Low-complexity stochastic modeling of turbulent flows

Mihailo Jovanović
www.umn.edu/~mihailo

joint work with:
Armin Zare
Tryphon Georgiou

UNIVERSITY
OF MINNESOTA

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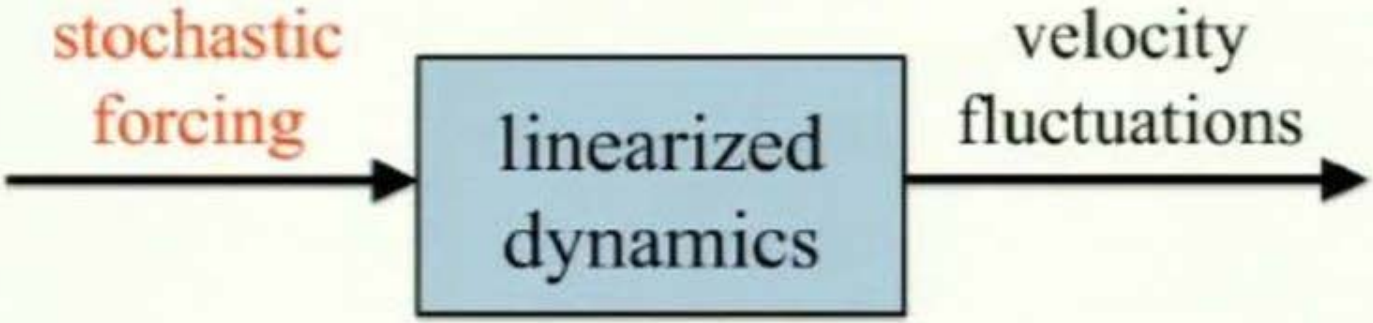
Control-oriented modeling of turbulent flows



- MOTIVATION
  - ⋆ forcing statistics influence performance of flow estimators

    Chevalier, Hœpffner, Bewley, Henningson, J. Fluid Mech. '06

  - ⋆ embed observed statistical features in control-oriented models

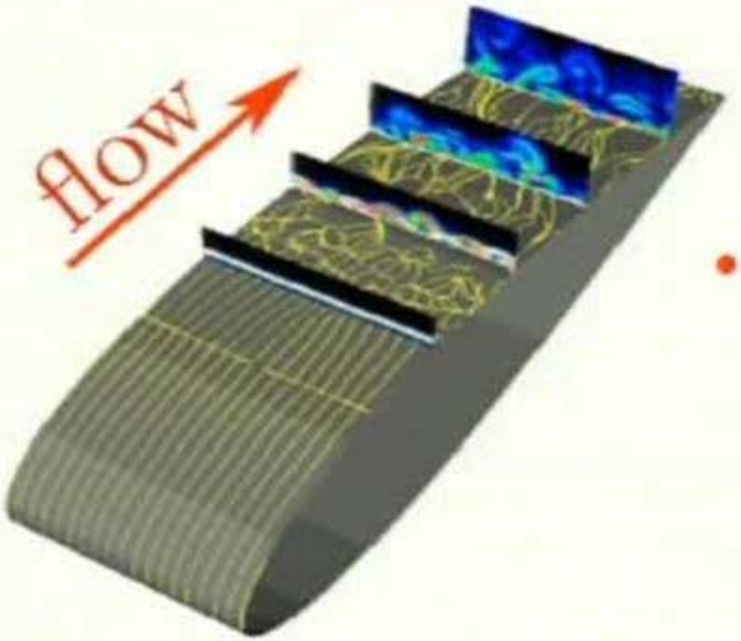
- PROPOSED APPROACH
  - ⋆ view second-order statistics as data for an inverse problem

- KEY QUESTIONS
  - ⋆ Can we identify statistics of forcing to reproduce available statistics?
  - ⋆ Can this be done by white in-time stochastic process?

OUR CONTRIBUTION:

systematic way of turbulence modeling as an inverse problem

Transition to turbulence

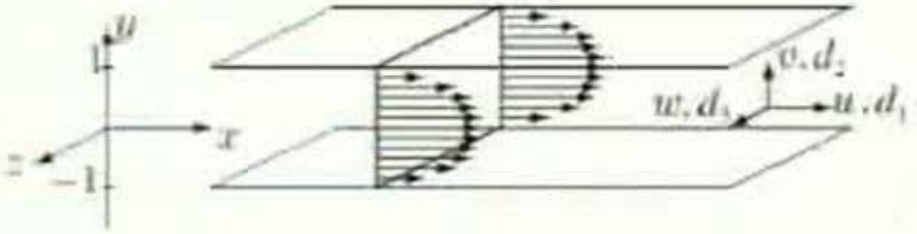


- EXPERIMENTAL ONSET OF TURBULENCE
  - ⋆ much before instability
  - ⋆ depends on experimental conditions

Input-output analysis

- TOOL FOR QUANTIFYING SENSITIVITY
  - ⋆ **spatio-temporal frequency responses**

$\mathbf{d}$ { free-stream turbulence; surface roughness } → **Linearized Dynamics** → fluctuating velocity field $\mathbf{v}$



$$\underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{\mathbf{d}} \xrightarrow{\text{amplification}} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\mathbf{v}}$$

IMPLICATIONS FOR:

transition: insight into mechanisms

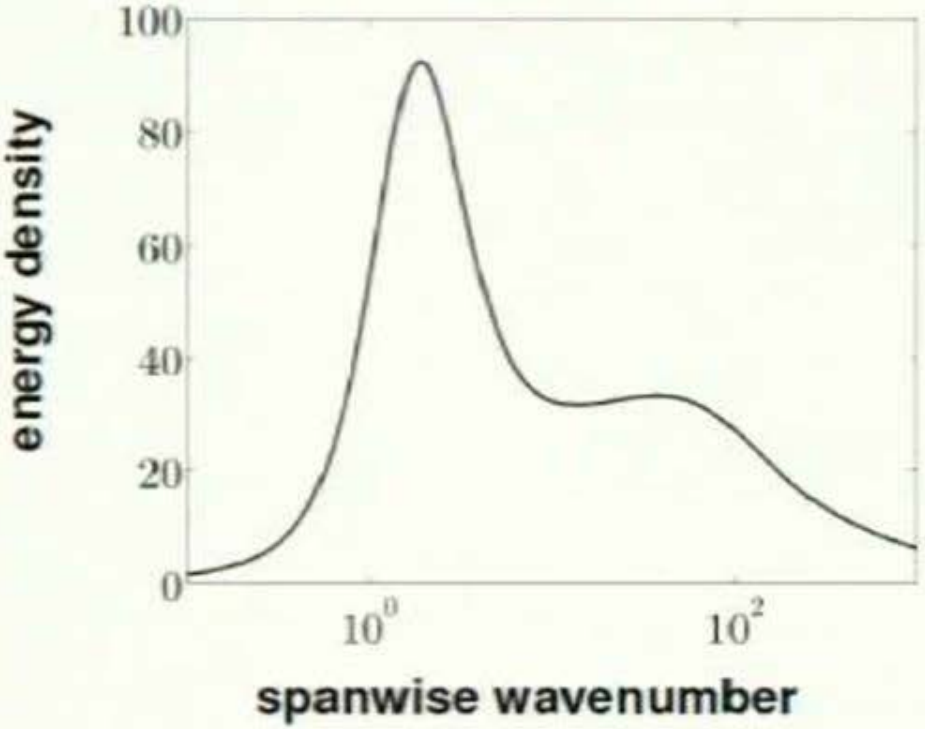
control: control-oriented modeling

Jovanović & Bamieh, J. Fluid Mech. '05

Input-output analysis of turbulent flows

- STREAMWISE CONSTANT FLUCTUATIONS

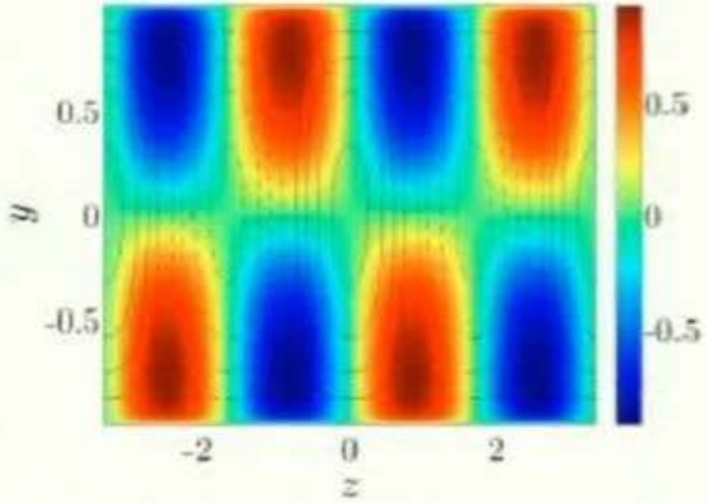
channel flow with $R_\tau = 547$



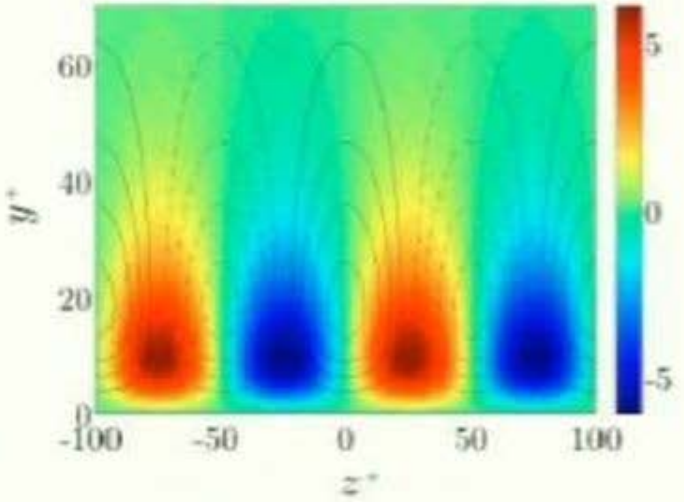
del Álamo & Jiménez, J. Fluid Mech. '06

Cossu & coworkers

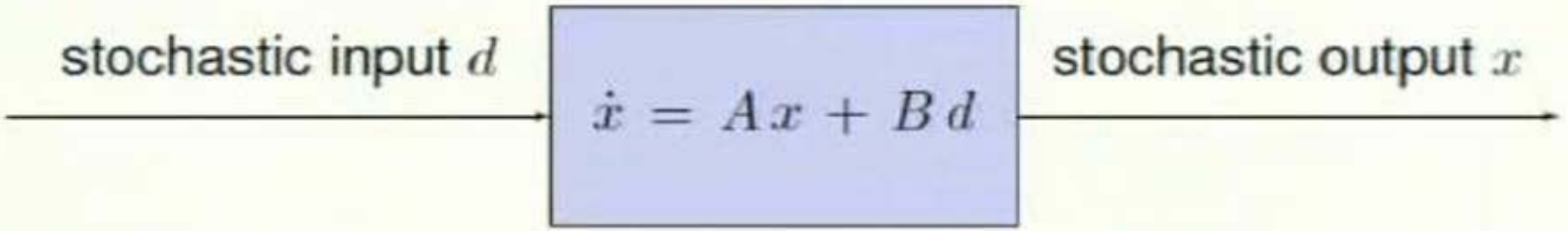
channel-wide streaks



near-wall streaks



Response to stochastic forcing



- LYAPUNOV EQUATION
  - ⋆ propagates white correlation of d into colored statistics of x

$$A\,X \;+\; X\,A^* \;=\; -\,B\,W\,B^*$$

  - ⋆ colored-in-time d

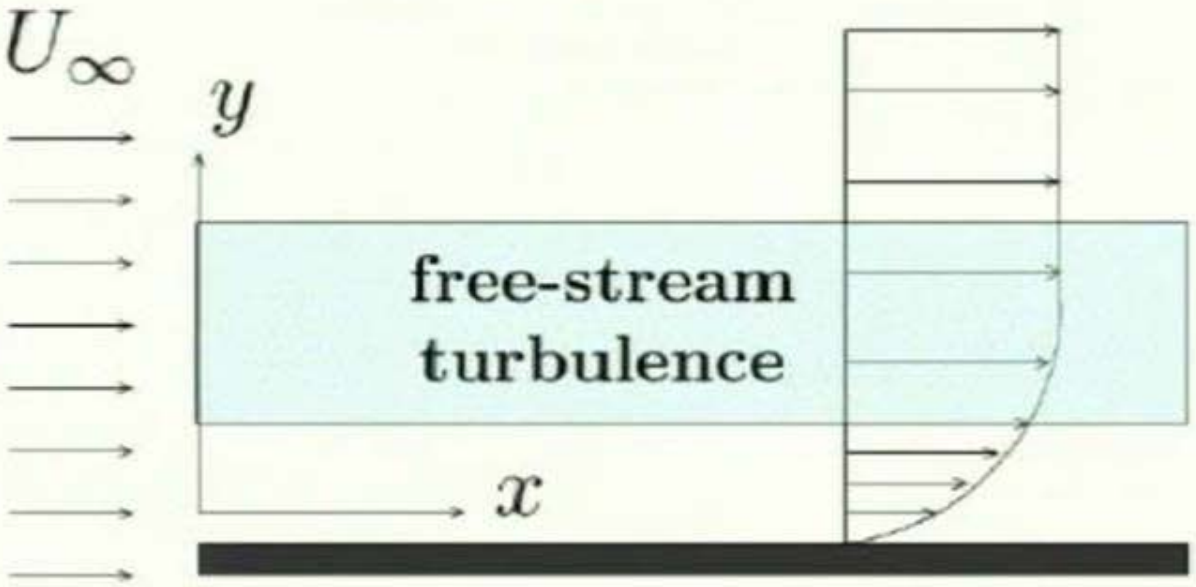
$$A\,X \;+\; X\,A^* \;=\; -\overbrace{(B\,H^* \;+\; H\,B^*)}^{Z}$$

white forcing: $H \;=\; (1/2)\,B\,W$

Georgiou, IEEE TAC '02

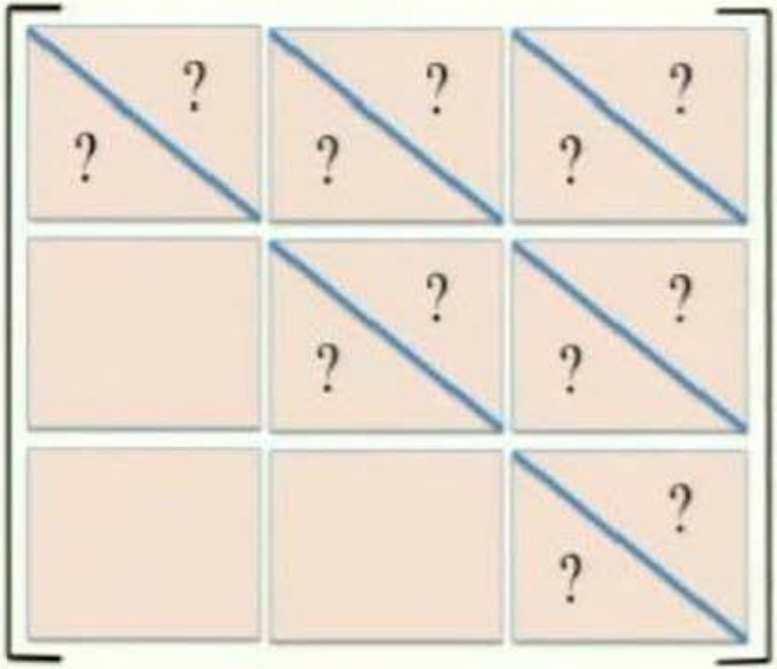
An example

- Response of a boundary layer to free-stream turbulence



Problem setup

- PROBLEM DATA
  - ⋆ **dynamical generator** A
  - ⋆ **partial second-order statistics**



$\mathcal{T}(X_{\mathrm{dns}})$

- UNKNOWNS
  - ⋆ **unavailable statistics of** x
  - ⋆ **disturbance dynamics**
    - **input matrix** B
    - **input power spectrum**

Inverse problem

- CONVEX OPTIMIZATION PROBLEM

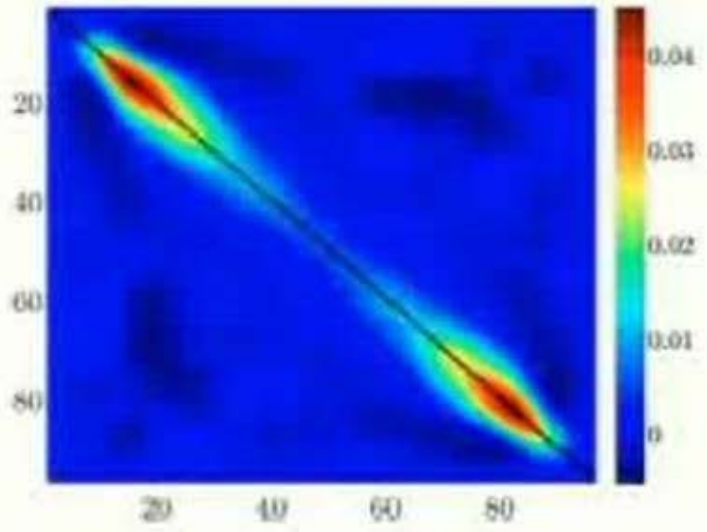
$$\begin{aligned} \underset{X,\,Z}{\text{minimize}} \quad & -\log\det(X) + \gamma\,\|Z\|_* \\ \text{subject to} \quad & AX + XA^* + Z = 0 \\ & \mathcal{T}(X) - \mathcal{T}(X_{\text{dns}}) = 0 \end{aligned}$$

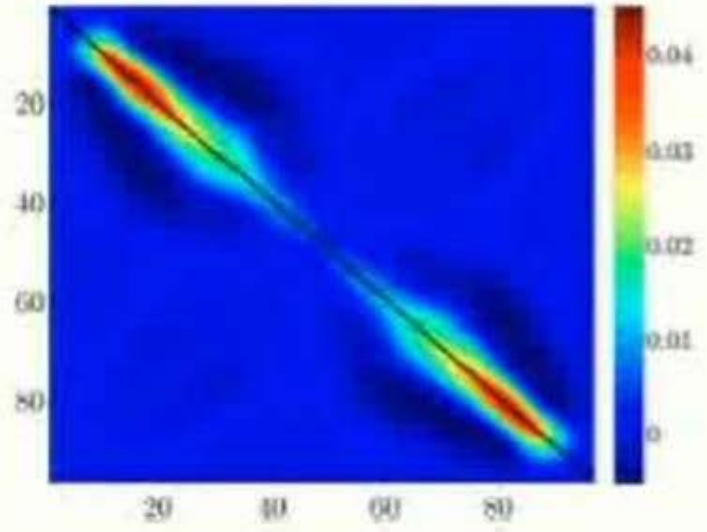
Two-point correlations

DNS

covariance completion

uu





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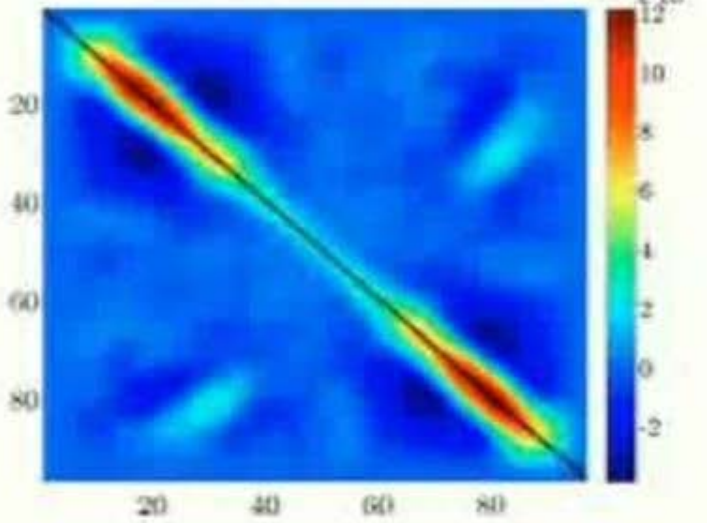
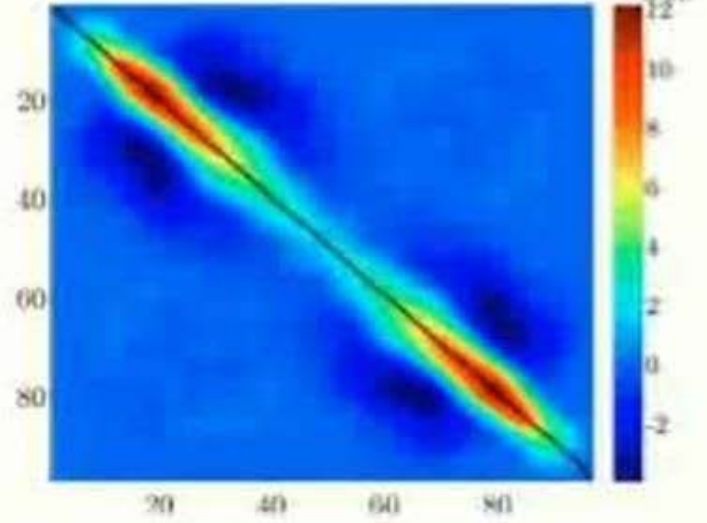
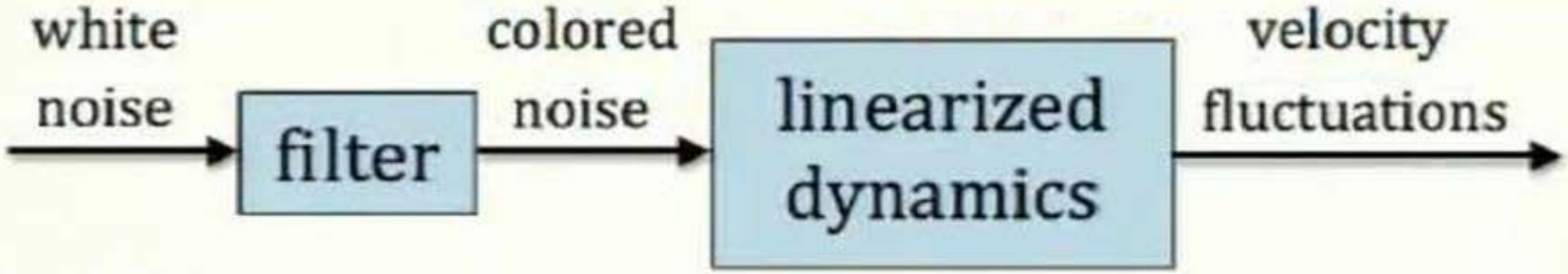




$R_\tau = 180;\ k_x = 2.5,\ k_z = 7$

Colored-in-time forcing

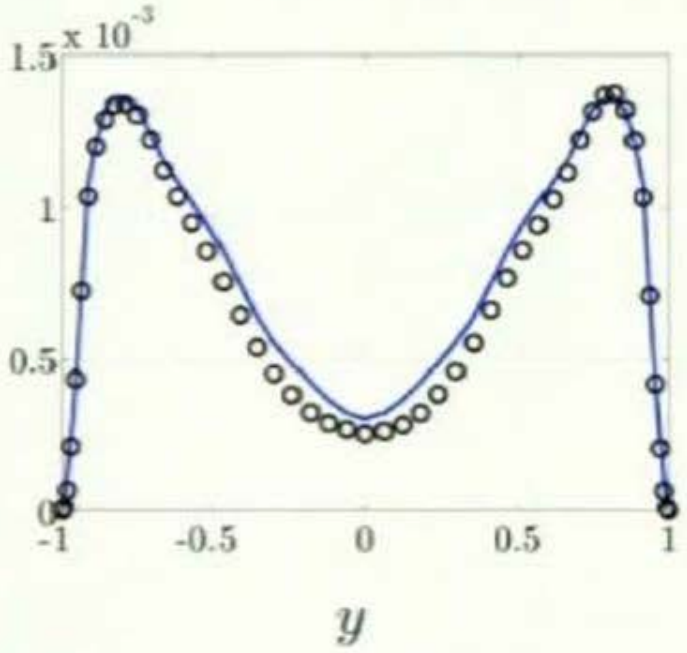


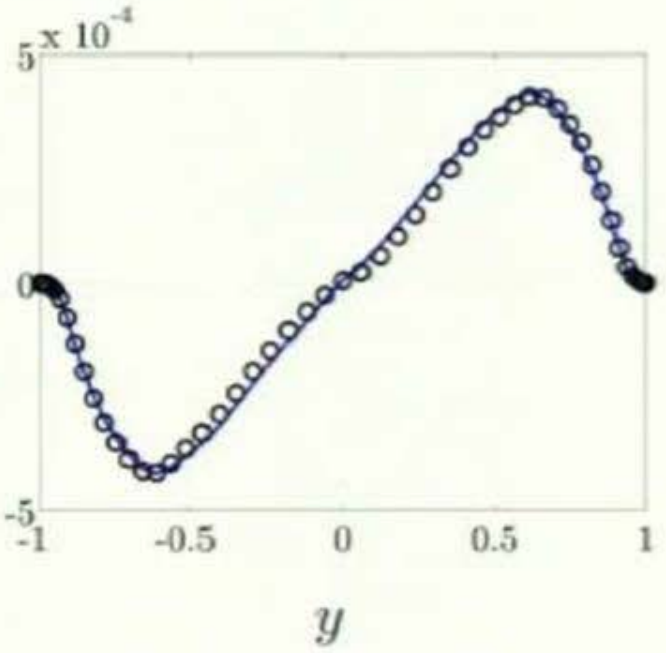
key result:

filter design using { **linearized dynamics**, **completed correlations** }

Zare, Chen, Jovanović, Georgiou, arXiv:1412.3399

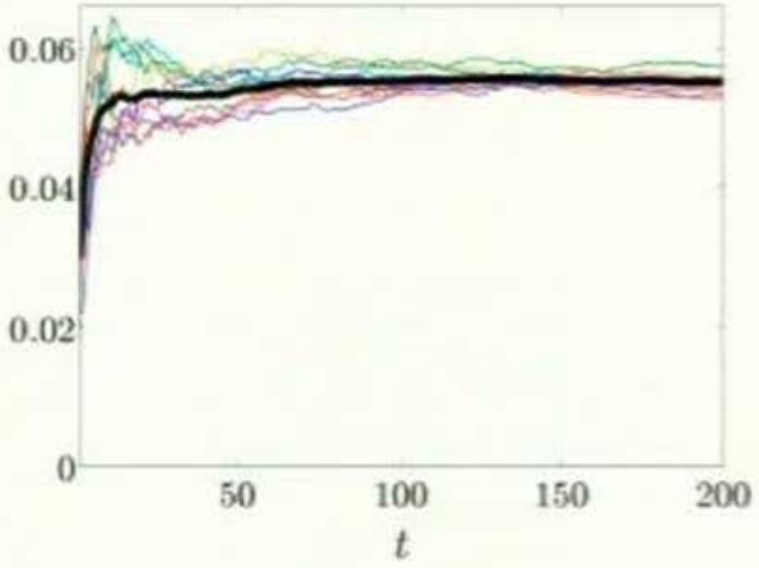
Verification in stochastic simulations



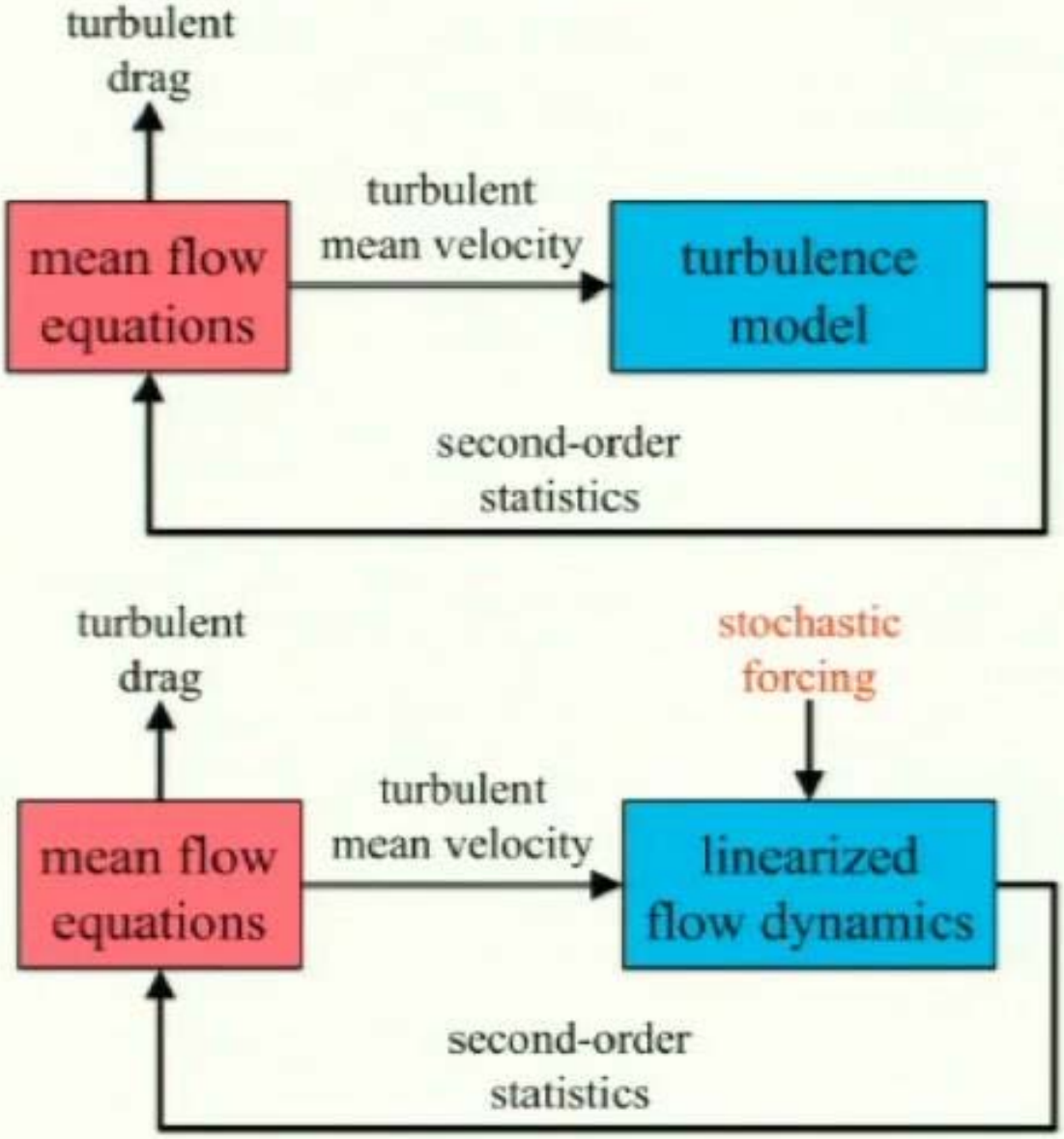


Direct Numerical Simulations —

Linear Stochastic Simulations o



New class of stochastically-forced closure models?



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