Estimation and Control Problems in Systems with Moreau's Sweeping Process

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TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

INRIA RHÔNE-ALPES

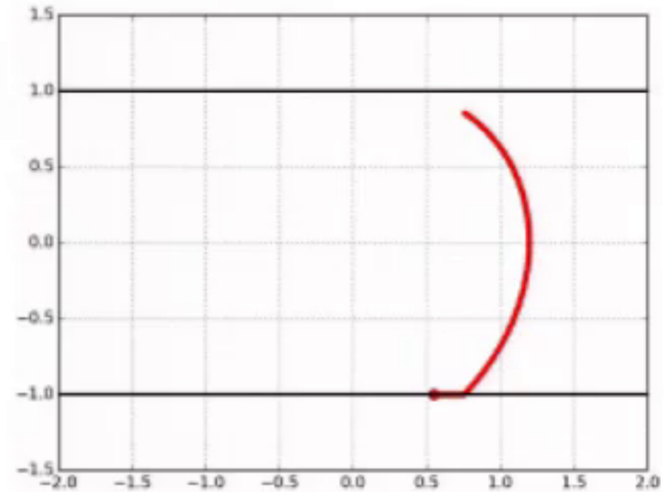
Perturbed Sweeping Processes

First Oder Systems

$\mathcal{S} : \mathbb{R}_+ \rightrightarrows \mathbb{R}^n$; $\mathcal{S}(t)$ closed.

$$dx \in f(t,x)dt - \mathcal{N}_{\mathcal{S}(t)}(x)$$

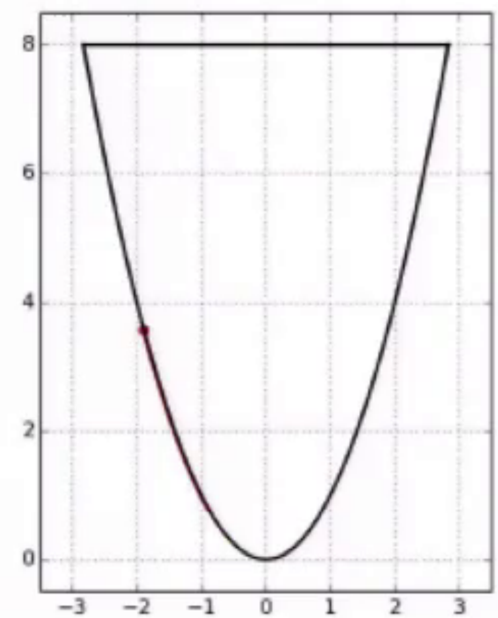
$\mathcal{N}_{\mathcal{S}}(x)$ is the *normal* cone to $\mathcal{S}$ at x.



Second Order Systems

$V : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is state-dependent.

$$dq = v\,dt$$
$$dv \in F(t,q,v)dt - \mathcal{N}_{V(q)}(v)$$



Outline

1. Generalized First Order Sweeping Process
2. Output Regulation with Convex Sets
3. State Estimation in Second Order Sweeping Process

Generalizations of First Order Sweeping Process

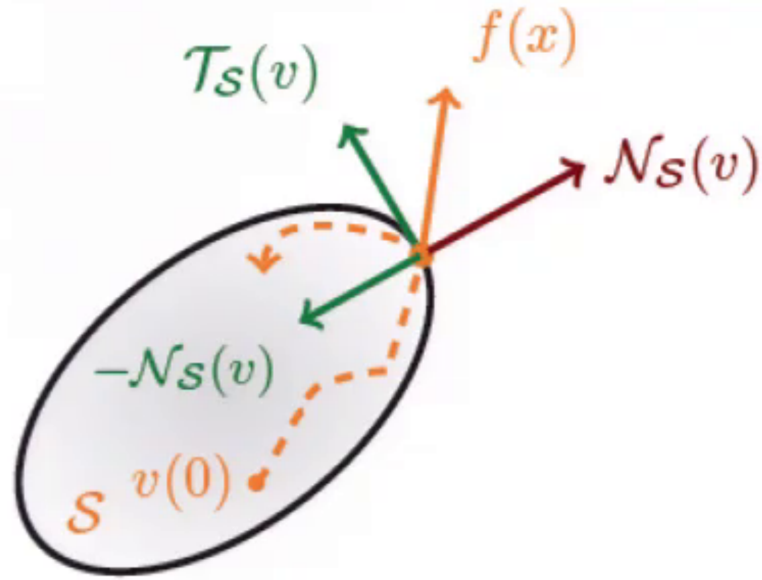
Differential Variational Ineq. (DVI)

Let $\mathcal{S} : \mathbb{R}_+ \rightrightarrows \mathbb{R}^s$ be closed and convex.

$$\dot{x} = f(t, x) + G\eta$$

$$v = Hx + J\eta, \quad v(t) \in \mathcal{S}(t)$$

$$\eta \in -\mathcal{N}_{\mathcal{S}}(v)$$



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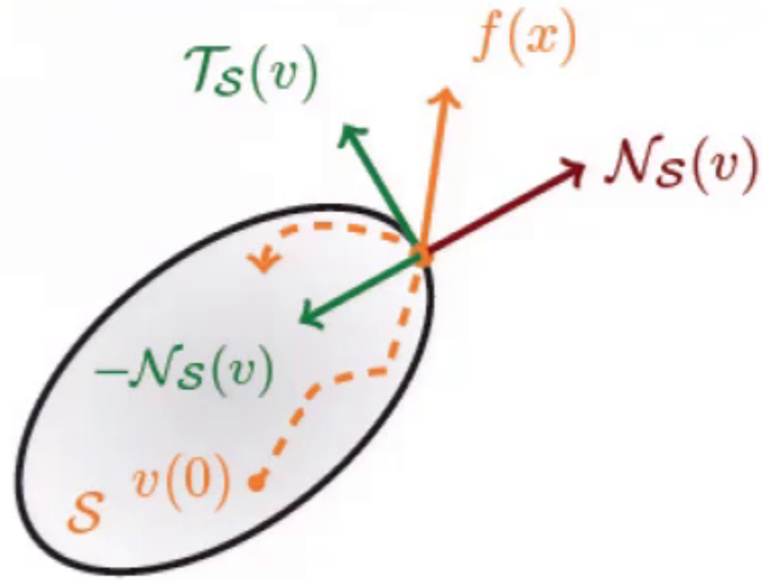
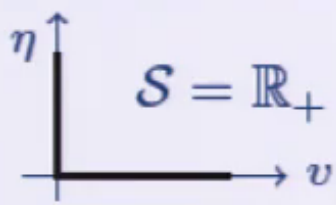
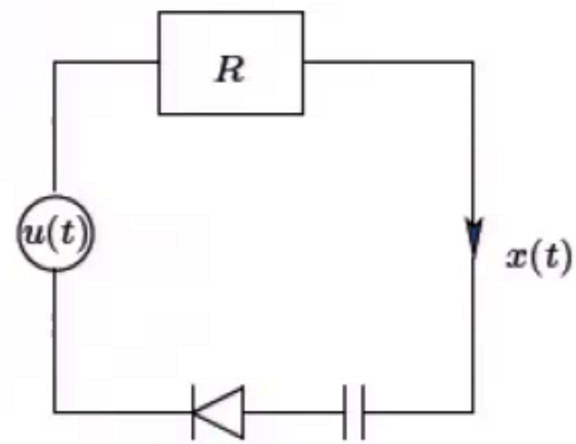


Linear Complementarity



$$0 \leq \eta \perp v \geq 0$$



- Sweeping Process [Moreau; Monteiro Marques; Colombo; Thibault] and [Participants of MS78, MS91 at SIAM DS'15]
- Evolution Inclusions [Aubin; Celina]
- DVIs & complementarity [Cottle; Pang; Stone; Stewart]

Existence and Uniqueness of Solutions for DVIs [CDC '14]

(DVI) : $\dot{x} = f(t,x) + G\eta, \quad \eta \in -\mathcal{N}_{\mathcal{S}}(Hx + J\eta)$

Theorem (Sufficient Conditions for Well-posedness of DVI)

There exists a unique absolutely continuous (a.c.) solution of **(DVI)**, *if*

(A1) $J \geq 0$ *and* $\exists P = P^\top > 0$ *s.t.* $\ker(J + J^\top) \subseteq \ker(PG - H^\top)$.

(A2) *range* $H \cap (\mathcal{S}(t) +$ *range* $J) \neq \emptyset$, $\forall t \geq 0$.

(A3) *The mapping* $t \mapsto \mathcal{S}(t)$ *is a.c., i.e.,*$\exists$ *an a.c. function* $\nu(\cdot) : [0,\infty) \to \mathbb{R}_+$ *s.t.*

$$|d(v, \mathcal{S}(t_1)) - d(v, \mathcal{S}(t_2))| \leq |\nu(t_1) - \nu(t_2)|, \ \forall\, t_1, t_2 \geq 0.$$

(A4) $f(t,x)$ *is is globally Lipschitz in* x *and a.c. in* t.

- There exists an operator Φ s.t. $\dot{x}(t) \in f(t,x) - G\,\Phi(t, Hx(t))$.
- Under **(A1)**, **(A2)**, $\exists$ a maximal monotone operator $\Psi(t,\cdot)$, a Lipschitz continuous function $g(t,\cdot)$, and a transformation $z = Rx$ s.t.

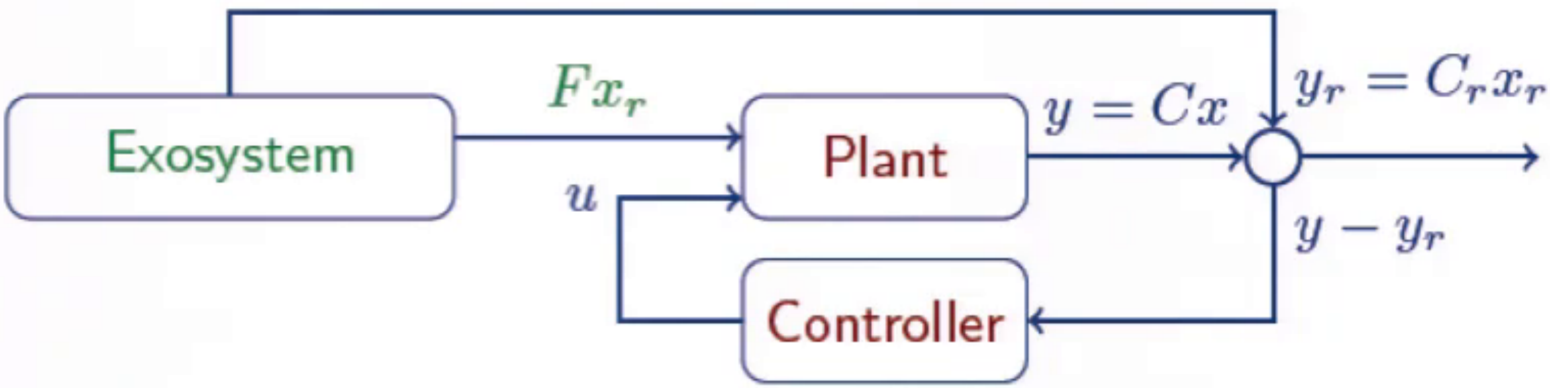
$$\dot{z}(t) \in g(t,z) - \Psi(t,z)$$

- Earlier results only consider the case $J = 0$ or J positive definite.

Outline

Control design: Exchange information between systems so that a subset of System 1 trajectories track a subset of System 2 trajectories.

Regulation Problem



Exosystem (Ref. Generator)

$$\dot{x}_r = A_r x_r + G_r \eta_r$$
$$\eta_r \in -\mathcal{N}_{\mathcal{S}}(H_r x_r + J_r \eta_r)$$

Plant Dynamics

$$\dot{x} = Ax + Bu + Fx_r + G\eta$$
$$\eta \in -\mathcal{N}_{\mathcal{S}}(Hx + J\eta)$$

Motivation: Such exosystems can generate **nonsmooth** trajectories.

Output Regulation (OR) Problem

Design the control input u such that

- The resulting closed loop system is well-posed
- The regulation error $w = Cx - C_r x_r$ converges asymp. to zero
- The overall closed-loop system is asymp. stable

Regulation with Static Feedback [CDC '14]

$$\dot{x}_r = A_r x_r + G_r \eta_r \qquad \dot{x} = Ax + Bu + Fx_r + G\eta$$
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Theorem (Regulator Synthesis Conditions)

Under **(A2)**, **(A3)**, *if there exist matrices* M, K, Π *and* $P > 0$ *s.t.*

- **Passivity of** $(A + BK, G, H, J)$**:**

$$\begin{bmatrix} (A+BK)^\top P + P(A+BK) + \gamma P & PG - H^\top \\ G^\top P - H & -(J + J^\top) \end{bmatrix} \leq 0$$

- **Internal model (IM):** $\Pi A_r = A\Pi + BM + F \quad \wedge \quad C\Pi - C_r = 0$
- **Constraint matching (CM):** $\Pi G_r = G \;\wedge\; H\Pi = H_r \;\wedge\; J_r = J$

hold, then **(OR)** *is solvable with* $\boxed{u = Kx + (M - K\Pi)x_r}$

Tools and Analysis:

- Well-posedness $\Leftarrow$ **(A1)** $\Leftarrow$ Passivity
- Regulation error converges $\Leftarrow$ Internal model principle
- Closed-loop Stability $\Leftarrow$ Constraint matching and passivity

Regulation with Error Feedback [CDC '14]

Premise: Only $w := y - y_r$ is measurable, and full state (x, x_r) is not available.

Dynamic Compensator

For some matrices $\overline{A}, \overline{B}, \overline{C}, \overline{G}, \overline{H}, \overline{J}$, and the set $\overline{\mathcal{S}}$, we let

$$\begin{aligned} \dot{\xi} &= (\overline{A} - L\overline{C})\xi + Lw + \overline{B}u + \overline{G}\,\overline{\eta}; \quad \overline{\eta} \in -\mathcal{N}_{\mathcal{S}}(\overline{H}\xi + \overline{J}\overline{\eta}) \\ u &= K\xi_x + (M - K\Pi)\xi_r \end{aligned} \qquad \textbf{(DC)}$$

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Theorem (Regulation with Dynamic Compensator)

- *If* $(A + BK, G, H, J)$ *and* $(\overline{A} - L\overline{C}, \overline{G}, \overline{H}, \overline{J})$ *are* **passive**,
- *and the matrices* M, Π *satisfy* **(IM)**, **(CM)** *hold*

then **(DC)** *solves* **(OR)**.

- Dynamic compensator is based on certainty equivalence principle, and $\xi := (\xi_x, \xi_r)$ estimates (x, x_r), which are used by control u.
- Well-posedness and stability follows from similar tools.

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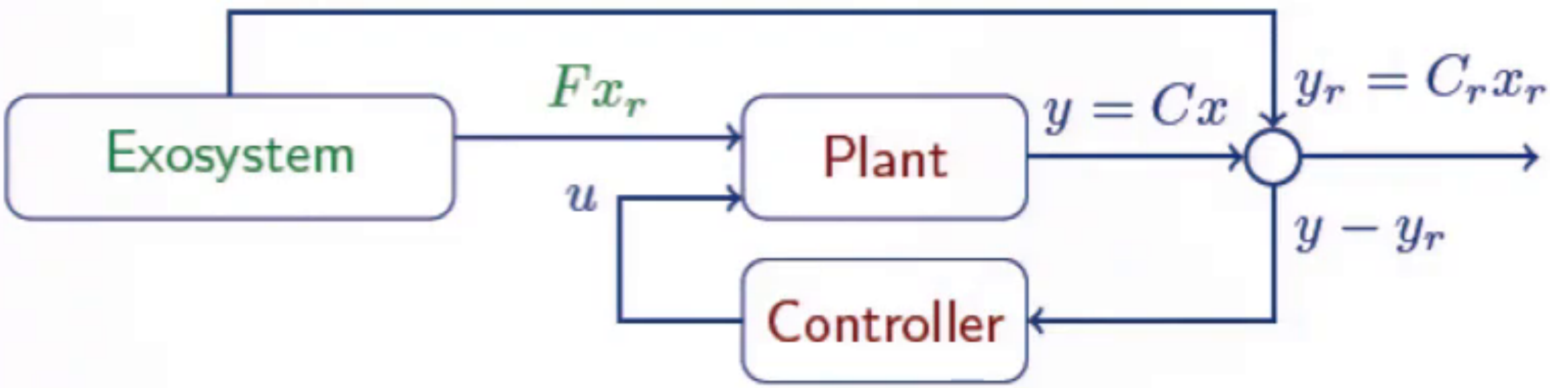

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A Case Study: Regulation with Constraints

Control design for regulation with polyhedral constraints

Plant: $\dot{x} = Ax + Bu$

Constraint set: $\mathcal{S}(t) := \{z \,|\, z + g(t) \geq 0\}$

Objective: Find u such that $Cx - C_r x_r \to 0$ and $Hx(t) \in \mathcal{S}(t)$.

Decompose $u = u_{IM} + u_\eta$

Choose $u_{IM} = Kx + (M - K\Pi)x_r$ for regulation using IM principle.

Choose u_η for constraints using **complementarity relations**

$$u_\eta(t) \in -\mathcal{N}_{\mathcal{S}(t)}(Hx(t))$$

$$\Leftrightarrow 0 \leq u_\eta(t) \perp y(t) = Hx(t) + g(t) \geq 0$$

$$\Leftrightarrow \begin{cases} u_\eta(t) = 0 & \text{if } Hx(t) \in \text{int } \mathcal{S}(t) \\ 0 \leq u_\eta(t) \perp \dot{y}(t) \geq 0 & \text{if } Hx(t) \in \text{bd } \mathcal{S}(t) \end{cases}$$

Limitation: Relative degree 1 between input and constrained states

Example: Track Clipped Sinusoid with Norm Constraints

Plant: $\dot{x}_1 = -0.1x_1 + x_2, \qquad \dot{x}_2 = u.$

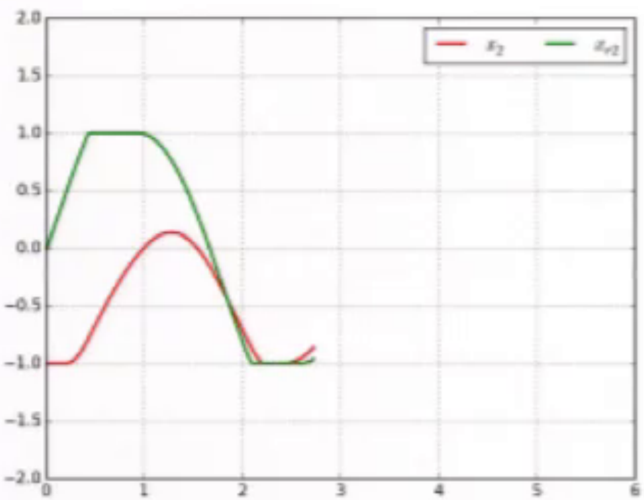
Objective: $x_2(t) \to x_{r2}(t)$ and $x_2(t) \in \mathcal{S}(t) := \{z : |z| \le 1\}$

Exosystem (LCS): $\begin{pmatrix} \dot{x}_{r1} \\ \dot{x}_{r2} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} x_r + \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} \eta_r;$

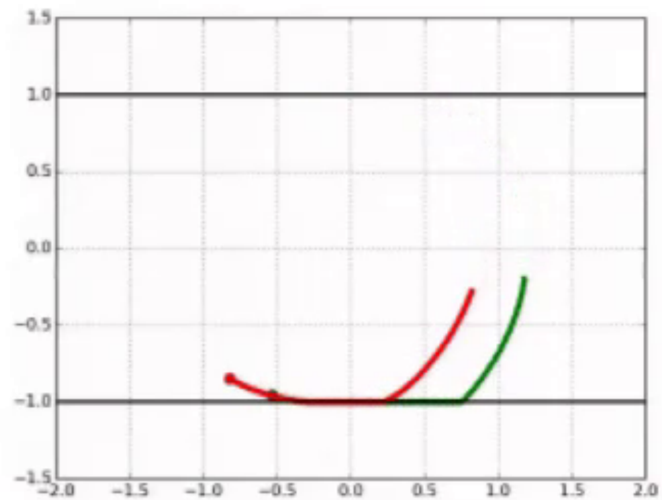
$$0 \le \eta_r \perp \begin{pmatrix} -x_{r2} \\ x_{r2} \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ge 0$$

Control: $u = u_{IM} + u_{\eta 1} + u_{\eta 2}$, where $u_{IM} = K_1 x + K_2 x_r$, and

$$0 \le \begin{pmatrix} -u_{\eta 1} \\ u_{\eta 2} \end{pmatrix} \perp \begin{pmatrix} -x_2 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ge 0$$



(a) Time evolution



(b) Phase portrait

Second Order Processes: Mechanical Systems with Impacts

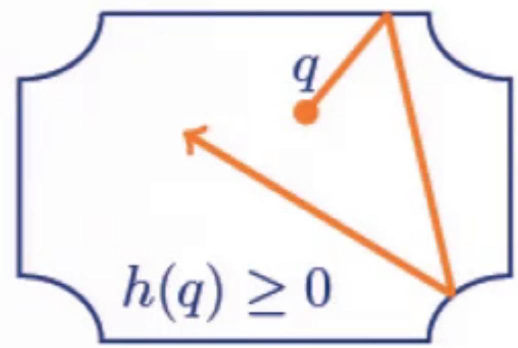
- **Unconstrained motion:**

$$M(q)\dot{v} + F(t,q,v) = 0$$

- **Constraints:**

$$h_i(q) \geq 0$$

- **An impact (jump) rule.**



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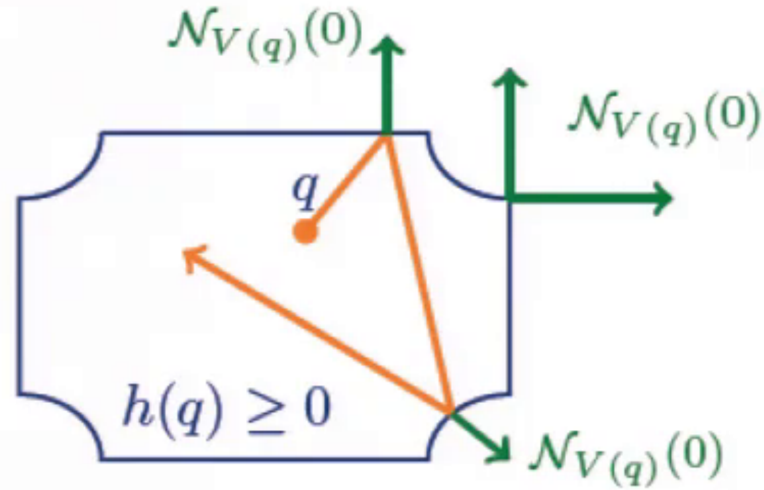


Sweeping Process Formulation

Velocity set at $h_i(q) = 0$:

$$V(q) := \{w \in \mathbb{R}^n \mid \langle w, \nabla_q h_i(q)\rangle \geq 0\}$$

System description:

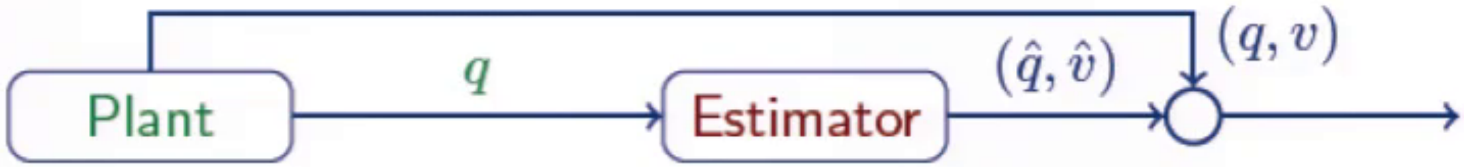
$$\dot{q} = v$$

$$M(q)dv + F(t,q,v)dt \in -\mathcal{N}_{V(q)}(v_e),$$

$$v_e = \frac{v^+ + ev^-}{1+e}$$

- v is *rcbv* with countably many discontinuities.
- Existence (and uniqueness) holds under mild assumptions (analyticity).
- Solution, in general, is not continuous with respect to initial conditions.

Estimator Dynamics [CDC '13; Submitted to TAC '14]



Our proposed estimator is a **first order** sweeping process:

$$\begin{aligned} \dot{z}_1 &= F_1(t, q, z) \\ M(q)dz_2 + F_2(t, q, z)dt &\in -\mathcal{N}_{\boldsymbol{V(q)}}(\hat{\boldsymbol{v}}_{\boldsymbol{e}}) \qquad \textbf{(obs)} \\ \hat{q} = g_1(z_1, q), &\qquad \hat{v} = z_2 + g_2(z_1, q). \end{aligned}$$

- In (**obs**), the set-valued map $V(\cdot)$ is parameterized by the external state q of the plant.
- Position estimated by the estimator $\hat{q}$ **does not** satisfy the constraints: $h_i(\hat{q}) \geq 0$.
- $V(\cdot)$ is lower semi-continuous and does not satisfy the "usual" nonempty interior condition.
- The (**obs**) admits a unique solution for an observed trajectory $q(\cdot)$, even if the plant does not have unique solutions.

Results on State Estimation

Let $x := (q, v)$ and $\hat{x} := (\hat{q}, \hat{v})$.

Theorem (Design of Estimator)

Under certain regularity assumption on system data, we can calculate the functions F_1, F_2 and g_1, g_2 such that

- **Well-posedness:** (**obs**) *admits a unique solution*
- **Exponential convergence:** $|\hat{x}(t) - x(t)| \leq ce^{-\beta t}|x(0) - \hat{x}(0)|$

Error Analysis:

$$\dot{\tilde{q}} = v - \widehat{F}_1(t, x, \hat{x})$$

$$M(q)d\tilde{v} + F(t, x) - \widehat{F}_2(t, x, \hat{x}) \in -(\eta - \hat{\eta})$$

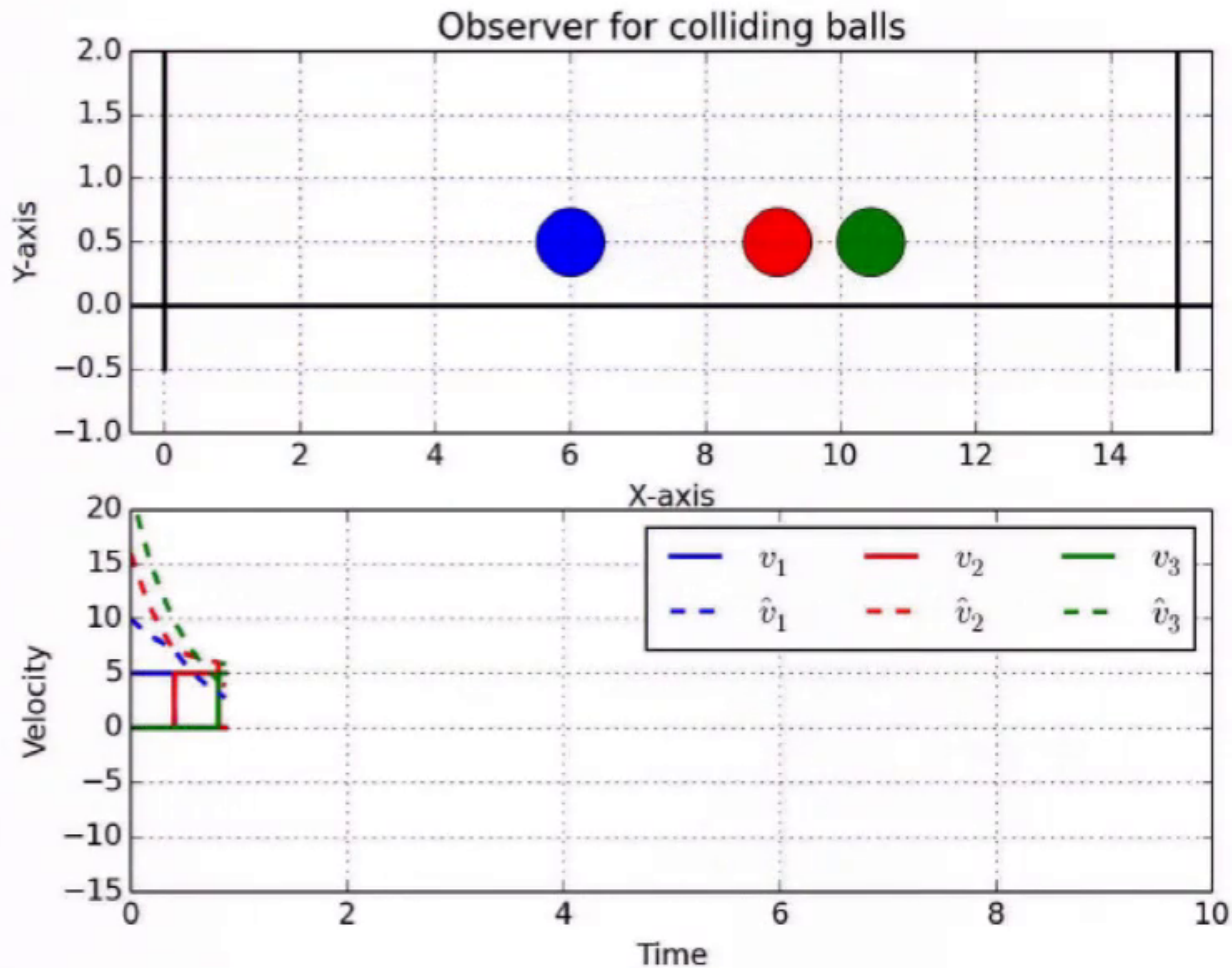
$$\eta \in \mathcal{N}_{V(q)}(v_e) \quad \text{and} \quad \hat{\eta} \in \mathcal{N}_{V(q)}(\hat{v}_e)$$

Induce dissipation in error dynamics (during flows) w.r.t. quadratic Lyapunov function

$$W(q) := \tilde{q}^\top R\tilde{q} + \tilde{v}^\top M(q)\tilde{v}.$$

- Well-posedness result is more involved.

Example: 3-Ball Chain with Walls



Example: A Nonconvex Biliard