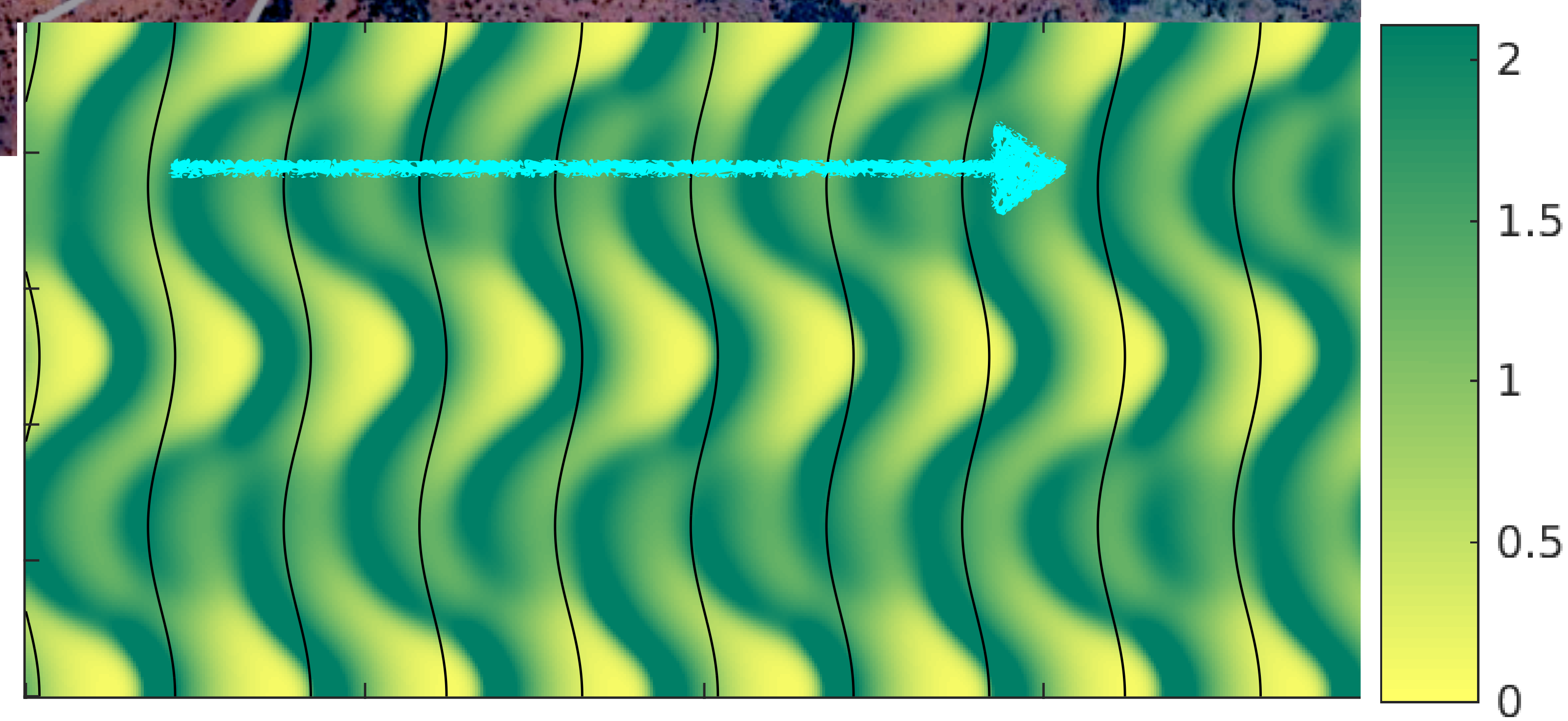
Vegetation Arcs, Western Australia



from Punit

Ridge Line

Valley Line



Vegetation Patterns in Mathematical Models:

Observations → Models → Analysis, Mechanisms, Predictions

and Vegetation Patterns in the Horn of Africa:

Observations → Models → New Observations, Conjectures

Early Warning Signs:

Human land use impacts can’t be ignored.

Topographic influences shouldn’t be ignored.

Thanks to

Karna Gowda, Sarah Iams (Tues. MS93)
+Punit Gandhi, Lucien Werner (Wed. poster)
Yuxin Chen, Hermann Riecke
Chad Topaz, Jake Ramthun, Elle Weeks
Andy Bernoff, Jordan Haack, Dan Schmidt

NSF
DigitalGlobe FOUNDATION
MCRN Mathematics and Climate Research Network