Asynchronous Optimized Schwarz Methods

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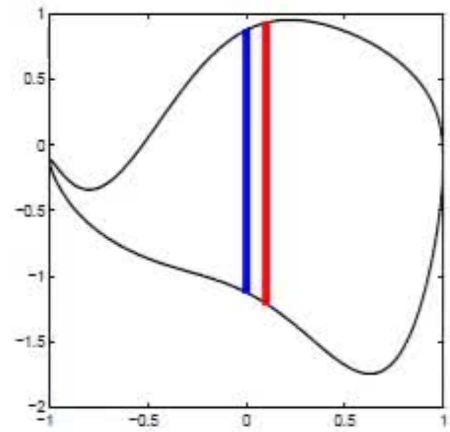
SIAM ALA, 27 October 2015

Thanks to NSF

Outline of the talk
Asynchronous Optimized Schwarz Methods

- Schwarz Methods (Domain Decomposition)
- Optimized Schwarz Methods
- Asynchronous Optimized Schwarz Methods
- New theorems
- Application. Numerical Results
- New paradigm, new philosophy

Alternating Schwarz (aka multiplicative Schwarz)



- General idea of *alternating* Schwarz method: solve on left domain using as Dirichlet data for red line previous approx. of soln. in right domain; solve on right domain using as Dirichlet data for blue line previous approx. of soln. in left domain
- Very good as a preconditioner for CG or other Krylov subspace methods
- Additive/multiplicative Schwarz can be interpreted as Block Jacobi/Gauss-Seidel with overlap. Thus convergence depends on spectral radius (or norm) of iteration operator

Optimized Schwarz Methods (OSM)

- Robin transmission conditions - say $\partial_\nu u(x) + \alpha u(x)$
  Optimal convergence is obtained by changing α
  (this is called OO0)
- Second order transmission conditions: $\dfrac{\partial u}{\partial \nu} + \alpha u + \beta \dfrac{\partial^2 u}{\partial \tau^2}$
  (two parameters, callled OO2).
- Usual tools for optimality: Fourier analysis
  (restricted to certain PDEs and simple domains)
- More recently, optimal parameter calculated using covariance adaptation evolution strategy (CMA-ES)
  [Magoulès, Ahamed, Putanowicz, 2015]
- Algebraic version (no restriction on domain shape or PDE)
  (Block Gauss-Seidel with overlap and changing some entries in overlap)

Some references

- for Domain Decomposition and Schwarz Methods:
  [Smith, Bjørstad, Gropp, 1996], [Quarteroni, Valli, 1999],
  [Toselli, Widlund, 2005], [Mathew, 2008]
- for OSM: [Gander, Halpern, Nataf, 2001],
  [Japhet, Nataf, Rogier, 2001], [Dolean, Lanteri, Nataf, 2002],
  [Côté, Gander, Laayouni, Loisel, 2004], [Gander, 2006],
  [Chevalier, Nataf, 2007], [Loisel, S., 2010]
  [Dubois, Gander, Loisel, St-Cyr, S., 2012]
  [Maday, Magoulés, 2006, 2007], [Magoulés, Roux, Salmon, 2004],
  [Magoulés, Roux, Series, 2005, 2006], [Nier, 1998/9]
- 23 DD conferences, hundreds of papers; see ddm.org

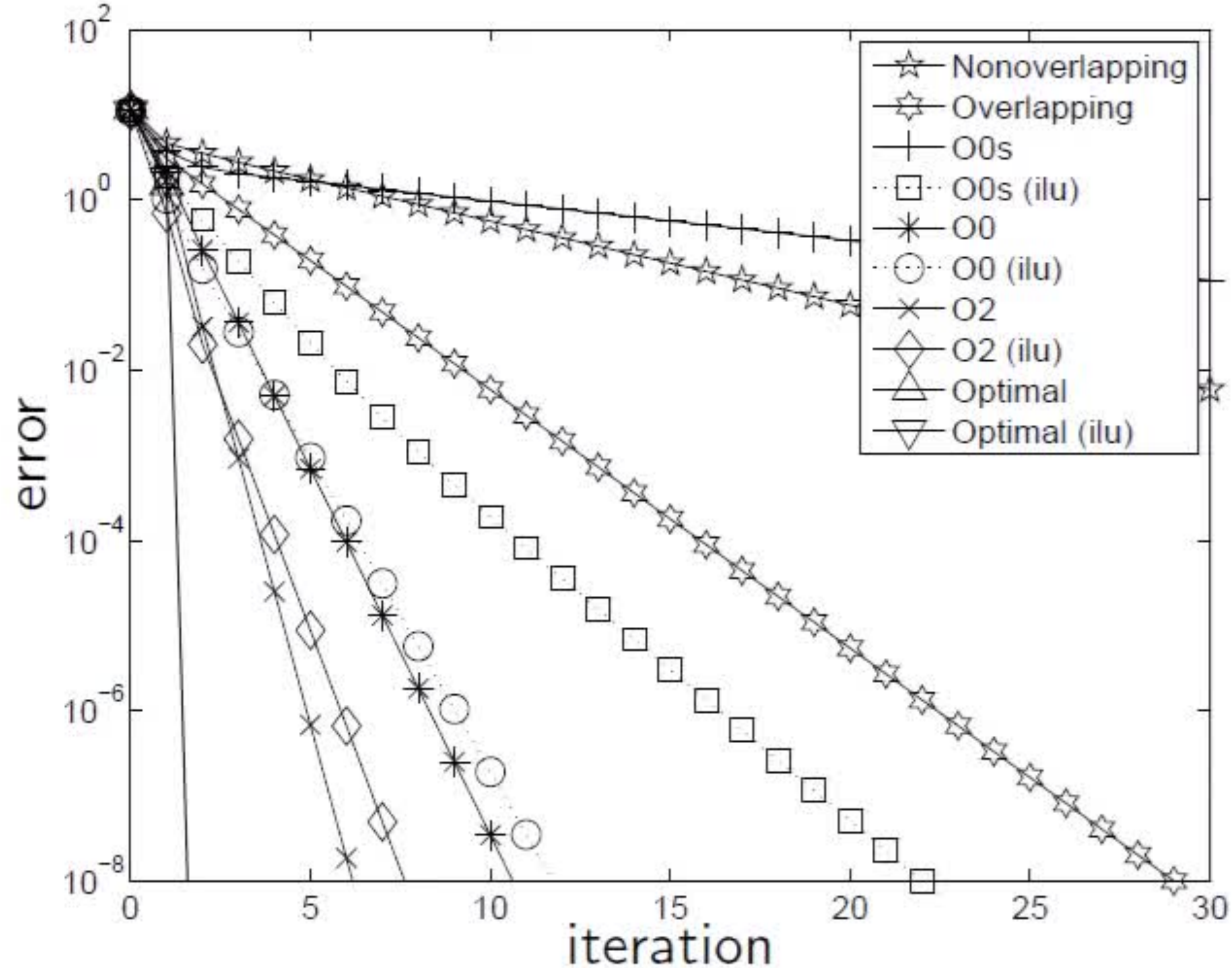
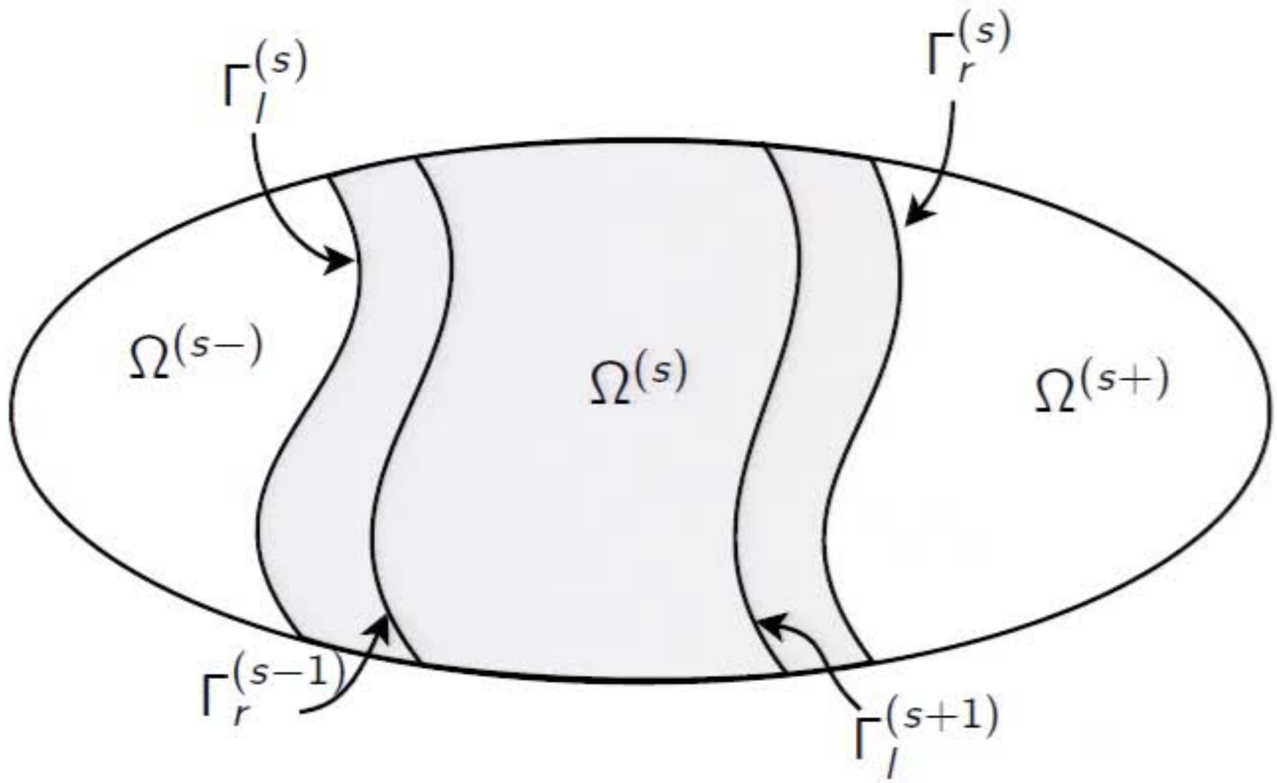
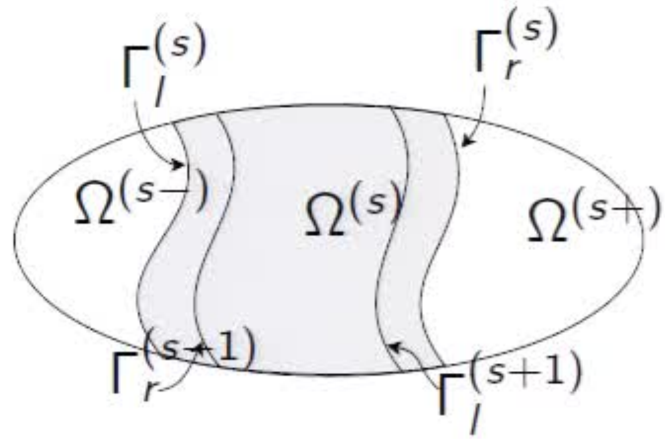


Figure : Square domain, two subdomains, alternating Schwarz
From [Gander, Loisel, S., 2012]

Continuous formulation in a one-way domain splitting



for $s = 2, \ldots, p-1$



For $s = 1, \ldots, p$, given $u(0)$, $u^{(s)}(0) = u(0)$ in $\Omega^{(s)}$

$$\begin{cases} \mathcal{L}\left(u^{(s)}(n+1)\right) = 0 \text{ in } \Omega^{(s)}, \\ \mathcal{C}(u^{(s)}(n+1)) = 0 \text{ on } \partial\Omega \cap \partial\Omega^{(s)}, \\ \left(\frac{\partial}{\partial \nu_l^{(s)}} - \Lambda^{(s-)}\right) u^{(s)}(n+1) = \left(\frac{\partial}{\partial \nu_l^{(s)}} - \Lambda^{(s-)}\right) u^{(s-1)}(n) \text{ on } \Gamma_l^{(s)}, \\ \left(\frac{\partial}{\partial \nu_r^{(s)}} - \Lambda^{(s+)}\right) u^{(s)}(n+1) = \left(\frac{\partial}{\partial \nu_r^{(s)}} - \Lambda^{(s+)}\right) u^{(s+1)}(n) \text{ on } \Gamma_r^{(s)}. \end{cases}$$

With overlap, it was shown that with optimal conditions, convergence is achieved in $p-1$ iterations.

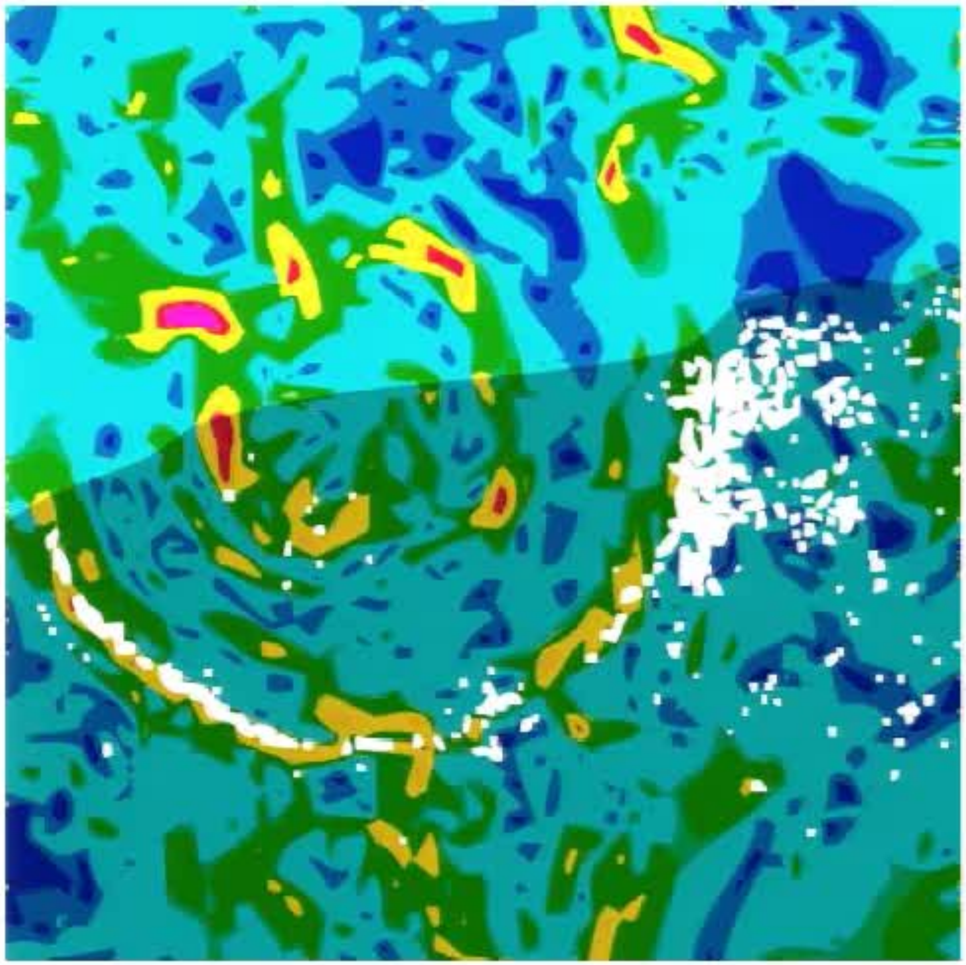
Two new theorems

In one-way domain splittings, with overlap, asynchronous optimized Schwarz methods converge

Shown:

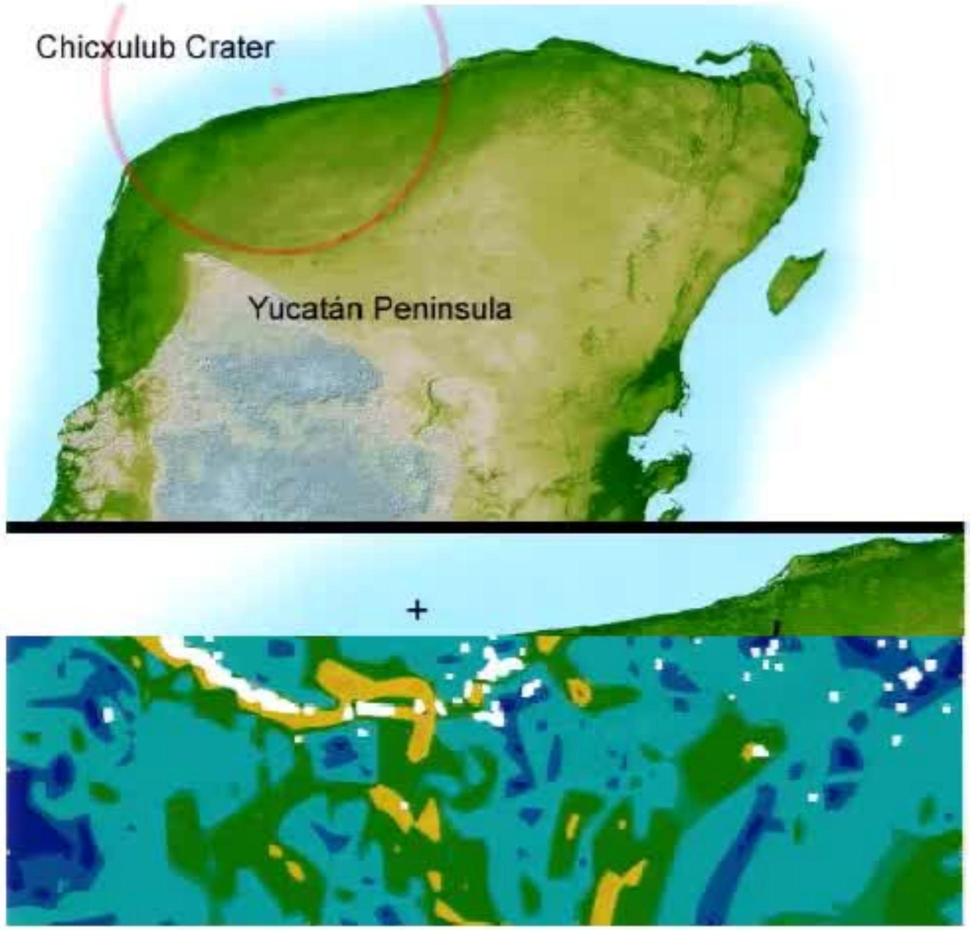
1. For the optimal case
2. For Laplacian operator on the plane, approximate optimized conditions

Artistic rendering of the gravity anomaly map (NASA, 2010). Different colors represent different gravity measurements, except the white dots, which are sinkholes called cenotes.



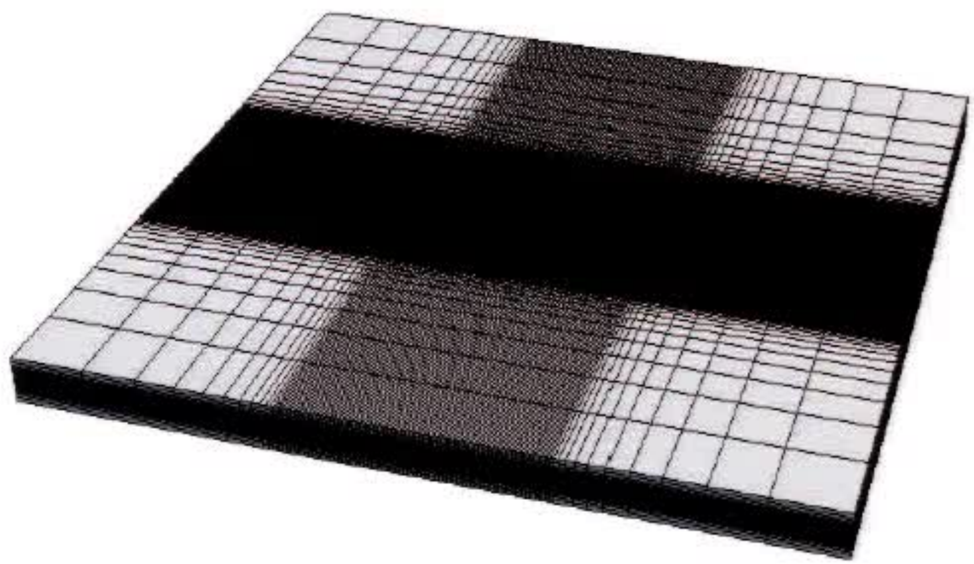
An application. Numerical experiments

Chicxulub Crater, created by a collision of an asteroid approx. 66 million years ago: Cretaceous-Paleogen boundary: extintion of dinosaurs, approx. diameter 180km (pictures NASA, 2010)



Our experiments

We want to compute the gravitational potential Φ on a parallelepiped geometric domain of dimensions $250km \times 250km \times 15km$.

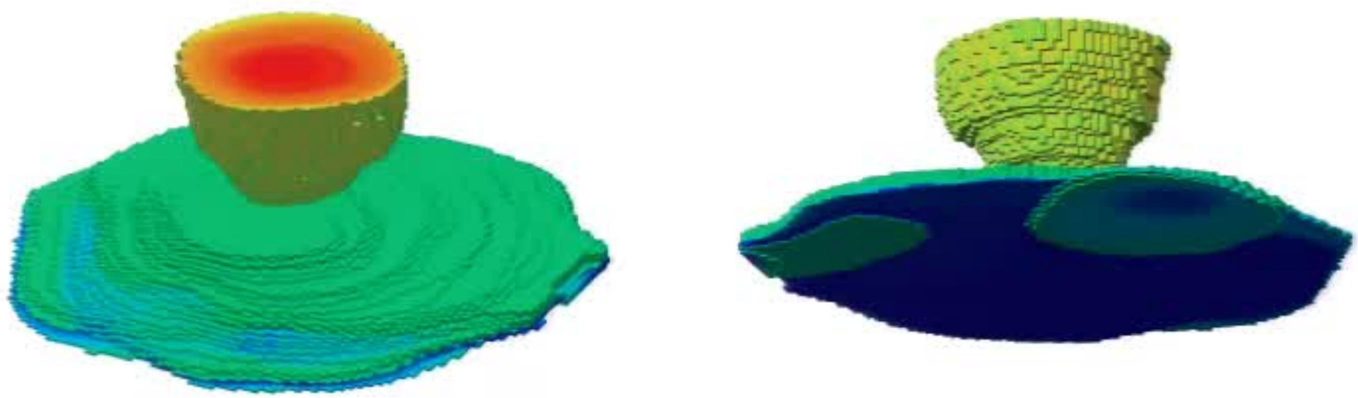


Finite element mesh

Equation to solve

$$\Delta\Phi = -4\pi G \delta\rho$$

- $G = 6.672 \times 10^{-11} m^3 kg^{-1} s^{-2}$ gravitational constant
- $\delta\rho$ anomaly density distribution computed from data aquisition on a salt dome (produced by the impact)



Close up of the salt dome geometry

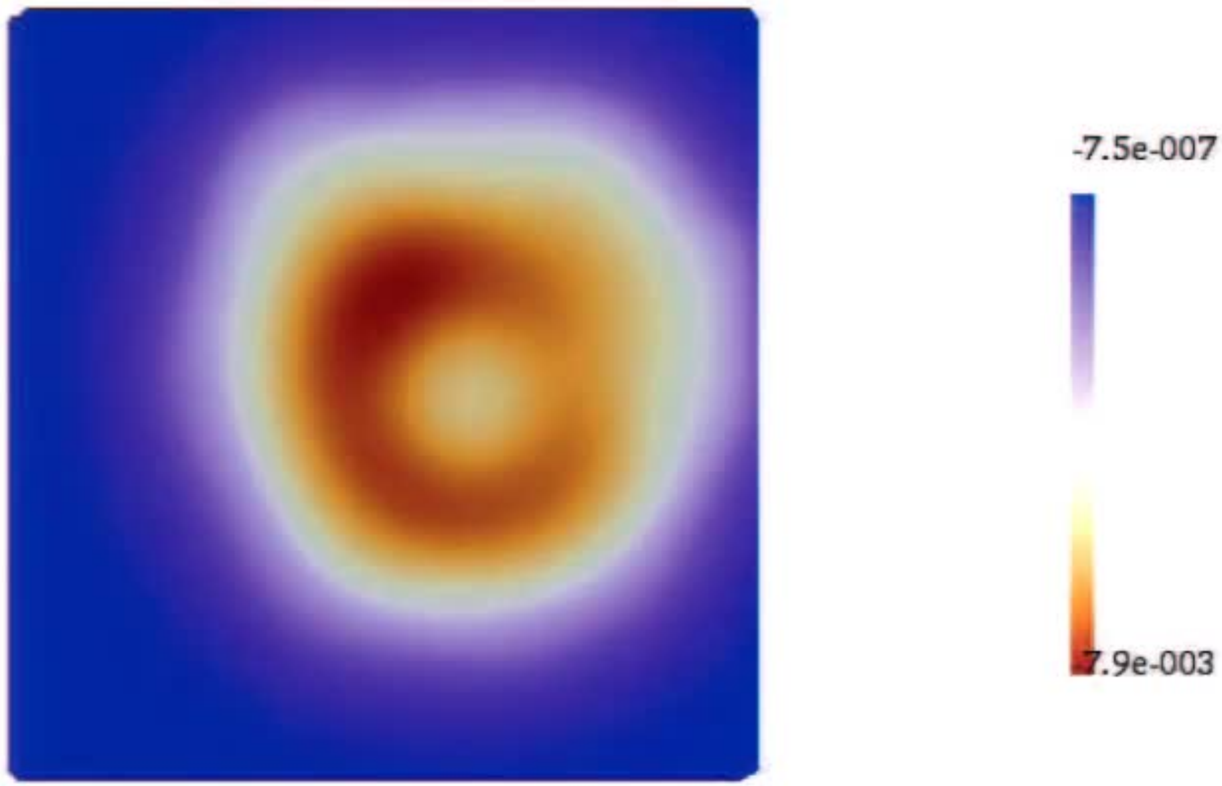
Three discretizations of box

- case I has 327 697 nodes (16 subdomains)
- case II has 2 491 632 nodes (256 and 512 subdomains)
- case III has 19 933 056 nodes (512 subdomains)
- all 512 processors are identical
- (Synchronous) OSM and asynchronous OSM
- Compute optimal parameters using CMA-ES
- In each subdomain solve linear system directly

case	iter	time	it min	it max	time
I (16)	848	53	1133	2164	42.1
II (256)	1722	43	1030	1917	40.2
II (512)	2482	25	1353	1854	19.1
III (512)	3379	777	2257	4438	590

- About 30% gain on an homogeneous network!!

One slice of box



Further comments

- Where are Krylov subspace iterative methods?
- In the past, asynchronous block Jacobi too slow, since block Jacobi too slow
- Asynchronous OSM fast, since OSM is fast

Conclussions / summary so far

- Asynchronous (parallel) optimized Schwarz method shown to work (theory and experiments)
- Asynchronous (parallel) (and even synchronous) optimized Schwarz method shown can be effective on very large problems
- About 30% gain on an homogeneous network!!
- Larger gains expected on inhomogeneous networks, and/or different shapes/types of subdomains

Report can be found at: `www.math.temple.edu/szyld`