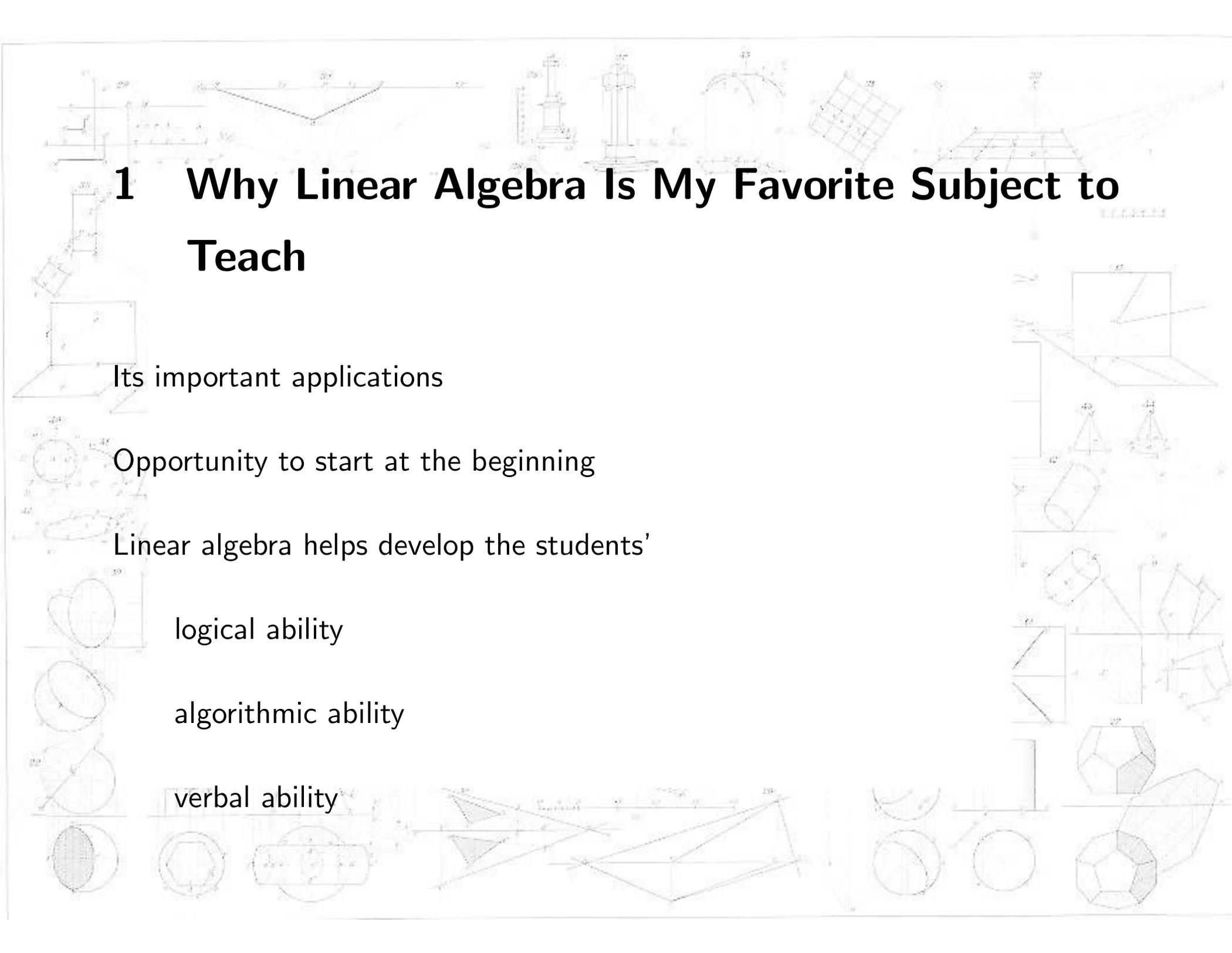
The background of the slide is a collage of various technical drawings and geometric diagrams. These include perspective views of buildings and structures, orthographic projections, and various geometric shapes like circles, ellipses, and polygons. Some diagrams are labeled with numbers such as 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100. The drawings are rendered in a light gray color, creating a subtle, textured background.

Linear Algebra as a Template for Applied Mathematics

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1 Why Linear Algebra Is My Favorite Subject to Teach

Its important applications

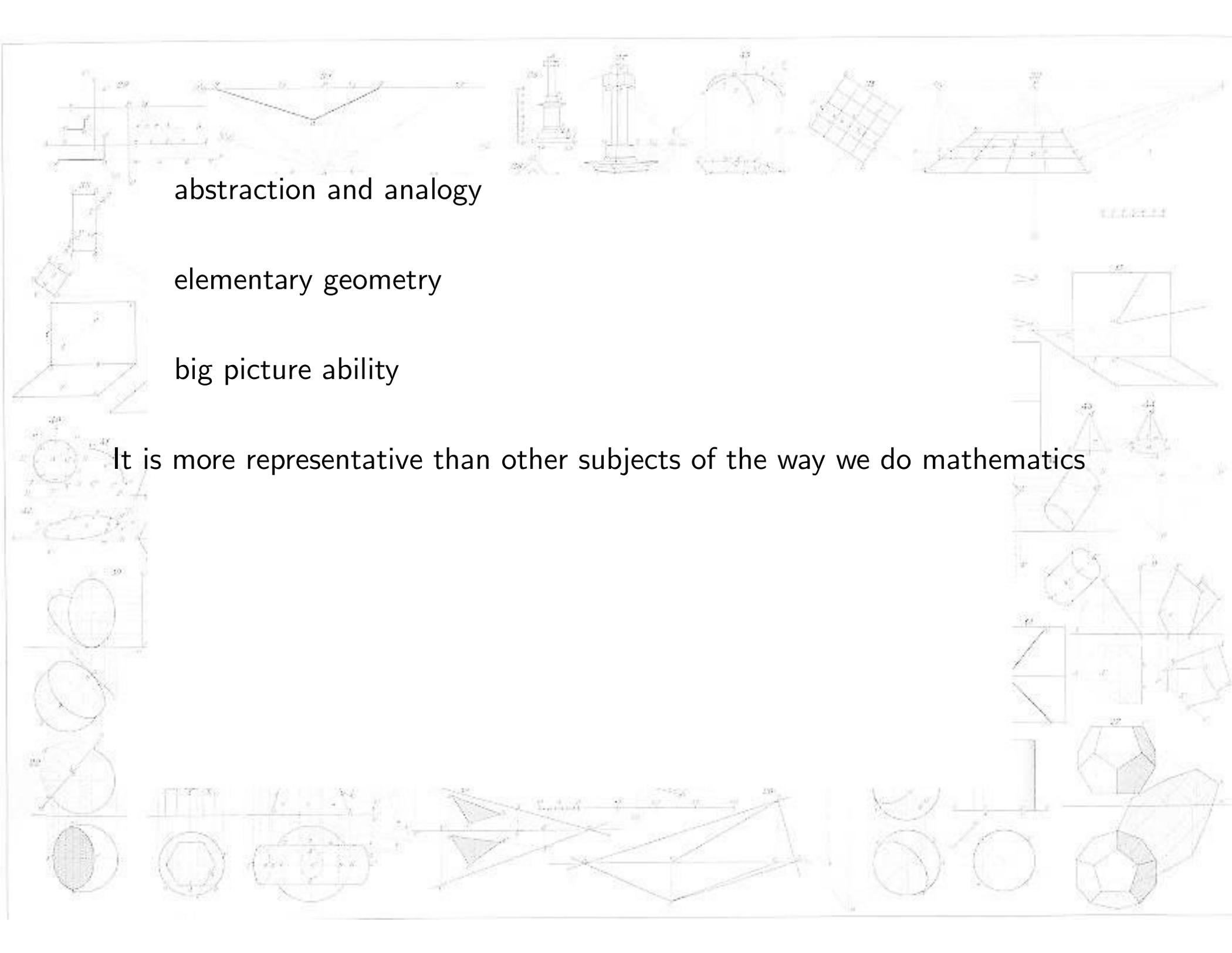
Opportunity to start at the beginning

Linear algebra helps develop the students'

logical ability

algorithmic ability

verbal ability

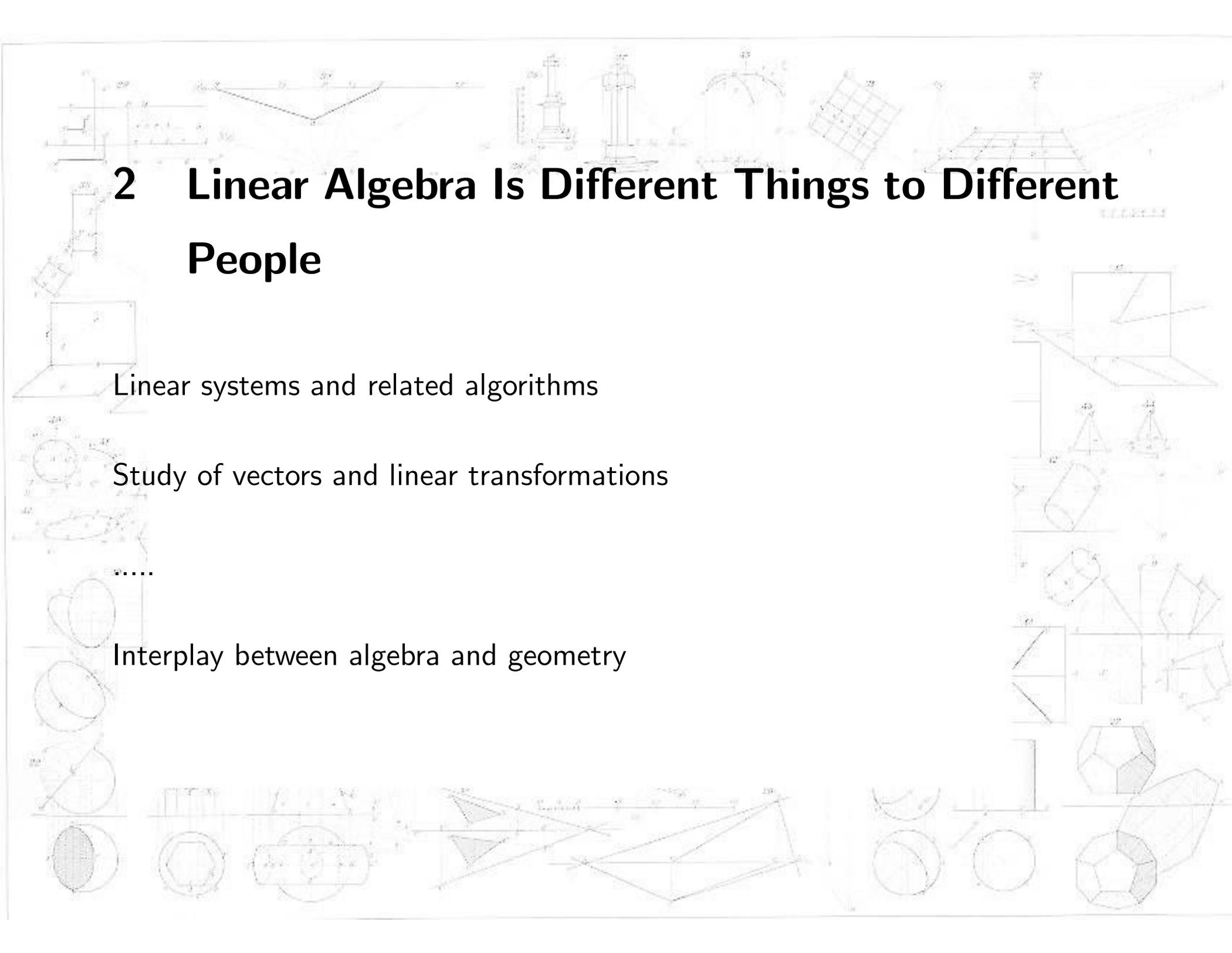


abstraction and analogy

elementary geometry

big picture ability

It is more representative than other subjects of the way we do mathematics

The background of the slide is filled with various architectural drawings, including floor plans, elevations, and sections of buildings and structures. These drawings are rendered in a light, faded style, serving as a decorative backdrop for the text.

2 Linear Algebra Is Different Things to Different People

Linear systems and related algorithms

Study of vectors and linear transformations

.....
Interplay between algebra and geometry

3 A Typical Calculus Question

What is the derivative of $\sin^2 x / (1 + \cos x)$?

$$\begin{aligned}\left(\frac{\sin^2 x}{1 + \cos x}\right)' &= \frac{(\sin^2 x)'(1 + \cos x) + \sin^2 x(1 + \cos x)'}{(1 + \cos x)^2} \\ &= \frac{2 \sin x \sin' x (1 + \cos x) + \sin^2 x (-\sin x)}{(1 + \cos x)^2} \\ &= \frac{2 \sin x \cos x (1 + \cos x) - \sin^3 x}{(1 + \cos x)^2}\end{aligned}$$

4 A Typical Linear Algebra Question

Are vectors

$$\begin{bmatrix} 21 \\ 22 \\ 23 \end{bmatrix}, \begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix}, \begin{bmatrix} 97 \\ 100 \\ 103 \end{bmatrix}$$

linearly dependent?

5 Expressions Are More Alive

The subspace from the previous slide can be captured by the expression

$$\begin{bmatrix} \alpha \\ \frac{\alpha+\beta}{2} \\ \beta \end{bmatrix}$$

But also by the expression

$$\begin{bmatrix} \alpha \\ \beta \\ 2\beta - \alpha \end{bmatrix}.$$

So

$$\begin{bmatrix} \alpha \\ \frac{\alpha+\beta}{2} \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ 2\beta - \alpha \end{bmatrix}!$$

6 ...Which Is Good Training

for many situations

$$\frac{\partial \frac{1}{2} x^T A x}{dx} = Ax$$
$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$
$$\int_{\Omega} df = \int_{\partial \Omega} f$$
$$|A| = \frac{1}{3!} \delta_{rst}^ijk A_i^r A_j^s A_k^t$$

7 Abstraction and Analogy

The differential equation

$$x^2 u'' + x u' + (x^2 - n^2) u = f(x)$$

is

$$Ax = b.$$

Therefore

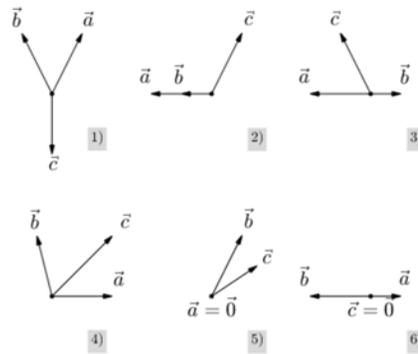
$$u(x) = u_p(x) + N(x)$$

More specifically, since $\dim N = 2$,

$$u(x) = u_p(x) + \alpha J_n(x) + \beta Y_n(x)$$

8 Overstepping analogy

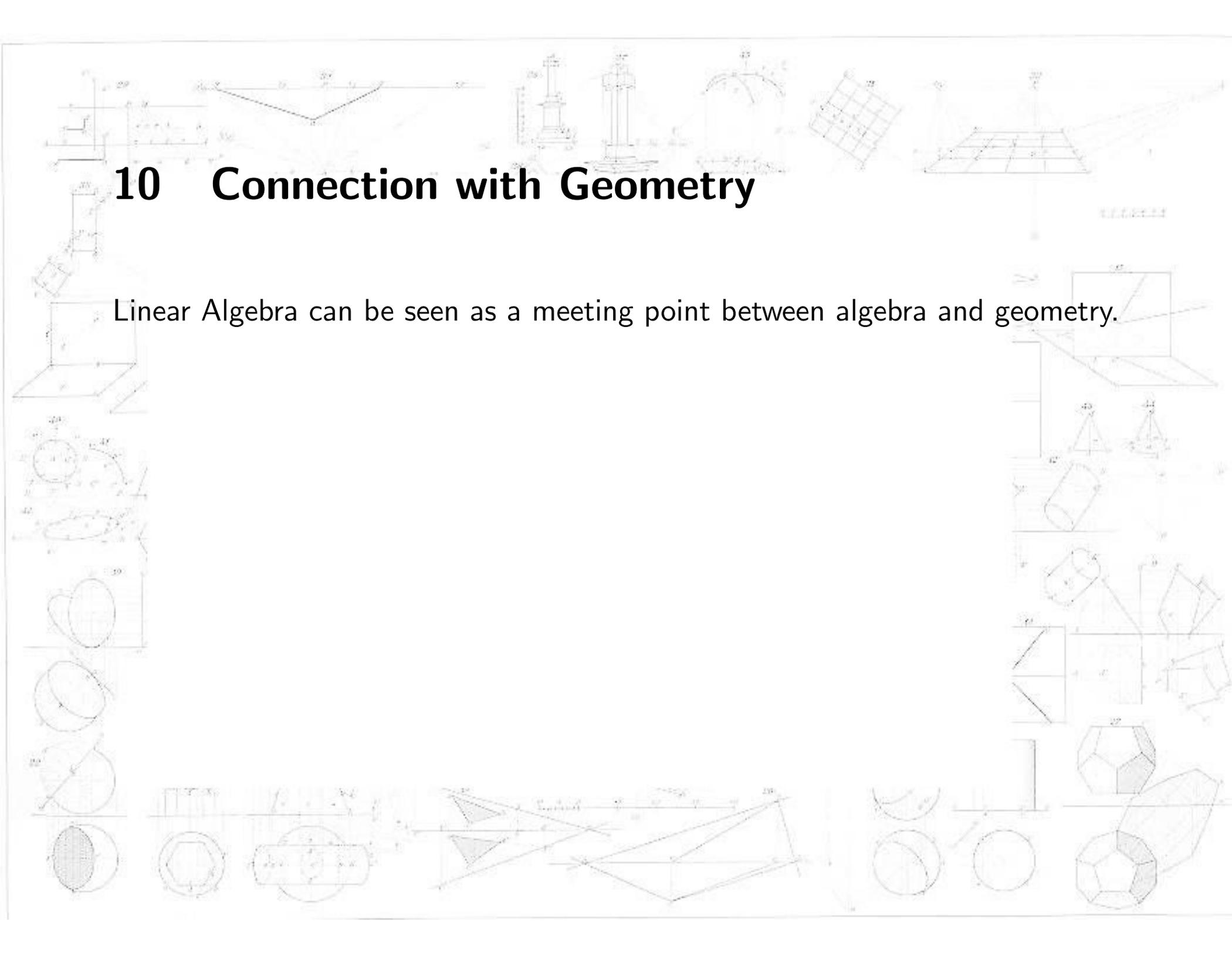
Come up with three linearly independent vectors in the plane.



9 Overstepping the Analogy

Are the following polynomials linearly dependent?

$$\begin{cases} x^2 - 72x + 9 \\ 13x^2 + 54x - 113 \\ \pi x^2 - 17x + \sqrt{2} \\ ex^2 + \sqrt{19}x - 4 \end{cases}$$

The background of the slide is filled with various architectural and geometric drawings. These include perspective views of buildings, technical drawings of columns and arches, and various geometric diagrams such as grids, circles, and polygons. The drawings are rendered in a light, faded style, serving as a decorative backdrop for the text.

10 Connection with Geometry

Linear Algebra can be seen as a meeting point between algebra and geometry.

11 My Favorite Problems

1. Construct a matrix with the following column space and null space:

$$R = \alpha \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 0 \\ 3 \\ 1 \end{bmatrix} \quad N = \alpha \begin{bmatrix} 7 \\ 8 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 4 \\ 2 \\ 1 \\ 2 \end{bmatrix}$$

2. Evaluate

$$\begin{bmatrix} 110 & 55 & -164 \\ 42 & 21 & -62 \\ 88 & 44 & -131 \end{bmatrix}^{2017}$$